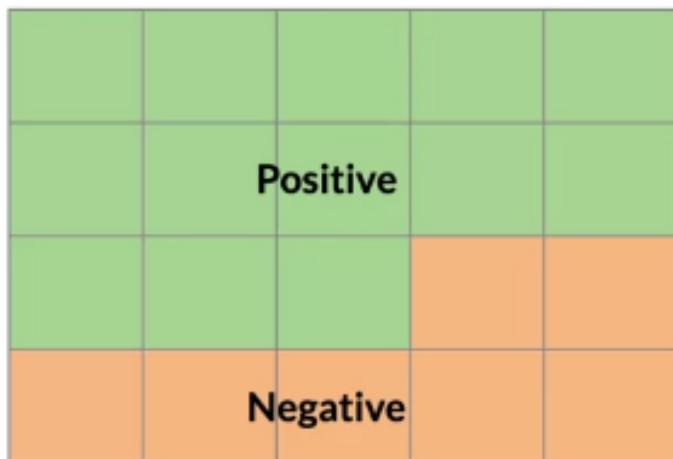


# Outline

- Probabilities
- Bayes' rule (Applied in different fields, including NLP)
- Build your own Naive-Bayes tweet classifier!

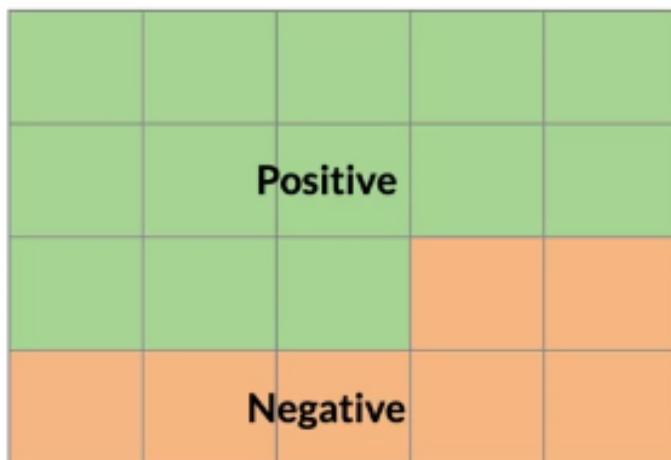
# Introduction

Corpus of tweets



# Introduction

Corpus of tweets

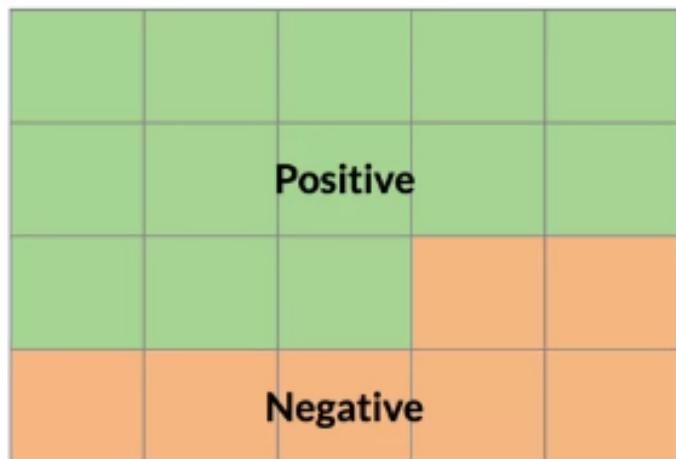


Tweets containing the word  
“happy”



# Probabilities

Corpus of tweets



$A \rightarrow$  Positive tweet

$$P(A) = P(\text{Positive}) = N_{\text{pos}} / N$$

# Probabilities

Corpus of tweets

|  |  |  |  |  |
|--|--|--|--|--|
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

Positive

Negative

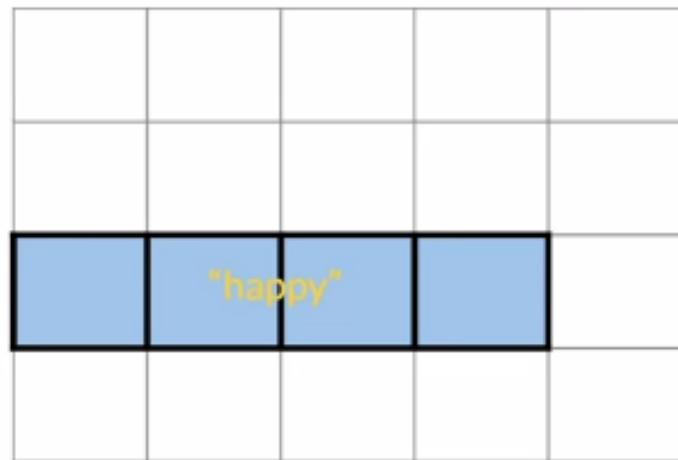
A → Positive tweet

$$P(A) = N_{\text{pos}} / N = 13 / 20 = 0.65$$

$$P(\text{Negative}) = 1 - P(\text{Positive}) = 0.35$$

# Probabilities

Tweets containing the word  
“happy”

