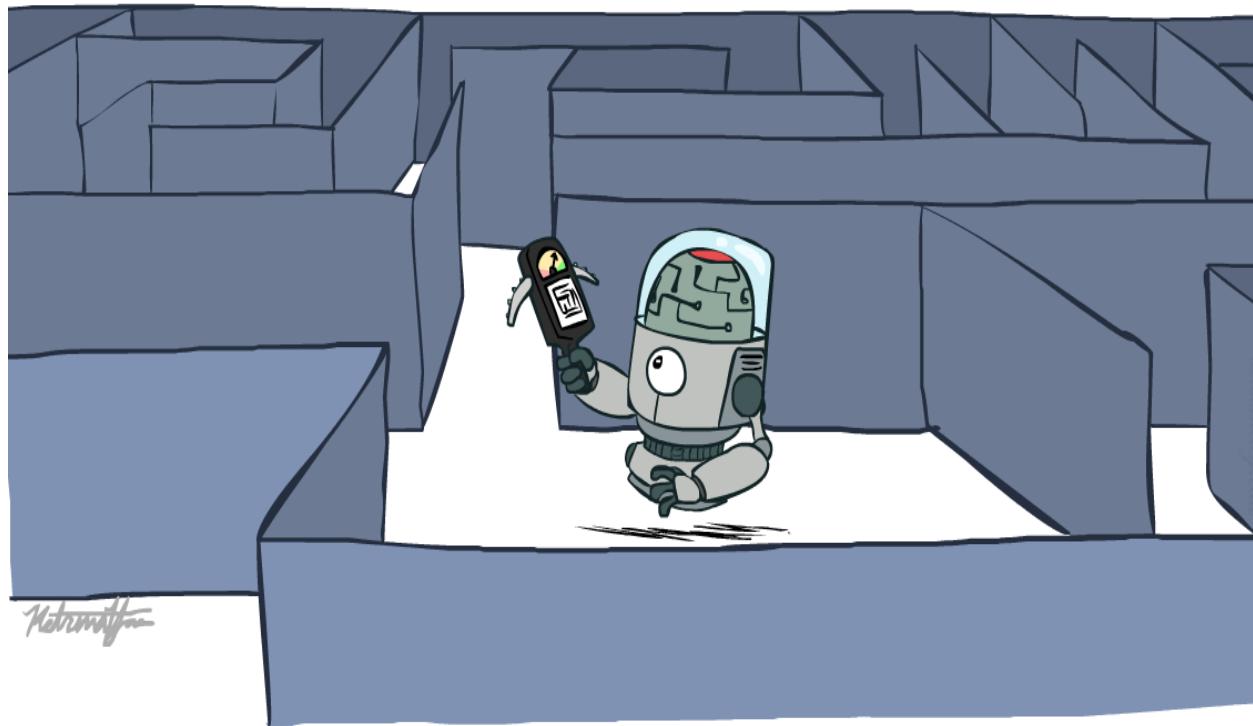


Artificial Intelligence

Informed Search



Instructors: David Suter and Qince Li

Course Delivered for HIT

[Many slides adapted from those created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. Some others from colleagues at Adelaide University.]

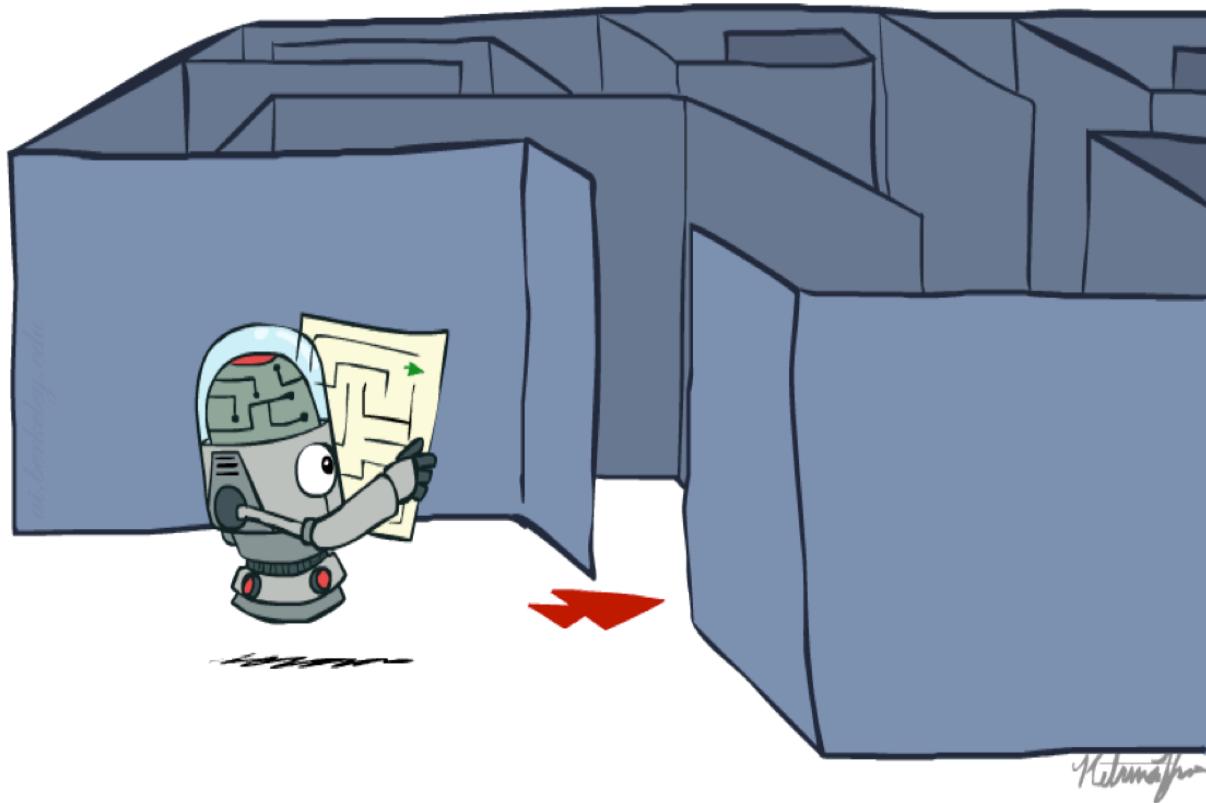
Topics

- Informed Search
 - Heuristics
 - Greedy Search
 - A* Search

- Graph Search

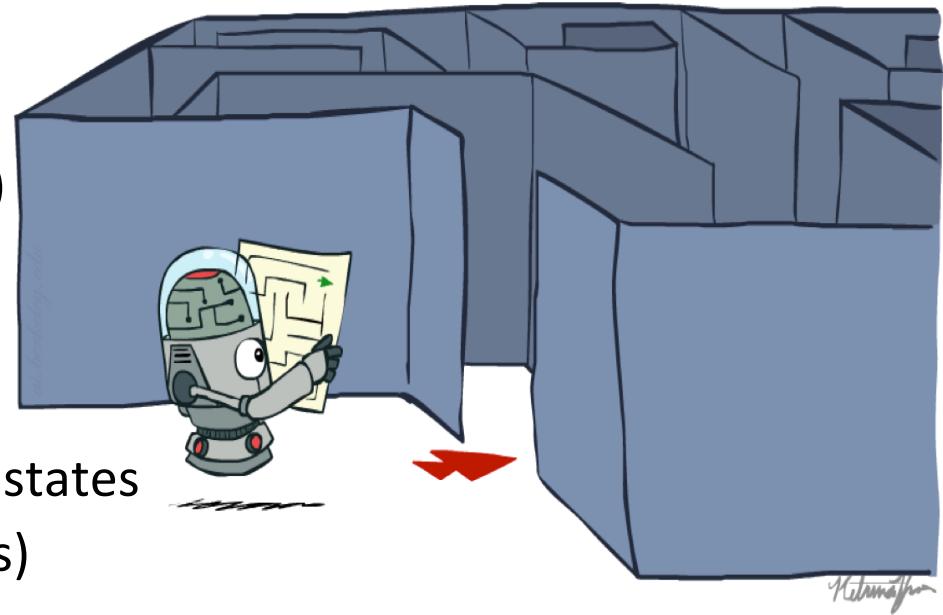


Recap: Search



Recap: Search

- **Search problem:**
 - States (configurations of the world)
 - Actions and costs
 - Successor function (world dynamics)
 - Start state and goal test
- **Search tree:**
 - Nodes: represent plans for reaching states
 - Plans have costs (sum of action costs)
- **Search algorithm:**
 - Systematically builds a search tree
 - Chooses an ordering of the fringe (unexplored nodes)
 - Optimal: finds least-cost plans

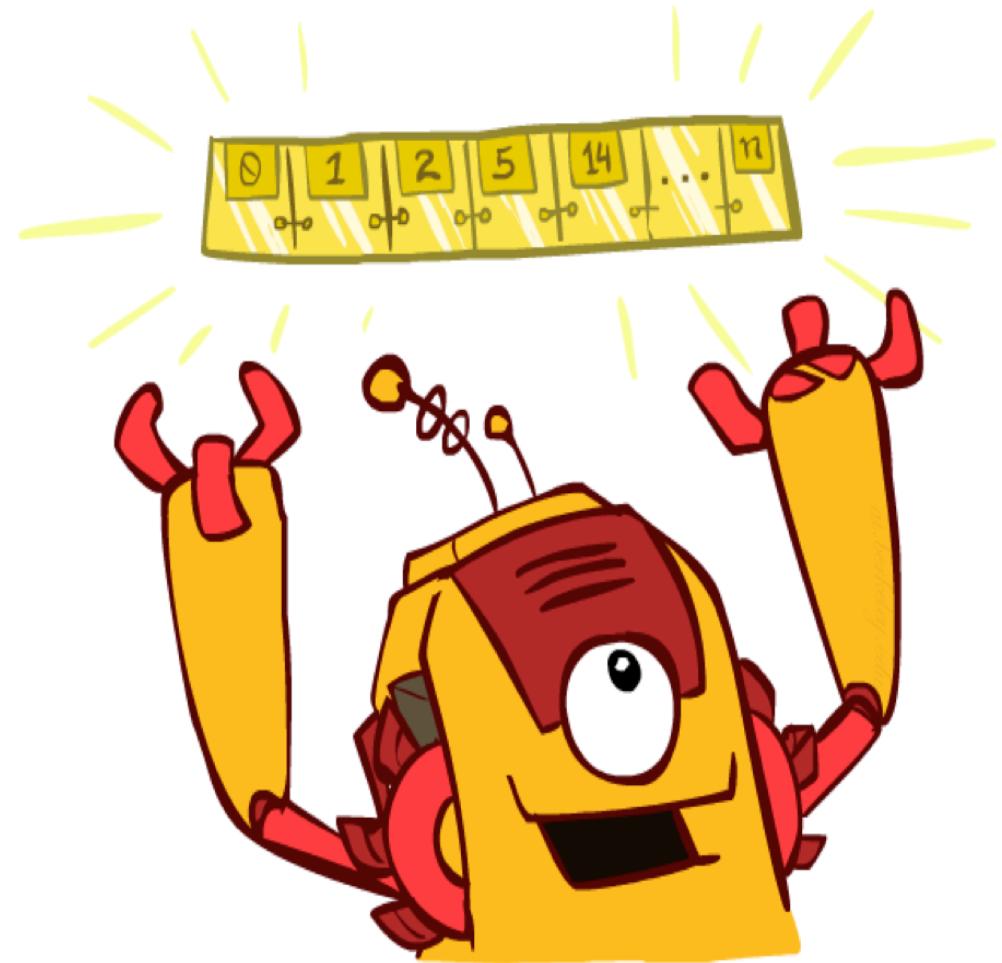


General Tree Search

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

The One Queue

- All these search algorithms are the same except for fringe strategies
 - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
 - Practically, for DFS and BFS, you can avoid the $\log(n)$ overhead from an actual priority queue, using stacks and queues
 - Can even code one implementation that takes a variable queuing object



