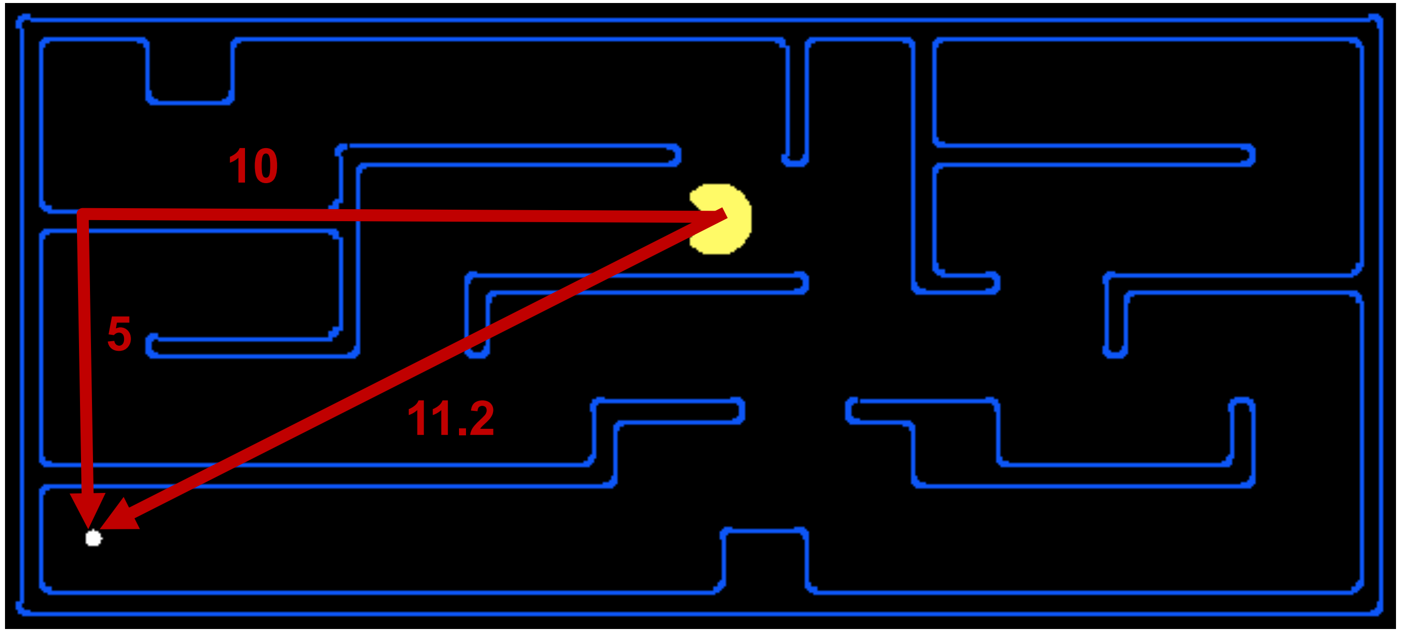
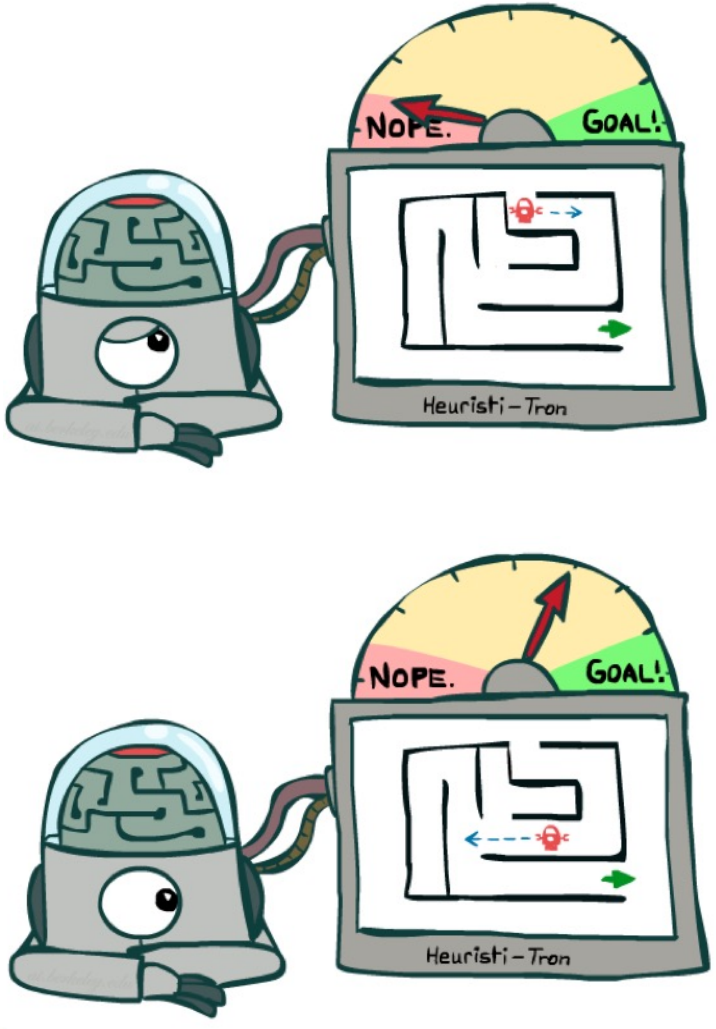
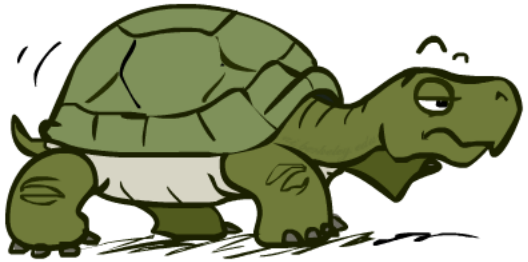
Recap: Search Heuristics

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing

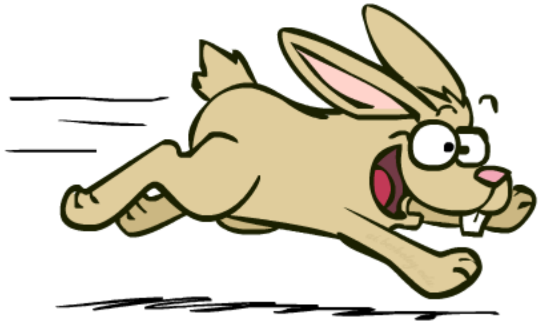




Recap: Cost- vs. Heuristic-Guided Search



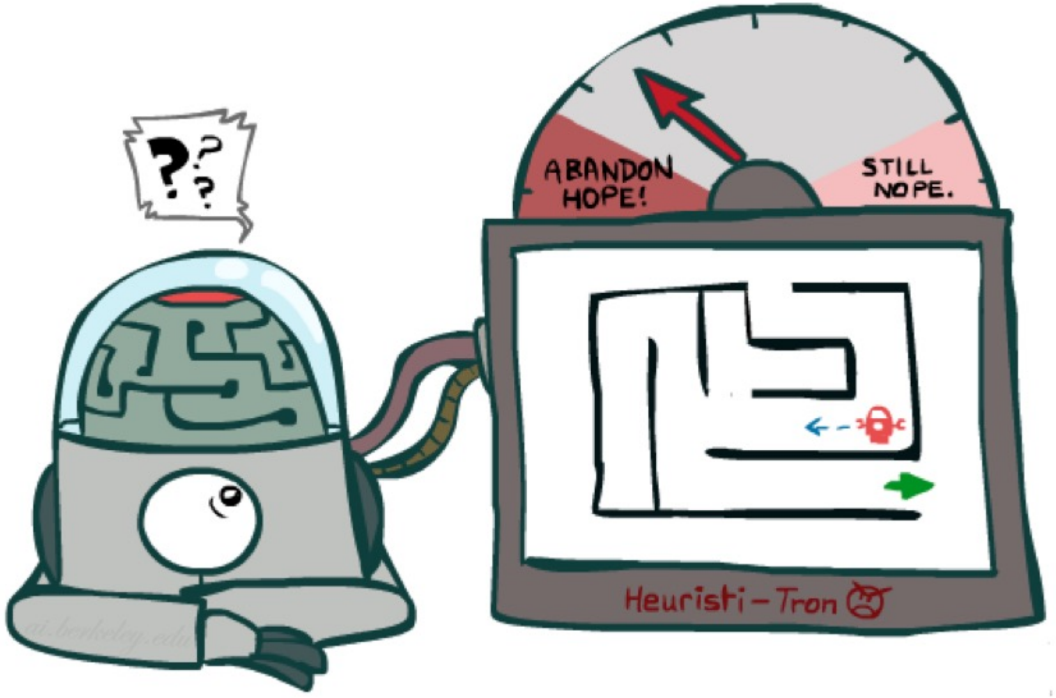
Uniform-Cost Search
(only costs, g)



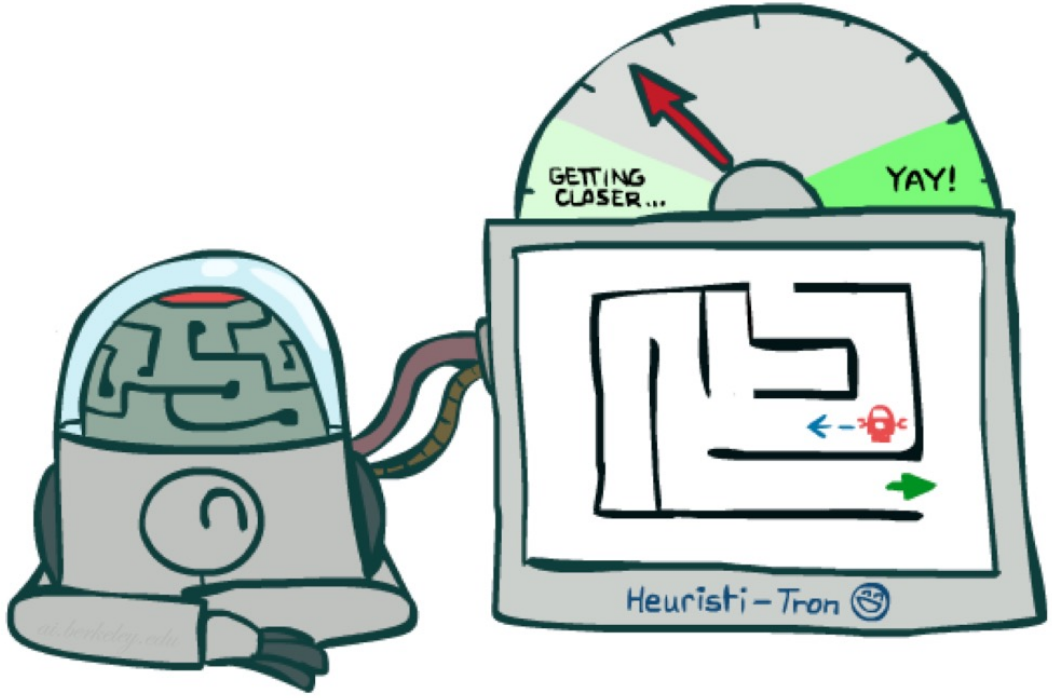
Greedy Best-First Search
(only heuristic, h)

A* Search
(both, f=g+h)

