Rules of Differentiation

1. $\frac{d}{dx}[C] = \text{Zero}$.
2. $\frac{d}{dx}[x^n] = n\,x^{n-1}$.
3. $\frac{d}{dx}[f(x)]^n = n\,[f(x)]^{n-1}\,f'(x)$.
4. $\frac{d}{dx}[f.g] = f.g' + g.f'$
5. $\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{g f' - f.g'}{g^2}$.
6. $\frac{d}{dx}\left[\sqrt{f(x)}\right] = \frac{f'(x)}{2\sqrt{f(x)}}$

7. $\frac{d}{dx}[\sin x] = \cos x$
8. $\frac{d}{dx}[\cos x] = -\sin x$
9. $\frac{d}{dx}[\tan x] = \sec^2 x$
10. $\frac{d}{dx}[\sec x] = \sec x \tan x$
11. $\frac{d}{dx}[\operatorname{cosec} x] = -\operatorname{cosec} x \cot x$
12. $\frac{d}{dx}[\cot x] = -\operatorname{cosec}^2 x$.

13. $\frac{d}{dx}[\sinh x] = \cosh x$
14. $\frac{d}{dx}[\cosh x] = \sinh x$
15. $\frac{d}{dx}[\tanh x] = \operatorname{sech}^2 x$
16. $\frac{d}{dx}[\operatorname{sech} x] = -\operatorname{sech} x \tanh x$
17. $\frac{d}{dx}[\operatorname{cosech} x] = -\operatorname{cosech} x \coth x$
18. $\frac{d}{dx}[\coth x] = -\operatorname{cosech}^2 x$

$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$

19. $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$
20. $\frac{d}{dx}[\cos^{-1} x] = \frac{-1}{\sqrt{1-x^2}}$
21. $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$
22. $\frac{d}{dx}[\cot^{-1} x] = \frac{-1}{1+x^2}$
23. $\frac{d}{dx}[\sec^{-1} x] = \frac{1}{x\sqrt{x^2-1}}$
24. $\frac{d}{dx}[\operatorname{cosec}^{-1} x] = \frac{-1}{x\sqrt{x^2-1}}$

25. $\frac{d}{dx}[\sinh^{-1} x] = \frac{1}{\sqrt{1+x^2}}$
26. $\frac{d}{dx}[\cosh^{-1} x] = \frac{1}{\sqrt{x^2-1}}$
27. $\frac{d}{dx}[\tanh^{-1} x] = \frac{1}{1-x^2}$
28. $\frac{d}{dx}[\coth^{-1} x] = \frac{1}{1-x^2}$
29. $\frac{d}{dx}[\operatorname{cosech}^{-1} x] = \frac{-1}{x\sqrt{x^2+1}}$
30. $\frac{d}{dx}[\operatorname{sech}^{-1} x] = \frac{-1}{x\sqrt{1-x^2}}$

31. $\frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)}.\,f'(x)$.
32. $\frac{d}{dx}\left[a^{f(x)}\right] = a^{f(x)}.\,f'(x).\ln a$.
33. $\frac{d}{dx}[\ln f(x)] = \frac{f'(x)}{f(x)}$.
34. $\frac{d}{dx}[\log_a f(x)] = \frac{f'(x)}{f(x)}.\frac{1}{\ln a}$

$e \simeq 2.7$, $\ln e = 1$, $e^{\ln 0} = 0$

$\ln e^0 = 0$, $\ln(\infty) = \infty$, $e^\infty = \infty$

Some properties of $\ln$.

35. $\ln(a.b) = \ln a + \ln b$.
36. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$.
37. $\ln(a^r) = r\ln a$.

