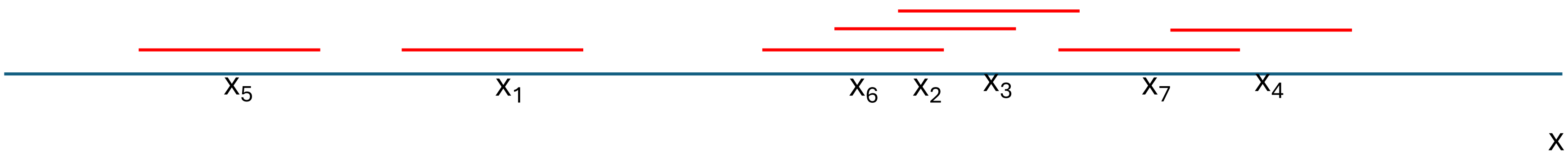
Geometric (Hyper)Graphs

1-D “Estimation of Location”

Measurements x_i with some ”tolerance” (maximum error”)

1-D “Estimation of Location”



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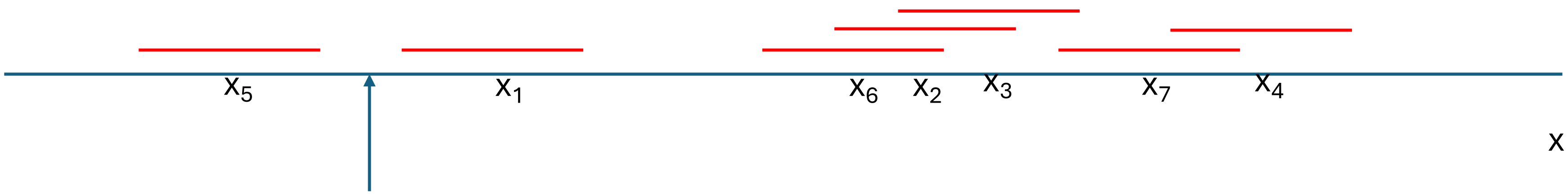

Hypothesis – this is the location – how many measurements support this?

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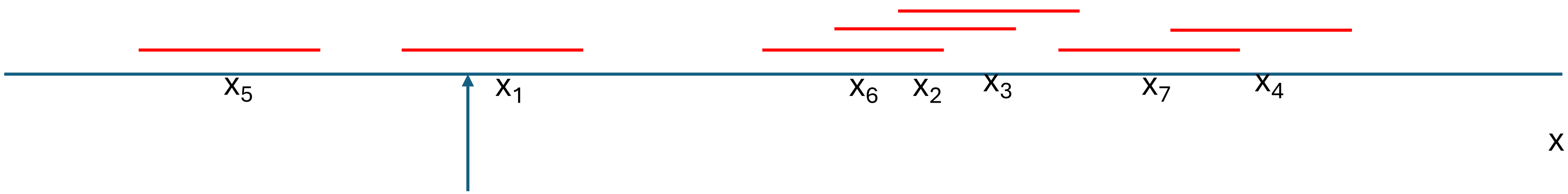

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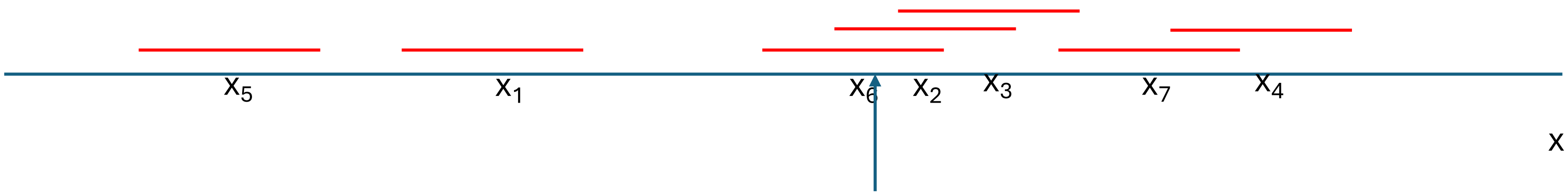

