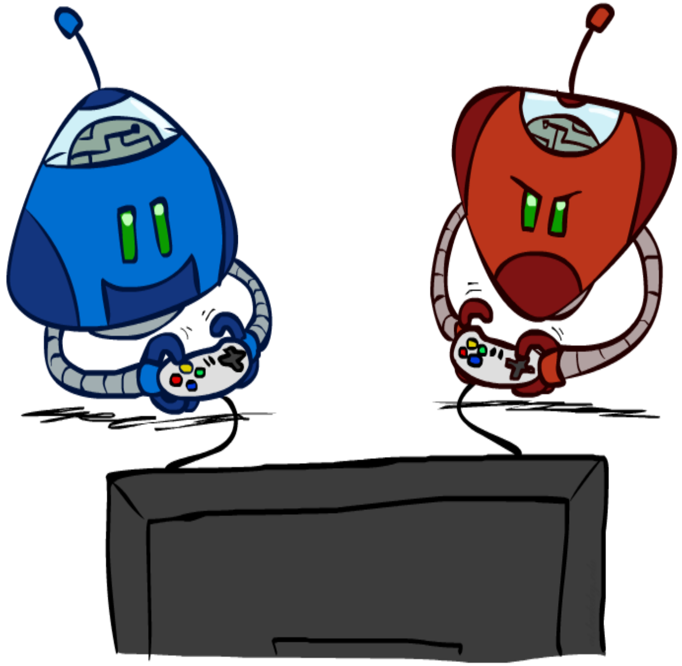
Adversarial Search



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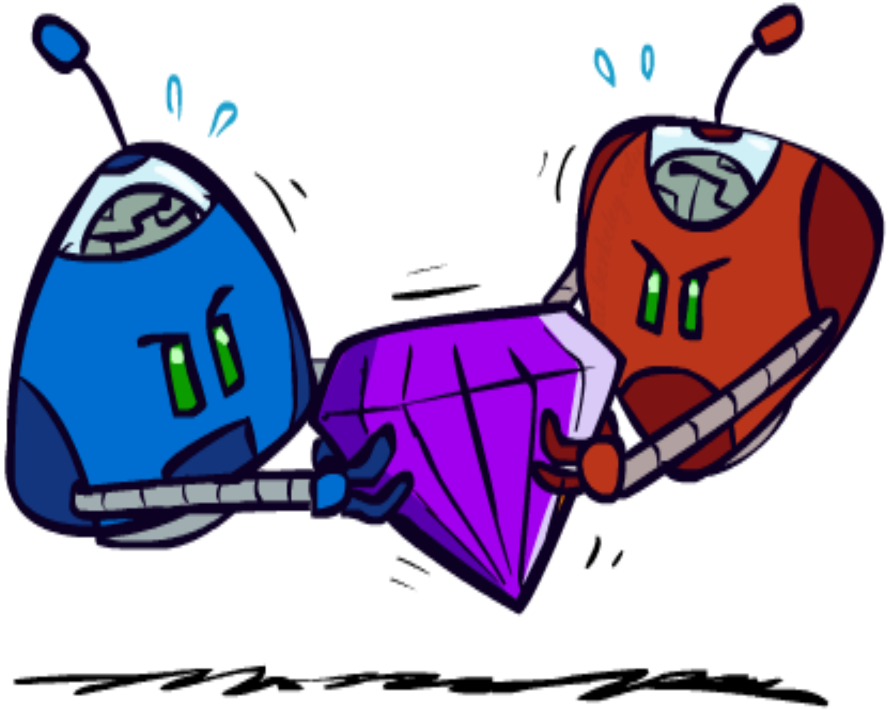
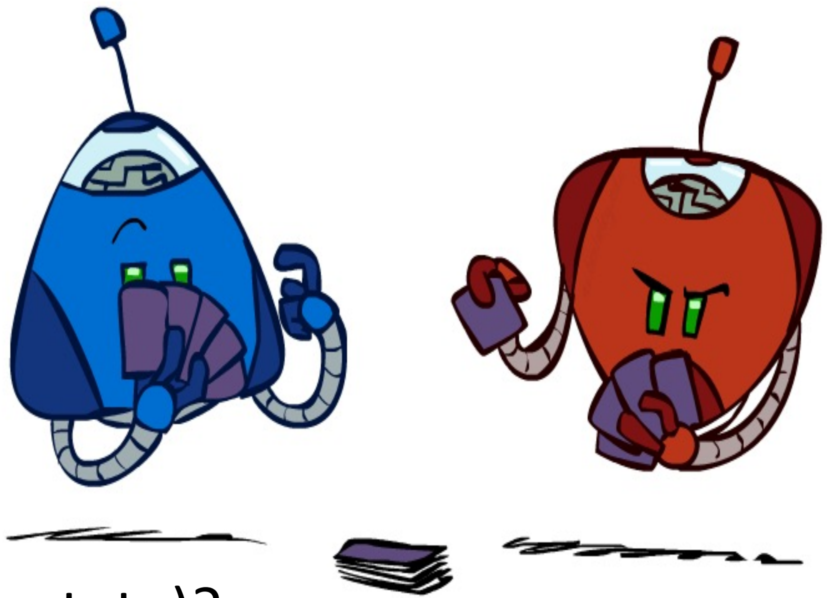
Course Delivered for Xidian

[Many slides adapted from those created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. Some others from colleagues at Adelaide University.]

Adversarial Games



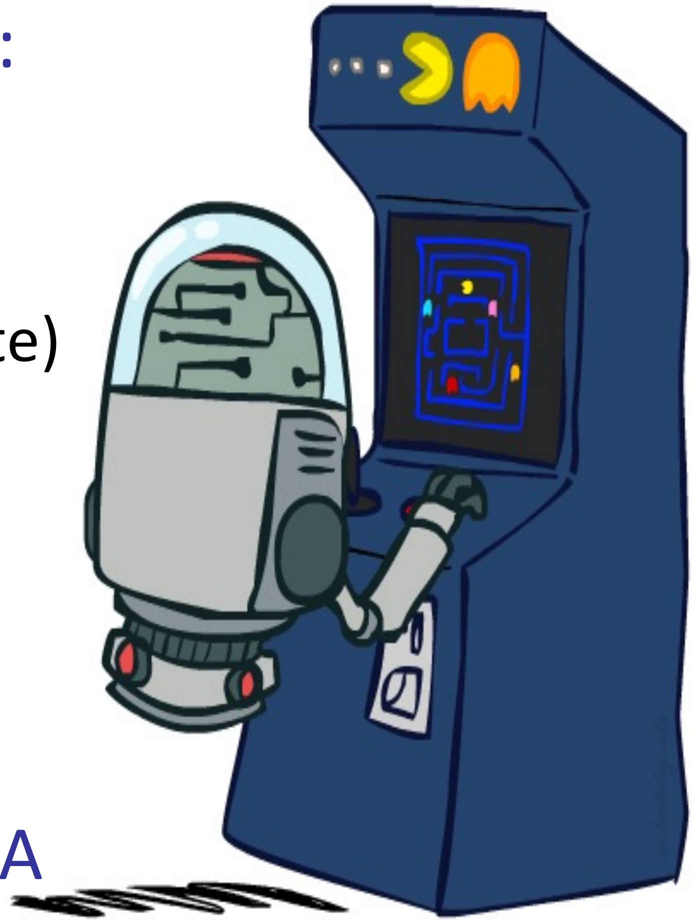
Types of Games



- Many different kinds of games!
- Axes:
  - Deterministic or stochastic?
  - One, two, or more players?
  - Zero sum?
  - Perfect information (can you see the state)?
- Want algorithms for calculating a strategy (policy) which recommends a move from each state

