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Electro-quasistatics in the frequency domain

For time-harmonic problems in the electro-quasistatic limit, the potential Φ satisfies the equation

$$\nabla \cdot [(\sigma + \mathrm{i}\omega\varepsilon)\nabla\Phi] = 0$$

where σ is the electric conductivity, $\mathrm{i}^2 = -1$ is the imaginary unit, $\omega = 2\pi f$ is the angular frequency, and $\varepsilon = \epsilon_0\epsilon$ is the permittivity. The quantity $\sigma_{\mathbb{C}} = \sigma + \mathrm{i}\omega\varepsilon$ can be interpreted as a complex-valued conductivity and in this context, the electro-quasistatic equation is a time-harmonic generalization of the steady currents equation, $\nabla \cdot \sigma_{\mathbb{C}}\nabla\Phi = 0$. Here, consider the cable termination model that is depicted in the figure below; the geometry exhibits rotational symmetry with respect to the x axis. The conductor is not part of the computational domain; it is modeled with a Dirichlet boundary condition $\Phi = 4$ kV on the boundary that is tagged «high-voltage». To simplify the problem, the conductivity is assumed to vanish everywhere, while the relative permittivity values $\epsilon_{\mathrm{I}} = 1$, $\epsilon_{\mathrm{II}} = 2$, $\epsilon_{\mathrm{IV}} = 1.2$ are assumed[1]. The relative permittivity of the stress control tube (III: FGM) is a nonlinear function of the field strength $E = |\nabla\Phi|$; in particular,

E [V/m] :	0	$6 \cdot 10^5$	10^6	$1.5 \cdot 10^6$	$2 \cdot 10^6$	$3 \cdot 10^6$	$5 \cdot 10^6$	in V/m
ϵ_{III} :	10	100	40	20	10	4	3,	

and hence, a cubic spline interpolation needs to be constructed in order to obtain $\varepsilon(E)$.

Instructions

After deriving both the strong problem (starting from Maxwell equations, setting $\partial B/\partial t = 0$, and using the time harmonic ansatz $\phi(x,t) = \Re[\mathrm{e}^{\mathrm{i}\omega t}\Phi(x)]$) and the variational form, solve the problem in FreeFEM++, using complex P_1 elements for Φ and P_0 elements for ε, on a mesh with more nodes close to the the tube[2]. Solve the problem with fixed point iterations and Newton's method and present convergence

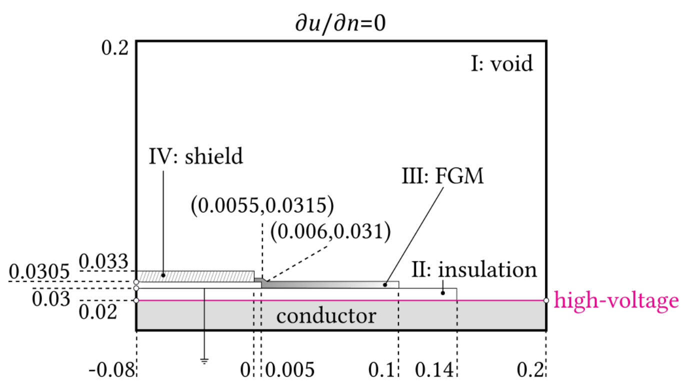

Figure 1: Computational domain design and materials (units in m).

[1] $\varepsilon_0 = 10^{-9}/(36\pi)$ F/m

[2] Make sure that region III contains at least three elements in the y direction

history plots in a common axis for both methods. Provide potential, field strength, and electric displacement plots. Present a sufficient comparative discussion for these methods, in terms of convergence, error, and solution time. Explain possible reasons for one method performing better than another.

Tip. To learn how to impose radial symmetry study the example Axisymmetry: 3D Rod with circular section. As an example of obtaining derivatives study the example Optimal Control. To obtain the cubic spline for $\varepsilon(E)$ study the gsl example. In short, you need to
(1) load the gsl library,
(2) create an array real[int,int] data(2,7) with data(0,), data(1,) being the given data, respectively,
(3) generate cubic spline with gslspline spline(gslinterpcspline, data), and
(4) define a function func real[int] csinter(real[int] & u) that calls the spline for each node of the mesh.

Reporting

The report should be submitted through email to the instructor, as a compressed file that contains tex, pdf/eps, and edp/m code files. The reports must be individual. You are allowed to use information from any source you like, but a reference should be given for each such statement. The report should consist of the following parts; title, abstract, introduction, problem statement, the discrete setting, numerical results, conclusions, references. Include only pdf/eps vector graphics for your figures; no pixel-based formats, such as png, are allowed. You have to typeset inline formulas as $<formula>$ to match the display formulas and hence, preserve a consistent notation; «a» is different than «a» and different than «α». All mathematical symbols have to be defined in words or formulas. All equations, figures, and tables must be numbered and properly referenced using label–ref/eqref. Do not express your opinion and do not speculate; just the facts. Make sure your report reads good, looks good, and it is scientifically sound.