Uninformed Search

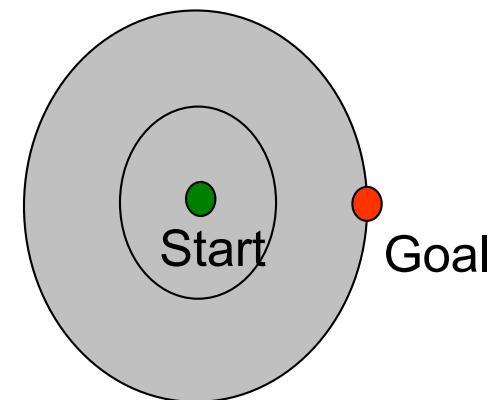
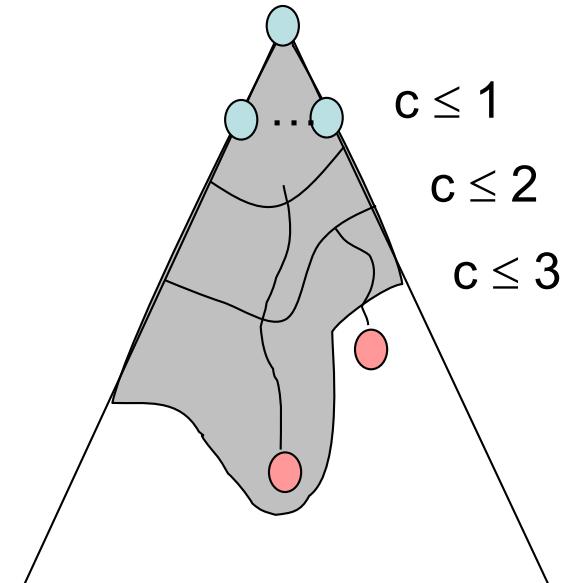


Uniform Cost Search

- Strategy: expand lowest path cost

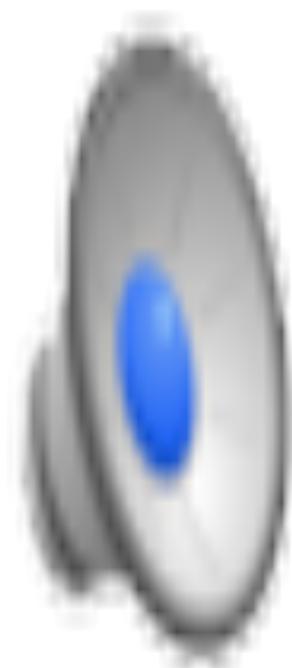
- The good: UCS is complete and optimal!

- The bad:
 - Explores options in every “direction”
 - No information about goal location

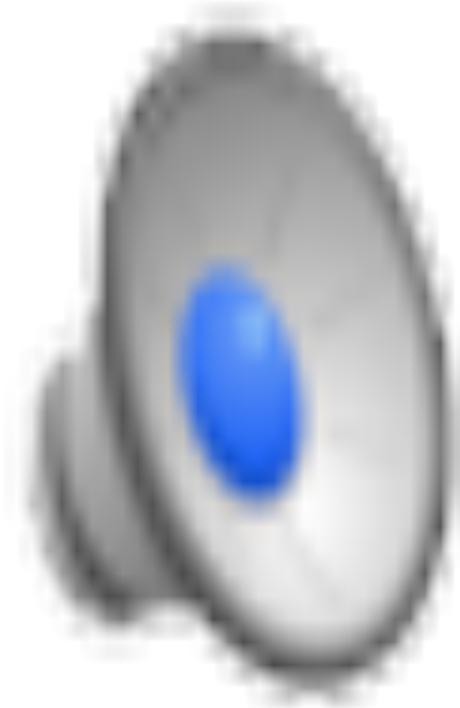


[Demo: contours UCS empty (L3D1)]
[Demo: contours UCS pacman small]

Video of Demo Contours UCS Empty



Video of Demo Contours UCS Pacman Small Maze



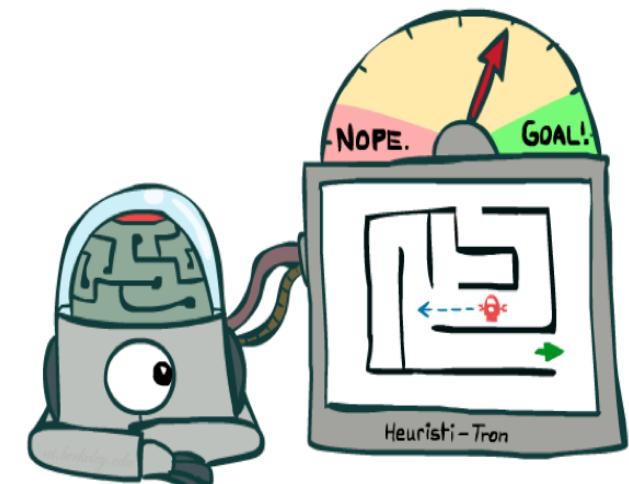
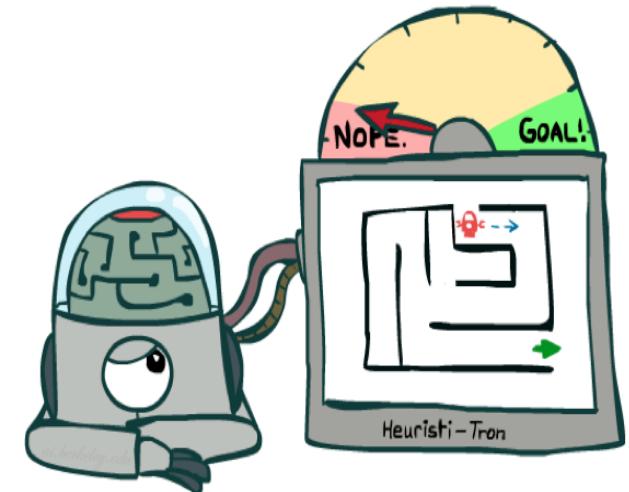
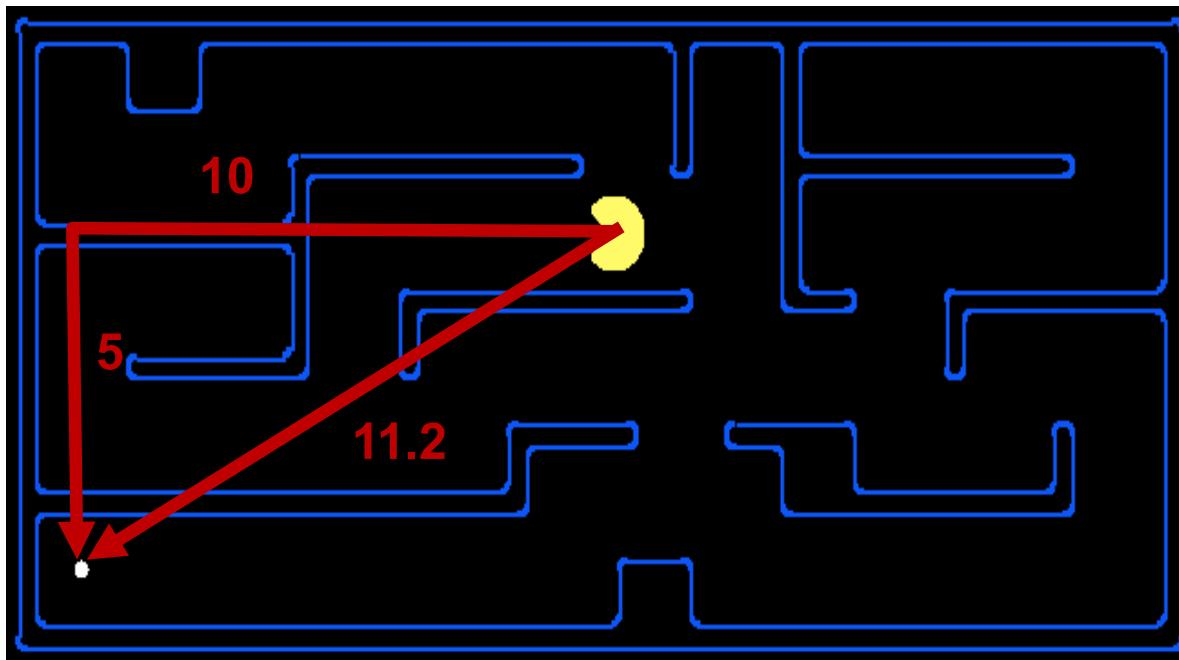
Informed Search



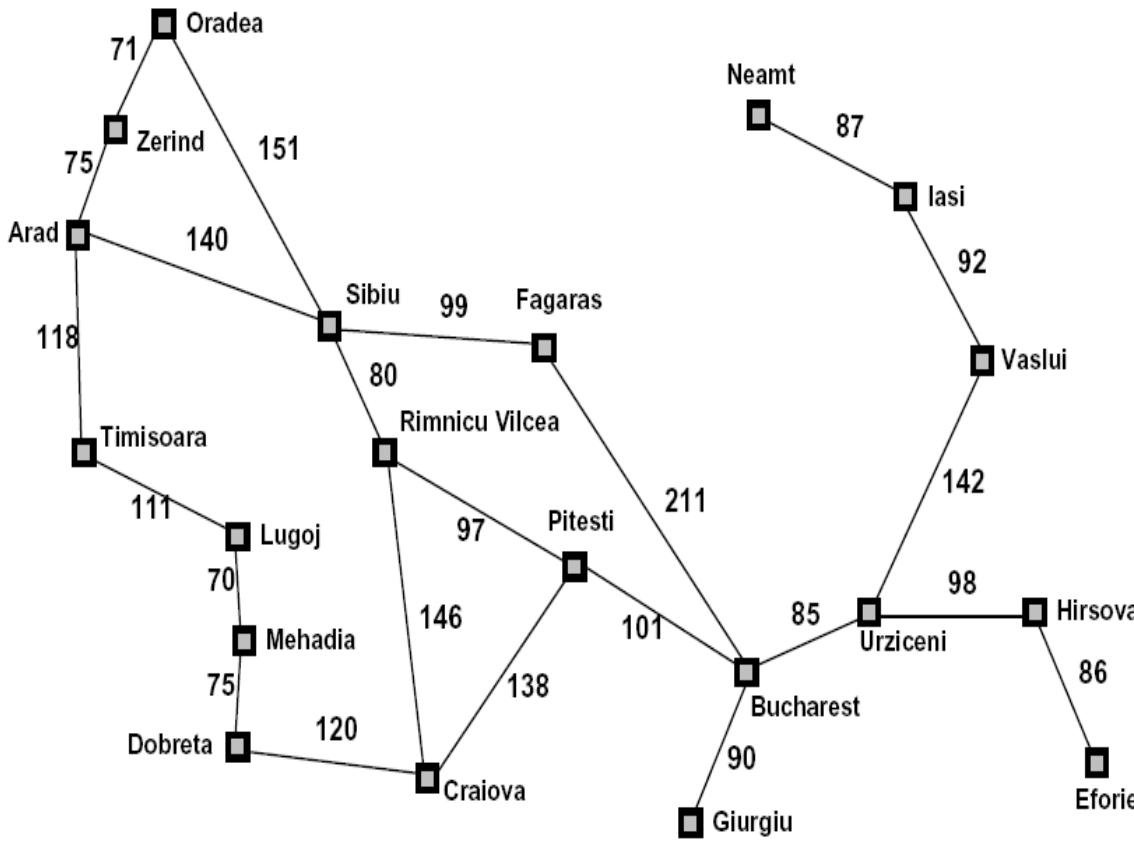
Search Heuristics

- A heuristic is:

- A function that *estimates* how close a state is to a goal
- Designed for a particular search problem
- Examples: Manhattan distance, Euclidean distance for path finding



Example: Heuristic Function



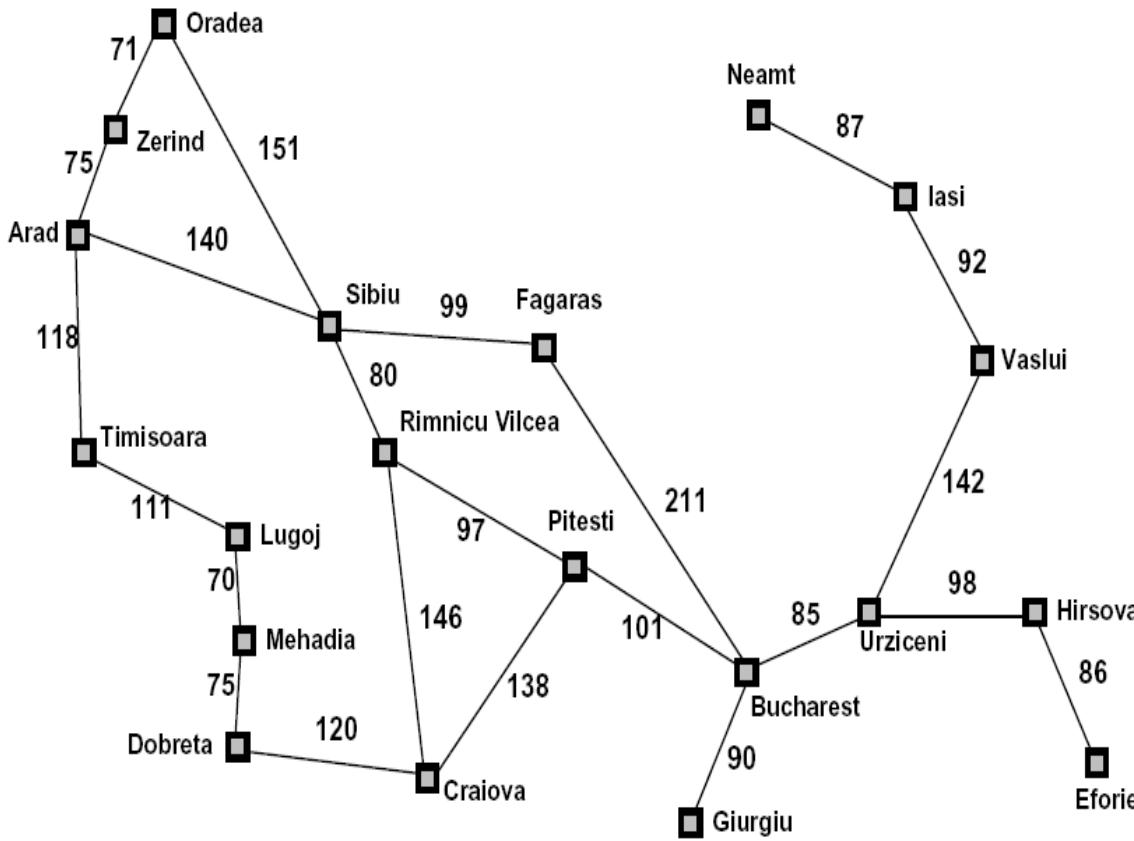
Straight-line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

$h(x)$

Greedy Search



Example: Heuristic Function

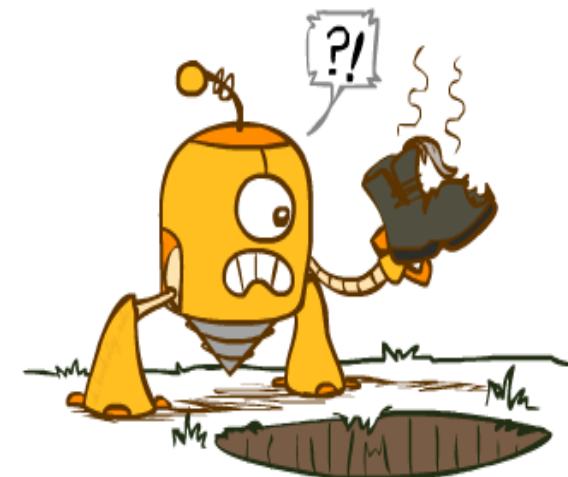
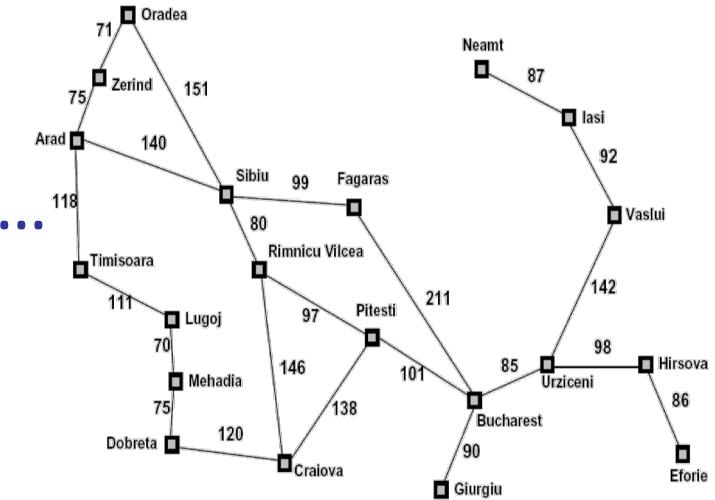
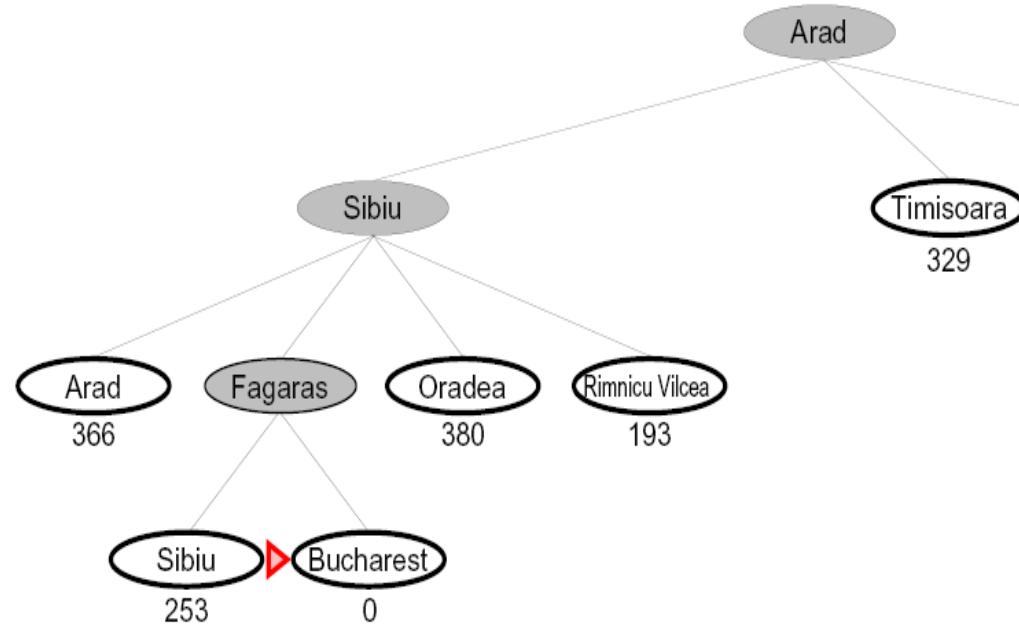


Straight-line distance to Bucharest	
Arad	366
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$h(x)$

Greedy Search

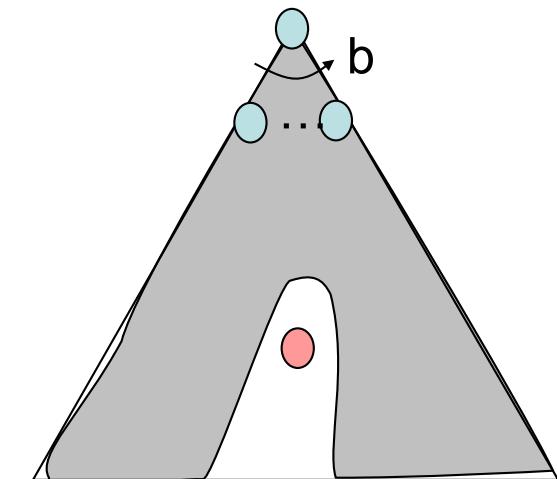
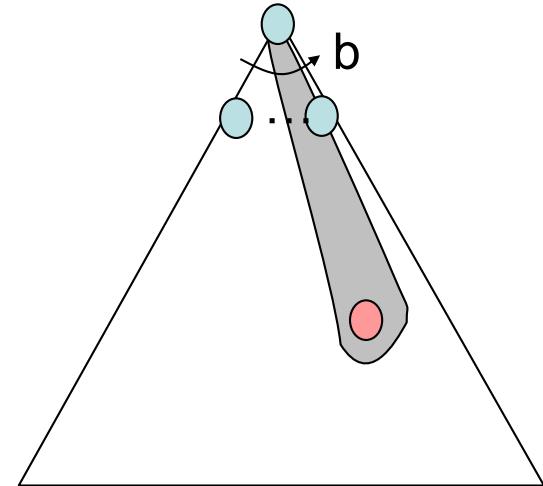
- Expand the node that seems closest...



- What can go wrong?