$\sec x = \frac{1}{\cos x}$

$\operatorname{cosec} x = \frac{1}{\sin x}$

$\cot x = \frac{1}{\tan x}$

$\tan x = \frac{\sin x}{\cos x}$

- $\log_a b = c \Leftrightarrow a^c = b$
- $a^x . a^y = a^{x+y}$.
- $a^x / a^y = a^{x-y}$.
- $(ab)^x = a^x b^x$.
- $\sin^2 x + \cos^2 x = 1$
- $\cosh^2 x - \sinh^2 x = 1$
- $\sin^{-1} x \neq \frac{1}{\sin x}$.

Techniques of Integration.

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1.$
2. $\int [f(x)]^n \cdot f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c, \quad n \neq -1$
3. $\int e^{f(x)} \cdot f'(x) dx = e^{f(x)} + c$
4. $\int a^{f(x)} \cdot f'(x) \cdot \ln a \, dx = a^{f(x)} + c$
5. $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$
6. $\int \sin x \, dx = -\cos x + C$
7. $\int \cos x \, dx = \sin x + C$
8. $\int \sec^2 x \, dx = \tan x + C$
9. $\int \operatorname{cosec}^2 x \, dx = -\cot x + C$
10. $\int \sec x \tan x \, dx = \sec x + C$
11. $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$
12. $\int \sinh x \, dx = \cosh x + C.$
13. $\int \cosh x \, dx = \sinh x + C.$
14. $\int \operatorname{sech}^2 x \, dx = \tanh x + C.$
15. $\int \operatorname{cosech}^2 x \, dx = -\coth x + C.$
16. $\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$
17. $\int \operatorname{cosech} x \coth x \, dx = -\operatorname{cosech} x + C$
18. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}(x/a) + C.$
19. $\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C.$
20. $\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}(x/a) + C.$
21. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
22. $\int \frac{1}{a^2-x^2} dx = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + c$
23. $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$
24. $\int \frac{1}{x\sqrt{a^2-x^2}} dx = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right) + c$
25. $\int \frac{1}{x\sqrt{a^2+x^2}} dx = -\frac{1}{a}\operatorname{cosech}^{-1}\left(\frac{x}{a}\right) + c$

Integration by parts. (26)

$\int u \, dv = u.v - \int v \, du.$

27. $\int \sin^m x \cos^n x \, dx$
   1) if m is odd let $u = \cos x$
   2) if n is odd let $u = \sin x$
   3) if m, n are both odd, m > n let u = (under m)
   4) if m, n are both even, use $\sin^2 x = \frac{1}{2}(1-\cos 2x)$, $\cos^2 x = \frac{1}{2}(1+\cos 2x)$.

28. $\int \tan^m x \sec^n x \, dx$

   $\int \cot^m x \operatorname{cosec}^n x \, dx$

   notice: 1) $1+\tan^2 x = \sec^2 x$
   2) $1+\cot^2 x = \operatorname{cosec}^2 x$

29. Trigonometric Substitution

1) $\int \sqrt{a^2-x^2} \, dx$

let $x = a\sin\theta, x = a\cos\theta, x = a\tanh\theta$

2) $\int \sqrt{x^2-a^2} \, dx$

let $x = a\sec\theta, x = a\cosh\theta$

3) $\int \sqrt{a^2+x^2} \, dx$

let $x = a\tan\theta, x = a\sinh\theta.$

30. Trigonometric Substitution

$\tan(x/2) = t, \sin(x/2) = \frac{t}{\sqrt{1+t^2}}$

$\cos(x/2) = \frac{1}{\sqrt{1+t^2}}, dx = \frac{2}{1+t^2}dt$

Numerical Integration.

1) Rectangular rule.

$\text{Area} = \int_a^b f(x)dx \simeq h[f_0 + f_1 + \cdots + f_{n-1}]$

$\simeq h[f_1 + f_2 + \cdots + f_n].$

2) Trapzoidal rule.

$\text{Area} = \int_a^b f(x)dx$

$\simeq \frac{h}{2}[f_0 + 2f_1 + 2f_2 + \cdots + 2f_{n-1} + f_n]$

3) Simpson rule.

