Robust Fitting in Computer Vision (and elsewhere)

Maximum Consensus - MaxCon

Already seen in course…

The CVPR2023 prize winning paper was essentially on this topic..
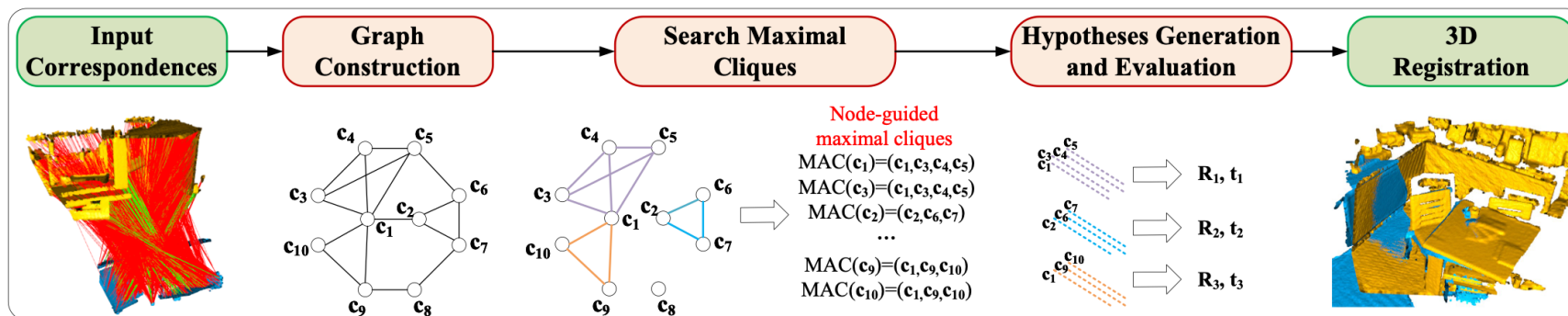

Figure 2. **Pipeline of MAC**. **1.** Construct a graph for the initial correspondence set. **2.** Select a set of maximal cliques from the graph as the consistent sets. **3.** Generate and evaluate the hypotheses according to the consistent sets. **4.** Select the best hypothesis to perform 3D registration.

Robust Fitting - MaxCon

(Permission to use photo via Daniel Barath)

As long ago as the 1970's,early 80's , image Analysis and computer vision people realized that need robust fitting methods: Hough Transform and RANSAC were algorithms devised for this and are still used today…

manually aligned picture with scene. CVPR2023 best paper was automatic alignment of point clouds.

Computer vision/Image analysis/Robotics etc….need *automatic methods* for many tasks – alignment of images and pointclouds, extraction of lines/planes/cylinders from the data, 3-d reconstruction from multiple images etc. etc.

These tasks are *fitting a model/equation* do data *when there are outliers* - data points that don't belong to your model.

Old studied problem…..without invoking

MACHINE ANALYSIS OF BUBBLE CHAMBER PICTURES

P. V. C. Hough

The University of Michigan, Ann Arbor, Mich.

Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959(!!!!) – though Duda and Hart generalised and popularised for the PR community in early 70's….

RANDOM SAMPLE CONSENSUS: A PARADIGM FOR MODEL FITTING WITH APPLICATIONS TO IMAGE ANALYSIS AND AUTOMATED CARTOGRAPHY

Technical Note 213

March 1980

By: Martin A. Fischler, Senior Computer Scientist
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    Menlo Park, California 94025

RANSAC - A contemporary of LMedS algorithm in Stats community

Hough works directly in "parameter space" - and is more or less inherently multi-structural, RANSAC works in "data subset space" and is more or less single structural in nature.
Technically they don't solve the same problem – they solve related problems – due to not optimizing the same criteria of model fit…but………..

Aligning Point Clouds

Maptek expertise helps with Chile rescue

Thursday, October 14th, 2010

On 11 August, 2010, Maptek received a call from the team that was working on the rescue in San José mine, asking for help on the drilling control and 3D display to map the drillholes.

We immediately sent one of our Mine Engineers, Alvaro Quezada, to the site to help. Following this, Estíbaliz Echevarría from the Maptek I-Site team travelled to the site to conduct surveys of the region.

