

# Robust Fitting in Computer Vision (and elsewhere)

Maximum Consensus - MaxCon

# Already seen in course...

The CVPR2023 prize winning paper was essentially on this topic..

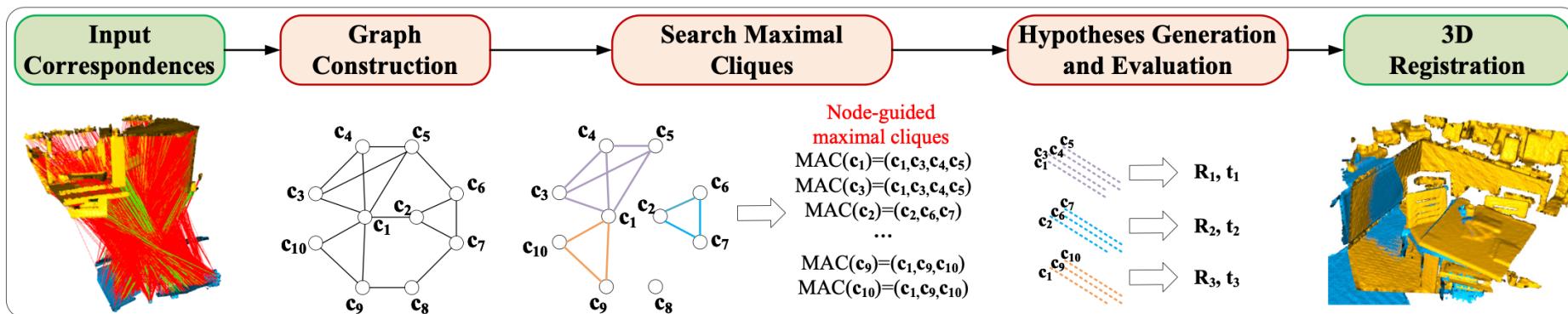


Figure 2. **Pipeline of MAC.** 1. Construct a graph for the initial correspondence set. 2. Select a set of maximal cliques from the graph as the consistent sets. 3. Generate and evaluate the hypotheses according to the consistent sets. 4. Select the best hypothesis to perform 3D registration.

# Robust Fitting - MaxCon

(Permission to use  
photo via Daniel  
Barath)

As long ago as the 1970's, early 80's, image Analysis and computer vision people realized that need robust fitting methods: Hough Transform and RANSAC were algorithms devised for this and are still used today...



\*manually aligned\* picture with scene. CVPR2023 best paper was automatic alignment of point clouds.

Computer vision/Image analysis/Robotics etc....need \*automatic methods\* for many tasks – alignment of images and pointclouds, extraction of lines/planes/cylinders from the data, 3-d reconstruction from multiple images etc. etc.

These tasks are \*fitting a model/equation\* do data \*when there are outliers\* - data points that don't belong to your model.

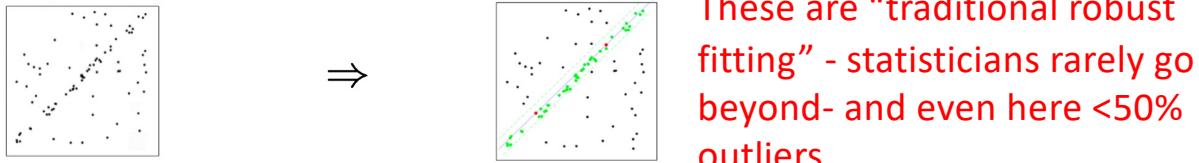
# Taxonomy of Geometric Estimation Problems

- Standard Single Class Single Instance Fitting Problem (SCSI)



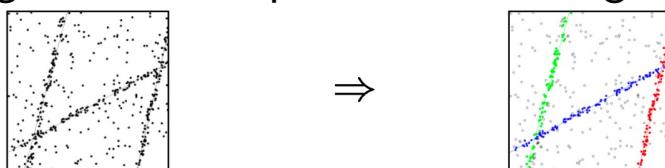
We are never going to be concerned with this – can solve by least squares

- Robust Single Class Single Instance Fitting Problem (R-SCSI)



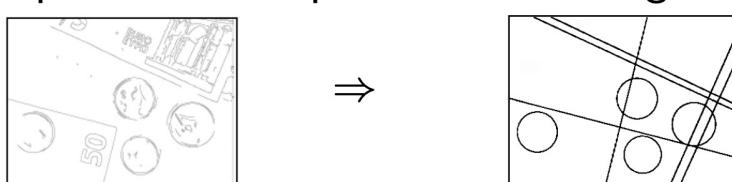
We are going to mostly talk about this problem – Least squares won’t work because of outliers.

- Single Class Multiple Instance Fitting Problem (SCMI)



These problems are also a major motivation – EVEN IF we only want to extract one of the structures.

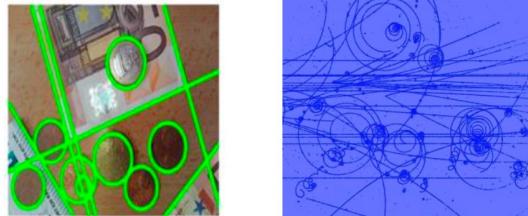
- Multiple Class Multiple Instance Fitting Problem (MCMI)



This becomes VERY hard – need model selection – a notoriously difficult topic ON TOP OF ROBUST fitting.

# Single/Multi-Class S/M-Instance Fitting Applications

- detection of geometric primitives



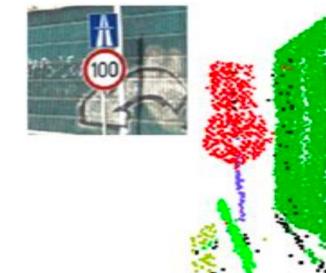
- epipolar geometry estimation
- detection of planar surfaces



- multiple motion segmentation

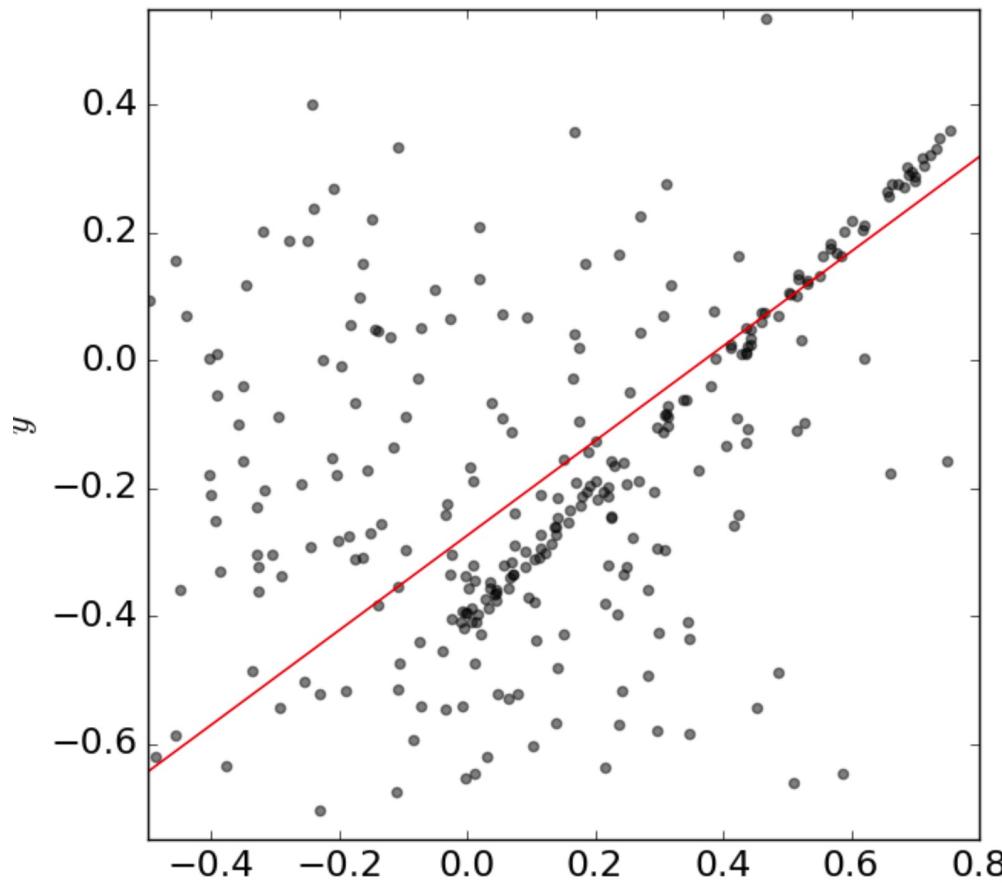


- Interpretation of lidar scans



# Line Fitting with Outliers

## Example 1



Least squares fit

# Old studied problem....without invoking

## MACHINE ANALYSIS OF BUBBLE CHAMBER PICTURES

**P. V. C. Hough**

The University of Michigan, Ann Arbor, Mich.

Proc. Int. Conf. High Energy Accelerators and  
Instrumentation, 1959(!!!!) – though Duda and  
Hart generalised and popularised for the PR  
community in early 70's....

