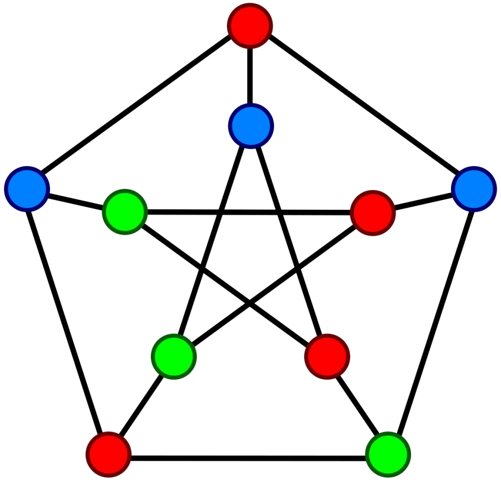
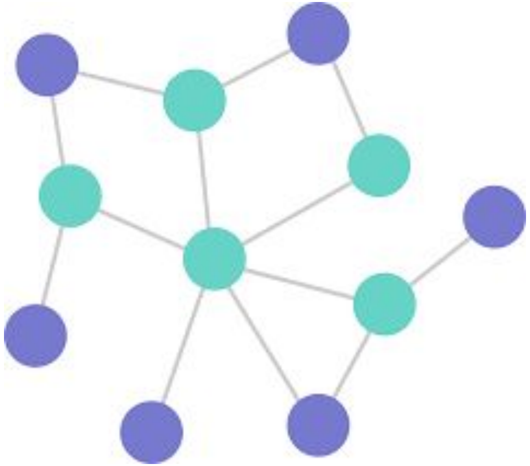
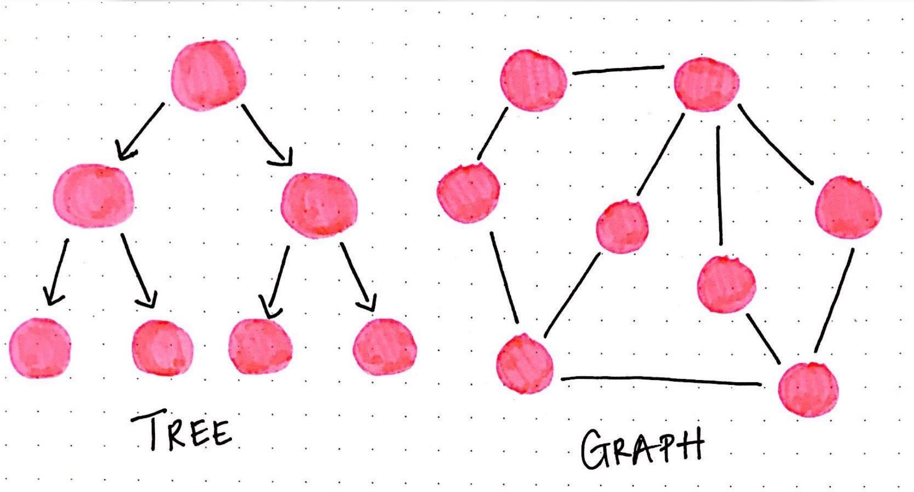


Graph Theory

AI Inspire

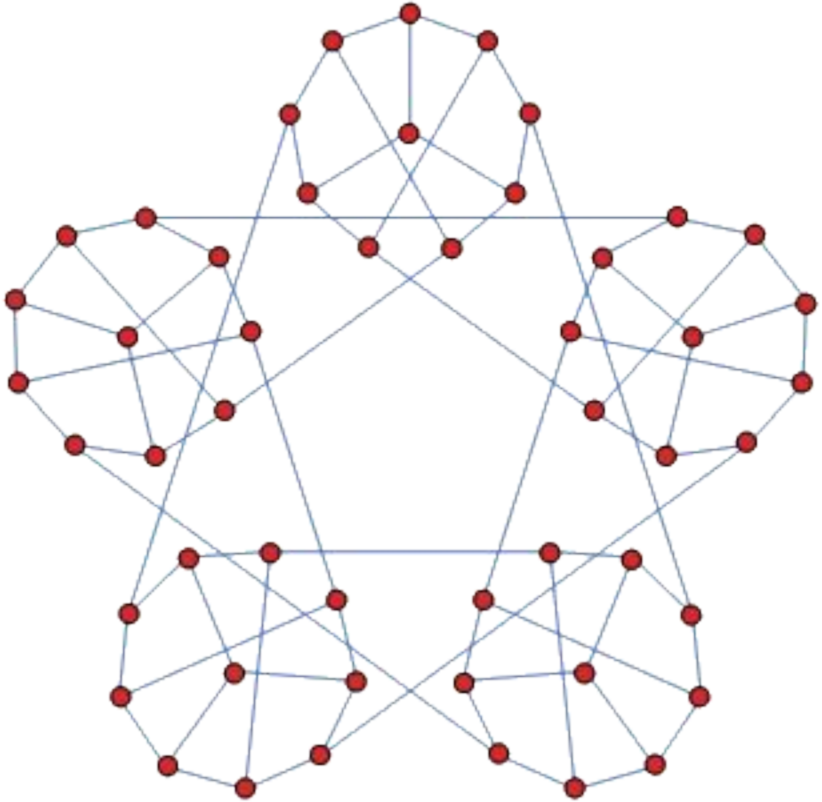


Basics



What is Graph Theory?

