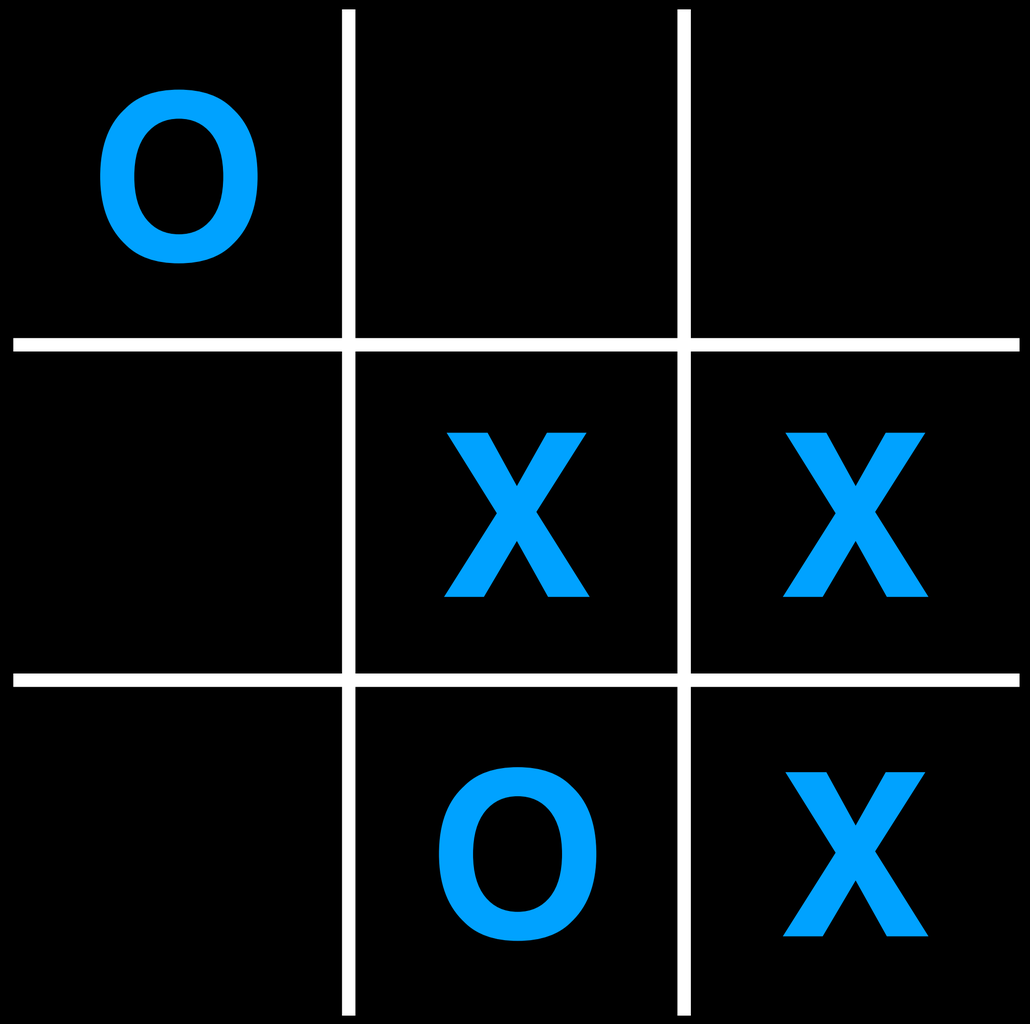
Introduction to
Artificial Intelligence
with Python