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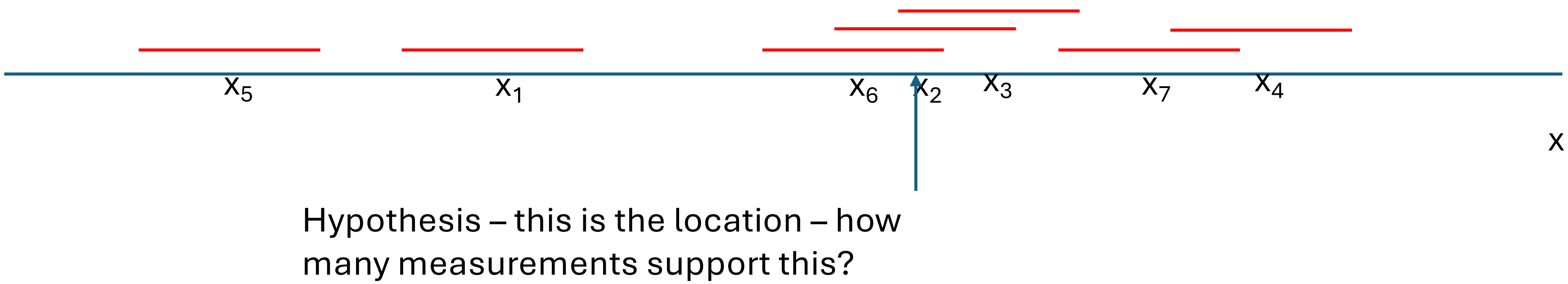

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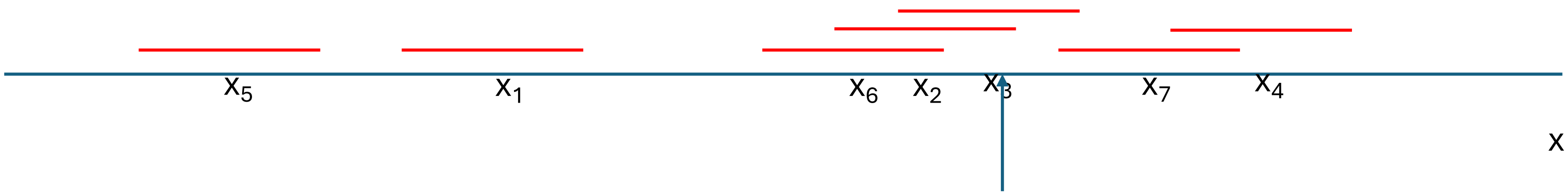

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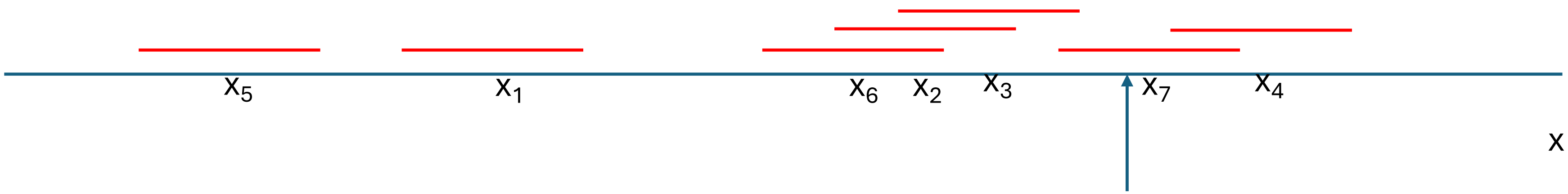

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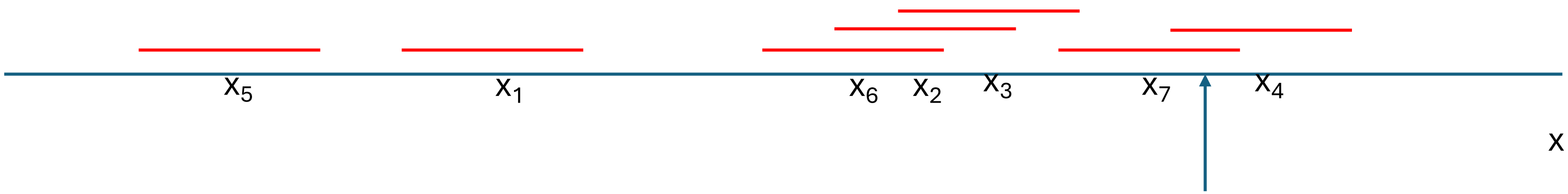