Recap: Admissibility

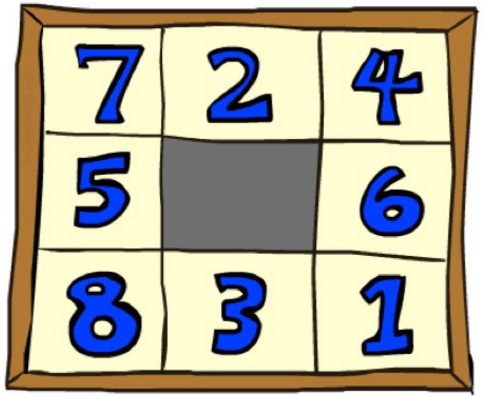


Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe

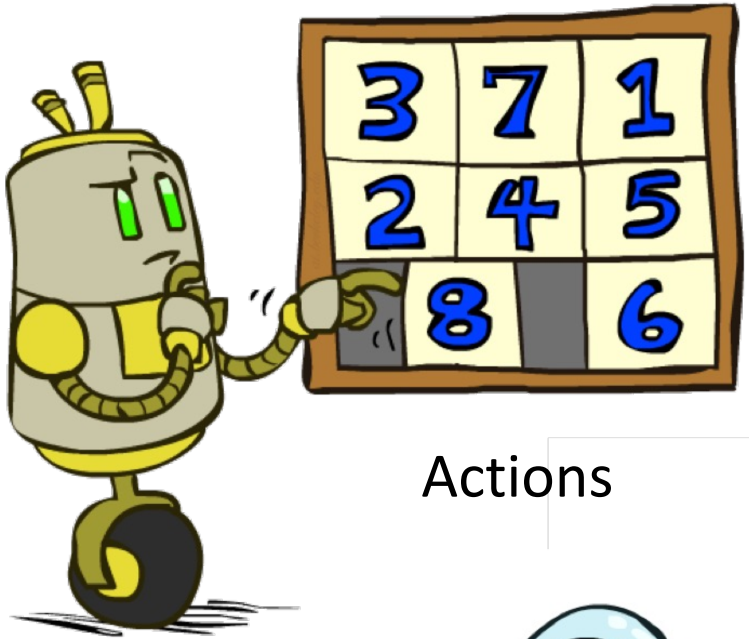


Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

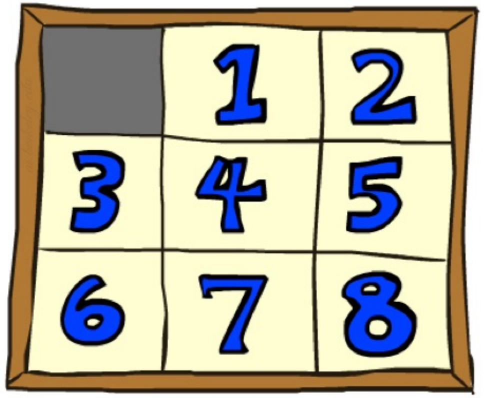
Recap: 8-Puzzle



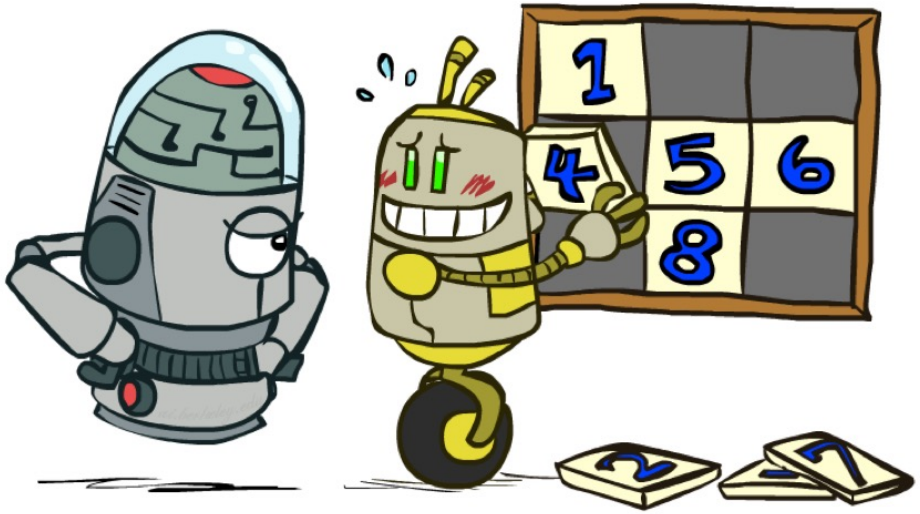
Start State



Actions

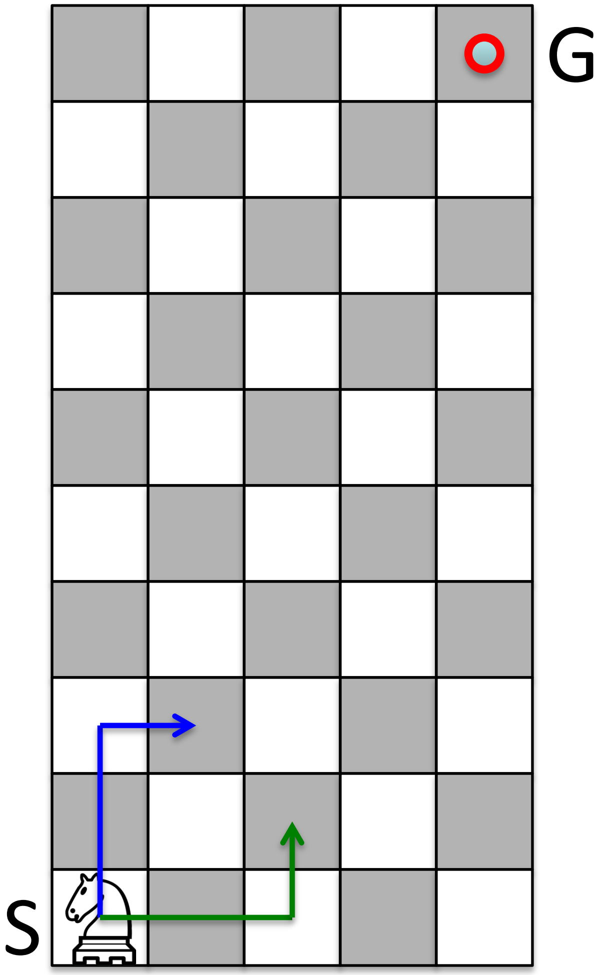


Goal State

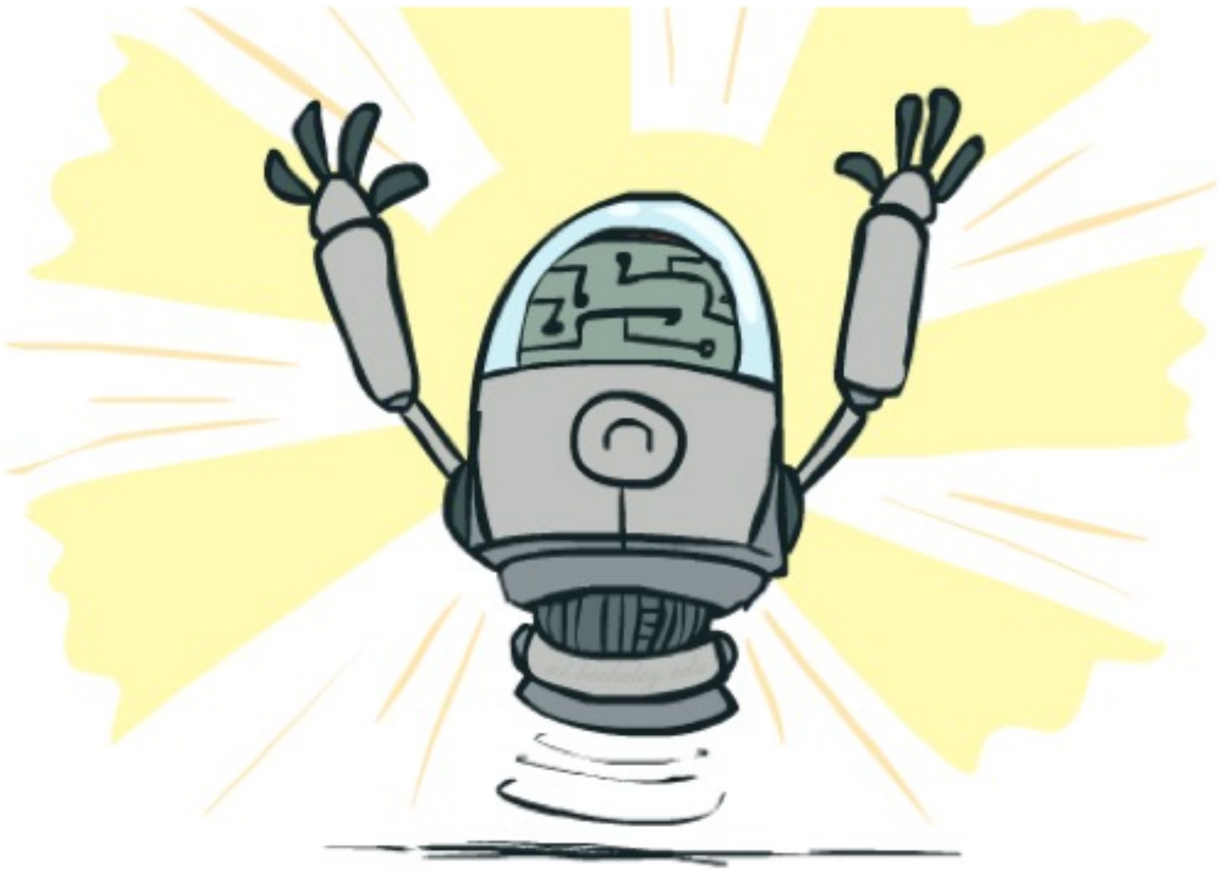


Designing a Heuristic: Knight's moves

- Minimum number of knight's moves to get from S to G?
  - h_1 = (Manhattan distance)/3
    - h_1' = h_1 rounded up to correct parity (even if S, G same color, odd otherwise)
  - h_2 = (Euclidean distance)/$\sqrt{5}$
    - h_2' = h_2 rounded up to correct parity
  - h_3 = (maximum horizontal or vertical distance)/2
    - h_3' = h_3 rounded up to correct parity
- $h(n) = \max(h_1'(n), h_2'(n), h_3'(n))$ is admissible!



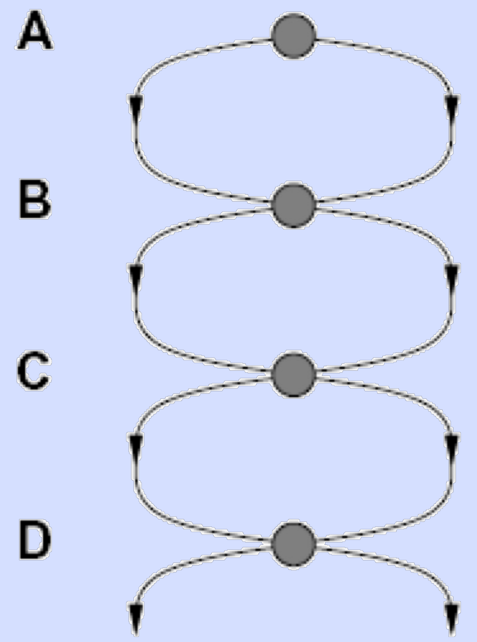
Recap: Optimality of A* Tree Search



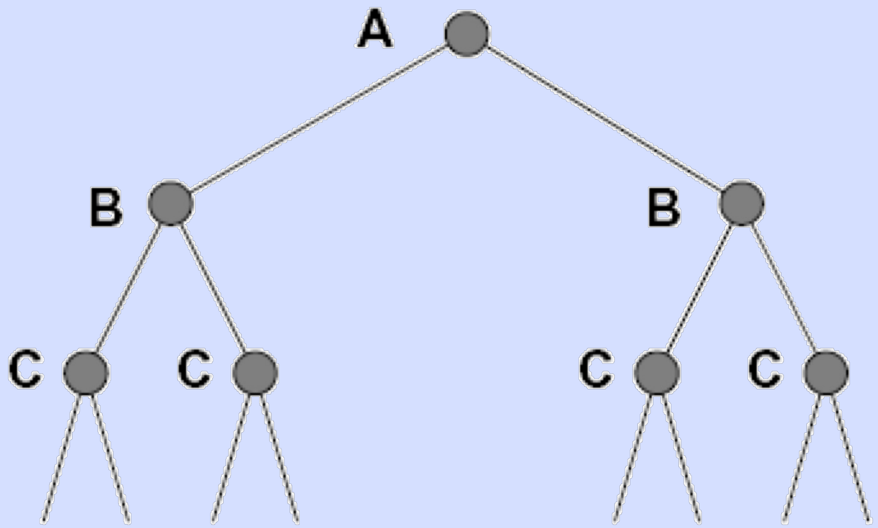
Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.