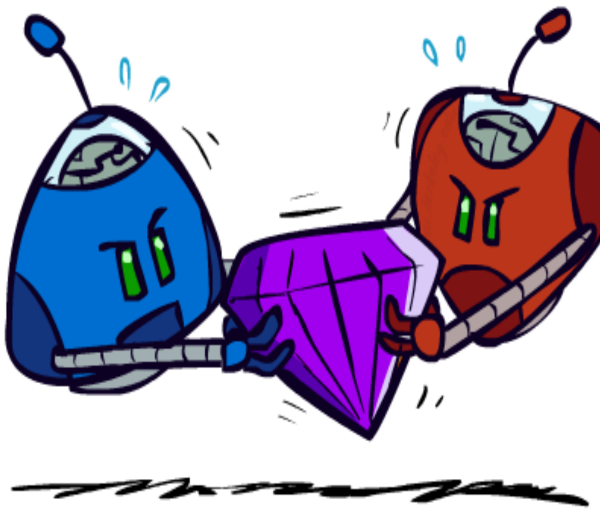
Deterministic Games

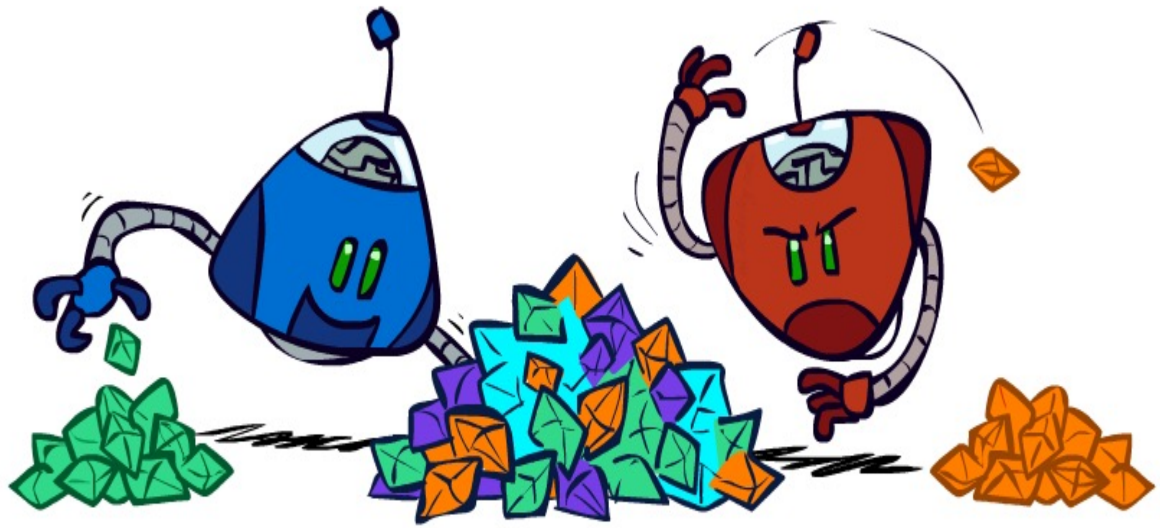
- Many possible formalizations, one is:
  - States: S (start at s_0)
  - Players: P={1...N} (usually take turns)
  - Actions: A (may depend on player / state)
  - Transition Function: SxA $\rightarrow$ S
  - Terminal Test: S $\rightarrow$ {t,f}
  - Terminal Utilities: SxP $\rightarrow$ R



