Edith Cowan University
Centre for Artificial Intelligence and Machine Learning

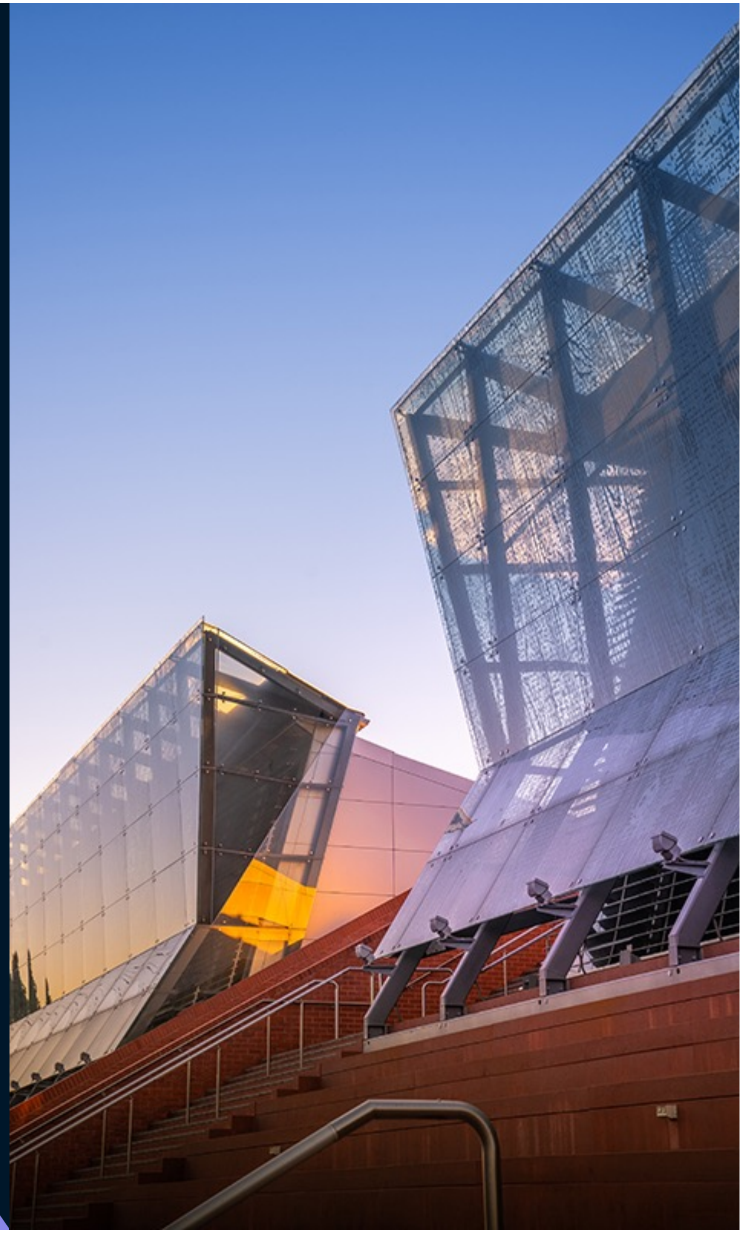
School of Science

Geometric Hypergraphs

David Suter

International Workshop on Optical Perception

Xidian University , September 2024



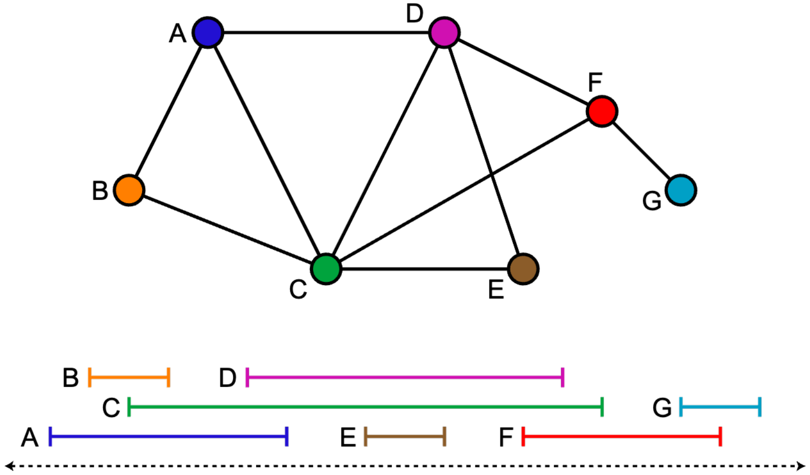
Simple Example – Interval Graphs

Graphs are relationships between PAIRS of items

Hypergraphs are relationships between ANY NUMBER of items

Geometric (Hyper)Graph arise through GEOMETRIC relationships of Items

- Perhaps simplest example is an interval graph
    - Items (vertices in (hyper)graph)
    - Relationship (pairwise) intersection – "overlap"
    - Applications – scheduling, and more!