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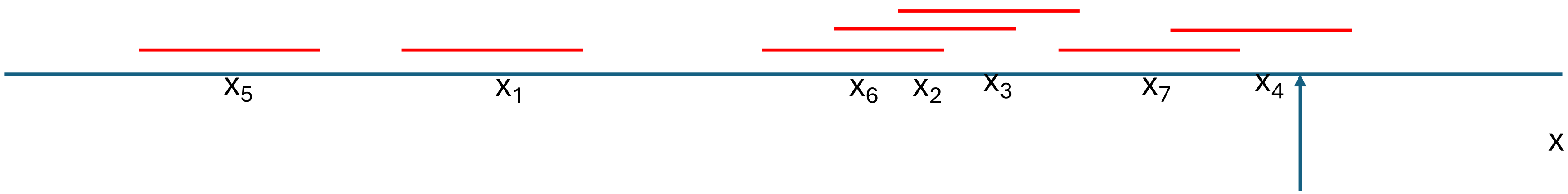

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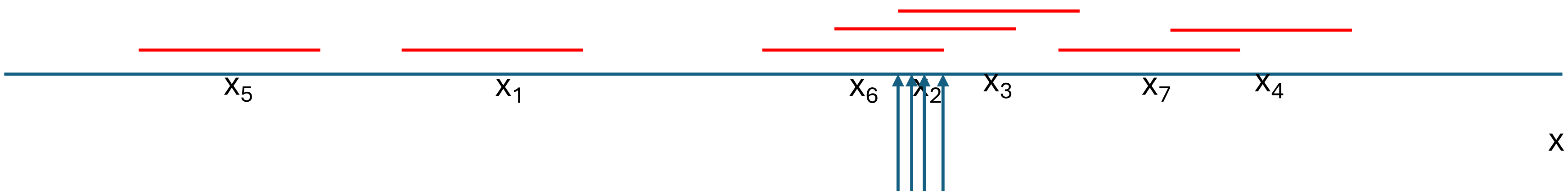

MaxCon solutions (infinitely many of them – supported by 3 measurements)

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Maximum Consensus solution – pick a solution with maximum agreement/consensus. In this example there are a range of solutions with support of 3 data points. All other hypotheses have 2 or less supporting data points.

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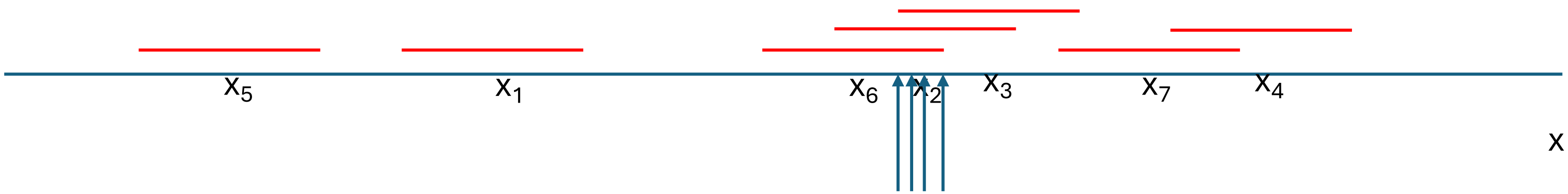

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This is an example of “interval stabbing” – think of the hypotheses as vertical stabbing “pins” and how many tolerance intervals stabbed as a given position is the measure of how good the solution is.

