# Geometric (Hyper)Graphs pt2

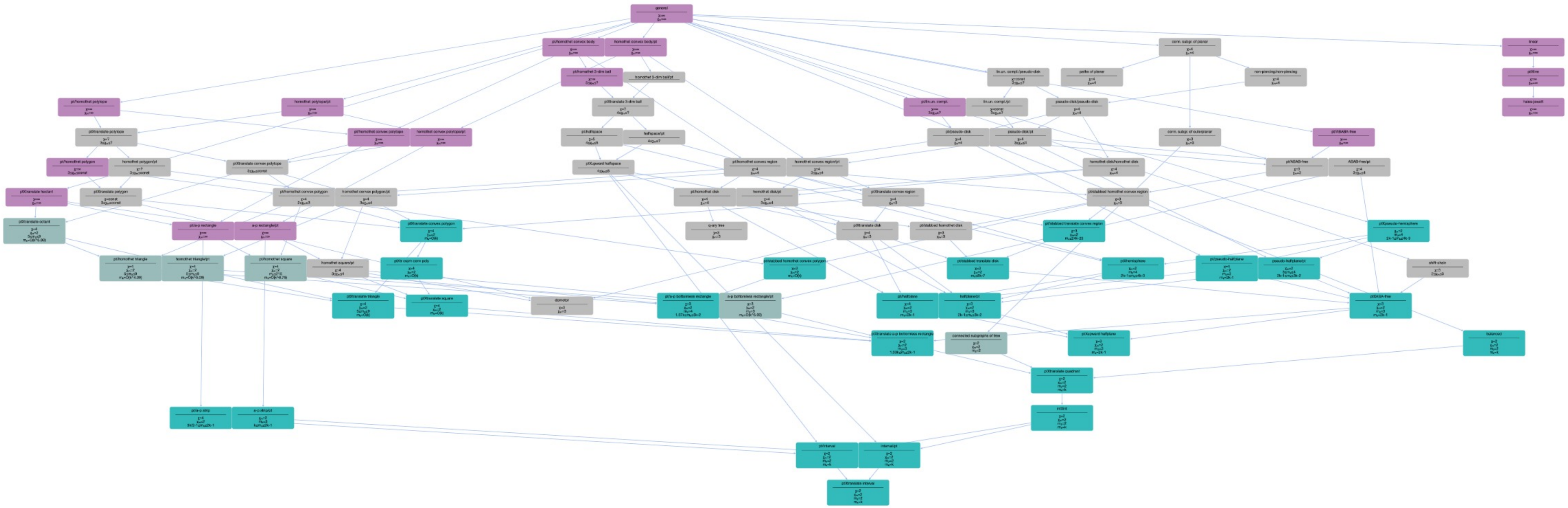
# There are a whole lot of geometric properties one can use to define geometric (hyper)graphs

- We just described a problem (MaxCon formulation of 1D location estimation) – where the tolerance intervals (more precisely their intersection patterns) lead to a (geometric) graph – and the MaxCon solution is the MaxClique of that graph.
- This is but ONE example of a general pattern – a finite collection of objects (e.g., rectangles in 2D, circles in 2D) and one defines a graph by assigning a vertex to every object and the graph captures the (pairwise) intersections of those objects. Then, depending on the problem you are trying to solve, your problem becomes a "prototypical" graph problem (MaxClique, Max independent set, Min Vertex cover….)
- Indeed a *part of* the possible zoo of such problems can be found at:

https://coge.elte.hu/cogezoo.html

# The geometric hypergraph zoo (beta)

## The geometric hypergraph zoo (definitions see below)



# Intersections, Containment, Proximity

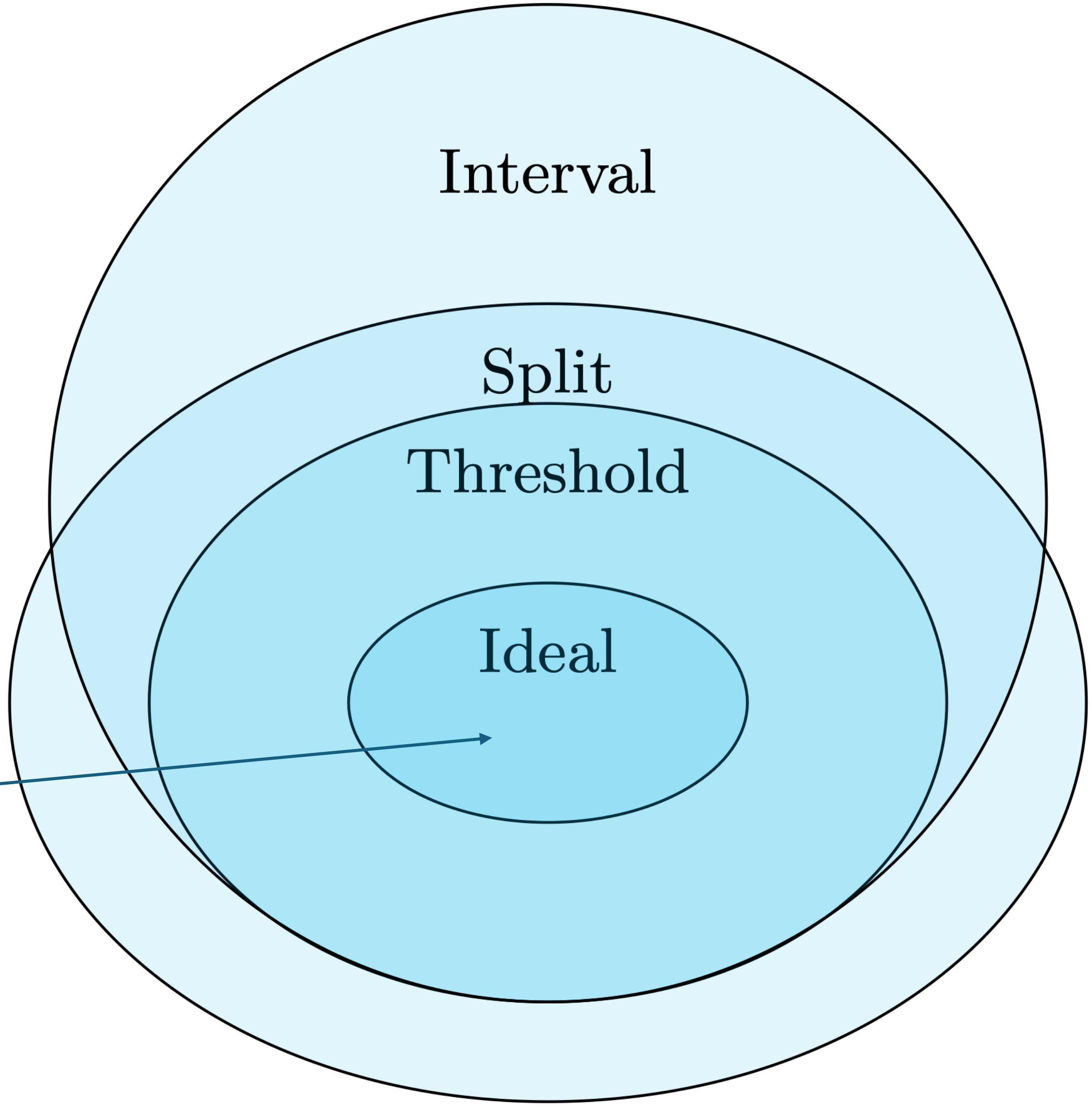
- This can be generalized in many many ways – replace intersection with containment or with proximity or with orthogonality etc. Set the problem in $R^d$ for various d, different type of geometric object, restrict how the objects can be placed – e.g., only placed on a grid, etc etc.
- So the previous website is only a small faction of the zoo.
- Actually, when many authors simply refer to Geometric (hyper)graph, they most commonly refer to proximity in $R^d$ or $S^d$ .

That is, for a collection of points in $R^d$ join a pair if their distance is less than some threshold. Even here, many variants can be defined depending on distance…