- ➔ Mathematical study/theory of graphs
  - ◆ Properties
  - ◆ Applications
- ➔ Graph is composed of
  - ◆ Vertices/Nodes/Points
  - ◆ Connected by Edges/Arcs/Lines
- ➔ Study of **relationships**



Importance/Impact

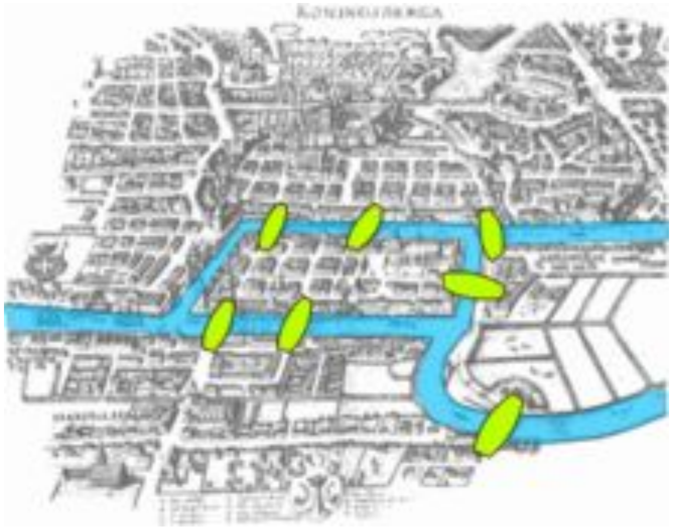
- ➔ Used in a multitude of diff. Comp. apps
  - ◆ Use algorithms to tackle diff. Problems
    - ● Ex. : Shortest path problem
- ➔ Simpler to express data in graph vs. table
  - ◆ See relationships between data
  - ◆ Better understanding of data

Where is it used?

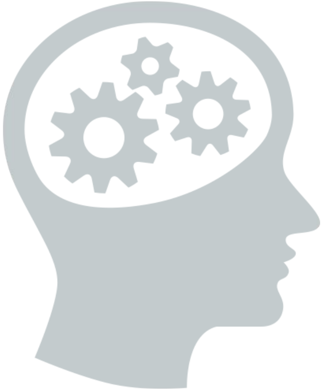
- Social media
  - Social networks
    - Facebook
    - Instagram
    - Snapchat, etc.
  - Connect Platforms
    - Connecting students or prof. With each other
- Shortest Route
  - GPS

History

- ➔ Leonhard Euler - 1736
  - ◆ Greatest mathematician
  - ◆ Founder of Graph Theory
  - ◆ 7 Bridges of Konigsberg Bridge Problem

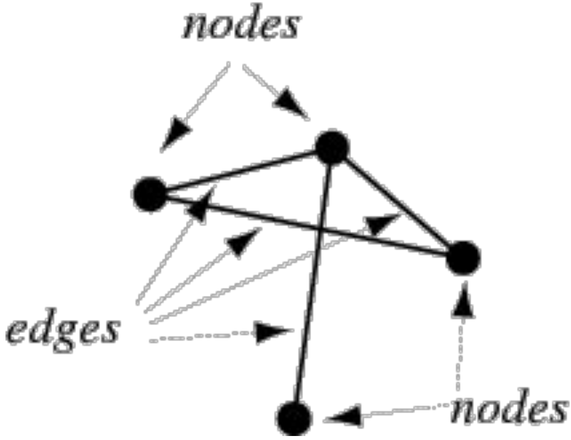


- ➔ Activity
  - ◆ https://www.mathsisfun.com/activity/seven-bridges-konigsberg.html