Artificial Intelligence

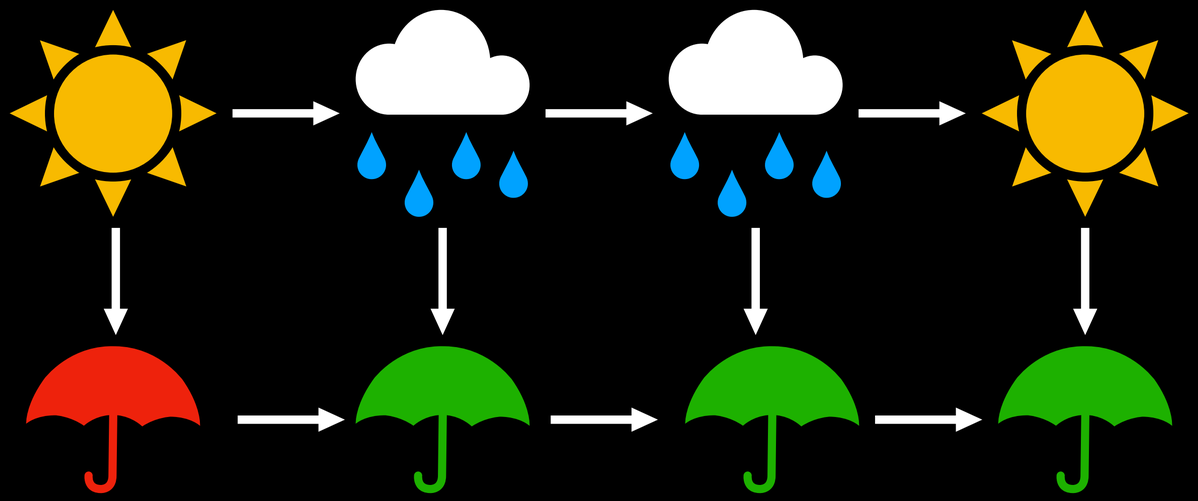
Search



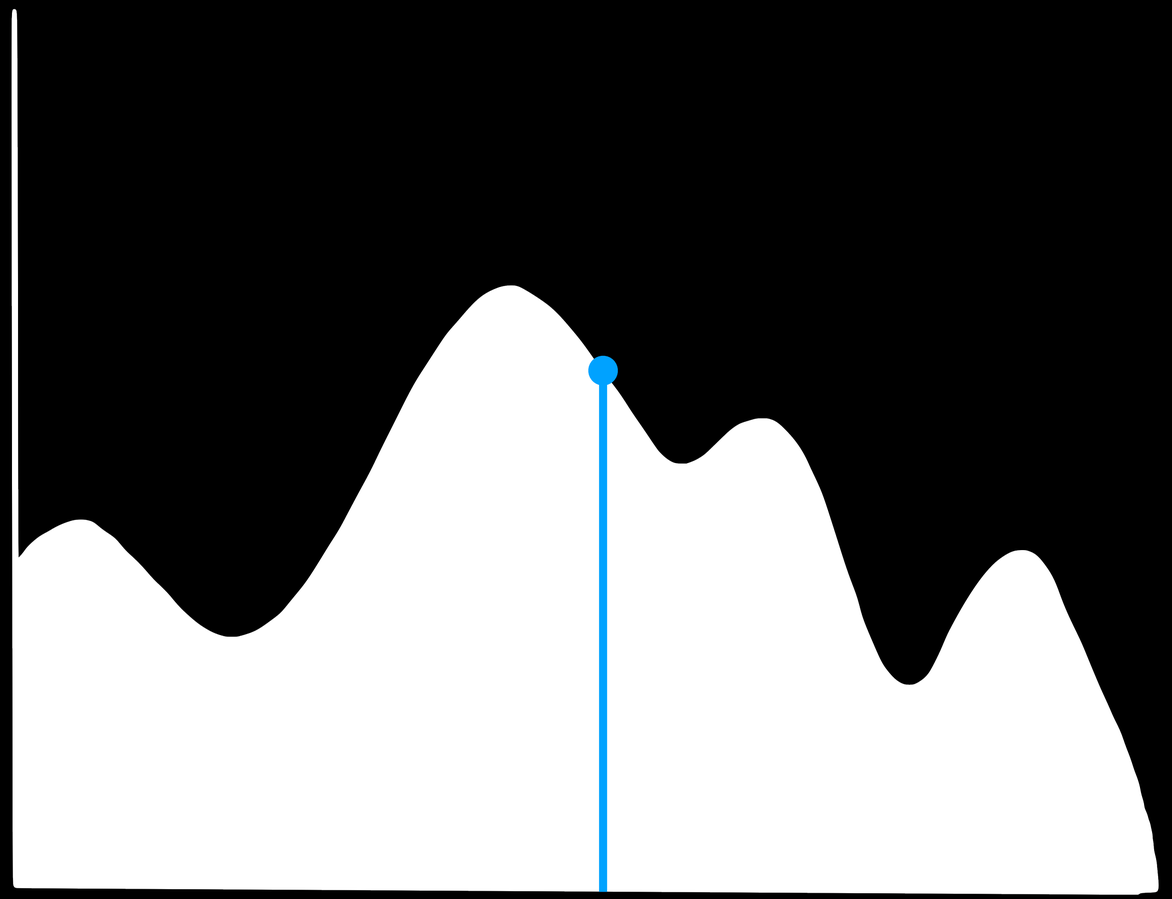
Knowledge

$$\frac{P \to Q \qquad P}{Q}$$

Uncertainty

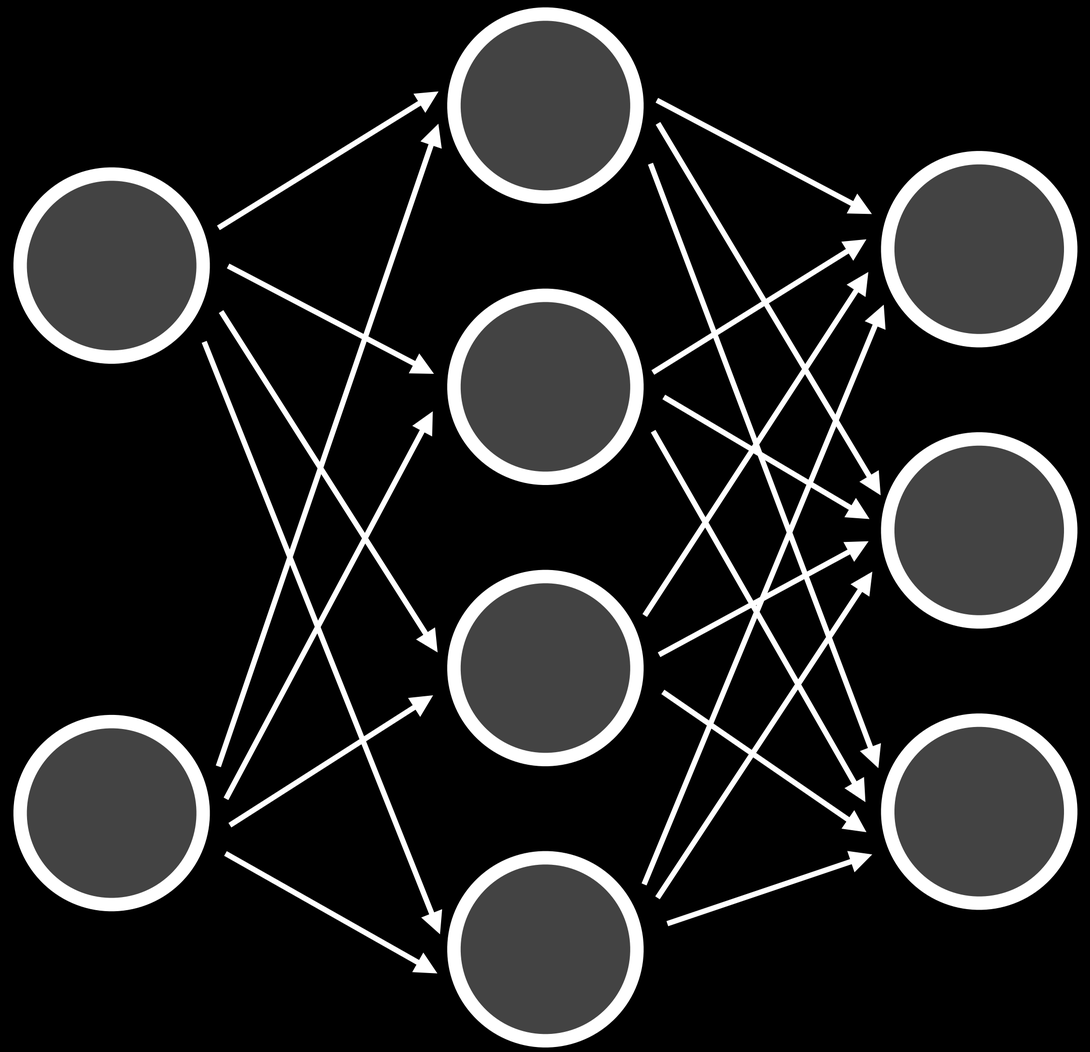


Optimization

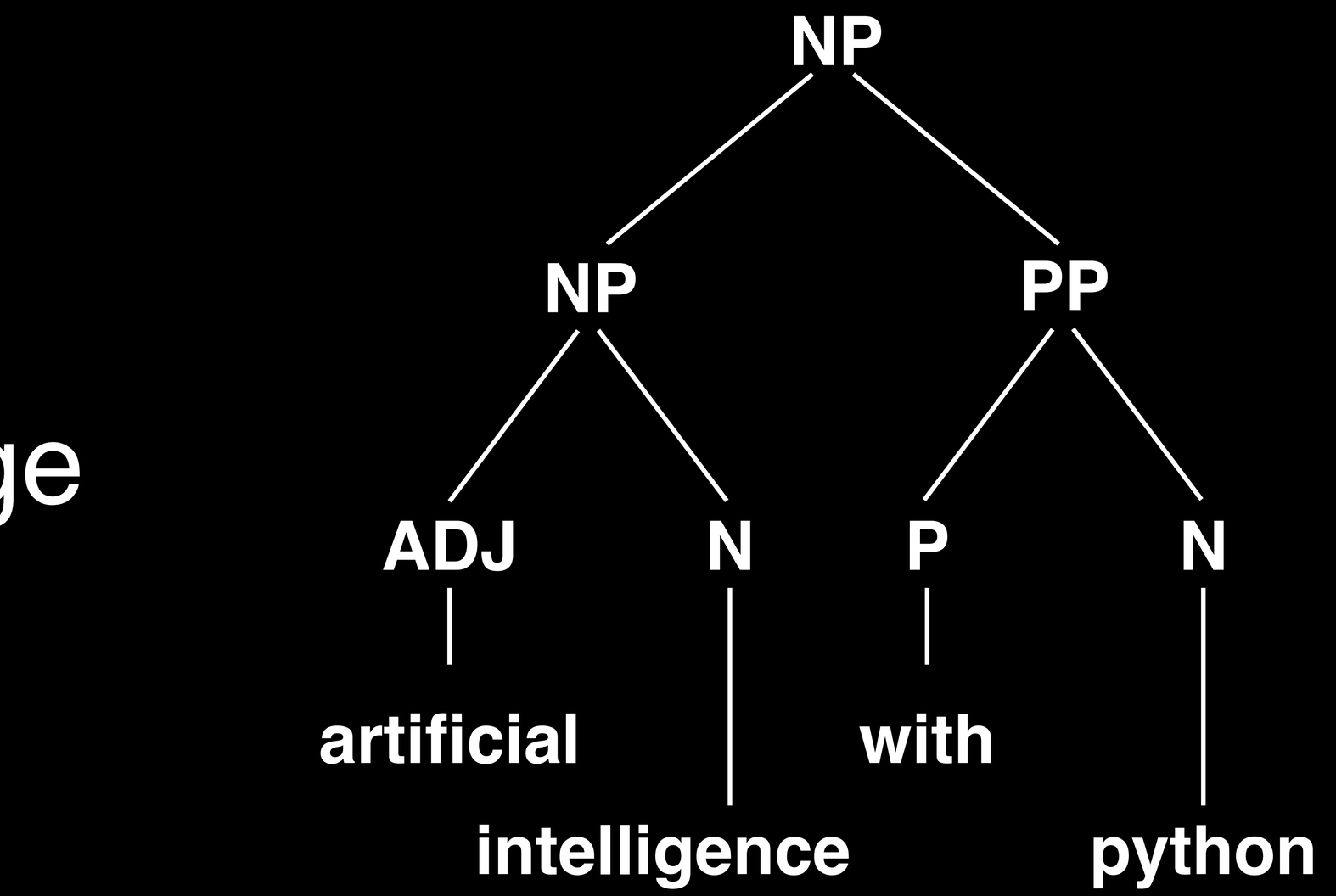


Learning

Neural Networks



Language



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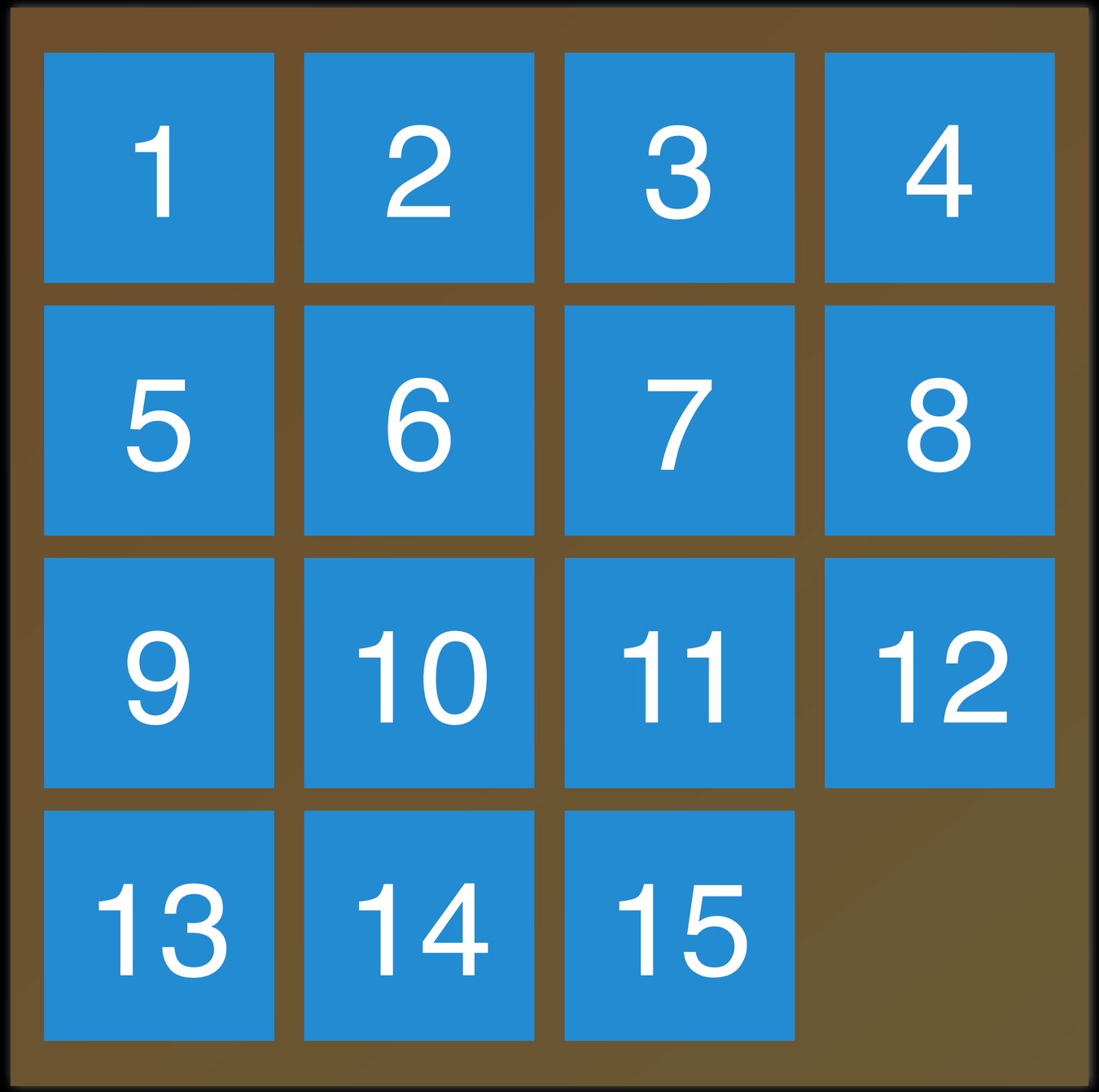
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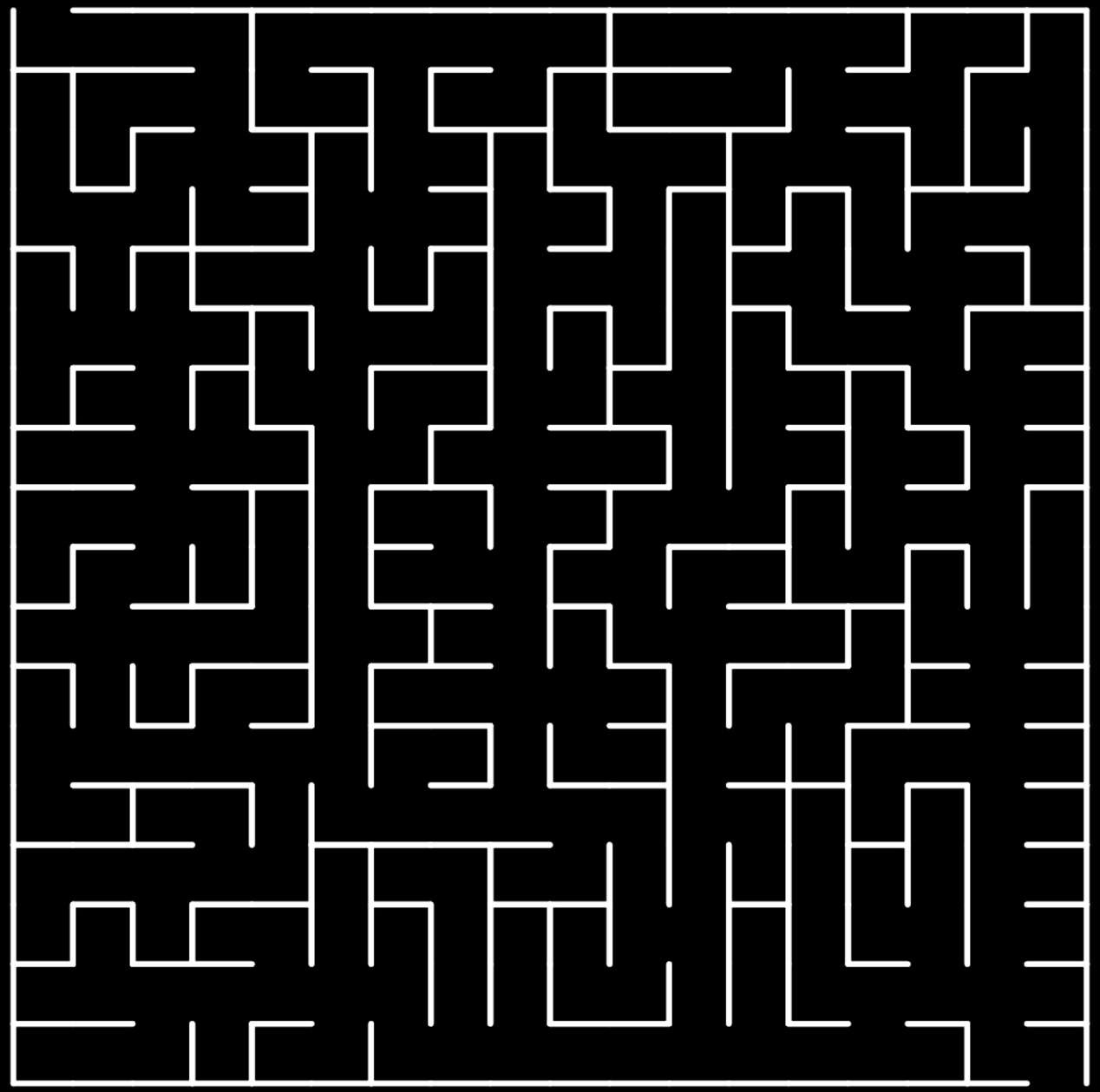