RANDOM SAMPLE CONSENSUS: A PARADIGM FOR MODEL  
FITTING WITH APPLICATIONS TO IMAGE ANALYSIS  
AND AUTOMATED CARTOGRAPHY

Technical Note 213

March 1980

By: Martin A. Fischler, Senior Computer Scientist  
Robert C. Bolles, Computer Scientist

Artificial Intelligence Center  
SRI International  
Menlo Park, California 94025

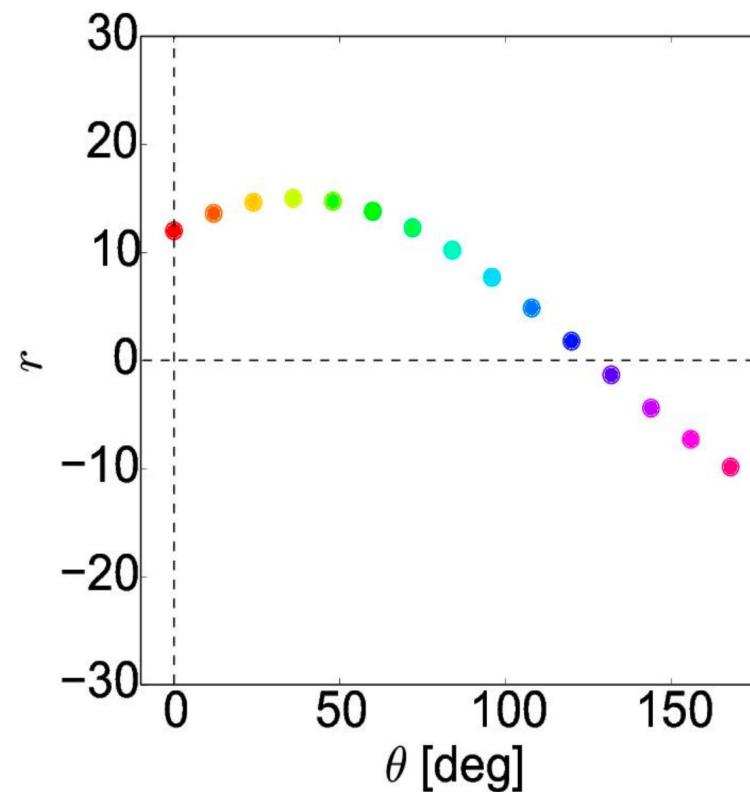
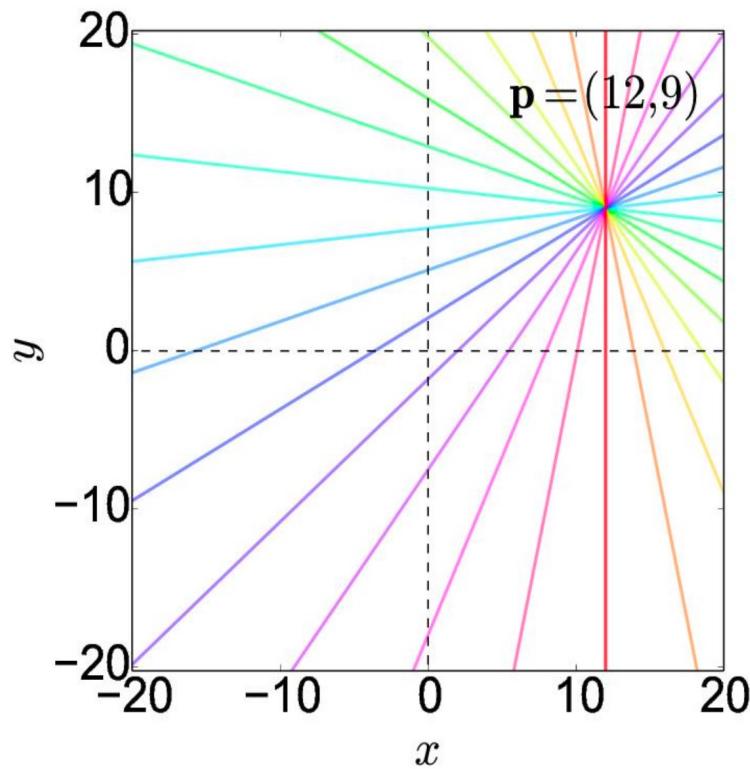
RANSAC - A contemporary of LMedS  
algorithm in Stats community

Hough works directly in “parameter space” - and is more or less inherently multi-structural, RANSAC works in “data subset space” and is more or less single structural in nature.

Technically they don’t solve the same problem – they solve related problems – due to not optimizing the same criteria of model fit...but.....

# Hough Transform Intro, Finding a Line

A point  $\mathbf{p}$  votes for all lines it can be incident with.



There are an infinite number of lines through a single point. Can parameterize these lines by theta and  $r$  ( $r$  perpendicular distance from origin to the line).

The infinite # lines trace a sinusoid curve in  $r,\theta$  space

These curves intersect in  $r,\theta$  corresponding to COMMON line between points.

Quantize  $r,\theta$  space into voting “bins” – helps deal with small noise: but more importantly makes it simple to implement

## Hough Transform, Finding a Line

A point  $p$  votes for all lines it can be incident with.

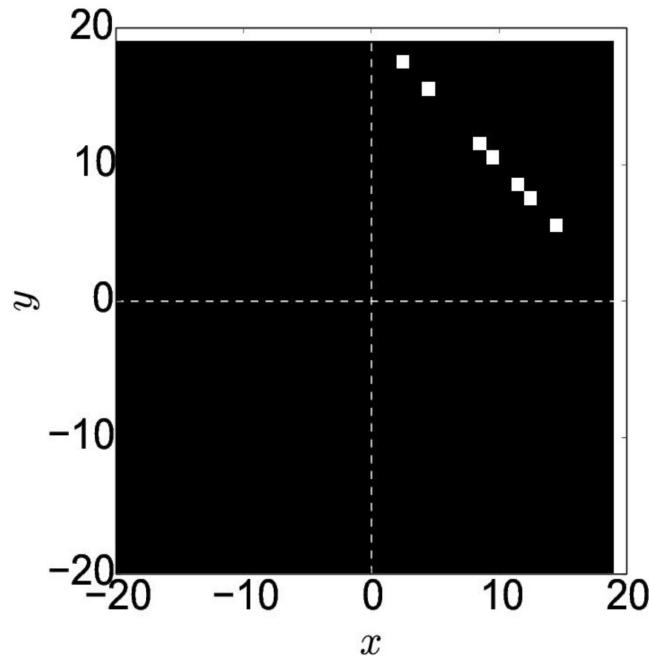
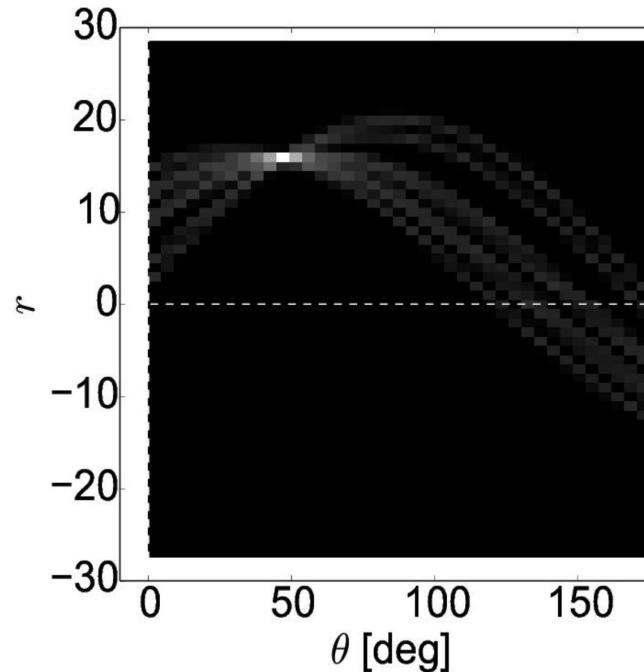


Image with a multiple points



Accumulated votes

# Ransac – most popular algorithm in computer vision for robust fitting

Set number of iterations (usually on some “back of the envelope calculations of how many iterations to get at least one clean \*minimal subset\* for the model (e.g., 2 points for a line) with some number of anticipated possible outliers...)

Loop

- randomly pick minimal subset

- “fit” model to subset

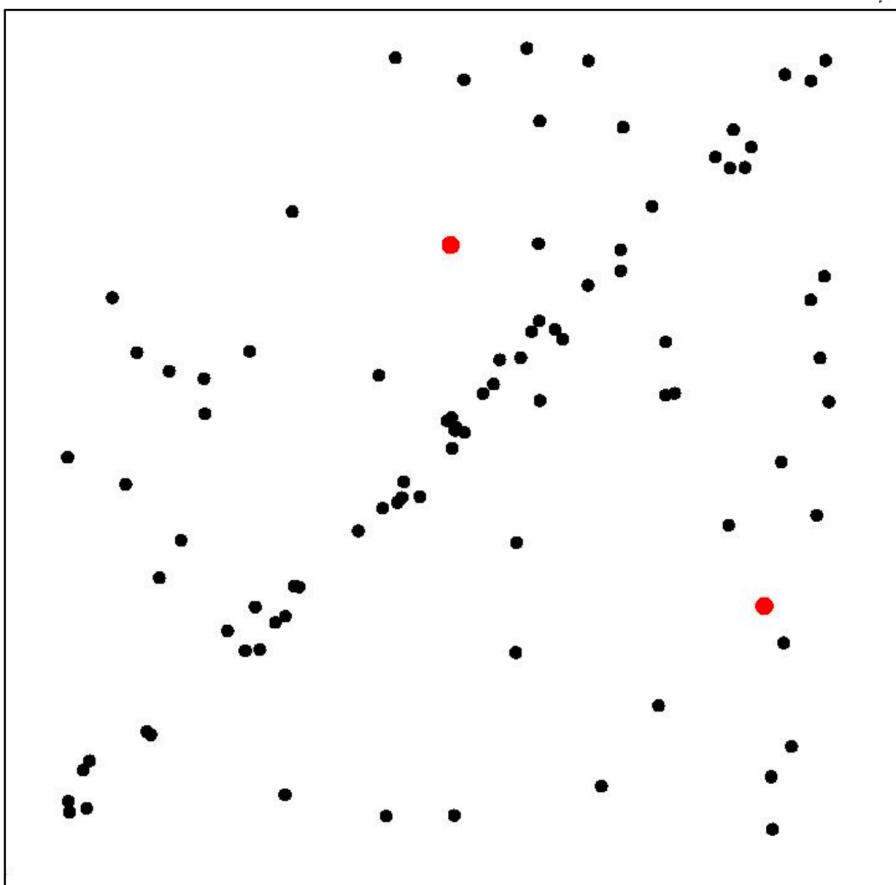
- count how many data points are within tolerance of model fit

