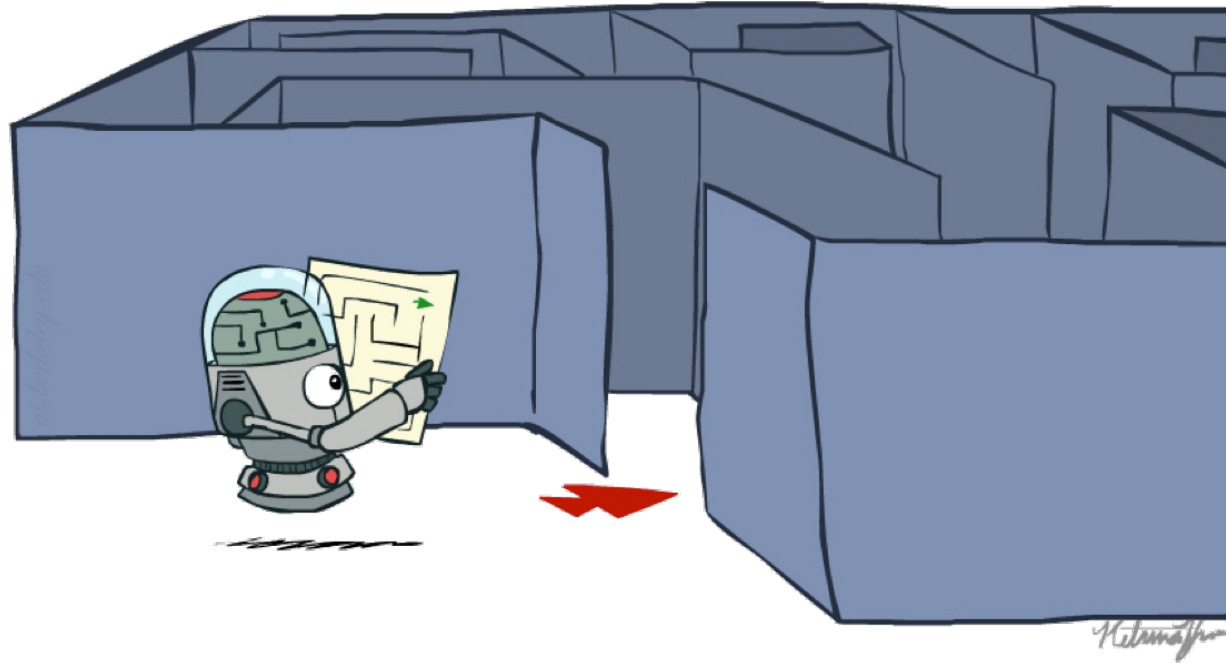


# CS 188: Artificial Intelligence

## Search



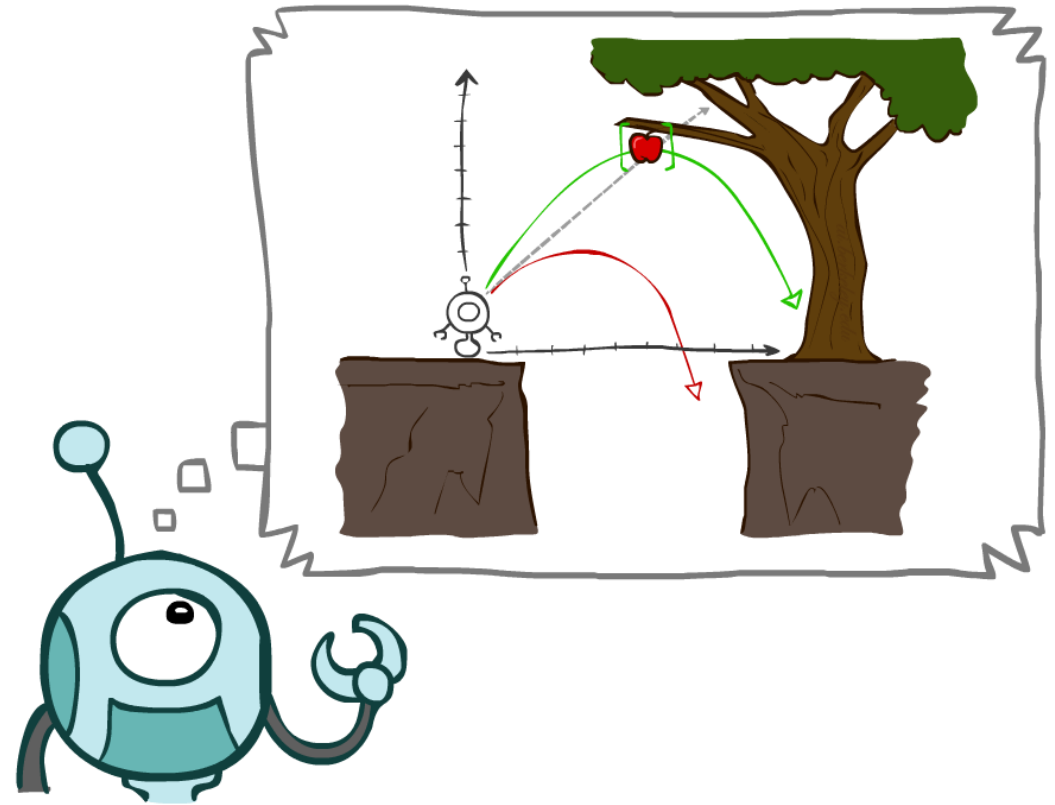
Spring 2024

University of California, Berkeley

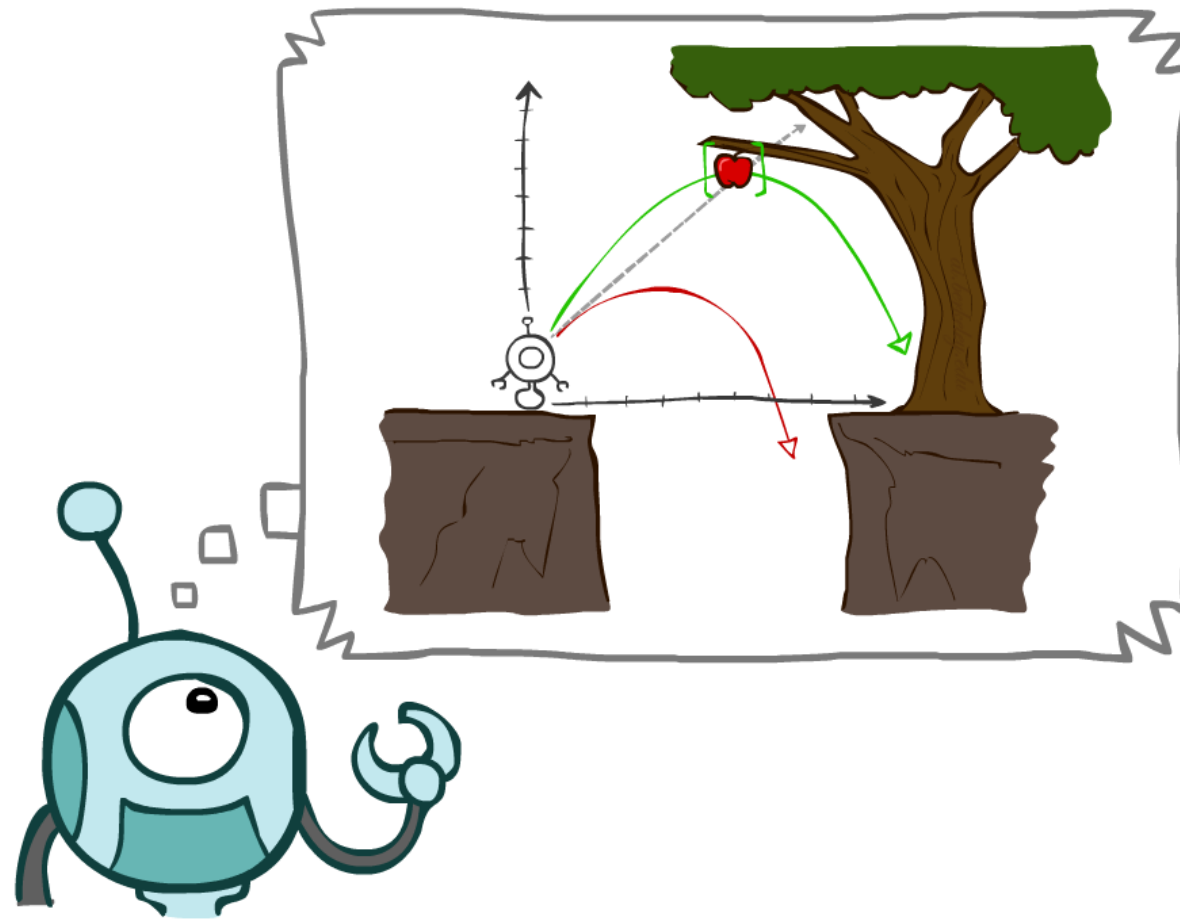
[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley ([ai.berkeley.edu](http://ai.berkeley.edu)).]

# Today

- Agents that Plan Ahead
- Search Problems
- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Uniform-Cost Search

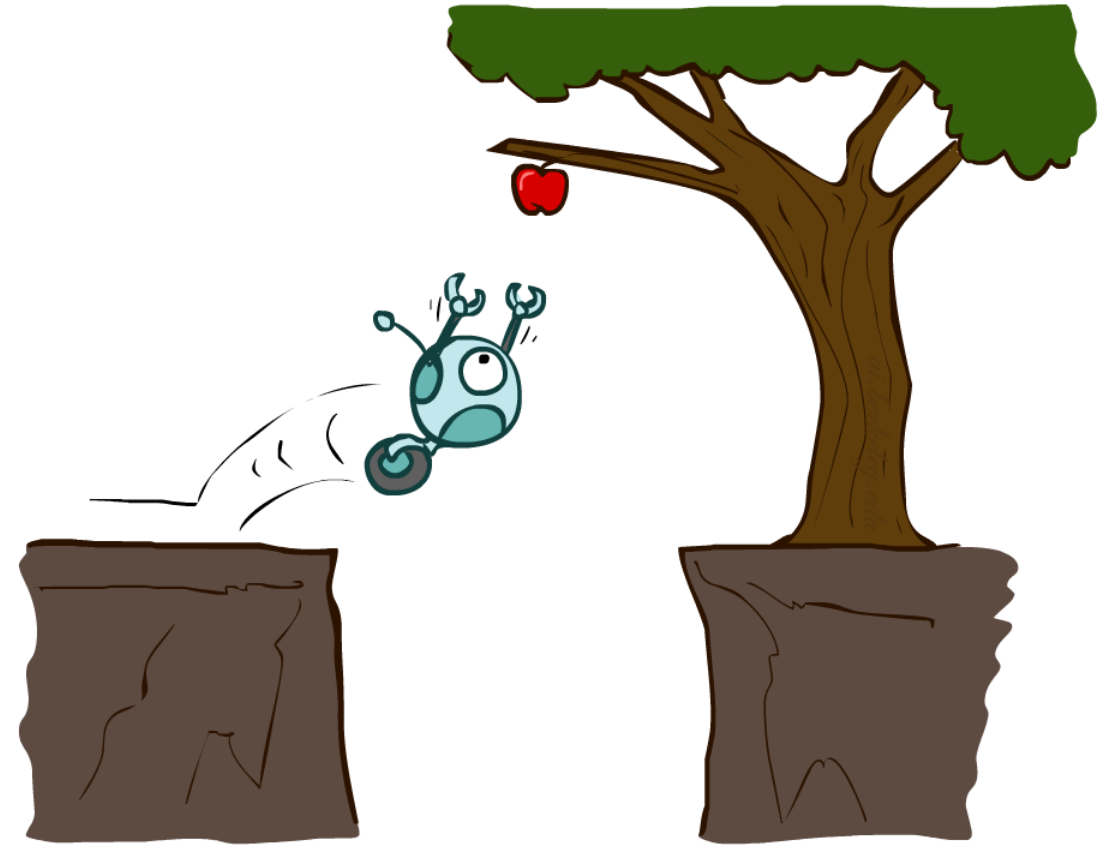


# Agents that Plan



# Reflex Agents

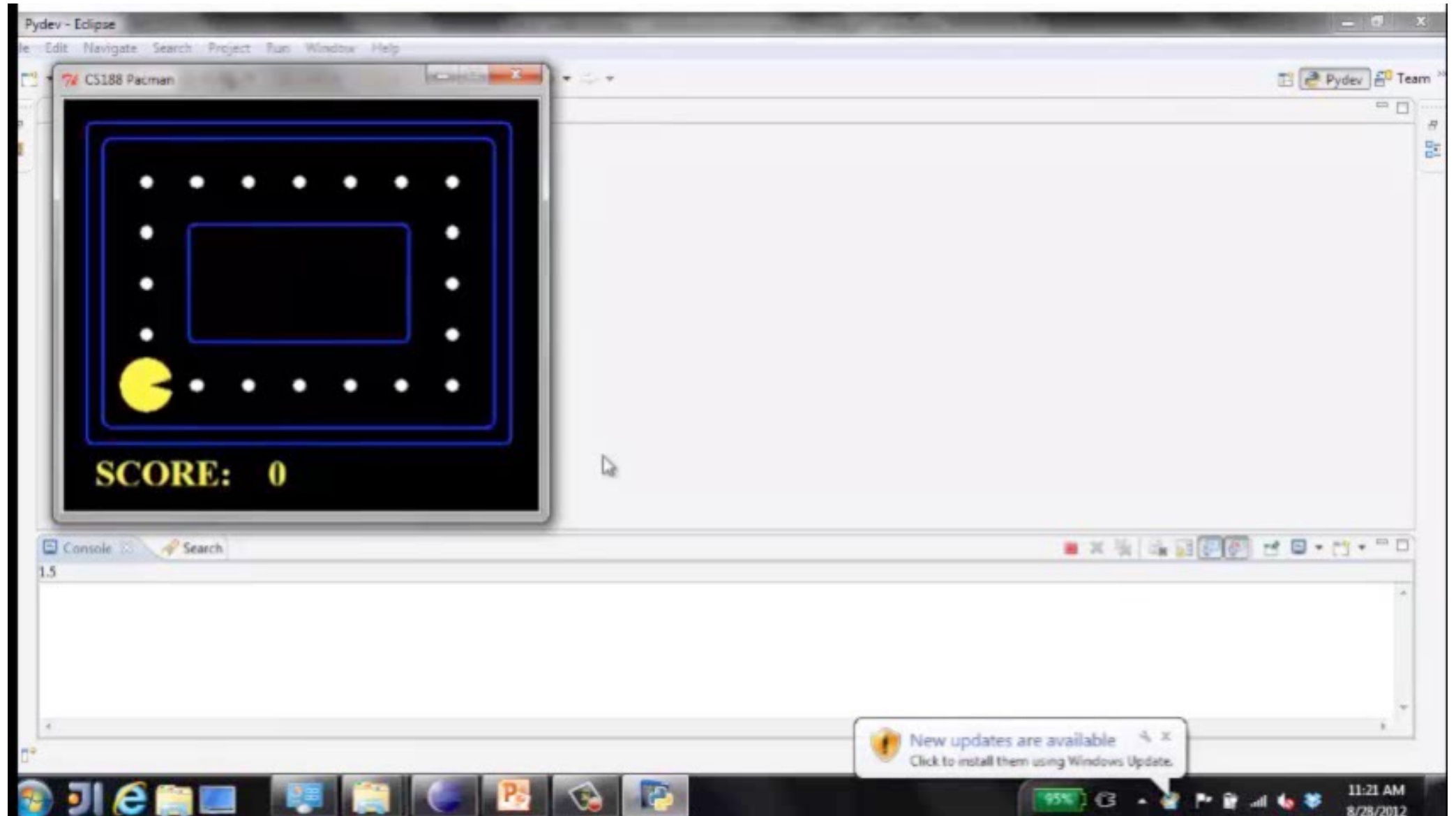
- Reflex agents:
  - Choose action based on current percept (and maybe memory)
  - May have memory or a model of the world's current state
  - Do not consider the future consequences of their actions
  - Consider how the world IS
- Can a reflex agent be rational?