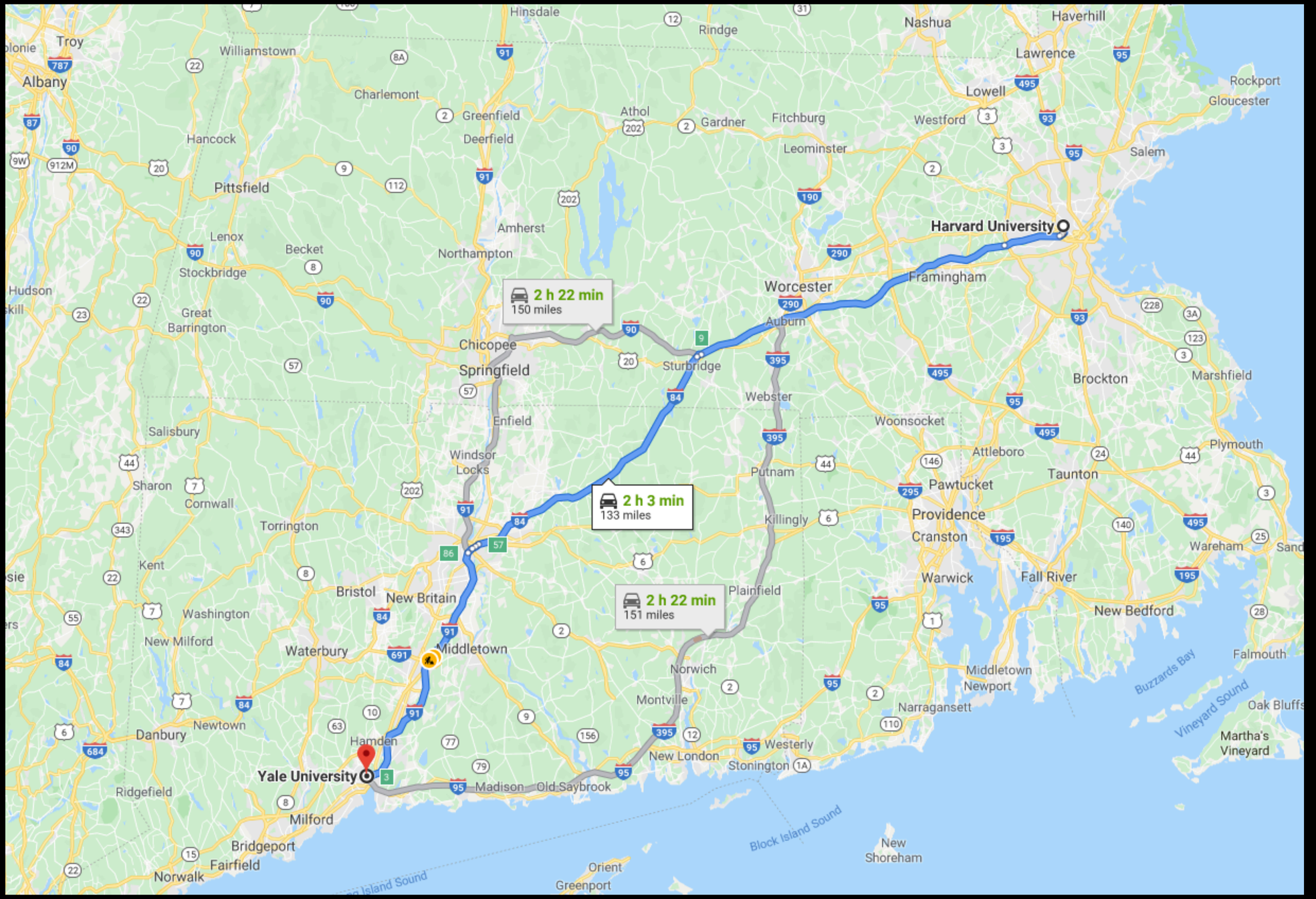

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Athol
Gardner
Fitchburg
Westford
Leominster
Salem
Hancock
Pittsfield
Amherst
Northampton
Lenox
Becket
Stockbridge
Great Barrington
Hudson
Worcester
Framingham
Auburn
Chicopee
Springfield
Sturbridge
Webster
Brockton
Marshfield
Enfield
Salisbury
Windsor Locks
Woonsocket
Attleboro
Plymouth
Sharon
Cornwall
Putnam
Pawtucket
Taunton
Torrington
Killingly
Providence
Cranston
Wareham
Kent
Bristol
New Britain
Plainfield
Warwick
Fall River
Washington
New Bedford
New Milford
Waterbury
Middletown
Norwich
Middletown
Newport
Falmouth
Buzzards Bay
Vineyard Sound
Oak Bluffs
Montville
Narragansett
Danbury
Newtown
Hamden
Westerly
New London
Stonington
Martha's Vineyard
Ridgefield
Madison
Old Saybrook
Milford
Block Island Sound
Bridgeport
Fairfield
Norwalk
Orient
Greenport
New Shoreham

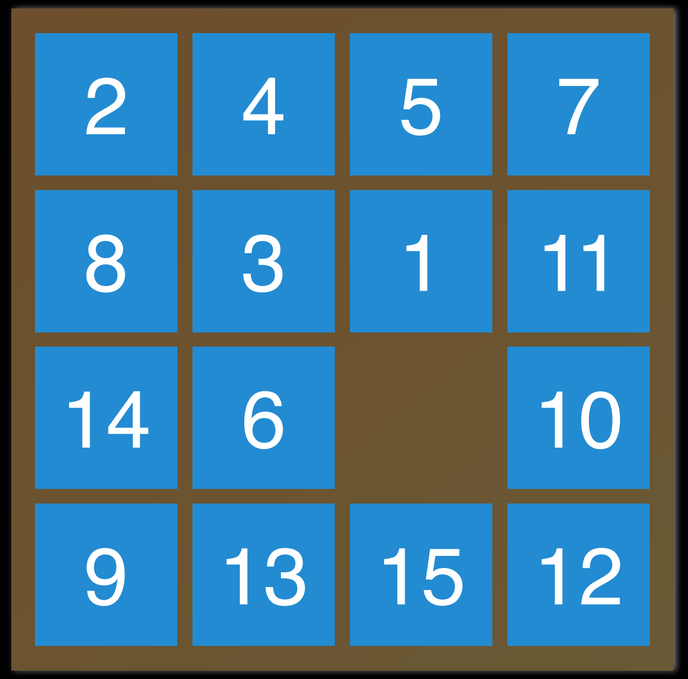
Search Problems

agent

entity that perceives its environment
and acts upon that environment

state

a configuration of the agent and
its environment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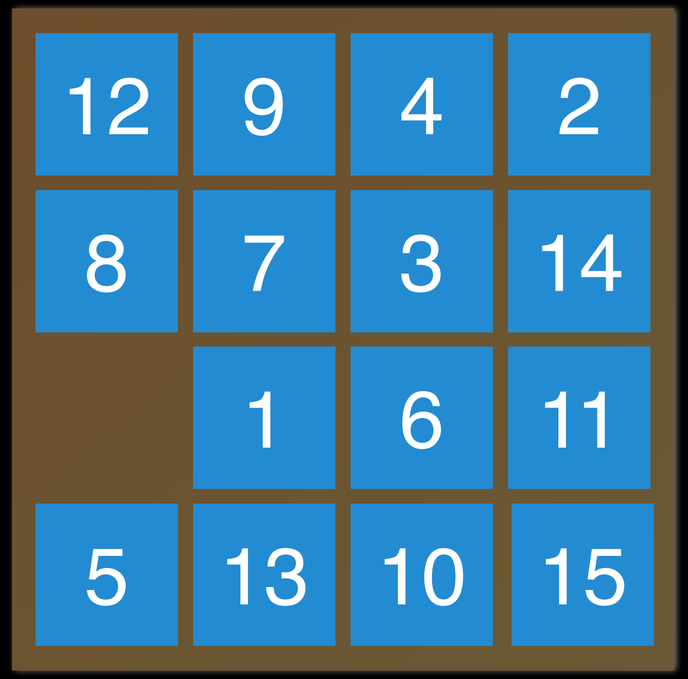
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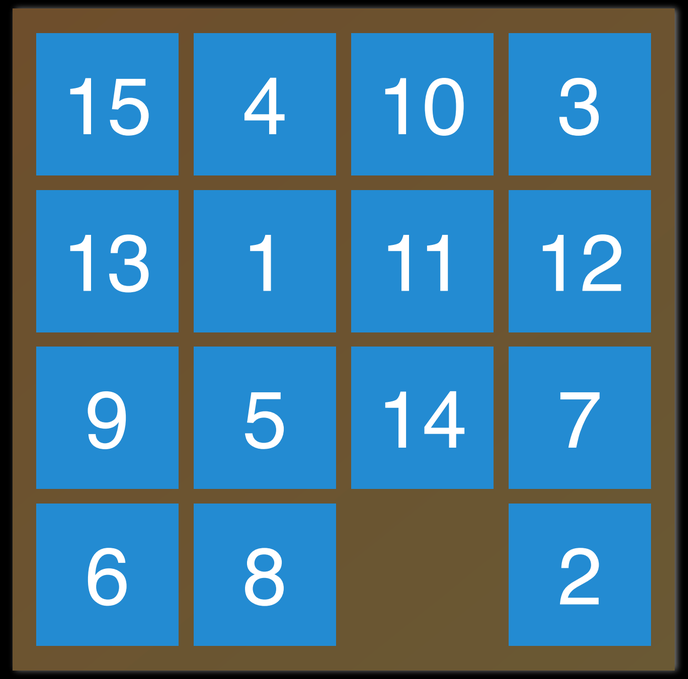
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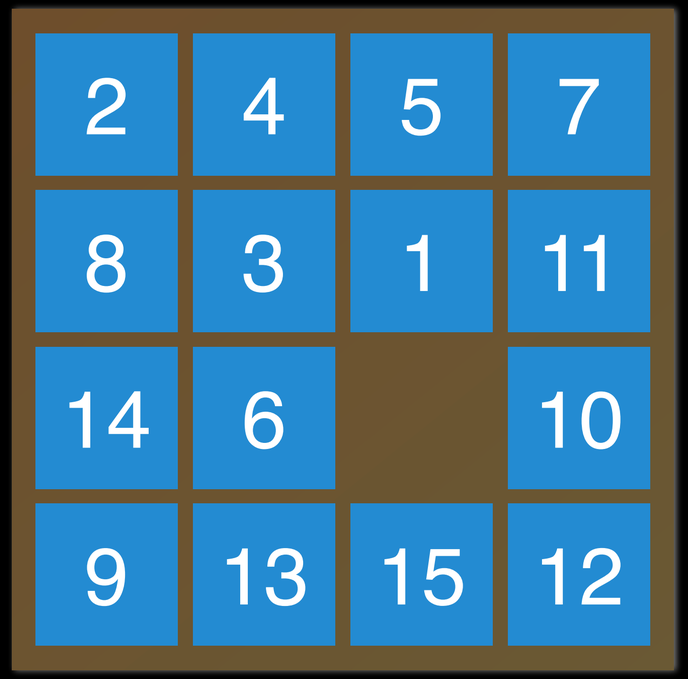
initial state