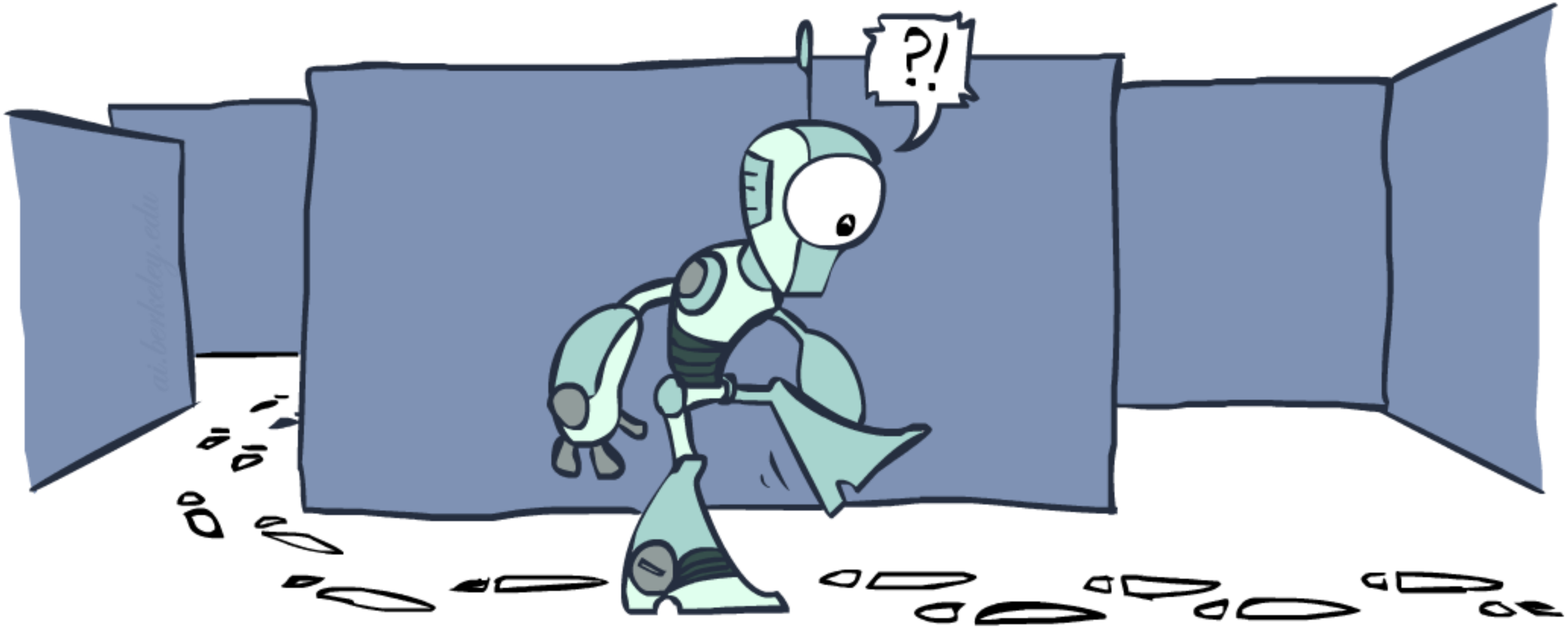
State Graph



Search Tree

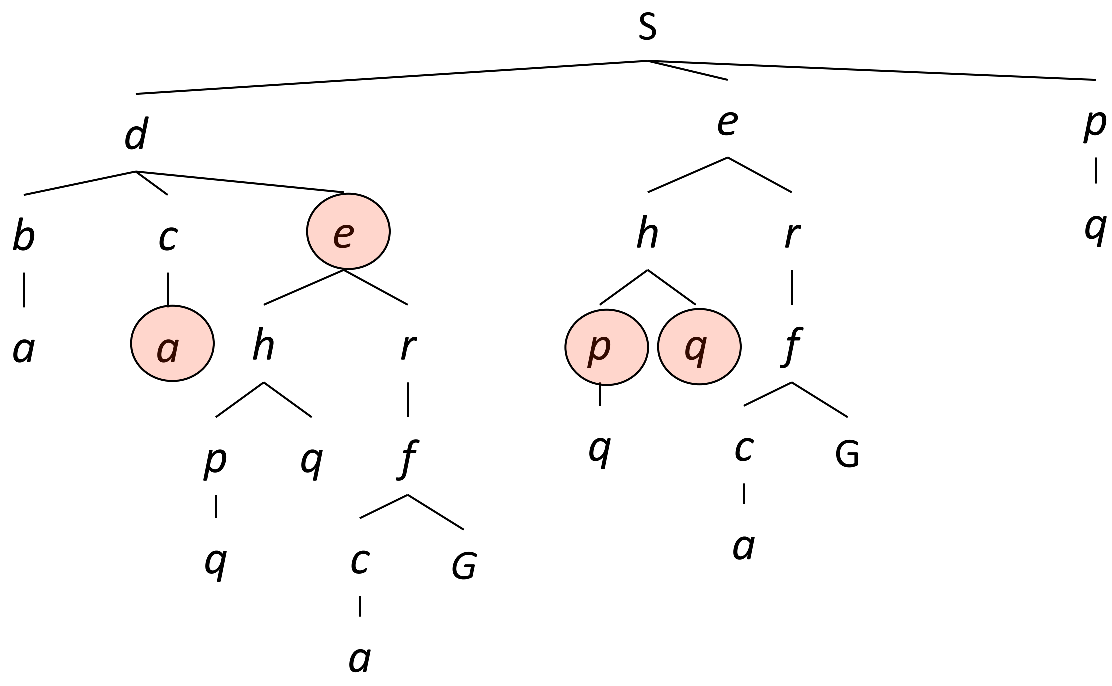


Graph Search



Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)

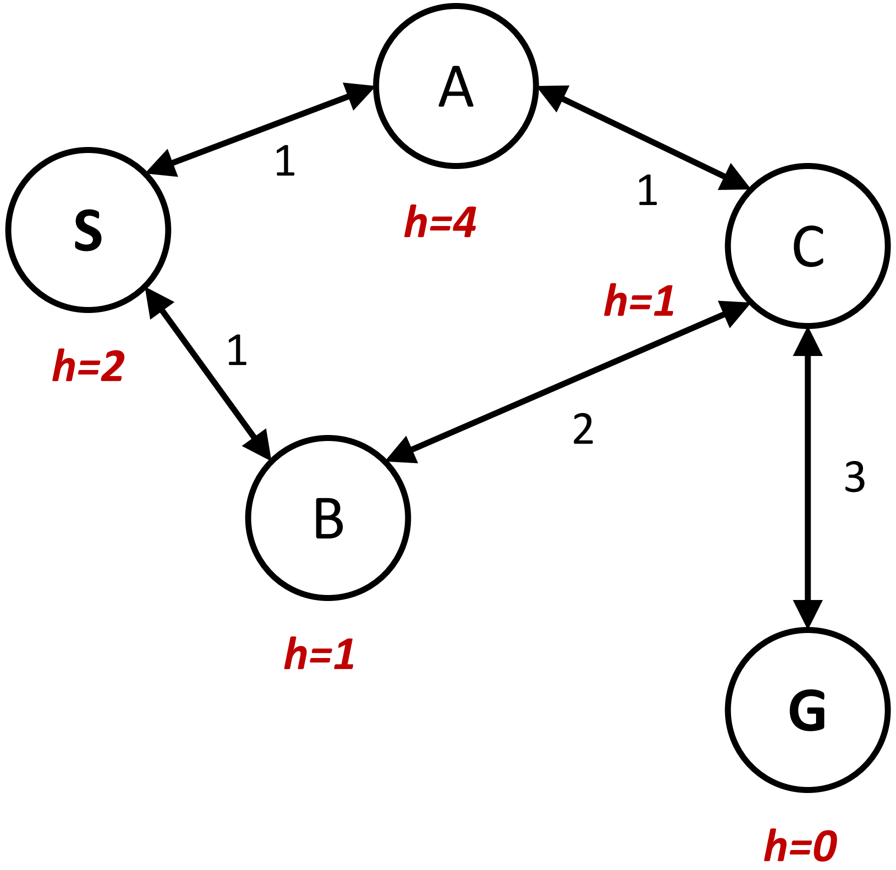


Graph Search

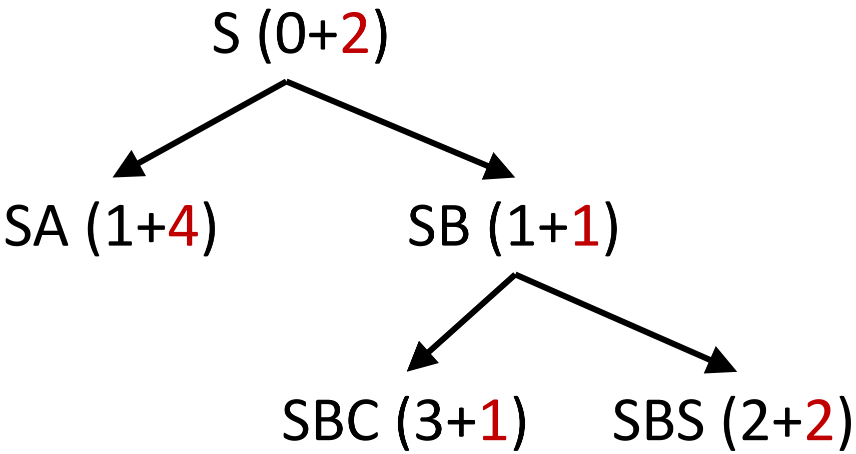
- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states (“closed set”)
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set
- Important: store the closed set as a set, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

A* Graph Search Gone Wrong?

State space graph



Search tree

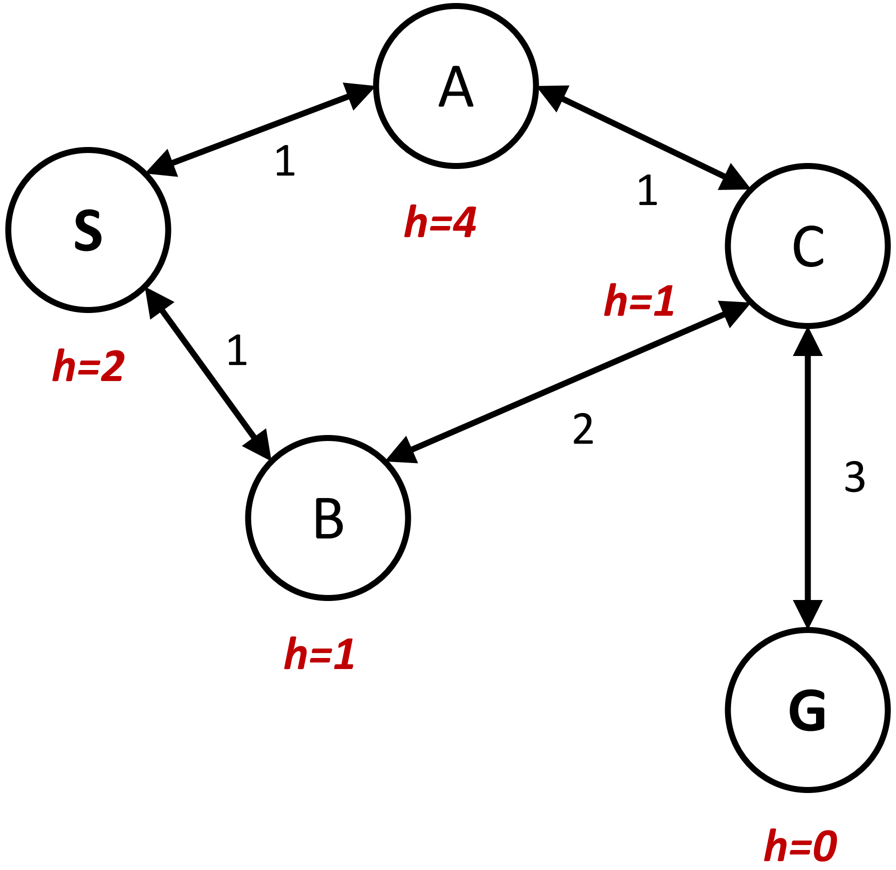


Closed set

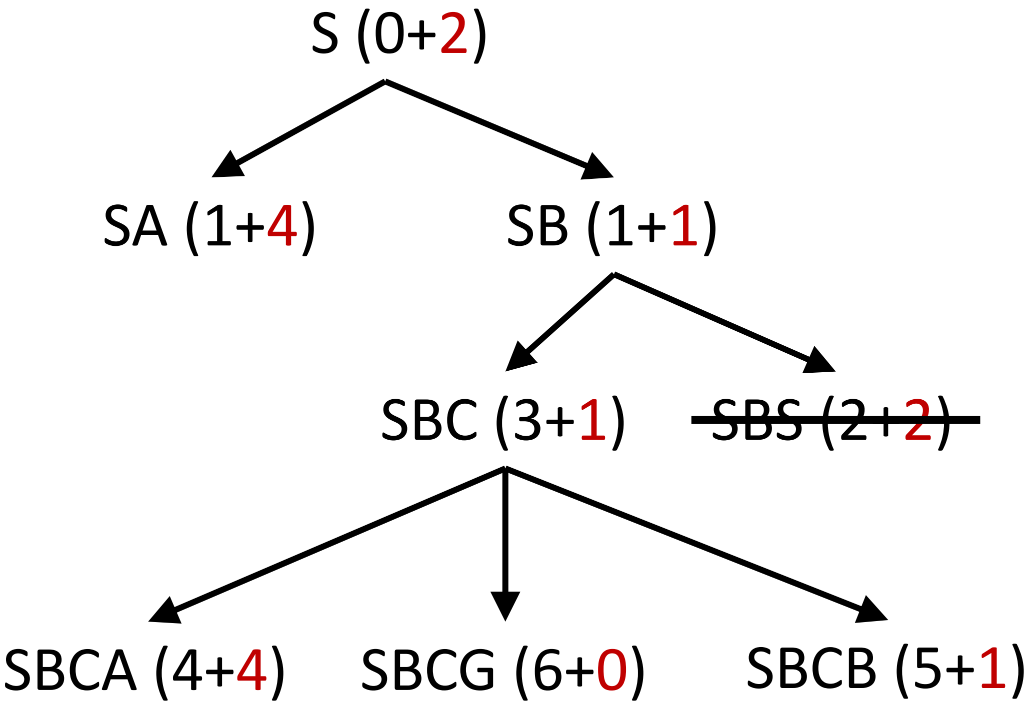
{ S B }

A* Graph Search Gone Wrong?