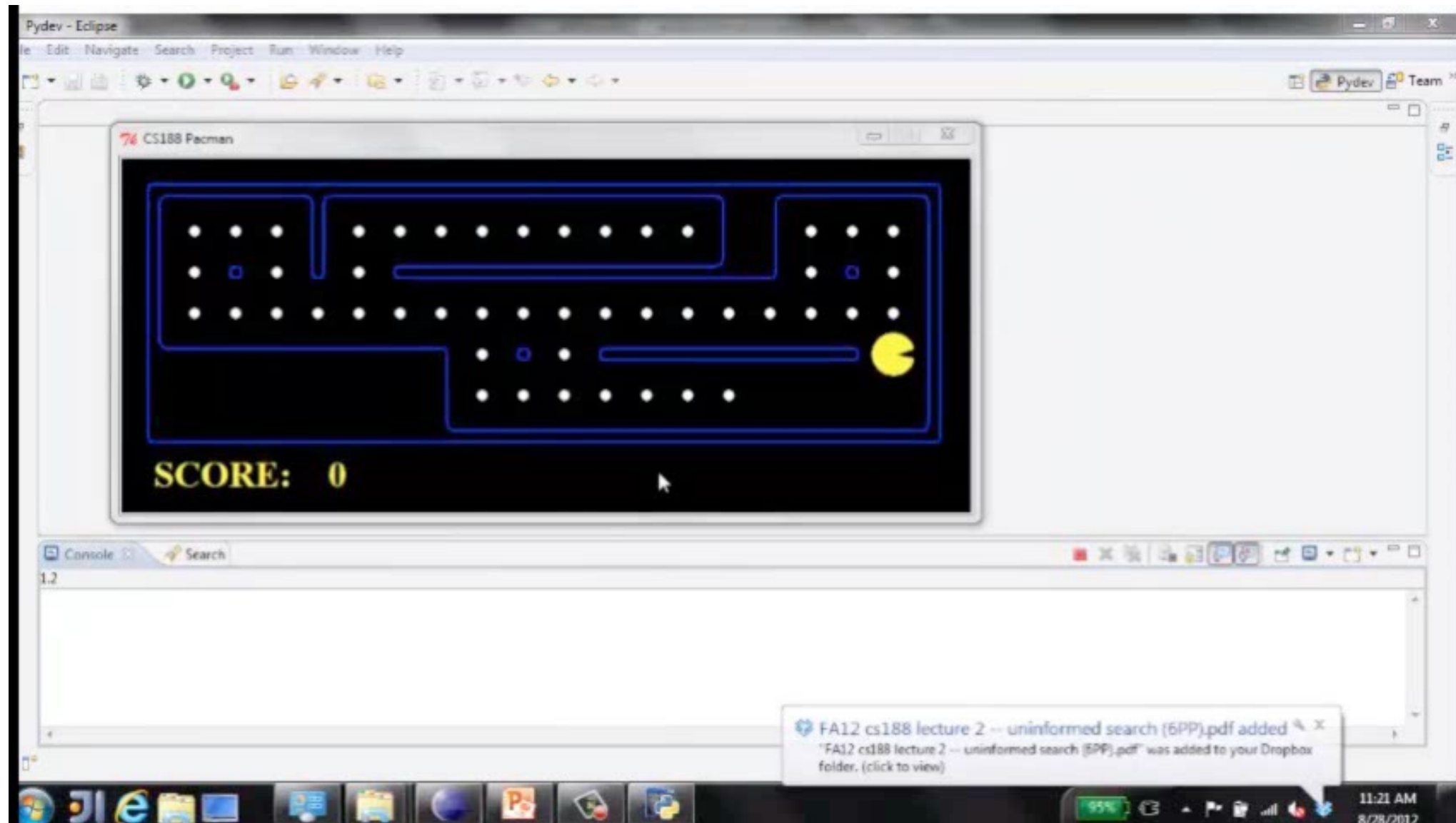
[Demo: reflex optimal (L2D1)]

[Demo: reflex optimal (L2D2)]

# Video of Demo Reflex Optimal

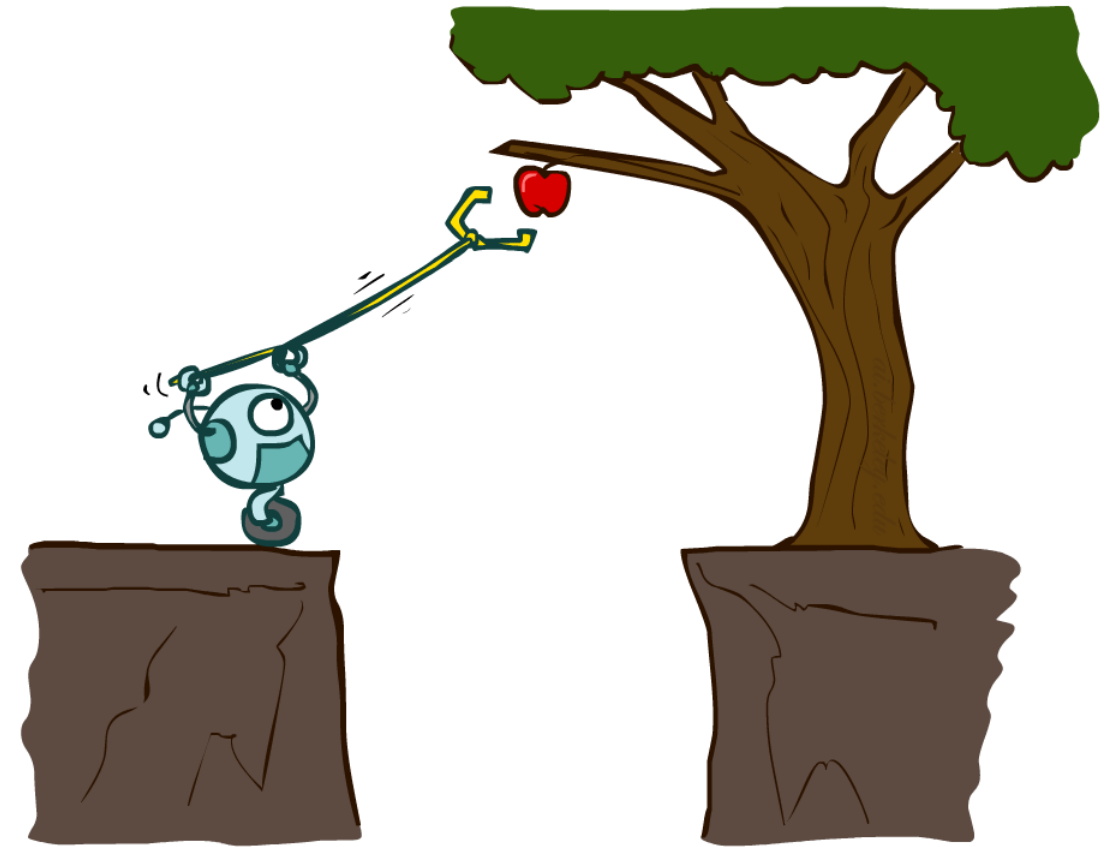


# Video of Demo Reflex Odd



# Planning Agents

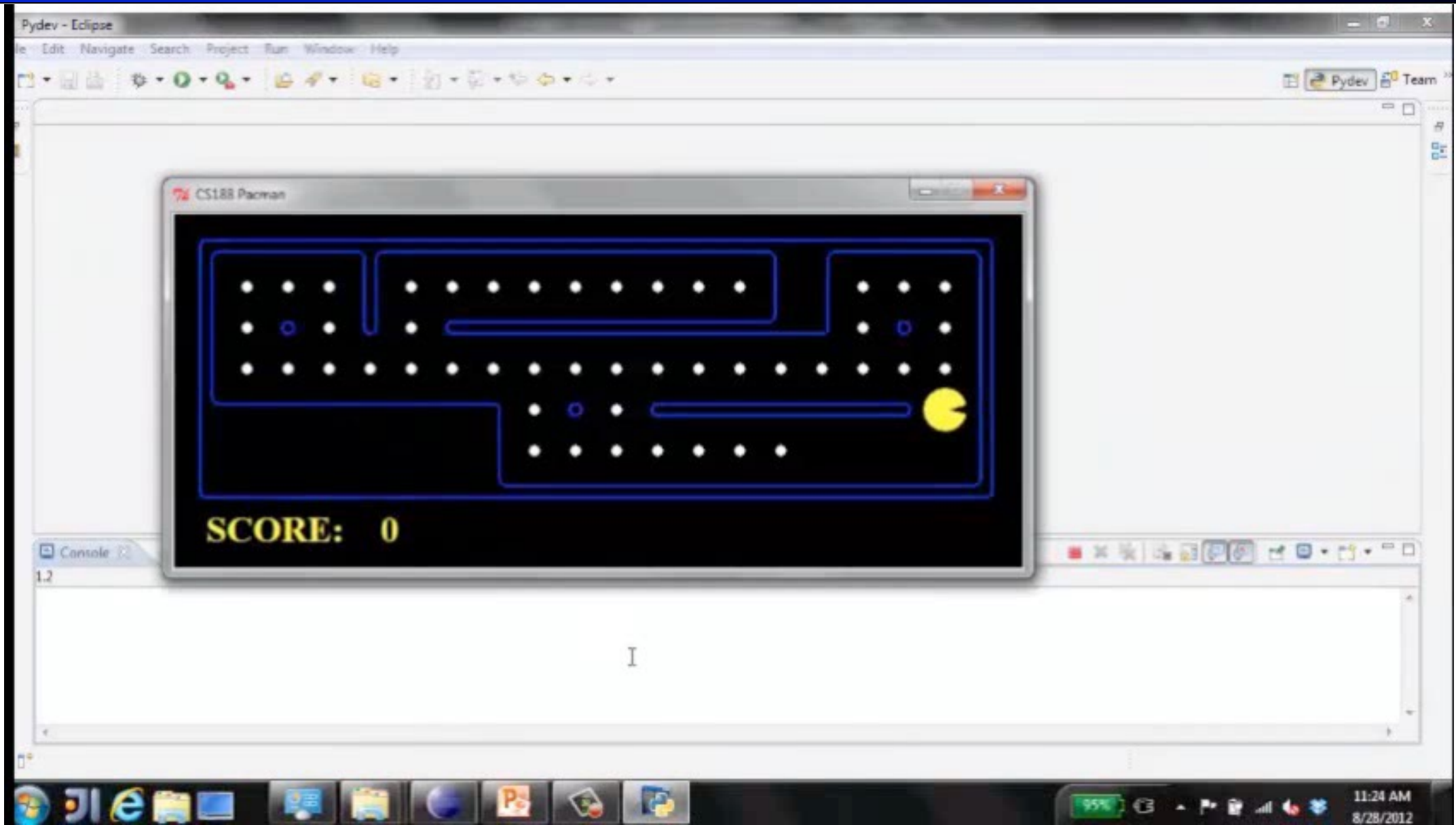
- Planning agents:
  - Ask “what if”
  - Decisions based on (hypothesized) consequences of actions
  - Must have a model of how the world evolves in response to actions
  - Must formulate a goal (test)
  - Consider how the world **WOULD BE**
- Optimal vs. complete planning
- Planning vs. replanning



[Demo: re-planning (L2D3)]