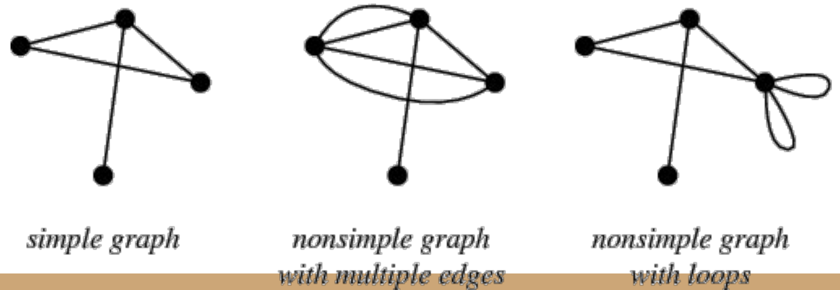


Vocabulary

Simple Vocab

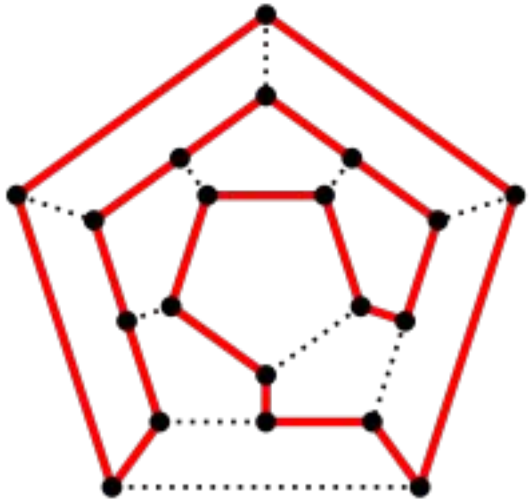


- ➔ Graph
  - ◆ Set of vertices/nodes connected with a set of edges
- ➔ Loop
  - ◆ Edge with same vertex at each end
- ➔ Degree of vertex
  - ◆ Number of edges which have that vertex as an endpoint
- ➔ Simple Graph
  - ◆ Graph with no loops
  - ◆ No more than one edge connecting a pair of vertices

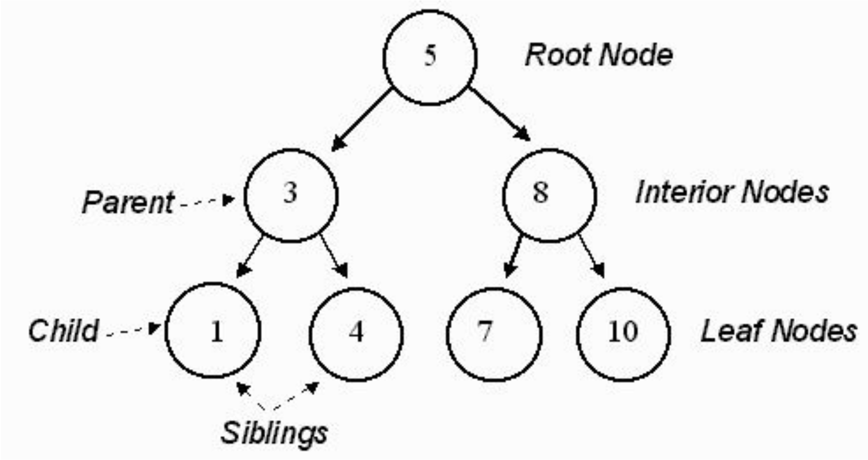


Simple Vocab (cont.)

- Walk
  - Sequence of edges where each edge is the beginning of the consecutively next edge (except last)
- Trail
  - No edge is repeated
- Path
  - No vertex is repeated
- Cycle
  - Closed path
- Hamiltonian cycle
  - Cycle which visits every vertex
    - Each vertex is only visited once
  - Cycle is a path



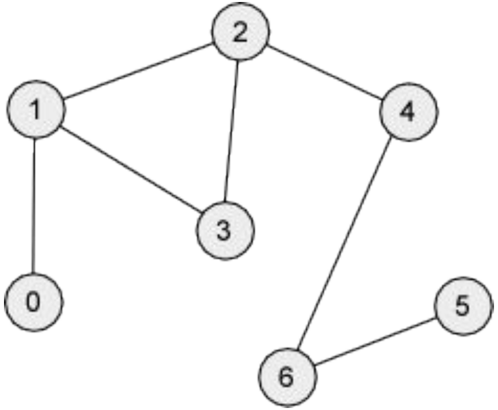
Terms & Definitions

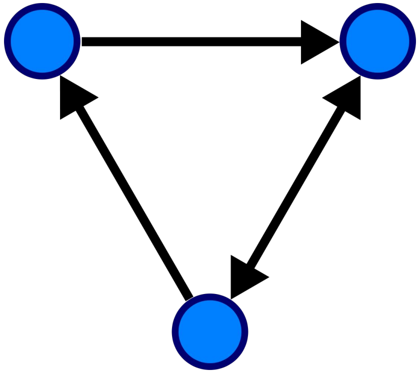


- Connected graph
  - Path exists between every pair of vertices
- Tree
  - Simple connected graph
  - **NO CYCLES!**
- Complete graph
  - Simple graph where every pair of vertices is connected by an edge

