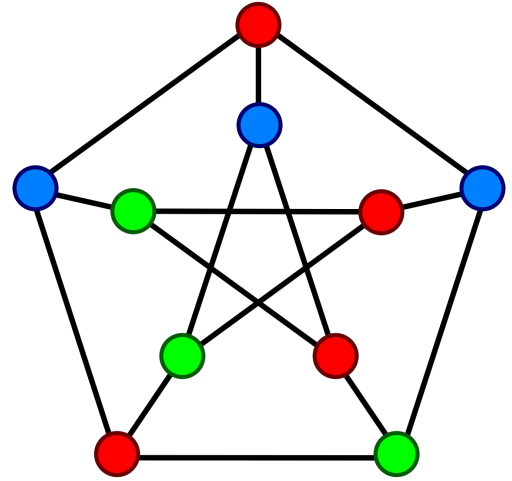
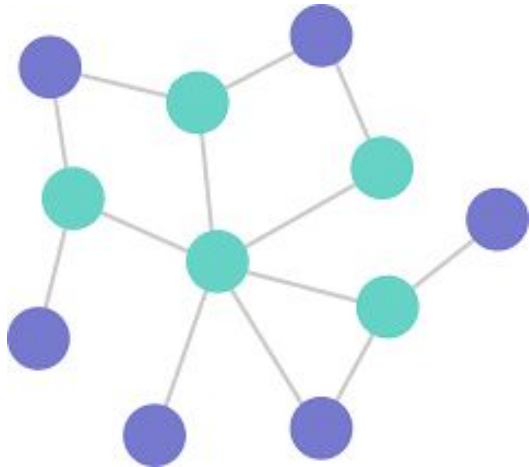
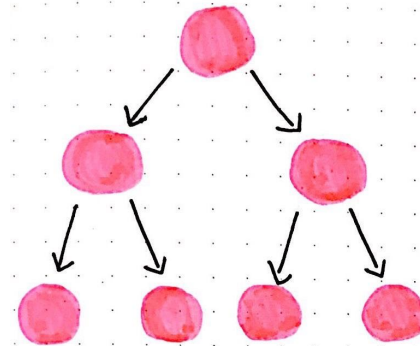


Graph Theory

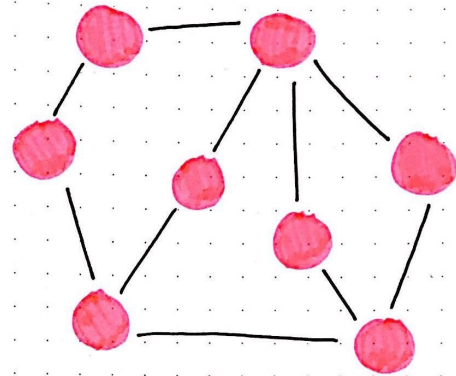
AI Inspire



Basics



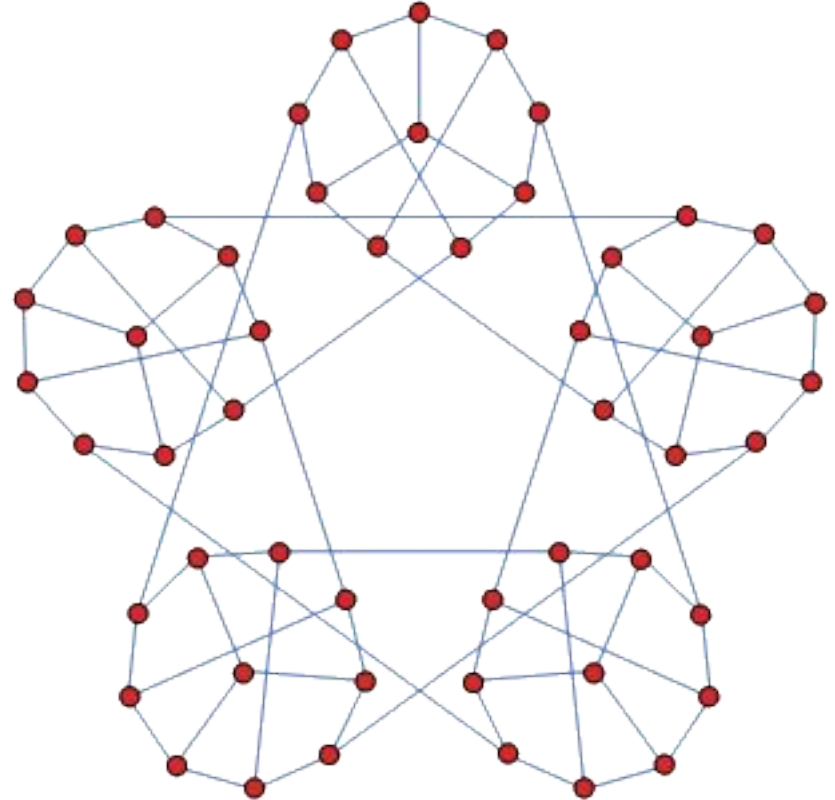
TREE



GRAPH

What is Graph Theory?