(Image source – wikepedia)

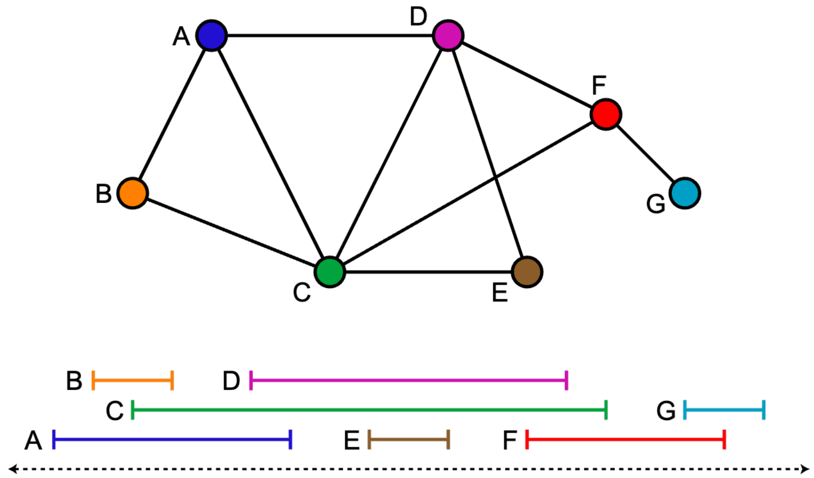
Simple Example – Interval Graphs

Graphs are relationships between PAIRS of items

Hypergraphs are relationships between ANY NUMBER of items

Geometric (Hyper)Graph arise through GEOMETRIC relationships of Items

- In scheduling applications – Independent sets (subsets of vertices not containing an edge) represent events that can be scheduled without overlap.
- Sometimes overlap is bad (conflicts – classes that cannot be at overlapping times for students) – other times good (allowing for passengers to go from train to train, plane to plane)



(Image source – wikepedia)

Why are Geometric (Hyper)graphs good?

Since they arise (naturally) from real world problems – they are often the underlying "concept" for your problem.

Since they arise from Geometry – they "benefit" for what is known about geometry and the restrictions of certain geometric relationships. E.g. Helly's theorem – if every (d+1) subset of CONVEX bodies in R^d intersect (overlap) then the whole set of bodies intersect/overlap (have a common point). Intervals are convex. d=1. Helly -> if every PAIR of intervals overlap then so does the whole collection. THAT IS, cliques in the interval graph of size k have to represent not only k intervals that PAIRWISE intersect, but K intervals that have a common point (their intersection collectively is non-empty).

Restrictions are good (though they sound bad ☺). Restrictions "tame the wild".

The set of possible graphs contains even the worst case graphs (for your algorithms/tasks).

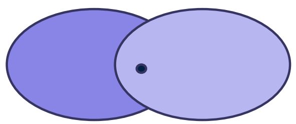
A RESTRICTED subclass of graphs could be good (for your algorithms/problem).

The restriction is a property that not all graphs obey, that property that graphs from YOUR problem obey – could be the thing your algorithm can exploit to run fast.

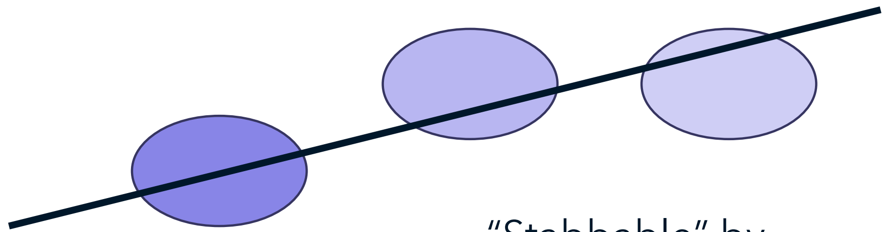
Generalizing Overlap/Intersection –stabbing etc
Universe of Geometric Hypergraphs

Intersection = "stabbable" by points. If sets of points in R^d intersect then they have at least one point in common – there is one point that "stabs" every set.

Generalise – "stabbable" by points -> stabbable by X (X could be lines – having at least one point in common becomes they have a common line that intersects them all)



Intersect <-> at least one point stabs them both

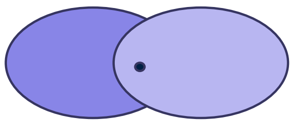


"Stabbable" by a common line

More technical/common terms – transversal/hitting set. Range spaces….etc.

Generalizing Overlap/Intersection –stabbing etc
Universe of Geometric Hypergraphs

Note – any PAIR of disks is ALWAYS stabbable by a single line. So that is an "information free" situation. Information only exists when 3 or more discs are stabble by a line. So this is not a "graph" setting – but a HYPERgraph setting.