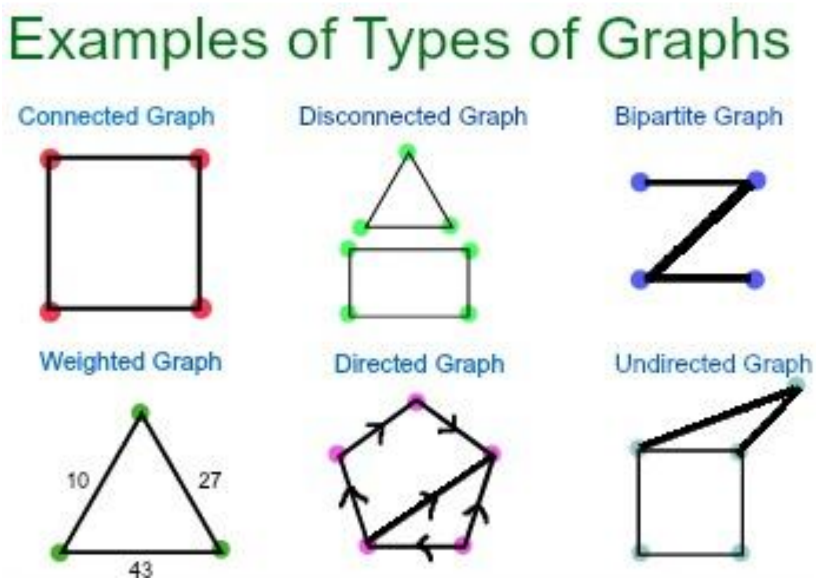
Types of Graphs

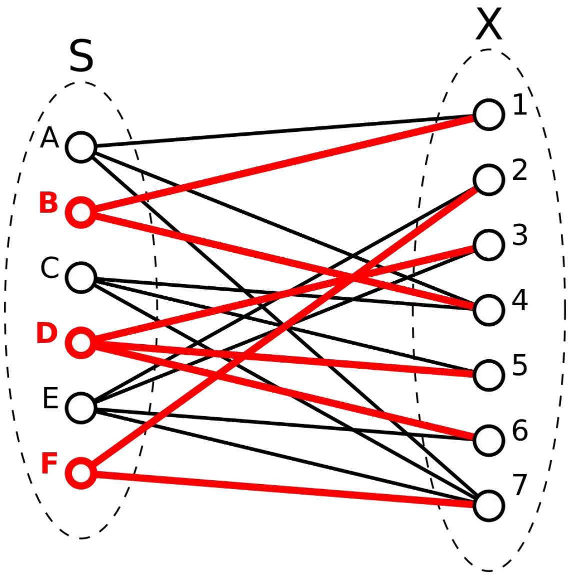
- ➔ Bipartite graph
  - ◆ 2 disjoint sets (no elements in common & no vertices between 2 sets are adj.)
  - ◆ Bigraph
- ➔ Directed graph
  - ◆ All edges are directed
  - ◆ Particular direction
- ➔ Undirected graph
- ➔ DAG (Directed Acyclic Graph)