This data was taken into Maptek Vulcan 3D software to create an accurate topographic model and 3D representation of the complex underground workings to understand where the 33 miners were trapped

Alvaro then helped to design the direction and orientation of the drillhole which targeted the tunnel, named the Esperanza Drillhole – Hope Drillhole. Maptek Geologist, Sandra Jara, working the shift on 22 August, witnessed the first contact with the miners when she heard the men tapping on the drill as it entered the chamber where they were trapped.

She immediately telephoned Maptek South American Vice-President Marcelo Arancibia and exclaimed "Marcelo, there is life down there!"

Sandra then helped to design and control the orientation of the next two drillholes that also successfully made contact with the 33 miners down below. This final drillhole was the one used for the Plan B access shaft that was used to rescue the men.

Sources of Outliers…

- Imperfections in sensors (multiple reflections of laser, noise in electrical wires, faulty equipment,….)

- Imperfections in preprocessing steps – a "matcher" matching parts of an image to parts of another may give wrong matches (similar image texture around the points even though entirely different points)

- Multiple structures – trying to find a line in an image when there are multiple lines…the data to one line are outliers to the other lines…
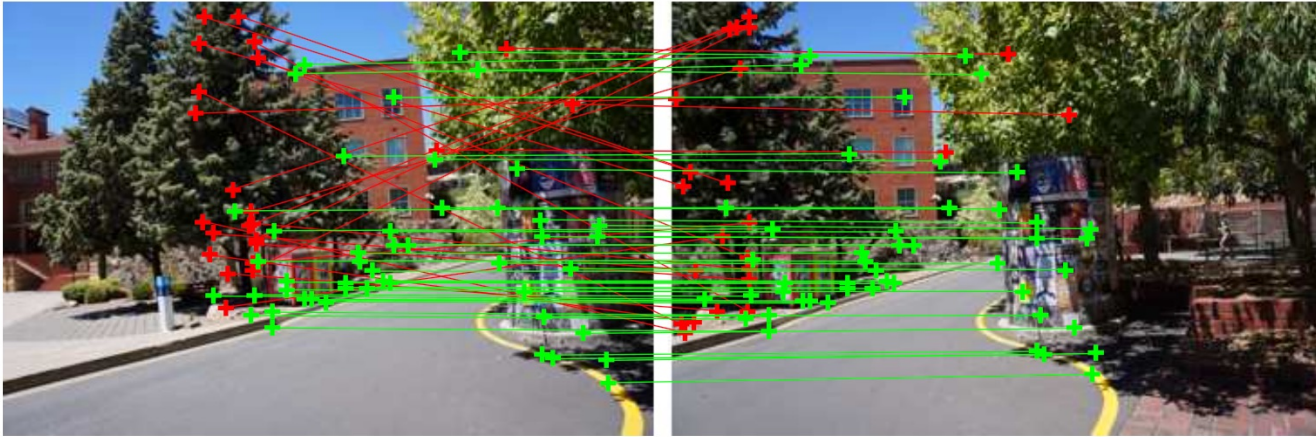
(multiple moving objects etc etc)

Figure 2.3: Applying RANSAC with an inlier threshold of $\epsilon = 3$ pixels for robust homography fitting on the data in Figure 1.2, where there are a total of 70 feature matches. RANSAC found a consensus set with 47 inlying feature matches (plotted in green) in a matter of seconds. Observe that the data identified as outliers (plotted in red) do correspond to incorrect feature matches.

Toy problem….

- The models/equations for most computer vision problems are a distraction to those not versed in the subject….so let's take a simple example that everyone can understand…fitting the line y=mx+b (i.e., finding m and b) to points in 2-D

Maximum Consensus line fitting –
the image at left is made by a
streak from a moving spaced
object against a stationary
background of stars…
Find the moving object…