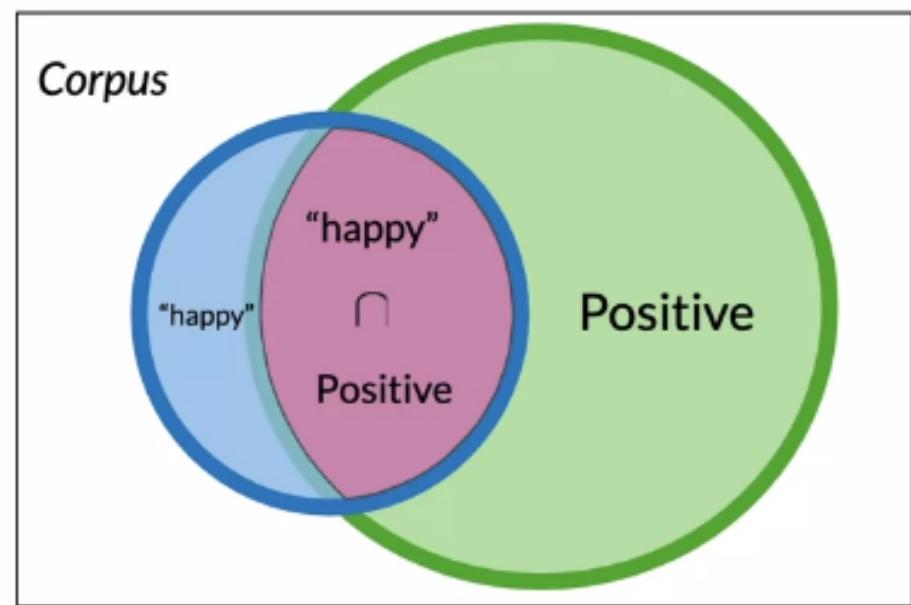


$B \rightarrow$  tweet contains “happy”.

$$P(B) = P(\text{happy}) = N_{\text{happy}} / N$$

$$P(B) = 4 / 20 = 0.2$$

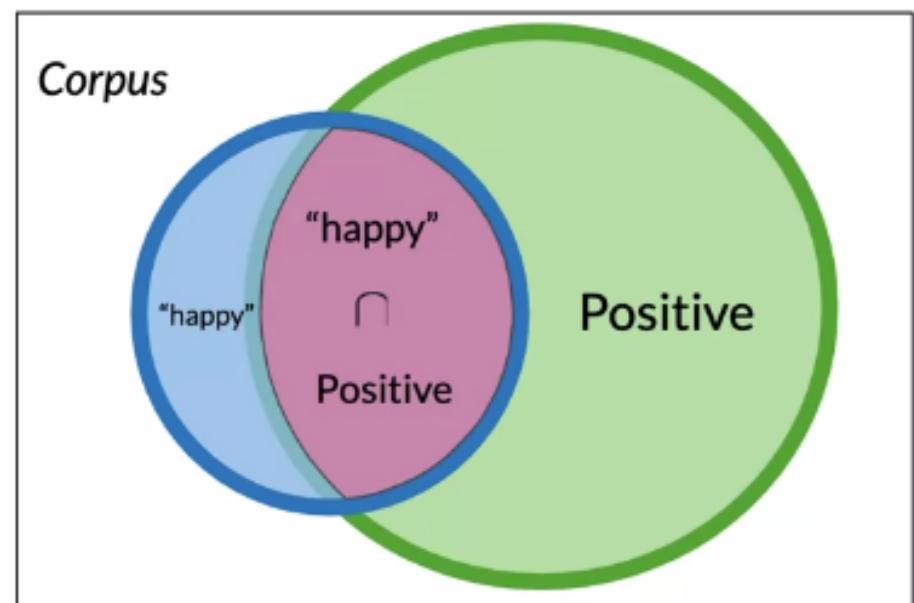
## Probability of the intersection



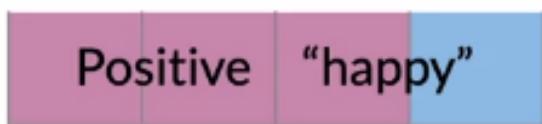
## Probability of the intersection

| Positive |  |  |  |  |
|----------|--|--|--|--|
| “happy”  |  |  |  |  |
|          |  |  |  |  |
|          |  |  |  |  |
|          |  |  |  |  |