- Mathematical study/theory of graphs
 - ◆ Properties
 - ◆ Applications
- Graph is composed of
 - ◆ Vertices/Nodes/Points
 - ◆ Connected by Edges/Arcs/Lines
- Study of **relationships**



Importance/Impact

- Used in a multitude of diff. Comp. apps
 - ◆ Use algorithms to tackle diff. Problems
 - Ex. : Shortest path problem
- Simpler to express data in graph vs. table
 - ◆ See relationships between data
 - ◆ Better understanding of data



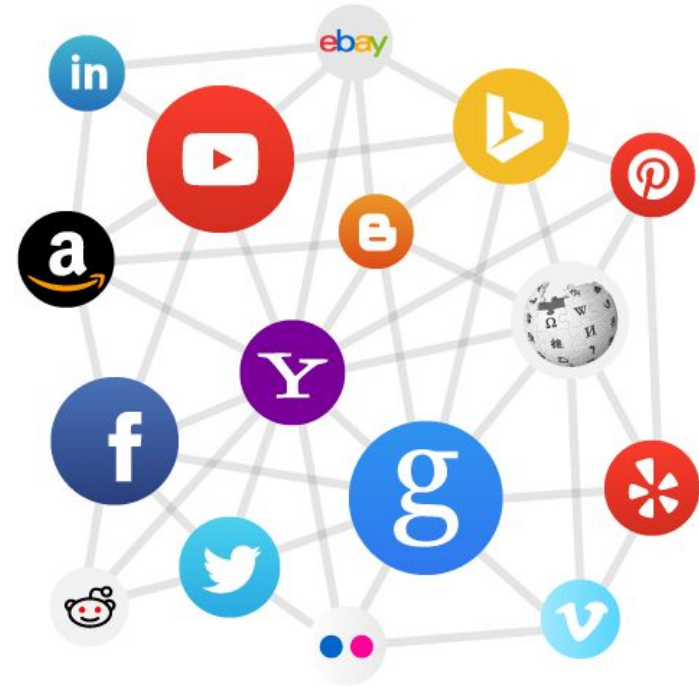
Where is it used?

→ Social media

- ◆ Social networks
 - Facebook
 - Instagram
 - Snapchat, etc.
- ◆ Connect Platforms
 - Connecting students or prof. With each other

→ Shortest Route

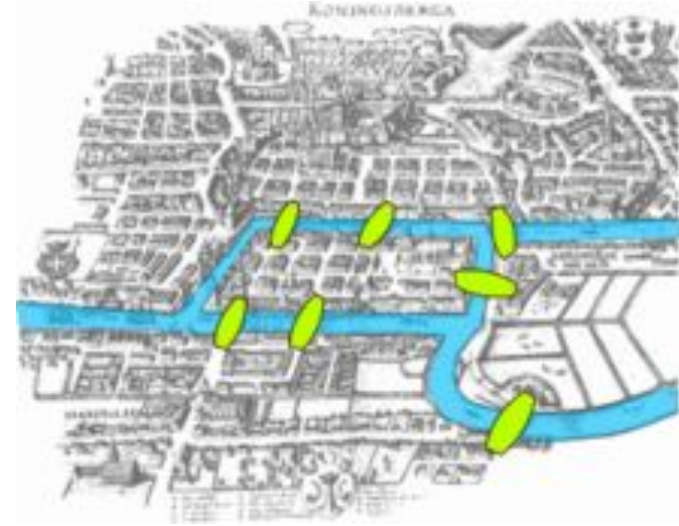
- ◆ GPS



History

→ Leonhard Euler - 1736

- ◆ Greatest mathematician
- ◆ Founder of Graph Theory
- ◆ 7 Bridges of Königsberg Bridge Problem