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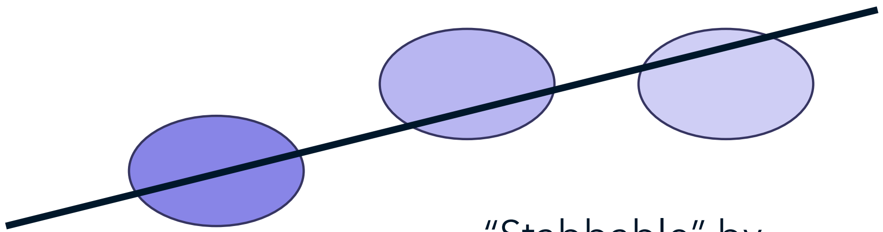

"Stabbable" by a common line

More technical/common terms – transversal/hitting set. Range spaces….etc.

But what does this have to do with AI, Machine Learning, Signal and Image Processing???

Things I am interested in – computer vision, AI, Machine Learning, Robust Fitting, Signal and Image Processing….

In some sense it is relatively obvious why Geometric Hypergraphs are relevant.

After all – geometry underlies a lot of computer vision (the "geometry of cameras, the visual world etc."). More abstractly, AI and Machine learning is about processing "features" extracted from the data to do a task. Features are often thought of as vectors in a space and the relationship between features becomes associated with geometric notions in that embedding space.

Indeed – what is clustering? That ubiquitous "engine" of a lot of data analysis, AI, Machine Learning. It is analysing the "associations <-> relationships " between points in feature space – their closeness to each other in some – often geometric sense/measure.

But what does this have to do with AI, Machine Learning, Signal and Image Processing???

My PRIMARY motivation currently is “robust fitting” (and robust fitting for computer vision).

Robust fitting is essentially like “fitting within tolerances”. Outliers “don’t belong to the fit” because they cannot be fit within tolerance.

Tolerances can be pictured as geometric shapes around the data points. The models must “stab” these geometric shapes to be within tolerance of the data points.

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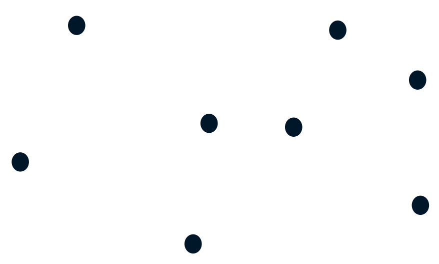

Data Points

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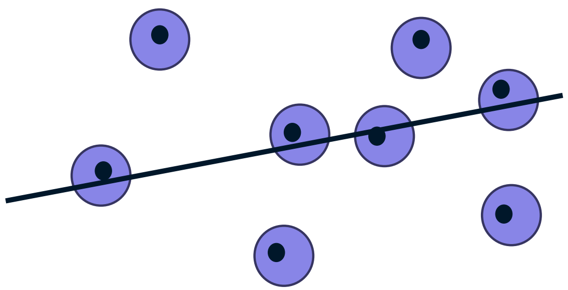
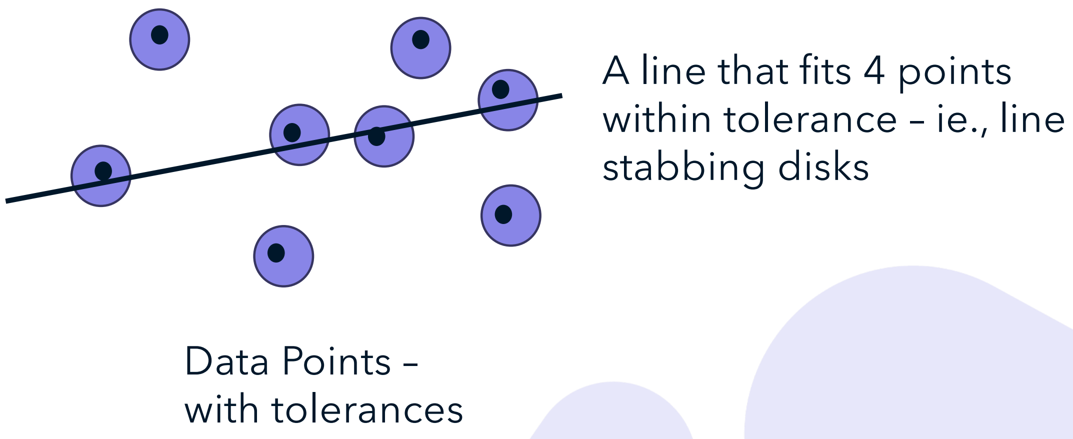


A line that fits 4 points within tolerance – ie., line stabbing disks

Data Points – with tolerances

But what does this have to do with AI, Machine Learning, Signal and Image Processing???

Maximum Consensus Robust fitting (MaxCon) – find the model that fits the most data points within tolerance.