the state in which the agent begins

initial state



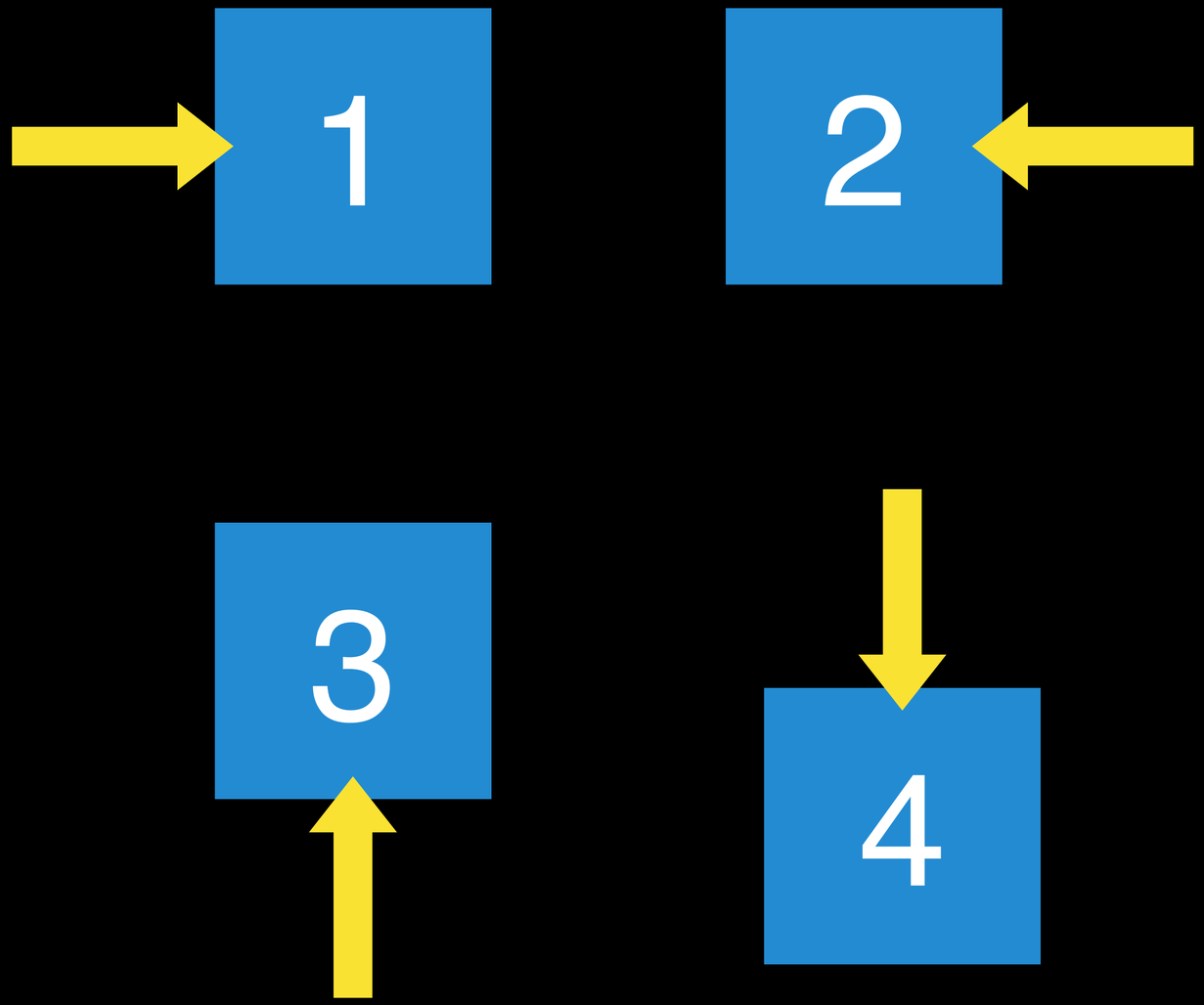
actions

choices that can be made in a state

actions

ACTIONS(s) returns the set of actions that can be executed in state s

actions



transition model

a description of what state results from performing any applicable action in any state

transition model

RESULT(s, a) returns the state resulting from performing action a in state s

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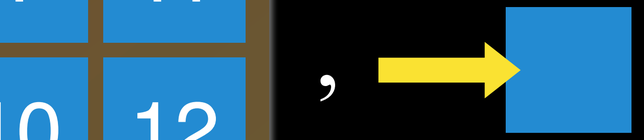
, ) =

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RESULT(

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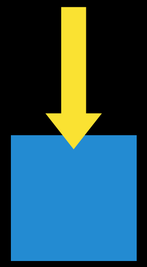
, ) =

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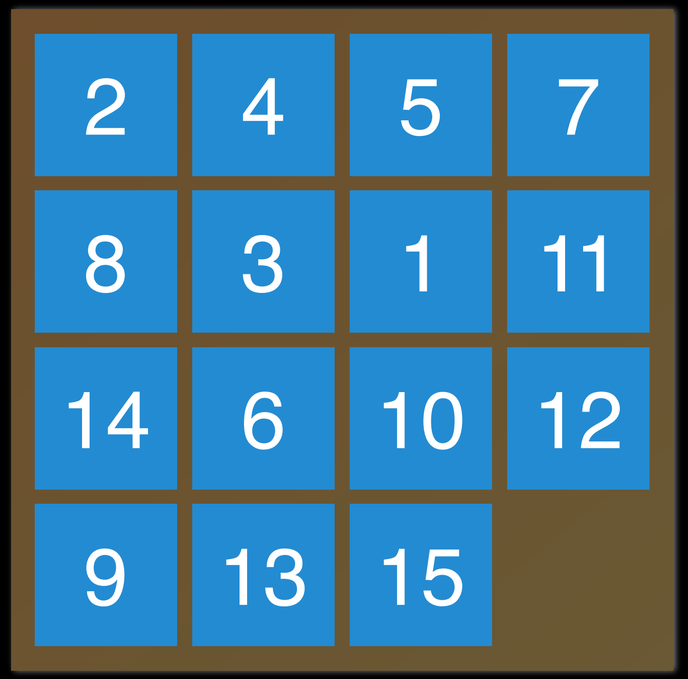
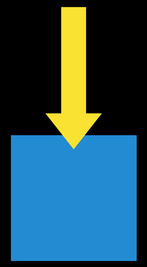
transition model

RESULT(

, ) =

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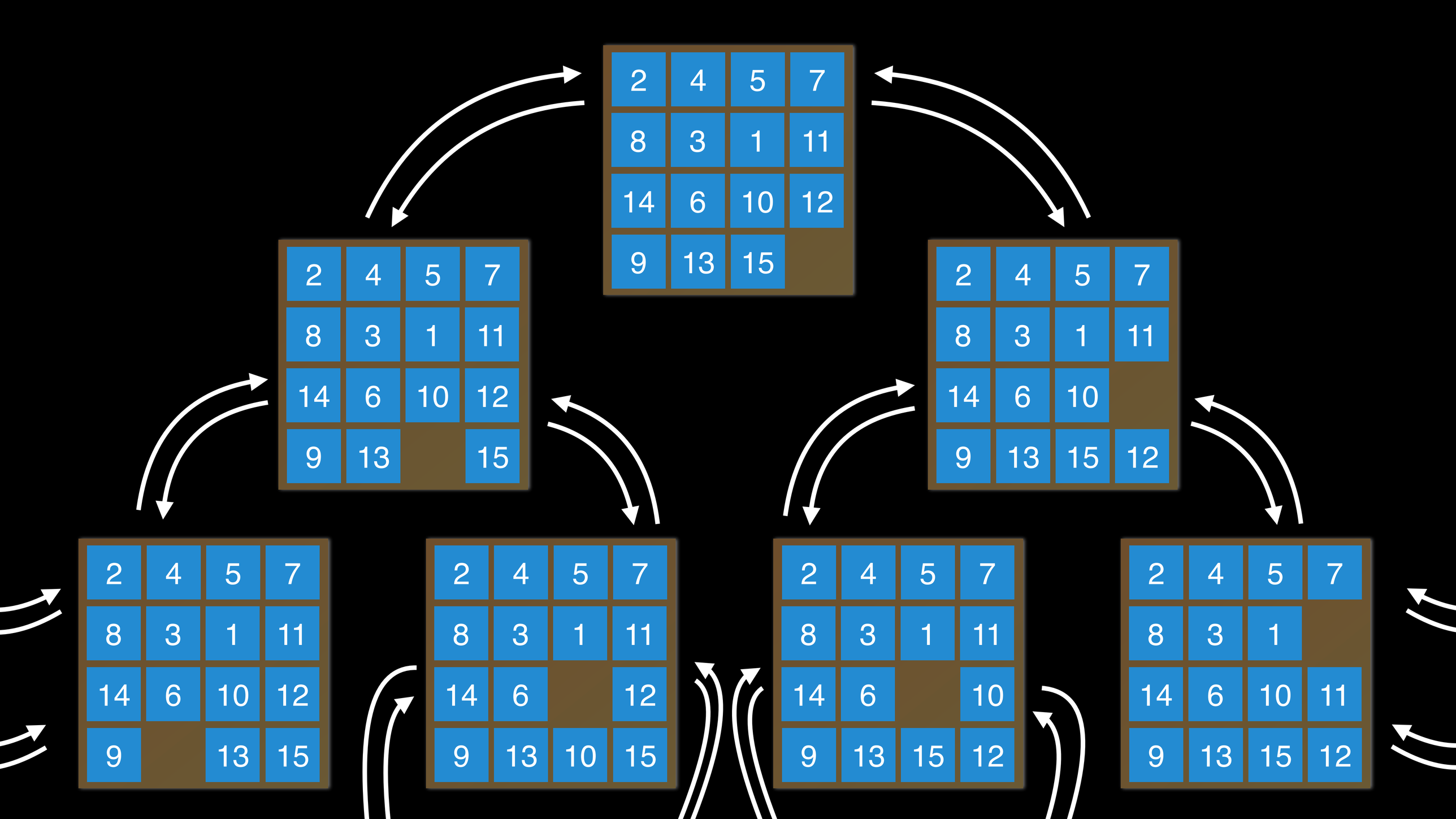
state space

