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An eddy current problem

For two-dimensional time-harmonic problems with imposed source currents of the form $\mathbf{J}^s = (0, 0, J_z^s)$, Ampère's law is

$$-\nabla \cdot \mu^{-1} \nabla A_z + \mathrm{i}\omega\sigma A_z = J_z^s,$$

where μ is the magnetic permeability, A_z is the z component of the complex-valued magnetic vector potential $\mathbf{A} = (0, 0, A_z)$, $\mathrm{i}^2 = -1$ is the imaginary unit, $\omega = 2\pi f$ is the angular frequency, and σ is the conductivity. Continuity of the normal component of $\mathbf{B}$ entails continuity of A_z, while continuity of $\mathbf{n} \times \mathbf{H}$ entails continuity of $\mu^{-1}\partial A_z/\partial n$. Here, consider the magnetic circuit that is depicted in the figure below; the geometry is symmetric with respect to the x axis. The left and right copper plates carry imposed source currents $J_z^s = \pm 10^5$ A/m^2, respectively. The material parameters are

$$\sigma_{\mathrm{I}} = 3 \cdot 10^{-15}, \quad \sigma_{\mathrm{II}} = 10^7, \quad \sigma_{\mathrm{III}} = 5.8 \cdot 10^7 \qquad \text{in S/m}$$

$$\mu_{\mathrm{I}} = 4\pi \cdot 10^{-7}, \quad \mu_{\mathrm{II}} = 6.3 \cdot 10^{-3}, \quad \mu_{\mathrm{III}} = 1.3 \cdot 10^{-6} \qquad \text{in H/m.}$$

Instructions

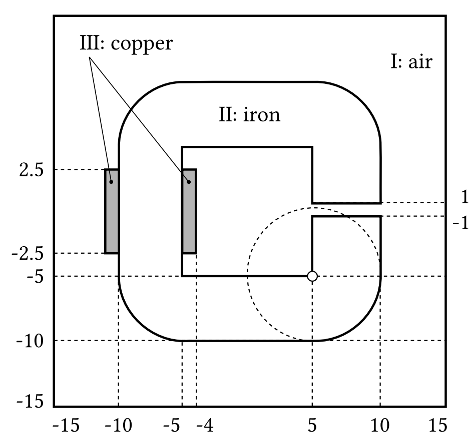

Figure 1: Computational domain design and materials (units in mm).

After deriving the variational form, solve the problem in FreeFEM++, using P_2 elements for A_z and P_0 elements for μ and σ, on a nearly uniform mesh; this result is only used to test your implementation. Save the mesh generated by FreeFEM++, assemble the matrices in FreeFEM++ using P_1 elements, and import both in GNU Octave. Solve the resulting linear system for $f_1 = 1$ Hz, $f_2 = 10$ Hz, and $f_3 = 100$ Hz using the direct solver `mldivide` and the iterative solvers `pcg`, `bicgstab`, `bicg`, `gmres`, `qmr`, `tfqmr`. Present convergence history plots in a common axis for the methods that converged. Provide relative errors between the direct solver solution and each solution obtained with an iterative solver. Present a sufficient comparative discussion for the iterative methods, in terms of convergence, error, and solution time. Explain possible reasons for one method performing better than another for a particular frequency. Present GNU Octave mesh plots and A_z plots for f_1, f_2, and f_3.

Reporting

The report should be submitted through email to the instructor, as a compressed file that contains `tex`, `pdf/eps`, and `edp/m` code files. The reports must be individual. You are allowed to use information from any source you like, but a reference should be given for each such statement. The report should consist of the following parts; title, abstract, introduction, problem statement, the discrete setting, numerical results, conclusions, references. Include only `pdf/eps` vector graphics for your figures; no pixel-based formats, such as `png`, are allowed. You have to typeset inline formulas as `$<formula>$` to match the display formulas and hence, preserve a consistent notation; «a» is different than «a» and different than «α». All mathematical symbols have to be defined in words or formulas. All equations, figures, and tables must be numbered and properly referenced using `label-ref/eqref`. Do not express your opinion and do not speculate; just the facts. Make sure your report reads good, looks good, and it is scientifically sound.