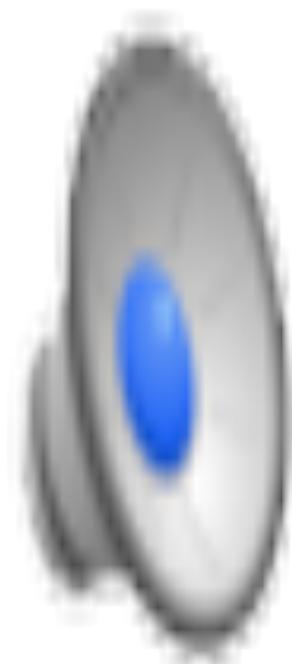
Greedy Search

- Strategy: expand a node that you think is closest to a goal state
 - Heuristic: estimate of distance to nearest goal for each state
- A common case:
 - Best-first takes you straight to the (wrong) goal
- Worst-case: like a badly-guided DFS

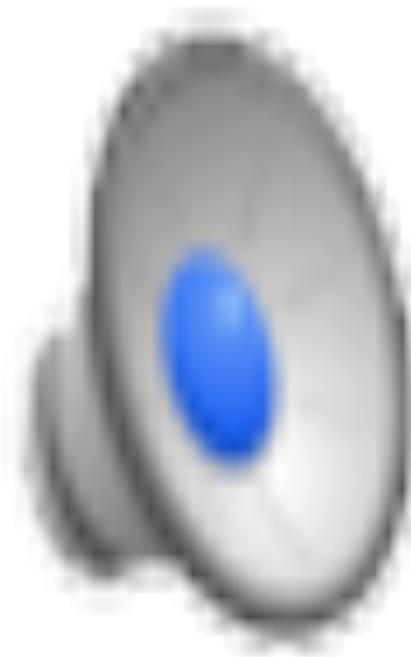


[Demo: contours greedy empty (L3D1)]
[Demo: contours greedy pacman small]

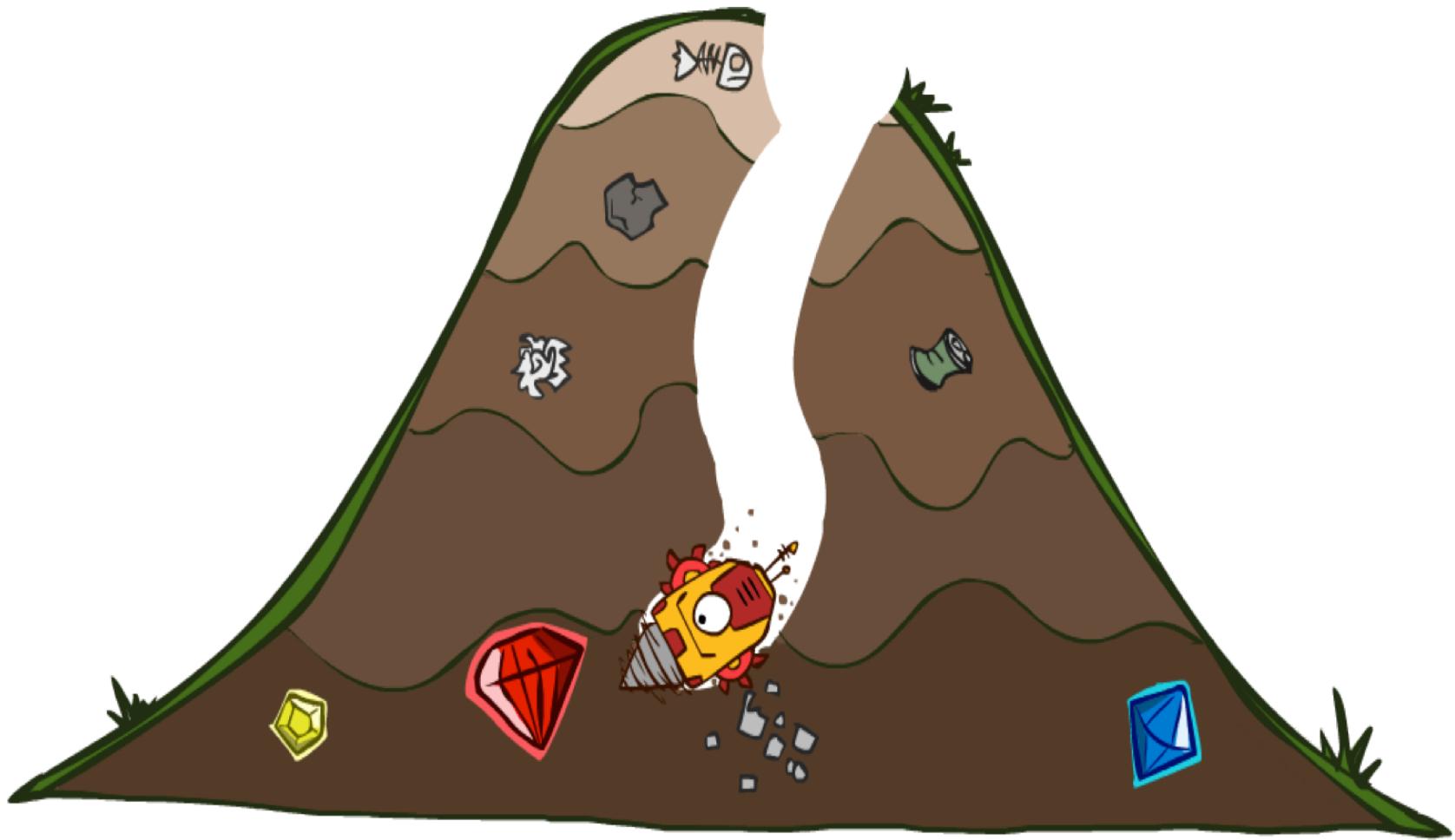
Video of Demo Contours Greedy (Empty)



Video of Demo Contours Greedy (Pacman Small Maze)

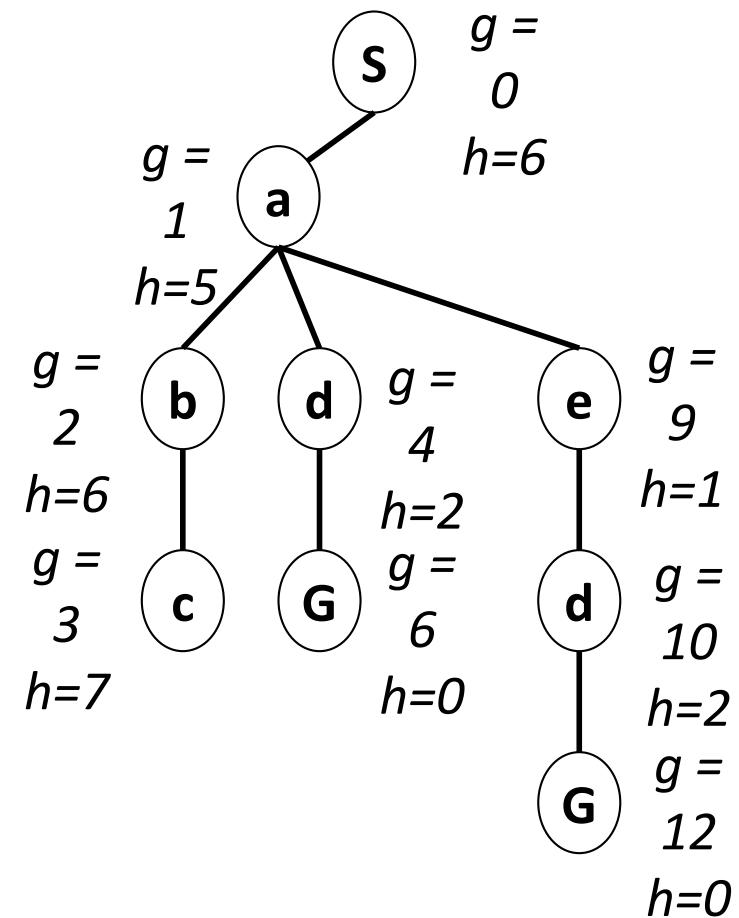
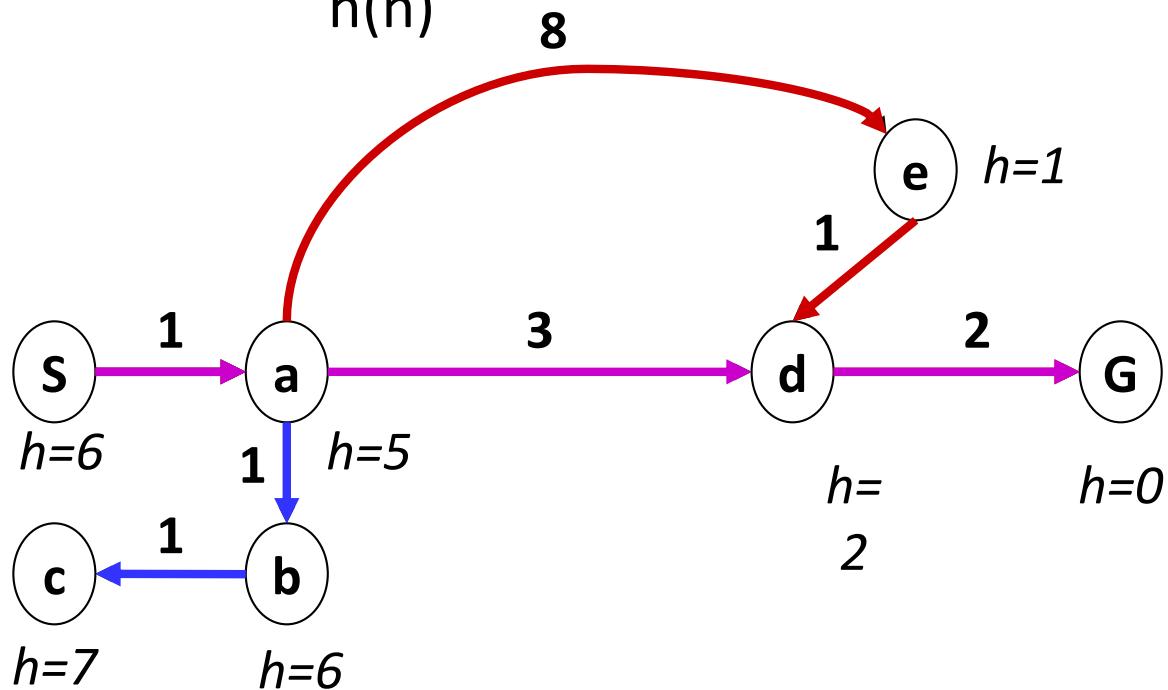


A* Search



Combining UCS and Greedy

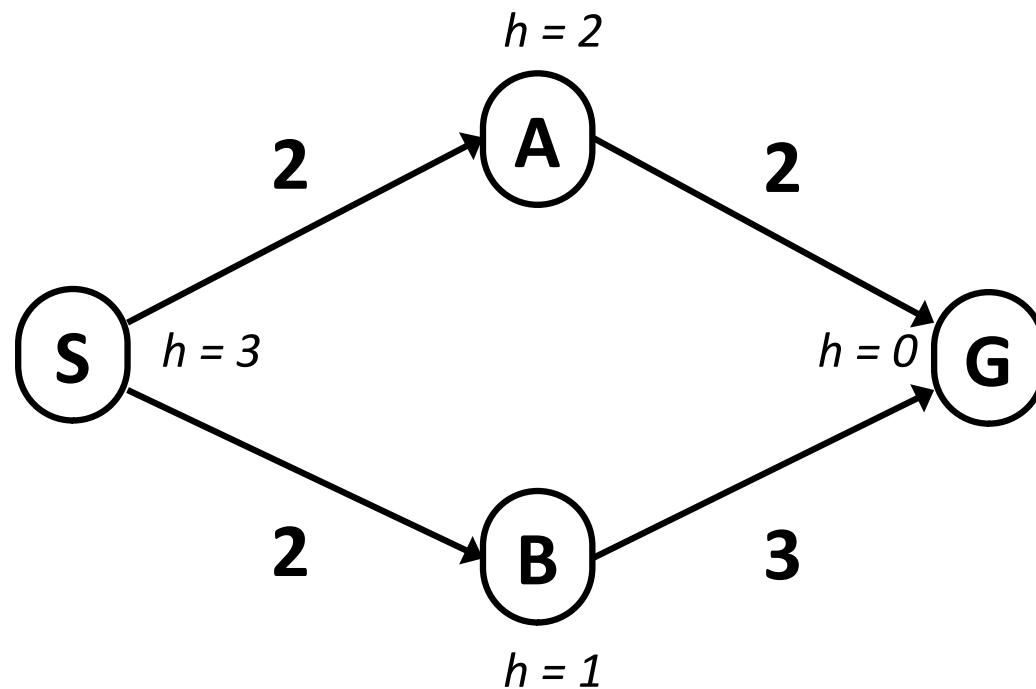
- Uniform-cost orders by path cost, or *backward cost* $g(n)$
- Greedy orders by goal proximity, or *forward cost* $h(n)$