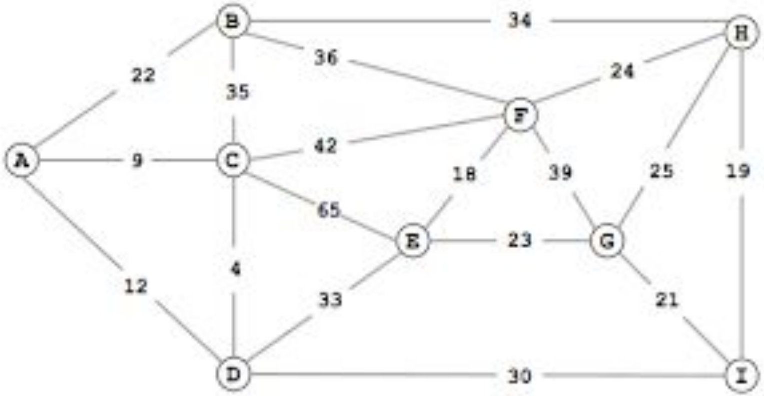




Types of Graphs

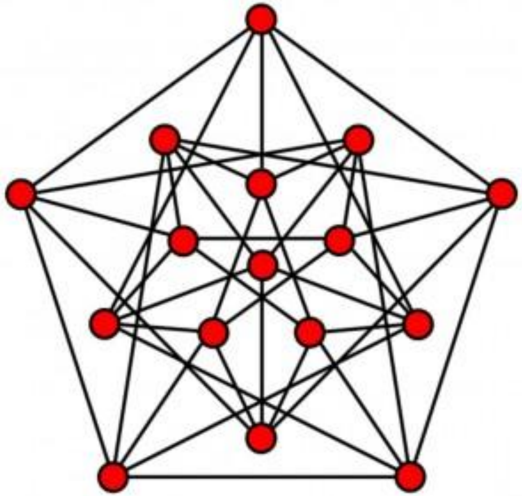




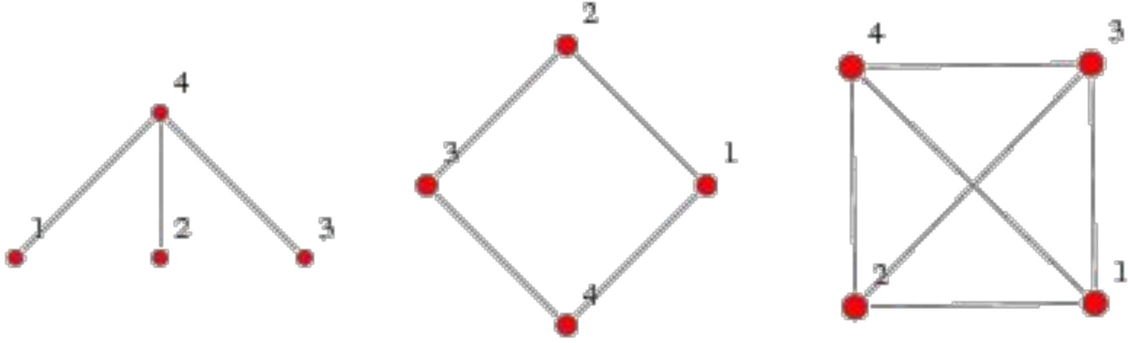


Representing Graphs

- ➔ Adjacency matrix
  - ◆ Square n * n matrix
  - ◆ Each number indicates whether vertices are adjacent or not
    - ● Diagonal : has 0s : vertice can't be adjacent to itself
  - ◆ 2D array in Java
- ➔ Adjacency list
  - ◆ Represents graph with <u>list</u>
  - ◆ Represents if vertices are adjacent to each other in graph
  - ◆ Represents length if directed graph between adjacent vertices