the set of all states reachable from the initial state by any sequence of actions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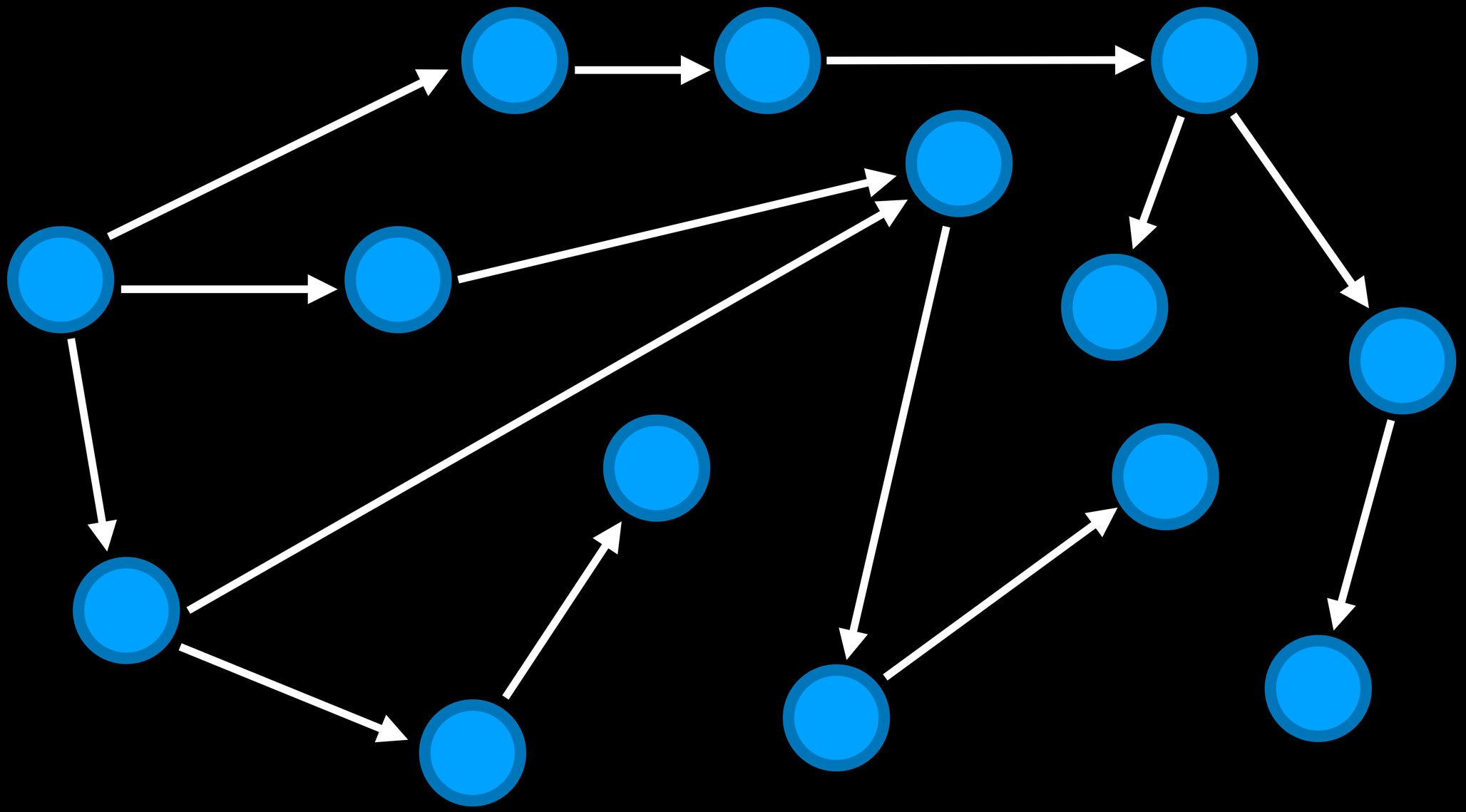
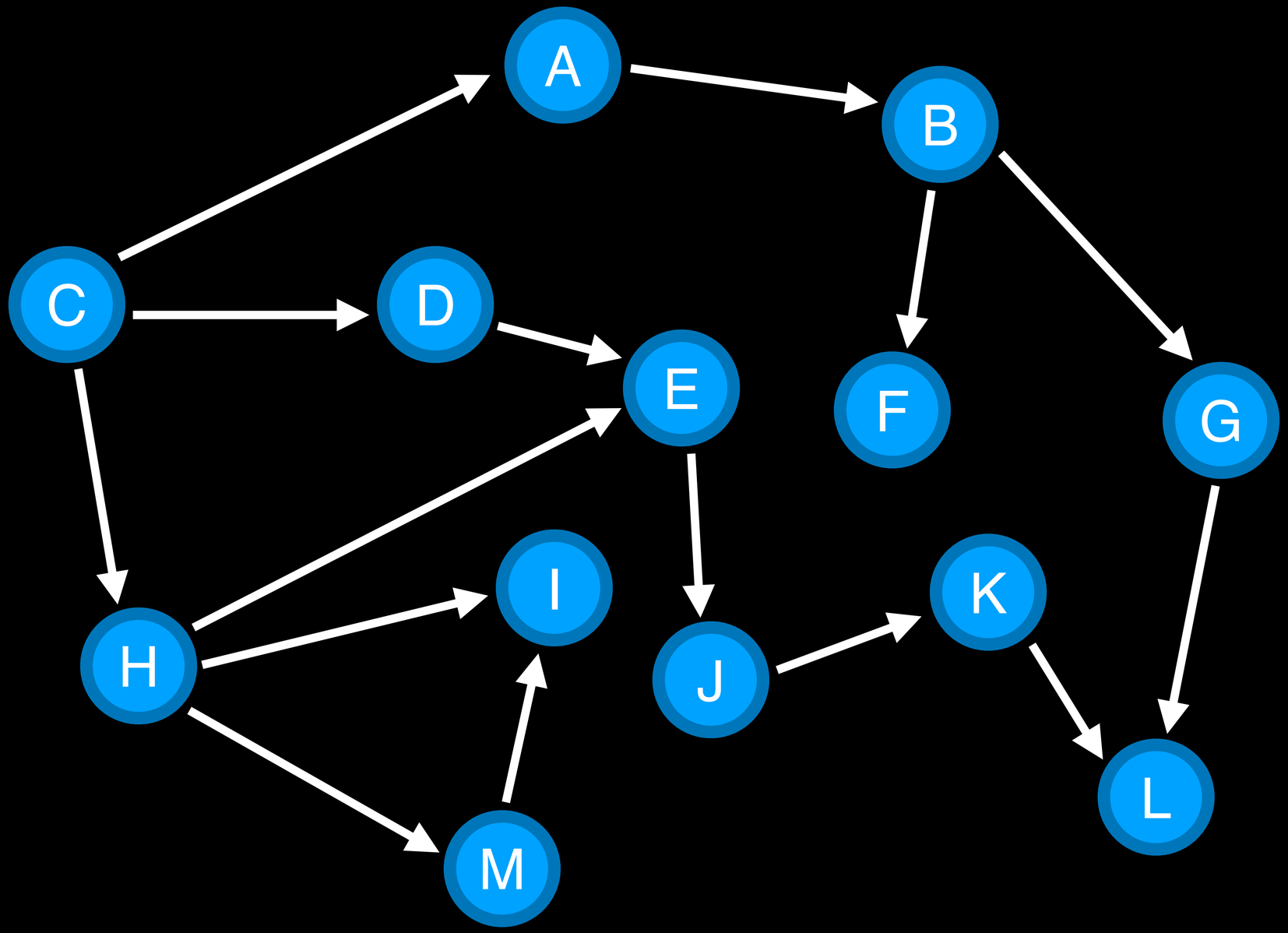


goal test

way to determine whether a given state
is a goal state

path cost

numerical cost associated with a given path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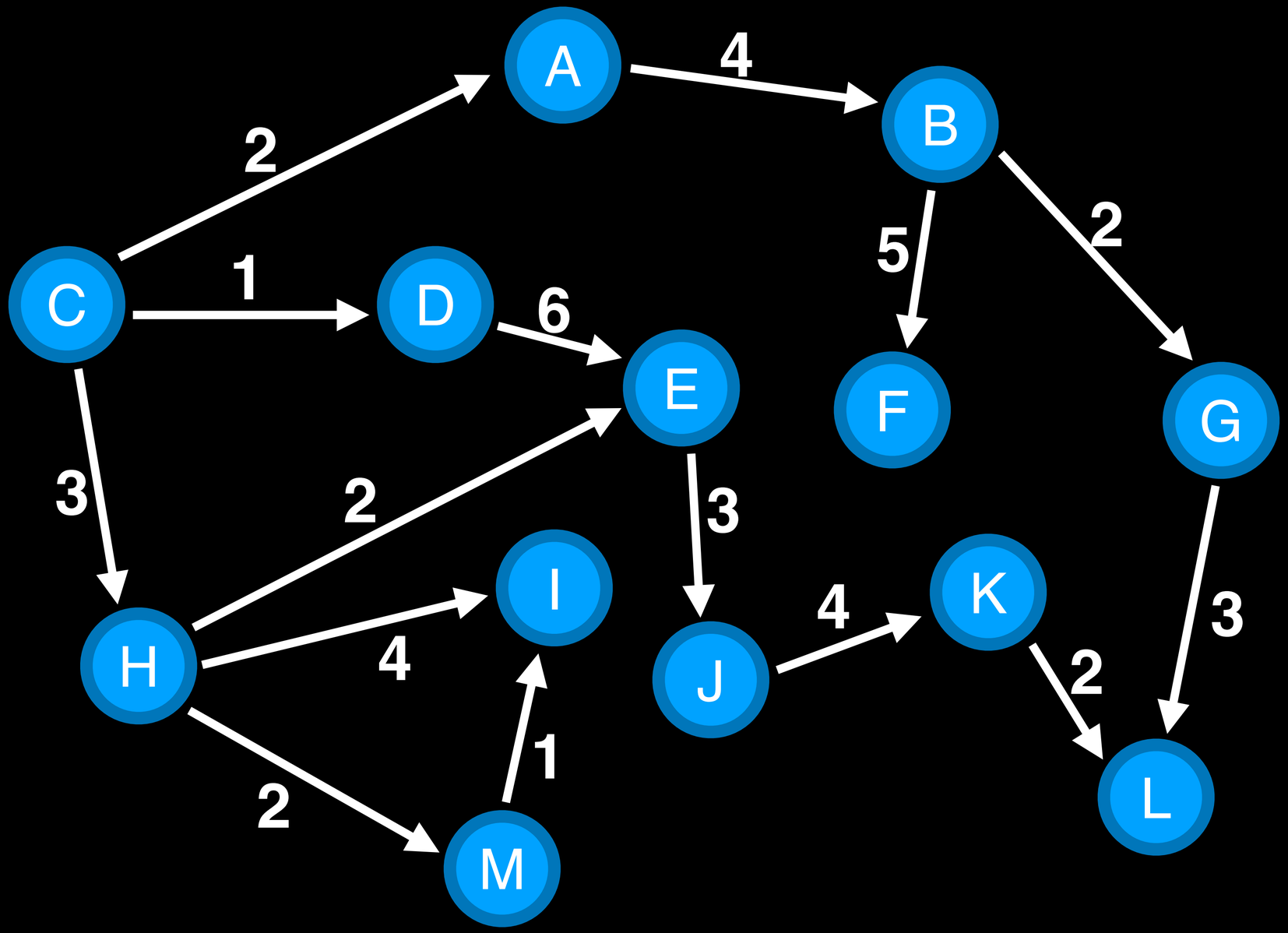
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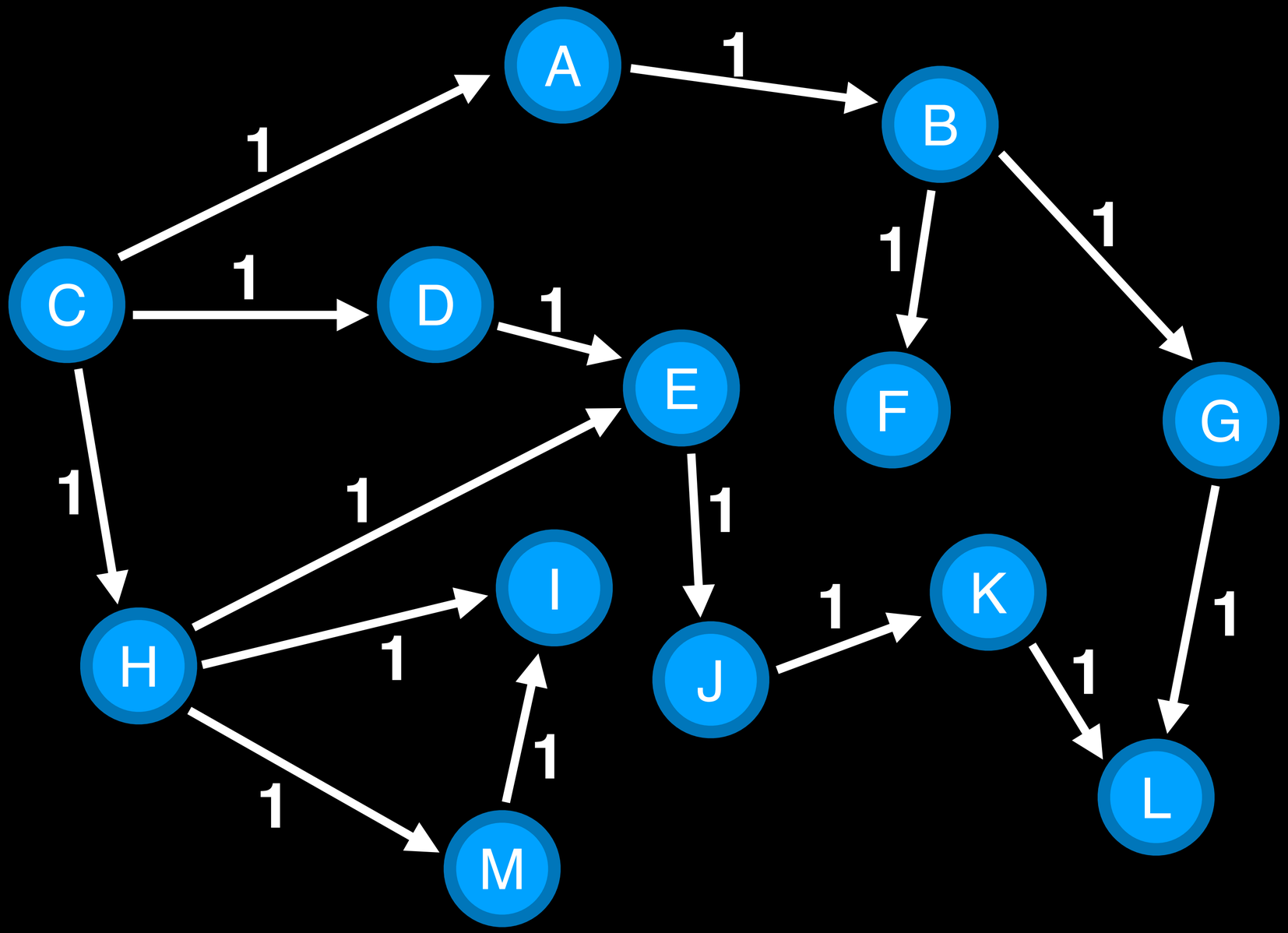
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Search Problems

