



Machine Learning

Linear Algebra review (optional)

Matrix-matrix multiplication

Example

$$\begin{array}{l} \begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 0 & 1 \\ \hline 5 & 2 \\ \hline \end{array} = \begin{bmatrix} 11 & 10 \\ 9 & 14 \end{bmatrix} \\ \textcircled{2 \times 3} \quad \textcircled{3 \times 2} \\ \begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline 5 \\ \hline \end{array} = \begin{bmatrix} 11 \\ 9 \end{bmatrix} \\ \begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{array}{|c|} \hline 3 \\ \hline 1 \\ \hline 2 \\ \hline \end{array} = \begin{bmatrix} 10 \\ 14 \end{bmatrix} \end{array}$$

Handwritten green annotations show the calculation of the 2x2 matrix result. The first row of the result is calculated as $1 \times 1 + 3 \times 0 + 2 \times 5 = 11$ and $1 \times 3 + 3 \times 0 + 2 \times 2 = 10$. The second row is calculated as $4 \times 1 + 0 \times 0 + 1 \times 5 = 9$ and $4 \times 3 + 0 \times 1 + 1 \times 2 = 14$. Arrows point from the intermediate 2x1 results [11, 9] and [10, 14] to the final 2x2 result matrix.

Details:

$$\underline{A} \times \underline{B} = \underline{C}$$

$m \times n$ matrix
(m rows,
 n columns)

$n \times o$ matrix
(n rows,
 o columns)

$m \times o$
matrix

The i^{th} column of the matrix C is obtained by multiplying A with the i^{th} column of B . (for $i = 1, 2, \dots, o$)

Example

$$\overset{2 \times 2}{\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}} \overset{2 \times 2}{\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}} =$$

$$\overset{2 \times 2}{\begin{bmatrix} 9 & 7 \\ 15 & 12 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} =$$

$$\begin{bmatrix} 1 \times 0 + 3 \times 3 \\ 2 \times 0 + 5 \times 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} =$$

$$\begin{bmatrix} 1 \times 1 + 3 \times 2 \\ 2 \times 1 + 5 \times 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

House sizes:

$$\begin{Bmatrix} \frac{2104}{1416} \\ \frac{1534}{852} \end{Bmatrix}$$

Matrix

$$\begin{bmatrix} 1 & \frac{2104}{1416} \\ 1 & \frac{1534}{852} \\ 1 & \frac{1534}{852} \\ 1 & \frac{1534}{852} \end{bmatrix} \times$$

Matrix

$$\begin{bmatrix} -40 \\ 0.25 \end{bmatrix} \begin{bmatrix} 200 \\ 0.1 \end{bmatrix} \begin{bmatrix} -150 \\ 0.4 \end{bmatrix} =$$

$$\begin{bmatrix} 486 \\ 314 \\ 344 \\ 173 \end{bmatrix} \begin{bmatrix} 410 \\ 342 \\ 353 \\ 285 \end{bmatrix} \begin{bmatrix} 692 \\ 416 \\ 464 \\ 191 \end{bmatrix}$$

Prediction
of 1st
h₀

Predictions
of 2nd
h₀

Have 3 competing hypotheses:

1. $h_{\theta}(x) = -40 + 0.25x$

2. $h_{\theta}(x) = 200 + 0.1x$

3. $h_{\theta}(x) = -150 + 0.4x$