State space graph



Search tree

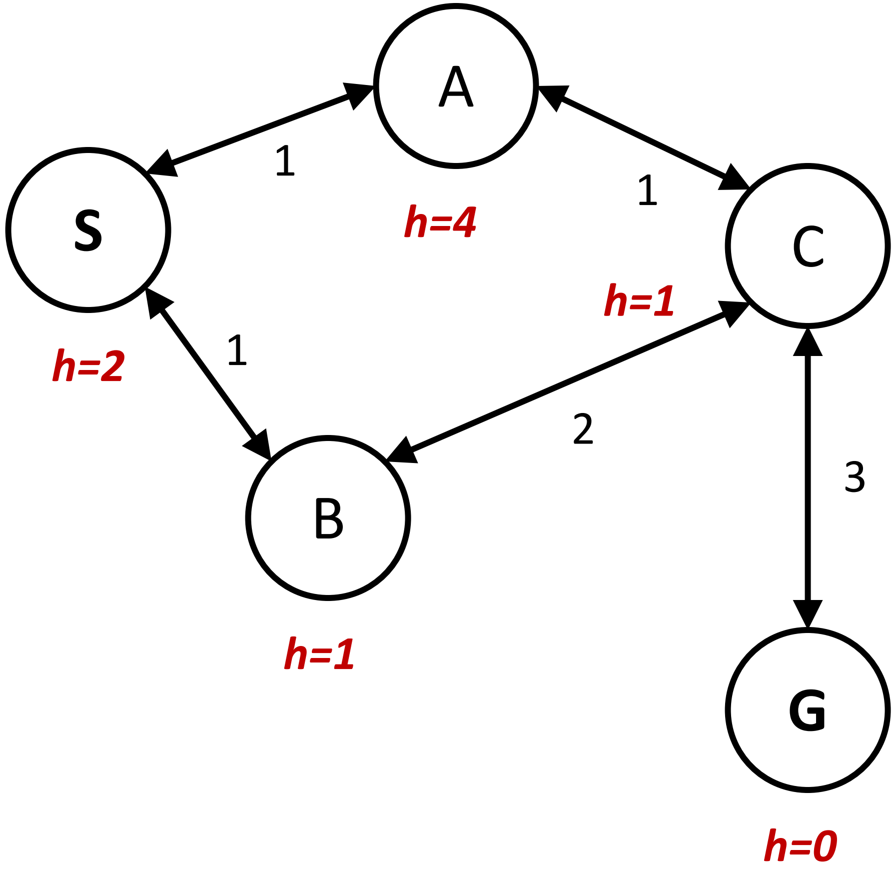


Closed set

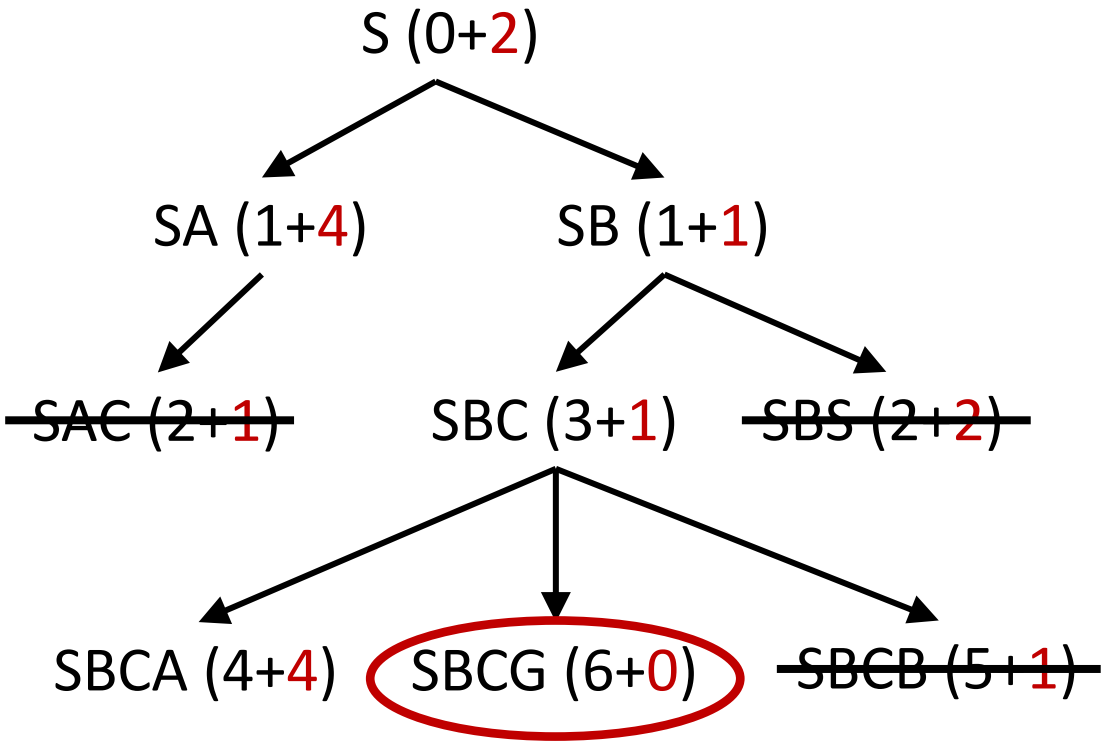
{ S B }

A* Graph Search Gone Wrong?

State space graph



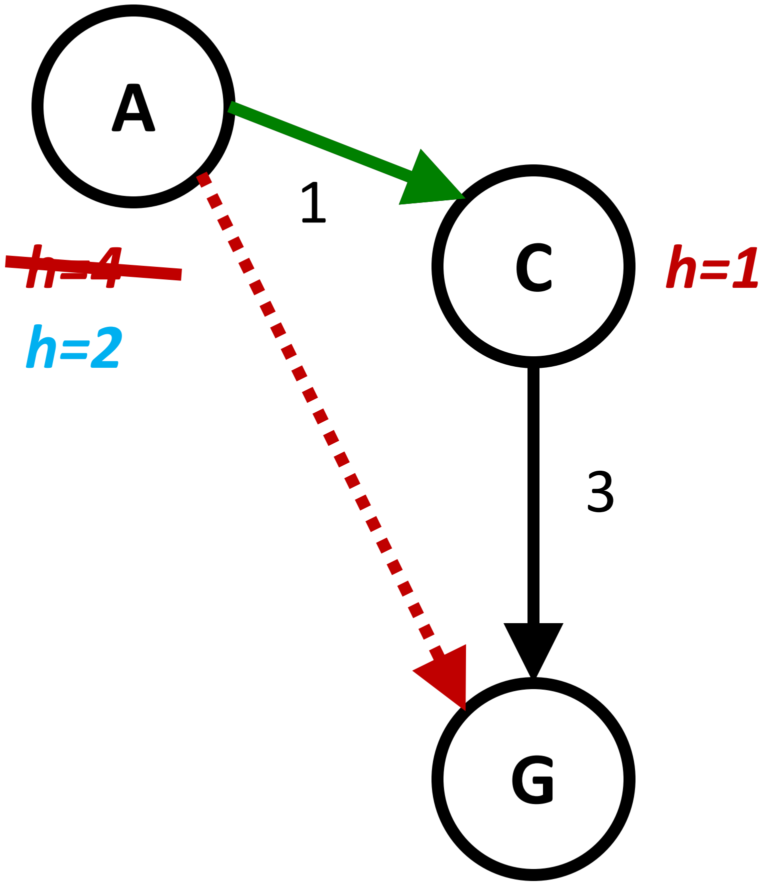
Search tree



Closed set

{ S B C A }

Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost *h** from A to G

  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

    h(A) – h(C) ≤ cost(A to C)

    - a.k.a. "triangle inequality": h(A) ≤ cost(A to C) + h(C)
    - Note: true cost *h** <u>necessarily</u> satisfies triangle inequality

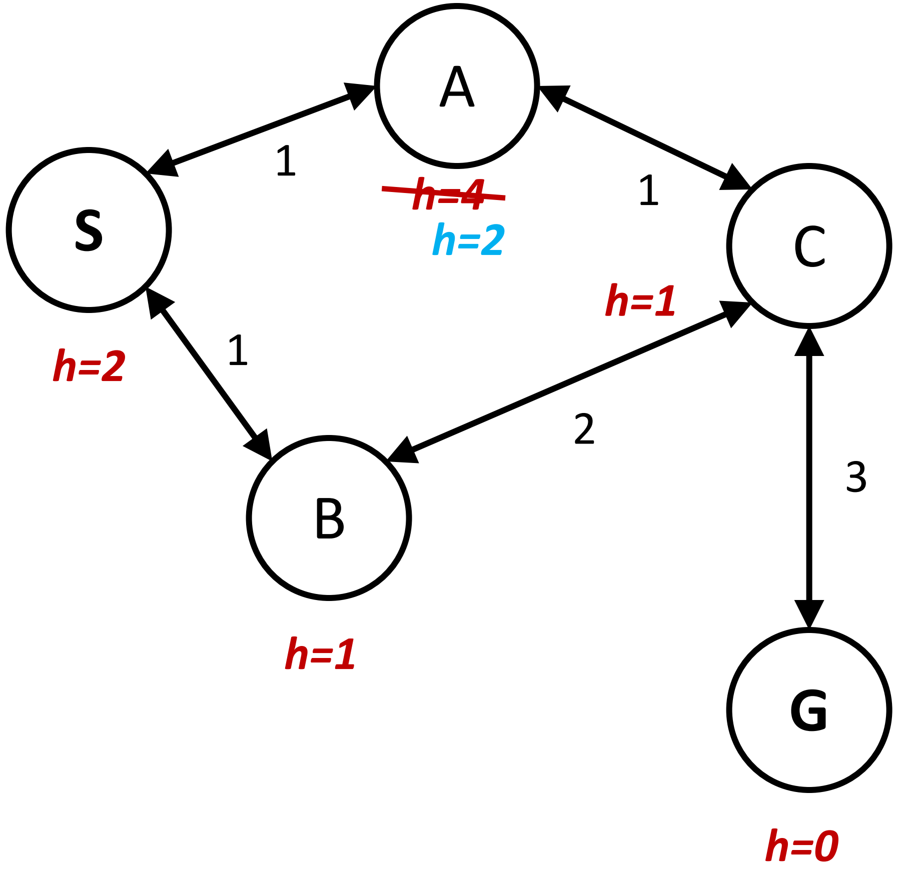
- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)