- Solution for a player is a policy: S $\rightarrow$ A

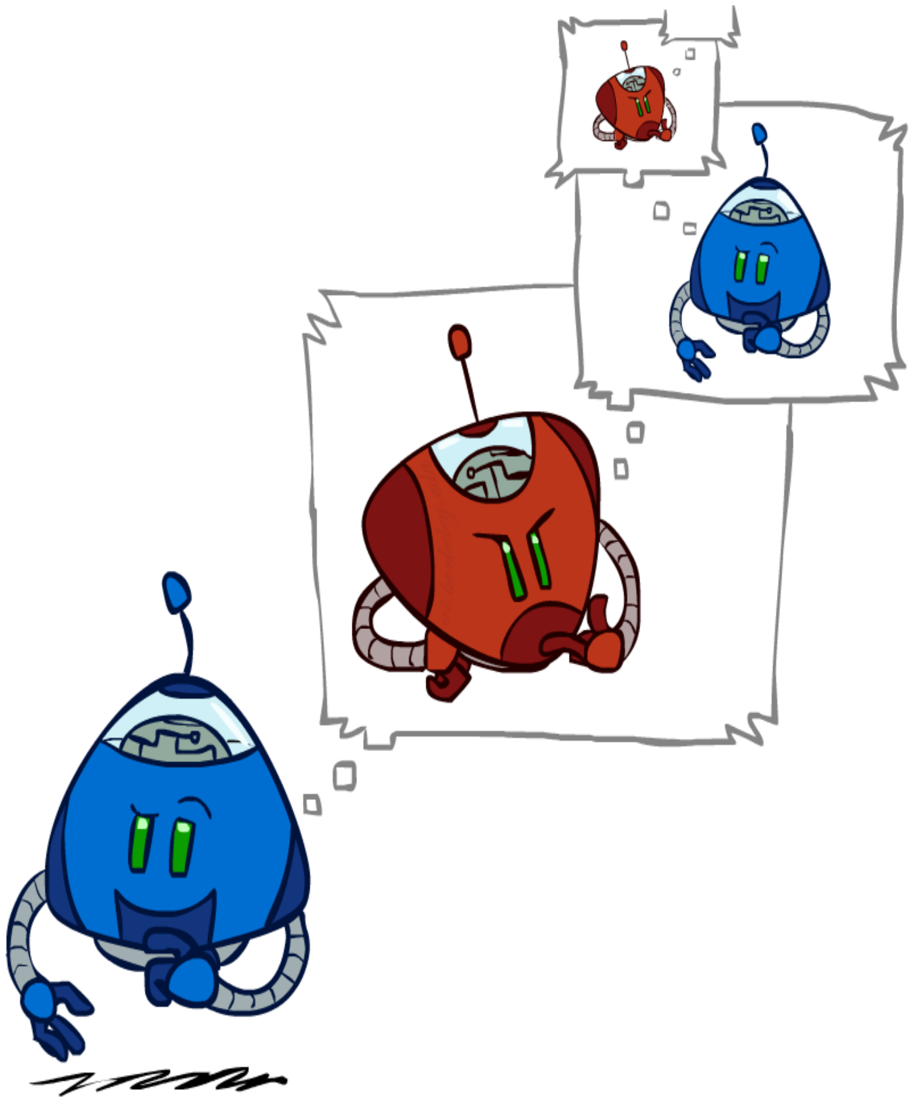
Zero-Sum Games



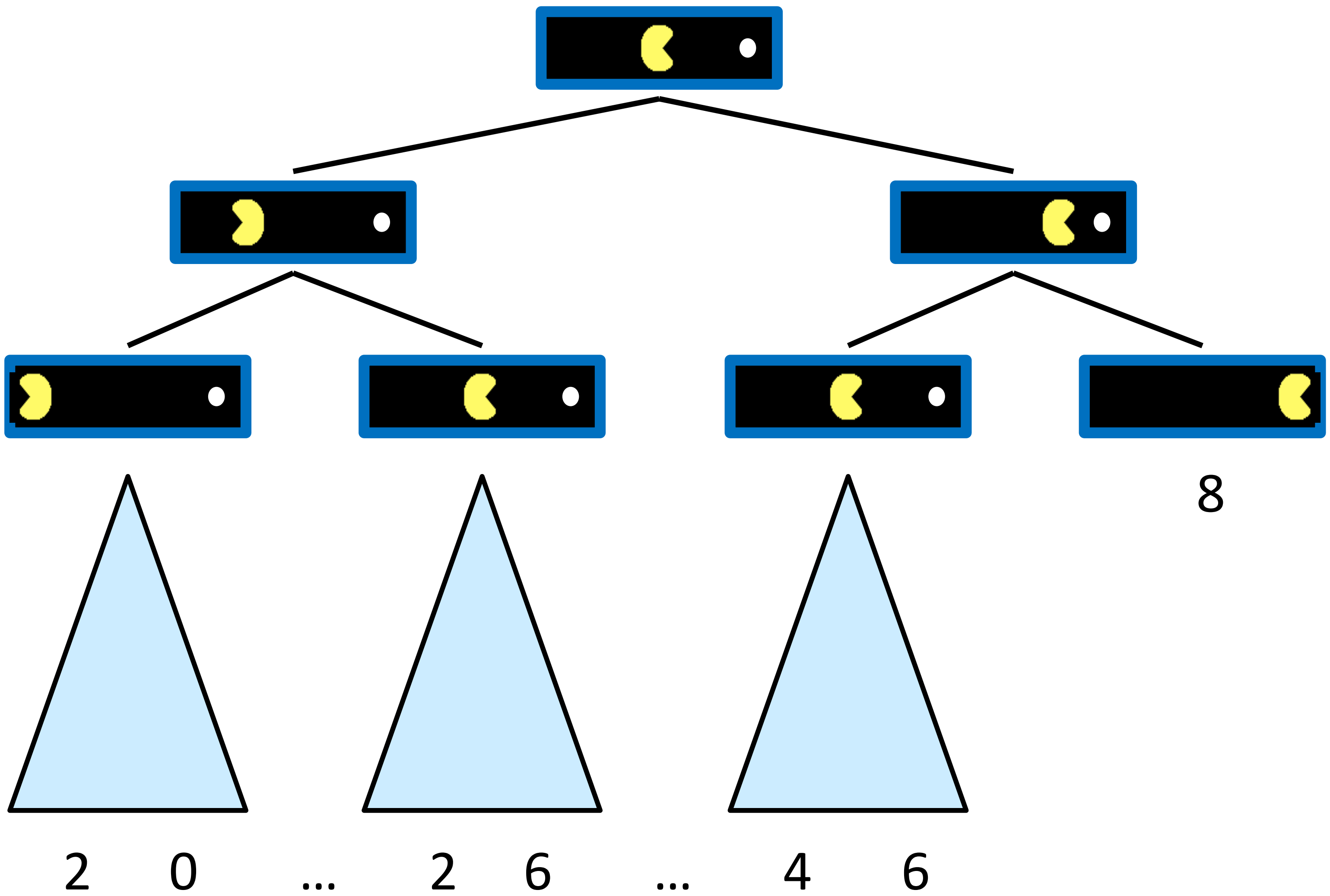


- Zero-Sum Games
  - Agents have opposite utilities (values on outcomes)
  - Lets us think of a single value that one maximizes and the other minimizes
  - Adversarial, pure competition
- General Games
  - Agents have independent utilities (values on outcomes)
  - Cooperation, indifference, competition, and more are all possible

Adversarial Search



Single-Agent Trees

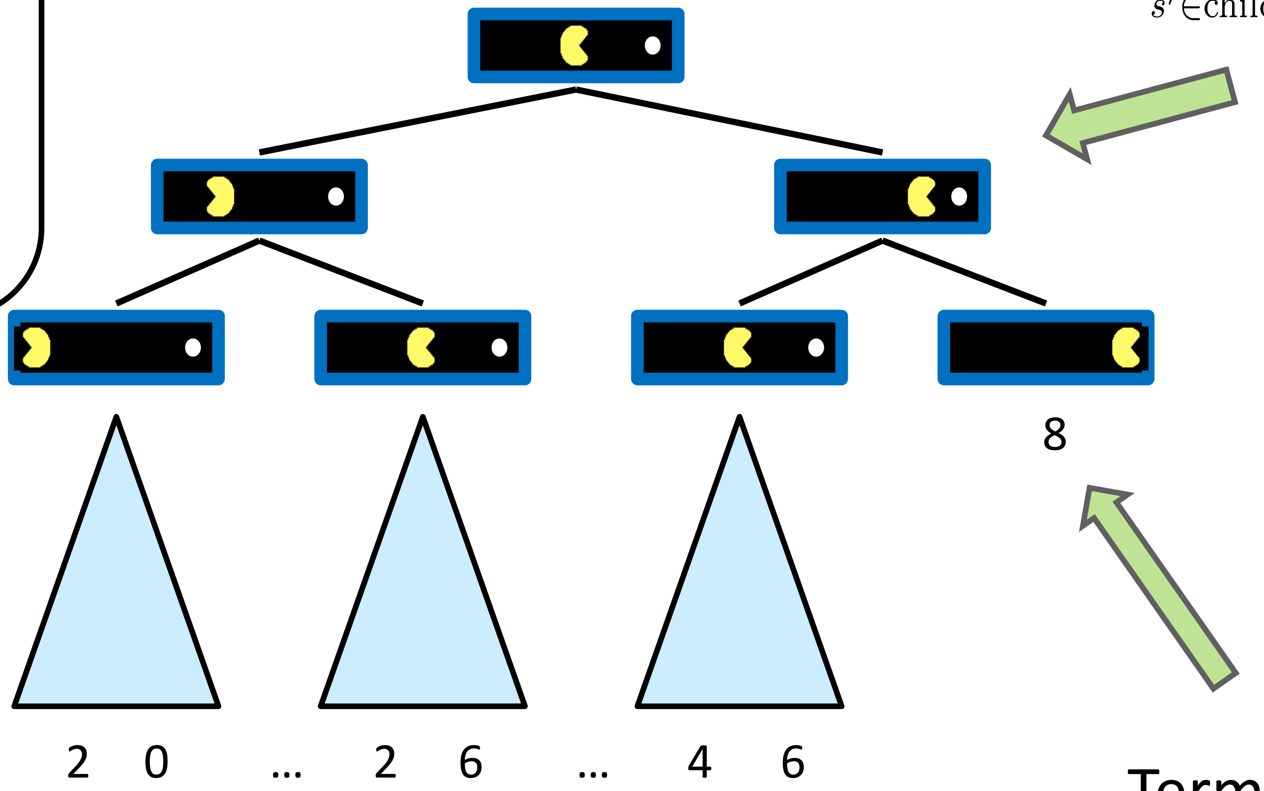


Value of a State

Value of a state:
The best achievable outcome (utility) from that state

Non-Terminal States:

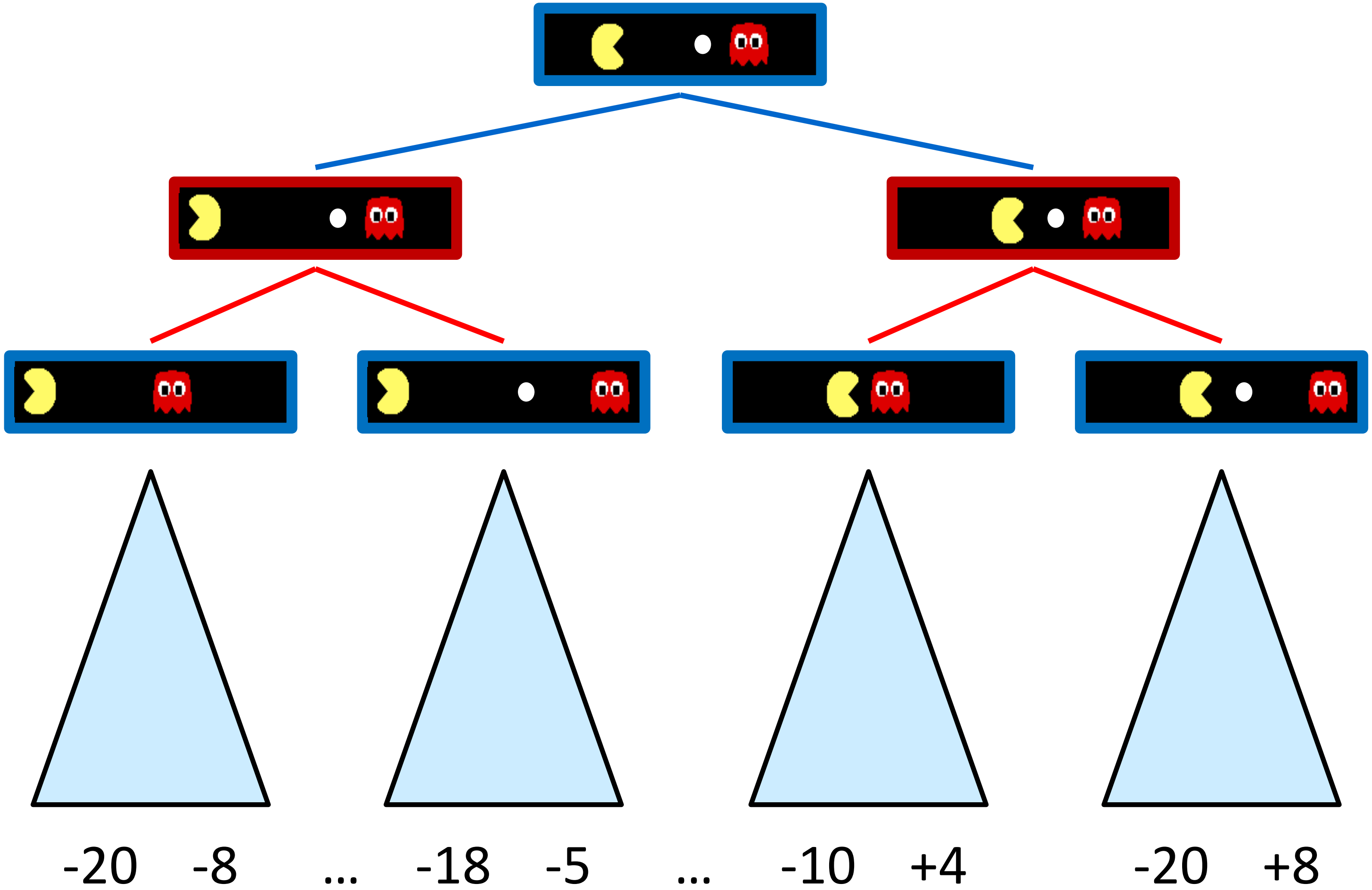
$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$



Terminal States:

$$V(s) = \text{known}$$

Adversarial Game Trees



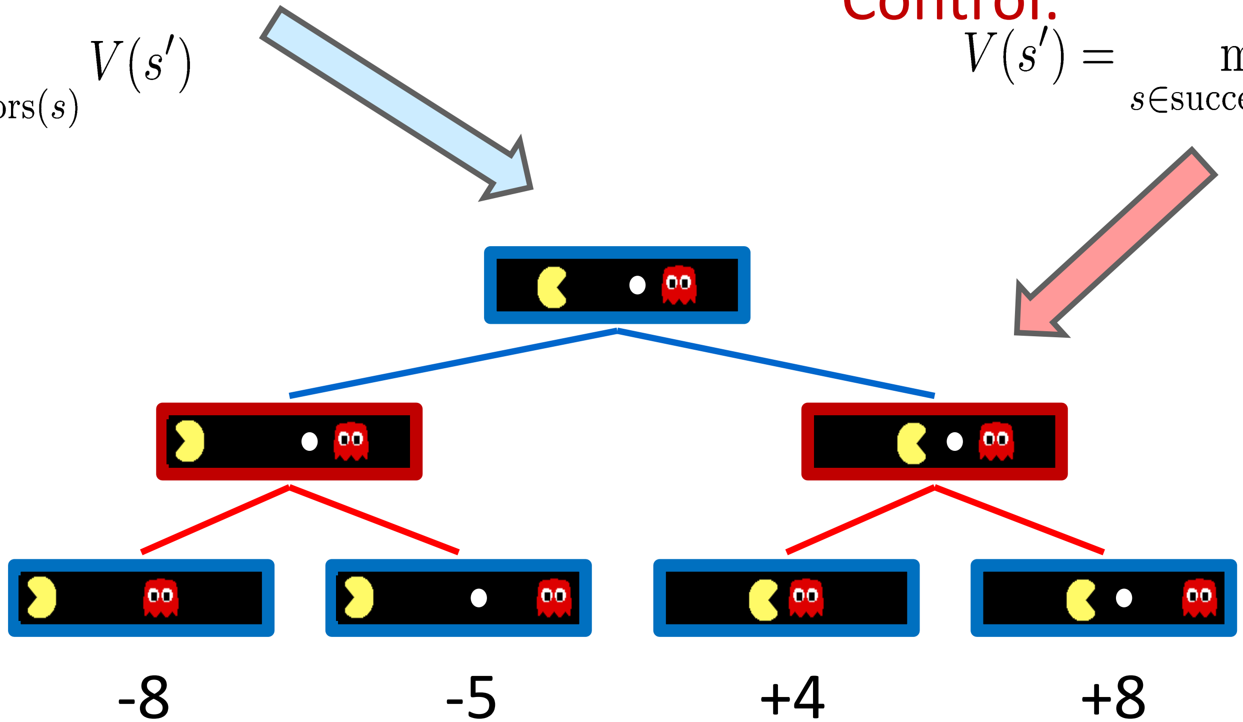
Minimax Values

States Under Agent's Control:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

States Under Opponent's Control:

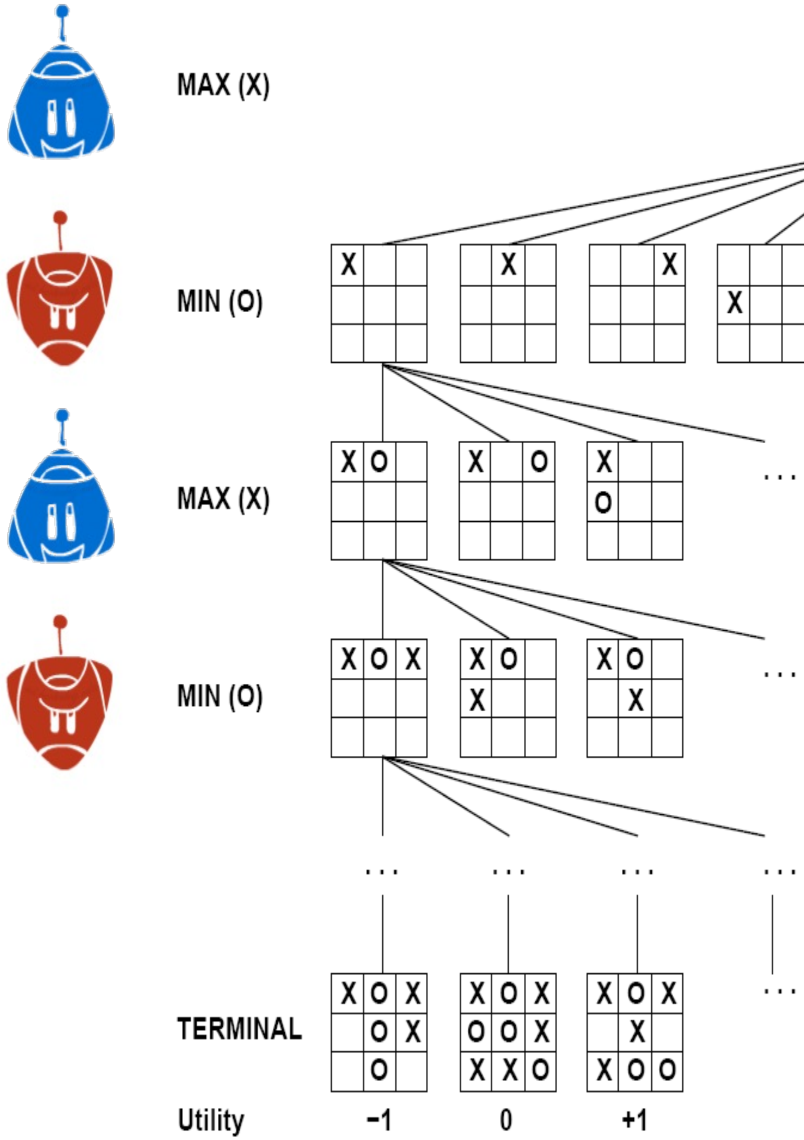
$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