- initial state
- actions
- transition model
- goal test
- path cost function

solution

a sequence of actions that leads from the initial state to a goal state

optimal solution

a solution that has the lowest path cost among all solutions

node

a data structure that keeps track of

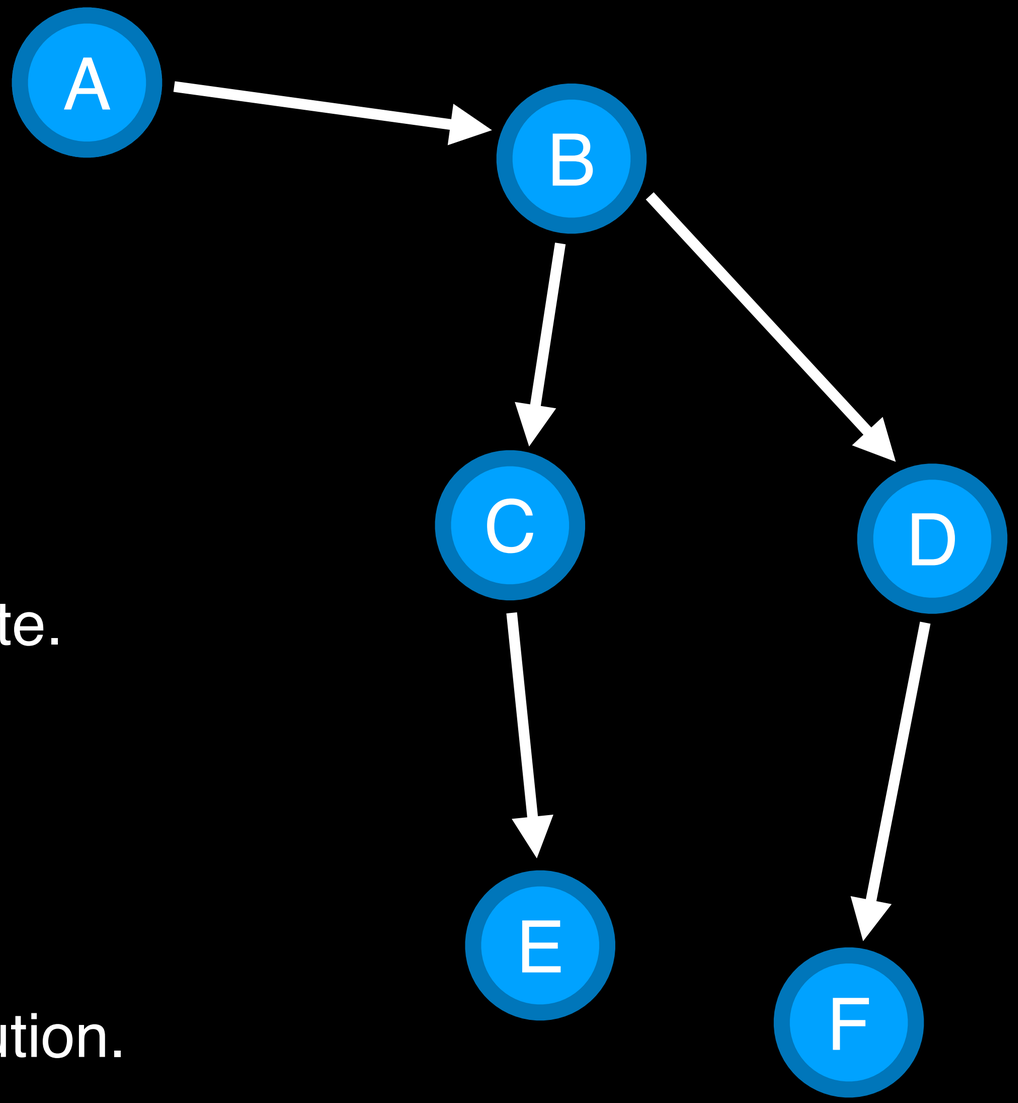
- a **state**
- a **parent** (node that generated this node)
- an **action** (action applied to parent to get node)
- a **path cost** (from initial state to node)

Approach

- Start with a **frontier** that contains the initial state.
- Repeat:
  - If the frontier is empty, then no solution.
  - Remove a node from the frontier.
  - If node contains goal state, return the solution.
  - **Expand** node, add resulting nodes to the frontier.

Find a path from A to E.

Frontier



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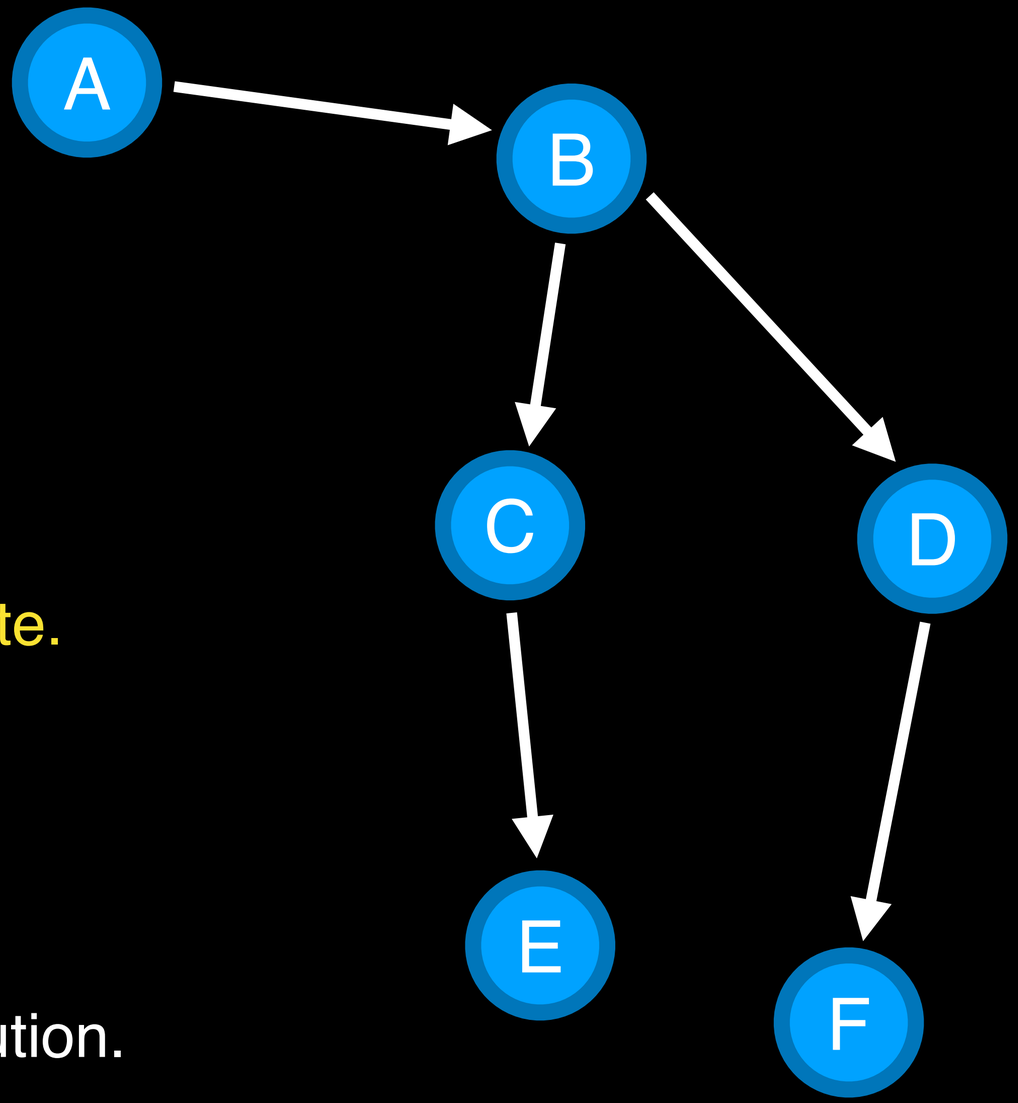

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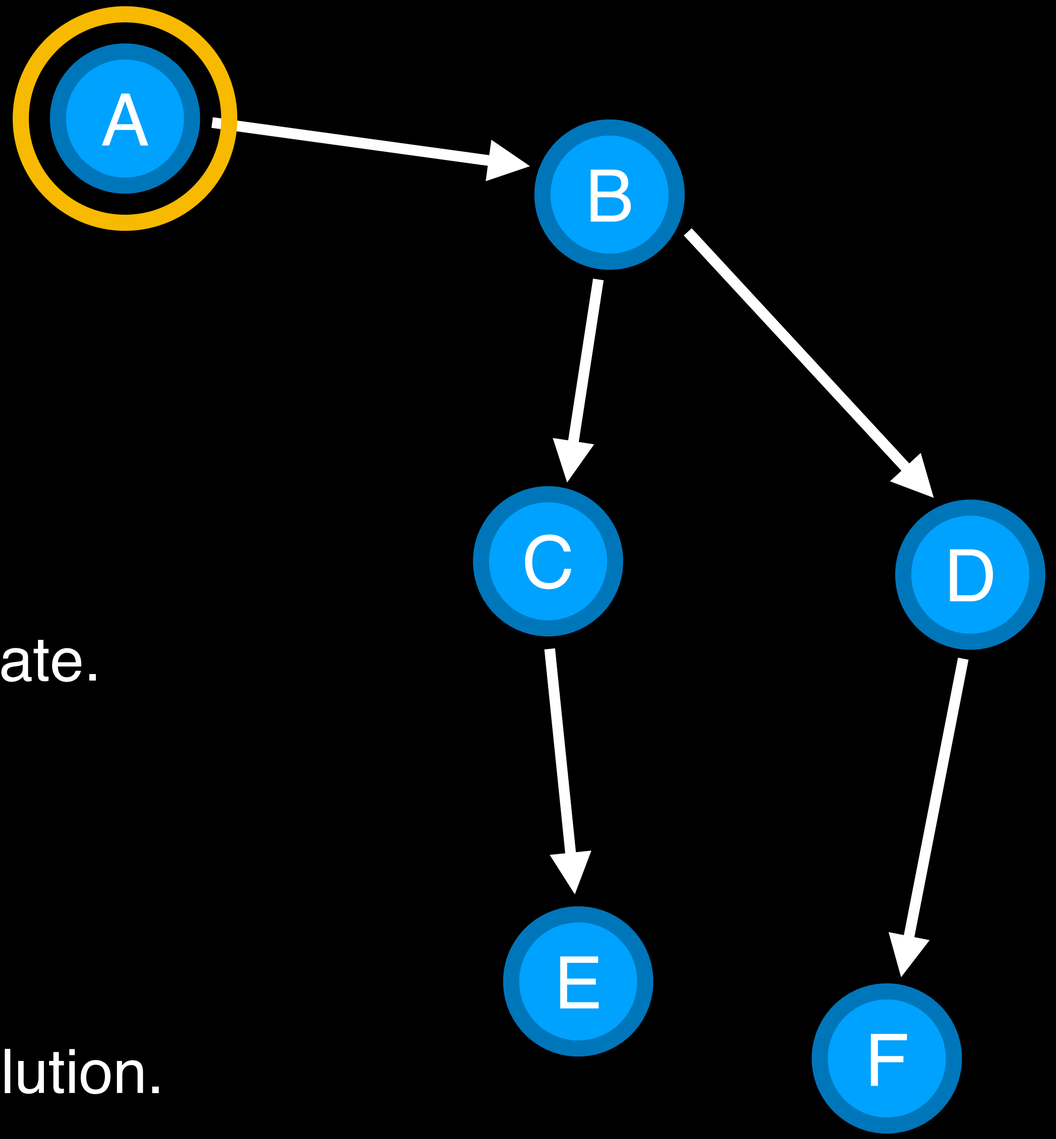