# Intersections, Containment, Proximity

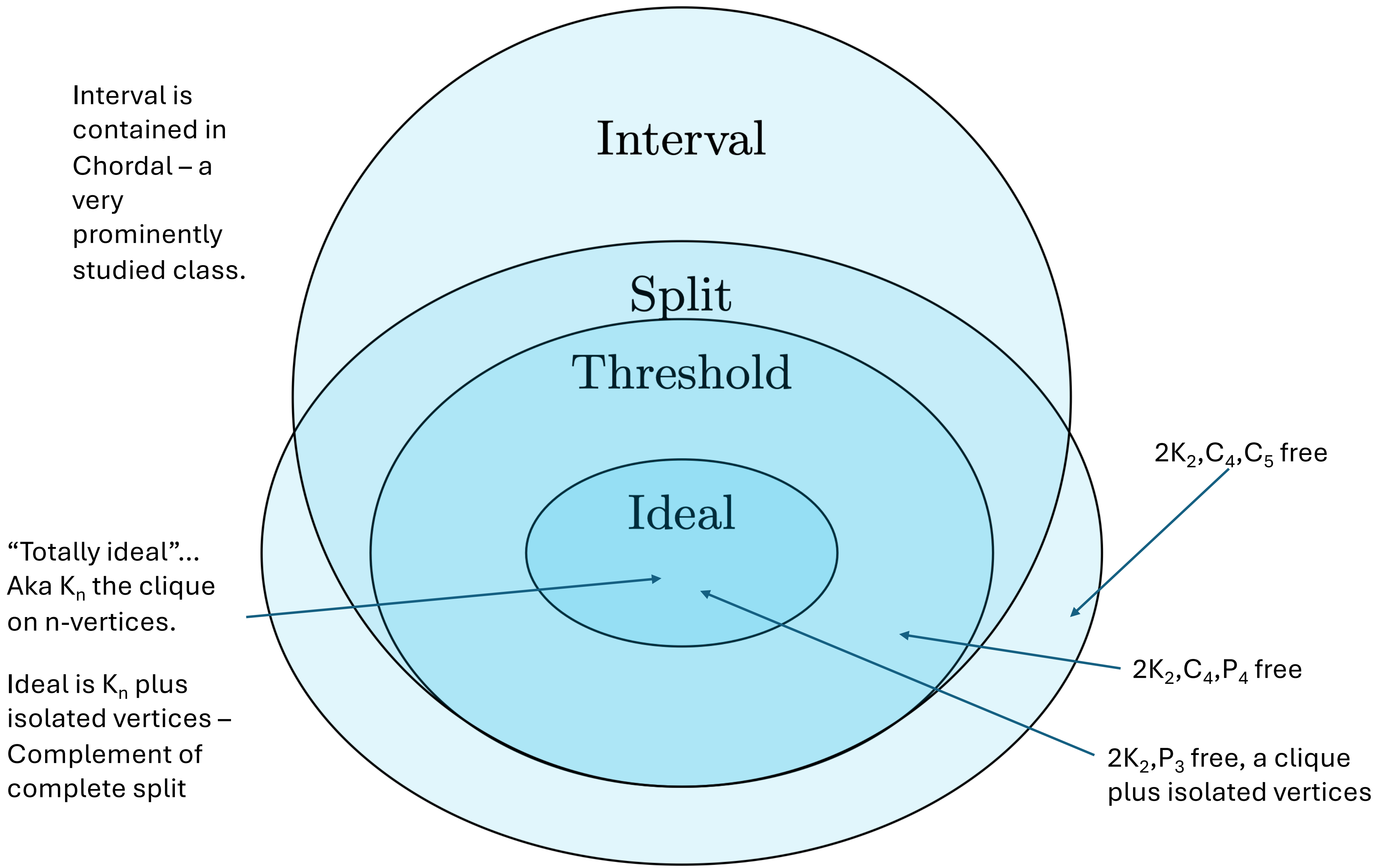
- Actually, when many authors simply refer to Geometric (hyper)graph, they most commonly refer to proximity in $R^d$ or $S^d$ .
- Perhaps the next most common is a family of problems generalizing the interval graphs – using any convex set (rather than interval) in $R^d$ . These (hyper)graphs inherit "nice" properties from the convexity and how this restricts the ways objects can intersect.
- Note: it turns out that ANY graph can be considered a geometric graph – for *some* dimension d and definition of "edge" for objects in that dimension. It is just that the dimension could be VERY HIGH. Thus, what we really mean by "geometric" (hyper)graph is those that arise "naturally" from geometric problems set in low dimensions. The latter is what gives then the nice properties (as well as even nicer if the the relevant objects are convex etc.).

Interval is contained in Chordal – a very prominently studied class.



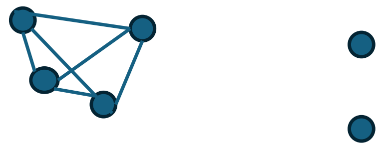

“Totally ideal”... Aka $K_n$ the clique on n-vertices.

Ideal is $K_n$ plus isolated vertices – Complement of complete split

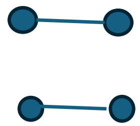

Interval is contained in Chordal – a very prominently studied class.
Interval
Split
Threshold
Ideal
$2K_2, C_4, C_5$ free
“Totally ideal”… Aka $K_n$ the clique on n-vertices.
$2K_2, C_4, P_4$ free
Ideal is $K_n$ plus isolated vertices – Complement of complete split
$2K_2, P_3$ free, a clique plus isolated vertices

# Why these forbidden (induced) graphs?

- Clique plus isolated vertices



$2P_2$



$P_3$

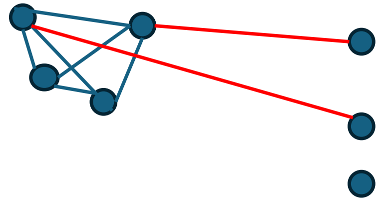
Where could the 2 edges of $2K_2$ ”live”? They can’t both be in the clique (because then edges missing). Neither of them can be in the isolated vertices (by definition no edges there!). One in clique and one between the clique and isolated - no, because the latter not allowed by definition.

Similar reasoning about the two edges in $P_3$ explains why that is forbidden.

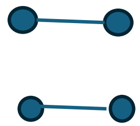
# Why these forbidden (induced) graphs?

- Threshold – clique, independent set

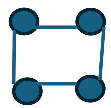
Possible edges between clique and Independent set.



$2P_2$



$P_4$



Where could the 2 edges of $2K_2$ ”live”? They can’t both be in the clique (because then edges missing). Neither of them can be in the independent vertices (by definition no edges there!). Both between the clique and isolated? - e.g., the red edges? – No because that gives a $P_4$ (can you see it)? So that only leaves one in the clique and one between… but that means 3 of the vertices must be in the clique – so those three must form a triangle.

You should convince yourself you can see why $P_4$ and $C_4$ are also not compatible with this type of graph. Proving That forbidding only these 3 forbidden induced subgraphs DEFINE this type of graph is quite a bit more complex!