- A* Search orders by the sum: $f(n) = g(n) + h(n)$ Example: Teg Grenager

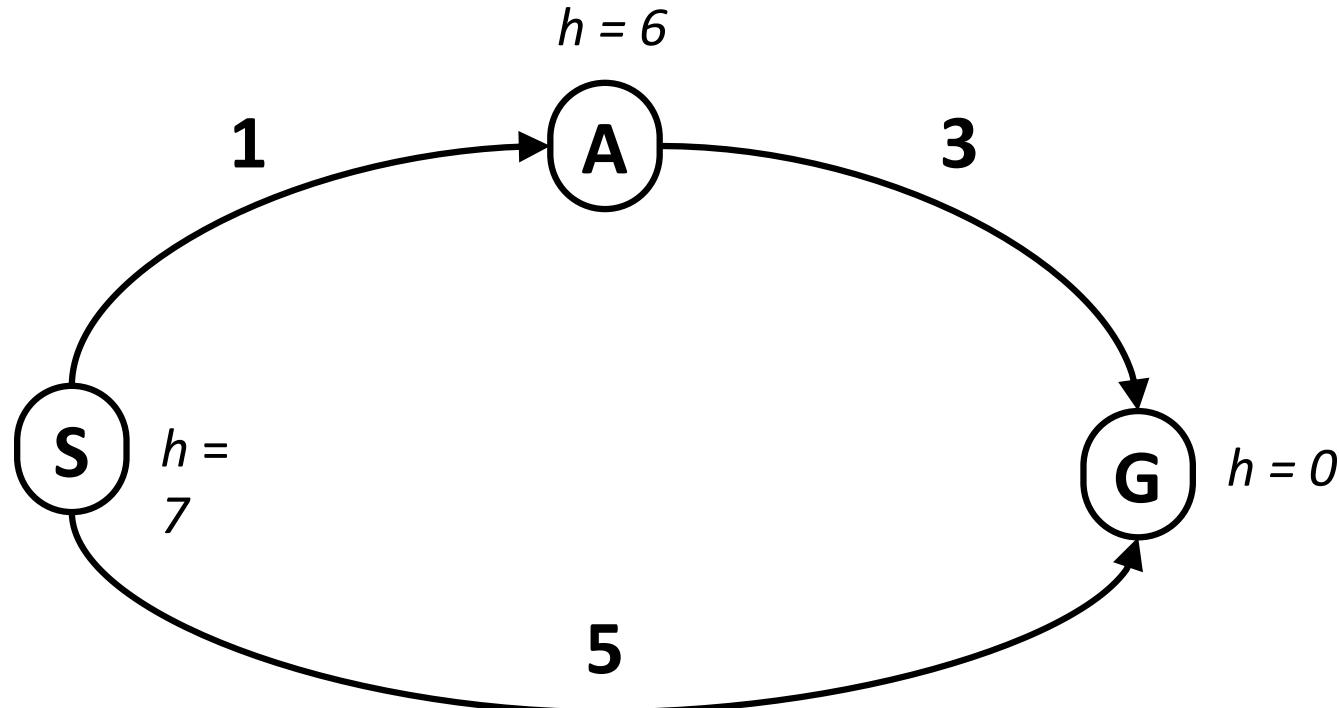
When should A* terminate?

- Should we stop when we enqueue a goal?



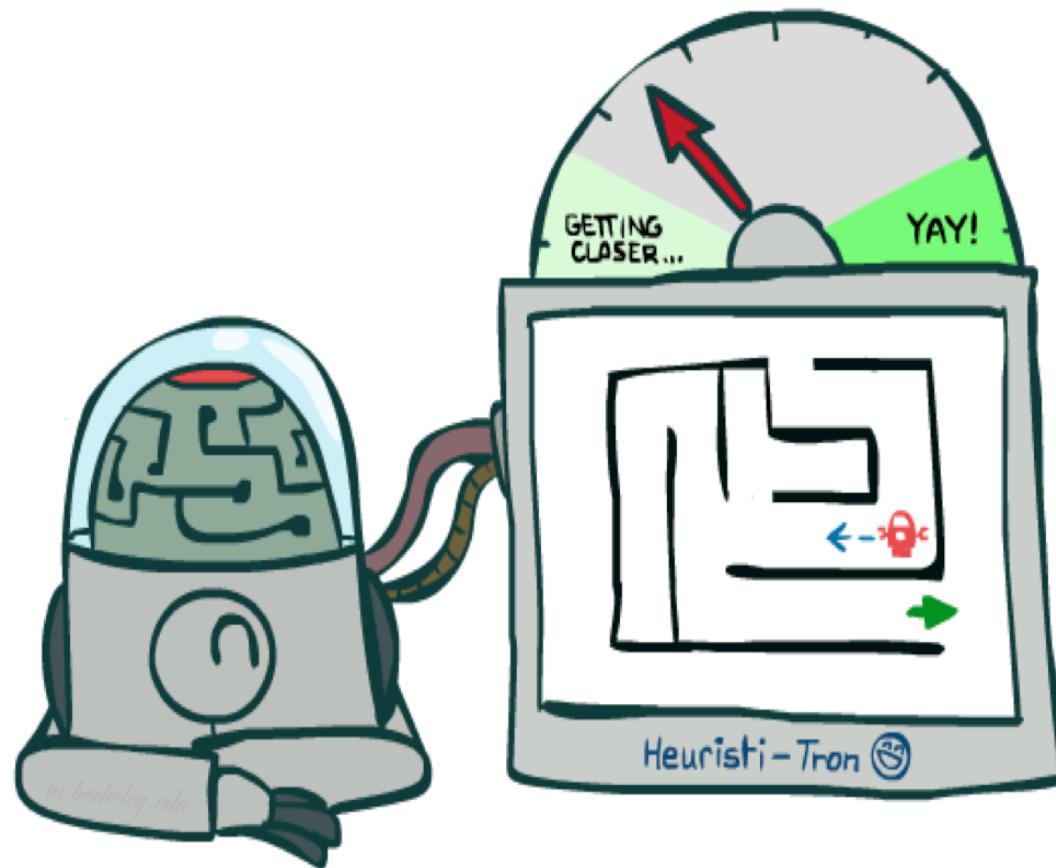
- No: only stop when we dequeue a goal

Is A* Optimal?



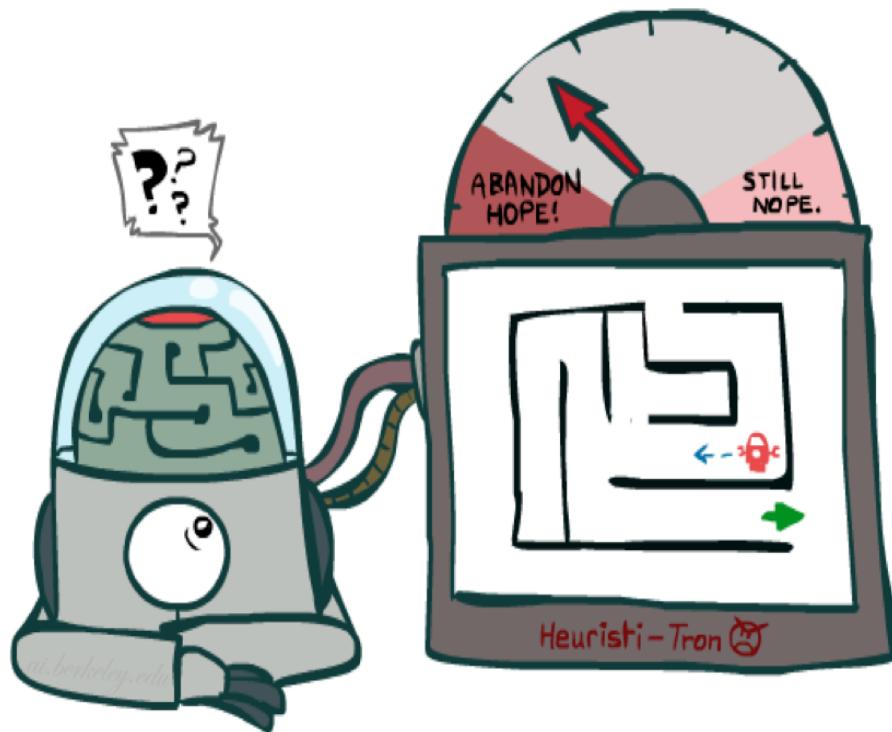
- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

Admissible Heuristics

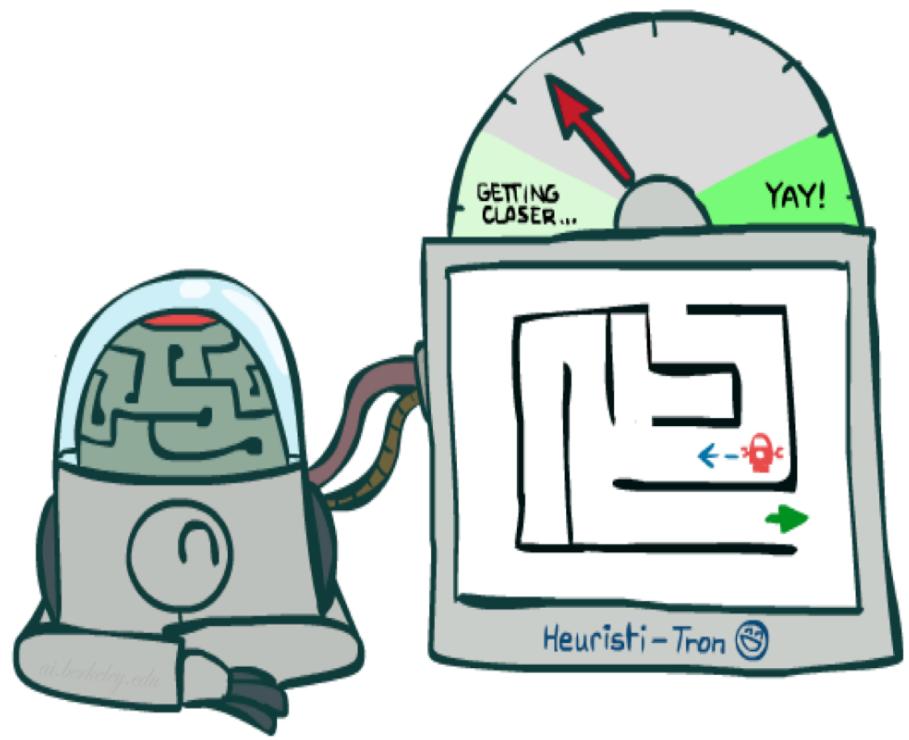


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Idea: Admissibility



Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe



Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

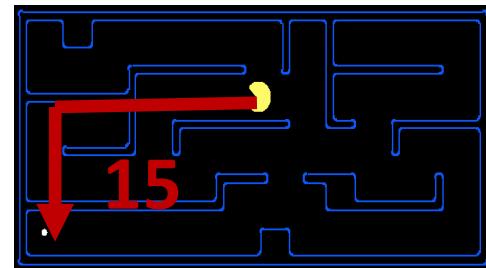
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