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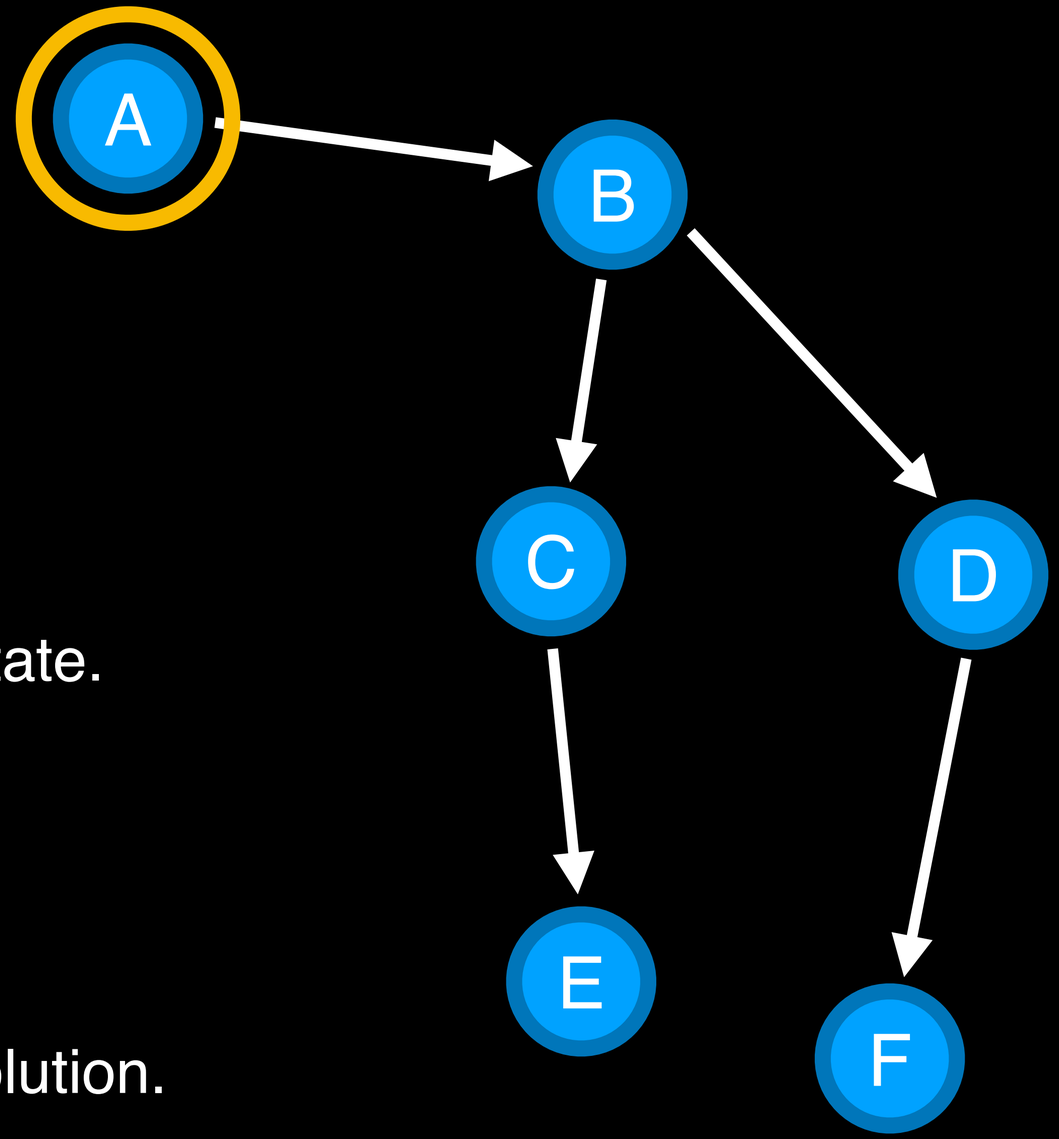
Frontier

B



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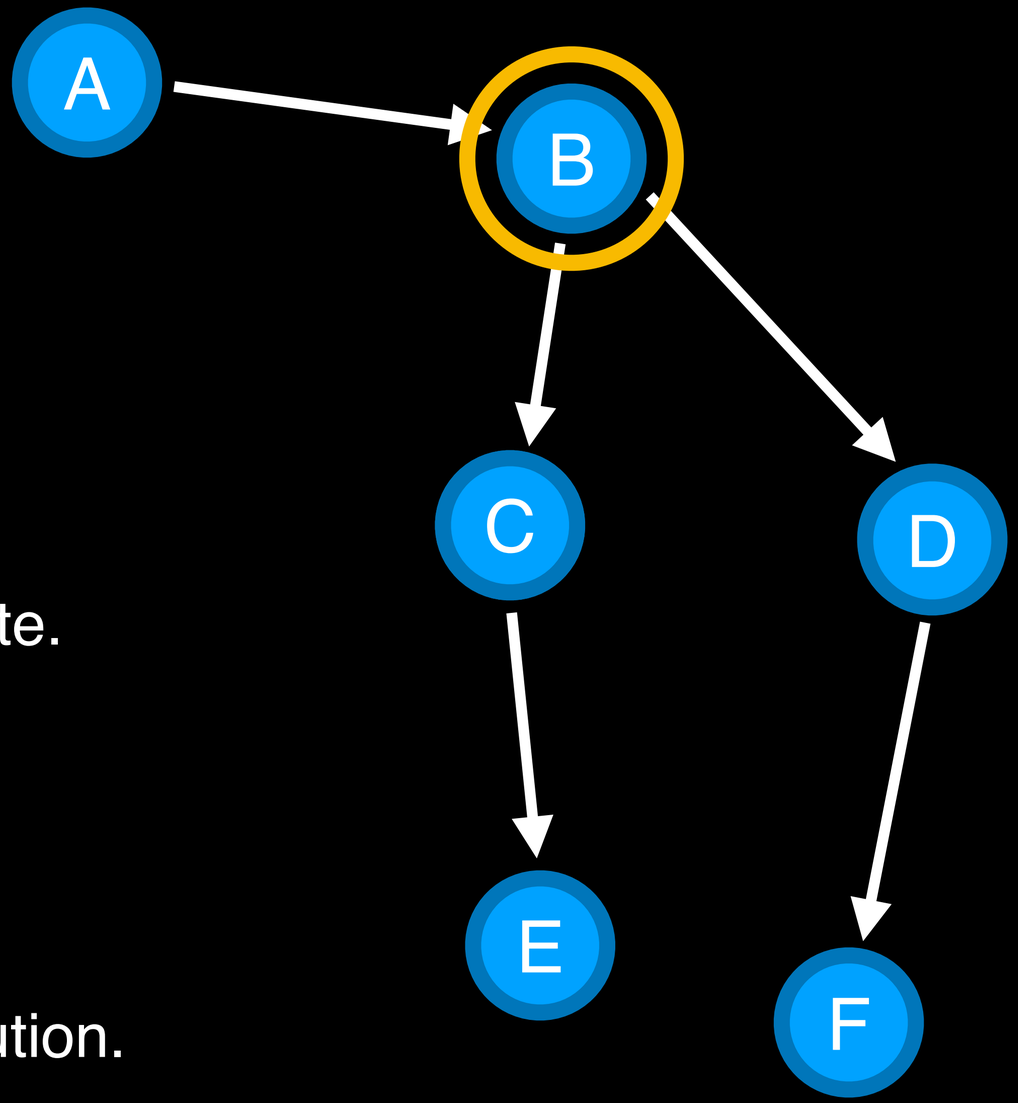

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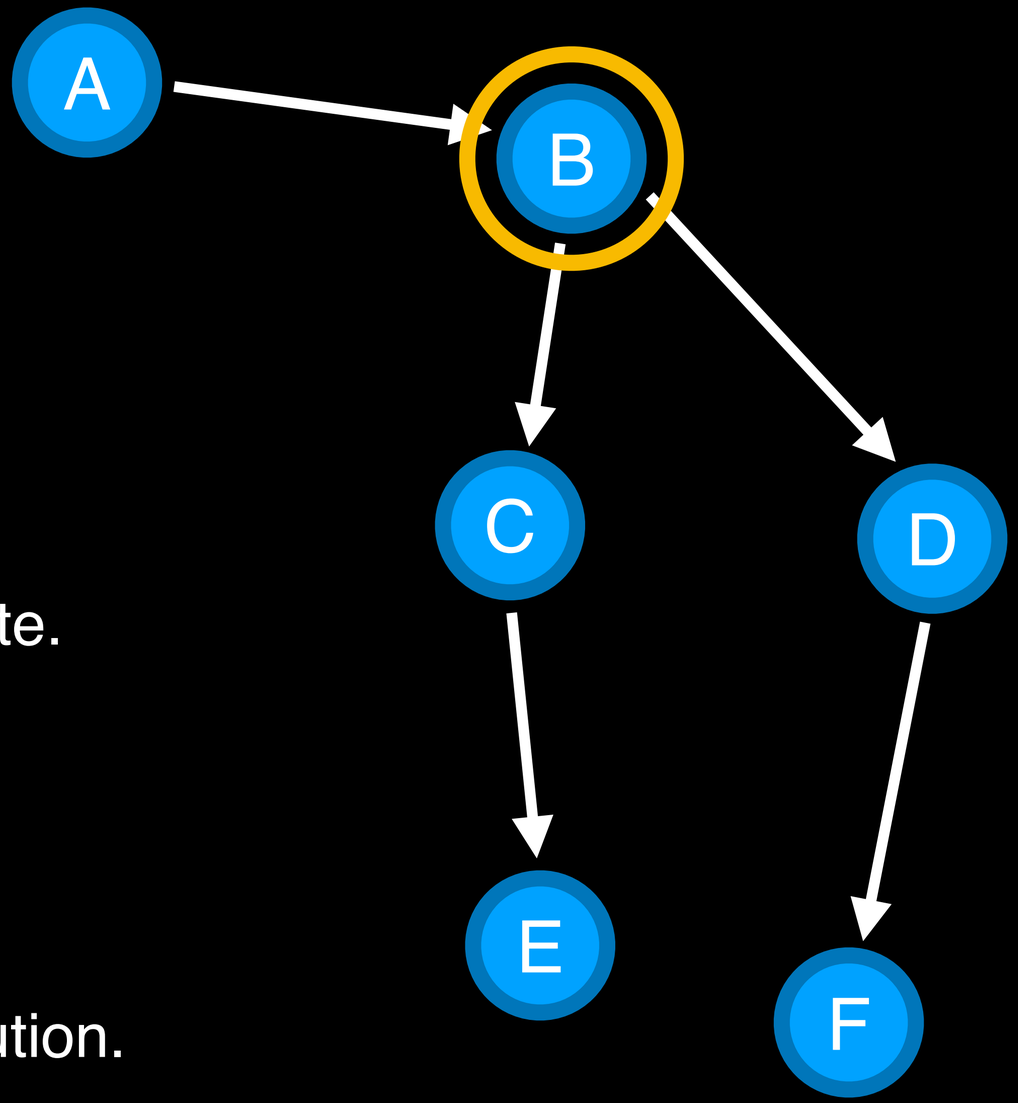
Frontier