- remember the model with the largest number of data points within tolerance

EndLoop

## Random Sample Consensus - RANSAC

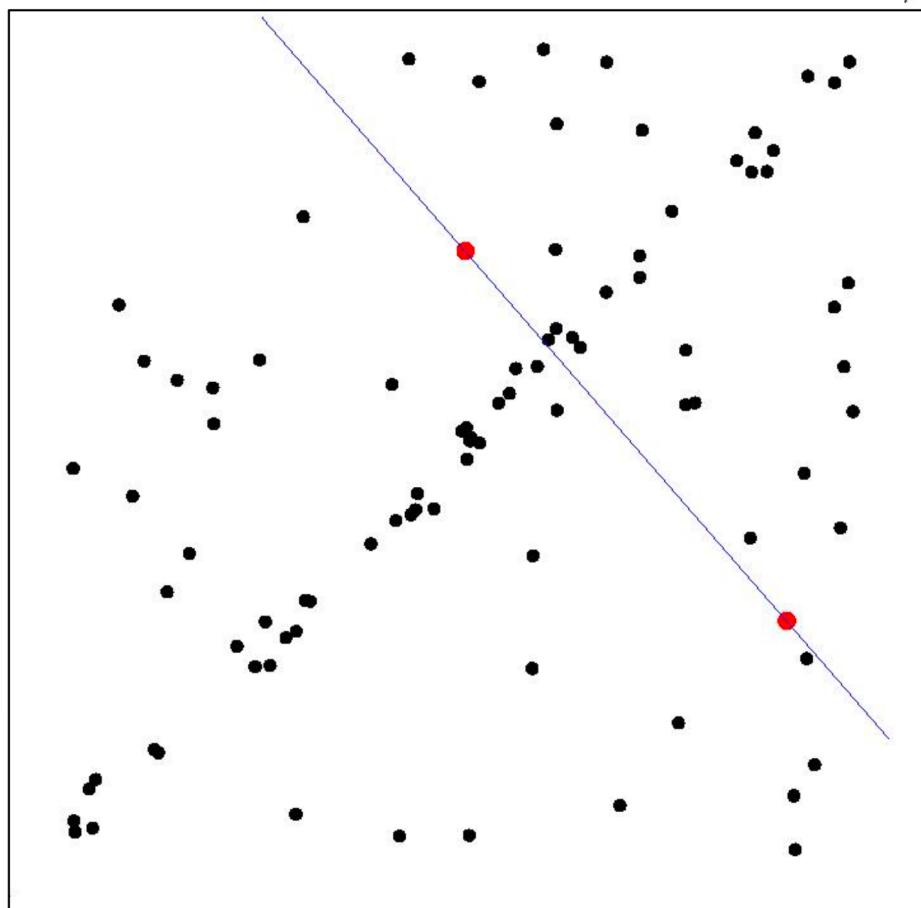
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Select sample of  $m$  points at random

# RANSAC

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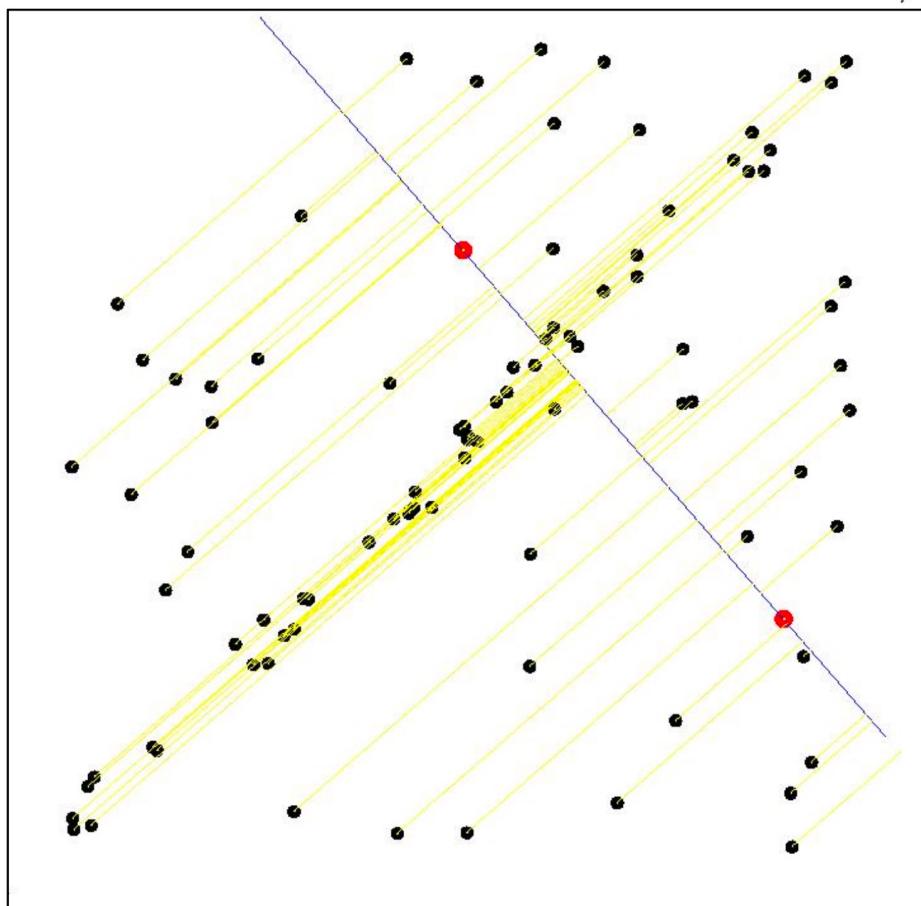


Select sample of  $m$  points  
at random

Estimate model parameters  
from the data in the sample

# RANSAC

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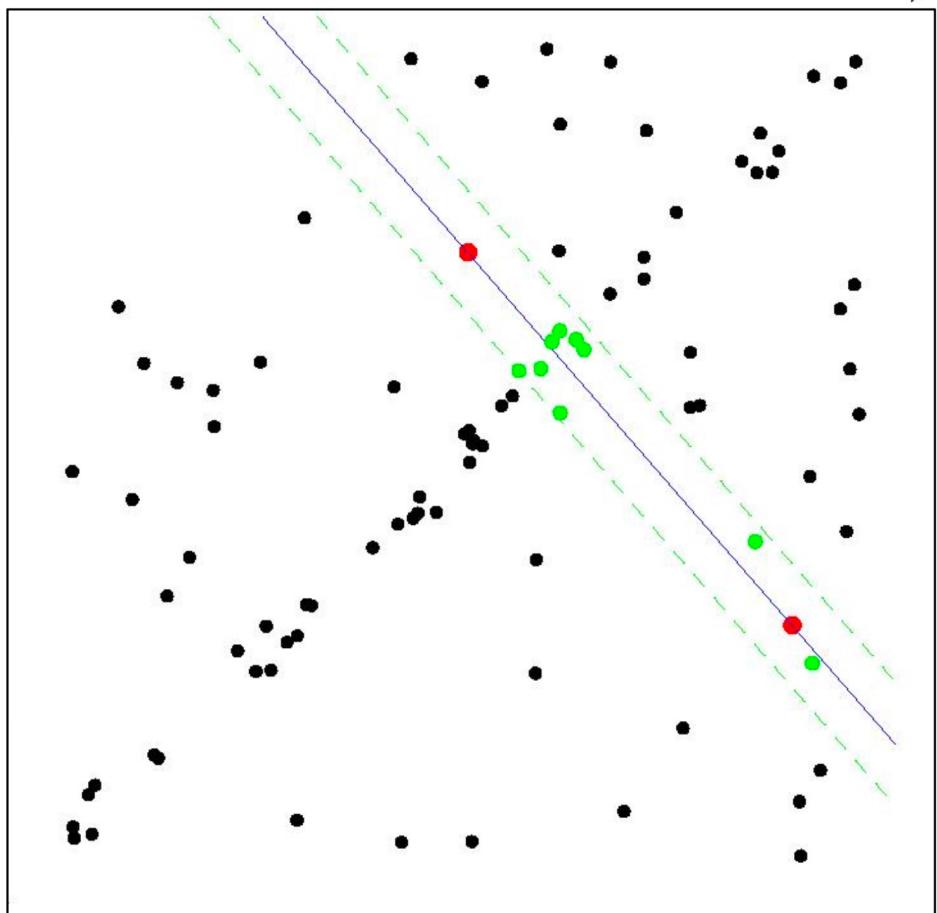


