

# Induced subgraphs (and isomorphic graphs)

# Subgraph

A graph is a set of vertices  $V$  and a set of edges  $E$  between them.

E.g.  $V=\{1,2,3,4\}$ ,  $E=\{\{1,3\},\{2,4\},\{2,3\}\}$  (draw it!)

A subgraph is any subset of vertices and any subset of edges between these. Informally, delete all other vertices (and of course the edges with the deleted vertices being at least one vertex of the edge), and keeping whichever of the remaining edges "you want to keep"

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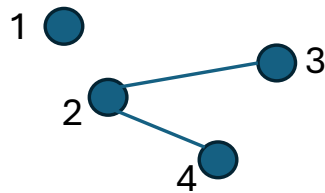
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There is another way of thinking of it that makes this perfectly natural. The vertices usually come from some data (they are the “objects”), the edges are the relationships between those data objects. An induced subgraph is the graph you would have gotten if instead you took a subset of the original data – not collected the whole data.

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For each of the following – are they subgraphs? Are they induced subgraphs? Draw them.

(a)  $V=\{1,2,3,4\}$ ,  $E=\{\{1,3\},\{2,4\},\{2,3\}\}$

(b)  $V=\{1,2,3,4\}$ ,  $E=\{\}$

(c)  $V=\{1,2,4\}$ ,  $E=\{\{2,4\}\}$

(d)  $V=\{1,2,4\}$ ,  $E=\{\}$

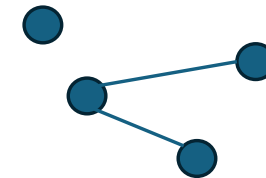
(e)  $V=\{1,2,3,4\}$ ,  $E=\{\{1,3\},\{2,3\}\}$

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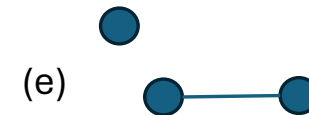
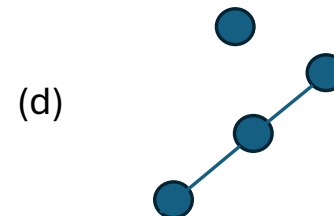
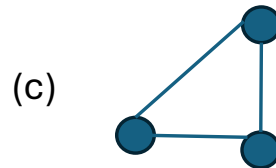
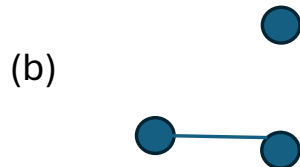
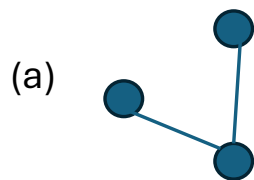
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E.g.  $V=[4]$ ,  $E$  given as drawn



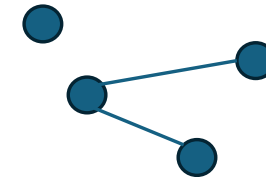
For each of the following – are they isomorphic to subgraphs? Are they isomorphic to induced subgraphs?



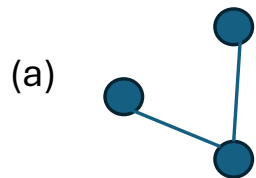
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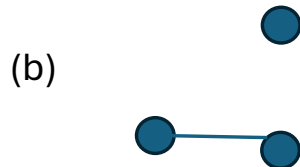
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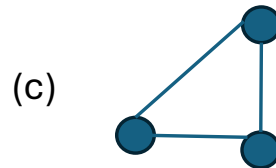
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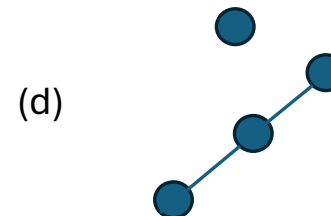
Yes to both.  
(there is only one possible  
copy - because vertex of  
degree 2



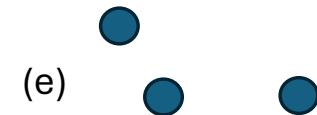
Yes to both.  
How many copies?



No to both.  
There are no  
triangles anywhere  
in original graph



Yes to both –  
isomorphic to  
original graph

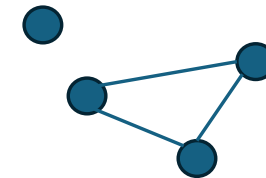


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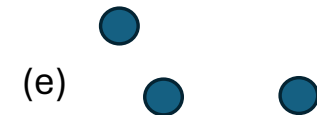
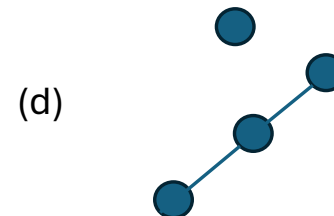
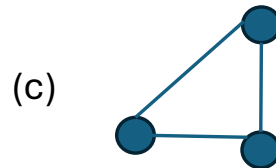
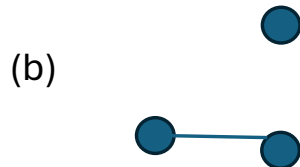
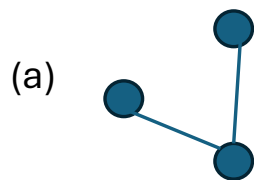
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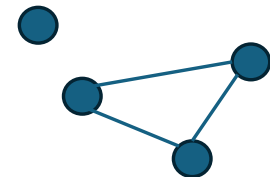




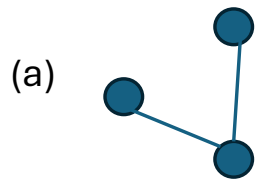
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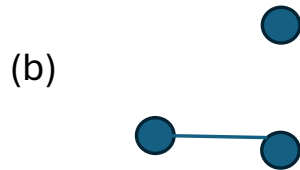
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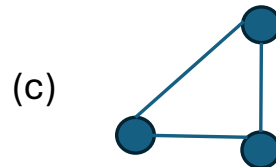
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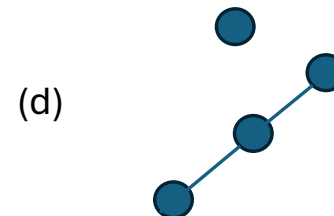
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How many copies?



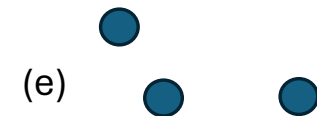
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How many copies?



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