$$P(A \cap B) = P(A, B) = \frac{3}{20} = 0.15$$

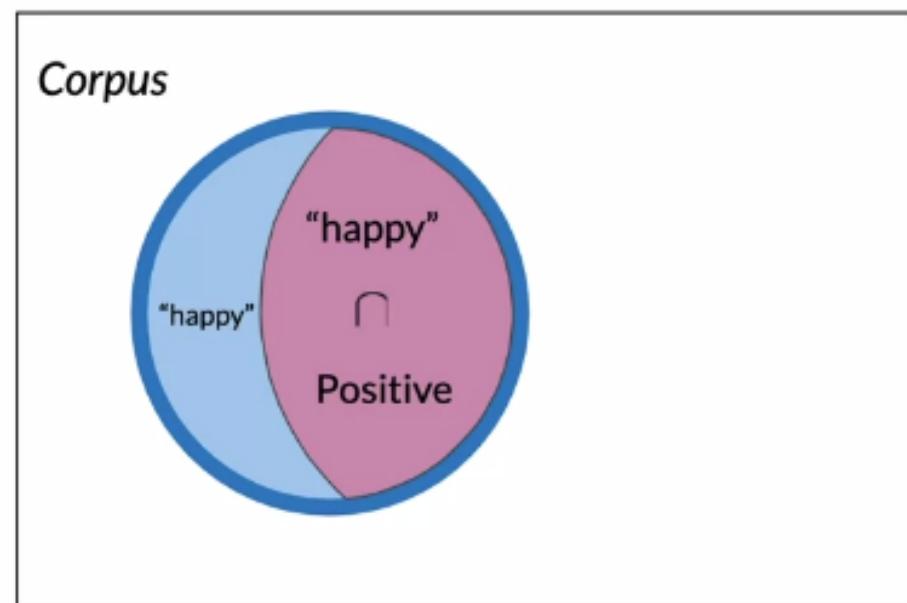


# Conditional Probabilities



$$P(A | B) = P(\text{Positive} | \text{"happy"})$$

$$P(A | B) = 3 / 4 = 0.75$$

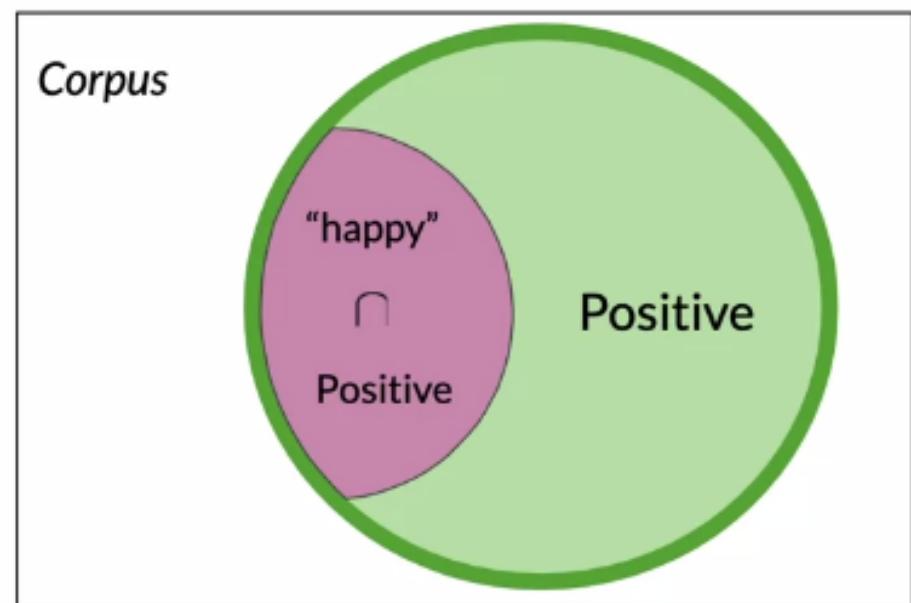


# Conditional Probabilities

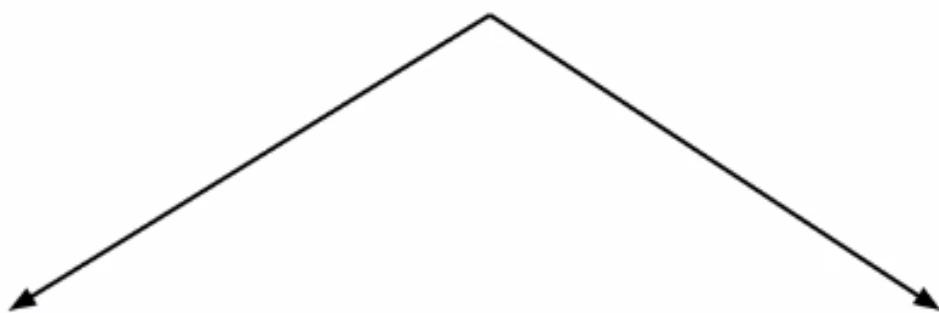
|         |  |  |  |          |
|---------|--|--|--|----------|
|         |  |  |  | Positive |
|         |  |  |  |          |
| “happy” |  |  |  |          |