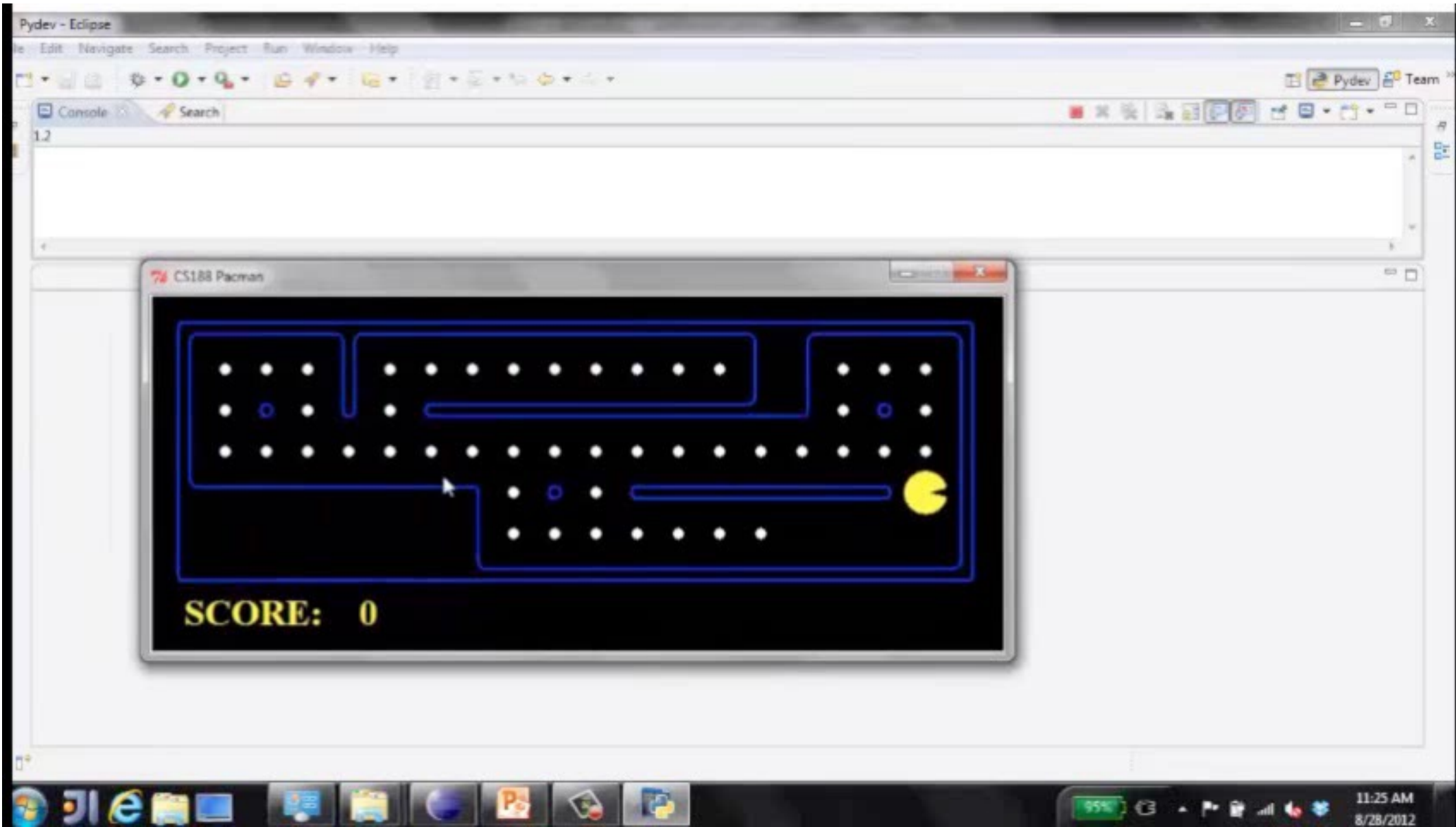
[Demo: mastermind (L2D4)]

# Video of Demo Replanning





# Video of Demo Mastermind



# Search Problems

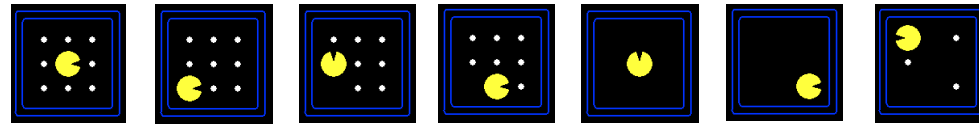
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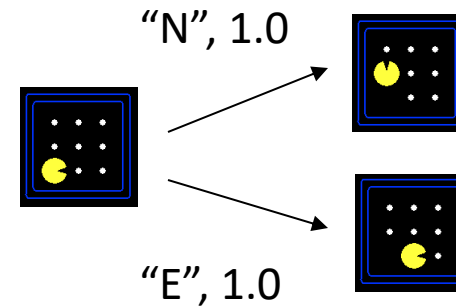
# Search Problems

- A **search problem** consists of:

- A state space



- A successor function  
(with actions, costs)



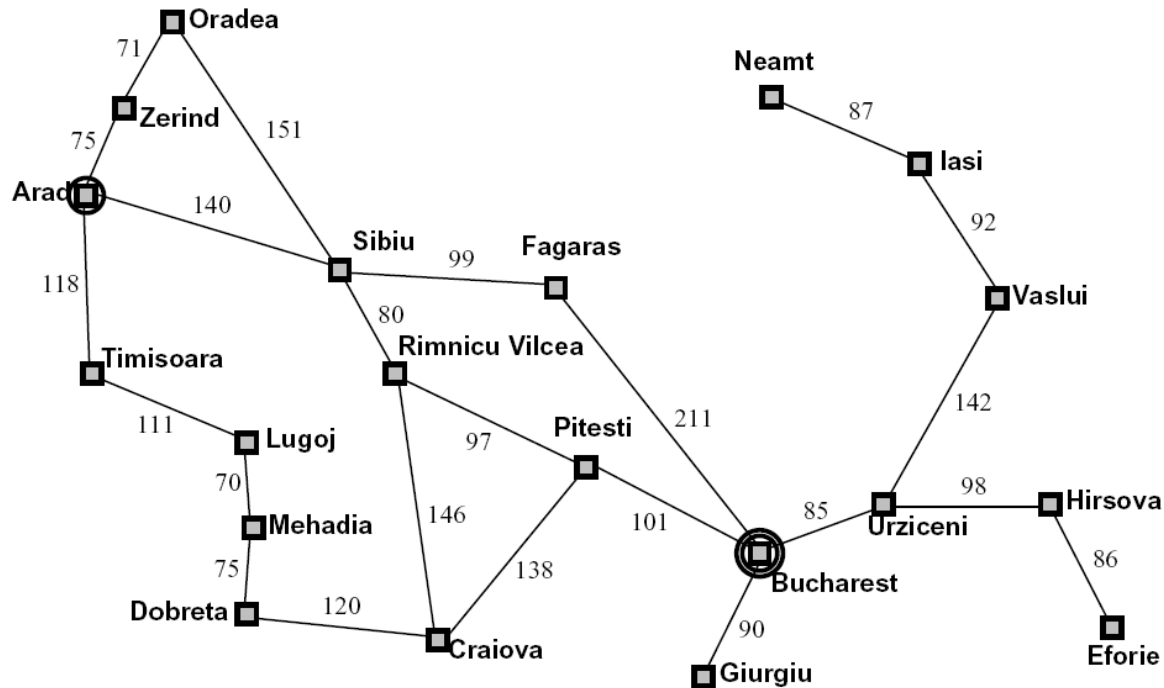
- A start state and a goal test
- A **solution** is a sequence of actions (a plan) which transforms the start state to a goal state

# Search Problems Are Models

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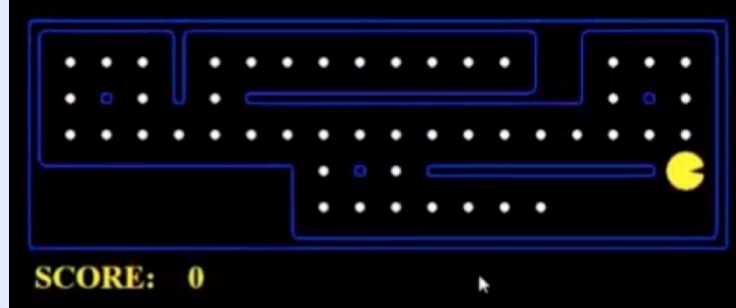
# Example: Traveling in Romania



- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?

# What's in a State Space?

The **world state** includes every last detail of the environment



A **search state** keeps only the details needed for planning (abstraction)

## ■ Problem: Pathing

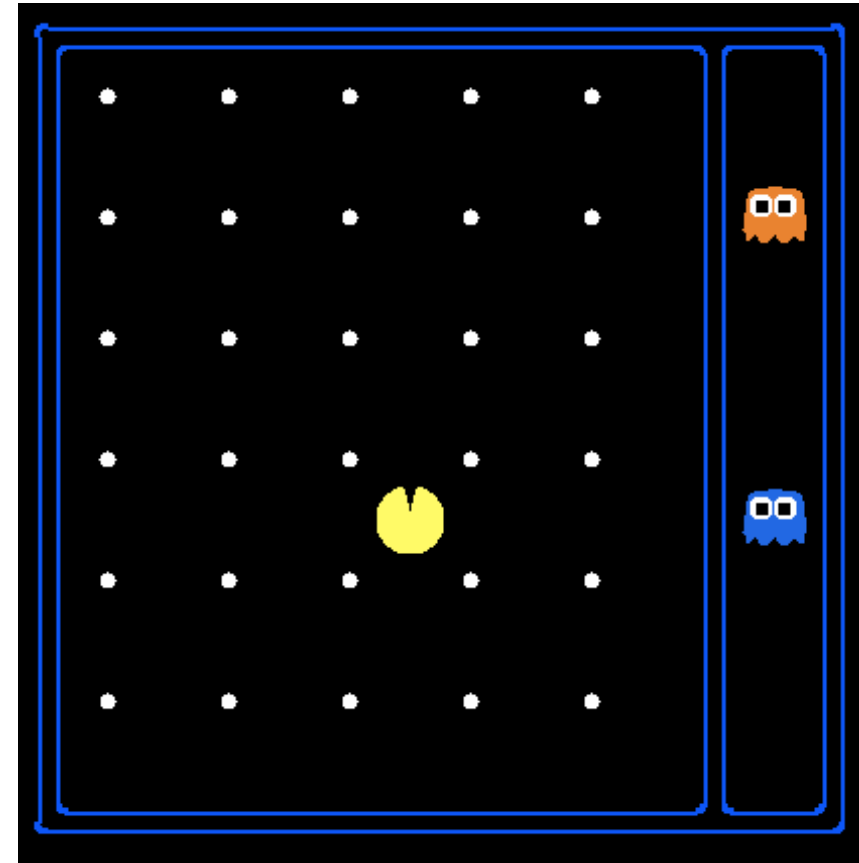
- States:  $(x,y)$  location
- Actions: NSEW
- Successor: update location only
- Goal test: is  $(x,y)=\text{END}$

## ■ Problem: Eat-All-Dots

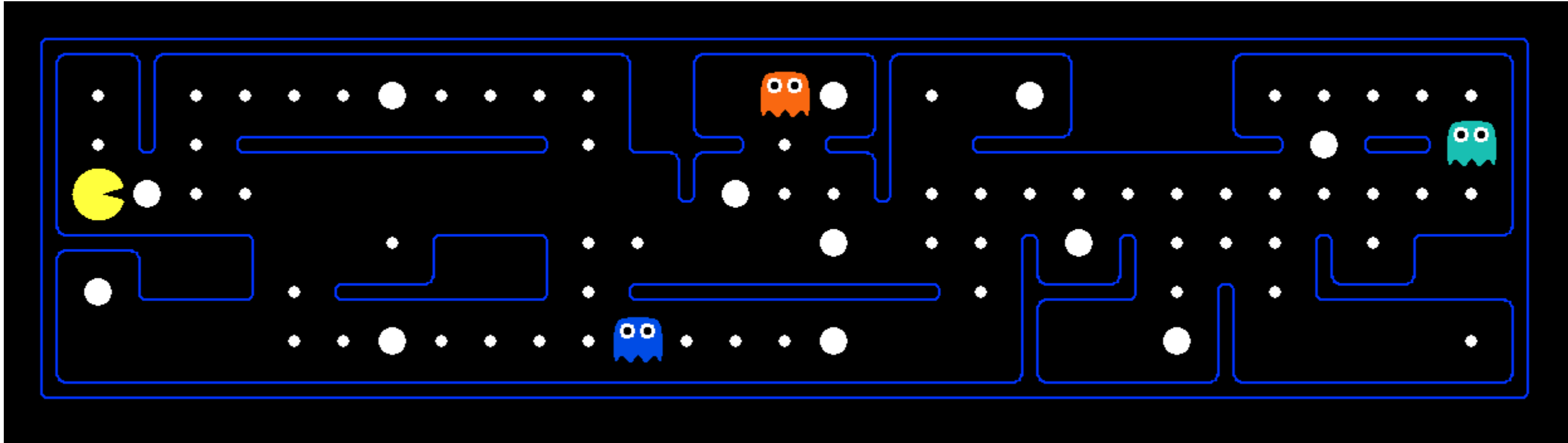
- States:  $\{(x,y), \text{dot booleans}\}$
- Actions: NSEW
- Successor: update location and possibly a dot boolean
- Goal test: dots all false

# State Space Sizes?

- World state:
  - Agent positions: 120
  - Food count: 30
  - Ghost positions: 12
  - Agent facing: NSEW
- How many
  - World states?  
 $120 \times (2^{30}) \times (12^2) \times 4$
  - States for pathing?  
120
  - States for eat-all-dots?  
 $120 \times (2^{30})$



# Quiz: Safe Passage

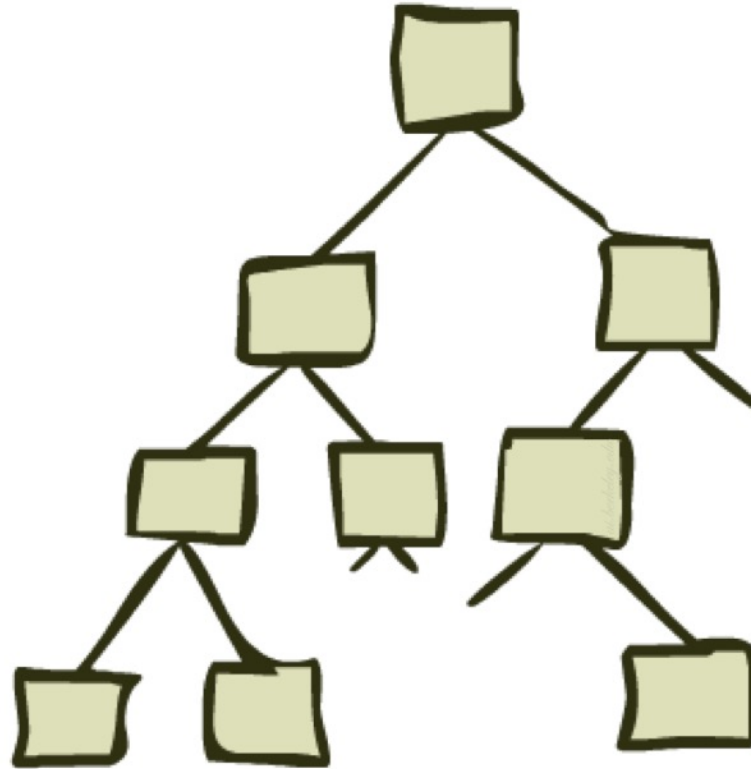


- Problem: eat all dots while keeping the ghosts perma-scared
- What does the state space have to specify?
  - (agent position, dot booleans, power pellet booleans, remaining scared time)