Select sample of  $m$  points  
at random

Estimate model parameters  
from the data in the sample

Evaluate the error (residual)  
for each data point

# RANSAC



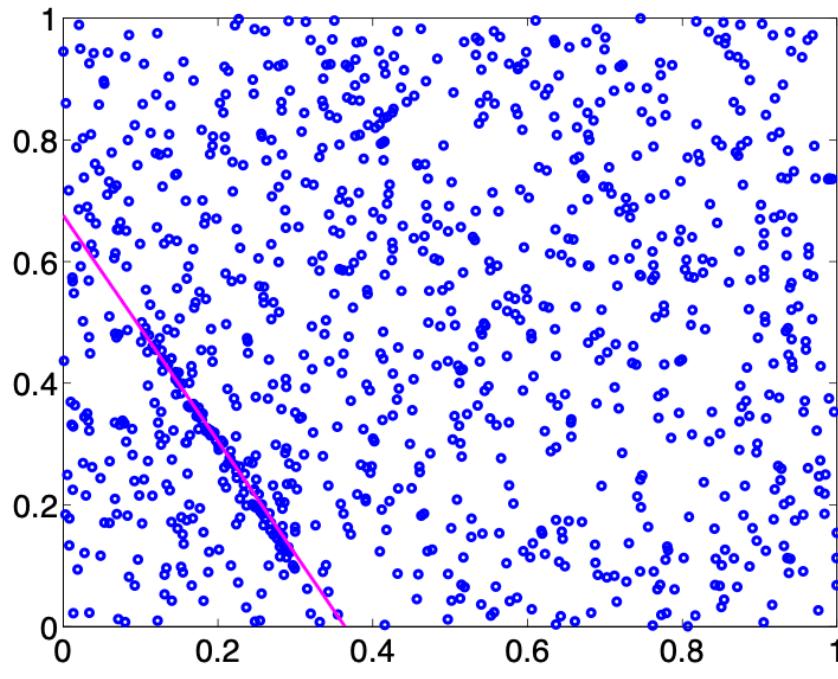
Select sample of  $m$  points  
at random

Estimate model parameters  
from the data in the sample

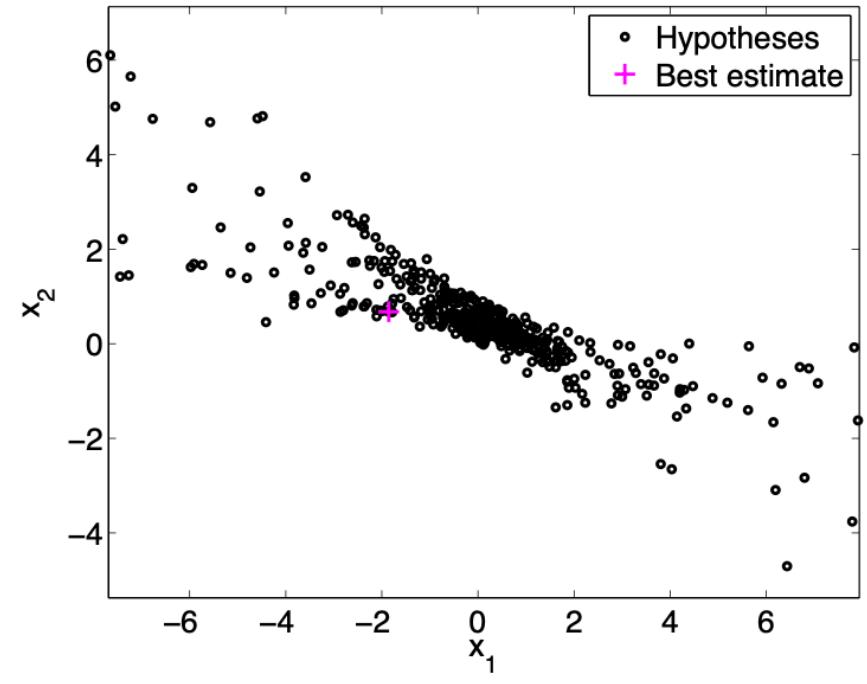
Evaluate the error (residual)  
for each data point

Select data that support  
the current hypothesis

And the COUNT of the data within tolerance is the  
score of the hypothesis – repeat – keeping always the  
currently highest “consensus” count.

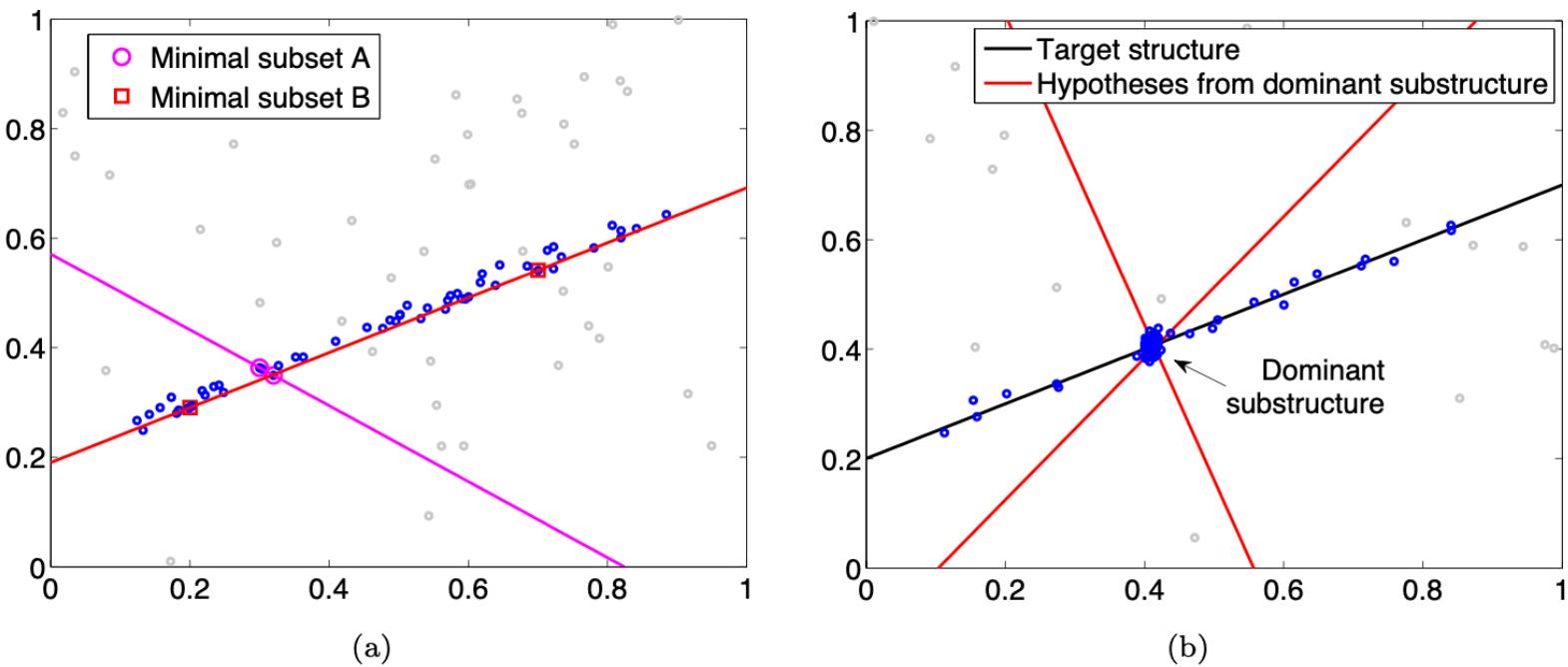


(a)