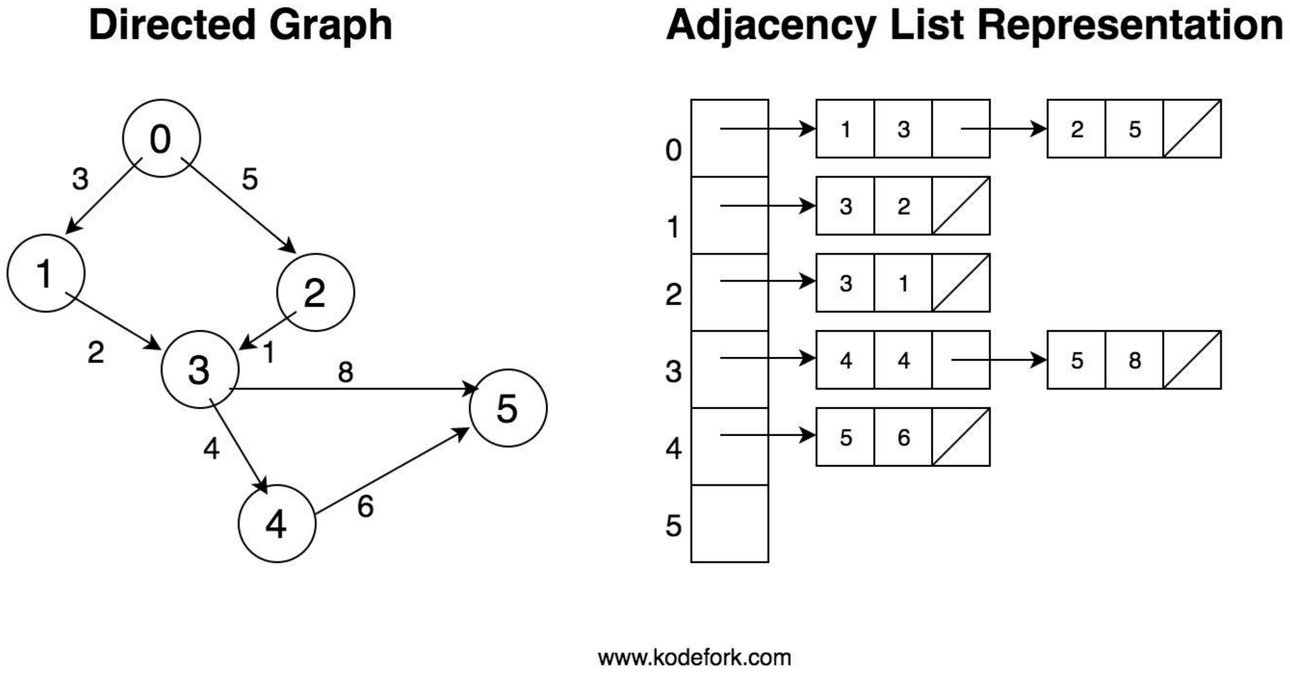


Adjacency Matrix

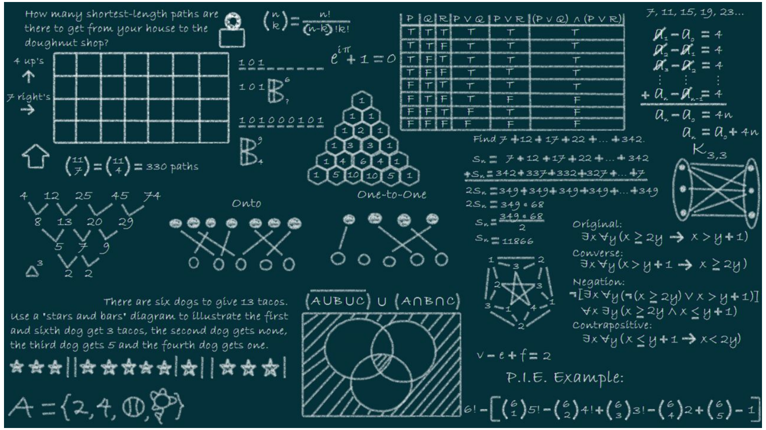


$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Adjacency List



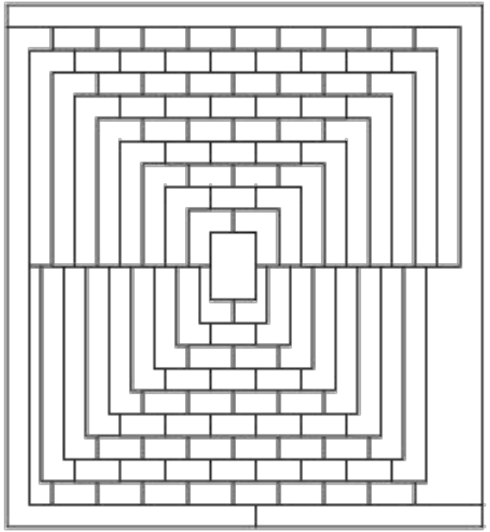
Algorithms & Activities



Activity 1 : How Many Colors?

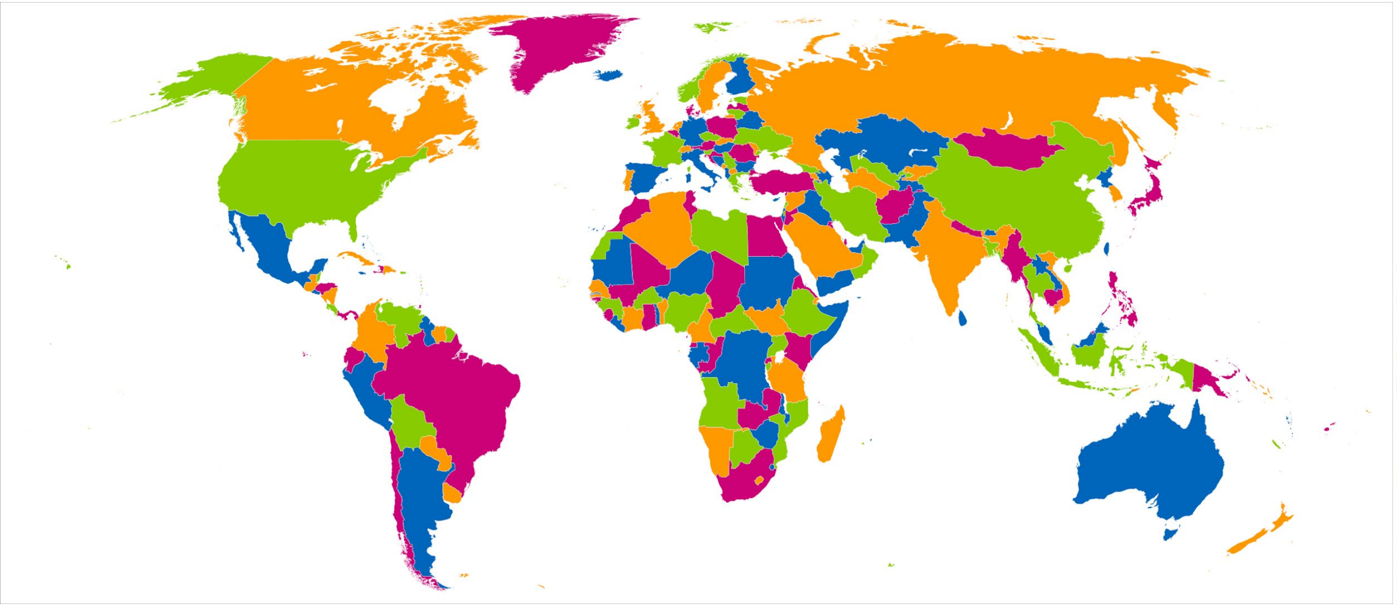
https://www.colorado.edu/education/DMP/activities/graph/ddgact03.html

