

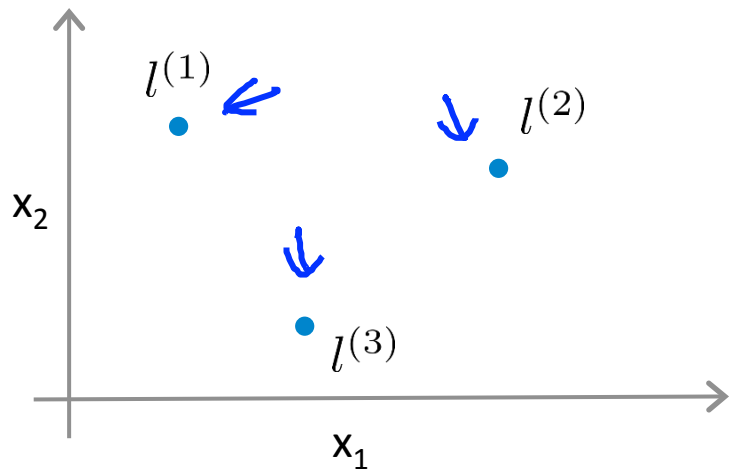


Machine Learning

Support Vector Machines

Kernels II

Choosing the landmarks

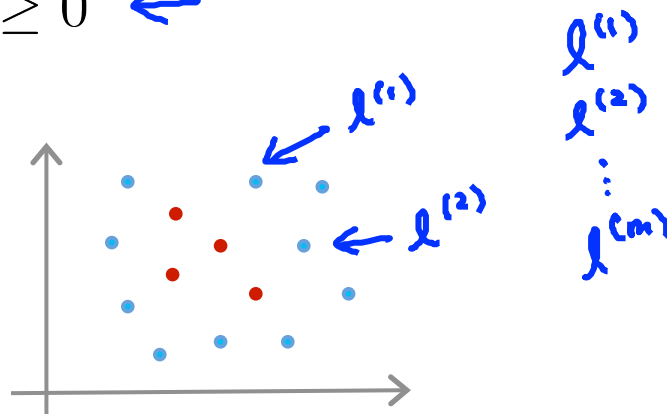
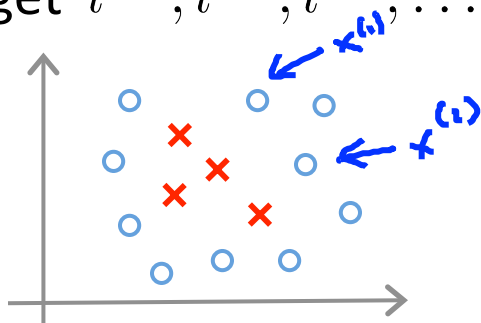


Given x :

$$\begin{aligned} \rightarrow f_i &= \text{similarity}(x, l^{(i)}) \\ &= \exp\left(-\frac{\|x - l^{(i)}\|^2}{2\sigma^2}\right) \leftarrow \end{aligned}$$

Predict $y = 1$ if $\theta_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_3 f_3 \geq 0$ \leftarrow

Where to get $l^{(1)}, l^{(2)}, l^{(3)}, \dots$?



SVM with Kernels

- Given $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})$,
- choose $l^{(1)} = x^{(1)}, l^{(2)} = x^{(2)}, \dots, l^{(m)} = x^{(m)}$.

Given example \underline{x} :

$$\begin{aligned} \rightarrow f_1 &= \text{similarity}(x, l^{(1)}) \\ \rightarrow f_2 &= \text{similarity}(x, l^{(2)}) \\ &\vdots \end{aligned}$$

$$f = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{bmatrix} \quad f_0 = 1$$

For training example $(x^{(i)}, y^{(i)})$:

$$\underline{x^{(i)}} \rightarrow \begin{bmatrix} f_1^{(i)} \\ f_2^{(i)} \\ \vdots \\ f_m^{(i)} \end{bmatrix} \quad \begin{aligned} f_1^{(i)} &= \text{sim}(x^{(i)}, l^{(1)}) \\ f_2^{(i)} &= \text{sim}(x^{(i)}, l^{(2)}) \\ &\vdots \\ f_i^{(i)} &= \text{sim}(x^{(i)}, l^{(i)}) = \exp\left(-\frac{0}{2\sigma^2}\right) = 1 \\ &\vdots \\ f_m^{(i)} &= \text{sim}(x^{(i)}, l^{(m)}) \end{aligned}$$

$$\underline{x^{(i)}} \in \mathbb{R}^{n+1} \quad (\text{or } \mathbb{R}^n)$$

$$f^{(i)} = \begin{bmatrix} f_0^{(i)} \\ f_1^{(i)} \\ f_2^{(i)} \\ \vdots \\ f_m^{(i)} \end{bmatrix} \quad f_0^{(i)} = 1$$

SVM with Kernels

Hypothesis: Given x , compute features $f \in \mathbb{R}^{m+1}$

→ Predict "y=1" if $\theta^T f \geq 0$

$$\theta_0 f_0 + \theta_1 f_1 + \dots + \theta_m f_m$$

$$\theta \in \mathbb{R}^{n+1}$$

Training:

$$\rightarrow \min_{\theta} C \sum_{i=1}^m y^{(i)} \text{cost}_1(\theta^T f^{(i)}) + (1 - y^{(i)}) \text{cost}_0(\theta^T f^{(i)}) + \frac{1}{2} \sum_{j=1}^m \theta_j^2$$

Handwritten notes: $\theta^T f^{(i)}$, θ_0 , $n=m$, θ_j

$$\sum_{j=1}^m \theta_j^2 = \theta^T \theta$$

Handwritten notes: $\theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_m \end{bmatrix}$, $\|\theta\|^2$, $\theta^T M \theta$, $(\text{ignore } \theta_0)$, $m = 10,000$

SVM parameters:

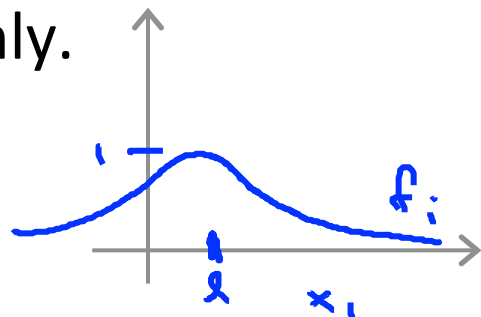
$C \left(= \frac{1}{\lambda} \right)$. \rightarrow Large C : Lower bias, high variance.
 \rightarrow Small C : Higher bias, low variance.

(small λ)

(large λ)

σ^2 Large σ^2 : Features f_i vary more smoothly.
 \rightarrow Higher bias, lower variance.

$$\exp\left(-\frac{\|x - \mu^{(i)}\|^2}{2\sigma^2}\right)$$



Small σ^2 : Features f_i vary less smoothly.
Lower bias, higher variance.