→ Activity

- ◆ <https://www.mathsisfun.com/activity/seven-bridges-konigsberg.html>



Vocabulary



Simple Vocab

→ Graph

- ◆ Set of vertices/nodes connected with a set of edges

→ Loop

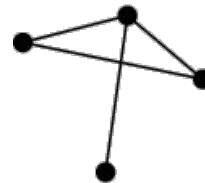
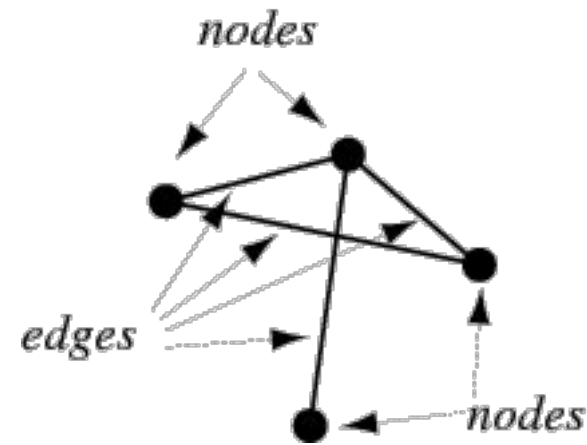
- ◆ Edge with same vertex at each end

→ Degree of vertex

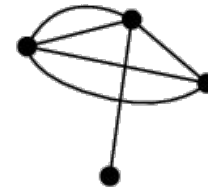
- ◆ Number of edges which have that vertex as an endpoint

→ Simple Graph

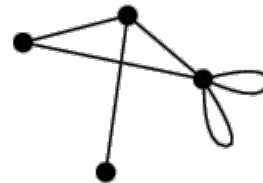
- ◆ Graph with no loops
- ◆ No more than one edge connecting a pair of vertices



simple graph



*nonsimple graph
with multiple edges*



*nonsimple graph
with loops*

Simple Vocab (cont.)

→ Walk

- ◆ Sequence of edges where each edge is the beginning of the consecutively next edge (except last)

→ Trail

- ◆ No edge is repeated

→ Path

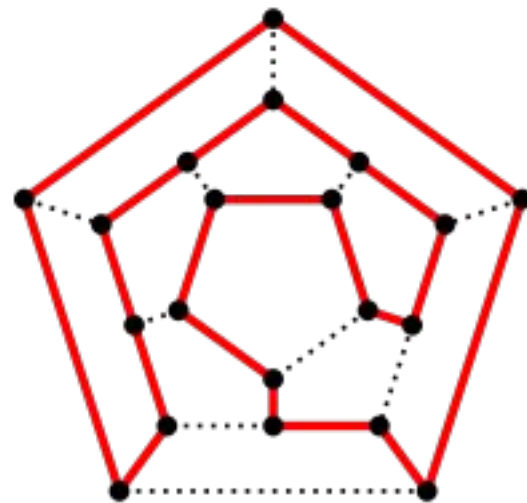
- ◆ No vertex is repeated

→ Cycle

- ◆ Closed path

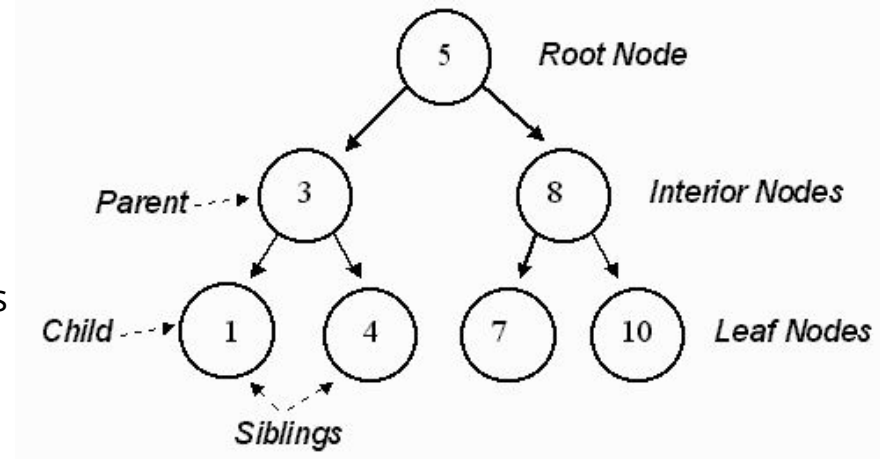
→ Hamiltonian cycle

- ◆ Cycle which visits every vertex
 - Each vertex is only visited once
- ◆ Cycle is a path



Terms & Definitions

- Connected graph
 - ◆ Path exists between every pair of vertices
- Tree
 - ◆ Simple connected graph
 - ◆ **NO CYCLES!**
- Complete graph
 - ◆ Simple graph where every pair of vertices is connected by an edge