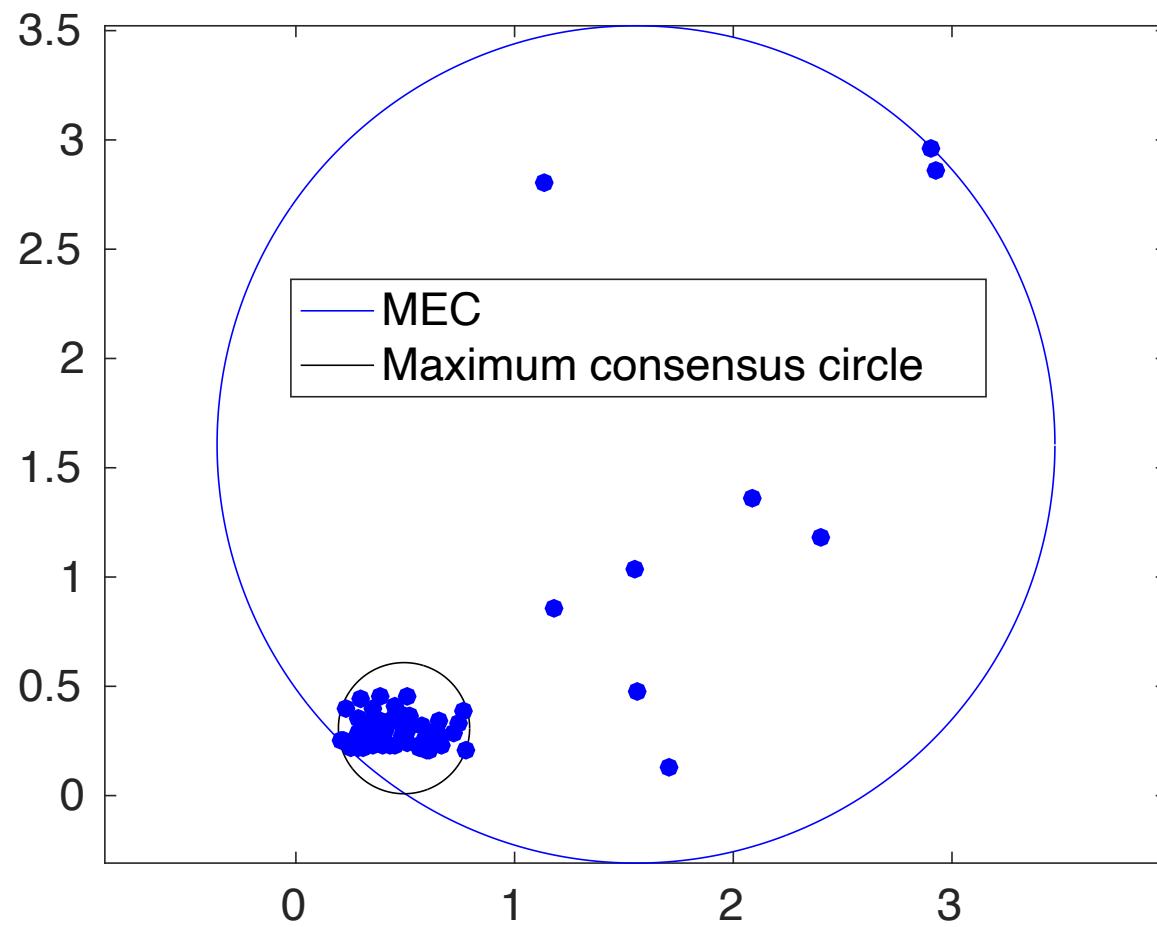
(b)

**Figure 2.2:** (a) An application of Algorithm 3 (RANSAC) on a line fitting problem with  $N = 1000$  points and 90% outliers ( $\eta = 0.1$ ). By setting confidence  $\gamma = 0.99$ , a total of 498 hypotheses were generated in this particular run before terminating in a matter of seconds. (b) A plot of a subset of the hypotheses generated in parameter space  $\mathbf{x} = [x_1, x_2]^T \in \mathbb{R}^2$ .



**Figure 2.4:** Illustrating the effects of data span and data degeneracy towards hypothesis generation from minimal subsets. (a) Not all minimal subsets that are composed purely of inliers give good estimates of the model. In particular, a minimal subset with a small span (*e.g.*, minimal subset A in the figure) can lead to a model hypothesis that is arbitrarily far from the target estimate. (b) A quasi-degenerate line fitting problem instance, where most of the inliers lie on a dominant substructure. If the minimal subsets are sampled randomly, most of the all-inlier minimal subsets will be retrieved from the dominant substructure.

Can fit any model by this criterion...



# Aligning Point Clouds



Home / News / Maptek expertise helps with Chile rescue

## Maptek expertise helps with Chile rescue

Thursday, October 14th, 2010

On 11 August, 2010, Maptek received a call from the team that was working on the rescue in San José mine, asking for help on the drilling control and 3D display to map the drillholes.

We immediately sent one of our Mine Engineers, Alvaro Quezada, to the site to help. Following this, Estíbaliz Echevarría from the Maptek I-Site team travelled to the site to conduct surveys of the region.

This data was taken into Maptek Vulcan 3D software to create an accurate topographic model and 3D representation of the complex underground workings to understand where the 33 miners were trapped

Alvaro then helped to design the direction and orientation of the drillhole which targeted the tunnel, named the Esperanza Drillhole – Hope Drillhole. Maptek Geologist, Sandra Jara, working the shift on 22 August, witnessed the first contact with the miners when she heard the men tapping on the drill as it entered the chamber where they were trapped.

She immediately telephoned Maptek South American Vice-President Marcelo Arancibia and exclaimed “Marcelo, there is life down there!”

Sandra then helped to design and control the orientation of the next two drillholes that also successfully made contact with the 33 miners down below. This final drillhole was the one used for the Plan B access shaft that was used to rescue the men.

# Sources of Outliers...

- Imperfections in sensors (multiple reflections of laser, noise in electrical wires, faulty equipment,...)
- Imperfections in preprocessing steps – a “matcher” matching parts of an image to parts of another may give wrong matches (similar image texture around the points even though entirely different points)
- Multiple structures – trying to find a line in an image when there are multiple lines...the data to one line are outliers to the other lines...  
(multiple moving objects etc etc)



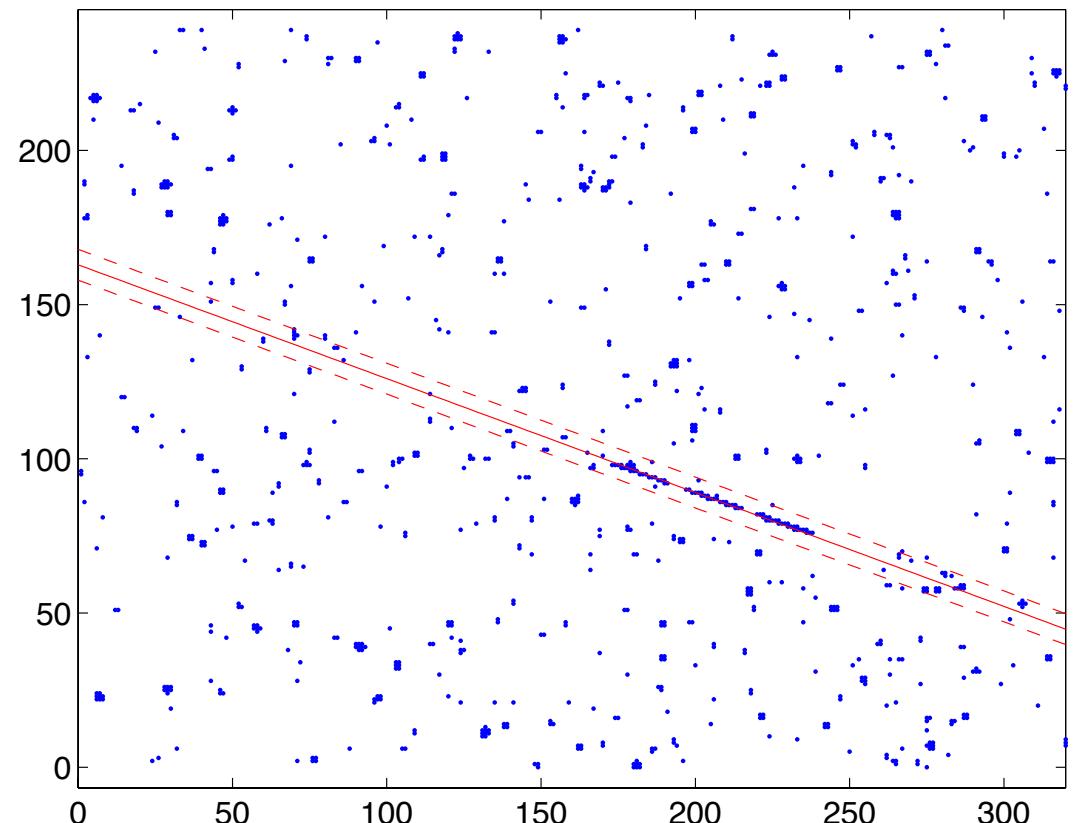
**Figure 2.3:** Applying RANSAC with an inlier threshold of  $\epsilon = 3$  pixels for robust homography fitting on the data in Figure 1.2, where there are a total of 70 feature matches. RANSAC found a consensus set with 47 inlying feature matches (plotted in green) in a matter of seconds. Observe that the data identified as outliers (plotted in red) do correspond to incorrect feature matches.

## Toy problem....

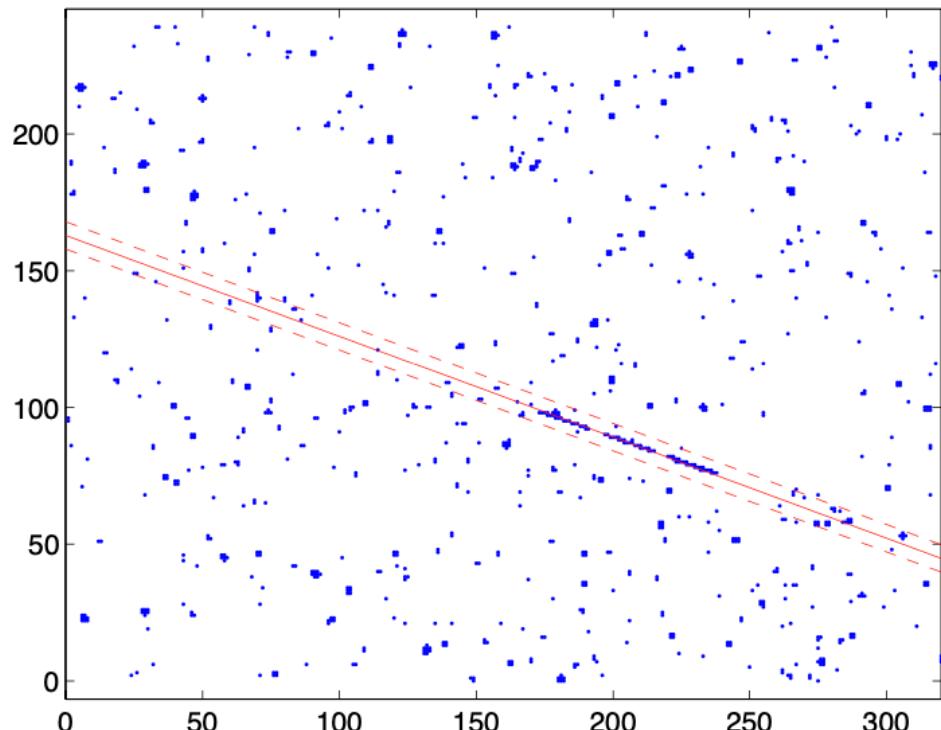
- The models/equations for most computer vision problems are a distraction to those not versed in the subject....so let's take a simple example that everyone can understand...fitting the line  $y=mx+b$  (i.e., finding  $m$  and  $b$ ) to points in 2-D

Maximum Consensus line fitting –  
the image at left is made by a  
streak from a moving spaced  
object against a stationary  
background of stars...  
Find the moving object...