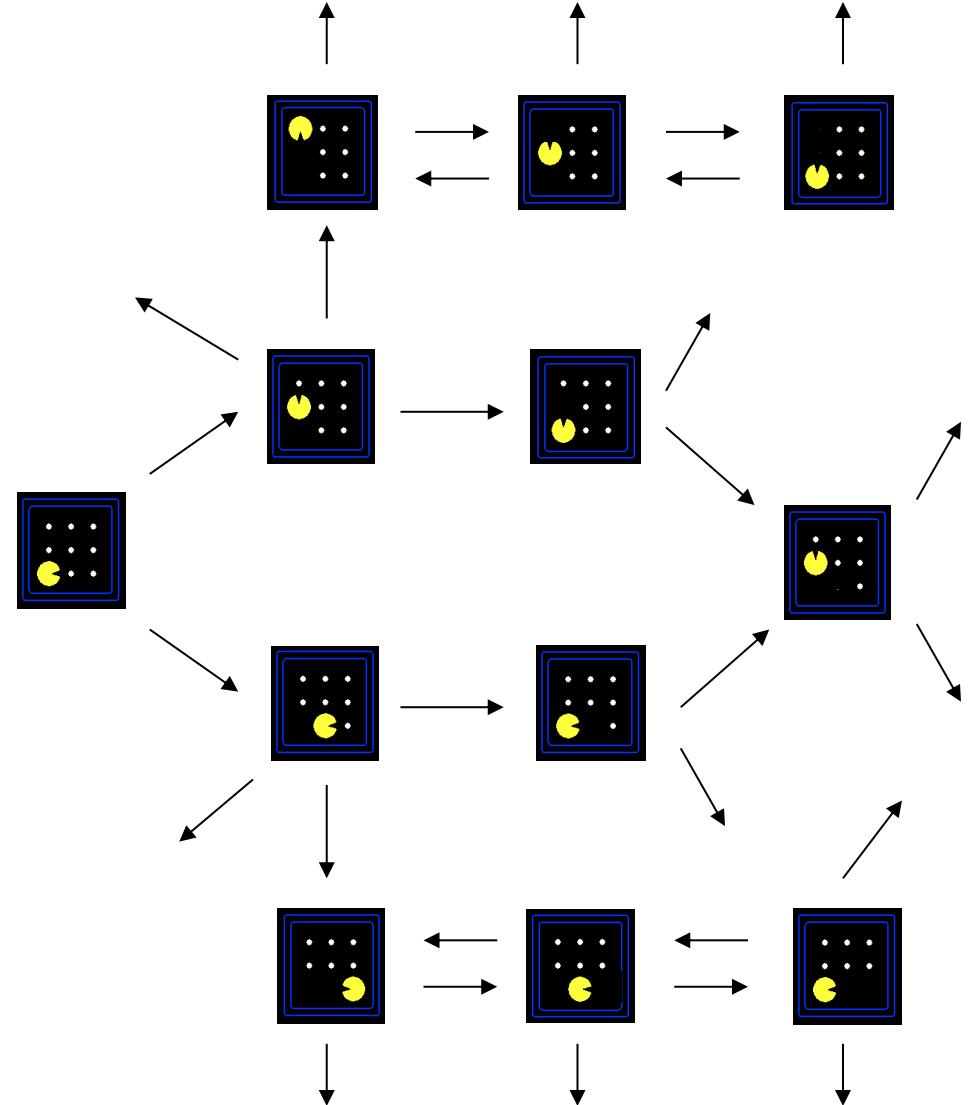
# State Space Graphs and Search Trees

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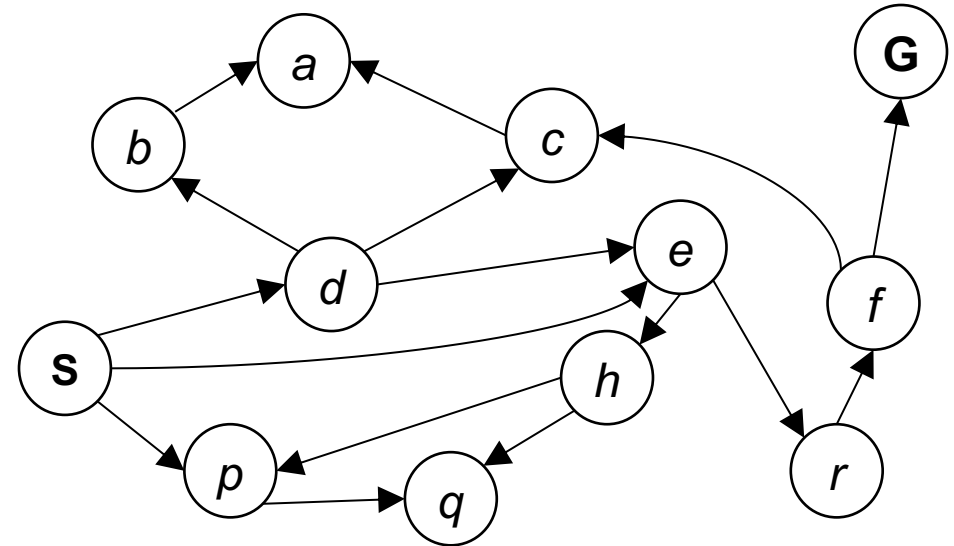
# State Space Graphs

- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations
  - Arcs represent successors (action results)
  - The goal test is a set of goal nodes (maybe only one)
- In a state space graph, each state occurs only once!
- We can rarely build this full graph in memory (it's too big), but it's a useful idea



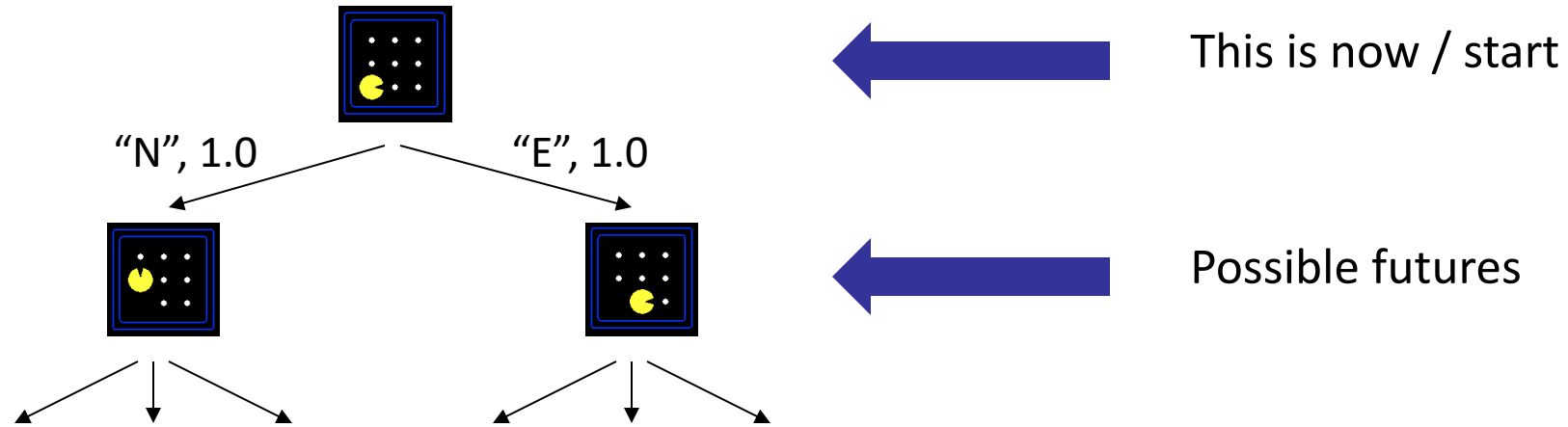
# State Space Graphs

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*Tiny state space graph for a tiny search problem*

# Search Trees

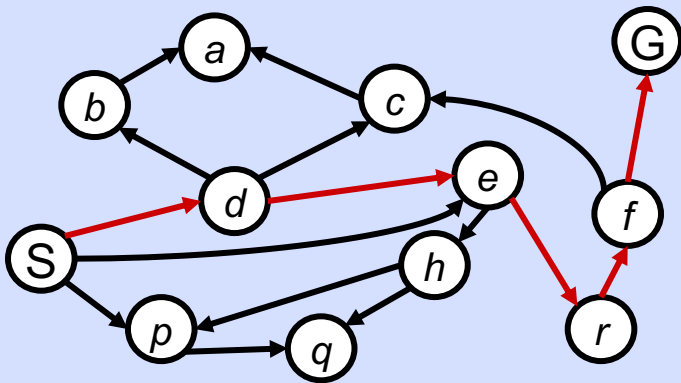


- A search tree:

- A “what if” tree of plans and their outcomes
- The start state is the root node
- Children correspond to successors
- Nodes show states, but correspond to PLANS that achieve those states
- For most problems, we can never actually build the whole tree