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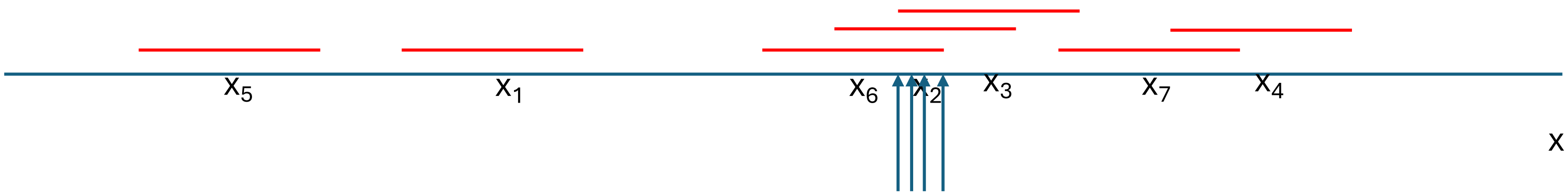

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Interval stabbing is a useful way of thinking of many problems – particularly problems that come from computational geometry.

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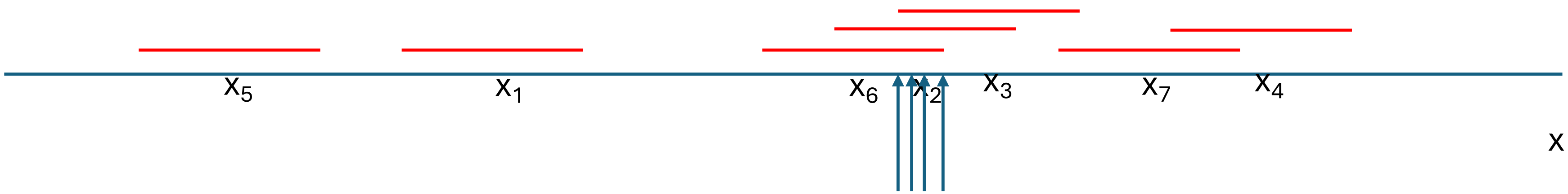

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But what has this to do with (hyper)graphs??

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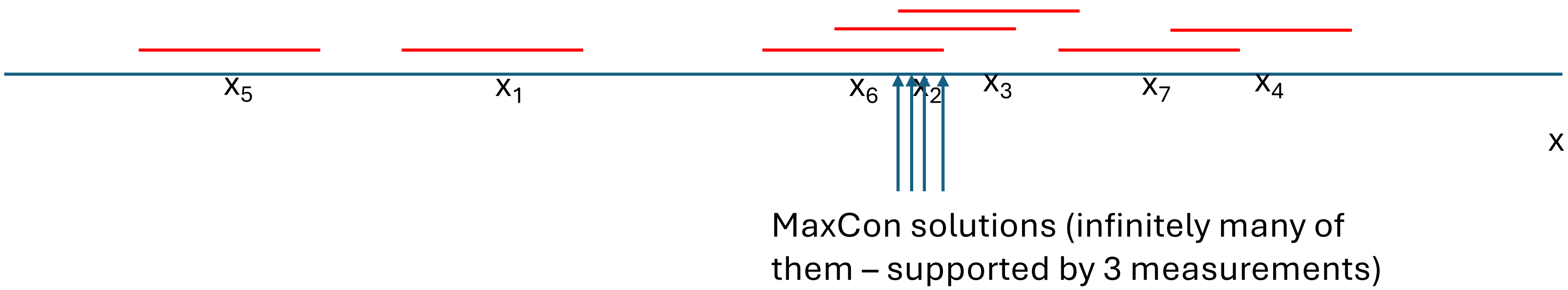

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Well multiple intervals can be stabbed iff they all overlap.
So…the space of possible solutions (with their scores) is described by the intersection pattern of the intervals.

Moreover *in this situation* three intervals overlap iff they *pairwise overlap* (this might seem like it is always a true statement but NOT for the intersection patterns of other object – in general – see later…)

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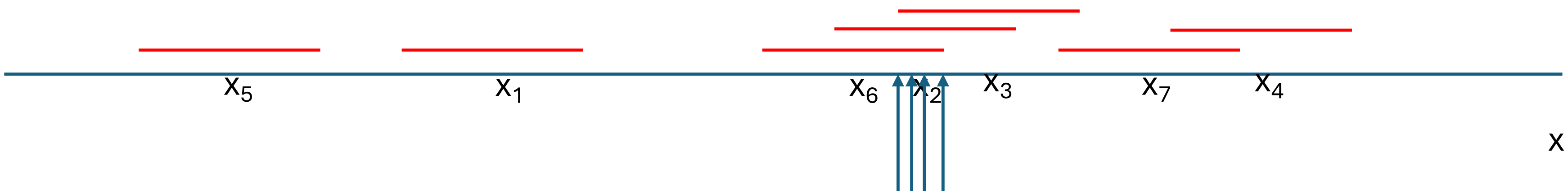

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So…..the *pairwise* intersections tell the whole story. I can “smell” a graph!
The graph of pairwise intersections….