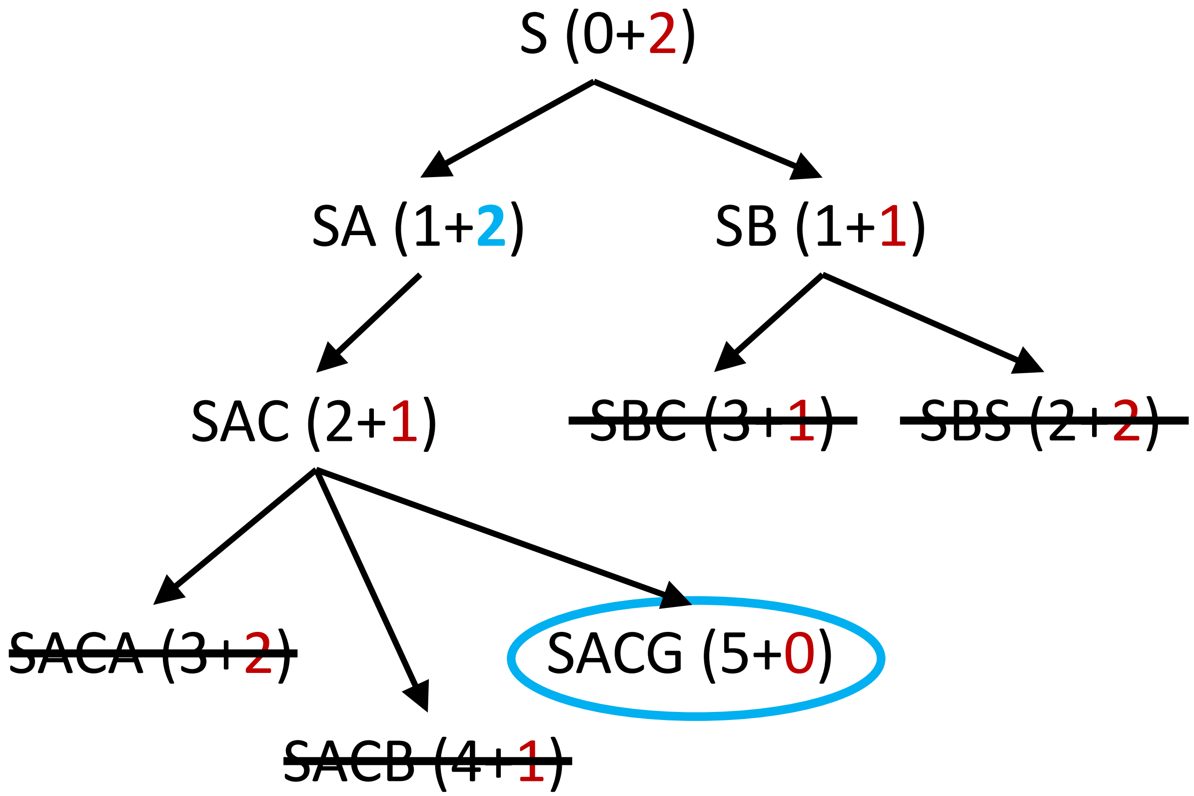
  - A* graph search is optimal

A* Graph Search with Consistent Heuristic

State space graph



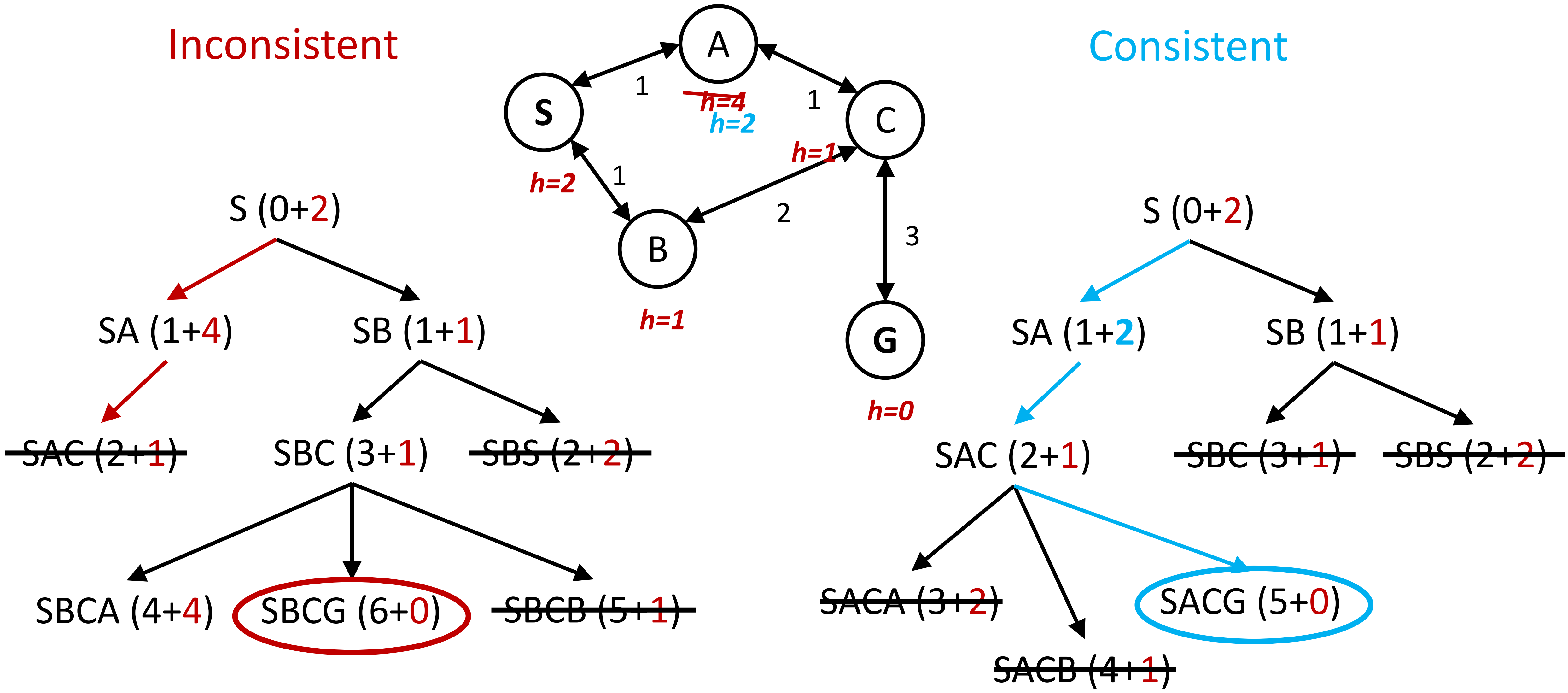
Search tree



Closed set

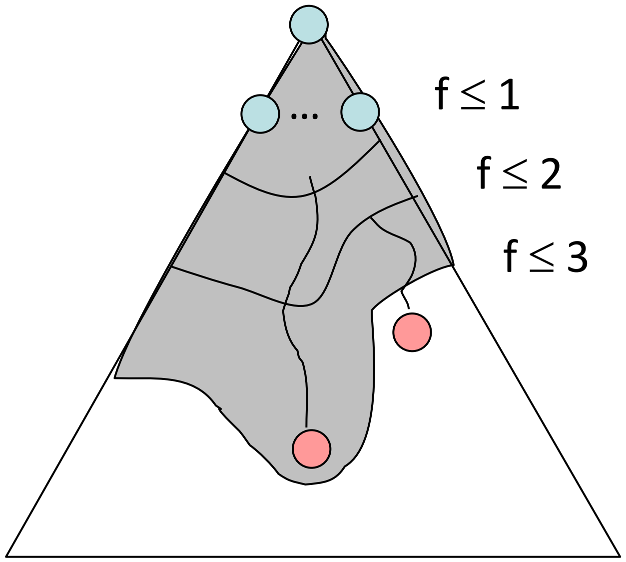
{ S B A C }

Consistency => non-decreasing f-score



Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
  - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
  - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally
  - Result: A* graph search is optimal



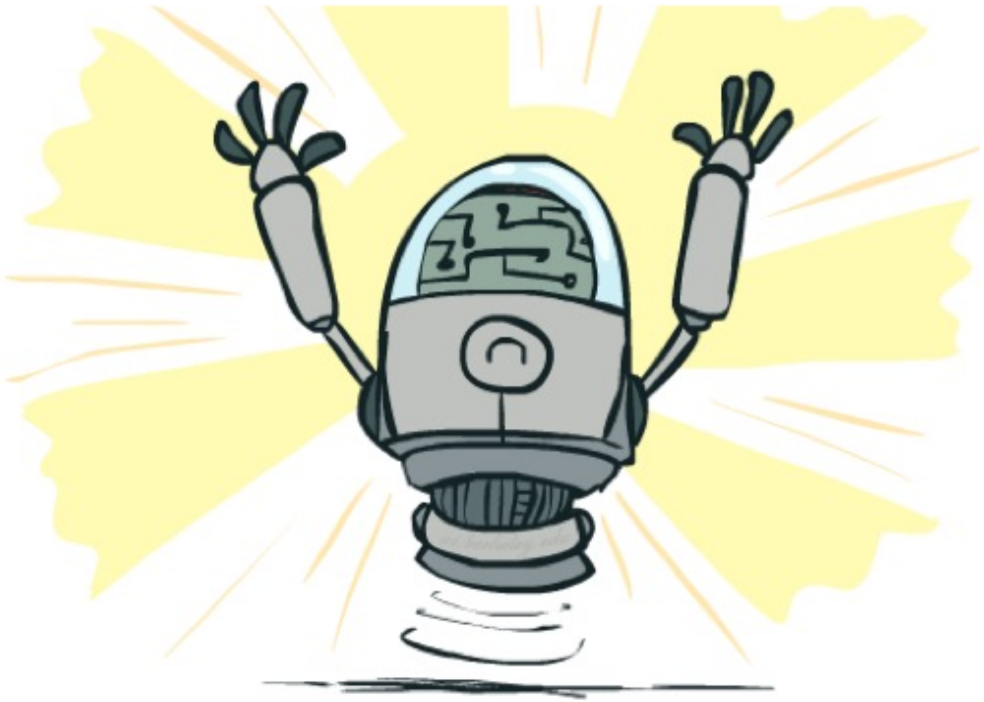
Optimality

- Tree search:
  - A* is optimal if heuristic is admissible
  - UCS is a special case (h = 0)

- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)

- Consistency implies admissibility

- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems

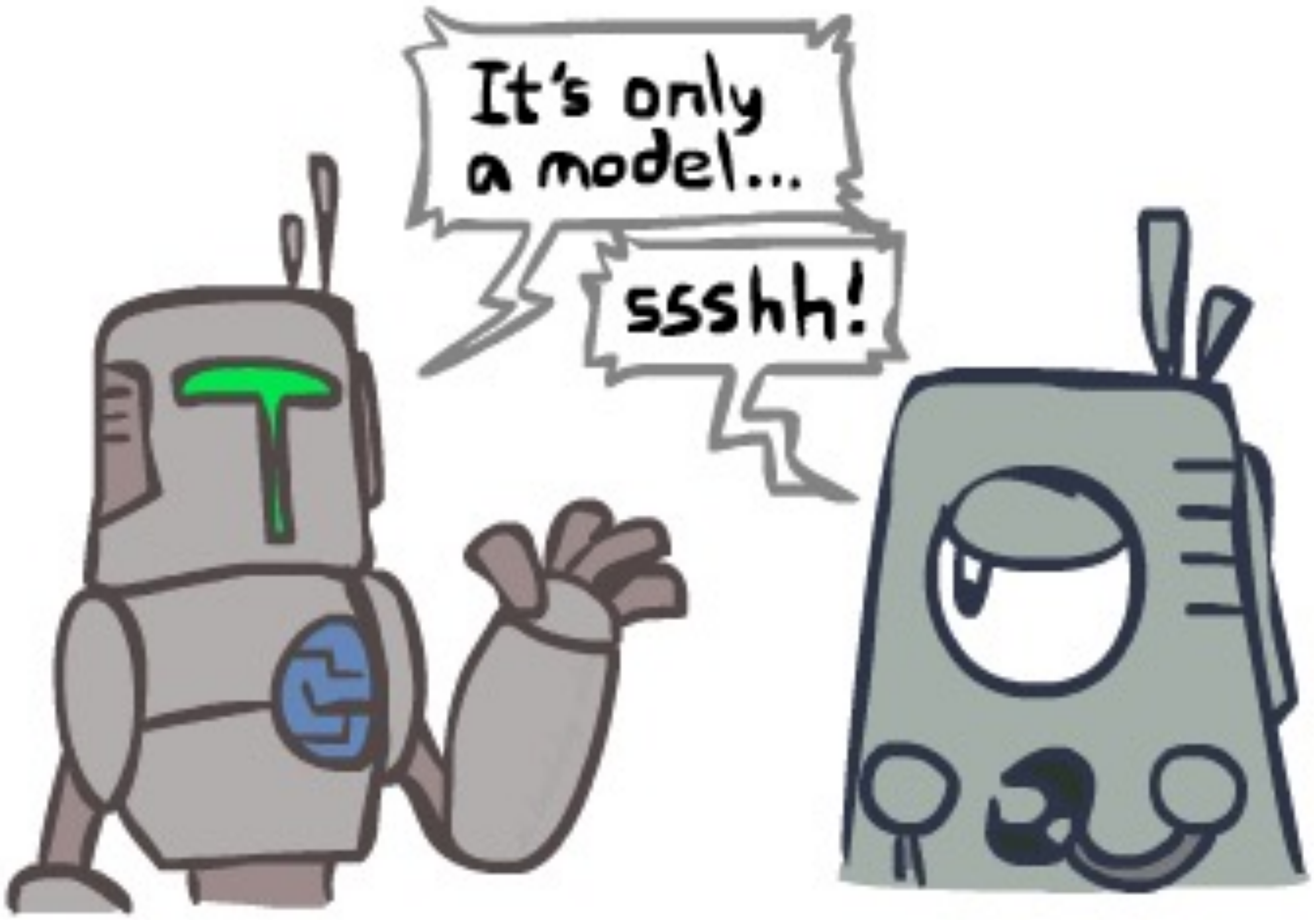


But…

- A* keeps the entire explored region in memory
- => will run out of space before you get bored waiting for the answer ☹️
- There are variants that use less memory (Section 3.5.5):
  - IDA* works like iterative deepening, except it uses an f-limit instead of a depth limit
    - On each iteration, remember the smallest f-value that exceeds the current limit, use as new limit
    - Very inefficient when f is real-valued and each node has a unique value
  - RBFS is a recursive depth-first search that uses an f-limit = the f-value of the best alternative path available from any ancestor of the current node
    - When the limit is exceeded, the recursion unwinds but remembers the best reachable f-value on that branch
  - SMA* uses *all available memory* for the queue, minimizing thrashing
    - When full, drop worst node on the queue but remember its value in the parent

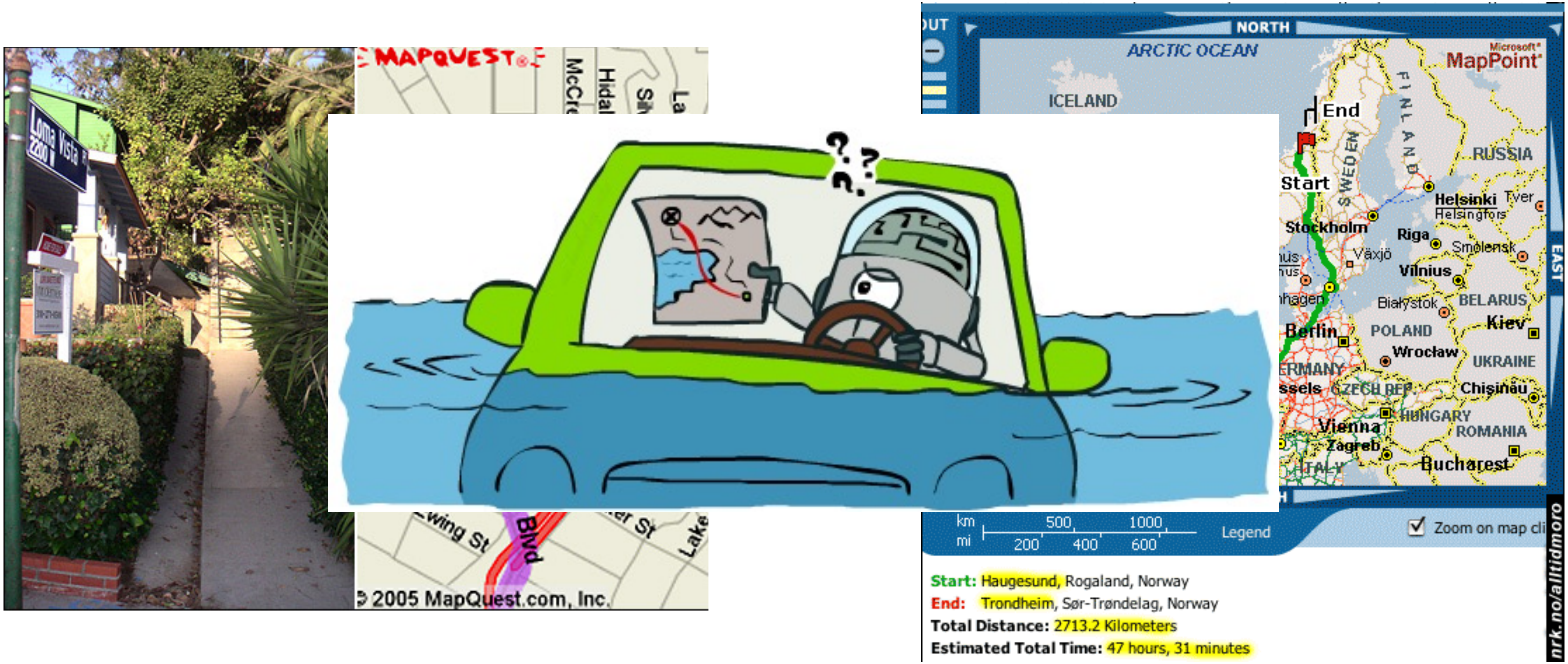
Search and Models

- Search operates over models of the world
  - The agent doesn't actually try all the plans out in the real world!
  - Planning is all "in simulation"
  - Your search is only as good as your models…



Search Gone Wrong?

