



Machine Learning

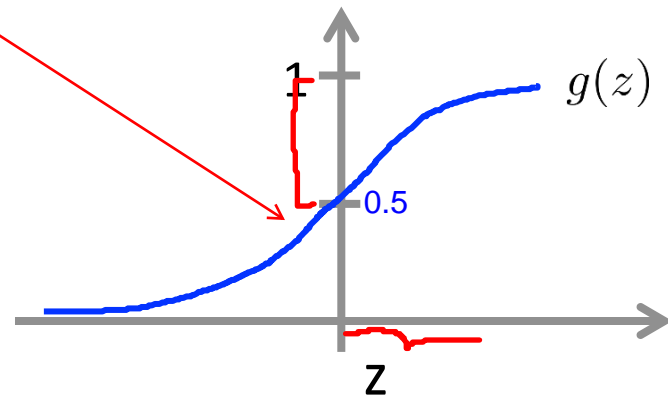
Logistic Regression

Decision boundary

Logistic regression

$$h_{\theta}(x) = g(\theta^T x) = P(y=1 | x; \theta)$$

$$g(z) = \frac{1}{1 + e^{-z}}$$



Suppose predict “ $y = 1$ ” if $h_{\theta}(x) \geq 0.5$

$$\theta^T x \geq 0$$

$$g(z) \geq 0.5 \text{ when } z \geq 0$$

$$h_{\theta}(x) = g(\theta^T x) \geq 0.5$$

predict “ $y = 0$ ” if $h_{\theta}(x) < 0.5$

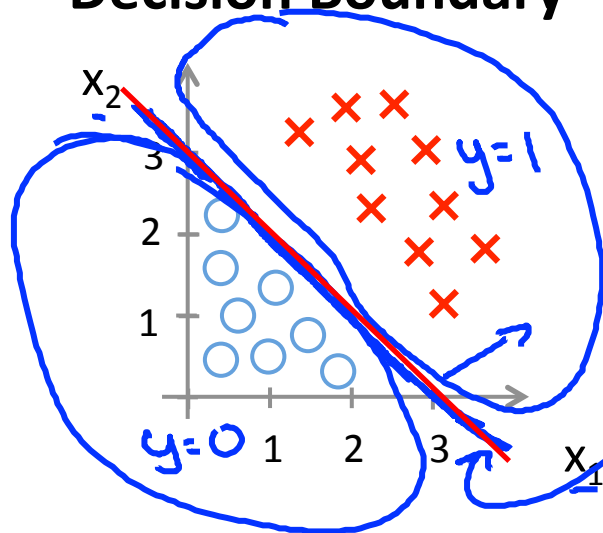
$$\theta^T x < 0$$

$$h_{\theta}(x) = g(\theta^T x)$$

$$\theta^T x < 0$$

$$g(z) < 0.5 \text{ when } z < 0$$

Decision Boundary



$$\theta = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \leftarrow$$

$$h_{\theta}(x) = g(\underbrace{\theta_0}_{-3} + \underbrace{\theta_1}_{1}x_1 + \underbrace{\theta_2}_{1}x_2)$$

Decision boundary

Predict " $y = 1$ " if $-3 + x_1 + x_2 \geq 0$

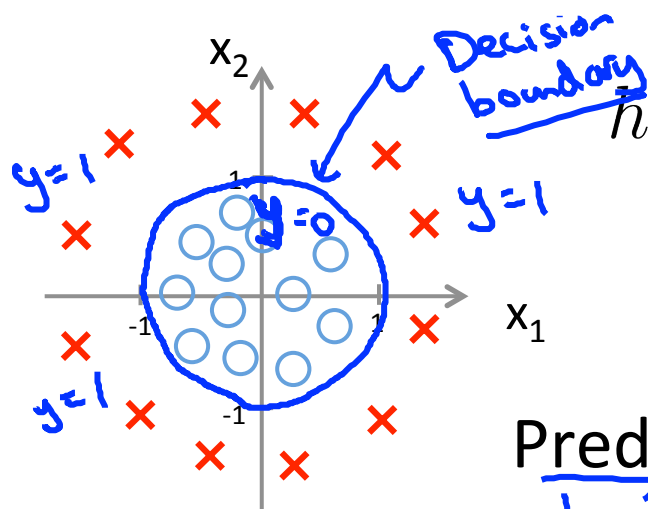
$$\theta^T x$$

$$\rightarrow \underline{x_1 + x_2 \geq 3}$$

x_1, x_2
 $\rightarrow h_{\theta}(x) = 0.5$
 $\boxed{x_1 + x_2 = 3}$

$x_1 + x_2 < 3$
 $\rightarrow y = 0$

Non-linear decision boundaries



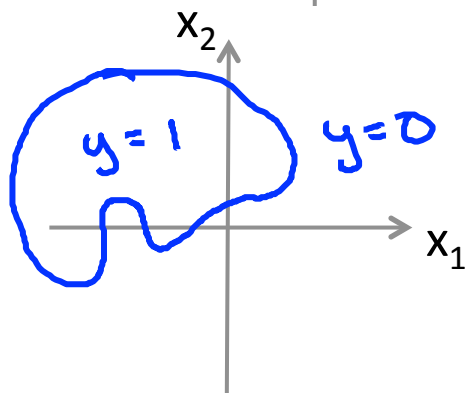
$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

$\begin{matrix} \text{---} \\ \text{---} \end{matrix}$ $\text{---} = 0$ $\text{---} = 0$
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$$\theta = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Predict "y = 1" if $-1 + x_1^2 + x_2^2 \geq 0$

$\boxed{x_1^2 + x_2^2 = 1}$ $\underbrace{x_1^2 + x_2^2 \geq 1}_{\text{circunferencia de raio 1}}$



$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 \underline{x_1^2} + \theta_4 \underline{x_1^2 x_2} + \theta_5 \underline{x_1^2 x_2^2} + \theta_6 \underline{x_1^3 x_2} + \dots)$$