Find the line that has more points
within some specified tolerance of
that line….
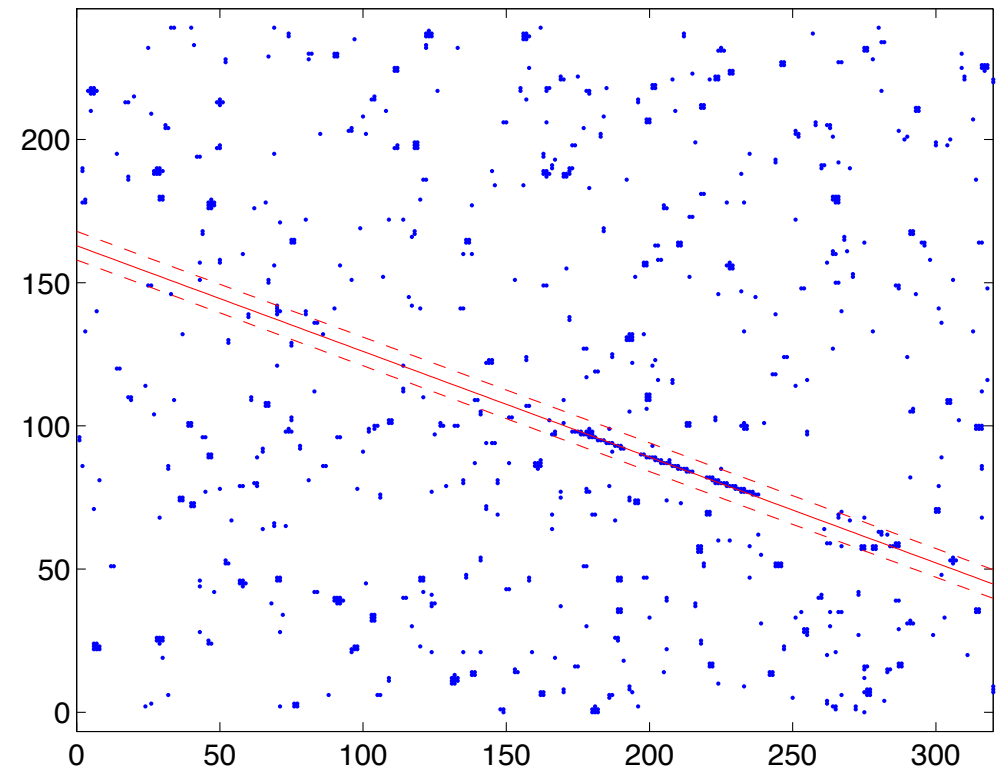
Surprisingly (even to me..☺) a lot of what I have done "boils down" to something akin to line fitting to noisy points (with outliers) – if you can do that **fully automatically** and without knowing things in advance (how many lines?, how many outliers?...) you can do many things…
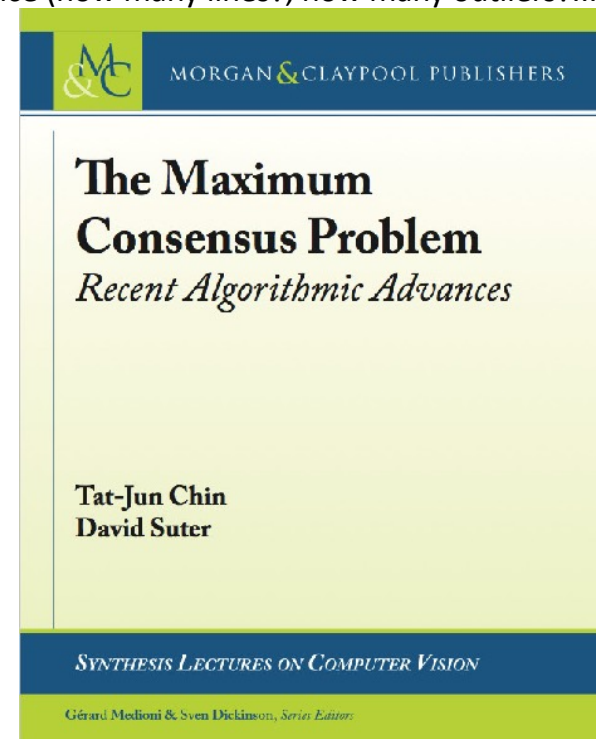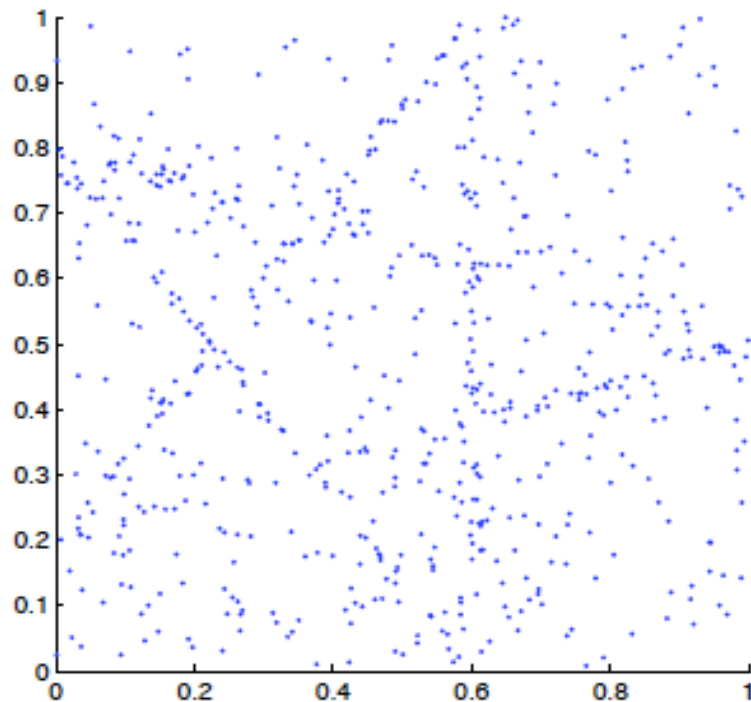
Strictly speaking, this is about single structure fitting….and with a particular measure of "fit"….