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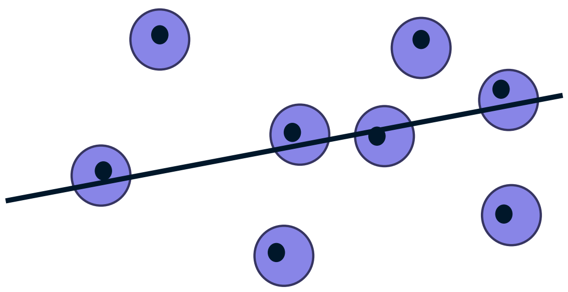


A line that fits 4 points within tolerance – ie., line stabbing disks

Data Points – with tolerances

But of course I am interested in problems more complex than fitting points (in 2D) to lines.

Models that relate camera positions, point cloud registrations, image stitching transformations, etc.

But what does this have to do with AI, Machine Learning, Signal and Image Processing???

(Permission to use photo via Daniel Barath)

Image 1 (x,y,1) <-> Image 2 (x′,y′,1)

(x′,y′,w)^T=H(x,y,1)^T
H – 3x3 homography matrix

(image stitching)

Other “models” R,t (point cloud aignment)
F (3x3 matrix relating geometry of stereo vision)
Etc.

But what does this have to do with AI, Machine Learning, Signal and Image Processing???

In fact, much of Machine Learning and related areas is essentially the geometry of "feature space".
e.g., image -> thousands of features (thought of as vectors in R^n). Then the task is reduced to something like clustering, linear separation (classifier) etc.
In fact, the most often hypergraph referred to as a geometric hypergraph is the one where edges are defined by proximity – spheres in feature space.

But what does this have to do with AI, Machine Learning, Signal and Image Processing???

Clustering then is finding large cliques in a such a geometric hypergraph.

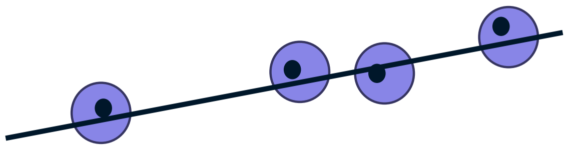
Many applications – community detection (social networks), preferences (commerce), etc etc

Structure and complexity in hypergraphs

Cliques are perhaps the most obvious type of structure in hypergraphs (and of course, therefore their complement – independent (stable) sets. Thus, the number of cliques and their sizes is one important measure of structure/complexity. More generally (hyper)graph classes capture a variety of “complexities”/structure.

Example (robust) line fitting

Data with no outliers – the simplest situation. (hyper)Graph is just one single clique (and complement (graph)hypergraph is a set of isolated vertices – one single independent set.



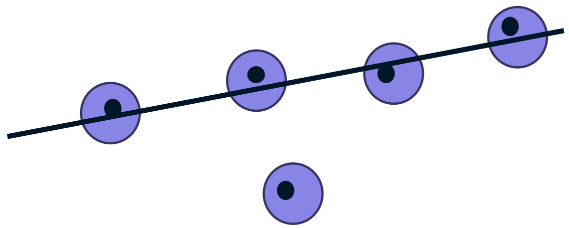
A line that fits 4 points within tolerance – ie., line stabbing disks

In graph theory denoted K_n

Data Points – with tolerances

Example (robust) line fitting

Next most simple – one clique and isolated vertices (one “structure” and “pure” outliers)

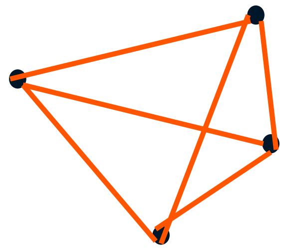


A line that fits 4 points within tolerance – ie., line stabbing disks

Data Points – with tolerances

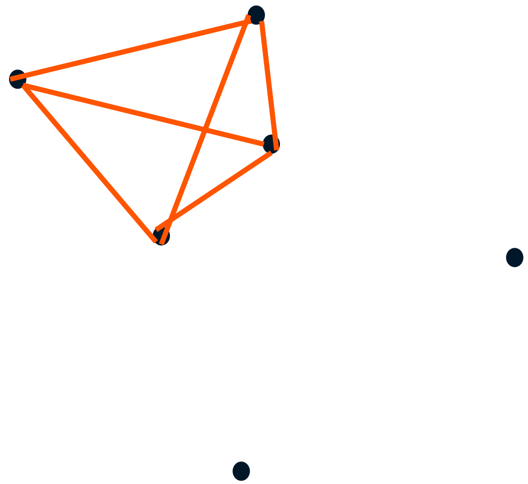
Complexity charcterization

Clique – “level 0 compolexity”



Complexity charcterization

Clique with isolated vertices – "level 1 complexity"