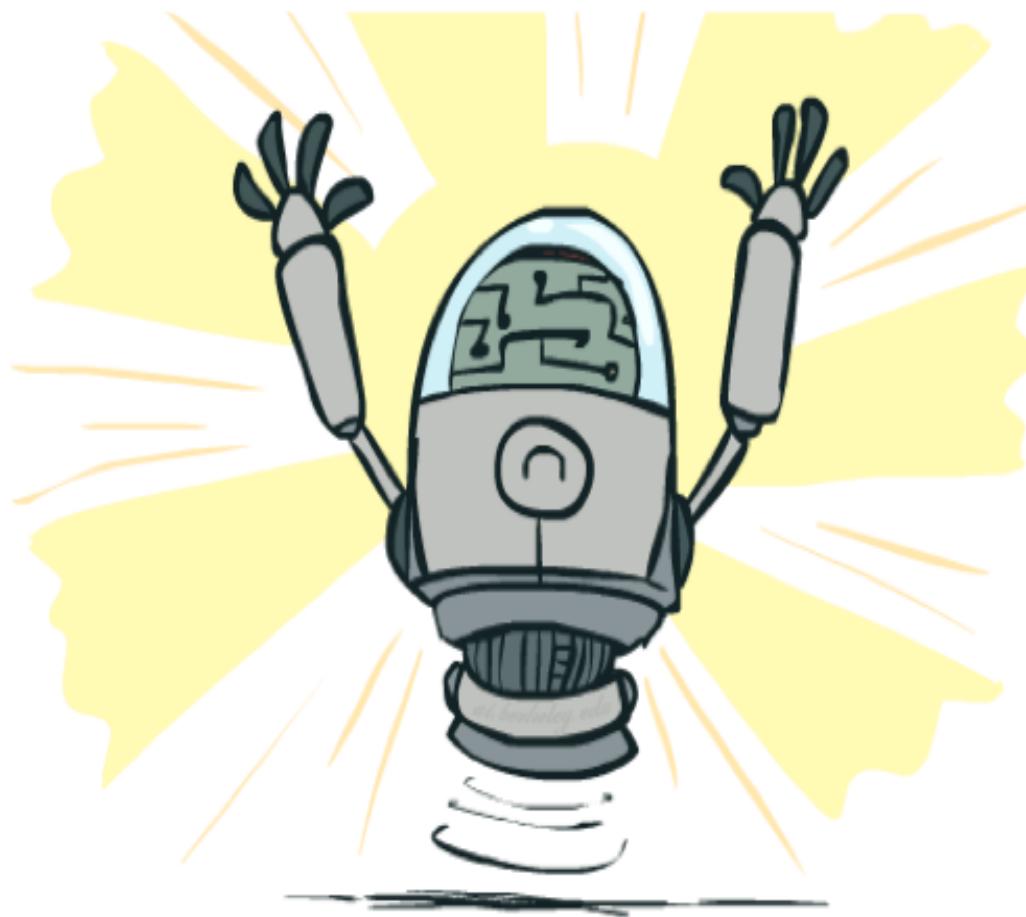
where $h^*(n)$ is the true cost to a nearest goal

- Examples:



- Coming up with admissible heuristics is most of what's involved in using A* in practice.

Optimality of A* Tree Search



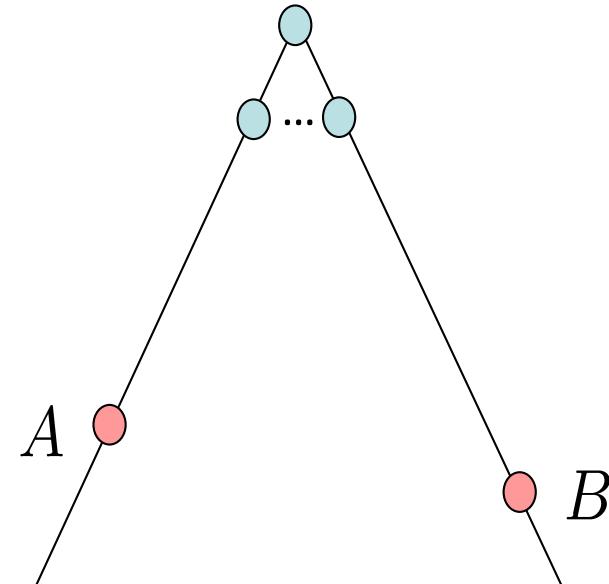
Optimality of A* Tree Search

Assume:

- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

Claim:

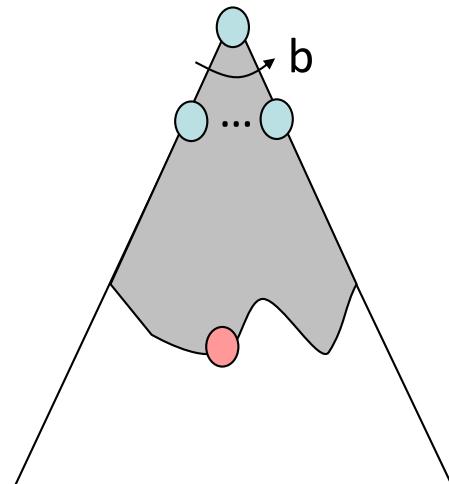
- A will exit the fringe before B



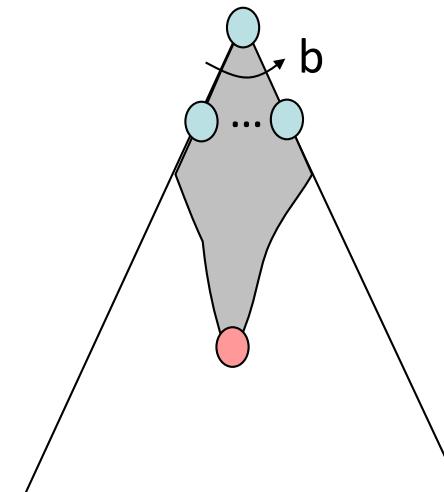
Properties of A*

Properties of A*

Uniform-
Cost

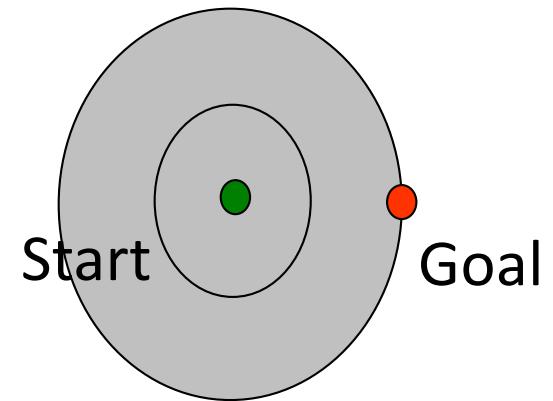


A*

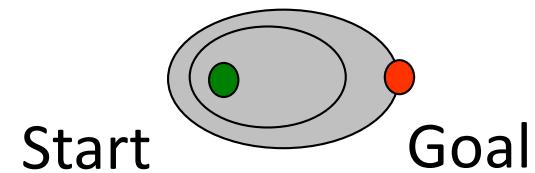


UCS vs A* Contours

- Uniform-cost expands equally in all “directions”



- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



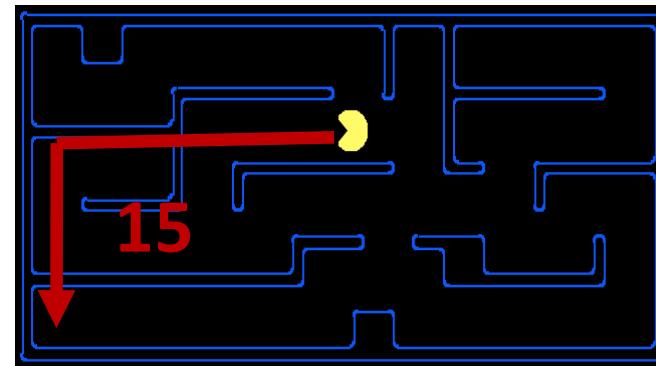
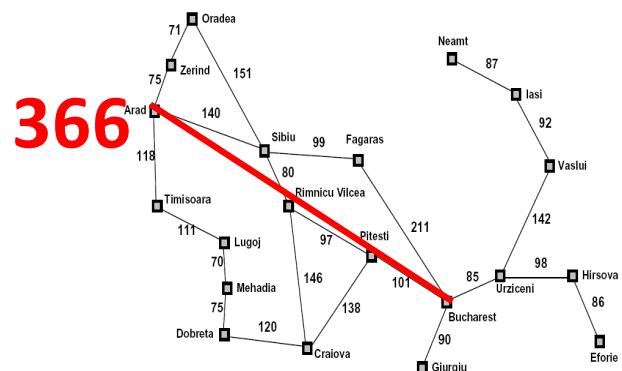
[Demo: contours UCS / greedy / A* empty (L3D1)]
[Demo: contours A* pacman small maze (L3D5)]

A* Applications



Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics
- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available

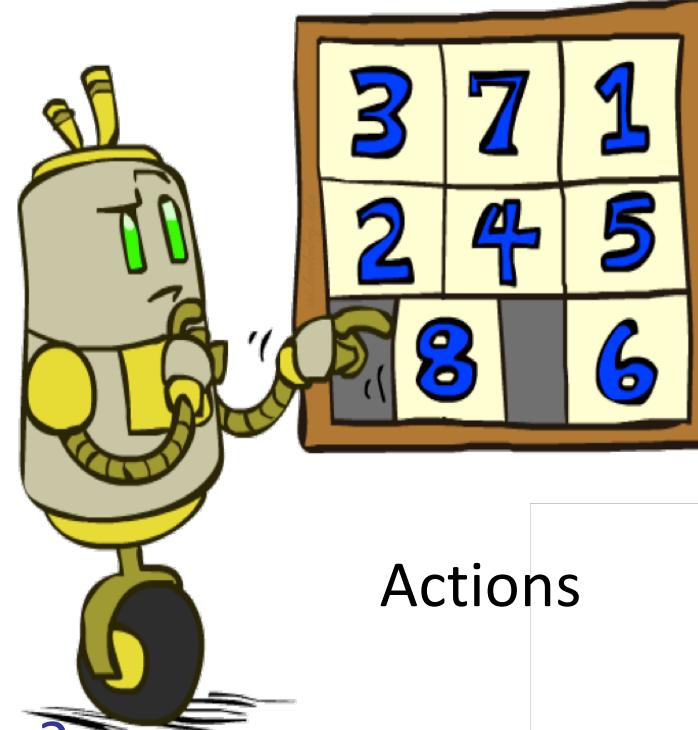


- Inadmissible heuristics are often useful too

Example: 8 Puzzle

7	2	4
5		6
8	3	1

Start State



Actions

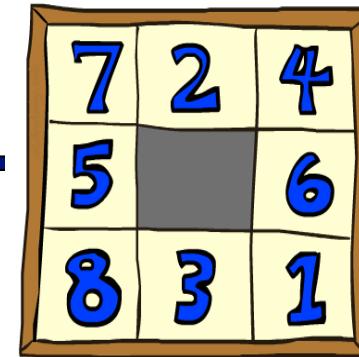
	1	2
3	4	5
6	7	8

Goal State

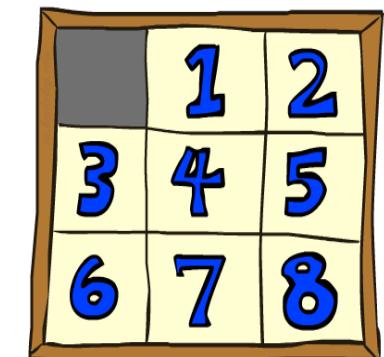
- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- $h(\text{start}) = 8$
- This is a *relaxed-problem* heuristic