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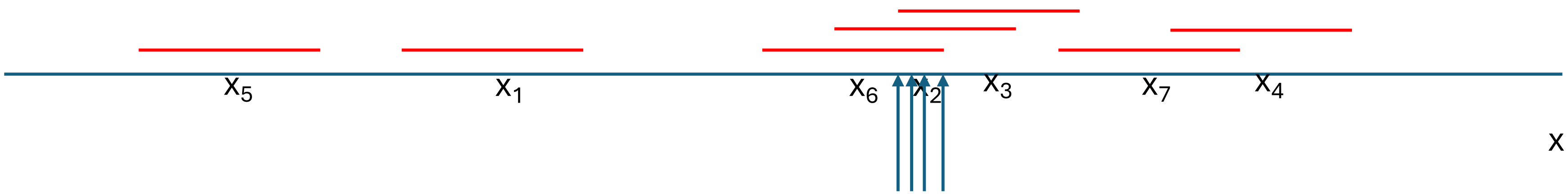

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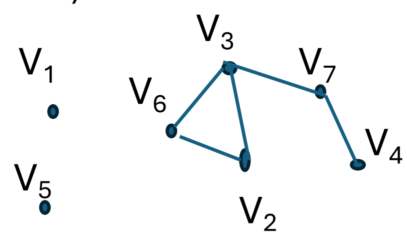

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Assign a vertex to each interval/data point. Join the vertices of the intervals pairwise intersect…

The MaxCon solutions are where the maximum number of vertices in the graph all pairwise are joined by edges (the intervals pairwise intersect). Maximum Clique problem!!!

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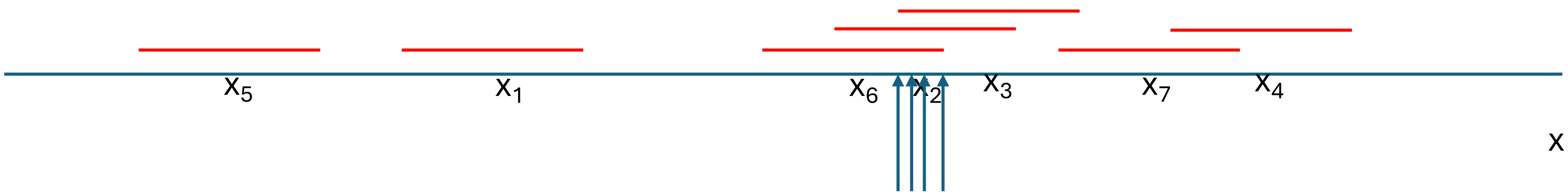

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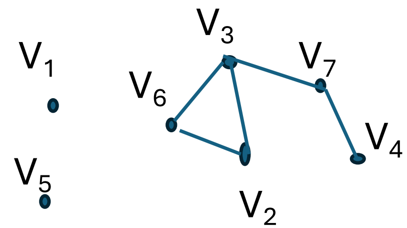

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Thus is a "dominant structure" case ($V_2V_3V_6$) with 3 inliers and 4 outliers. We will talk about the "ideal single structure case" where there is a single clique and the outliers (not part of the clique) are isolated vertices

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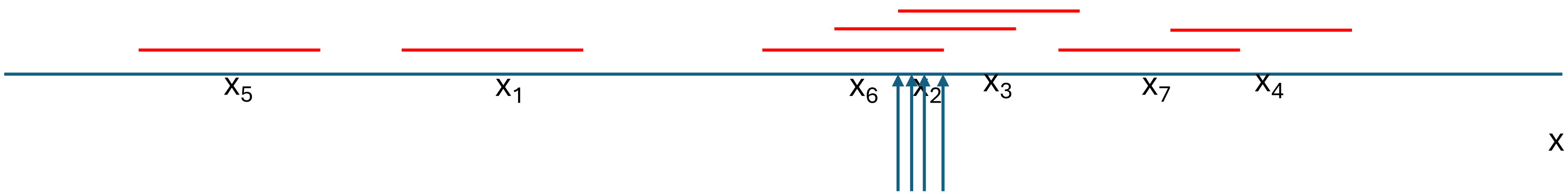