# State Space Graphs vs. Search Trees

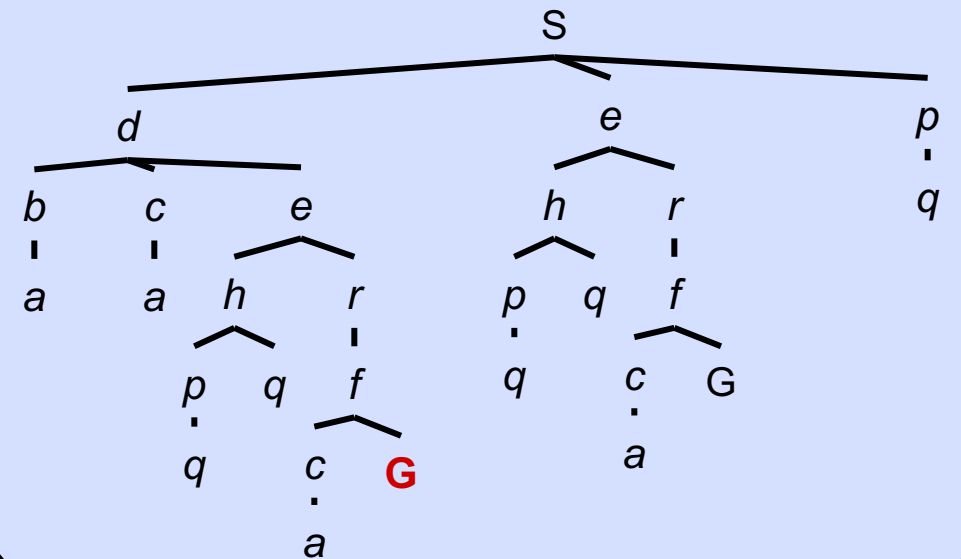
## State Space Graph



*Each NODE in the search tree is an entire PATH in the state space graph.*

*We construct both on demand – and we construct as little as possible.*

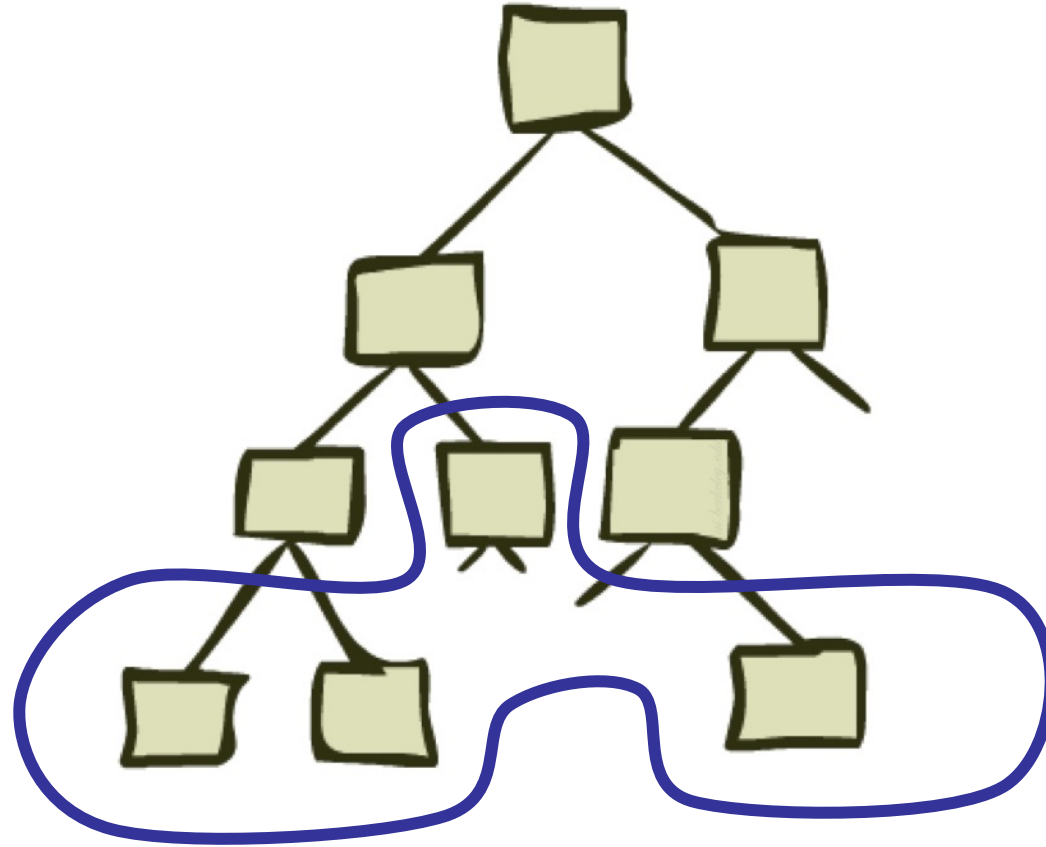
## Search Tree



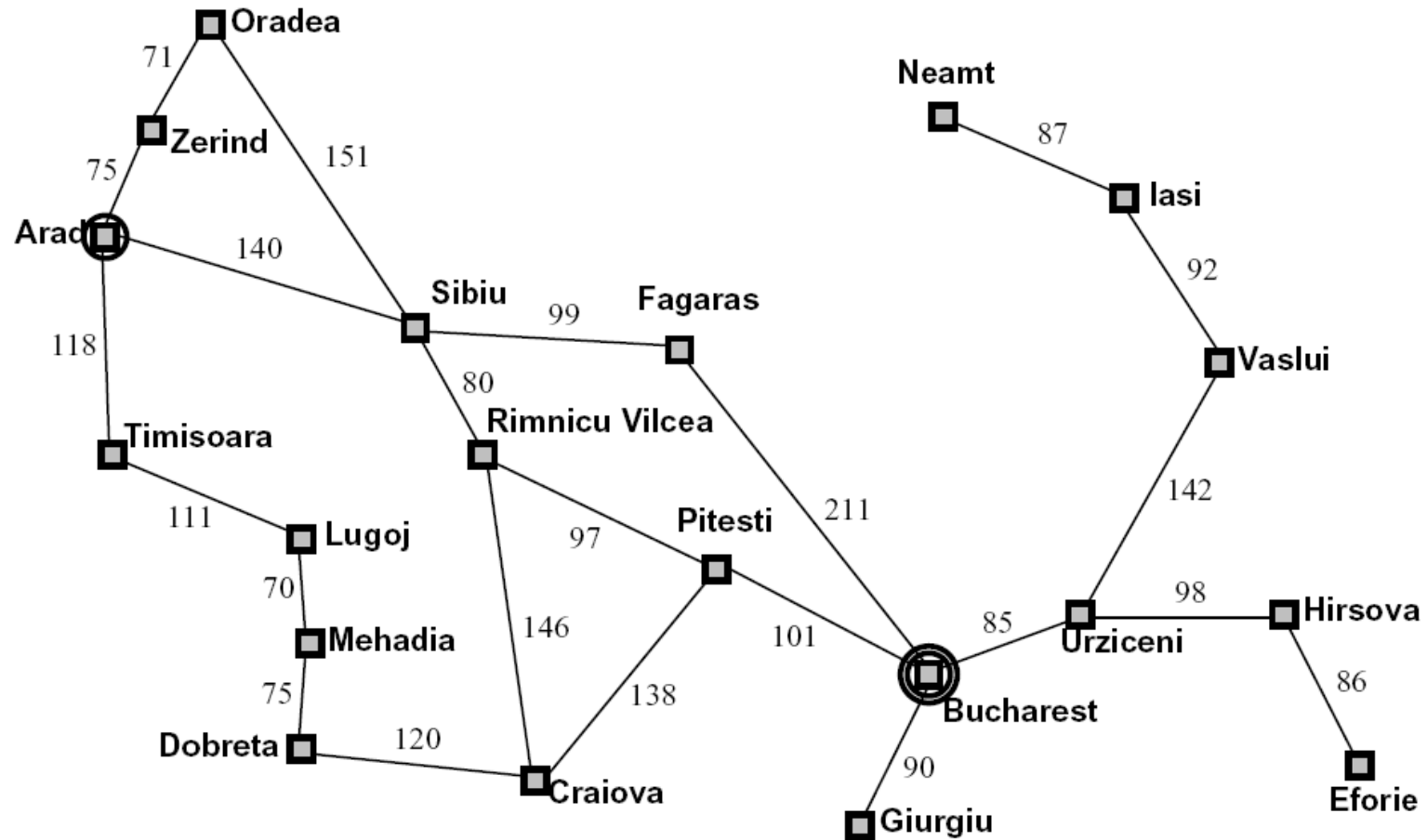


# Tree Search

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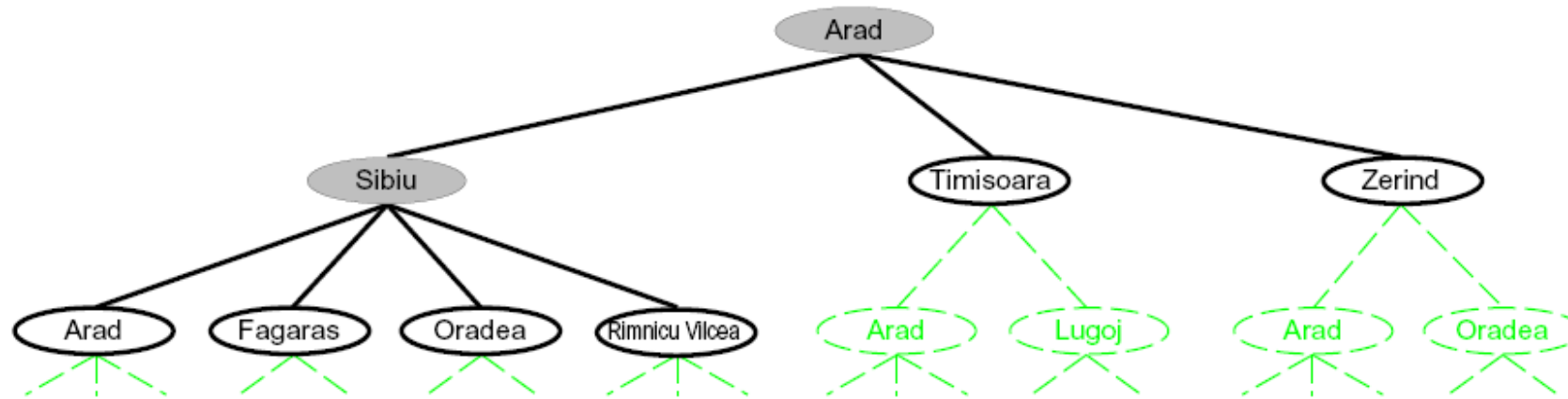


# Search Example: Romania





# Searching with a Search Tree



## ■ Search:

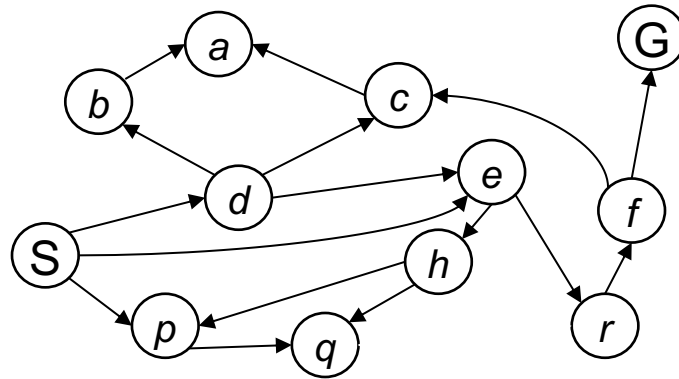
- Expand out potential plans (tree nodes)
- Maintain a **fringe** of partial plans under consideration
- Try to expand as few tree nodes as possible

# General Tree Search

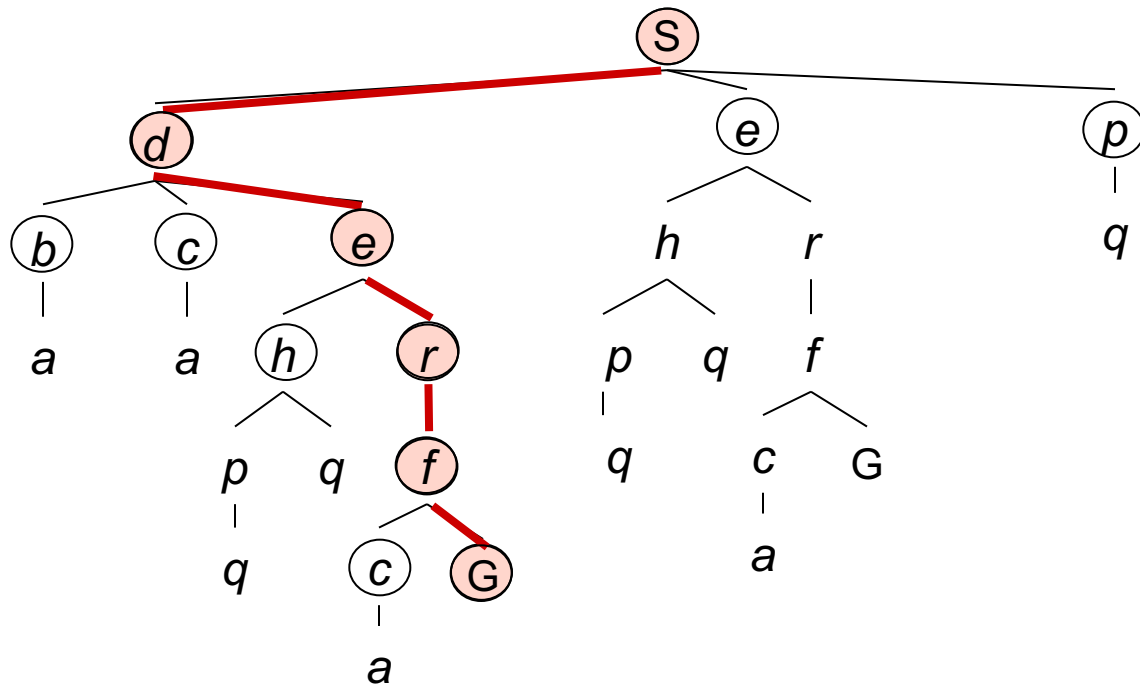
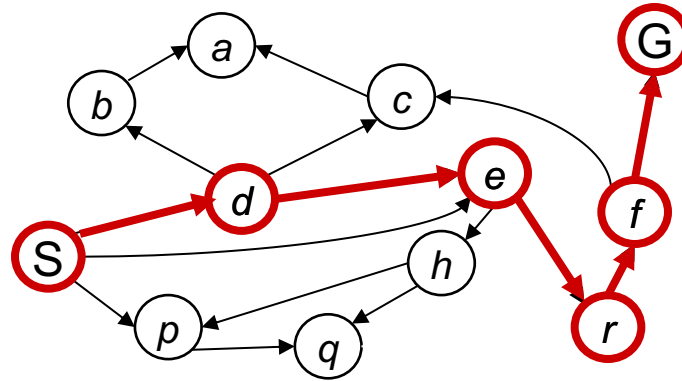
```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end
```

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy
- Main question: which fringe nodes to explore?

# Example: Tree Search



# Example: Tree Search



~~s~~  
~~s → d~~  
s → e  
s → p  
s → d → b  
s → d → c  
~~s → d → e~~  
s → d → e → h  
~~s → d → e → r~~  
~~s → d → e → r → f~~  
s → d → e → r → f → c  
~~s → d → e → r → f → G~~

# Depth-First Search

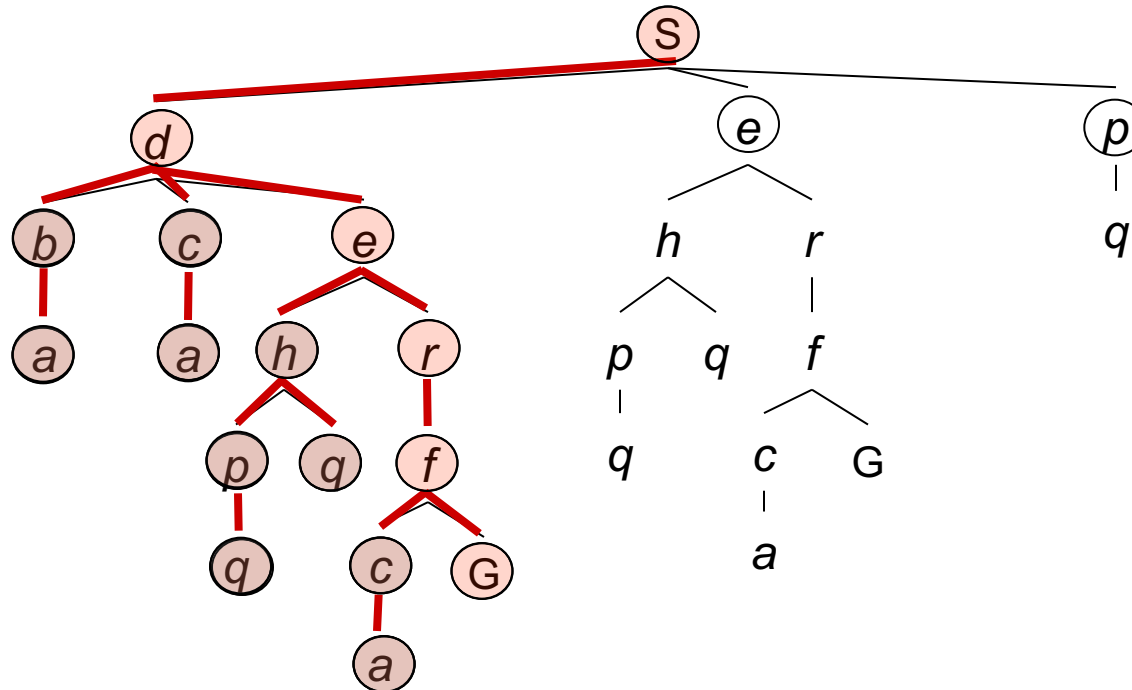
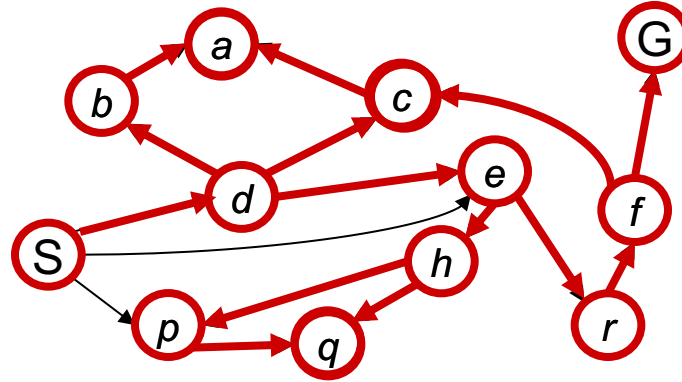
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# Depth-First Search