$$P(B | A) = P(\text{“happy”} | \text{Positive})$$

$$P(B | A) = 3 / 13 = 0.231$$



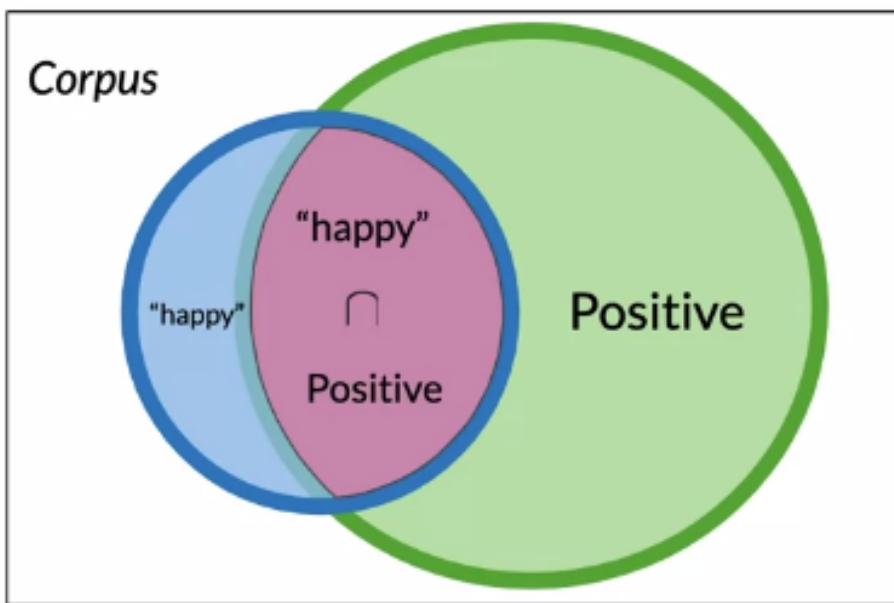
## Conditional probabilities



Probability of B, given A happened

Looking at the elements of set A, the chance that one also belongs to set B

## Conditional probabilities



$$P(\text{Positive} | \text{"happy"}) = \frac{P(\text{Positive} \cap \text{"happy"})}{P(\text{"happy"})}$$

## Bayes' rule

$$P(\text{Positive} | \text{"happy"}) = \frac{P(\text{Positive} \cap \text{"happy"})}{P(\text{"happy"})}$$

## Bayes' rule

$$P(\text{Positive} | \text{"happy"}) = \frac{P(\text{Positive} \cap \text{"happy"})}{P(\text{"happy"})}$$

$$P(\text{"happy"} | \text{Positive}) = \frac{P(\text{"happy"} \cap \text{Positive})}{P(\text{Positive})}$$

## Bayes' rule

$$P(\text{Positive} | \text{"happy"}) = P(\text{"happy"} | \text{Positive}) \times \frac{P(\text{Positive})}{P(\text{"happy"})}$$

$$P(X|Y) = P(Y|X) \times \frac{P(X)}{P(Y)}$$

## Summary

- Conditional probabilities → Bayes' Rule
- $P(X|Y) = P(Y|X) \times \frac{P(X)}{P(Y)}$

# Naïve Bayes for Sentiment Analysis

## Positive tweets

I am happy because I am learning NLP  
I am happy, not sad.

## Negative tweets

I am sad, I am not learning NLP  
I am sad, not happy

# Naïve Bayes for Sentiment Analysis

## Positive tweets

I am happy because I am learning NLP  
I am happy, not sad.

## Negative tweets

I am sad, I am not learning NLP  
I am sad, not happy

---

word

---

I

am

happy

because

learning

NLP

sad

not

---

# Naïve Bayes for Sentiment Analysis

## Positive tweets

I am happy because I am learning NLP

I am happy, not sad.

## Negative tweets

I am sad, I am not learning NLP

I am sad, not happy

| word                     | Pos       | Neg       |
|--------------------------|-----------|-----------|
| I                        | 3         | 3         |
| am                       | 3         | 3         |
| happy                    | 2         | 1         |
| because                  | 1         | 0         |
| learning                 | 1         | 1         |
| NLP                      | 1         | 1         |
| sad                      | 1         | 2         |
| not                      | 1         | 2         |
| <b>N<sub>class</sub></b> | <b>13</b> | <b>12</b> |

$P(w_i | \text{class})$

| word          | Pos       | Neg       |
|---------------|-----------|-----------|
| I             | 3         | 3         |
| am            | 3         | 3         |
| happy         | 2         | 1         |
| because       | 1         | 0         |
| learning      | 1         | 1         |
| NLP           | 1         | 1         |
| sad           | 1         | 2         |
| not           | 1         | 2         |
| <b>Nclass</b> | <b>13</b> | <b>12</b> |

$$p(I|Pos) = \frac{3}{13}$$

| word | Pos  | Neg |
|------|------|-----|
| I    | 0.24 |     |

$P(w_i | \text{class})$

| word          | Pos       | Neg       |
|---------------|-----------|-----------|
| I             | 3         | 3         |
| am            | 3         | 3         |
| happy         | 2         | 1         |
| because       | 1         | 0         |
| learning      | 1         | 1         |
| NLP           | 1         | 1         |
| sad           | 1         | 2         |
| not           | 1         | 2         |
| <b>Nclass</b> | <b>13</b> | <b>12</b> |