$\text{Area} = \int_a^b f(x)dx$

$\simeq \frac{h}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n].$

Area.

1) Area under the curve.

$\int_a^b y\,dx$ or $\int_a^b x\,dy$

2) Area between two curves

$\int_a^b (y_1-y_2)dx$ or $\int_a^b (x_1-x_2)dy$

3) Area in polar Co-ordinates

$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 d\theta.$

Volume of Solid of revolution

1) $V = \int_a^b \pi y^2 dx$ or $\int_a^b \pi x^2 dy$

or $\int_a^b \pi(y-k)^2 dx$ or $\int_a^b \pi(x-h)^2 dy$

or $\int_a^b \pi(y_1^2 - y_2^2)dx$ or $\int_a^b \pi(x_1^2 - x_2^2)dy$

The arc length of a graph.

1) $S = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2} dx$

2) $S = \int_a^b \sqrt{1+\left(\frac{dx}{dy}\right)^2} dy$

3) $S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

Area of a Surface of revolution

$S = \int_a^b 2\pi y \, ds$

Some properties of definite integral

1. $\int_a^b c\,dx = c(b-a)$
2. $\int_a^a f(x)dx = 0$
3. $\int_a^b f(x)dx = -\int_b^a f(x)dx.$
4. $\int_a^b [c_1 f(x) \pm c_2 g(x)]dx = c_1\int_a^b f(x)dx \pm c_2\int_a^b g(x)dx$
5. for the case $f(x) \geq 0$, $a < c < b$

   $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
6. If $f(x) \geq 0$, $a \leq x \leq b$ then $\int_a^b f(x)dx \geq 0$
7. If $f(x) \geq g(x)$, $a \leq x \leq b$ then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$
8. $\int_0^b f(x)dx = \int_0^b f(b-x)dx$
9. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
10. if $m \leq f(x) \leq M$, $a \leq x \leq b$ then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$
11. $\int_{-a}^a f(x)dx = \begin{cases} 0 & , f \text{ is odd} \\ 2\int_0^a f(x)dx & , f \text{ is even} \end{cases}$
12. $\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$

Important rules.

1. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$
2. $\cos^2 x = \frac{1}{2}(1+\cos 2x)$
3. $\sinh^2 x = \frac{1}{2}(\cosh 2x - 1)$
4. $\cosh^2 x = \frac{1}{2}(\cosh 2x + 1)$
5. $\tan^2 x = \sec^2 x - 1$
6. $\cot^2 x = \operatorname{cosec}^2 x - 1$
7. $\tanh^2 x = 1 - \operatorname{sech}^2 x$
8. $\coth^2 x = \operatorname{cosech}^2 x + 1$
9. $\sin ax \sin bx = \frac{1}{2}[\cos(a+b)x - \cos(a-b)x]$
10. $\sinh ax \sinh bx = \frac{1}{2}[\sinh(a+b)x - \sinh(a-b)x]$
11. $\sin ax \cos bx = \frac{1}{2}[\sin(a+b)x + \sin(a-b)x]$
12. $\sinh ax \cosh bx = \frac{1}{2}[\sinh(a+b)x + \sinh(a-b)x]$
13. $\cos ax \cos bx = \frac{1}{2}[\cos(a+b)x + \cos(a-b)x]$
14. $\cosh ax \cosh bx = \frac{1}{2}[\cosh(a+b)x + \cosh(a-b)x]$

De Moivre's Theorem.

1. $[r(\cos\theta + i\sin\theta)]^n, \quad n \in \mathbb{Z}$

   $= r^n(\cos(n\theta) + i\sin(n\theta))$
2. $[r(\cos\theta, \sin\theta)]^{p/q}$

   $= r^{p/q}\left(\cos\frac{p\theta+2\pi k}{q}, \sin\frac{p\theta+2\pi k}{q}\right)$

   $k = 0, 1, 2, \ldots, q-1.$