https://www.colorado.edu/education/DMP/activities/graph/ddghnd03.html



4 COLOR THEOREM

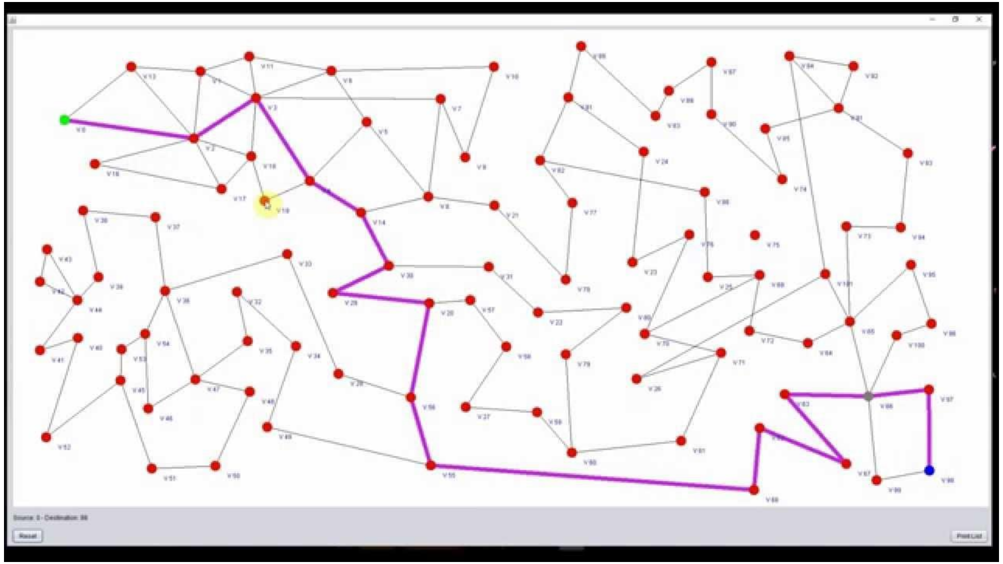
- ➔ No more than 4 colors are needed to color regions of map
  - ◆ No 2 adjacent regions have same color



Activity 2 : Shortest Route Problem

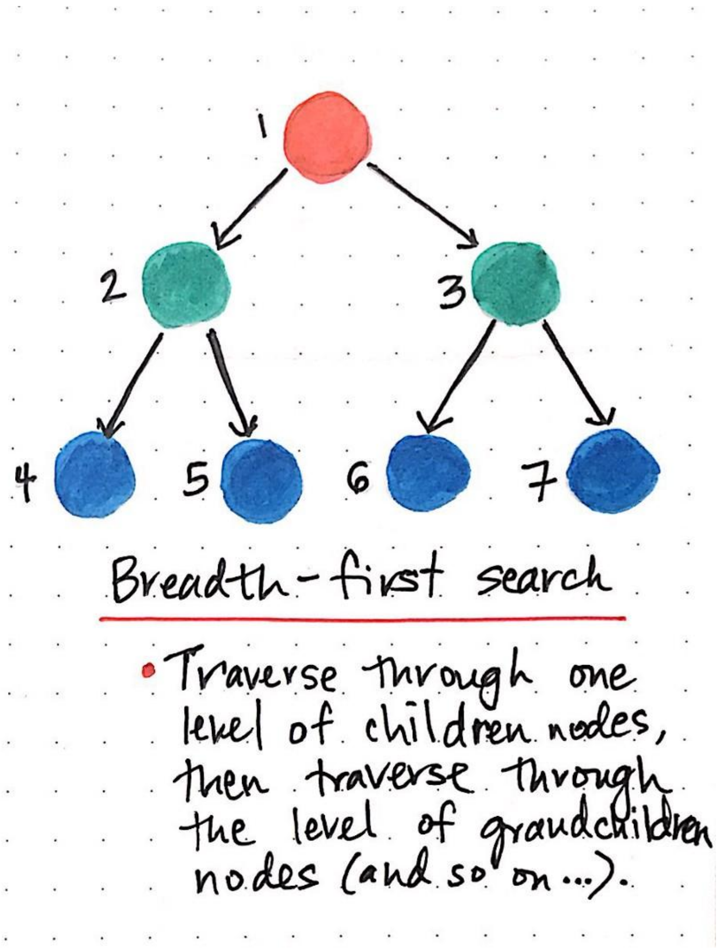
https://www.colorado.edu/education/DMP/activities/graph/ddgact04.html