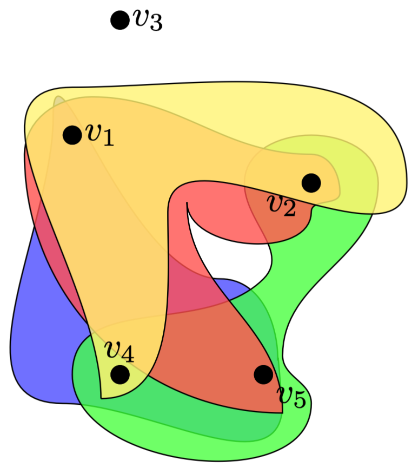
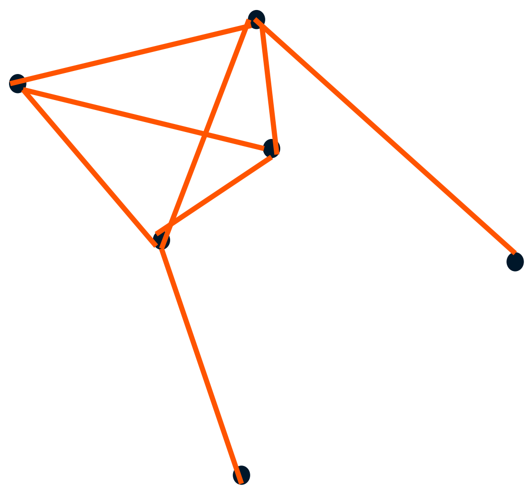