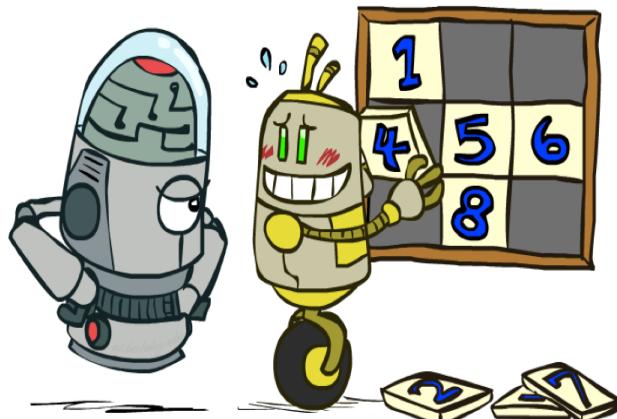
Start
State



Goal
State

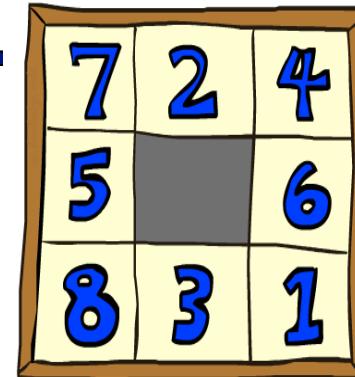
Average nodes expanded
when the optimal path
has...

	...4 steps	...8 steps	...12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

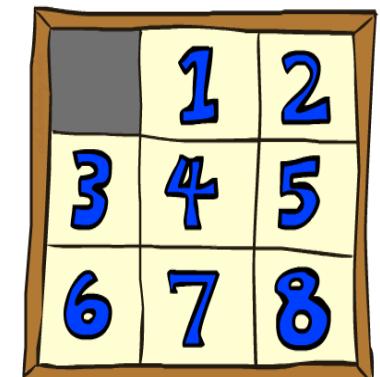


8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total *Manhattan* distance
- Why is it admissible?
- $h(\text{start}) = 3 + 1 + 2 + \dots = 18$



Start
State



Goal
State

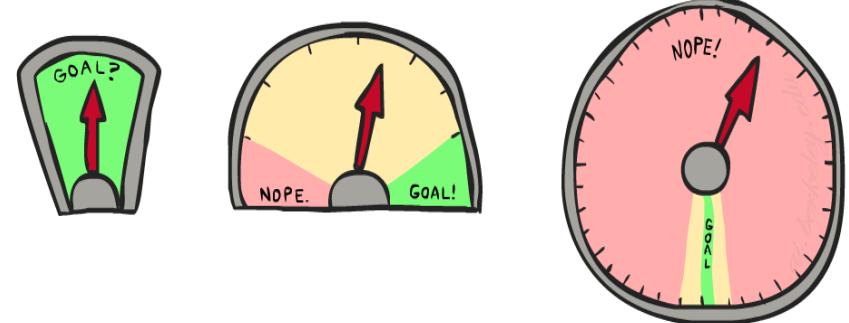
Average nodes expanded
when the optimal path
has...

	...4 steps	...8 steps	...12 steps
TILES	13	39	227
MANHATTAN	12	25	73

8 Puzzle III

- How about using the *actual cost* as a heuristic?

- Would it be admissible?
- Would we save on nodes expanded?
- What's wrong with it?



- With A*: a trade-off between quality of estimate and work per node
 - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Semi-Lattice of Heuristics

Trivial Heuristics, Dominance

- Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

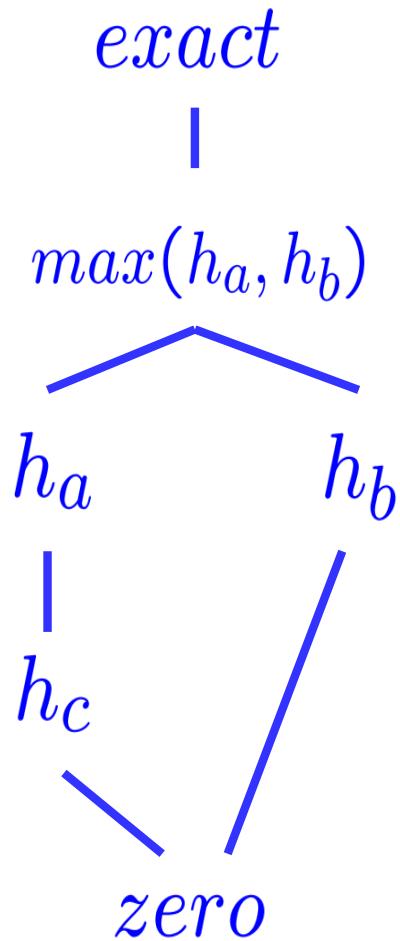
- Heuristics form a semi-lattice:

- Max of admissible heuristics is admissible

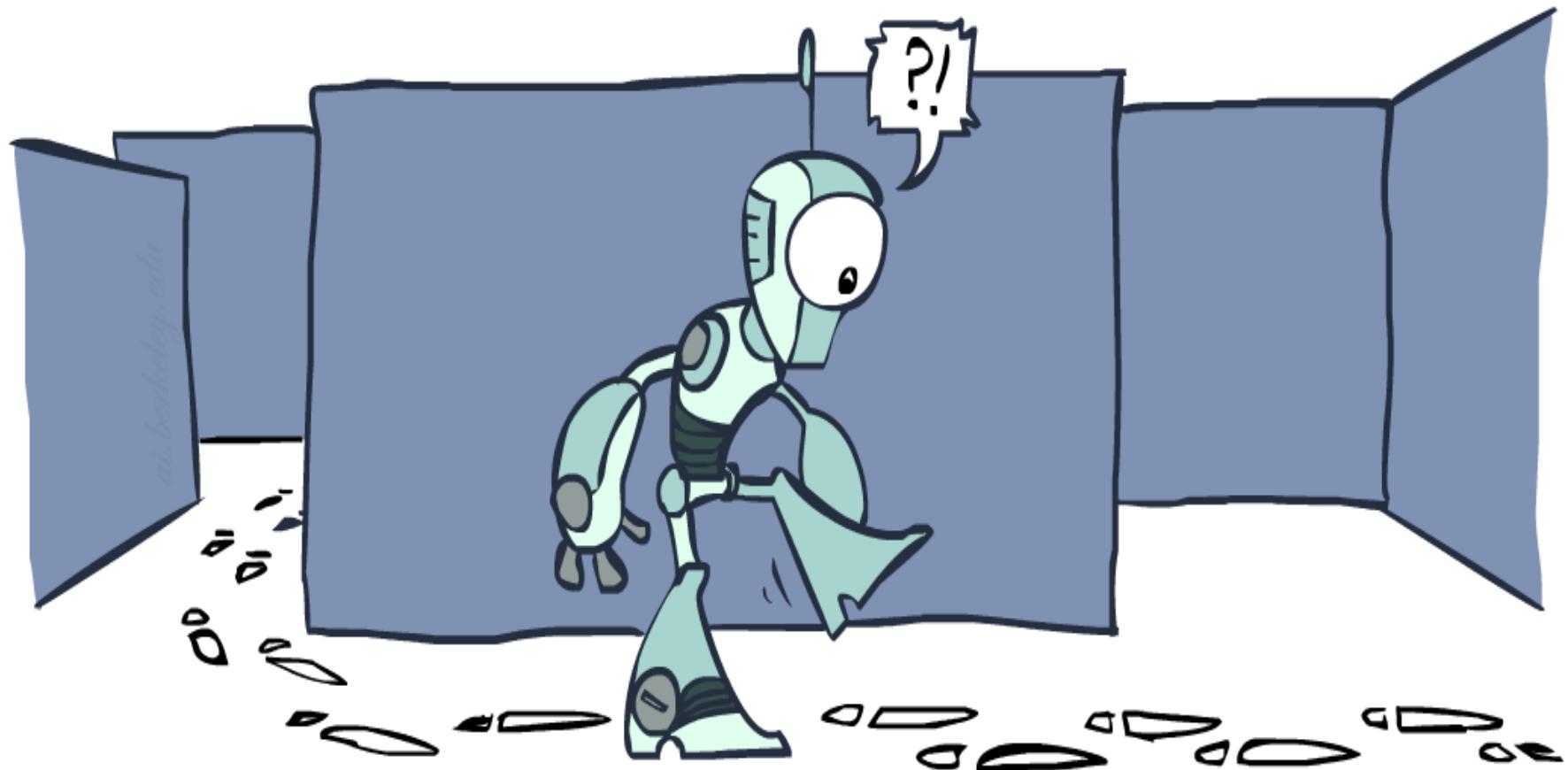
$$h(n) = \max(h_a(n), h_b(n))$$

- Trivial heuristics

- Bottom of lattice is the zero heuristic (what does this give us?)
 - Top of lattice is the exact heuristic