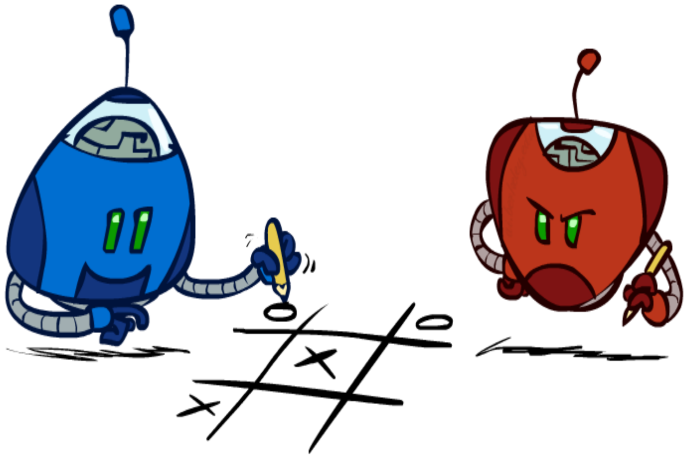


Terminal States:

$$V(s) = \text{known}$$

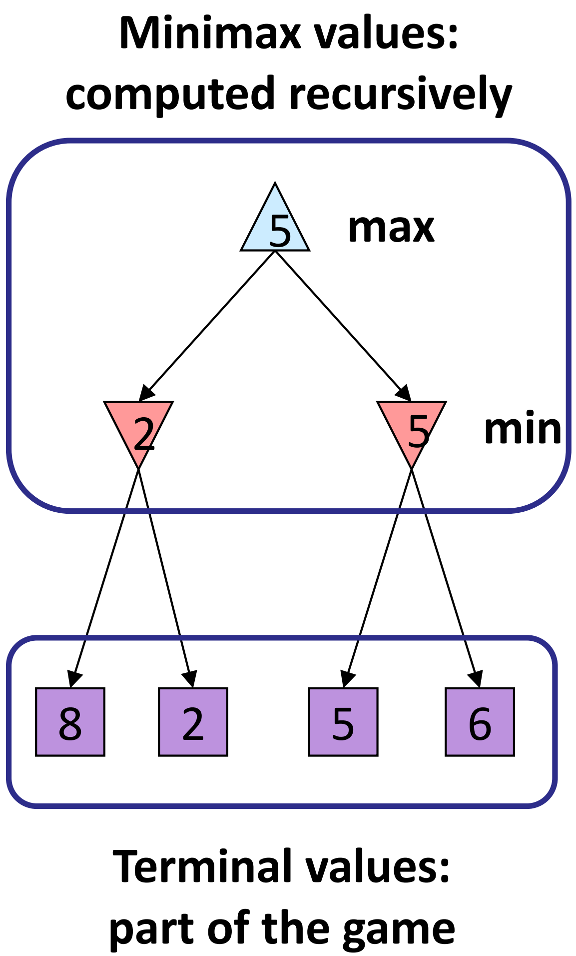
Tic-Tac-Toe Game Tree





Adversarial Search (Minimax)

- Deterministic, zero-sum games:
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result
- Minimax search:
  - A state-space search tree
  - Players alternate turns
  - Compute each node's minimax value: the best achievable utility against a rational (optimal) adversary



Minimax Implementation

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor))
    return v
```

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, max-value(successor))
    return v
```

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

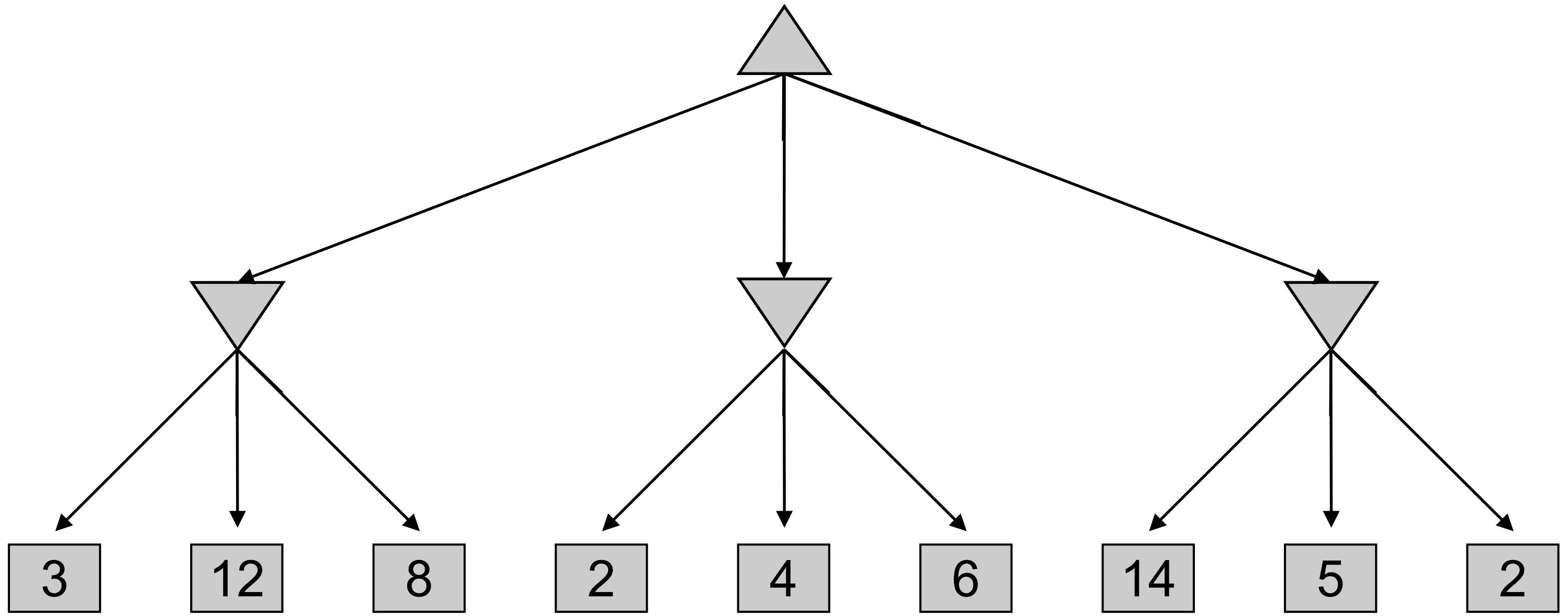
Minimax Implementation (Dispatch)

