



Machine Learning

Neural Networks: Learning

Backpropagation algorithm

Gradient computation

$$\rightarrow \underline{J(\Theta)} = -\frac{1}{m} \left[\sum_{i=1}^m \sum_{k=1}^K y_k^{(i)} \log h_{\theta}(x^{(i)})_k + (1 - y_k^{(i)}) \log(1 - h_{\theta}(x^{(i)})_k) \right] \\ + \frac{\lambda}{2m} \sum_{l=1}^{L-1} \sum_{i=1}^{s_l} \sum_{j=1}^{s_{l+1}} (\Theta_j^{(l)})^2$$

$$\rightarrow \min_{\Theta} J(\Theta)$$

Need code to compute:

$$\rightarrow - \underline{J(\Theta)}$$

$$\rightarrow - \underline{\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta)} \quad \leftarrow$$

$$\Theta_{ij}^{(l)} \in \mathbb{R}$$

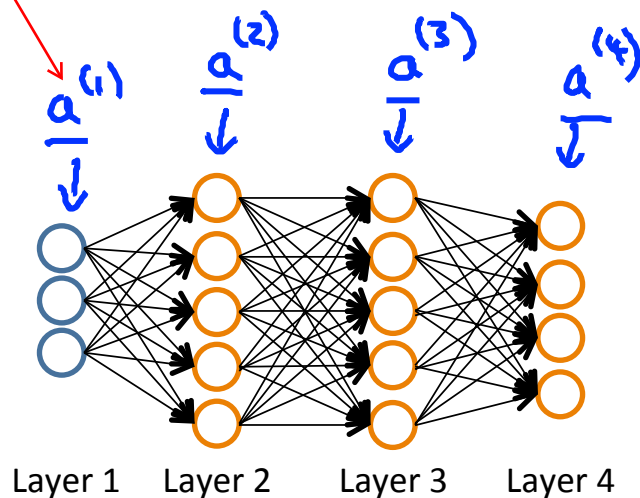
Gradient computation

activation values
of this first layer

Given **one** training example (x, y) :

Forward propagation:

$$\begin{aligned} &\rightarrow \underline{a^{(1)}} = \underline{x} \\ &\rightarrow z^{(2)} = \Theta^{(1)} a^{(1)} \\ &\rightarrow a^{(2)} = g(z^{(2)}) \quad (\text{add } \underline{a_0^{(2)}}) \\ &\rightarrow z^{(3)} = \Theta^{(2)} a^{(2)} \\ &\rightarrow a^{(3)} = g(z^{(3)}) \quad (\text{add } a_0^{(3)}) \\ &\rightarrow z^{(4)} = \Theta^{(3)} a^{(3)} \\ &\rightarrow \underline{a^{(4)}} = \underline{h_{\Theta}(x)} = g(z^{(4)}) \end{aligned}$$



IMPLEMENTAÇÃO VECTORIAL

Gradient computation: Backpropagation algorithm

para cada nó calculamos termo delta (l)j

Intuition: $\delta_j^{(l)}$ = "error" of node j in layer l.

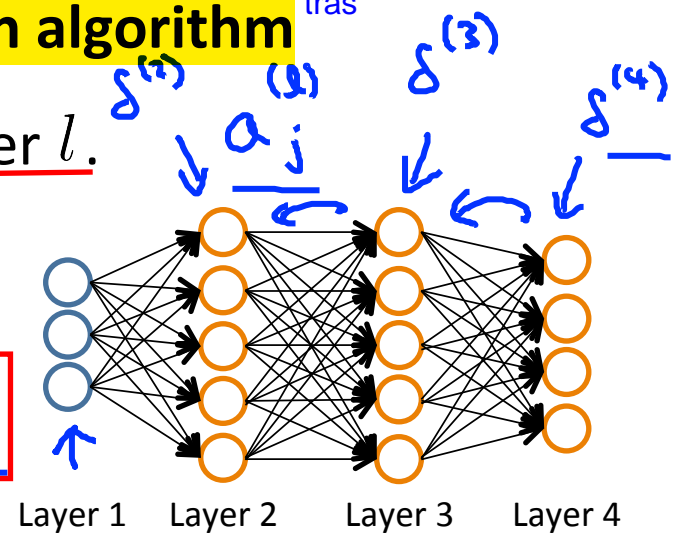
vai capturar o erro da activação do nó

For each output unit (layer L = 4)

$$\delta_j^{(4)} = a_j^{(4)} - y_j$$

implementação vectorial

$$\delta^{(4)} = a^{(4)} - y$$



$$\delta^{(3)} = (\Theta^{(3)})^T \delta^{(4)} \cdot g'(z^{(3)})$$

$$\delta^{(2)} = (\Theta^{(2)})^T \delta^{(3)} \cdot g'(z^{(2)})$$

$$a^{(3)} \cdot (1 - a^{(3)})$$

$$a^{(2)} \cdot (1 - a^{(2)})$$

$$\frac{\partial J(\Theta)}{\partial \Theta_{ij}^{(2)}} = a_j^{(1)} \delta_i^{(2)}$$

(ignoring λ ; if $\lambda = 0$)

derivada funcao g calculada com dados z3

vao funcionar como acumuladores para
calcular derivadas

Backpropagation algorithm

→ Training set $\{(x^{(1)}, y^{(1)}), \dots, (x^{(m)}, y^{(m)})\}$

Set $\Delta_{ij}^{(l)} = 0$ (for all l, i, j).

(used to compute $\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta)$)

For $i = 1$ to m ← $(\underline{x}^{(i)}, \underline{y}^{(i)})$

Set $\underline{a}^{(1)} = \underline{x}^{(i)}$

percorrer training set

→ Perform forward propagation to compute $\underline{a}^{(l)}$ for $l = 2, 3, \dots, L$

→ Using $\underline{y}^{(i)}$, compute $\delta^{(L)} = \underline{a}^{(L)} - \underline{y}^{(i)}$

→ Compute $\delta^{(L-1)}, \delta^{(L-2)}, \dots, \delta^{(2)}$

→ $\Delta_{ij}^{(l)} := \Delta_{ij}^{(l)} + \underline{a}_j^{(l)} \delta_i^{(l+1)}$

$\Delta^{(l)} := \Delta^{(l)} + \delta^{(l+1)} (\underline{a}^{(l)})^T$

→ $D_{ij}^{(l)} := \frac{1}{m} \Delta_{ij}^{(l)} + \lambda \Theta_{ij}^{(l)}$ if $j \neq 0$

→ $D_{ij}^{(l)} := \frac{1}{m} \Delta_{ij}^{(l)}$ if $j = 0$

$$\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta) = D_{ij}^{(l)}$$