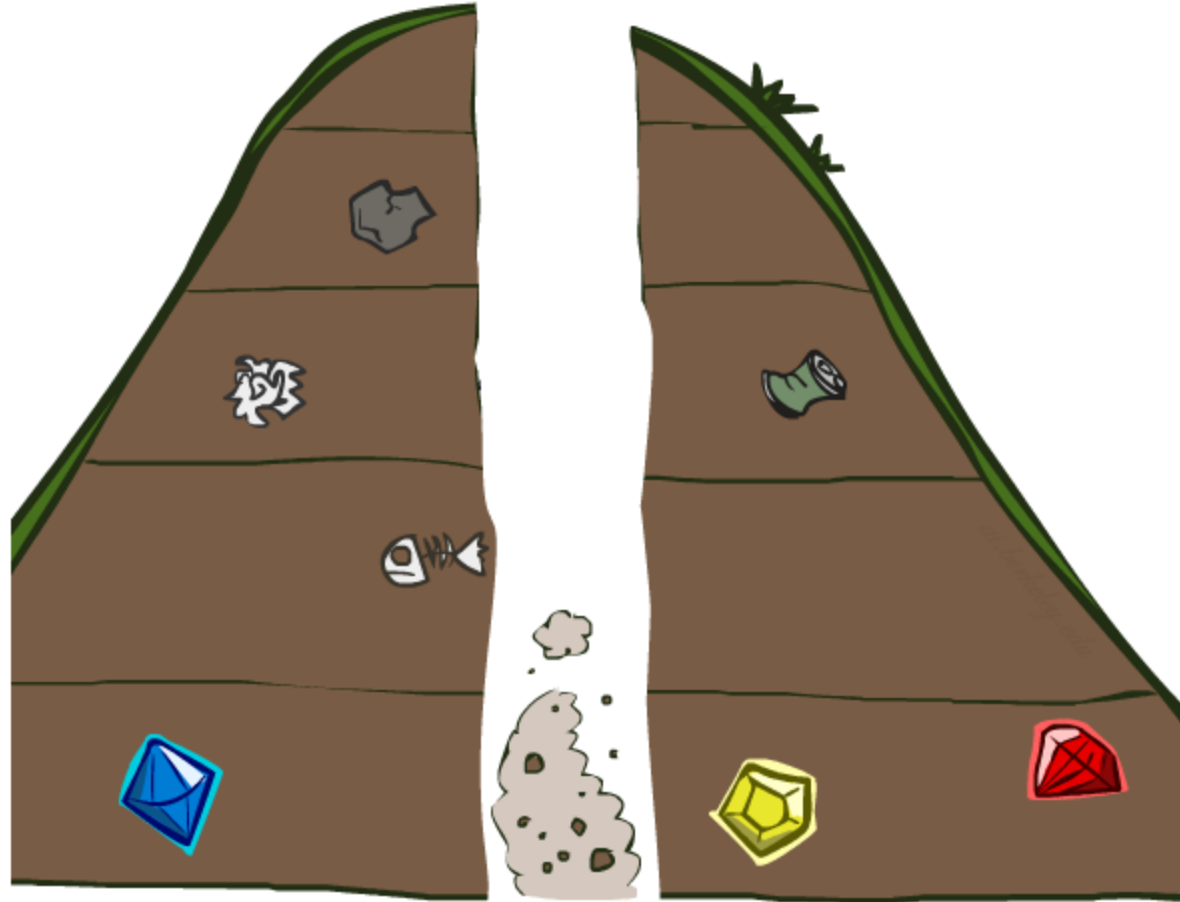
*Strategy: expand a  
deepest node first*

*Implementation:  
Fringe is a LIFO stack*



# Search Algorithm Properties

---



# Search Algorithm Properties

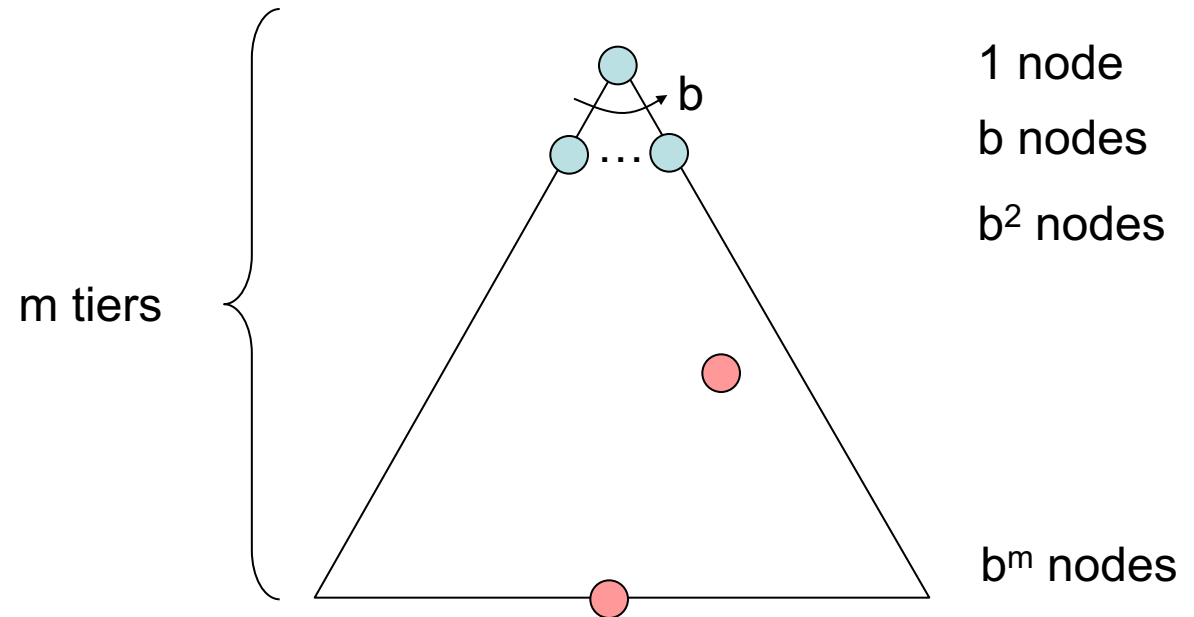
- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?

- Cartoon of search tree:

- $b$  is the branching factor
- $m$  is the maximum depth
- solutions at various depths

- Number of nodes in entire tree?

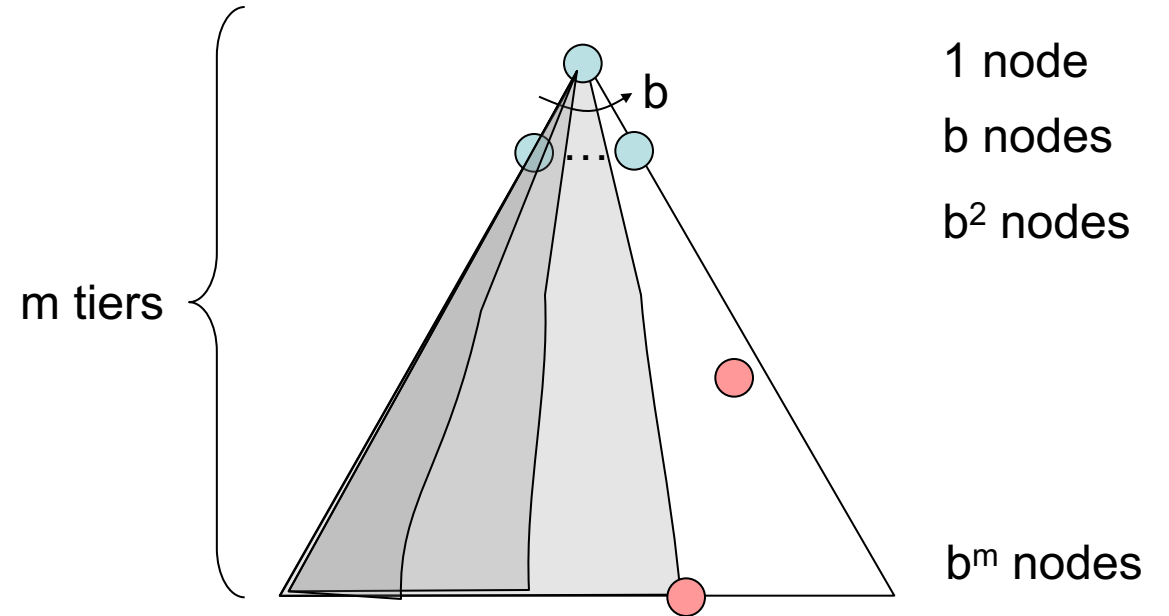
- $1 + b + b^2 + \dots + b^m = O(b^m)$





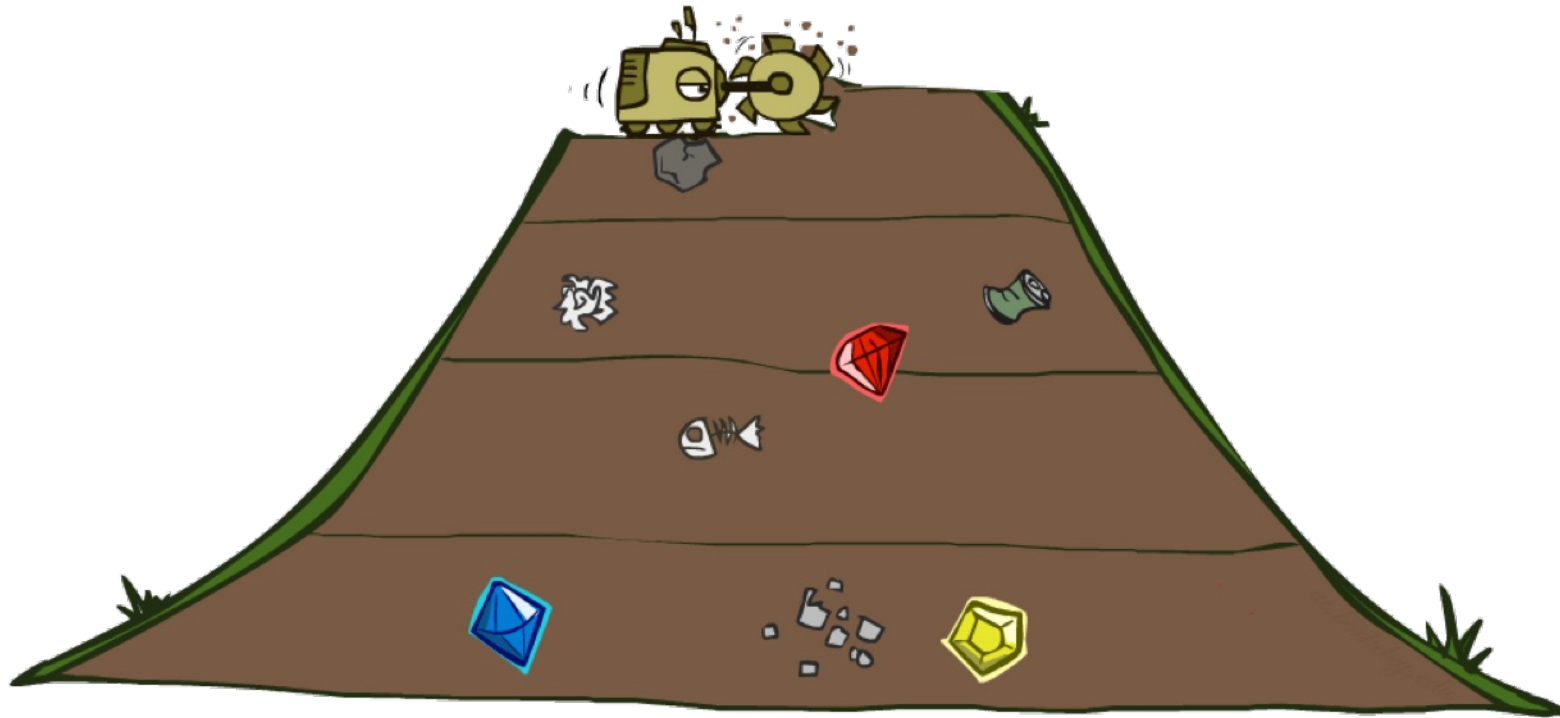
# Depth-First Search (DFS) Properties

- What nodes DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If  $m$  is finite, takes time  $O(b^m)$
- How much space does the fringe take?
  - Only has siblings on path to root, so  $O(bm)$
- Is it complete?
  - $m$  could be infinite, so only if we prevent that
- Is it optimal?
  - No, it finds the “leftmost” solution, regardless of depth or cost



# Breadth-First Search

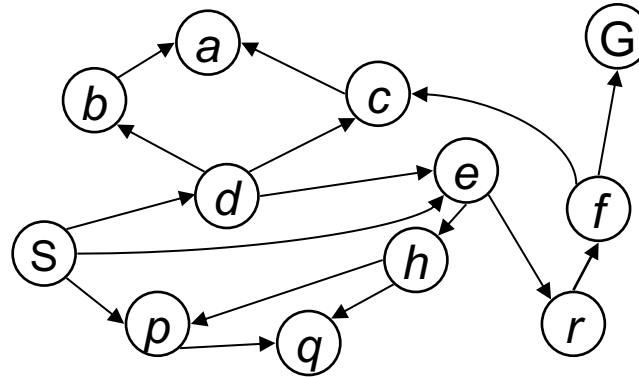
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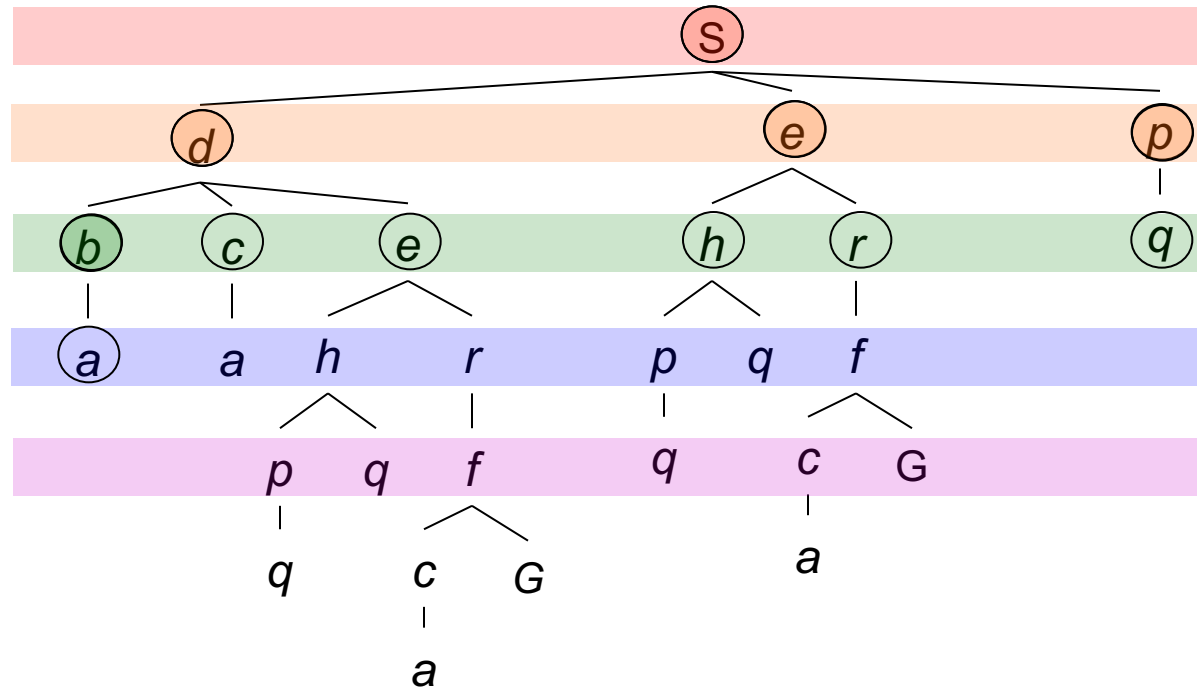
# Breadth-First Search