Search Gone Wrong?



Tree Search Pseudo-Code

```
function TREE-SEARCH(problem, fringe) return a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        for child-node in EXPAND(STATE[node], problem) do
            fringe ← INSERT(child-node, fringe)
        end
    end
```

Graph Search Pseudo-Code

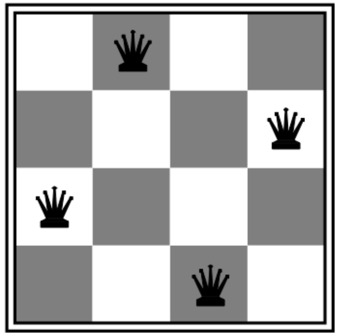
```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            for child-node in EXPAND(STATE[node], problem) do
                fringe ← INSERT(child-node, fringe)
            end
    end
```

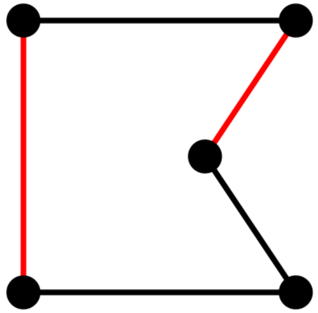
Local Search

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]
[Updated slides from: Stuart Russell and Dawn Song]

Local search algorithms

- In many optimization problems, ***path*** is irrelevant; the goal state ***is*** the solution
- Then state space = set of "complete" configurations;
  find ***configuration satisfying constraints***, e.g., n-queens problem; or, find ***optimal configuration***, e.g., travelling salesperson problem





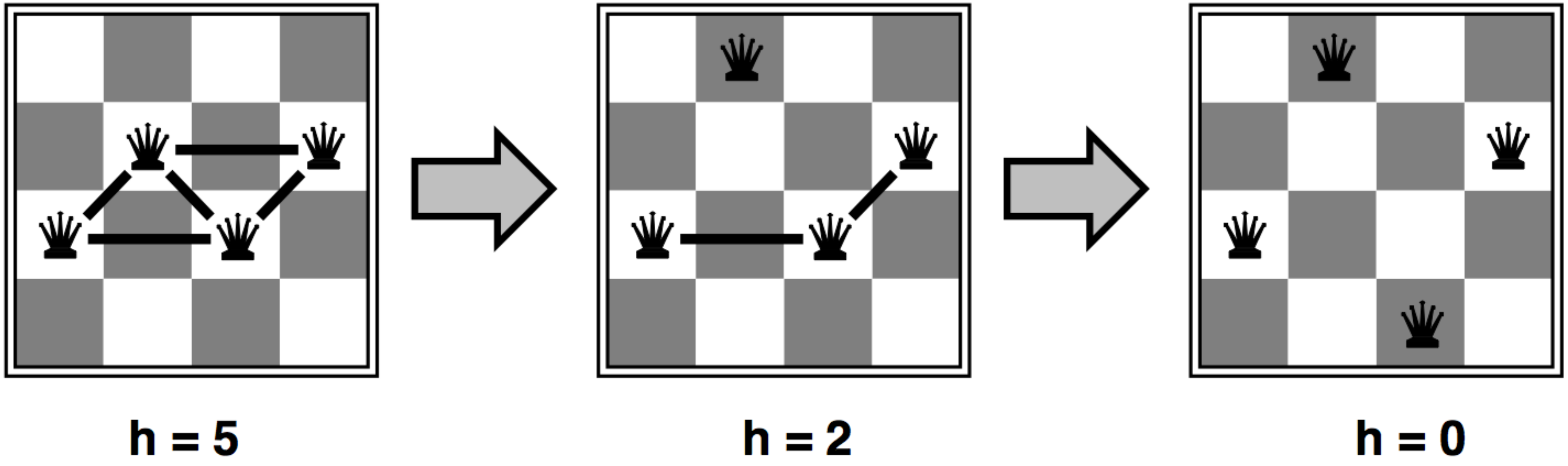
- In such cases, can use ***iterative improvement*** algorithms: keep a single "current" state, try to improve it
- Constant space, suitable for online as well as offline search
- More or less unavoidable if the "state" is yourself (i.e., learning)

Hill Climbing

- Simple, general idea:
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit

Heuristic for *n*-queens problem

- Goal: n queens on board with no ***conflicts***, i.e., no queen attacking another
- States: n queens on board, one per column
- Actions: move a queen in its column
- Heuristic value function: number of conflicts

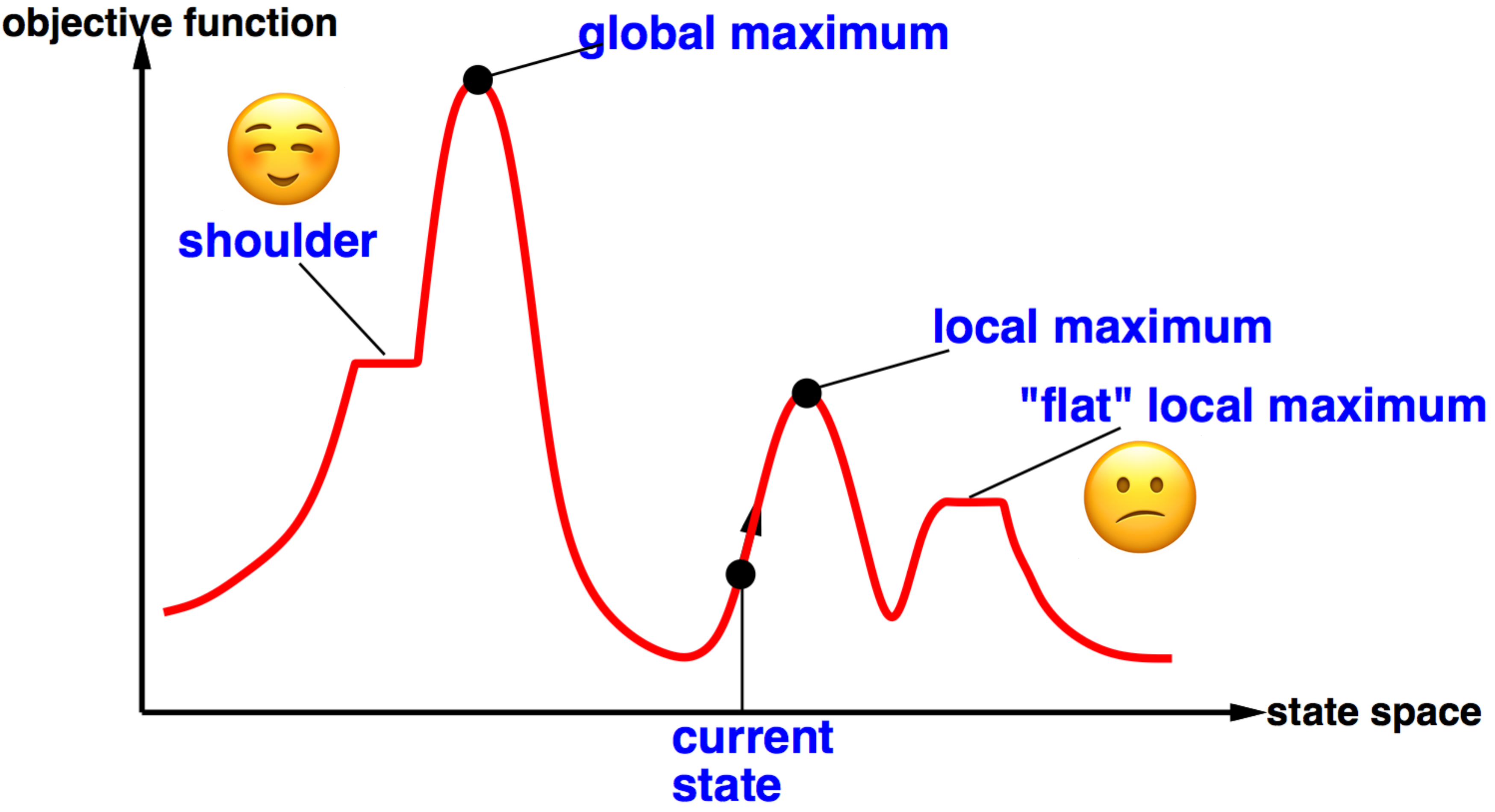


Hill-climbing algorithm

```
function HILL-CLIMBING(problem) returns a state
    current ← make-node(problem.initial-state)
    loop do
        neighbor ← a highest-valued successor of current
        if neighbor.value ≤ current.value then
            return current.state
        current ← neighbor
```

"Like climbing Everest in thick fog with amnesia"

Global and local maxima



Random restarts

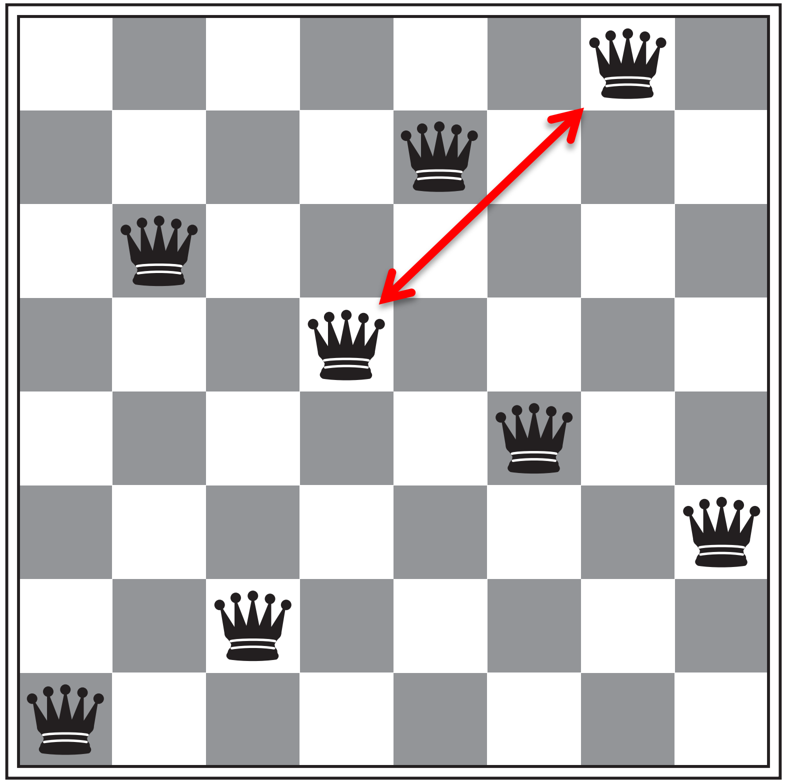
- find global optimum
- duh

Random sideways moves

- Escape from shoulders
- Loop forever on flat local maxima

Hill-climbing on the 8-queens problem

- No sideways moves:
  - Succeeds w/ prob. 0.14
  - Average number of moves per trial:
    - 4 when succeeding, 3 when getting stuck
  - Expected total number of moves needed:
    - 3(1-p)/p + 4 =~ 22 moves
- Allowing 100 sideways moves:
  - Succeeds w/ prob. 0.94
  - Average number of moves per trial:
    - 21 when succeeding, 65 when getting stuck
  - Expected total number of moves needed:
    - 65(1-p)/p + 21 =~ 25 moves



Moral: algorithms with knobs to twiddle are irritating

Simulated annealing

- Resembles the annealing process used to cool metals slowly to reach an ordered (low-energy) state
- Basic idea:
  - Allow “bad” moves occasionally, depending on “temperature”
  - High temperature => more bad moves allowed, shake the system out of its local minimum
  - Gradually reduce temperature according to some schedule
  - Sounds pretty flaky, doesn’t it?