Figure 5.1: A 3-uniform hypergraph containing a 4-clique

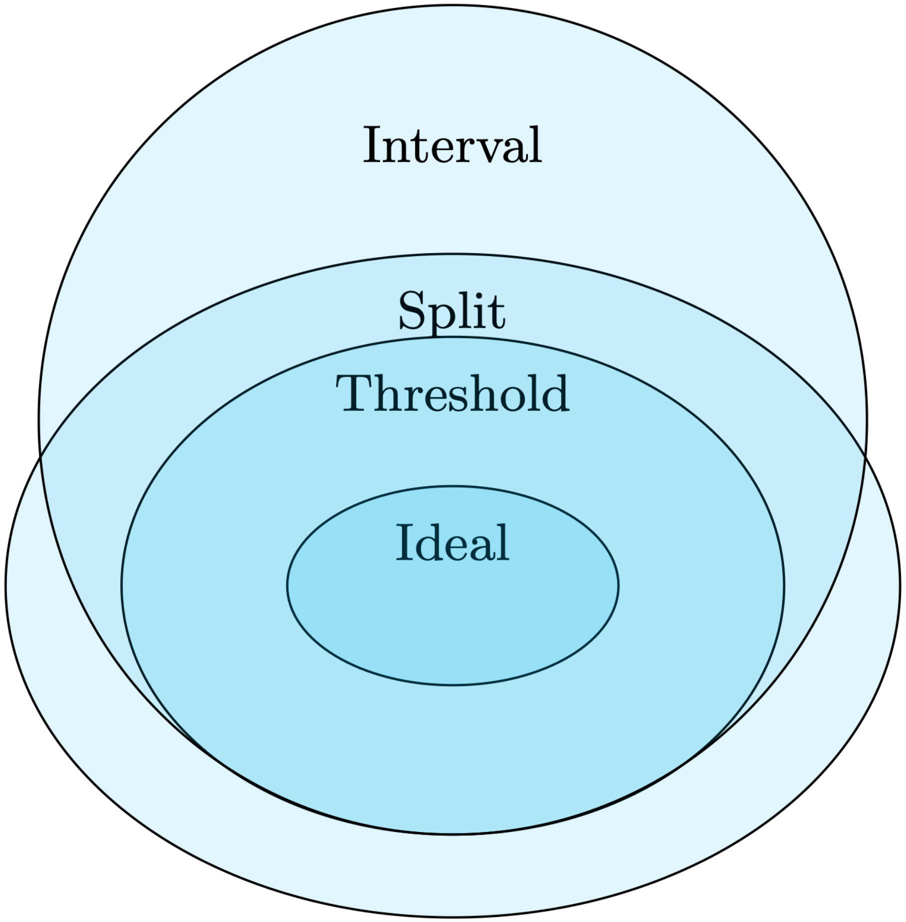
Complexity charcterization

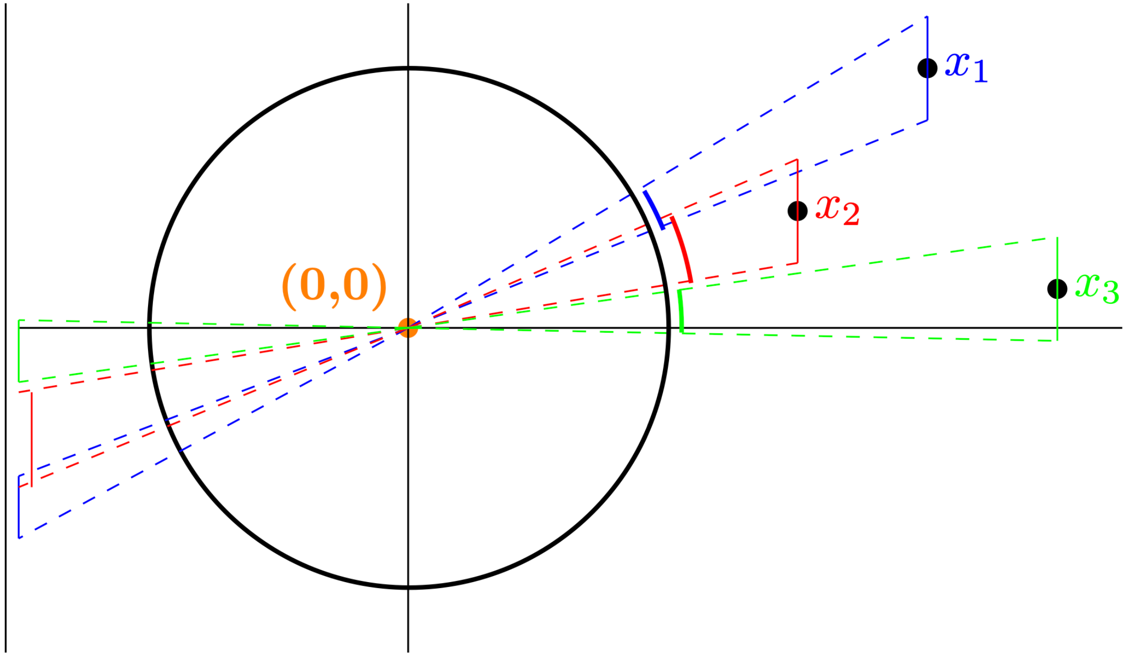
Clique with set of independent vertices and some edges between – “level 2 complexity”



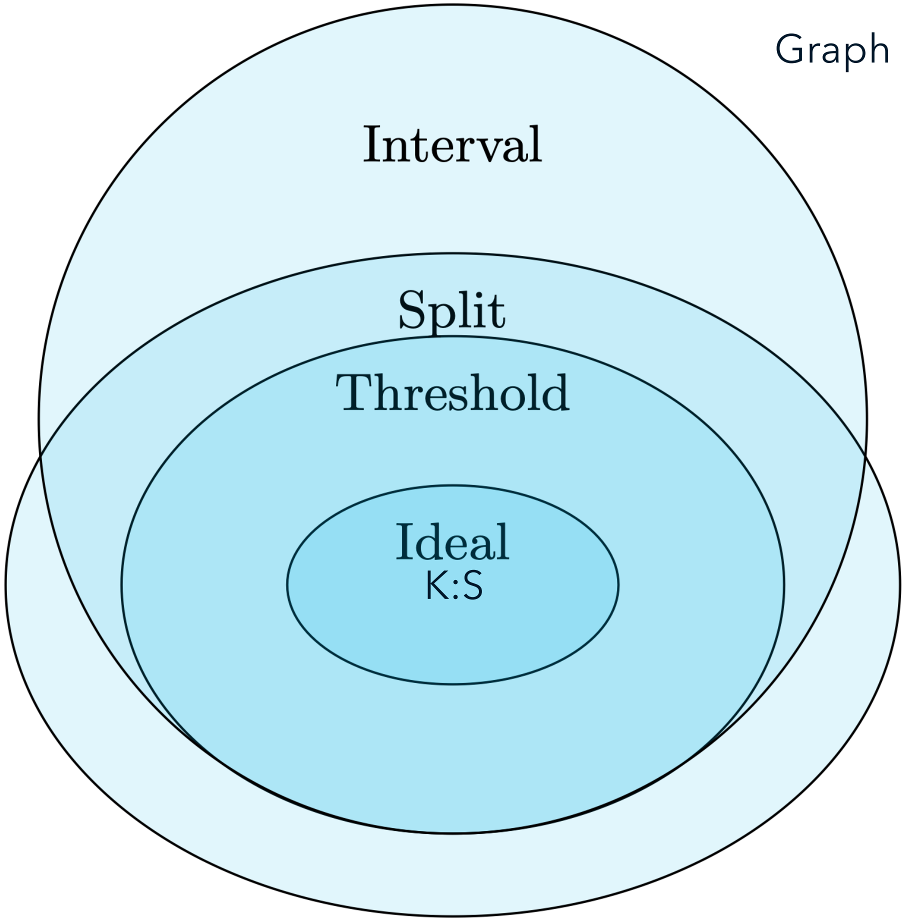
K:S aka “Split Graph”

Complexity characterization – robust linear subspace in 2D (line) regression





Complexity characterization – robust linear subspace in 2D (line) regression



Graph

1D location,
1D subspace in 2D

Hypergraph

?

(affine) line fitting in 2D, Homography fitting, Fundamental Matrix fitting, Rigid Motion fitting…..

Complexity characterization – why does this matter?

Problems Interval Class

Problems in italics have no summary page and are only listed when ISC class.