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

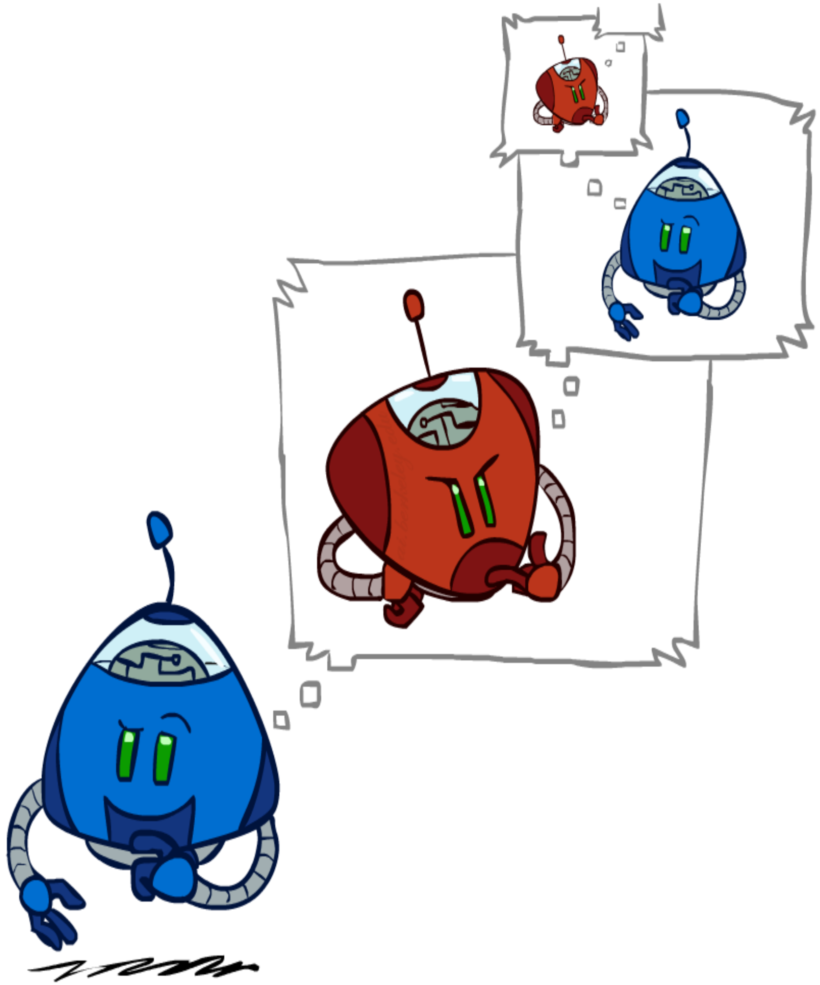
```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor))
    return v
```

Minimax Example



Minimax Efficiency

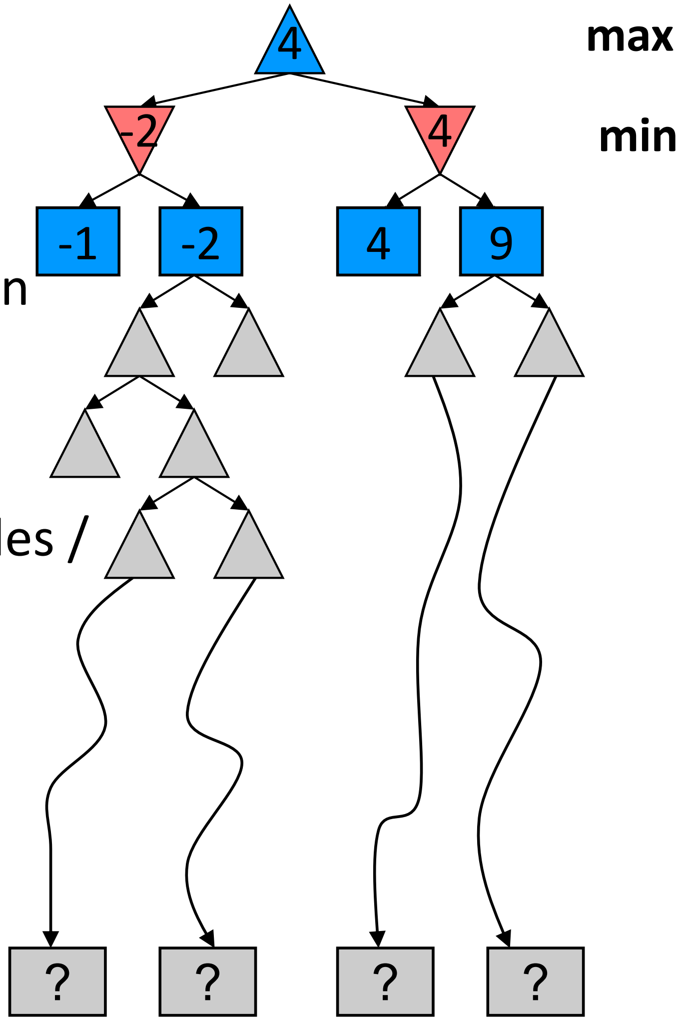
- How efficient is minimax?
  - Just like (exhaustive) DFS
  - Time: $O(b^m)$
  - Space: $O(bm)$
- Example: For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?



Resource Limits

Resource Limits

- Problem: In realistic games, cannot search to leaves!
- Solution: Depth-limited search
  - Instead, search only to a limited depth in the tree
  - Replace terminal utilities with an evaluation function for non-terminal positions
- Example:
  - Suppose we have 100 seconds, can explore 10K nodes / sec
  - So can check 1M nodes per move
  - α-β reaches about depth 8 – decent chess program
- Guarantee of optimal play is gone
- More plies makes a BIG difference
- Use iterative deepening for an anytime algorithm



Depth Matters

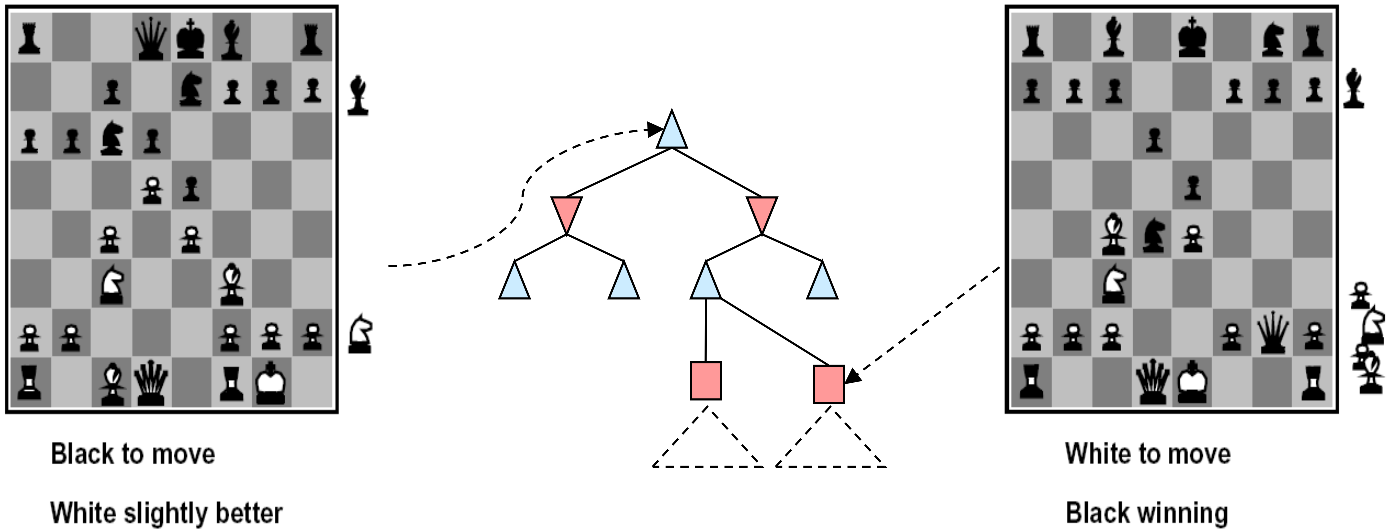
- Evaluation functions are always imperfect
- The deeper in the tree the evaluation function is buried, the less the quality of the evaluation function matters
- An important example of the tradeoff between complexity of features and complexity of computation