Simulated annealing algorithm

function SIMULATED-ANNEALING(problem,schedule) **returns** a state

current ← problem.initial-state

for t = 1 **to** ∞ **do**

  T ←schedule(t)

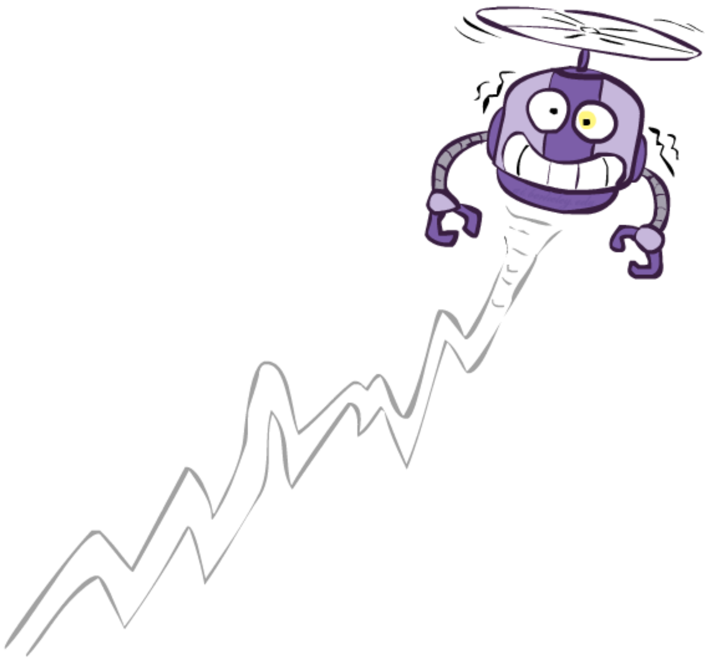
  if T = 0 **then return** current

  next ← a randomly selected successor of current

  ΔE ← next.value – current.value

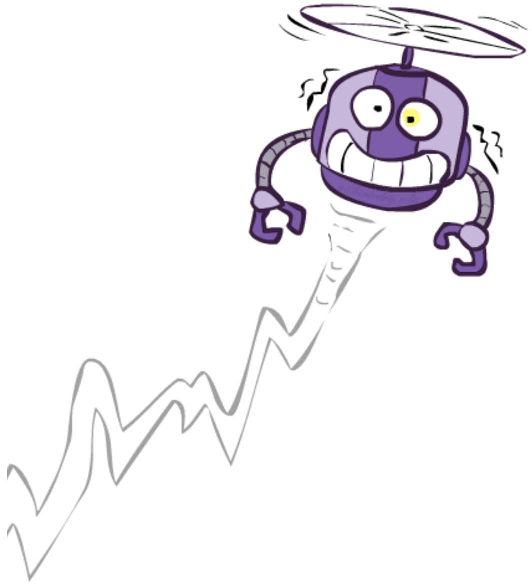
  if ΔE > 0 **then** current ← next

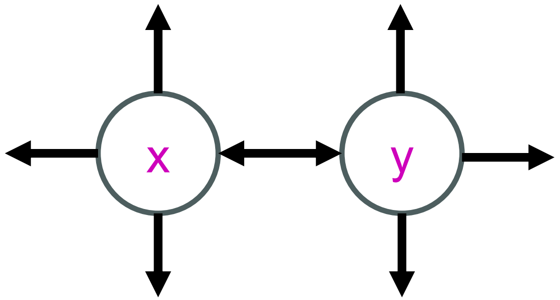
    else current ← next only with probability $e^{\Delta E/T}$



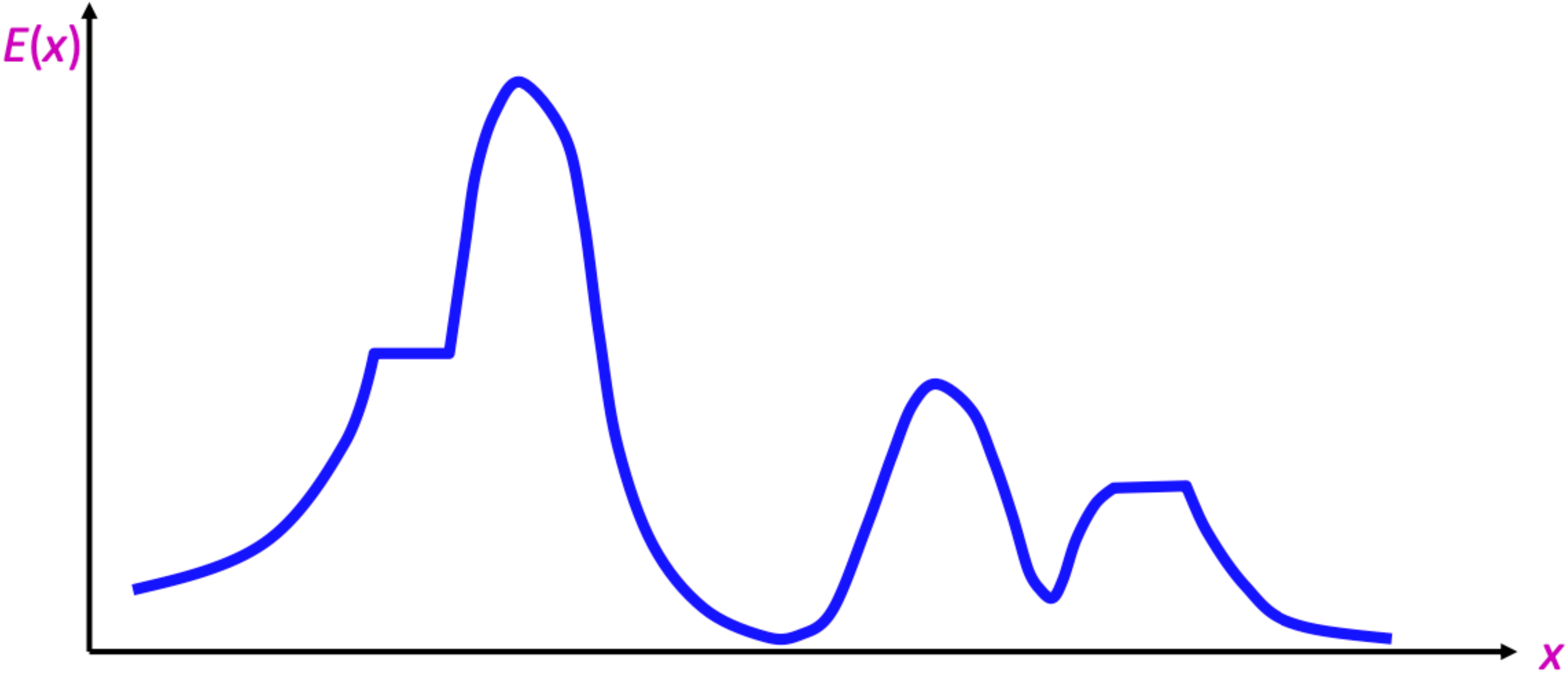
Simulated Annealing

- Theoretical guarantee:
  - Stationary distribution (Boltzmann): $P(x) \propto e^{E(x)/T}$
  - If T decreased slowly enough, will converge to optimal state!
- Proof sketch
  - Consider two adjacent states x, y with $E(y) > E(x)$ [high is good]
  - Assume $x \rightarrow y$ and $y \rightarrow x$ and outdegrees $D(x) = D(y) = D$
  - Let $P(x)$, $P(y)$ be the equilibrium occupancy probabilities at T
  - Let $P(x \rightarrow y)$ be the probability that state x transitions to state y



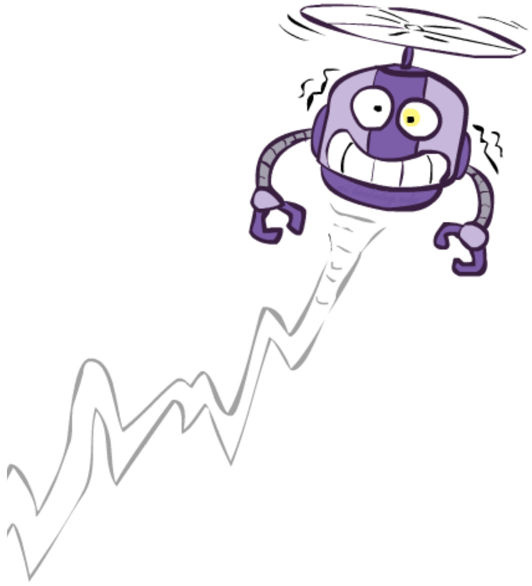


Occupation probability as a function of T



Simulated Annealing

- Is this convergence an interesting guarantee?
- Sounds like magic, but reality is reality:
  - The more downhill steps you need to escape a local optimum, the less likely you are to ever make them all in a row
  - “Slowly enough” may mean exponentially slowly
  - Random restart hillclimbing also converges to optimal state…
- Simulated annealing and its relatives are a key workhorse in VLSI layout and other optimal configuration problems

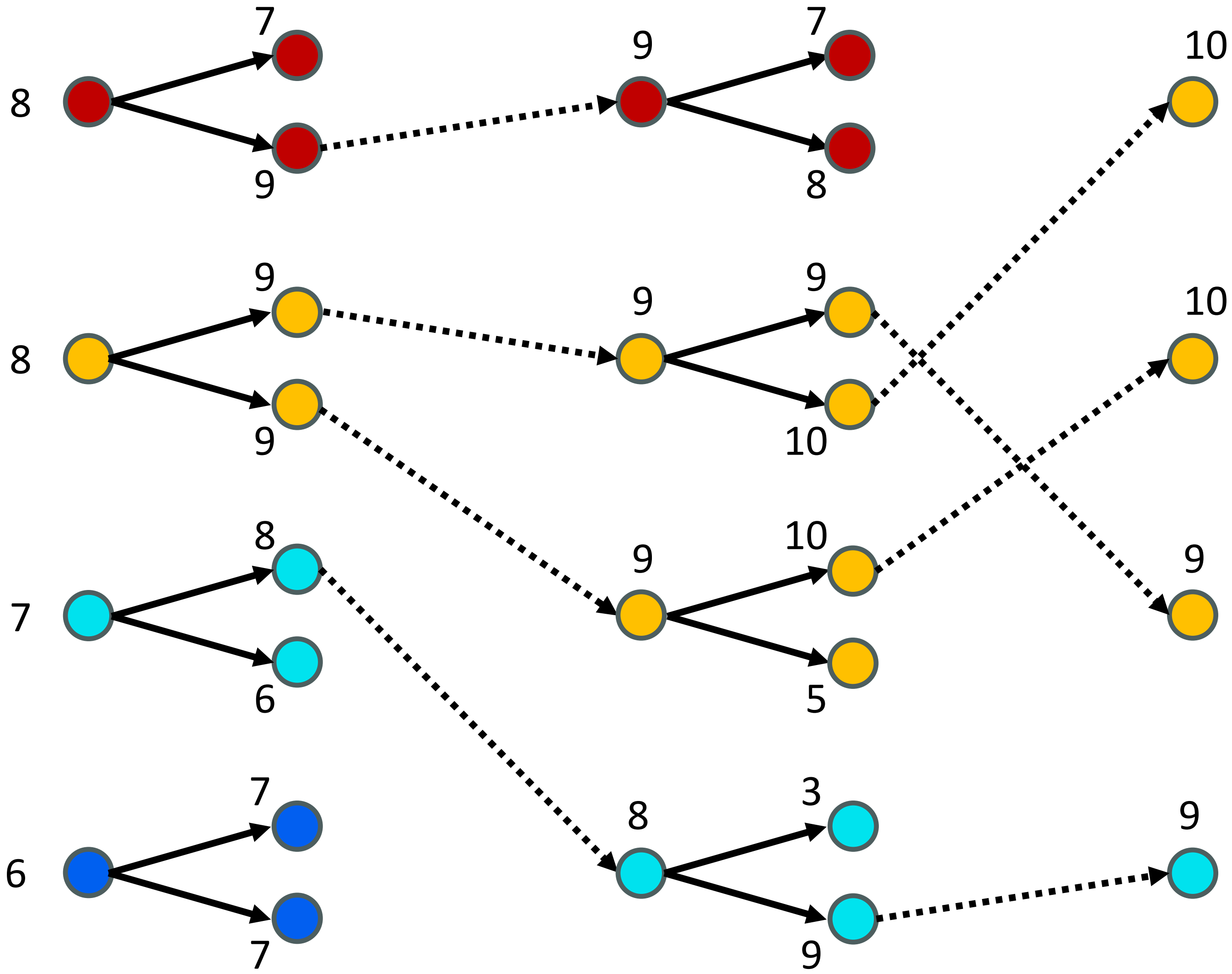


Local beam search

- Basic idea:
  - *K* copies of a local search algorithm, initialized randomly
  - For each iteration
    - Generate ALL successors from *K* current states
    - Choose best *K* of these to be the new current states