(published Feb 2017)

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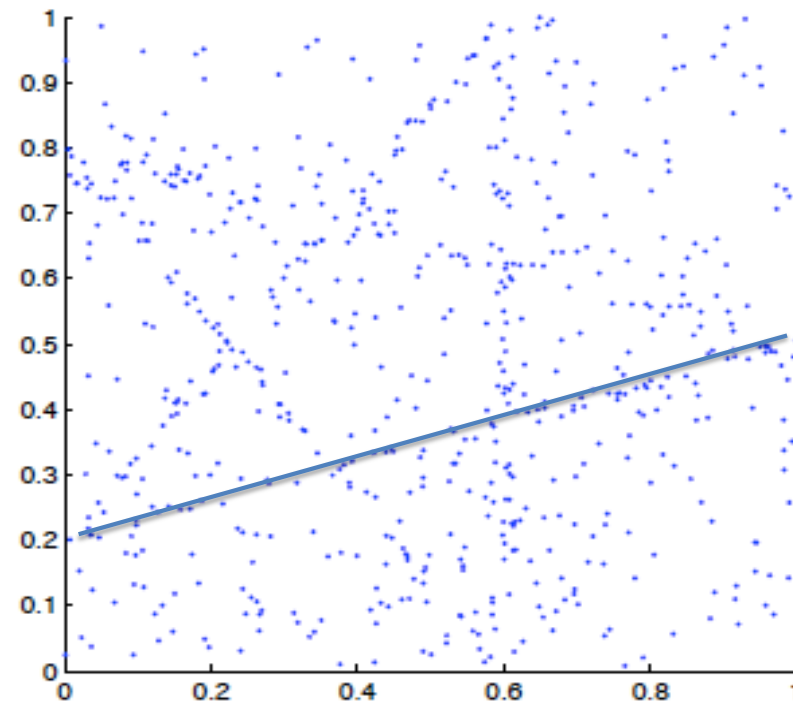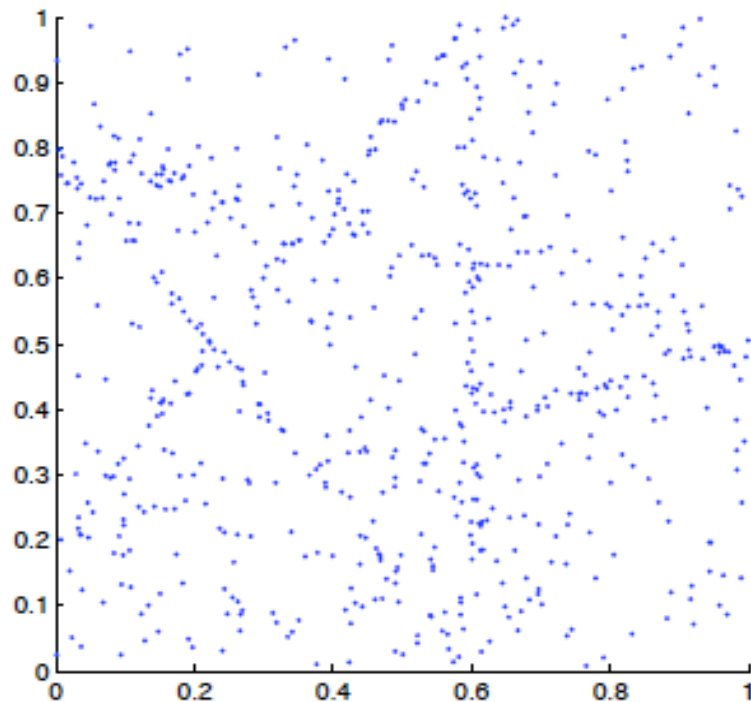# Maximum Consensus