[Demo: depth limited (L6D4,

Evaluation Functions

Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search

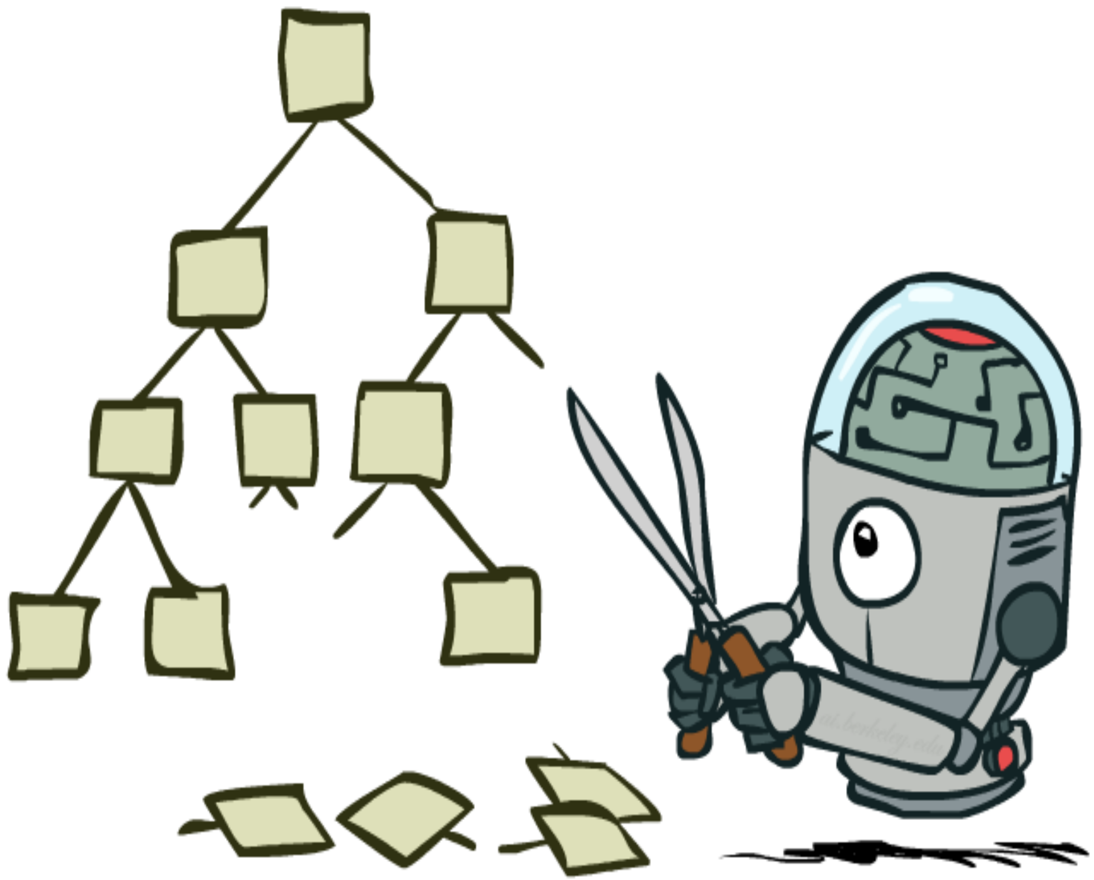


- Ideal function: returns the actual minimax value of the position
- In practice: typically weighted linear sum of features:

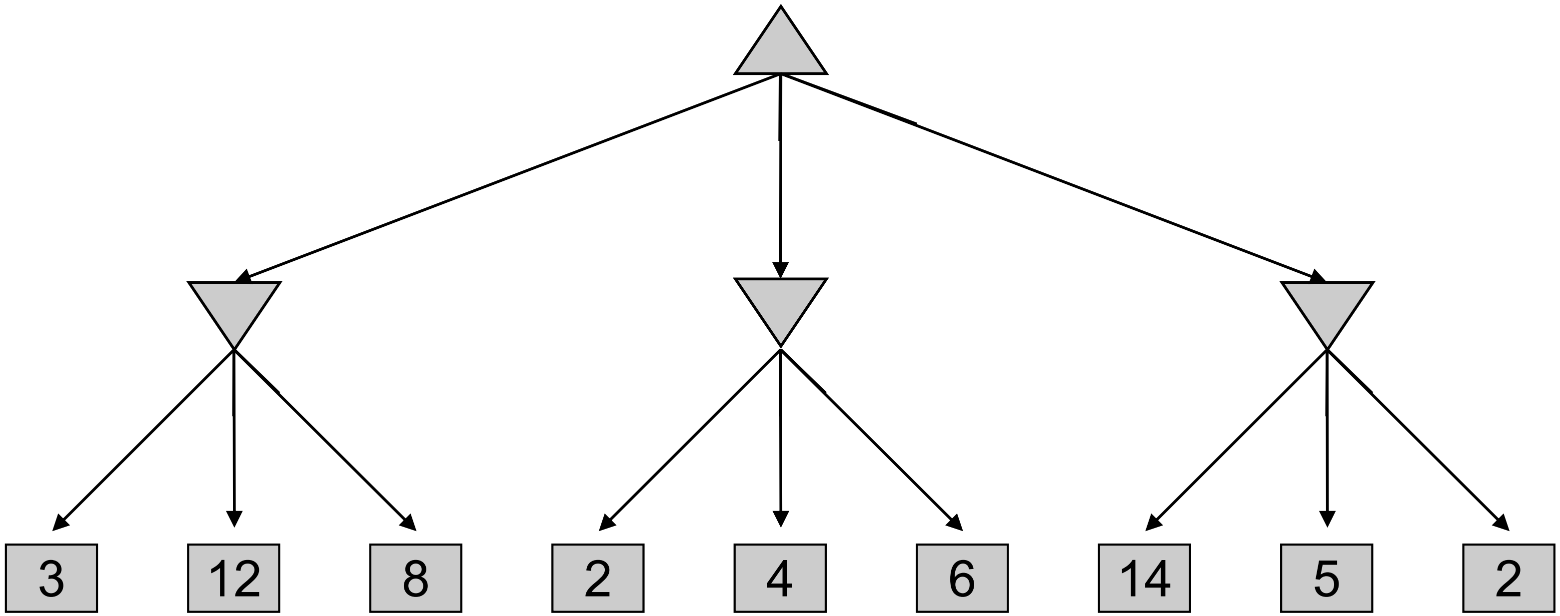
$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- e.g. $f_1(s)$ = (num white queens – num black queens), etc.