Parameter decomposition

book thickness decomposition [?]	Unknown to ISGCI
booleanwidth decomposition [?]	Polynomial
cliquewidth decomposition [?]	Unknown to ISGCI
cutwidth decomposition [?]	Unknown to ISGCI
treewidth decomposition [?]	Polynomial

Unweighted problems

3-Colourability [?]	Linear
Clique [?]	Polynomial
Clique cover [?]	Linear
Colourability [?]	Linear
Domination [?]	Linear
Feedback vertex set [?]	Linear
Graph isomorphism [?]	Linear
Hamiltonian cycle [?]	Linear
Hamiltonian path [?]	Polynomial
Independent dominating set [?]	Linear
Independent set [?]	Linear
Maximum cut [?]	NP-complete
Monopolarity [?]	Linear
Polarity [?]	Polynomial
Recognition [?]	Linear

Weighted problems

Weighted clique [?]	Polynomial
Weighted feedback vertex set [?]	Linear
Weighted independent dominating set [?]	Polynomial
Weighted independent set [?]	Linear
Weighted maximum cut [?]	NP-complete

Problems Threshold Class

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Colourability [?]	Linear
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Weighted problems

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Weighted feedback vertex set [?]	Linear
Weighted independent dominating set [?]	Linear
Weighted independent set [?]	Linear
Weighted maximum cut [?]	NP-complete

Ideal Class (K:S)

Many problems trivial – any greedy algorithm finds max clique, max independent set…

"Totally" Ideal Class (K)

No algorithm needed!!!
No robust fitting needed…

Complexity charcterization

The “full story” is much more rich and complicated…
graphclasses.org
(for **hyper**graphs much less is documented/known –
There is a beta “geometric hypergraph zoo page”
but for a very small part of the hypergraph zoo…)

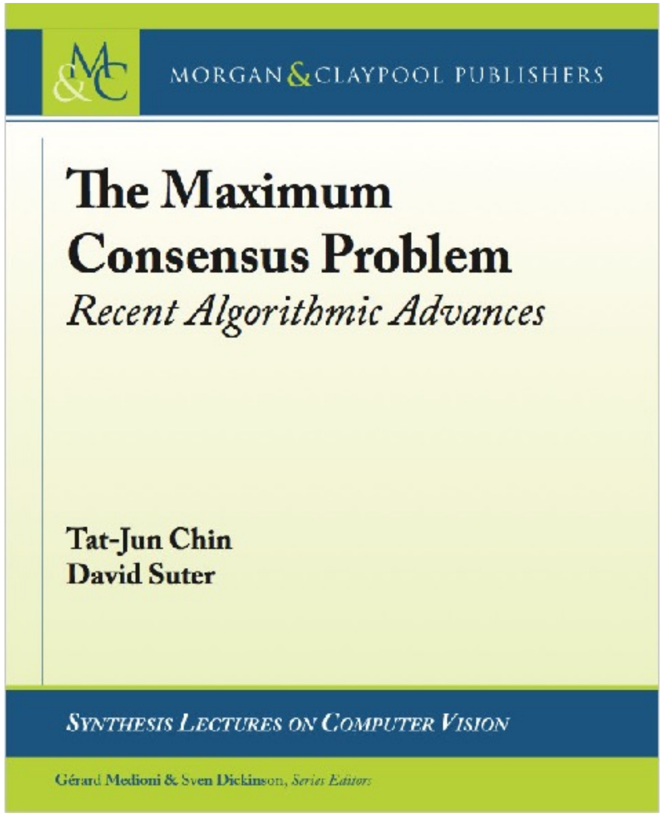
Much less is known (yet) about geometric
hypergraphs – and a class taxonomy

Only a peak into the whole story…..

This area is an interesting (and still being discovered) story at the intersection of combinatorics, discrete/computational maths, discrete topology, theoretical computer science, Boolean function theory, etc.

I've only been able to give a "taste/glimpse" of both the area and my interests in application to robust fitting.

Some of this material will be in 2nd edition……



1st ed 2017
2nd ed under contract

Related publications:

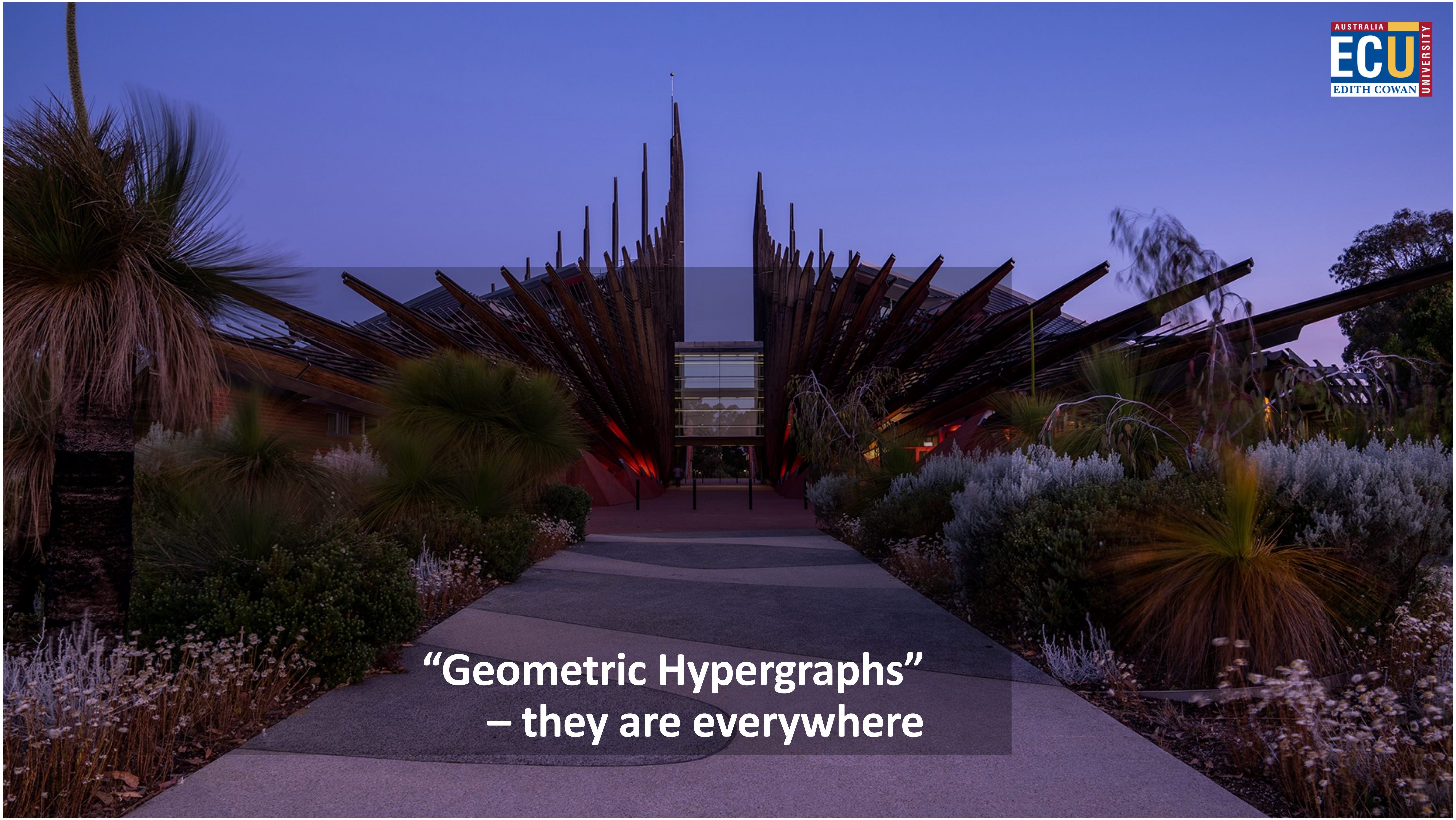
Giang Truong, Huu Le, Erchuan Zhang, David Suter, and Syed Zulqarnain Gilani. "Unsupervised Learning for Max- imum Consensus Robust Fitting: A Reinforcement Learning Approach". In: IEEE Transactions on Pattern Analysis and Machine Intelligence 45.3 (2023), pp. 3890–3903. doi: 10.1109/TPAMI.2022.3178442.

Erchaun Zhang, David Suter, Ruwan Tennakoon, Tat-Jun Chin, Alireza Bab-Hadiashar, Giang Truong, and Syed Zulqarnain Gilani. "Maximum Consensus by Weighted Influences of Monotone Boolean Functions". In: 2022 IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR). 2022, pp. 8954–8962. doi: 10.1109/CVPR52688. 2022.00876.

Tat-Jun Chin, David Suter, Shin-Fang Ch'ng, and James Quach. "Quantum Robust Fitting". In: Computer Vision – ACCV 2020. Ed. by Hiroshi Ishikawa, Cheng-Lin Liu, Tomas Pajdla, and Jianbo Shi. Cham: Springer International Publishing, 2021, pp. 485–499. isbn: 978-3-030-69525-5. doi: 10.1007/978-3-030-69525-5_29.

Ruwan Tennakoon, David Suter, Erchuan Zhang, Tat-Jun Chin, and Alireza Bab-Hadiashar. "Consensus Maximisation Using Influences of Monotone Boolean Functions". In: 2021 IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR). Oral Presentation (17% of accepted papers, roughly 3% of sumitted papers). 2021, pp. 2865– 2874. doi: 10.1109/CVPR46437.2021.00289.

Robust Fitting on a Gate Quantum Computer – ECCV2024 to appear – colleagues at UoAdelaide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“Geometric Hypergraphs”
– they are everywhere

Dilworth number of graph

Maximum number of chains (of vicinal order) necessary to cover the graph…
Threshold graphs are Dilworth1

Permutation graph

Permutation includes cograph which includes threshold