Types of Graphs

→ Bipartite graph

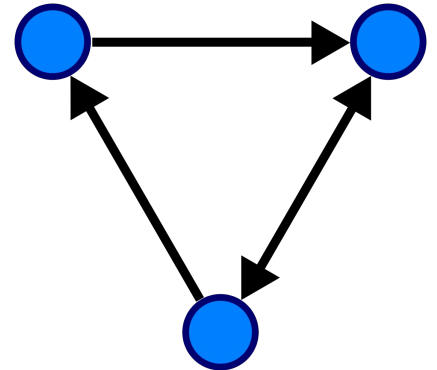
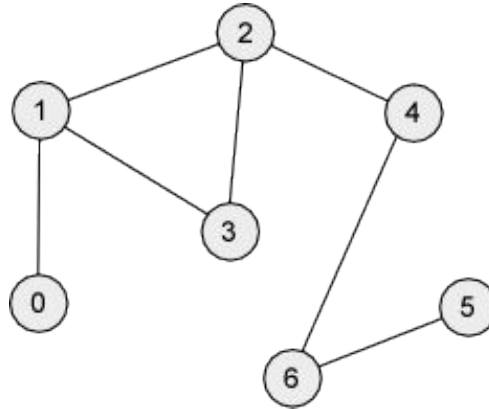
- ◆ 2 disjoint sets (no elements in common & no vertices between 2 sets are adj.)
- ◆ Bigraph

→ Directed graph

- ◆ All edges are directed
- ◆ Particular direction

→ Undirected graph

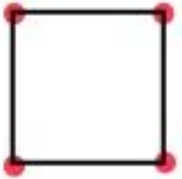
→ DAG (Directed Acyclic Graph)



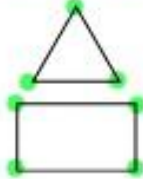
Types of Graphs

Examples of Types of Graphs

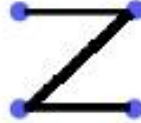
Connected Graph



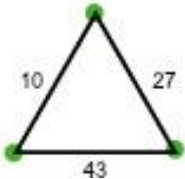
Disconnected Graph



Bipartite Graph



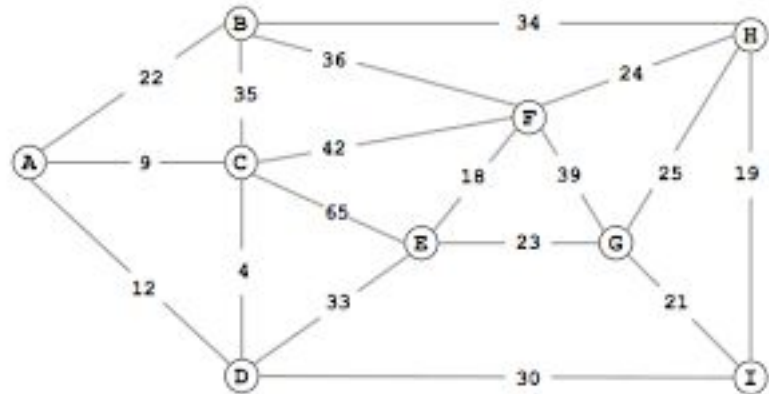
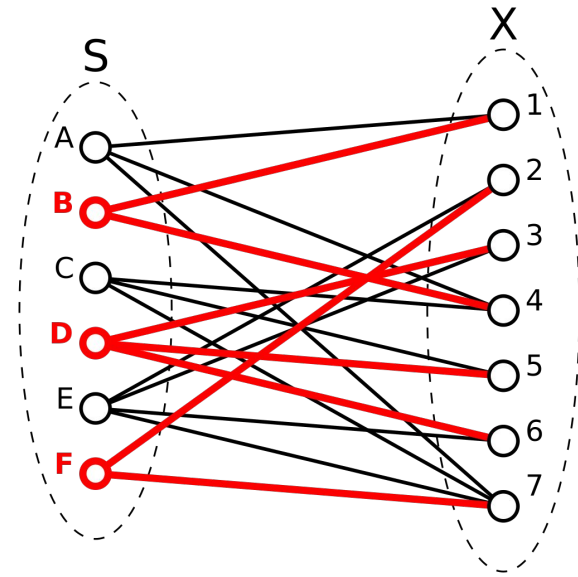
Weighted Graph