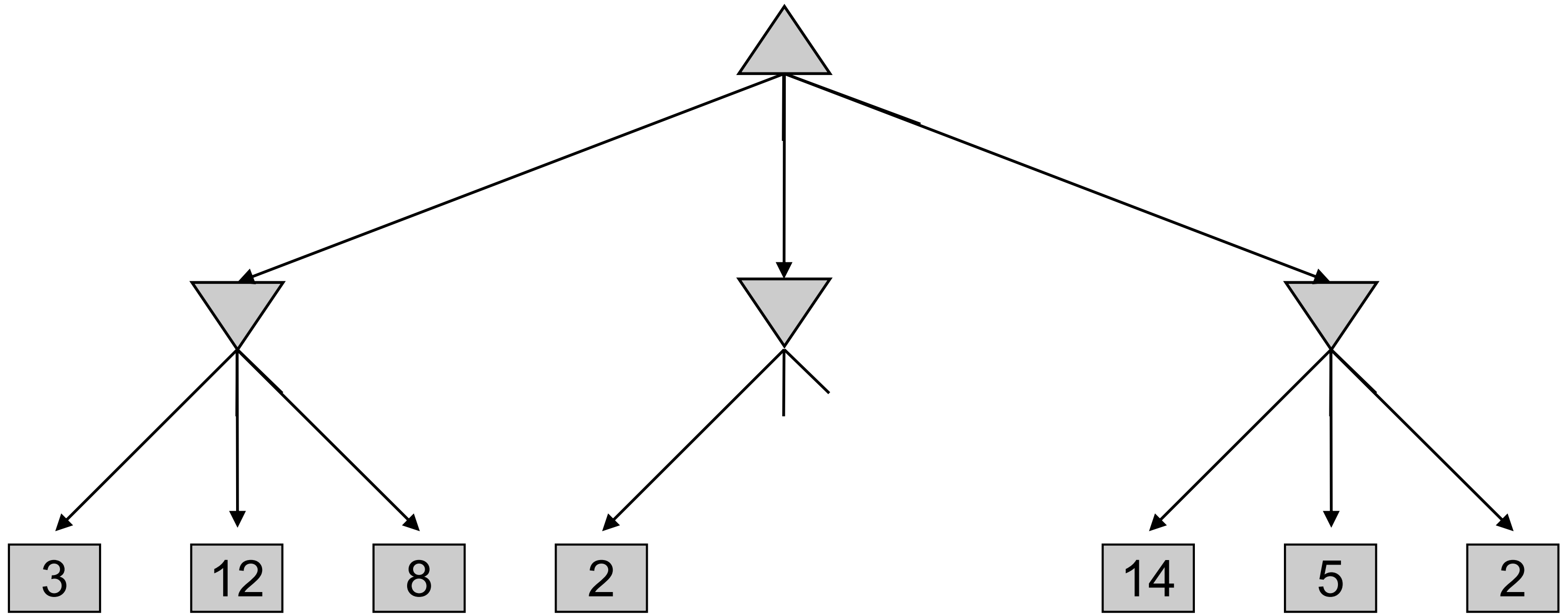
Game Tree Pruning



Minimax Example

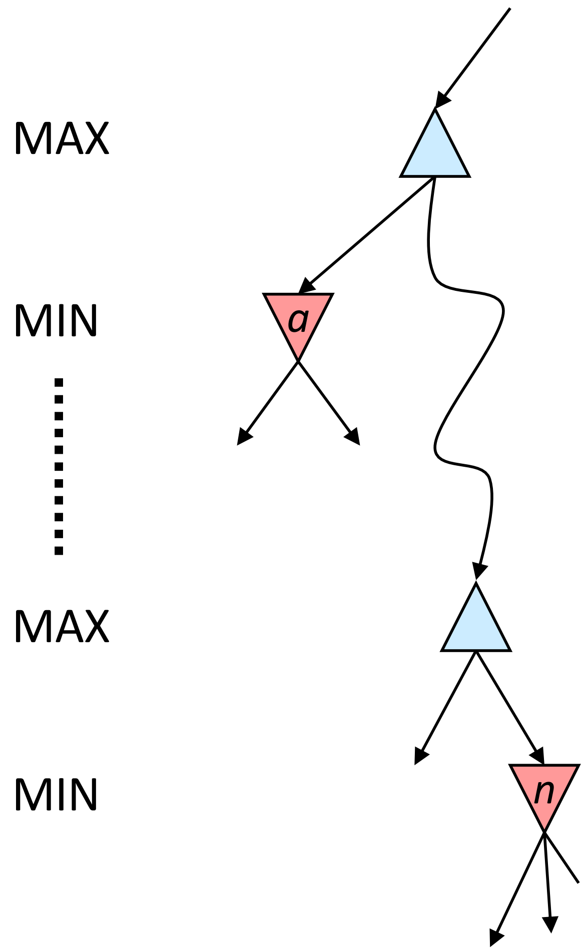


Minimax Pruning



Alpha-Beta Pruning

- General configuration (MIN version)
  - We're computing the MIN-VALUE at some node *n*
  - We're looping over *n*'s children
  - *n*'s estimate of the childrens' min is dropping
  - Who cares about *n*'s value? MAX
  - Let *a* be the best value that MAX can get at any choice point along the current path from the root
  - If *n* becomes worse than *a*, MAX will avoid it, so we can stop considering *n*'s other children (it's already bad enough that it won't be played)



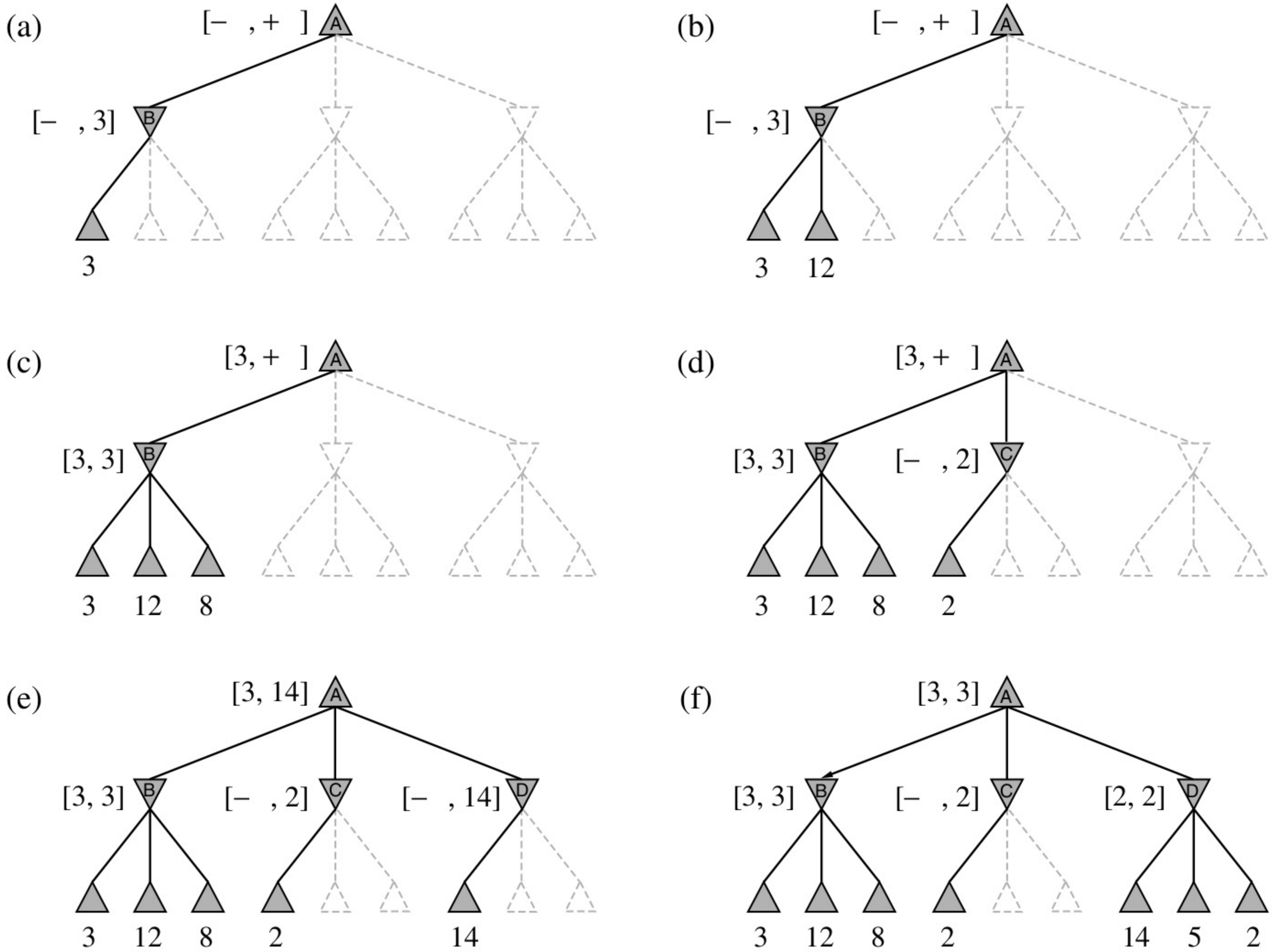
Alpha-Beta Implementation

α: MAX's best option on path to root

β: MIN's best option on path to root

```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v,
          value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v,
          value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```



(a)
[− , +]
A
[− , 3]
B
3
(b)
[− , +]
A
[− , 3]
B
3
12
(c)
[3, +]
A
[3, 3]
B
3
12
8
(d)
[3, +]
A
[3, 3]
B
[− , 2]
C
3
12
8
2
(e)
[3, 14]
A
[3, 3]
B
[− , 2]
C
[− , 14]
D
3
12
8
2
14
(f)
[3, 3]
A
[3, 3]
B
[− , 2]
C
[2, 2]
D
3
12
8
2
14
5
2

Alpha-Beta Pruning Properties

- This pruning has no effect on minimax value computed for the root!

- Good child ordering improves effectiveness of pruning

- With "perfect ordering":
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth!
  - Full search of, e.g. chess, is still hopeless…

- This is a simple example of metareasoning (computing about what to compute)

Alpha-Beta

Step-By-Step Examples