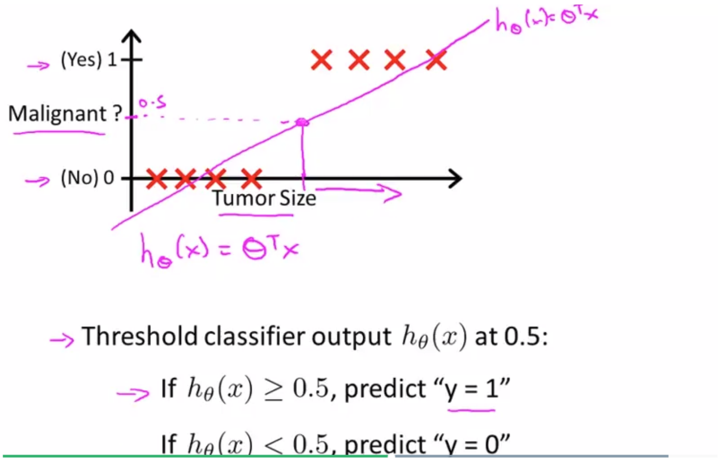
Classification
To attempt classification, one method is to use linear regression and map all predictions greater than 0.5 as a 1 and all less than 0.5 as a 0:



However, this method doesn't work well because classification is not actually a linear function.
The classification problem is just like the regression problem, except that the values we now want to predict take on only a small number of discrete values. For now, we will focus on the binary classification problem in which y can take on only two values, 0 and 1.

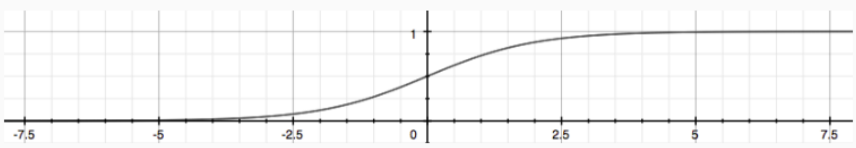
For instance, if we are trying to build a spam classifier for email, then:
→ x(i) may be some features of a piece of email,
→ y may be 1, if it is a piece of spam, and 0 otherwise. Hence, y∈{0,1}.

0 is also called the negative class, and 1 the positive class, and they are sometimes also denoted by the symbols "-" and "+."
Given x(i), the corresponding y(i) is also called the label for the training example.

Hypothesis Representation
Let's change the form for our hypotheses hθ(x) to satisfy 0≤hθ(x)≤1. This is accomplished by plugging $\theta^T x$ into the Logistic Function. Our new form uses the "Sigmoid Function," also called the "Logistic Function":

$$h_\theta(x) = g(\theta^T x)$$
$$z = \theta^T x$$
$$g(z) = \frac{1}{1 + e^{-z}}$$

The following image shows us what the sigmoid function looks like:



The function *g(z)*, maps any real number to the (0, 1) interval, making it useful for transforming an arbitrary-valued function into a function better suited for classification.

hθ(x) will give us the probability that our output is 1. For example, hθ(x)=0.7 gives us a probability of 70% that our output is 1.

$$h_\theta(x) = P(y = 1|x;\theta) = 1 - P(y = 0|x;\theta)$$
$$P(y = 0|x;\theta) + P(y = 1|x;\theta) = 1$$

Decision Boundary

In order to get our discrete 0 or 1 classification, we can translate the output of the hypothesis function as follows:

$$h_\theta(x) \geq 0.5 \rightarrow y = 1$$
$$h_\theta(x) < 0.5 \rightarrow y = 0$$

The way our logistic function g behaves is that when its input is greater than or equal to zero, its output is greater than or equal to 0.5:

$$g(z) \geq 0.5$$
$$when\ z \geq 0$$

Remember:

$$\begin{aligned} z = 0, e^{0} = 1 &\Rightarrow g(z) = 1/2 \\ z \rightarrow \infty, e^{-\infty} \rightarrow 0 &\Rightarrow g(z) = 1 \\ z \rightarrow -\infty, e^{\infty} \rightarrow \infty &\Rightarrow g(z) = 0 \end{aligned}$$

$$h_\theta(x) = g(\theta^T x) \geq 0.5$$
$$when\ \theta^T x \geq 0$$

$$\theta^T x \geq 0 \Rightarrow y = 1$$
$$\theta^T x < 0 \Rightarrow y = 0$$

Example: x2=y

$$\theta = \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix}$$
$$y = 1\ if\ 5 + (-1)x_1 + 0x_2 \geq 0$$
$$5 - x_1 \geq 0$$
$$-x_1 \geq -5$$
$$x_1 \leq 5$$