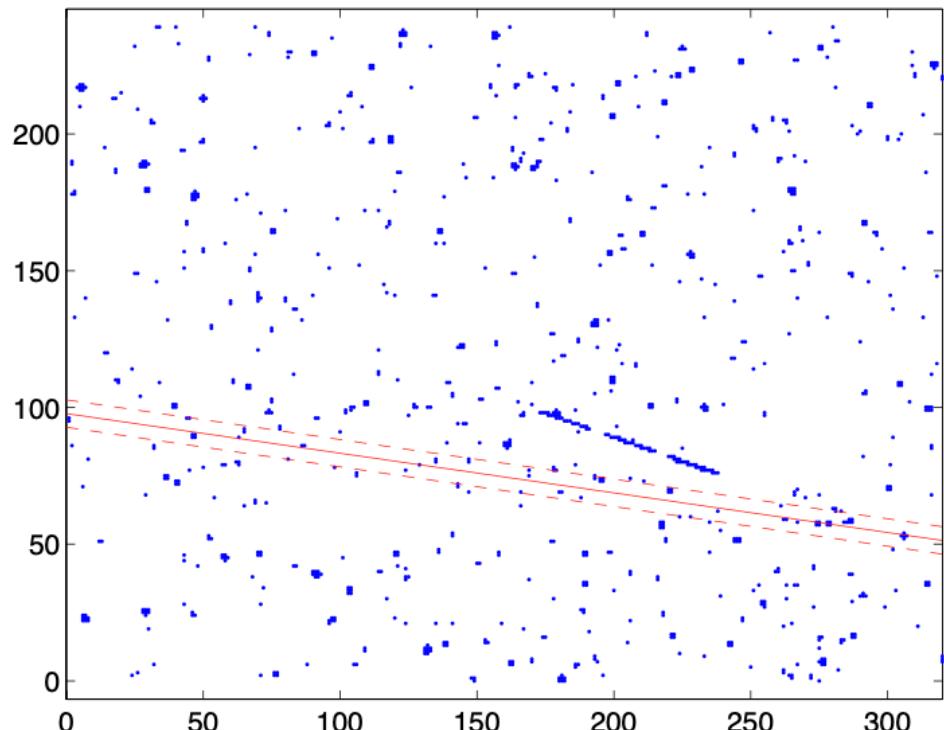
Find the line that has more points  
within some specified tolerance of  
that line....



## 1. THE MAXIMUM CONSENSUS PROBLEM



(a) Consensus = 131.

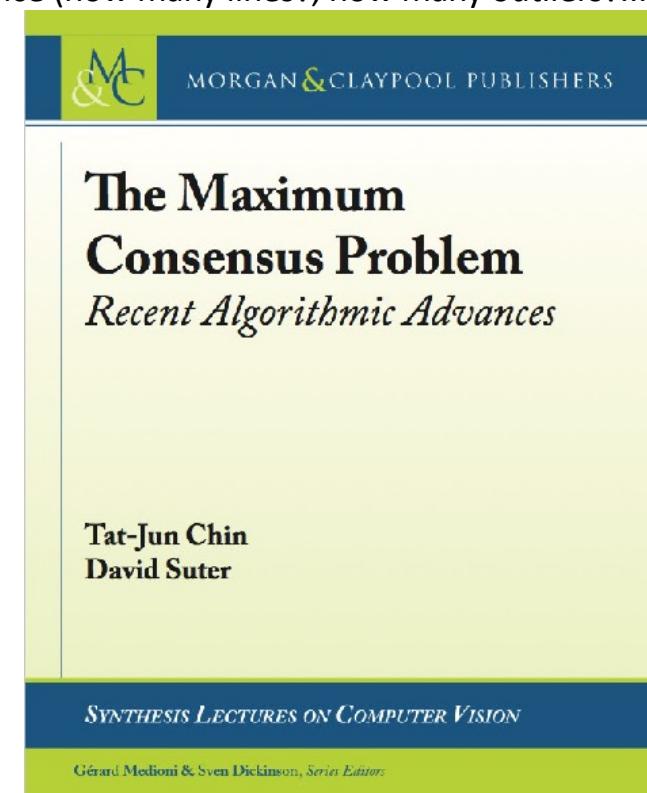
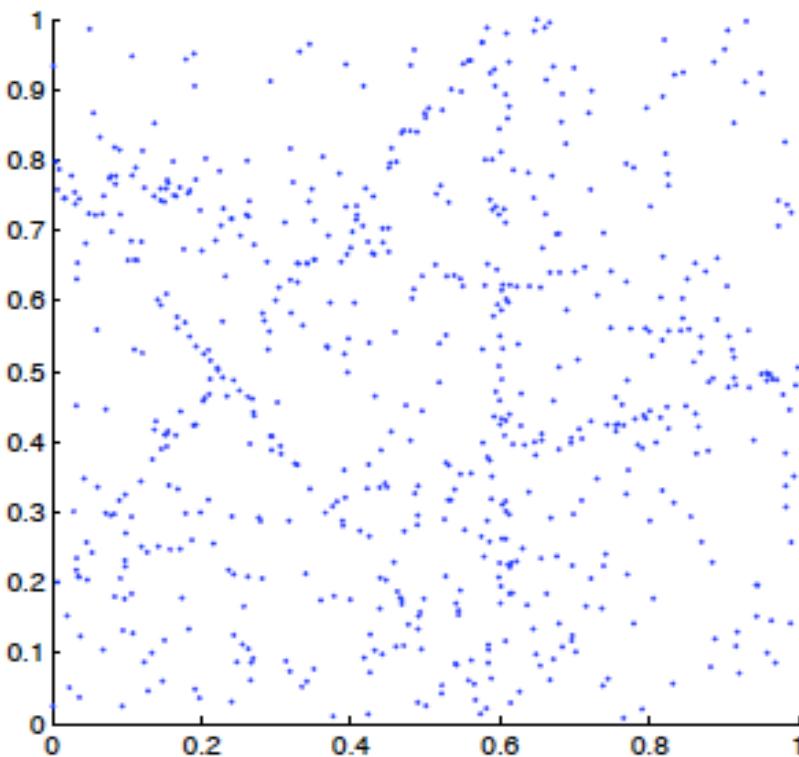


(b) Consensus = 61.

**Figure 1.3:** Lines with respectively 131 and 61 consensus with  $\epsilon = 5$  pixels. The dashed lines indicate the boundary of the inlier threshold  $\epsilon$ .

So – what do you do? Tracking, Face Recognition, Activity Detection, Human Motion Modelling, Medical Image Analysis, Film Restoration, Forensic Image Analysis, 3D Shape Extraction,...but.....

Surprisingly (even to me..😊) a lot of what I have done “boils down” to something akin to line fitting to noisy points (with outliers) – if you can do that **fully automatically** and without knowing things in advance (how many lines?, how many outliers?...) you can do many things...