Directed Graph



Undirected Graph



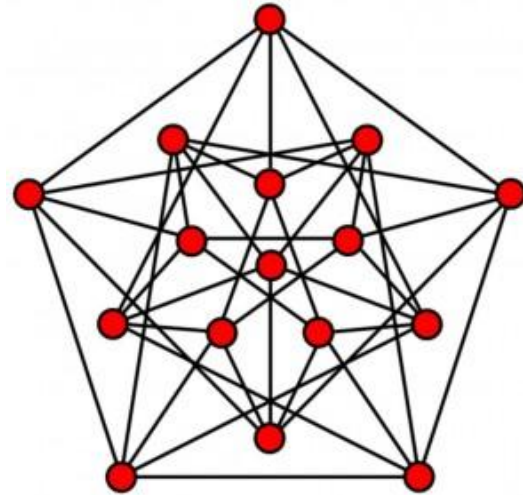
Representing Graphs

→ Adjacency matrix

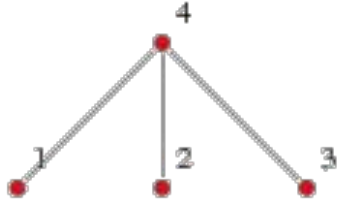
- ◆ Square $n * n$ matrix
- ◆ Each number indicates whether vertices are adjacent or not
 - Diagonal : has 0s : vertex can't be adjacent to itself
- ◆ 2D array in Java

→ Adjacency list

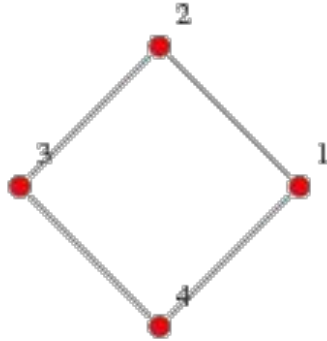
- ◆ Represents graph with list
- ◆ Represents if vertices are adjacent to each other in graph
- ◆ Represents length if directed graph between adjacent vertices



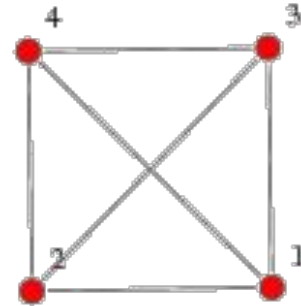
Adjacency Matrix



$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$



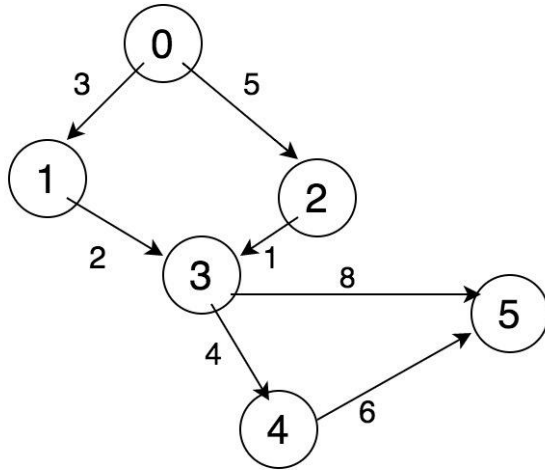
$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$



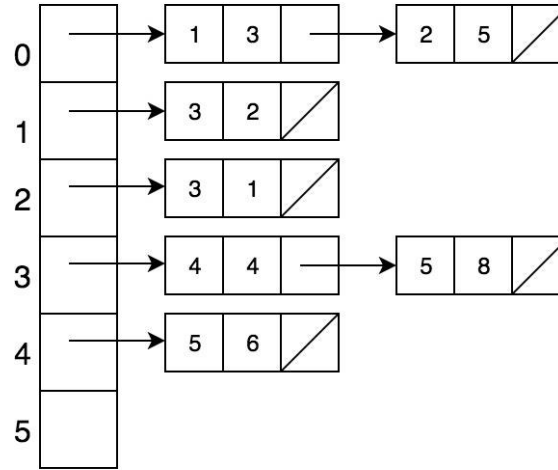
$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

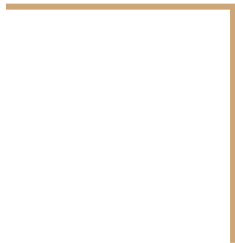
Adjacency List

Directed Graph



Adjacency List Representation





How many shortest-length paths are there to get from your house to the doughnut shop?

4 ups
7 rights

$\binom{11}{7} = \binom{11}{4} = 330$ paths

Onto

One-to-One

There are six dogs to give 13 tacos. Use a stars and bars diagram to illustrate the first and sixth dog gets a taco, the second dog gets none, the third dog gets 5 and the fourth dog gets one.