When solving a problem one generally formulates an objective function (measuring how well a proposed solution does), and then finds a way to optimize the objective function.

The objective function of RANSAC is "how many data points are within some specified tolerance distance away from a proposed model/solution". This is called consensus as it is like these data points agreeing/voting for the proposed solution.

(aside – given the above…you might be thinking that sounds like Influence and the theory outlined earlier in the course…and that is true – but I discovered the connections and details the long way around….for me the journey of discovery started around 10 years ago..work we published in 2015 CVPR)

Been studying Maxcon (well, more generally, robust fitting) for a while…..

It all "started" a long time ago…

International Journal of Computer Vision
August 1998, Volume 29, Issue 1, pp 59–77 | Cite as

Robust Optic Flow Computation

Authors	Authors and affiliations

Alireza Bab-Hadiashar, David Suter

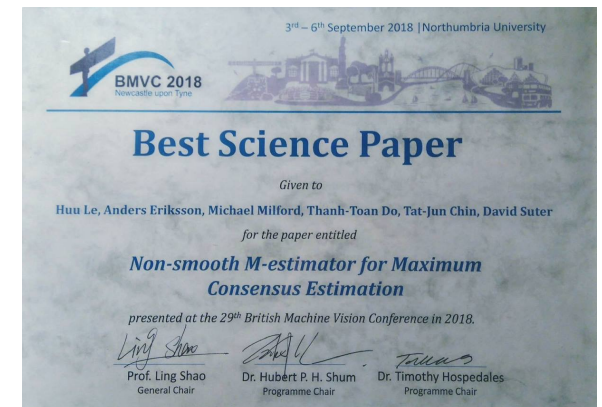Although that worked with LMedianSquares which has similar complexity issues….

Along the journey CVPR2015 best paper honorable mention award for A* optimal MaxCon

"The CVPR2015 paper"

And BMVC 2018 Best Science Paper award

"The BMVC paper"

3rd – 6th September 2018 | Northumbria University

BMVC 2018
Newcastle upon Tyne

Best Science Paper

Given to

Huu Le, Anders Eriksson, Michael Milford, Thanh-Toan Do, Tat-Jun Chin, David Suter

for the paper entitled

Non-smooth M-estimator for Maximum Consensus Estimation

presented at the 29th British Machine Vision Conference in 2018.

Prof. Ling Shao
General Chair

Dr. Hubert P. H. Shum
Programme Chair

Dr. Timothy Hospedales
Programme Chair

Why MaxCon is so hard..EVEN FOR SINGLE STRUCTURE AND EVEN IN LOW DIM?

$$\max_{\theta,\ \mathcal{I} \subseteq \mathcal{X}} |\mathcal{I}|$$

$$\text{subject to} \quad r_i(\theta) \leq \epsilon \ \ \forall \mathbf{x}_i \in \mathcal{I},$$

Usually θ is continuous – "so looks like" an optimization/search over a continuous space (albeit with "nasty" local optima structure of objective)

But actually, can also be seen/shown to be a DISCRETE optimization (albeit over a combinatorially huge discrete space).

"Two ways" to "see" this – Theory of LP-type problems (see "our" 2017 book/CVPR 2015 paper…..)
                  - "Chicken and egg" type problem: if know θ then finding and eliminating outliers is trivial, if know outliers/inliers then finding θ is trivial.