$$p(I|Neg) = \frac{3}{12}$$

| word | Pos  | Neg  |
|------|------|------|
| I    | 0.24 | 0.25 |

$P(w_i | \text{class})$ 

| word          | Pos       | Neg       |
|---------------|-----------|-----------|
| I             | 3         | 3         |
| am            | 3         | 3         |
| happy         | 2         | 1         |
| because       | 1         | 0         |
| learning      | 1         | 1         |
| NLP           | 1         | 1         |
| sad           | 1         | 2         |
| not           | 1         | 2         |
| <b>Nclass</b> | <b>13</b> | <b>12</b> |

| word       | Pos      | Neg      |
|------------|----------|----------|
| I          | 0.24     | 0.25     |
| am         | 0.24     | 0.25     |
| happy      | 0.15     | 0.08     |
| because    | 0.08     | 0.00     |
| learning   | 0.08     | 0.08     |
| NLP        | 0.08     | 0.08     |
| sad        | 0.08     | 0.17     |
| not        | 0.08     | 0.17     |
| <b>Sum</b> | <b>1</b> | <b>1</b> |

$P(w_i \mid \text{class})$ 

| word     | Pos  | Neg  |
|----------|------|------|
| I        | 0.24 | 0.25 |
| am       | 0.24 | 0.25 |
| happy    | 0.15 | 0.08 |
| because  | 0.08 | 0    |
| learning | 0.08 | 0.08 |
| NLP      | 0.08 | 0.08 |
| sad      | 0.08 | 0.17 |
| not      | 0.08 | 0.17 |

# Naïve Bayes

Tweet: I am happy today; I am learning.

$$\prod_{i=1}^m \frac{P(w_i|pos)}{P(w_i|neg)}$$

| word     | Pos  | Neg  |
|----------|------|------|
| I        | 0.20 | 0.20 |
| am       | 0.20 | 0.20 |
| happy    | 0.14 | 0.10 |
| because  | 0.10 | 0.05 |
| learning | 0.10 | 0.10 |
| NLP      | 0.10 | 0.10 |
| sad      | 0.10 | 0.15 |
| not      | 0.10 | 0.15 |

# Naïve Bayes

Tweet: I am happy today; I am learning.

$$\prod_{i=1}^m \frac{P(w_i|pos)}{P(w_i|neg)}$$

$$\frac{0.20}{0.20}$$

| word     | Pos  | Neg  |
|----------|------|------|
| I        | 0.20 | 0.20 |
| am       | 0.20 | 0.20 |
| happy    | 0.14 | 0.10 |
| because  | 0.10 | 0.05 |
| learning | 0.10 | 0.10 |
| NLP      | 0.10 | 0.10 |
| sad      | 0.10 | 0.15 |
| not      | 0.10 | 0.15 |

# Naïve Bayes

Tweet: I am happy today; I am learning.

$$\prod_{i=1}^m \frac{P(w_i|pos)}{P(w_i|neg)}$$

$$\frac{0.20}{0.20} * \frac{0.20}{0.20}$$

| word     | Pos  | Neg  |
|----------|------|------|
| I        | 0.20 | 0.20 |
| am       | 0.20 | 0.20 |
| happy    | 0.14 | 0.10 |
| because  | 0.10 | 0.05 |
| learning | 0.10 | 0.10 |
| NLP      | 0.10 | 0.10 |
| sad      | 0.10 | 0.15 |
| not      | 0.10 | 0.15 |

# Naïve Bayes

Tweet: I am happy today; I am learning.

$$\prod_{i=1}^m \frac{P(w_i|pos)}{P(w_i|neg)}$$

$$\frac{0.20}{0.20} * \frac{0.20}{0.20} * \frac{0.14}{0.10}$$

| word     | Pos  | Neg  |
|----------|------|------|
| I        | 0.20 | 0.20 |
| am       | 0.20 | 0.20 |
| happy    | 0.14 | 0.10 |
| because  | 0.10 | 0.05 |
| learning | 0.10 | 0.10 |
| NLP      | 0.10 | 0.10 |
| sad      | 0.10 | 0.15 |
| not      | 0.10 | 0.15 |

# Naïve Bayes

Tweet: I am happy today; I am learning.

$$\prod_{i=1}^m \frac{P(w_i|pos)}{P(w_i|neg)}$$

$$\frac{0.20}{0.20} * \frac{0.20}{0.20} * \frac{0.14}{0.10}$$

| word     | Pos  | Neg  |
|----------|------|------|
| I        | 0.20 | 0.20 |
| am       | 0.20 | 0.20 |
| happy    | 0.14 | 0.10 |
| because  | 0.10 | 0.05 |
| learning | 0.10 | 0.10 |
| NLP      | 0.10 | 0.10 |
| sad      | 0.10 | 0.15 |
| not      | 0.10 | 0.15 |