☆☆☆|☆☆☆☆|☆|☆☆☆|

$A = \{2, 4, \emptyset, \{2\}\}$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$e^{\pi i} + 1 = 0$

P	Q	R	P ∨ Q	P ∨ R	(P ∨ Q) ∧ (P ∨ R)	(P ∨ R) ∧ (P ∨ Q)
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	F	F
F	T	T	T	T	T	T
F	T	F	T	F	F	F
F	F	T	F	T	F	F
F	F	F	F	F	F	F

Find $7 + 12 + 17 + 22 + \dots + 342$.

$S_1 = 7 + 12 + 17 + 22 + \dots + 342$
 $+ S_2 = 342 + 337 + 332 + 327 + \dots + 7$
 $2S_1 = 349 + 349 + 349 + 349 + \dots + 349$
 $2S_1 = 349 \cdot 62$
 $S_1 = \frac{349 \cdot 62}{2}$
 $S_1 = 10861$

Original:
 $\exists x \forall y (x \geq y) \rightarrow x > y + 1$
 Converse:
 $\exists x \forall y (x > y + 1 \rightarrow x \geq y)$
 Negation:
 $\neg [\exists x \forall y (x < y) \vee x > y + 1]$
 $\forall x \exists y (x \geq y \wedge x \leq y + 1)$
 Contrapositive:
 $\exists x \forall y (x \leq y + 1 \rightarrow x < y)$

$v = c + f = 2$

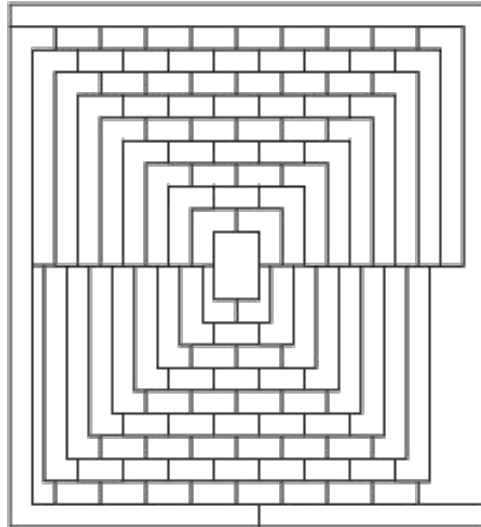
P.I.E. Example:

$6! - \left[\binom{6}{2} 5! - \binom{6}{2} + 1 + \binom{6}{3} 3! - \binom{6}{3} + 2 + \binom{6}{4} 2! - \binom{6}{4} + 3 \right]$

Activity 1 : How Many Colors?

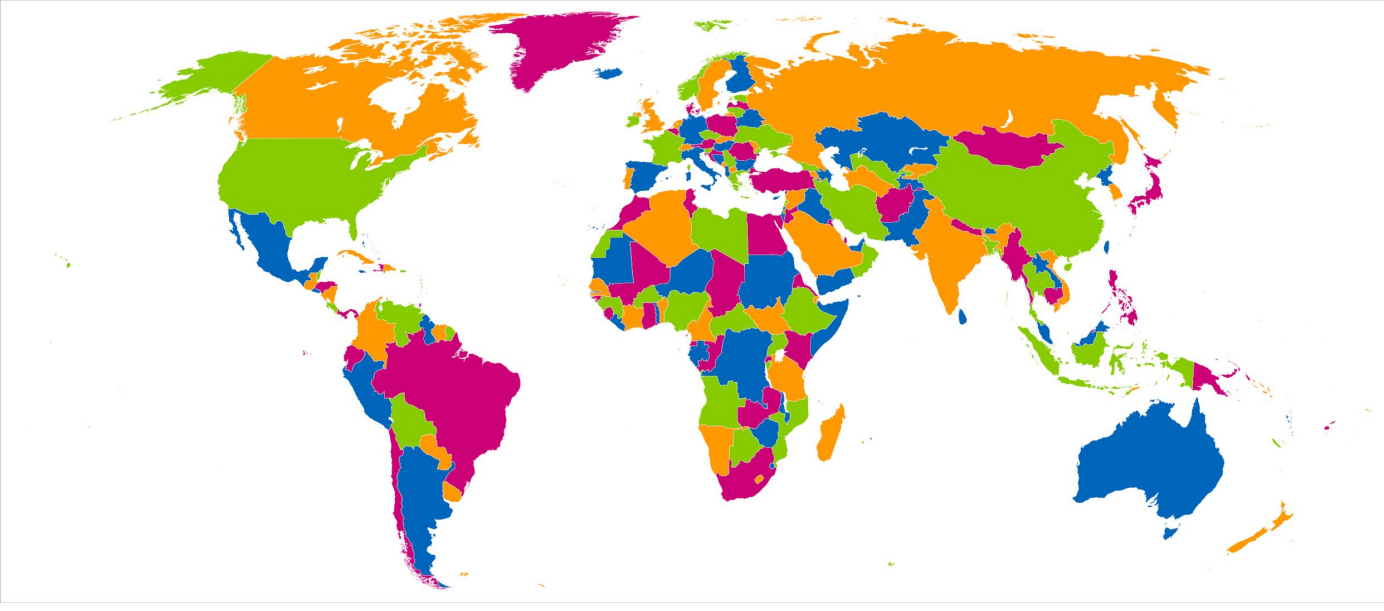
<https://www.colorado.edu/education/DMP/activities/graph/ddgact03.html>