*Strategy: expand a shallowest node first*

*Implementation: Fringe is a FIFO queue*

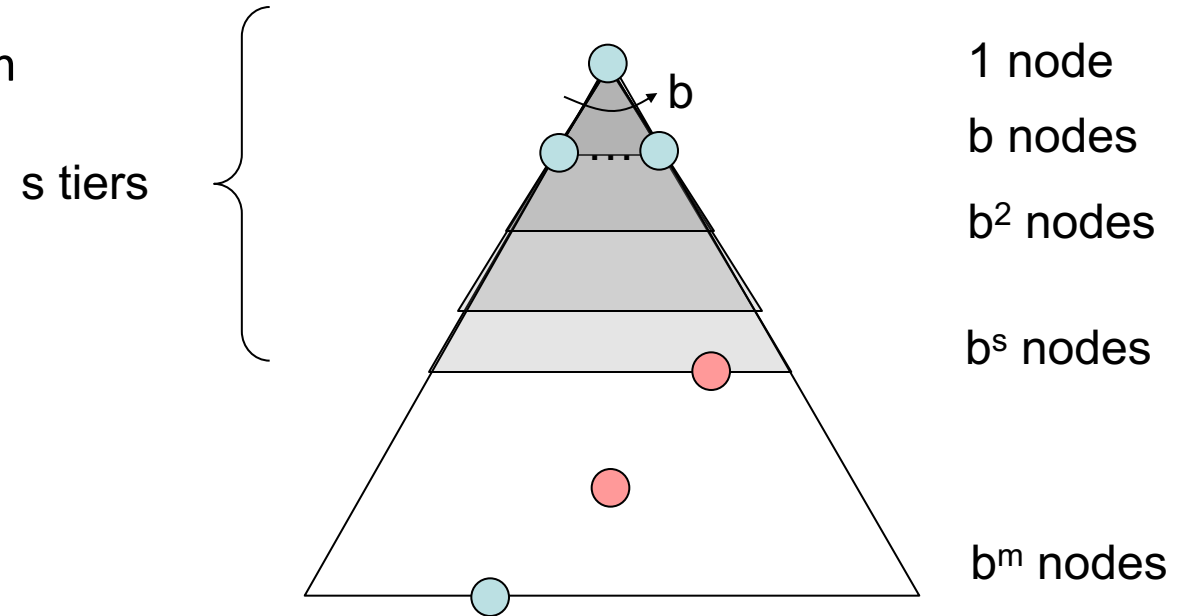


Search  
Tiers

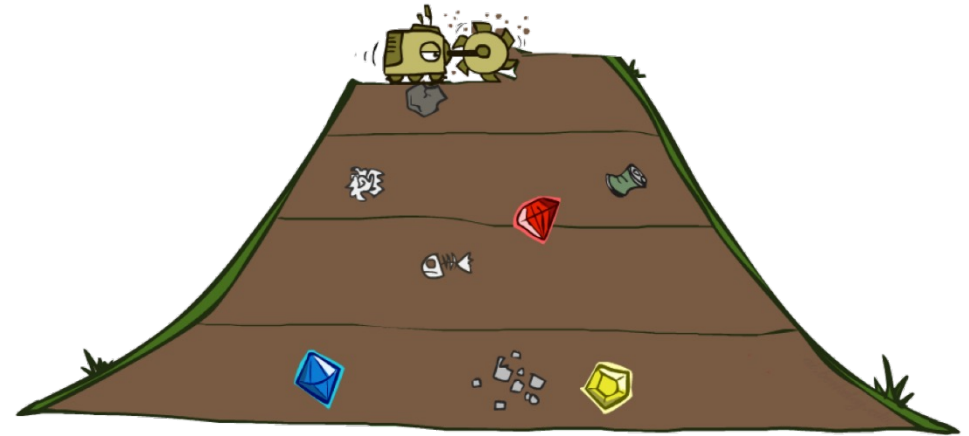


# Breadth-First Search (BFS) Properties

- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be  $s$
  - Search takes time  $O(b^s)$
- How much space does the fringe take?
  - Has roughly the last tier, so  $O(b^s)$
- Is it complete?
  - $s$  must be finite if a solution exists, so yes!
- Is it optimal?
  - Only if costs are all 1 (more on costs later)



# Quiz: DFS vs BFS



# Quiz: DFS vs BFS

---

- When will BFS outperform DFS?
- When will DFS outperform BFS?

# Video of Demo Maze Water DFS/BFS (part 1)

---



# Video of Demo Maze Water DFS/BFS (part 2)

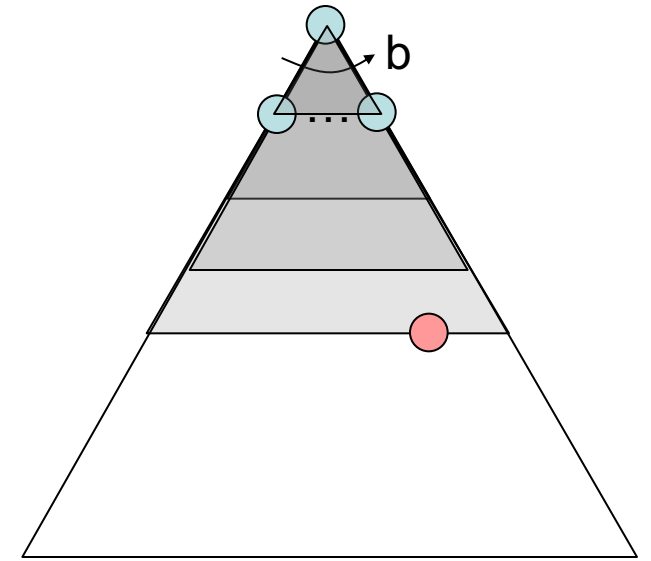
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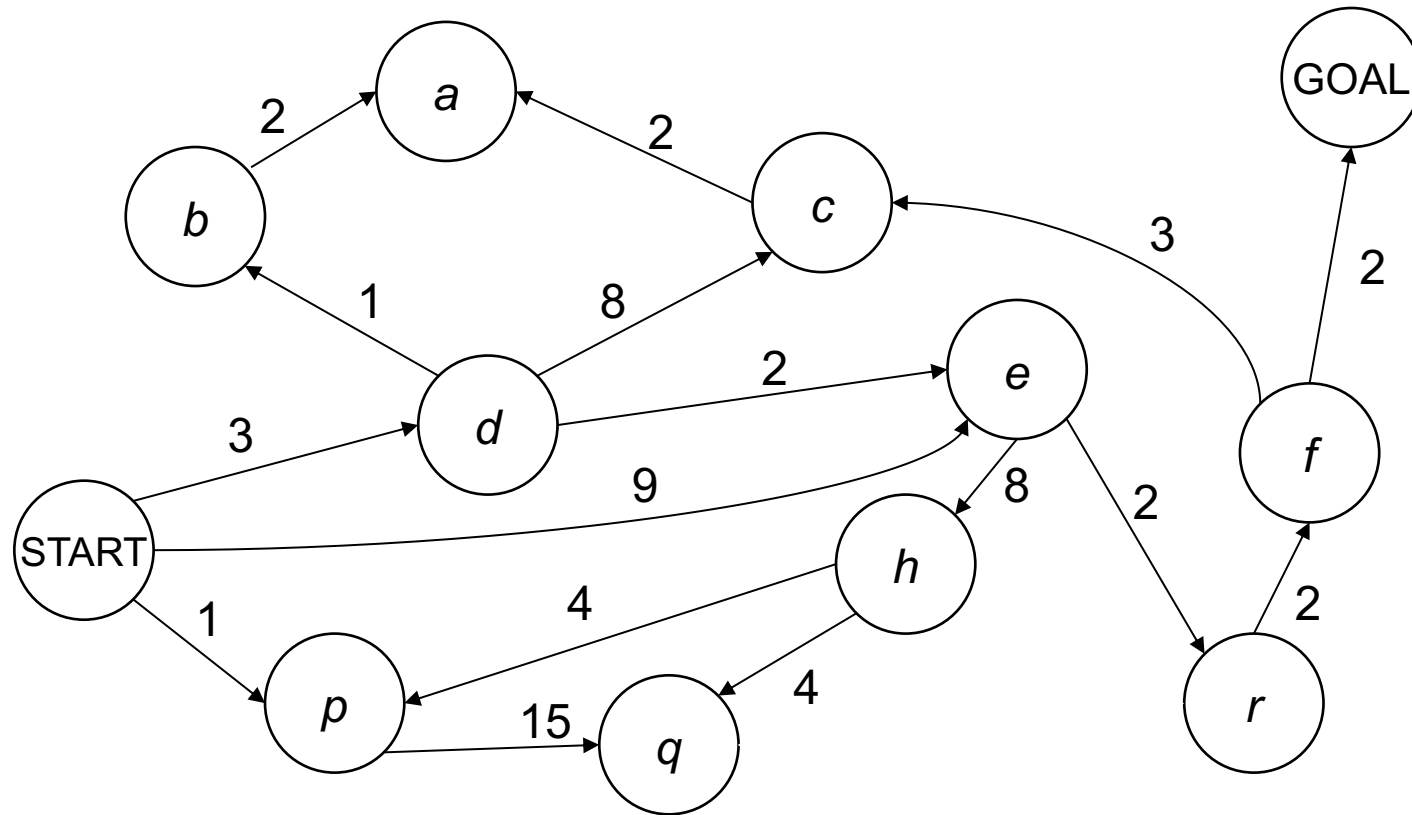


# Iterative Deepening

- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages
  - Run a DFS with depth limit 1. If no solution...
  - Run a DFS with depth limit 2. If no solution...
  - Run a DFS with depth limit 3. ....
- Isn't that wastefully redundant?
  - Generally most work happens in the lowest level searched, so not so bad!
  - Branching factor 10, solution 5 deep:
    - BFS:  $10 + 100 + 1,000 + 10,000 + 100,000 = 111,110$
    - IDS:  $50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$



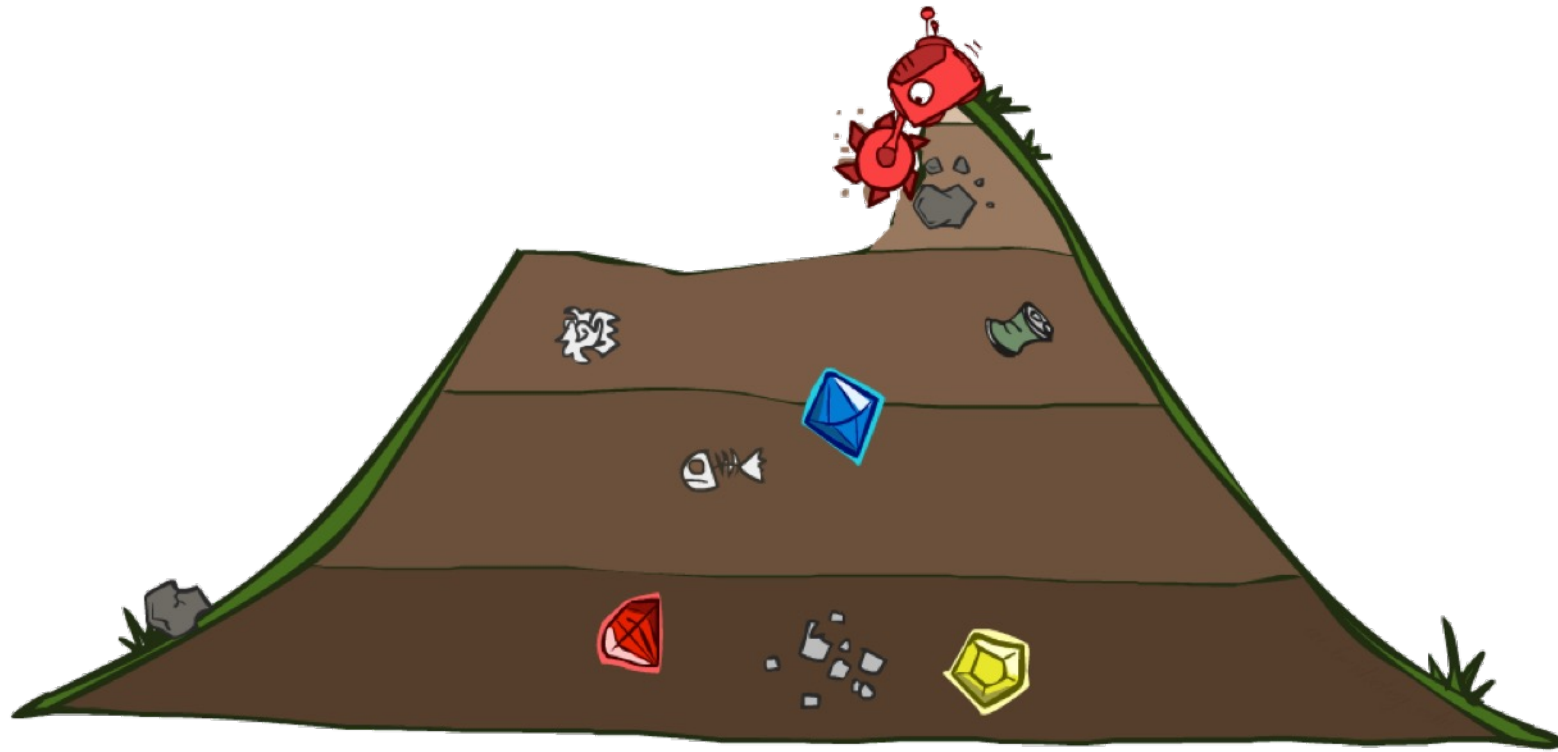
# Cost-Sensitive Search



BFS finds the shortest path in terms of number of actions.  
It does not find the least-cost path. We will now cover  
a similar algorithm which does find the least-cost path.