# Naïve Bayes

Tweet: I am happy today; I am learning.

$$\prod_{i=1}^m \frac{P(w_i|pos)}{P(w_i|neg)}$$

$$\frac{0.20}{0.20} * \frac{0.20}{0.20} * \frac{0.14}{0.10} * \frac{0.20}{0.20}$$

| word     | Pos  | Neg  |
|----------|------|------|
| I        | 0.20 | 0.20 |
| am       | 0.20 | 0.20 |
| happy    | 0.14 | 0.10 |
| because  | 0.10 | 0.05 |
| learning | 0.10 | 0.10 |
| NLP      | 0.10 | 0.10 |
| sad      | 0.10 | 0.15 |
| not      | 0.10 | 0.15 |

# Naïve Bayes

Tweet: I am happy today; I am learning.

$$\prod_{i=1}^m \frac{P(w_i|pos)}{P(w_i|neg)}$$

$$\frac{0.20}{0.20} * \frac{0.20}{0.20} * \frac{0.14}{0.10} * \frac{0.20}{0.20} * \frac{0.20}{0.20}$$

| word     | Pos  | Neg  |
|----------|------|------|
| I        | 0.20 | 0.20 |
| am       | 0.20 | 0.20 |
| happy    | 0.14 | 0.10 |
| because  | 0.10 | 0.05 |
| learning | 0.10 | 0.10 |
| NLP      | 0.10 | 0.10 |
| sad      | 0.10 | 0.15 |
| not      | 0.10 | 0.15 |

# Naïve Bayes

Tweet: I am happy today; I am learning.

$$\prod_{i=1}^m \frac{P(w_i|pos)}{P(w_i|neg)}$$

$$\frac{0.20}{0.20} * \frac{0.20}{0.20} * \frac{0.14}{0.10} * \frac{0.20}{0.20} * \frac{0.20}{0.20} * \frac{0.10}{0.10}$$

| word     | Pos  | Neg  |
|----------|------|------|
| I        | 0.20 | 0.20 |
| am       | 0.20 | 0.20 |
| happy    | 0.14 | 0.10 |
| because  | 0.10 | 0.05 |
| learning | 0.10 | 0.10 |
| NLP      | 0.10 | 0.10 |
| sad      | 0.10 | 0.15 |
| not      | 0.10 | 0.15 |

# Naïve Bayes

Tweet: I am happy today; I am learning.

$$\prod_{i=1}^m \frac{P(w_i|pos)}{P(w_i|neg)}$$
$$\frac{0.20}{0.20} * \frac{0.20}{0.20} * \frac{0.14}{0.10} * \frac{0.20}{0.20} * \frac{0.20}{0.20} * \frac{0.10}{0.10}$$

| word     | Pos  | Neg  |
|----------|------|------|
| I        | 0.20 | 0.20 |
| am       | 0.20 | 0.20 |
| happy    | 0.14 | 0.10 |
| because  | 0.10 | 0.05 |
| learning | 0.10 | 0.10 |
| NLP      | 0.10 | 0.10 |
| sad      | 0.10 | 0.15 |
| not      | 0.10 | 0.15 |

# Naïve Bayes

Tweet: I am happy today; I am learning.

$$\prod_{i=1}^m \frac{P(w_i|pos)}{P(w_i|neg)} = \frac{0.14}{0.10} = 1.4$$

$$\cancel{\frac{0.20}{0.20}} * \cancel{\frac{0.20}{0.20}} * \boxed{\frac{0.14}{0.10}} * \cancel{\frac{0.20}{0.20}} * \cancel{\frac{0.20}{0.20}} * \cancel{\frac{0.10}{0.10}}$$

| word     | Pos  | Neg  |
|----------|------|------|
| I        | 0.20 | 0.20 |
| am       | 0.20 | 0.20 |
| happy    | 0.14 | 0.10 |
| because  | 0.10 | 0.05 |
| learning | 0.10 | 0.10 |
| NLP      | 0.10 | 0.10 |
| sad      | 0.10 | 0.15 |
| not      | 0.10 | 0.15 |

# Naïve Bayes

Tweet: I am happy today; I am learning.

$$\prod_{i=1}^m \frac{P(w_i|pos)}{P(w_i|neg)} = \frac{0.14}{0.10} = 1.4 > 1$$

$$\cancel{\frac{0.20}{0.20}} * \cancel{\frac{0.20}{0.20}} * \boxed{\frac{0.14}{0.10}} * \cancel{\frac{0.20}{0.20}} * \cancel{\frac{0.20}{0.20}} * \cancel{\frac{0.10}{0.10}}$$

| word     | Pos  | Neg  |
|----------|------|------|
| I        | 0.20 | 0.20 |
| am       | 0.20 | 0.20 |
| happy    | 0.14 | 0.10 |
| because  | 0.10 | 0.05 |
| learning | 0.10 | 0.10 |
| NLP      | 0.10 | 0.10 |
| sad      | 0.10 | 0.15 |
| not      | 0.10 | 0.15 |