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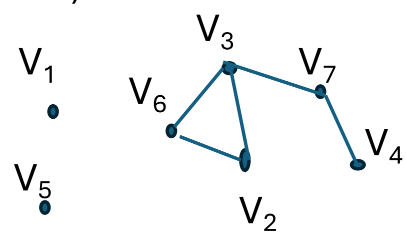

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Now that we have decided that a 1-D estimation of location problem (MaxCon) – can be described by a graph – a natural question is –
What type of graph can arise this way? Any graph? Only some graphs? If so *which* graphs and what are their properties?

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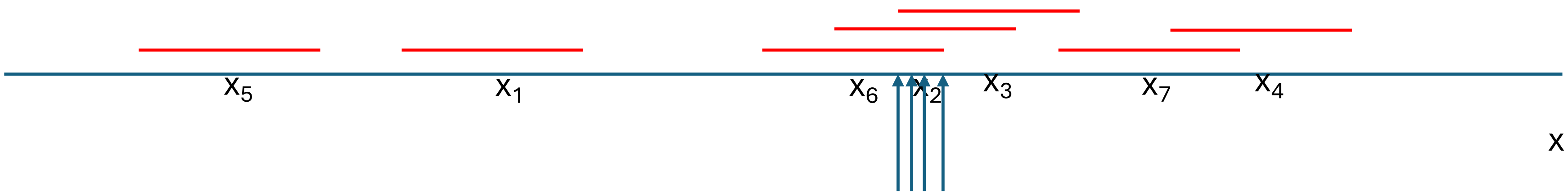

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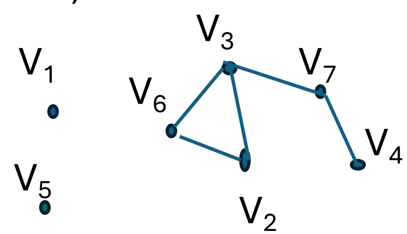

What type of graph can arise this way? Any graph? Only some graphs? If so *which* graphs and what are their properties?

It turns out that this type of graph arose from other considerations – e.g., timetabling – where one might want to *minimize* overlap. Or scheduling where one might want to ensure some overlaps (train arriving and another leaving so passengers can transfer…

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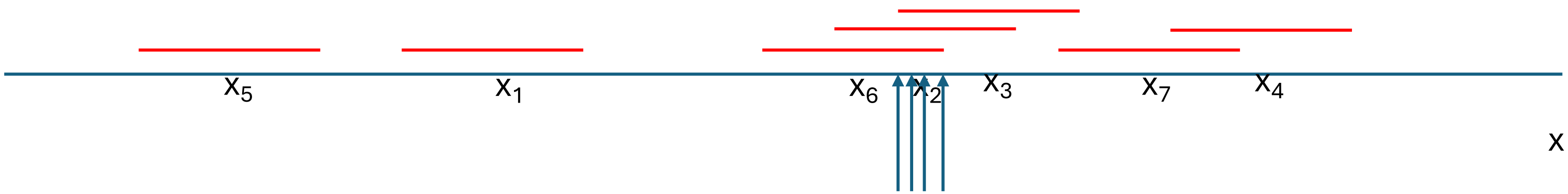

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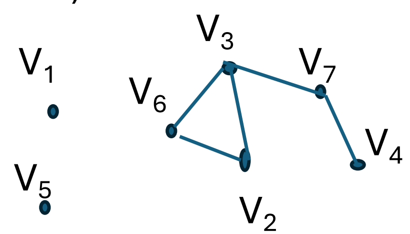

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The calls is called Interval class of graphs. Well, actually, since our intervals are equal sized, our examples are a subclass – “unit interval graphs”. Look them up!