# Uniform Cost Search

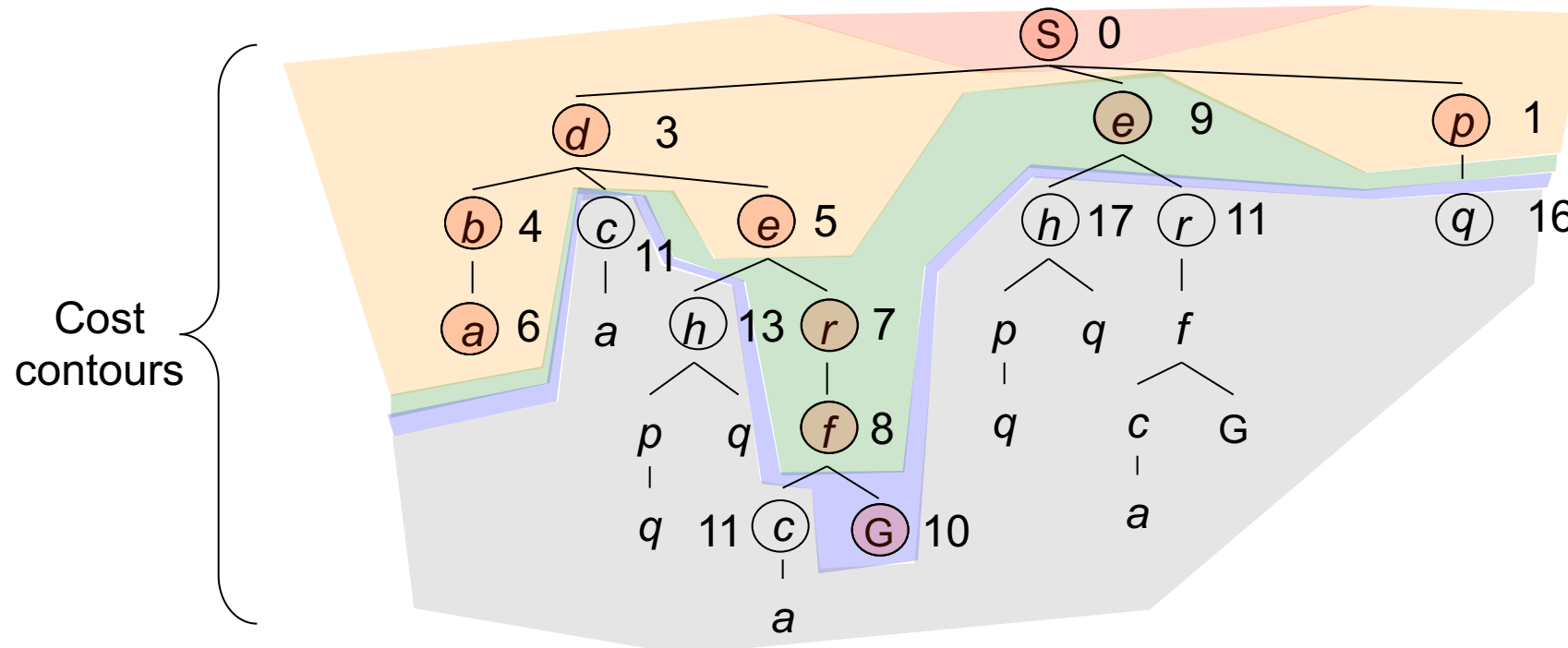
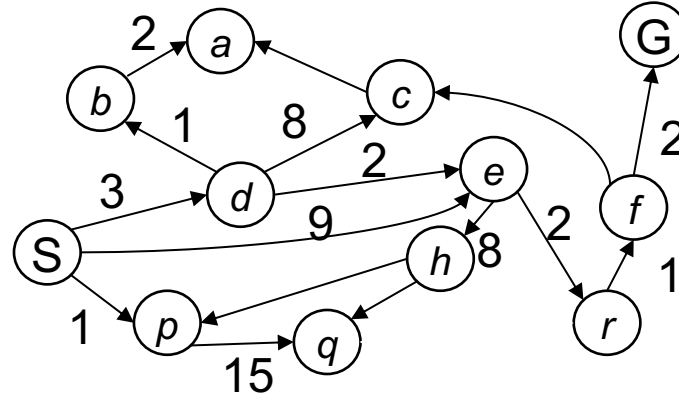
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# Uniform Cost Search

*Strategy: expand a  
cheapest node first:*

*Fringe is a priority queue  
(priority: cumulative cost)*



# Uniform Cost Search (UCS) Properties

## ■ What nodes does UCS expand?

- Processes all nodes with cost less than cheapest solution!
- If that solution costs  $C^*$  and arcs cost at least  $\varepsilon$ , then the “effective depth” is roughly  $C^*/\varepsilon$
- Takes time  $O(b^{C^*/\varepsilon})$  (exponential in effective depth)

## ■ How much space does the fringe take?

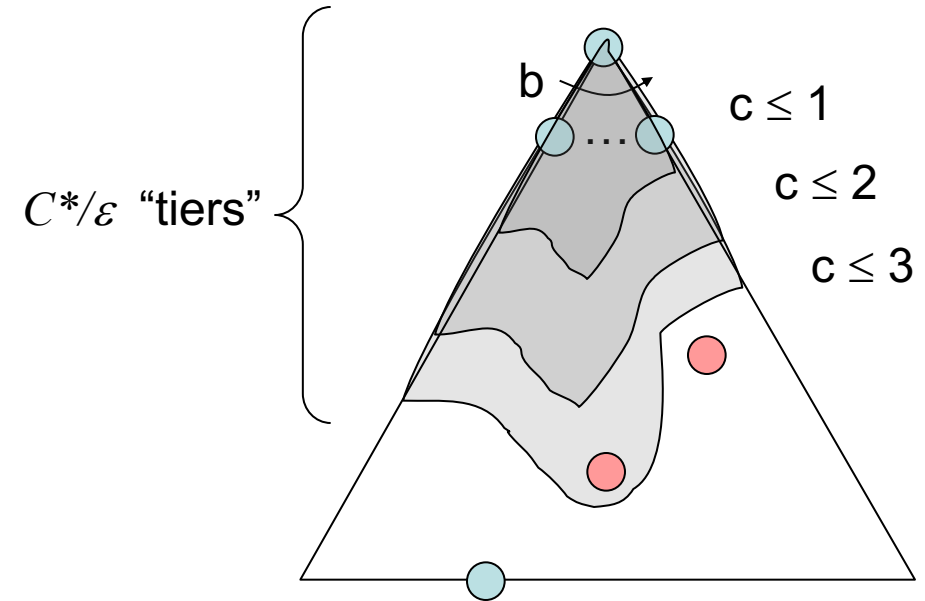
- Has roughly the last tier, so  $O(b^{C^*/\varepsilon})$

## ■ Is it complete?

- Assuming best solution has a finite cost and minimum arc cost is positive, yes!

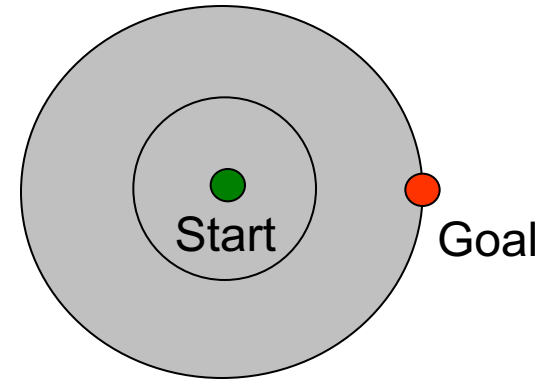
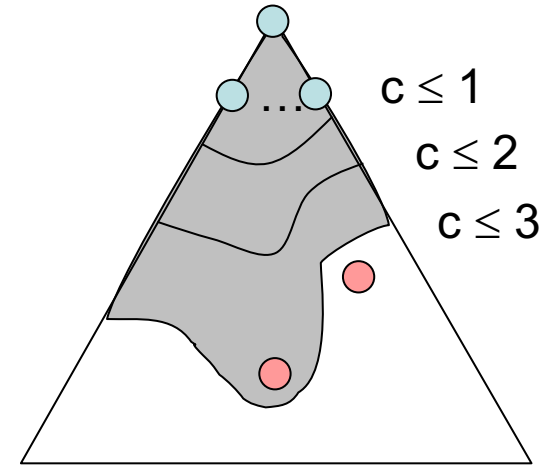
## ■ Is it optimal?

- Yes! (Proof next lecture via  $A^*$ )



# Uniform Cost Issues

- Remember: UCS explores increasing cost contours
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every “direction”
  - No information about goal location
- We'll fix that soon!



[Demo: empty grid UCS (L2D5)]  
[Demo: maze with deep/shallow  
water DFS/BFS/UCS (L2D7)]

# Video of Demo Contours UCS Pacman Small Maze

---



# Video of Demo Empty UCS

---





## Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 1)

---



## Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 2)

---



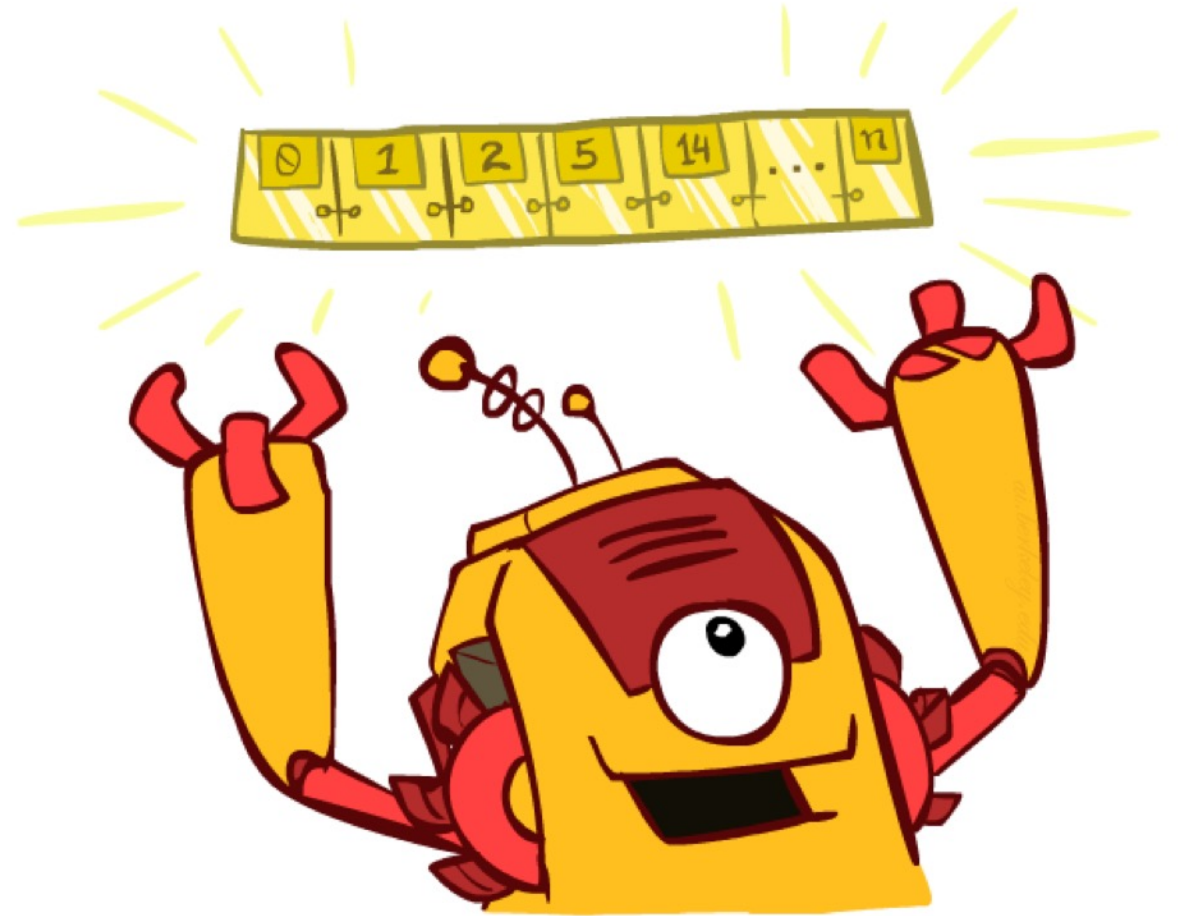
## Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 3)

---



# The One Queue

- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the  $\log(n)$  overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object



# Comparing uninformed search algorithms

## 3.4.6 Comparing uninformed search algorithms

Figure 3.15 compares uninformed search algorithms in terms of the four evaluation criteria set forth in Section 3.3.4. This comparison is for tree-like search versions which don't check for repeated states. For graph searches which do check, the main differences are that depth-first search is complete for finite state spaces, and the space and time complexities are bounded by the size of the state space (the number of vertices and edges,  $|V| + |E|$ ).

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes <sup>1</sup>	Yes <sup>1,2</sup>	No	No	Yes <sup>1</sup>	Yes <sup>1,4</sup>
Optimal cost?	Yes <sup>3</sup>	Yes	No	No	Yes <sup>3</sup>	Yes <sup>3,4</sup>
Time	$O(b^d)$	$O(b^{1+\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$

**Figure 3.15** Evaluation of search algorithms.  $b$  is the branching factor;  $m$  is the maximum depth of the search tree;  $d$  is the depth of the shallowest solution, or is  $m$  when there is no solution;  $\ell$  is the depth limit. Superscript caveats are as follows: <sup>1</sup> complete if  $b$  is finite, and the state space either has a solution or is finite. <sup>2</sup> complete if all action costs are  $\geq \epsilon > 0$ ; <sup>3</sup> cost-optimal if action costs are all identical; <sup>4</sup> if both directions are breadth-first or uniform-cost.