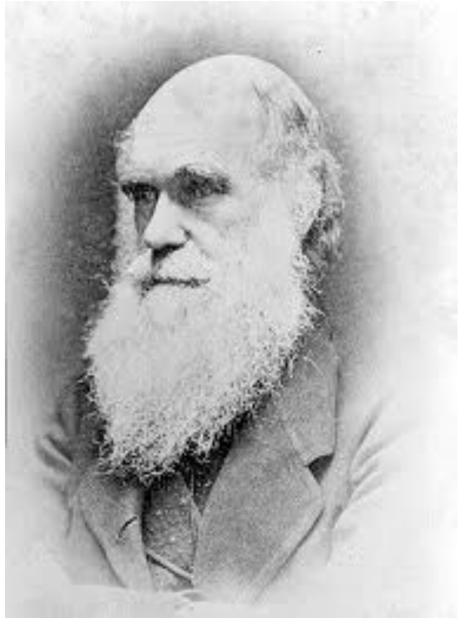
Or, K chosen randomly with
a bias towards good ones

Beam search example (*K*=4)

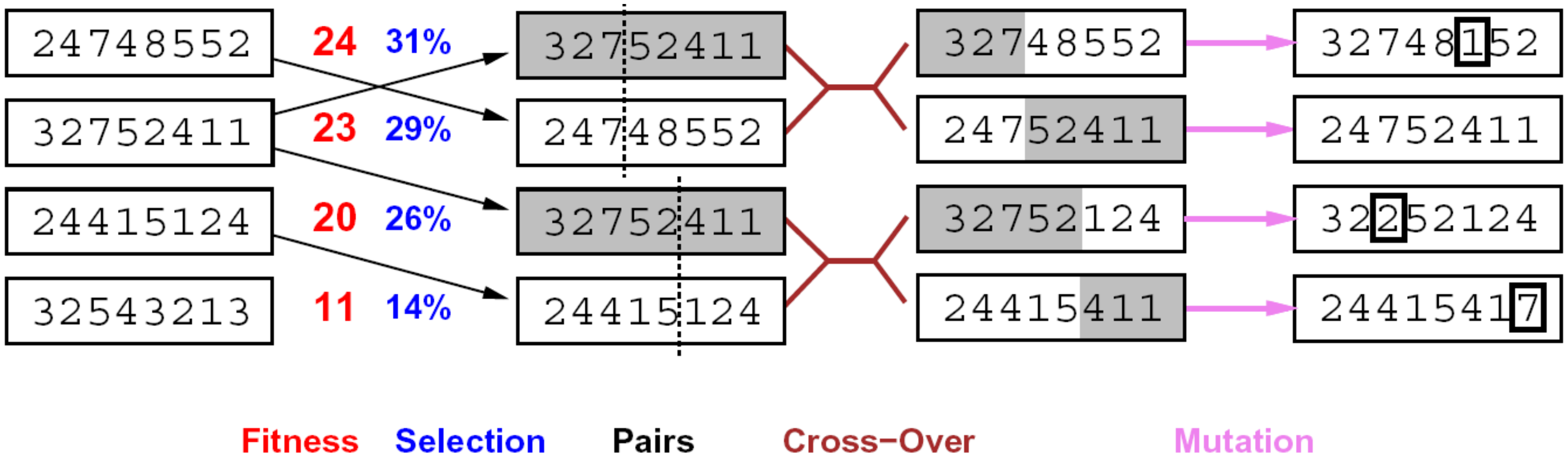


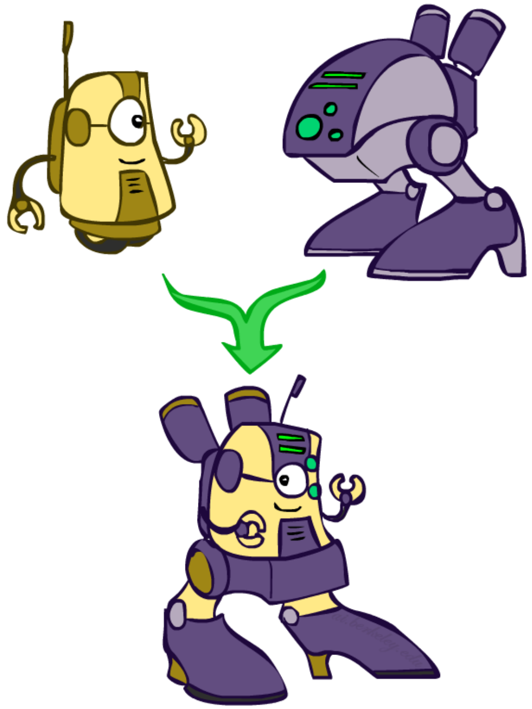
Local beam search

- Why is this different from *K* local searches in parallel?
  - The searches ***communicate***! “Come over here, the grass is greener!”
- What other well-known algorithm does this remind you of?
  - Evolution!



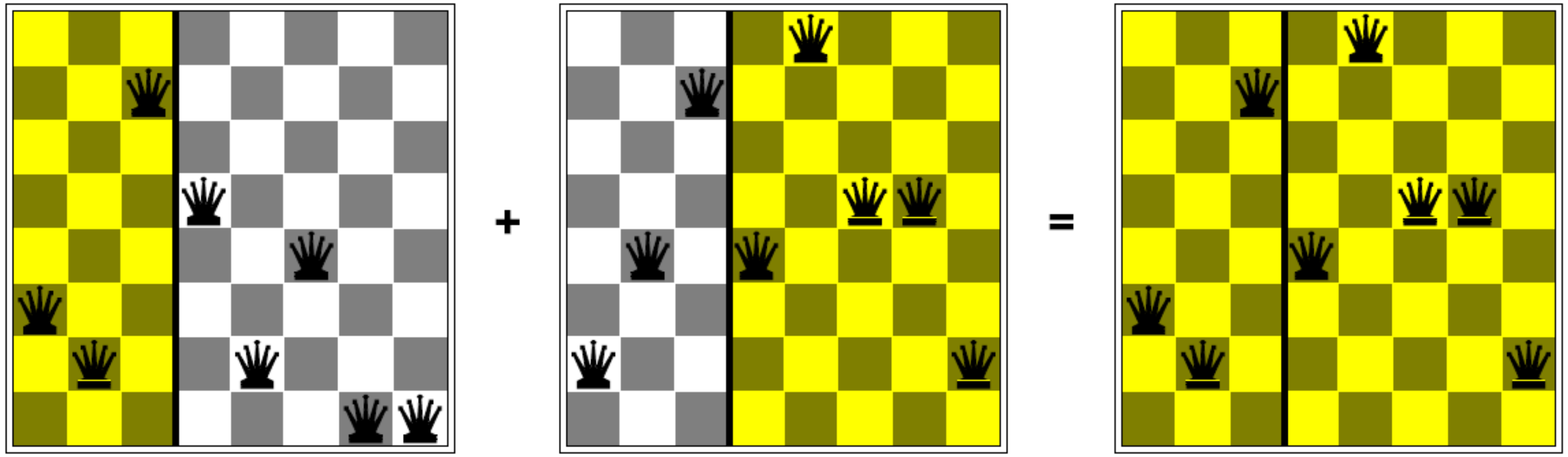
Genetic algorithms





- Genetic algorithms use a natural selection metaphor
  - Resample *K* individuals at each step (selection) weighted by fitness function
  - Combine by pairwise crossover operators, plus mutation to give variety

Example: N-Queens

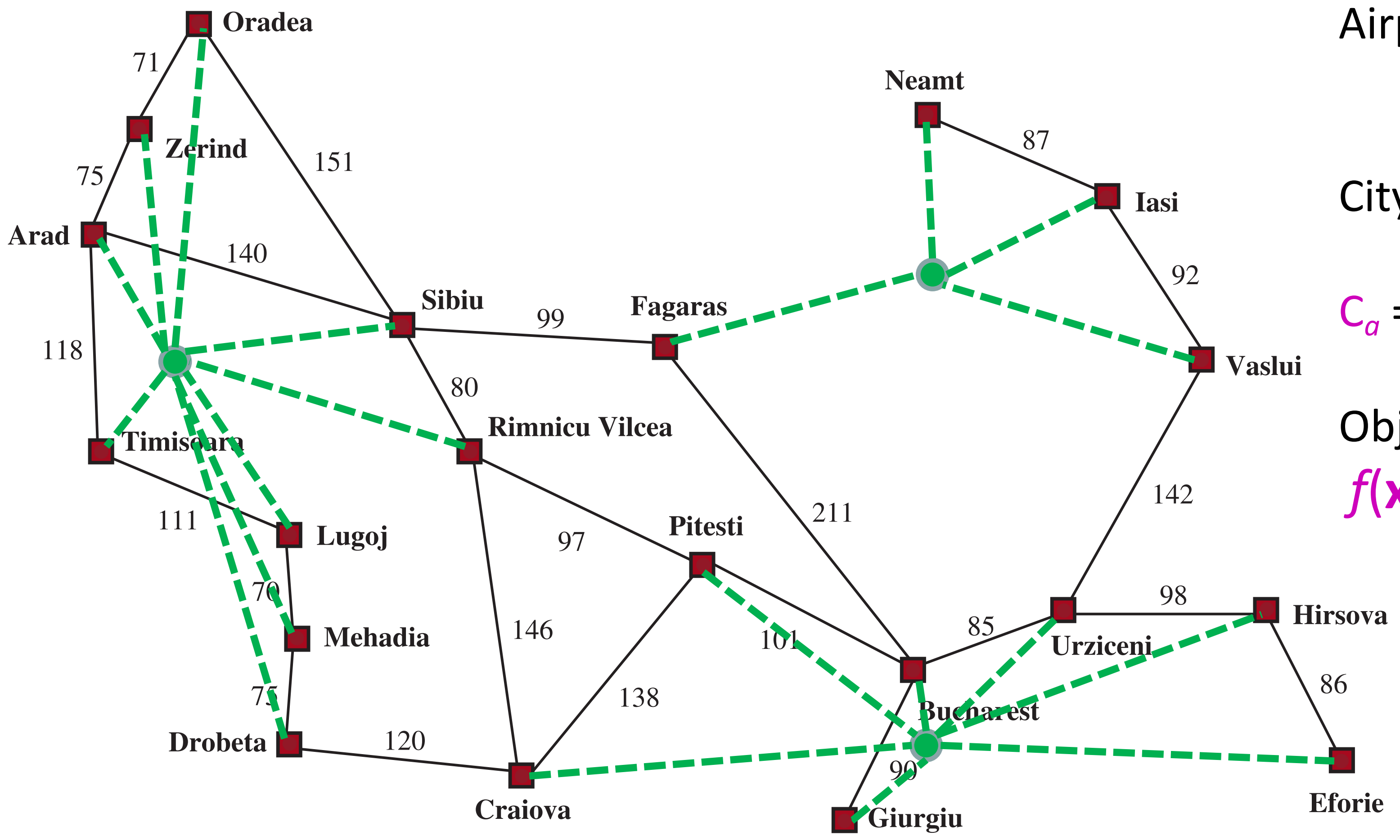


- Does crossover make sense here?
- What would mutation be?
- What would a good fitness function be?

Local search in continuous spaces

Example: Placing airports in Romania

Place 3 airports to minimize the sum of squared distances from each city to its nearest airport



Airport locations

$\mathbf{x} = (x_1,y_1), (x_2,y_2), (x_3,y_3)$

City locations (x_c,y_c)

C_a = cities closest to airport a

Objective: minimize

$$f(\mathbf{x}) = \Sigma_a \Sigma_{c \in C_a} (x_a - x_c)^2 + (y_a - y_c)^2$$

Handling a continuous state/action space

1. Discretize it!
   - Define a grid with increment δ, use any of the discrete algorithms
2. Choose random perturbations to the state
   a. First-choice hill-climbing: keep trying until something improves the state
   b. Simulated annealing
3. Compute gradient of $f(\mathbf{x})$ analytically

Finding extrema in continuous space

- Gradient vector $\nabla f(\mathbf{x}) = (\partial f/\partial x_1, \partial f/\partial y_1, \partial f/\partial x_2, \ldots)^T$
- For the airports, $f(\mathbf{x}) = \Sigma_a \Sigma_{c \in C_a} (x_a - x_c)^2 + (y_a - y_c)^2$
- $\partial f/\partial x_1 = \Sigma_{c \in C_1} 2(x_1 - x_c)$
- At an extremum, $\nabla f(\mathbf{x}) = 0$
- Can sometimes solve in closed form: $x_1 = (\Sigma_{c \in C_1} x_c)/|C_1|$
  - Is this a local or global minimum of f?
- If we can't solve $\nabla f(\mathbf{x}) = 0$ in closed form…
  - Gradient descent: $\mathbf{x} \leftarrow \mathbf{x} - \alpha \nabla f(\mathbf{x})$
- Huge range of algorithms for finding extrema using gradients

Summary

- Many configuration and optimization problems can be formulated as local search
- General families of algorithms:
  - Hill-climbing, continuous optimization
  - Simulated annealing (and other stochastic methods)
  - Local beam search: multiple interaction searches
  - Genetic algorithms: break and recombine states

Many machine learning algorithms are local searches