https://www.colorado.edu/education/DMP/activities/graph/ddghnd04.html



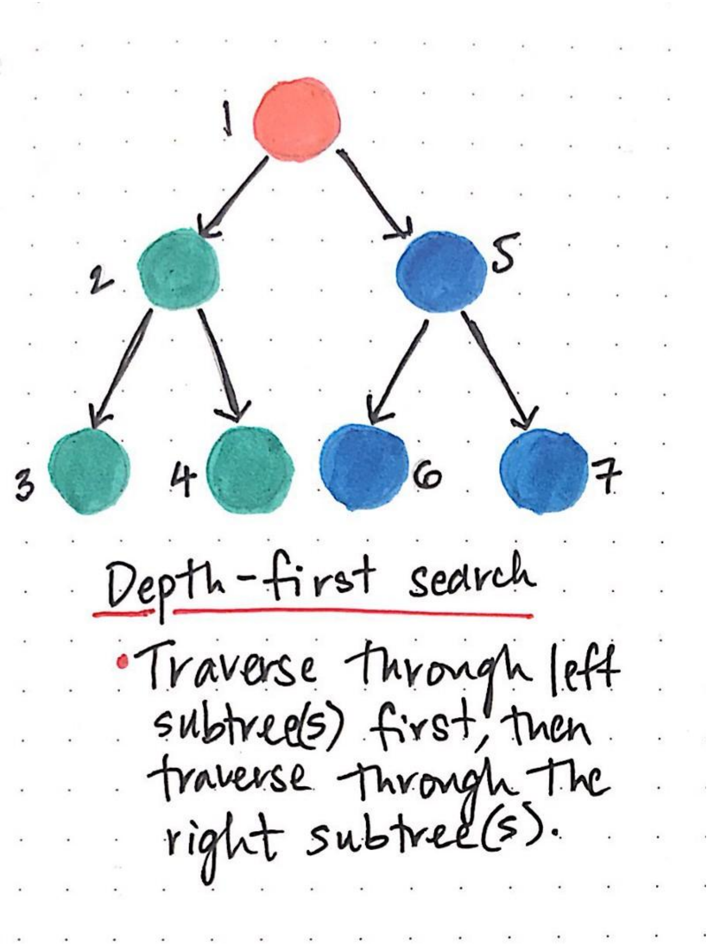
Breadth First Search

- Algorithm traversing/searching through graph
- Broad
- Visits siblings/neighbor nodes over children nodes



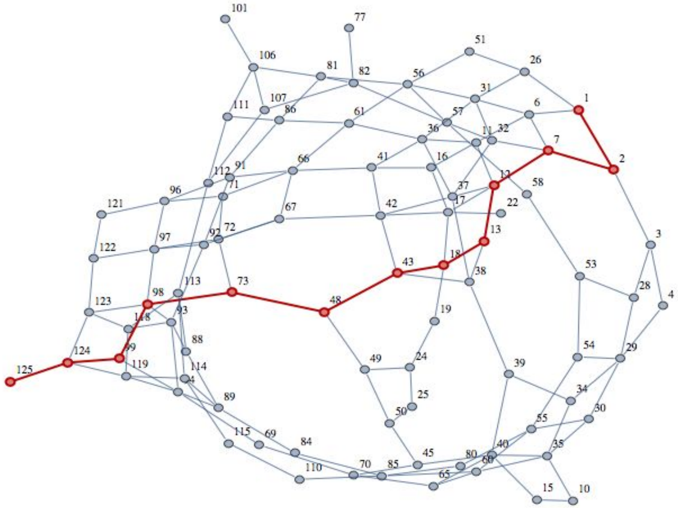
Depth First Search

- ➢ Algorithm traversing/searching through graph
- ➢ Deep
- ➢ Visits children nodes over sibling nodes



Dijkstra's Shortest Path Algorithm

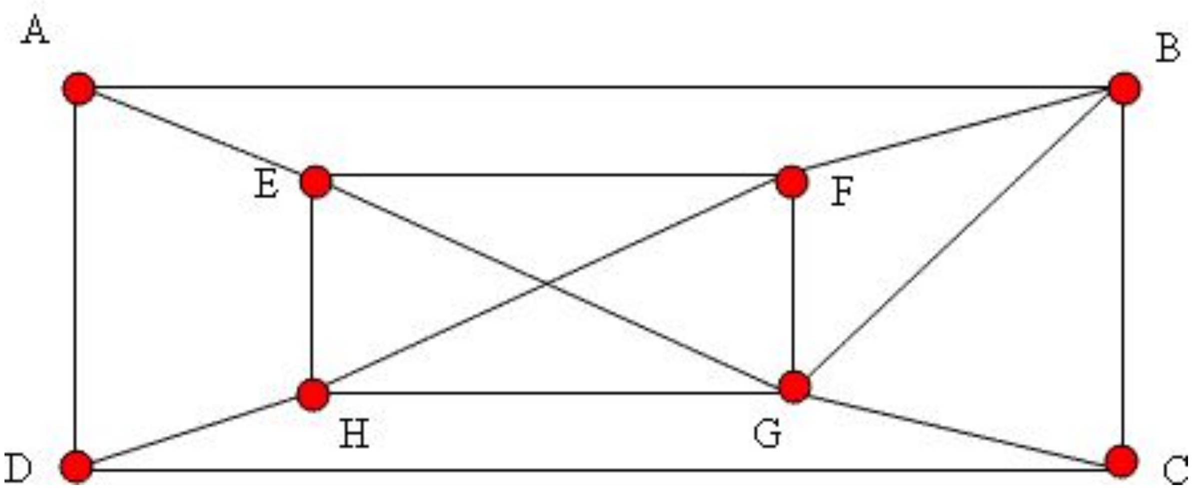
- ➔ Shortest path between nodes in a graph
  - ◆ Ex : road networks
- ➔ Used by GPS and other systems
- ➔ https://www.youtube.com/watch?v=McXPZz4c0FY



Bonus Activity 3 : Travelling Salesman Problem

https://nrich.maths.org/2325

- Visit all cities
- Optimal path
  - Shortest distance
  - Min travel time



Quick Review of Concepts Learned

https://www.youtube.com/watch?v=82zlRaRUsaY

https://www.youtube.com/watch?v=2iViaEAytxw