# Summary

- Naive Bayes inference condition rule for binary classification
- Table of probabilities

$$\prod_{i=1}^m \frac{P(w_i|pos)}{P(w_i|neg)}$$

## Laplacian Smoothing

$$P(w_i | \text{class}) = \frac{\text{freq}(w_i, \text{class})}{N_{\text{class}}} \quad \text{class} \in \{\text{Positive}, \text{Negative}\}$$

$N_{\text{class}}$  = frequency of all words in class

$V$  = number of unique words in vocabulary

## Laplacian Smoothing

$$P(w_i|class) = \frac{\text{freq}(w_i, \text{class})}{N_{\text{class}}} \quad \text{class} \in \{\text{Positive}, \text{Negative}\}$$

$$P(w_i|class) = \frac{\text{freq}(w_i, \text{class}) + 1}{N_{\text{class}} + V}$$

$N_{\text{class}}$  = frequency of all words in class

$V$  = number of unique words in vocabulary

## Introducing $P(w_i | \text{class})$ with smoothing

| word          | Pos       | Neg       |
|---------------|-----------|-----------|
| I             | 3         | 3         |
| am            | 3         | 3         |
| happy         | 2         | 1         |
| because       | 1         | 0         |
| learning      | 1         | 1         |
| NLP           | 1         | 1         |
| sad           | 1         | 2         |
| not           | 1         | 2         |
| <b>Nclass</b> | <b>13</b> | <b>12</b> |

| word | Pos | Neg |
|------|-----|-----|
| I    | -   | -   |

$$\nabla = 8$$

## Introducing $P(w_i | \text{class})$ with smoothing

| word          | Pos       | Neg       |
|---------------|-----------|-----------|
| I             | 3         | 3         |
| am            | 3         | 3         |
| happy         | 2         | 1         |
| because       | 1         | 0         |
| learning      | 1         | 1         |
| NLP           | 1         | 1         |
| sad           | 1         | 2         |
| not           | 1         | 2         |
| <b>Nclass</b> | <b>13</b> | <b>12</b> |

$$P(I|Pos) = \frac{3+1}{13+8}$$

$$\nu = 8$$

| word | Pos | Neg |
|------|-----|-----|
| I    | -   | -   |

## Introducing $P(w_i | \text{class})$ with smoothing

| word          | Pos       | Neg       |
|---------------|-----------|-----------|
| I             | 3         | 3         |
| am            | 3         | 3         |
| happy         | 2         | 1         |
| because       | 1         | 0         |
| learning      | 1         | 1         |
| NLP           | 1         | 1         |
| sad           | 1         | 2         |
| not           | 1         | 2         |
| <b>Nclass</b> | <b>13</b> | <b>12</b> |

$$P(I|Pos) = \frac{3+1}{13+8}$$

$$\nu = 8$$

| word | Pos  | Neg |
|------|------|-----|
| I    | 0.19 |     |

## Introducing $P(w_i | \text{class})$ with smoothing

| word          | Pos       | Neg       |
|---------------|-----------|-----------|
| I             | 3         | 3         |
| am            | 3         | 3         |
| happy         | 2         | 1         |
| because       | 1         | 0         |
| learning      | 1         | 1         |
| NLP           | 1         | 1         |
| sad           | 1         | 2         |
| not           | 1         | 2         |
| <b>Nclass</b> | <b>13</b> | <b>12</b> |

$$P(I|Neg) = \frac{3+1}{12+8}$$

$$\textcolor{blue}{V} = 8$$

| word | Pos  | Neg |
|------|------|-----|
| I    | 0.19 | -   |

## Introducing $P(w_i | \text{class})$ with smoothing

| word          | Pos       | Neg       |
|---------------|-----------|-----------|
| I             | 3         | 3         |
| am            | 3         | 3         |
| happy         | 2         | 1         |
| because       | 1         | 0         |
| learning      | 1         | 1         |
| NLP           | 1         | 1         |
| sad           | 1         | 2         |
| not           | 1         | 2         |
| <b>Nclass</b> | <b>13</b> | <b>12</b> |

$$P(I|Neg) = \frac{3+1}{12+8}$$

$$\Delta = 8$$

| word | Pos  | Neg  |
|------|------|------|
| I    | 0.19 | 0.20 |

## Introducing $P(w_i | \text{class})$ with smoothing

| word          | Pos       | Neg       |
|---------------|-----------|-----------|
| I             | 3         | 3         |
| am            | 3         | 3         |
| happy         | 2         | 1         |
| because       | 1         | 0         |
| learning      | 1         | 1         |
| NLP           | 1         | 1         |
| sad           | 1         | 2         |
| not           | 1         | 2         |
| <b>Nclass</b> | <b>13</b> | <b>12</b> |

$$\sqrt{ } = 8$$

| word     | Pos  | Neg  |
|----------|------|------|
| I        | 0.19 | 0.20 |
| am       | 0.19 | 0.20 |
| happy    | 0.14 | 0.10 |
| because  | 0.10 | 0.05 |
| learning | 0.10 | 0.10 |
| NLP      | 0.10 | 0.10 |
| sad      | 0.10 | 0.15 |
| not      | 0.10 | 0.15 |