<https://www.colorado.edu/education/DMP/activities/graph/ddghnd03.html>



4 COLOR THEOREM

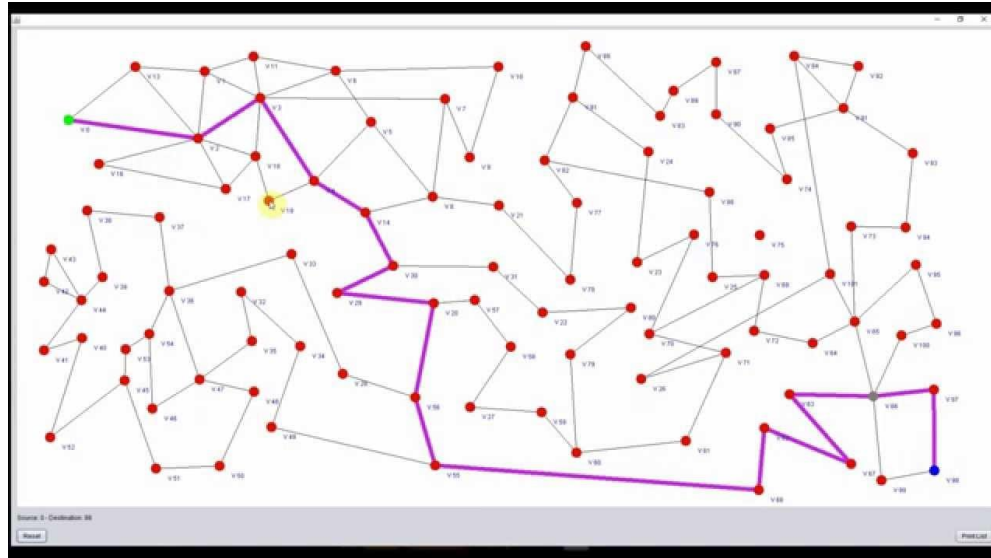
- No more than 4 colors are needed to color regions of map
 - ◆ No 2 adjacent regions have same color



Activity 2 : Shortest Route Problem

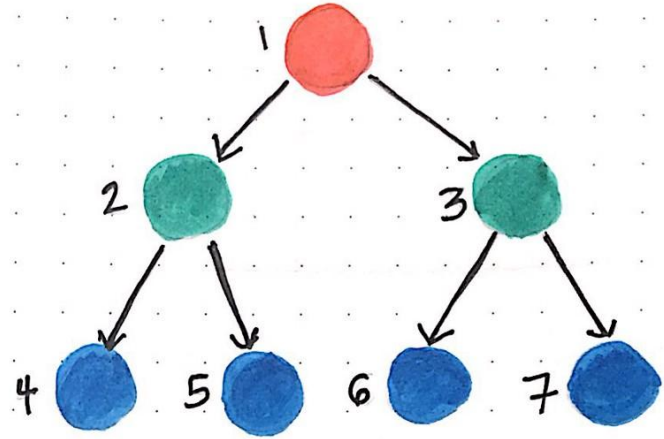
<https://www.colorado.edu/education/DMP/activities/graph/ddgact04.html>

<https://www.colorado.edu/education/DMP/activities/graph/ddghnd04.html>



Breadth First Search

- Algorithm traversing/searching through graph
- Broad
- Visits siblings/neighbor nodes over children nodes

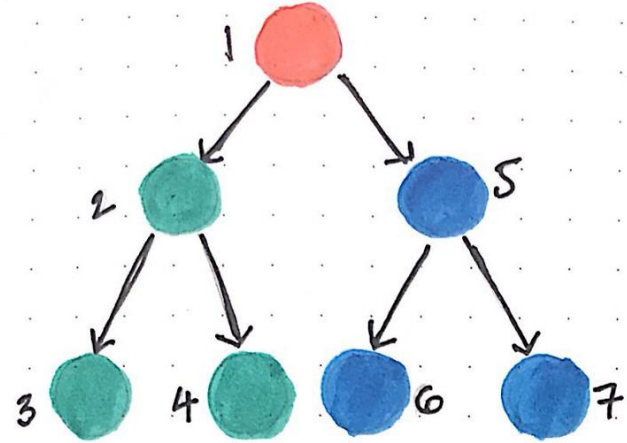


Breadth-first search

- Traverse through one level of children nodes, then traverse through the level of grandchildren nodes (and so on...).