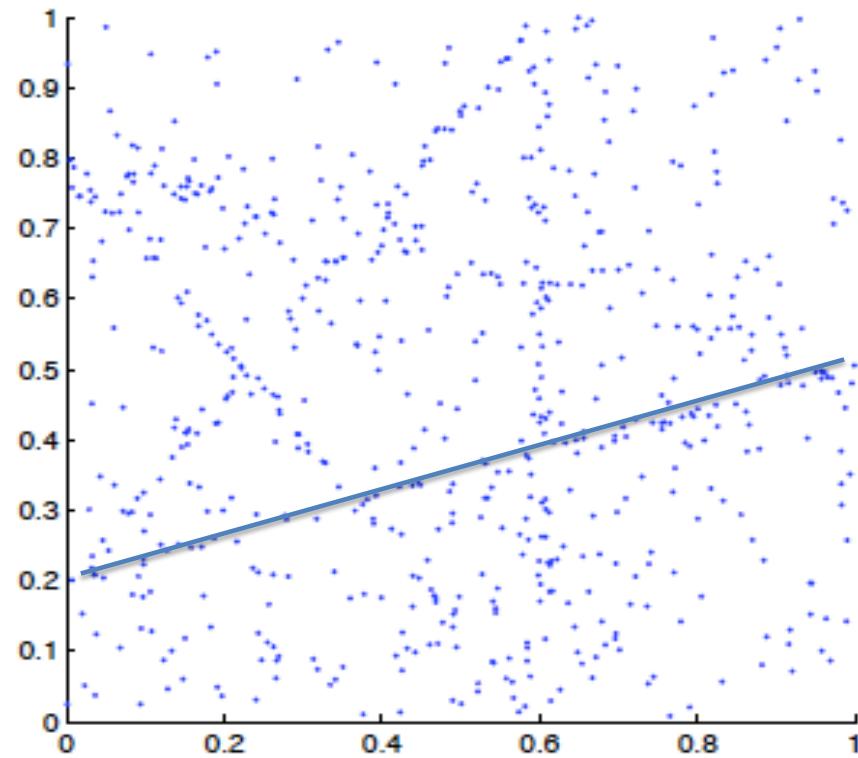
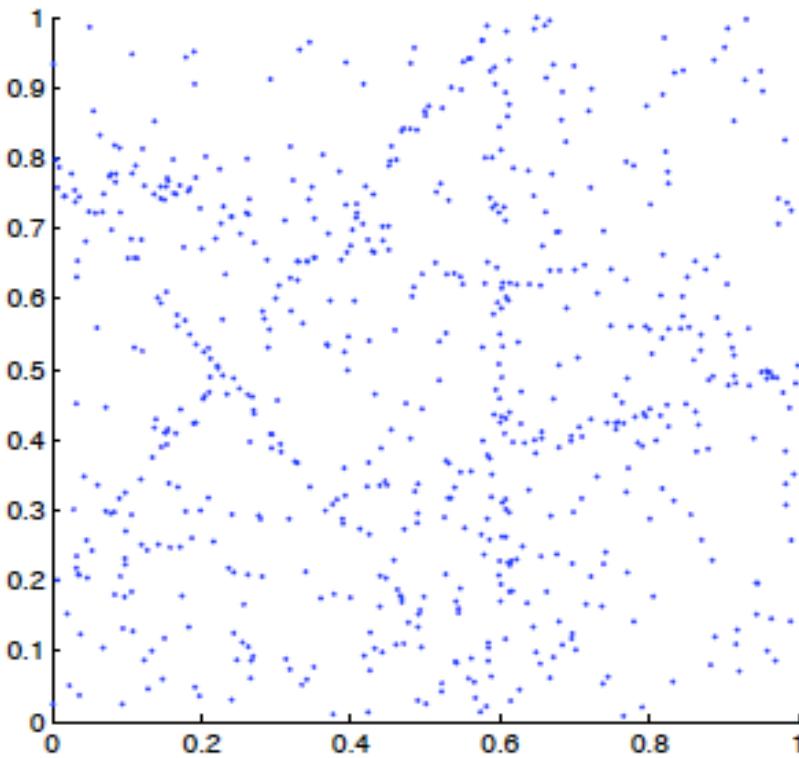


Strictly speaking, this is about single structure fitting....and with a particular measure of “fit”....

(published Feb 2017)

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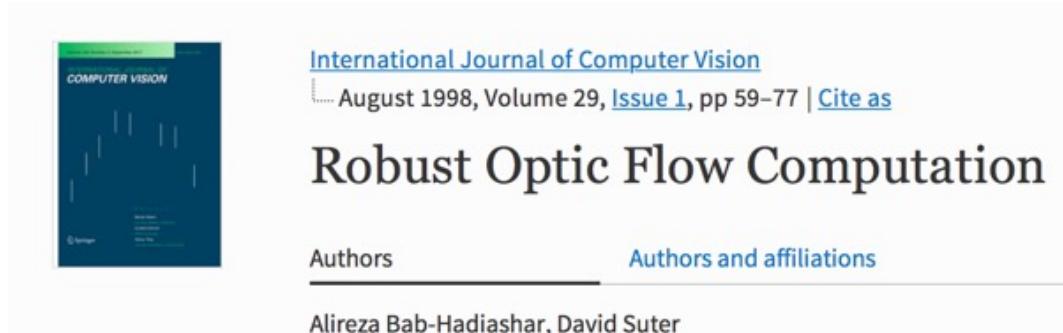
# Maximum Consensus

When solving a problem one generally formulates an objective function (measuring how well a proposed solution does), and then finds a way to optimize the objective function.

The objective function of RANSAC is “how many data points are within some specified tolerance distance away from a proposed model/solution”. This is called consensus as it is like these data points agreeing/voting for the proposed solution.

# Been studying Maxcon (well, more generally, robust fitting) for a while.....

It all “started” a long time ago...



The screenshot shows the journal's cover and the article details:

- Journal:** International Journal of Computer Vision
- Issue:** August 1998, Volume 29, Issue 1, pp 59–77 | [Cite as](#)
- Title:** Robust Optic Flow Computation
- Authors:** Alireza Bab-Hadiashar, David Suter
- Affiliations:** Authors and affiliations

Although that worked with LMedianSquares which has similar complexity issues....

Along the journey CVPR2015 best paper honorable mention award for A\* optimal MaxCon

“The CVPR2015 paper”



And BMVC 2018 Best Science Paper award

“The BMVC paper”



# WHY MAXCON IS SO NARROW EVEN FOR SINGLE STRUCTURE AND EVEN IN LOW DIM?

$$\max_{\theta, \mathcal{I} \subseteq \mathcal{X}} \quad |\mathcal{I}|$$

subject to  $r_i(\theta) \leq \epsilon \quad \forall \mathbf{x}_i \in \mathcal{I},$

Usually  $\theta$  is continuous – “so looks like” an optimization/search over a continuous space (albeit with “nasty” local optima structure of objective)

But actually, can also be seen/shown to be a DISCRETE optimization (albeit over a combinatorially huge discrete space).

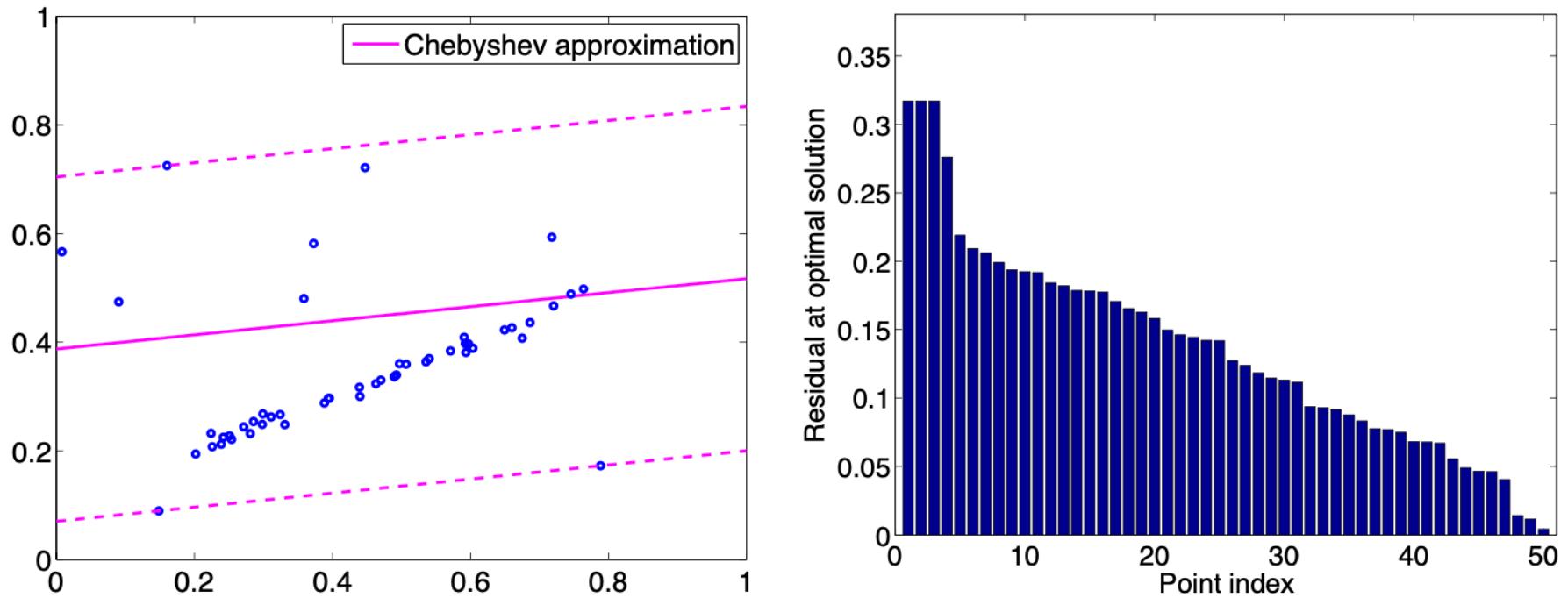
“Two ways” to “see” this – Theory of LP-type problems (see “our” 2017 book/CVPR 2015 paper....)

- “Chicken and egg” type problem: if know  $\theta$  then finding and eliminating outliers is trivial, if know outliers/inliers then finding  $\theta$  is trivial.

One key concept – for every model, there is a number  $p$  such that  $p$  data points can always be fit by that model. For line fitting in 2D –  $p=2$ . For ANY two points there is always a line that fits them perfectly. Hence, MaxCon solution  $\geq 2$ .

Moreover, for  $p+1$  points there may be NO model that fits all  $p+1$  points within the tolerance. So  $p+1$  is the size of the smallest possible infeasible (can’t be fit by the model within tolerance) subset.