## Introducing $P(w_i | \text{class})$ with smoothing

| word          | Pos       | Neg       |
|---------------|-----------|-----------|
| I             | 3         | 3         |
| am            | 3         | 3         |
| happy         | 2         | 1         |
| because       | 1         | 0         |
| learning      | 1         | 1         |
| NLP           | 1         | 1         |
| sad           | 1         | 2         |
| not           | 1         | 2         |
| <b>Nclass</b> | <b>13</b> | <b>12</b> |

$$\sqrt{ } = 8$$

| word       | Pos      | Neg      |
|------------|----------|----------|
| I          | 0.19     | 0.20     |
| am         | 0.19     | 0.20     |
| happy      | 0.14     | 0.10     |
| because    | 0.10     | 0.05     |
| learning   | 0.10     | 0.10     |
| NLP        | 0.10     | 0.10     |
| sad        | 0.10     | 0.15     |
| not        | 0.10     | 0.15     |
| <b>Sum</b> | <b>1</b> | <b>1</b> |

## Summary

- Laplacian smoothing to avoid  $P(w_i|class) = 0$
- Naïve Bayes formula

$$\prod_{i=1}^m \frac{P(w_i|pos)}{P(w_i|neg)}$$

## Ratio of probabilities

| word     | Pos  | Neg  | ratio |
|----------|------|------|-------|
| I        | 0.20 | 0.20 |       |
| am       | 0.20 | 0.20 |       |
| happy    | 0.14 | 0.10 |       |
| because  | 0.10 | 0.10 |       |
| learning | 0.10 | 0.10 |       |
| NLP      | 0.10 | 0.10 |       |
| sad      | 0.10 | 0.15 |       |
| not      | 0.10 | 0.15 |       |

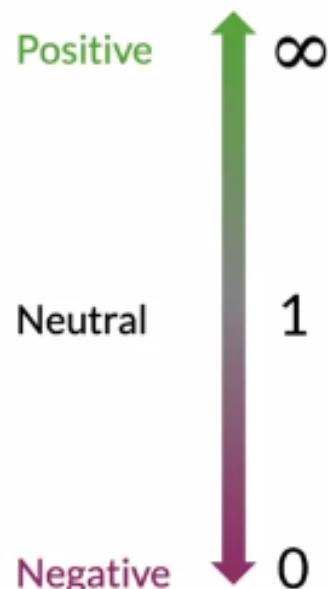
$$\text{ratio}(w_i) = \frac{P(w_i | \text{Pos})}{P(w_i | \text{Neg})}$$

## Ratio of probabilities

| word     | Pos  | Neg  | ratio |
|----------|------|------|-------|
| I        | 0.20 | 0.20 | 1     |
| am       | 0.20 | 0.20 | 1     |
| happy    | 0.14 | 0.10 | 1.4   |
| because  | 0.10 | 0.10 | 1     |
| learning | 0.10 | 0.10 | 1     |
| NLP      | 0.10 | 0.10 | 1     |
| sad      | 0.10 | 0.15 | 0.6   |
| not      | 0.10 | 0.15 | 0.6   |

$$\text{ratio}(w_i) = \frac{P(w_i | \text{Pos})}{P(w_i | \text{Neg})}$$

## Ratio of probabilities



| word     | Pos  | Neg  | ratio |
|----------|------|------|-------|
| I        | 0.20 | 0.20 | 1     |
| am       | 0.20 | 0.20 | 1     |
| happy    | 0.14 | 0.10 | 1.4   |
| because  | 0.10 | 0.10 | 1     |
| learning | 0.10 | 0.10 | 1     |
| NLP      | 0.10 | 0.10 | 1     |
| sad      | 0.10 | 0.15 | 0.6   |
| not      | 0.10 | 0.15 | 0.6   |

$$\text{ratio}(w_i) = \frac{P(w_i | \text{Pos})}{P(w_i | \text{Neg})}$$

$\approx$  
$$\frac{\text{freq}(w_i, 1) + 1}{\text{freq}(w_i, 0) + 1}$$

## Naïve Bayes' inference

$\text{class} \in \{\text{pos}, \text{neg}\}$

$w \rightarrow \text{Set of } m \text{ words in a tweet}$

$$\frac{P(\text{pos})}{P(\text{neg})} \prod_{i=1}^m \frac{P(w_i|\text{pos})}{P(w_i|\text{neg})} > 1$$

- A simple, fast, and powerful baseline
- A probabilistic model used for classification