One key concept – for every model, there is a number p such that p data point can always be fit by that model. For line fitting in 2D – p=2. For ANY two points there is always a line that fits them perfectly. Hence, MaxCon solution >=2.

Moreover, for p+1 points there may be NO model that fits all p+1 points within the tolerance. So p+1 is the size of the smallest possible infeasible (can't be fit by the model within tolerance) subset.

If we had a test for in feasibility….

- If given a set of points, we could test whether those point can be fit by the model (feasible) then we could search over all subsets (Boolean cube!) – from (111…..1) down to subsets of size p+1, and take the first one we find (highest in the cube – largest subset)!

- There is such a test! L-infinity fitting. Given a set of points find the fit with *maximum* possible largest residual. If this is less than our tolerance, then the set is feasible, else – infeasible.

- Even though L-infinity fitting is reasonably fast, the search space is typically huge and so need to be more clever…

- The CVPR2015 best paper honourable mention work used A* search which is guaranteed to give the true answer, often runs quite fast, but could still be exponential in runtime.,..

Discrete Search - Search over subsets of the data – Search over hypercube/Hasse Diagram
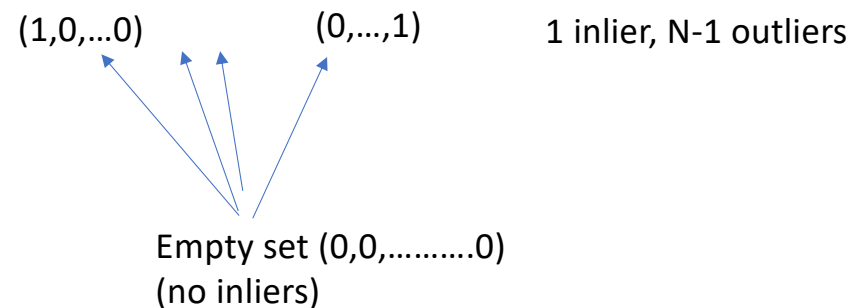
Find the highest feasible set in the Hasse Diagram

All inliers, 0 outliers

(1,1,.....1)

N-1 inlier, 1 outliers
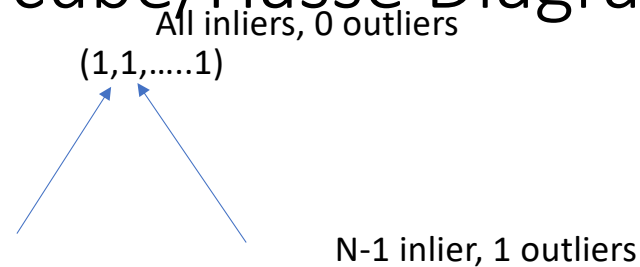
Lowest level that one needs to search downto....

P+1 inlier

p inlier, N-p outliers, p is minimal set to determine θ

(1,0,...0)   (0,...,1)   1 inlier, N-1 outliers

Empty set (0,0,..........0)
(no inliers)

Other searches...

- We tried bisection searches...(ECCV2018 paper)....each level of the cube is still usually large so still need clever additions to the idea...

New Ideas – CVPR2021 and CVPR2022 papers

Summary…

- Feasibility is a monotone Boolean function over the cube – if a set of points is feasible than delete a point must leave the set feasible. Conversely, if a set of points is infeasible then adding a point cannot make it feasible.

- In fact, the set of subsets of feasible points is a simplicial complex – downward closed set of subsets. The maxcon solution is the maximal face of the this simplicial complex.

- Moreover, the simplicial complex above is the independent set complex of the hypergraph of minimal sized infeasible (p+1 uniform hypergraph) subsets.

- Moreover ++ the influence of the monotone Boolean function (feasibility) can be related to the likelihood that a data point is an inlier….influences of inliers are generally lower then influences of outliers

- However…calculating Influences exactly is exponential in cost – need to be more clever and estimate influences efficiently…

Prof. Tat-Jun Chin (Adelaide U)

Prof. Hanzi Wang (Xiamen University)

Prof. Ali Bab-Hadiashar (RMIT U)

and, for various works around the topics mentioned

To around 20 former postgrad students, 5 former postdocs..