Depth First Search

- Algorithm traversing/searching through graph
- Deep
- Visits children nodes over sibling nodes

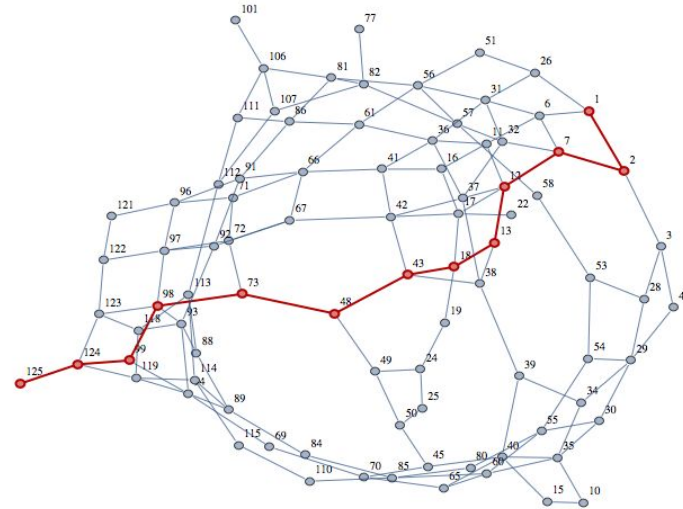


Depth-first search

- Traverse through left subtree(s) first, then traverse through the right subtree(s).

Dijkstra's Shortest Path Algorithm

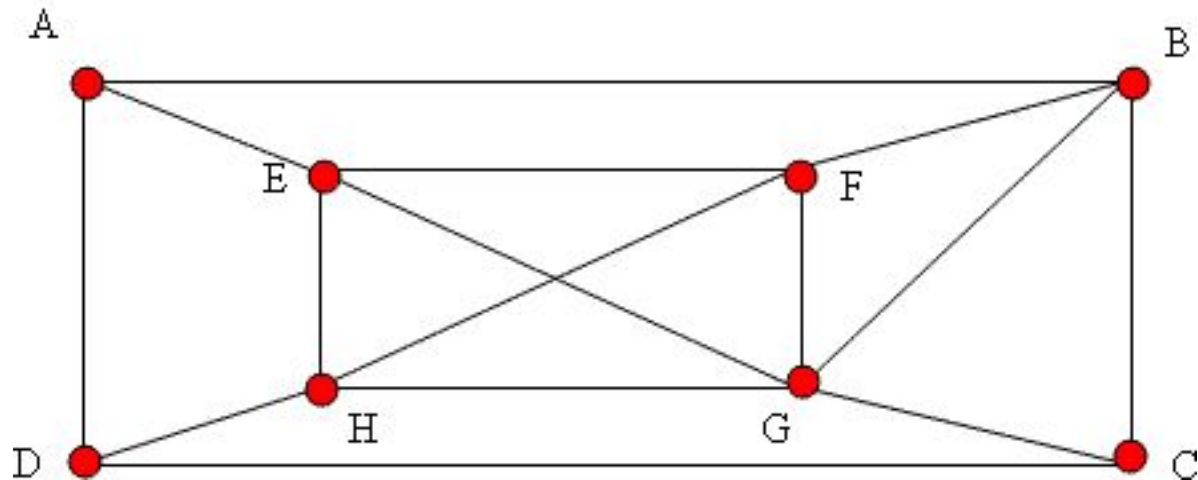
- Shortest path between nodes in a graph
 - ◆ Ex : road networks
- Used by GPS and other systems
- <https://www.youtube.com/watch?v=McXPZz4c0FY>



Bonus Activity 3 : Travelling Salesman Problem

<https://nrich.maths.org/2325>

- Visit all cities
- Optimal path
 - ◆ Shortest distance
 - ◆ Min travel time



Quick Review of Concepts Learned

<https://www.youtube.com/watch?v=82zIRaRUsaY>

<https://www.youtube.com/watch?v=2iViaEAytxw>