# L-infinity fitting a.k.a Chebyshev



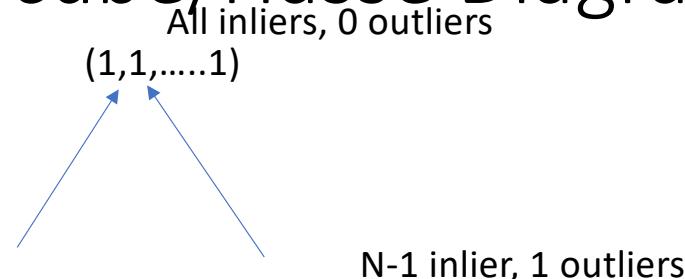
**Figure 2.11:** Chebyshev approximation (2.46) on line fitting with outliers. In conjunction to the Chebyshev estimate, the dashed lines indicate the optimised minimax residual value. The bar chart on the right shows the sorted residuals at the optimal solution.

# If we had a test for in feasibility....

- If given a set of points, we could test whether those point can be fit by the model (feasible) then we could search over all subsets (Boolean cube!) – from (111.....1) down to subsets of size  $p+1$ , and take the first one we find (highest in the cube – largest subset)!
- There is such a test! L-infinity fitting. Given a set of points find the fit with \*maximum\* possible largest residual. If this is less than our tolerance, then the set is feasible, else – infeasible.
- Even though L-infinity fitting is reasonably fast, the search space is typically huge and so need to be more clever...
- The CVPR2015 best paper honourable mention work used A\* search which is guaranteed to give the true answer, often runs quite fast, but could still be exponential in runtime.,..

# Discrete Search - Search over subsets of the data – Search over hypercube/Hasse Diagram

Find the highest feasible set in the  
Hasse Diagram

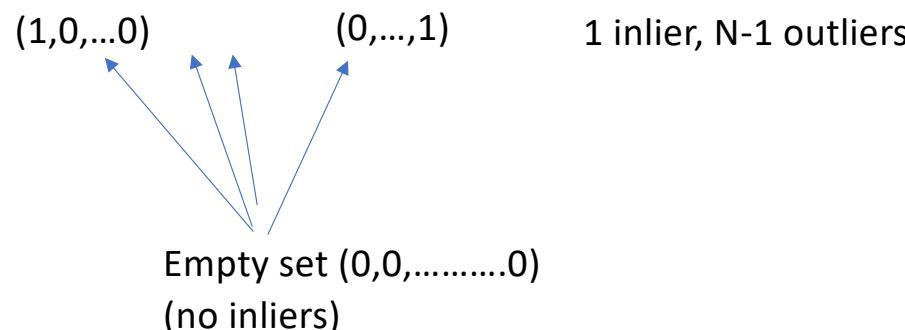


Lowest level that one needs to  
search down to....

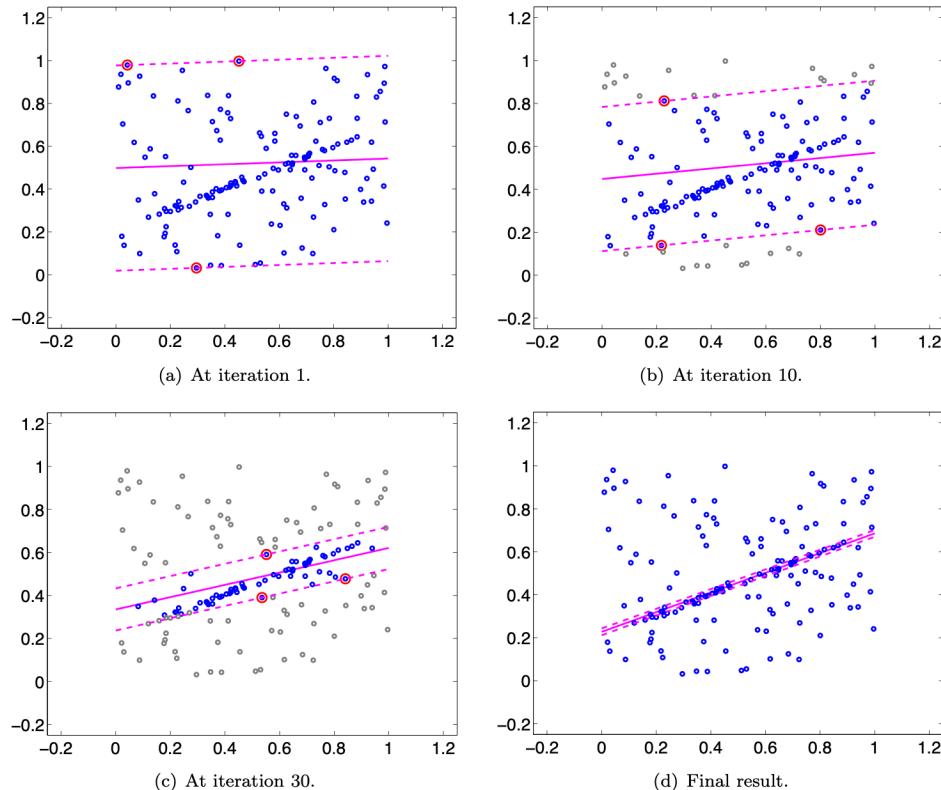


$p+1$  inlier

$p$  inlier,  $N-p$  outliers,  $p$  is minimal set to determine  $\theta$



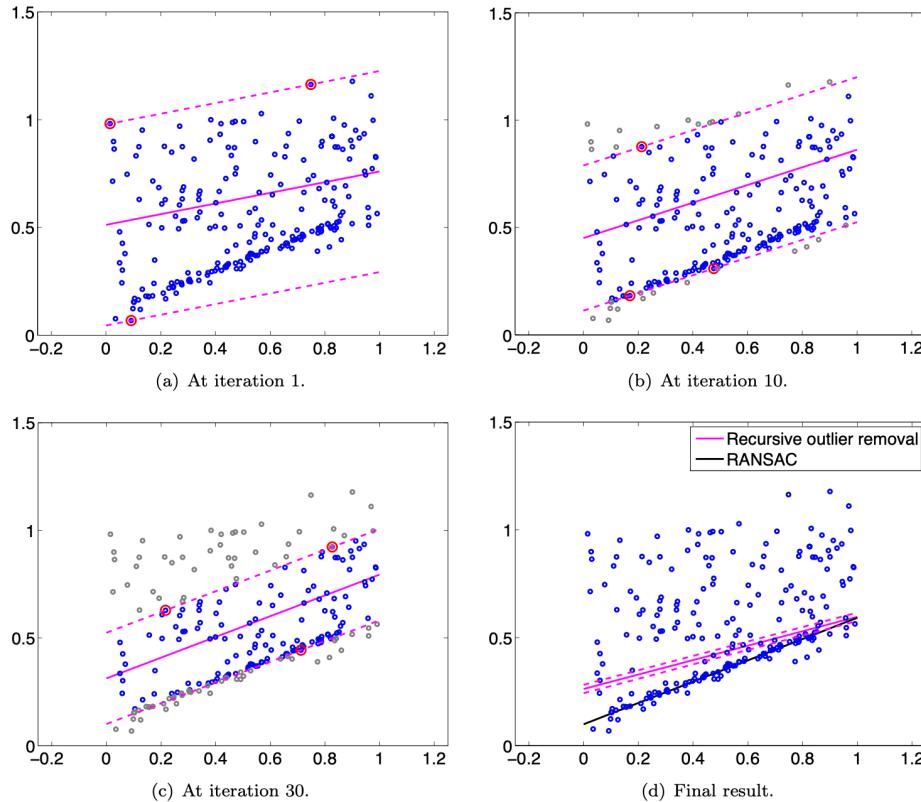
When a subset is infeasible it is clear \*at least one of the basis points must go...but which?



If you are lucky –  
removing all basis  
points might work...

**Figure 2.13:** Demonstration of outlier removal with  $\ell_\infty$  minimisation (Algorithm 4) on a line fitting problem. Since  $d = 2$  for line fitting, at most three points are removed at each iteration.

# Removing all basis points can fail!



**Figure 2.14:** Applying the  $\ell_\infty$  minimisation technique for outlier removal (Algorithm 4) on a line fitting problem with unbalanced data, *i.e.*, the structure of interest does not lie centrally in the spatial extent of the data. In such a data, the  $\ell_\infty$  technique tends to remove a significant amount of inliers before finding a consensus set. Thus, the maximum consensus solution found is far from ideal. Contrast this to the RANSAC solution, which is much more acceptable.

## Other searches...

- We tried bisection searches...(ECCV2018 paper)....each level of the cube is still usually large so still need clever additions to the idea...

# New Ideas – CVPR2021 and CVPR2022 papers

Summary...

- Feasibility is a monotone Boolean function over the cube – if a set of points is feasible than delete a point must leave the set feasible. Conversely, if a set of points is infeasible then adding a point cannot make it feasible.
- In fact, the set of subsets of feasible points is a simplicial complex – downward closed set of subsets. The maxcon solution is the maximal face of the this simplicial complex.
- Moreover, the simplicial complex above is the independent set complex of the hypergraph of minimal sized infeasible ( $p+1$  uniform hypergraph) subsets.
- Moreover ++ the influence of the monotone Boolean function (feasibility) can be related to the likelihood that a data point is an inlier....influences of inliers are generally lower then influences of outliers
- However...calculating Influences exactly is exponential in cost – need to be more clever and estimate influences efficiently...

Credits go to...a whole lost of colleagues...

Prof. Tat-Jun Chin (Adelaide U)

Prof. Hanzi Wang (Xiamen University)

Prof. Ali Bab-Hadiashar (RMIT U)

and, for various works around the topics mentioned

To around 20 former postgrad students, 5 former postdocs..