

## Lecture 1: Overview of Probability

① Definition of prob. space

$$(\Omega, \mathcal{F}, \mathbb{P})$$

② Random variables.

③ pmf, pdf,  $E[\cdot]$ ,  $\text{Var}(\cdot)$

④ Conditional probability and conditional expectation

⑤ Concentrations / Chernoff Bound /

Sub-Gaussian rvs. / MGF.

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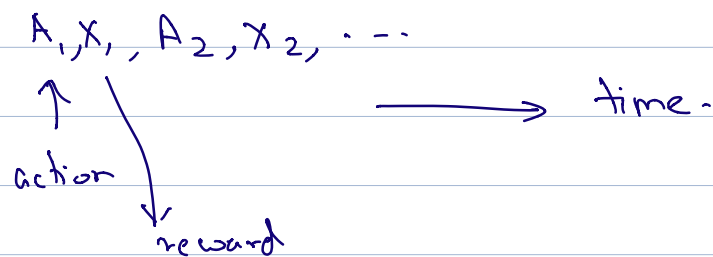
\* Module focuses on multiarmed bandits

Textbook: Bandit Algorithms, T. Lattimore and  
C. Szepesvári, Cambridge Univ.  
Press, 2019.

① Problem setting : explore vs. exploit

- Online advertising
- Recommendation systems
- Clinical trials
- Online resource allocation
- Online search (e.g. MCTS)

② Setup :



Key assumptions:

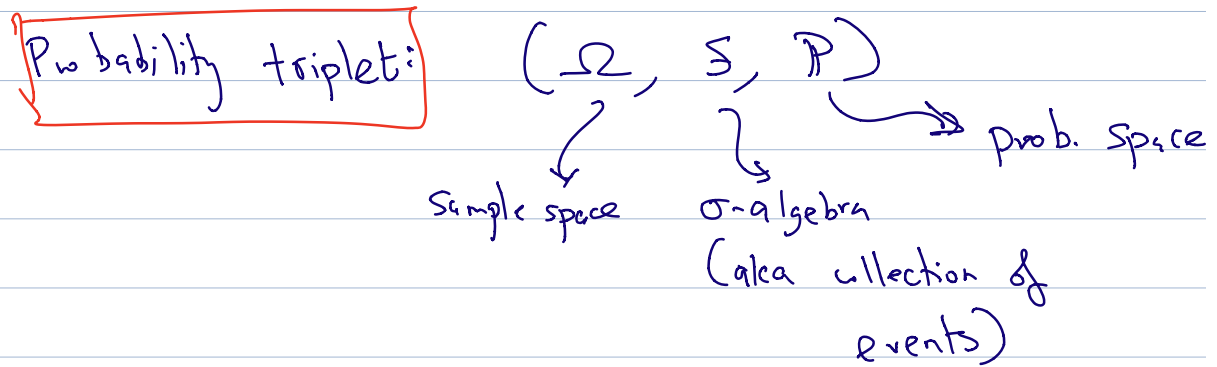
① Reward observed only corresponding to the action taken. (bandit feedback)

② Action does not alter the environment

③ Taking an action does not restrict future actions

Distinguishes bandits from RL

## Probability Review



(Bertsekas & Tsitsiklis, Introduction to Probability)  
Athena Scientific Press

e.g: ① Roll a pair of dies:

$$\Omega = \left\{ (1,1), (1,2), \dots, (1,6), (2,1), \dots, \right. \\ \left. \dots, (6,6) \right\}$$

$$\mathcal{F} = \left\{ \emptyset, \Omega, \{(1,1)\}, \{(1,1), (1,2)\}, \{(1,1), (3,2)\}, \dots \right\}$$

events are subsets of  $\Omega$

$\mathbb{P}: \mathcal{F} \mapsto [0,1]$  the probability "measures"

the "size" of an event.

Fair dies:  $P(\{(1,1)\}) = \frac{1}{36}$

$$P(\{(1,1), (2,2), (3,3)\}) = \frac{3}{36}$$

Random Variable: A map from outcomes of the experiment to the real line.

ex: Continuing roll of dies:

$X$  = Sum of the two rolls.

$X$  can take values:  $\{2, 3, 4, \dots, 12\}$ .

pmf: Probability mass function

$$P_X(x) = P(X=x)$$

example:  $P_X(2) = P(X=2) = \frac{1}{36}$

$$P_X(3) = P(X=3) = \frac{1}{36} + \frac{1}{36} = \frac{1}{18}$$

$\vdots$

$$P_x(12) = P(X=12) = \frac{1}{36}$$

pdf: Probability density function.

Used to model continuous random variables

$$f_x(x) : P(a < X \leq b) \\ = \int_a^b f_x(x) dx.$$

Expectation: (aka mean)

$$E[X] = \begin{cases} \int_{-\infty}^{\infty} x f_x(x) dx & \text{continuous} \\ \sum_{x=-\infty}^{\infty} x p_x(x) & \text{discrete} \end{cases}$$

Variance:  $\text{Var}(X) = \sigma^2 = E[(X - E[X])^2].$

## Conditioning

$A, B$  events; with  $P(B) > 0$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

(re-normalizing w.r.t the universe restricted to  $B$ )

## Joint probability:

$$P_{X,Y}(x,y) = P(X=x, Y=y).$$

Independence:  $P_{X,Y}(x,y) = P_X(x)P_Y(y).$

Notation:  $\{X_1, X_2, \dots, X_n\}$  a sequence of iid r.v.s.

↳ independent and identically distributed

i.e.,  $P_{X_1, X_2, \dots, X_n}(x_1, \dots, x_n)$

product form → independence.

$$= \underbrace{P_{X_1}(x_1)}_{\text{identically dist.}} \cdot \underbrace{P_{X_1}(x_2)}_{\text{all same pmf}} \cdot \underbrace{P_{X_1}(x_3)}_{\text{all same pmf}} \cdot \dots \cdot \underbrace{P_{X_1}(x_n)}_{\text{all same pmf}}$$

## Conditional Expectation

$$E[X|A] = \sum_x x P_{X|A}(x|A)$$

$$E[X|Y=y] = \sum_x x P_{X|Y}(x|y)$$

$$\text{where } P_{X|Y}(x|y) = \frac{P_{X,Y}(x,y)}{P_Y(y)}.$$