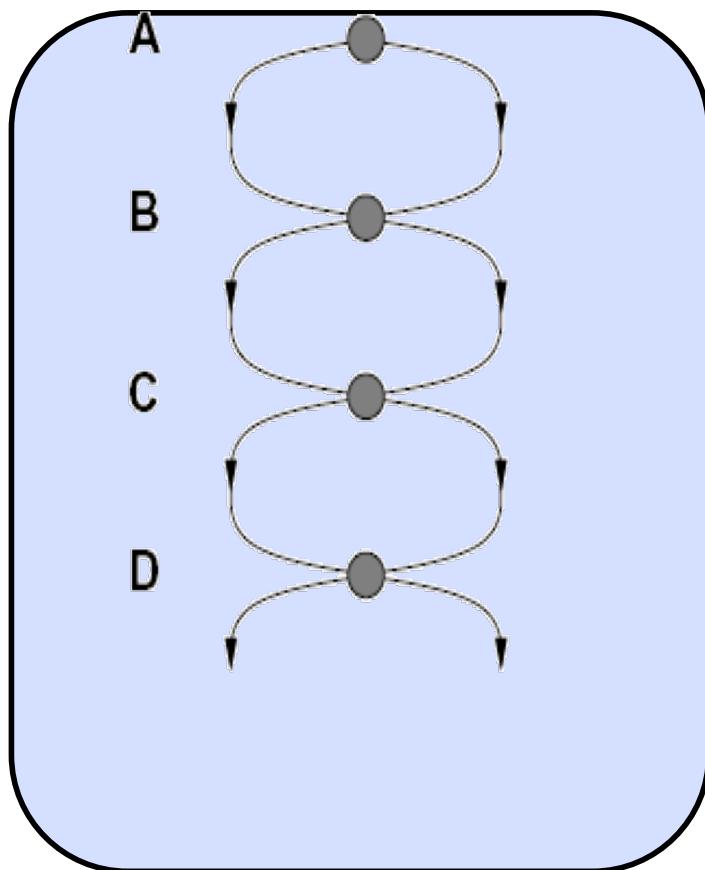
Graph Search



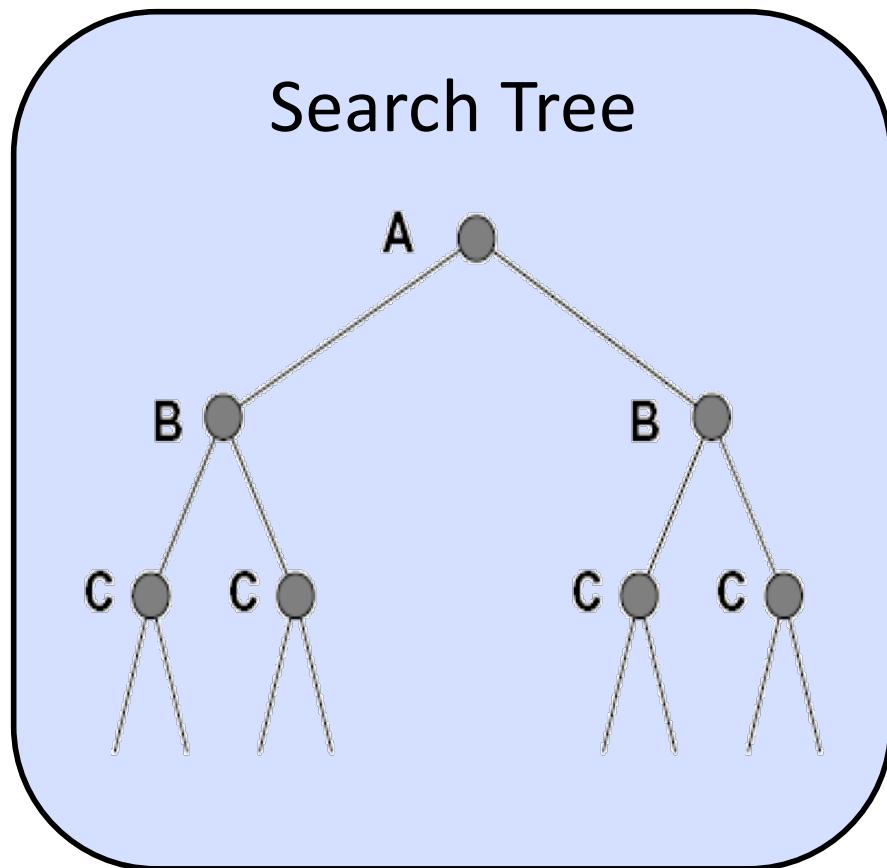
Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.

State Graph

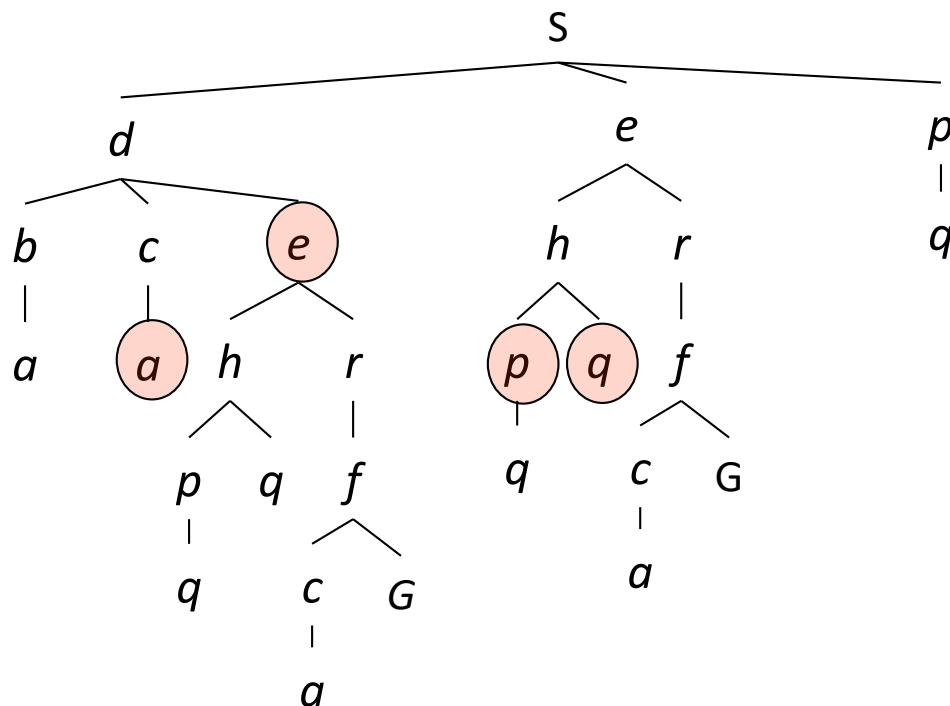


Search Tree



Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)



Graph Search

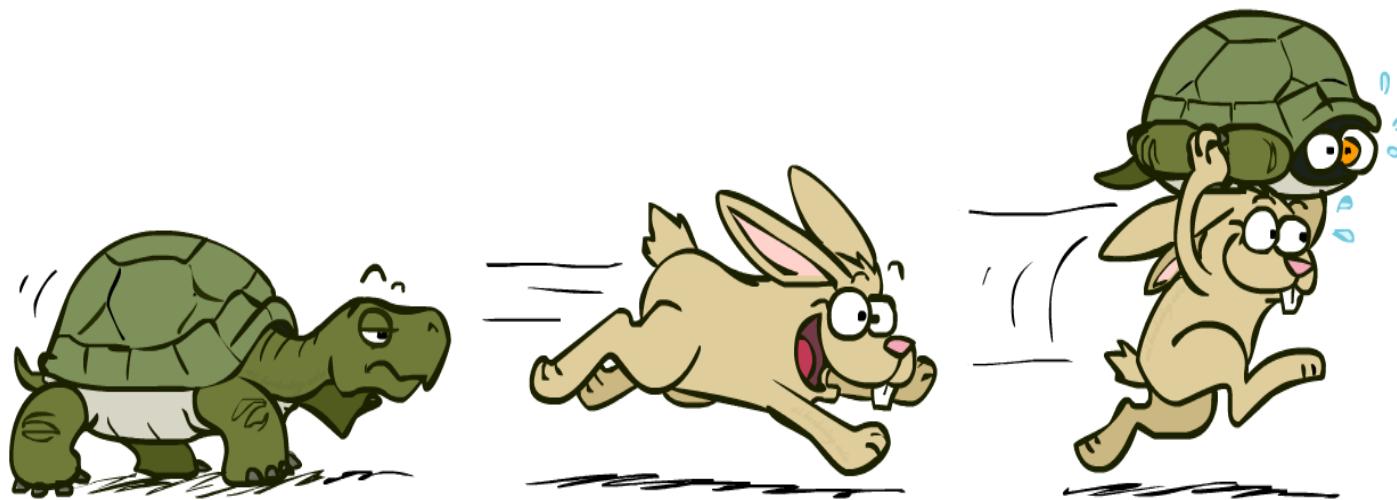
- Idea: never **expand** a state twice
- How to implement:
 - Tree search + set of expanded states (“closed set”)
 - Expand the search tree node-by-node, but...
 - Before expanding a node, check to make sure its state has never been expanded before
 - If not new, skip it, if new add to closed set
- Important: **store the closed set as a set, not a list**

A*: Summary



A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems



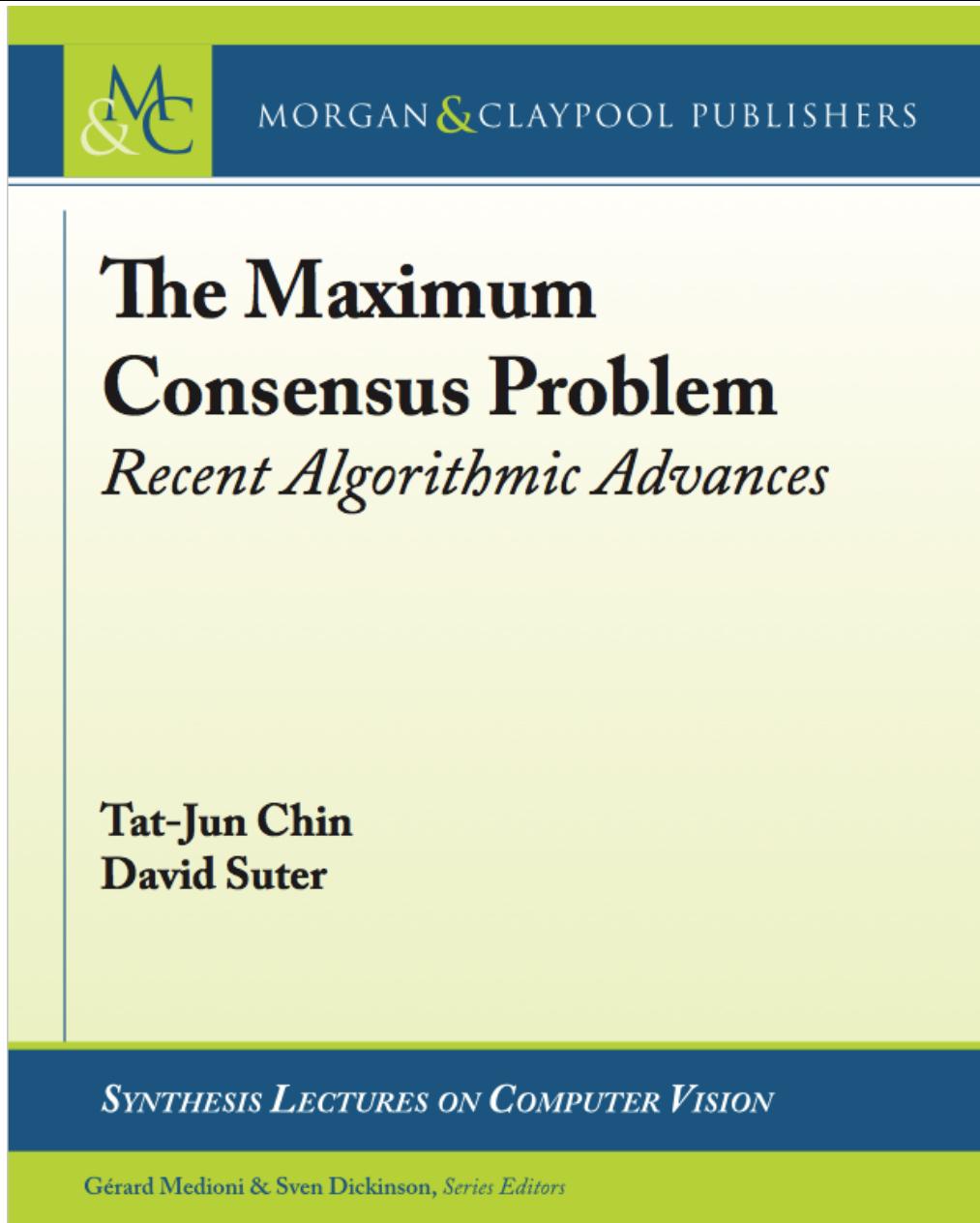
Tree Search Pseudo-Code

```
function TREE-SEARCH(problem, fringe) return a solution, or failure
  fringe  $\leftarrow$  INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node  $\leftarrow$  REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE[node]) then return node
    for child-node in EXPAND(STATE[node], problem) do
      fringe  $\leftarrow$  INSERT(child-node, fringe)
    end
  end
```

Graph Search Pseudo-Code

```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            for child-node in EXPAND(STATE[node], problem) do
                fringe ← INSERT(child-node, fringe)
        end
    end
```

A* turns up in surprising places!



A* Search

Prof. Abbeel steps through a few A* search examples