C D



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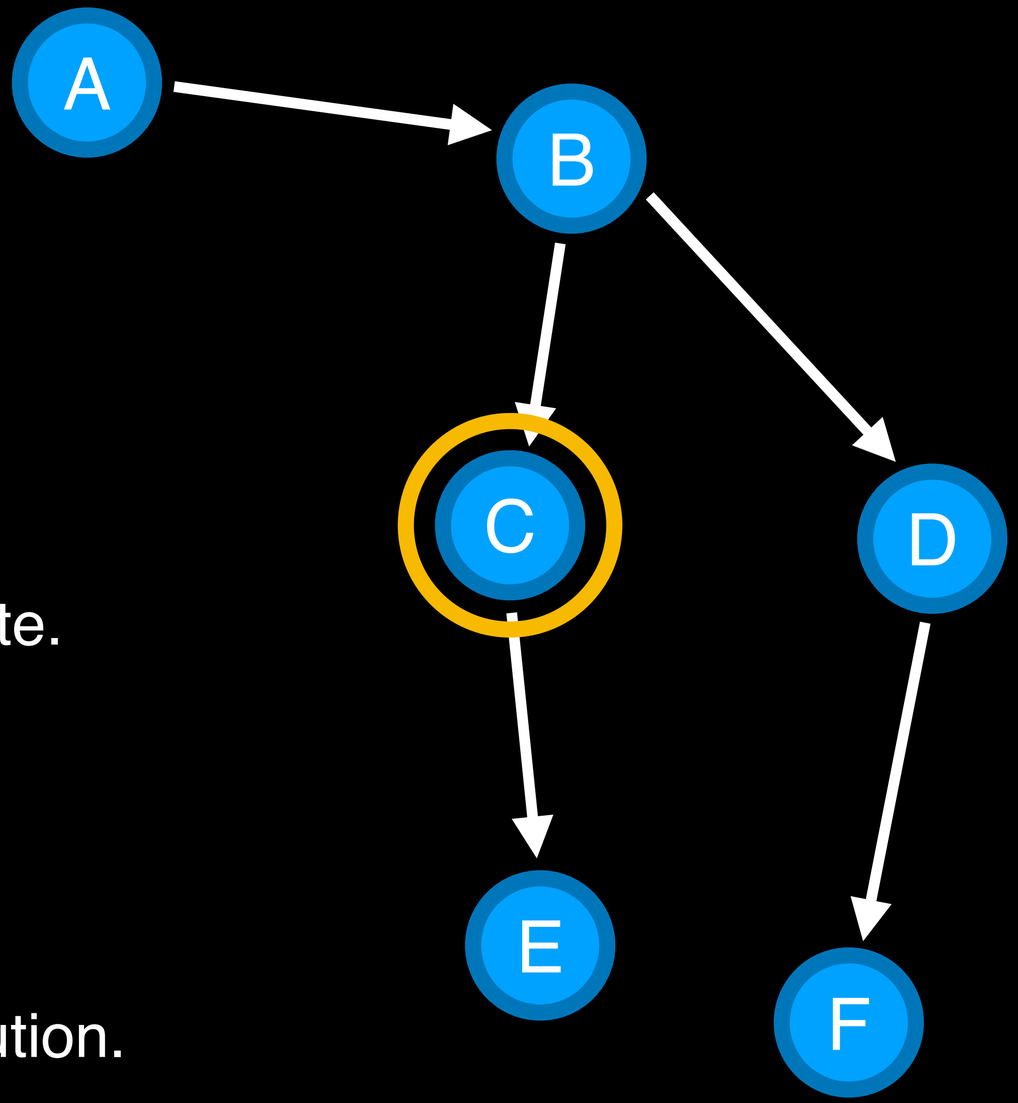
Frontier

D



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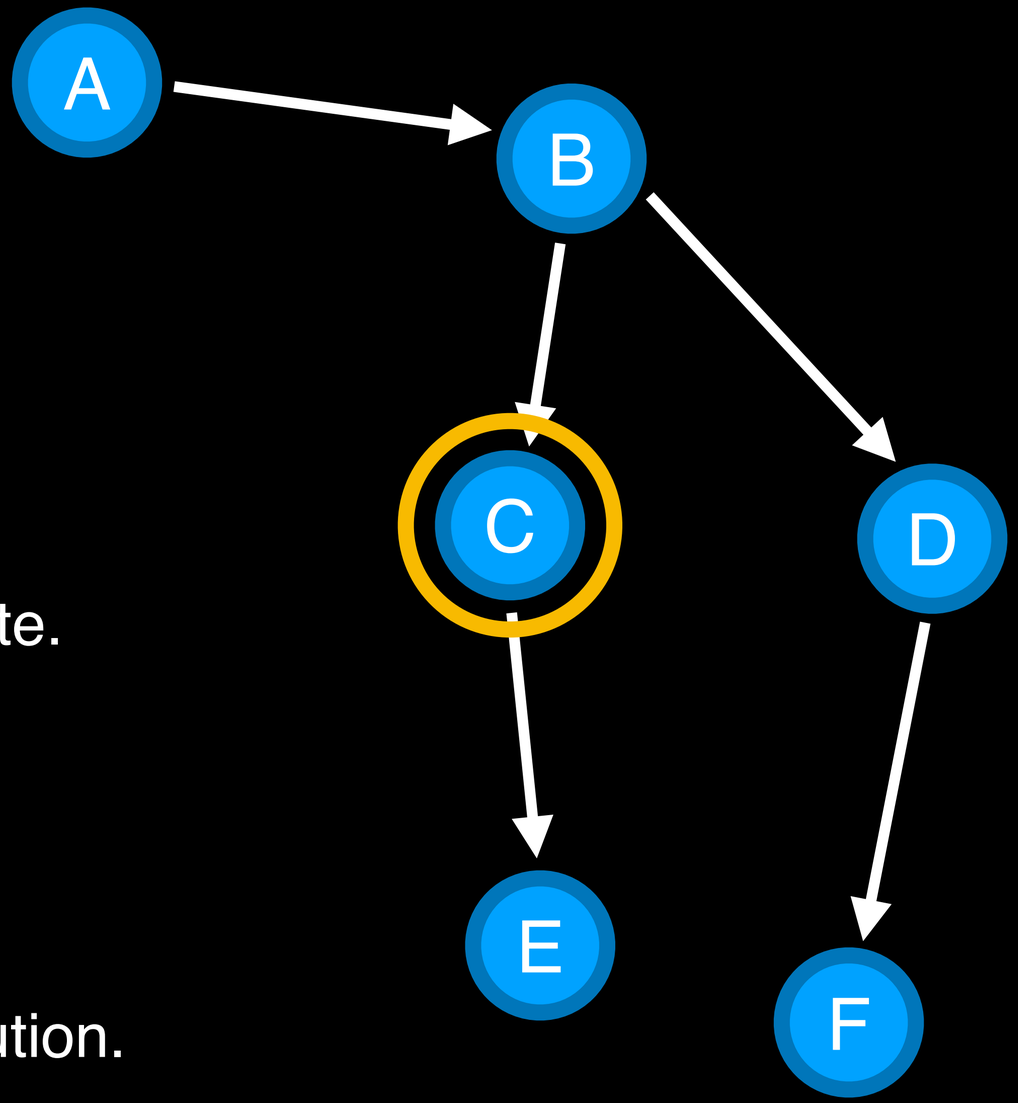
Frontier

E D



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Frontier

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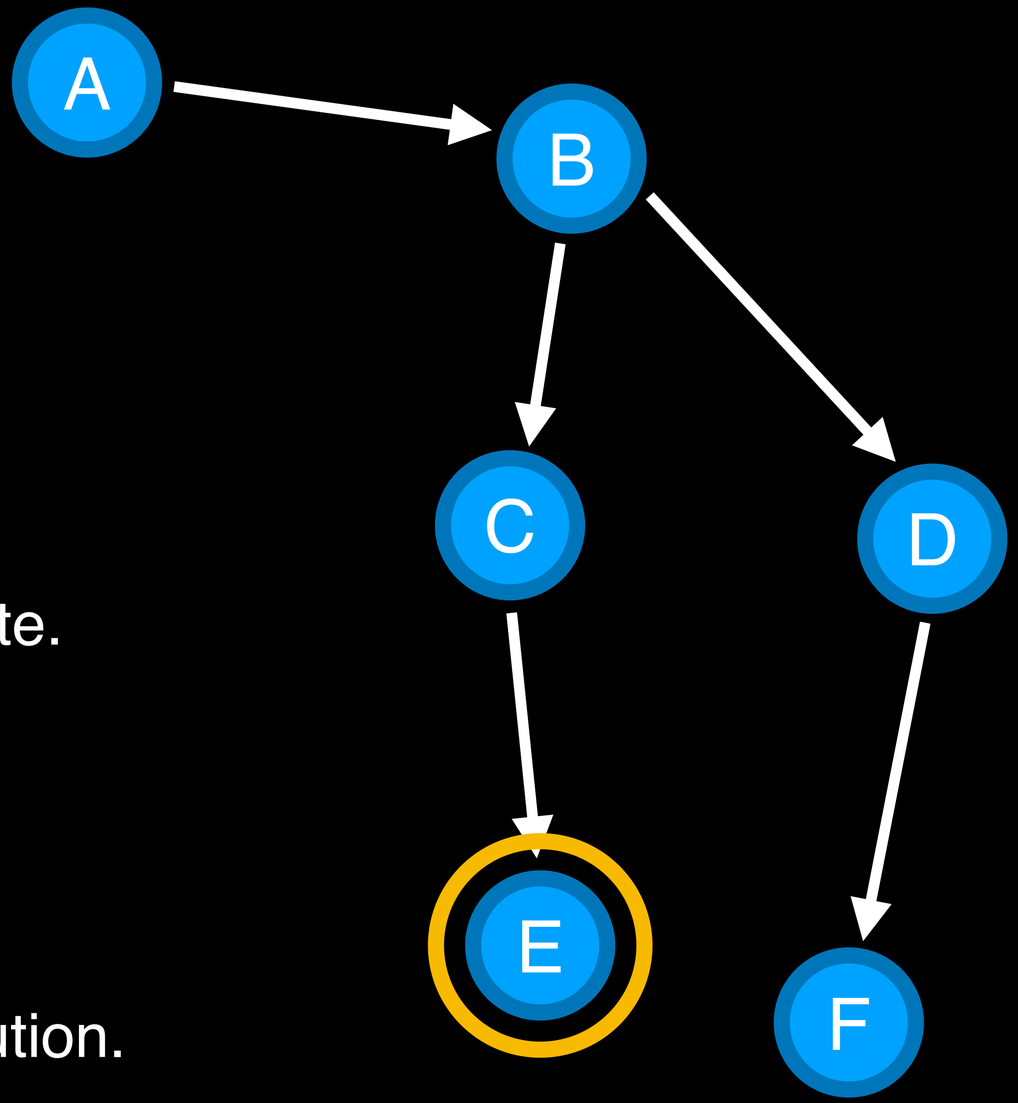

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Frontier

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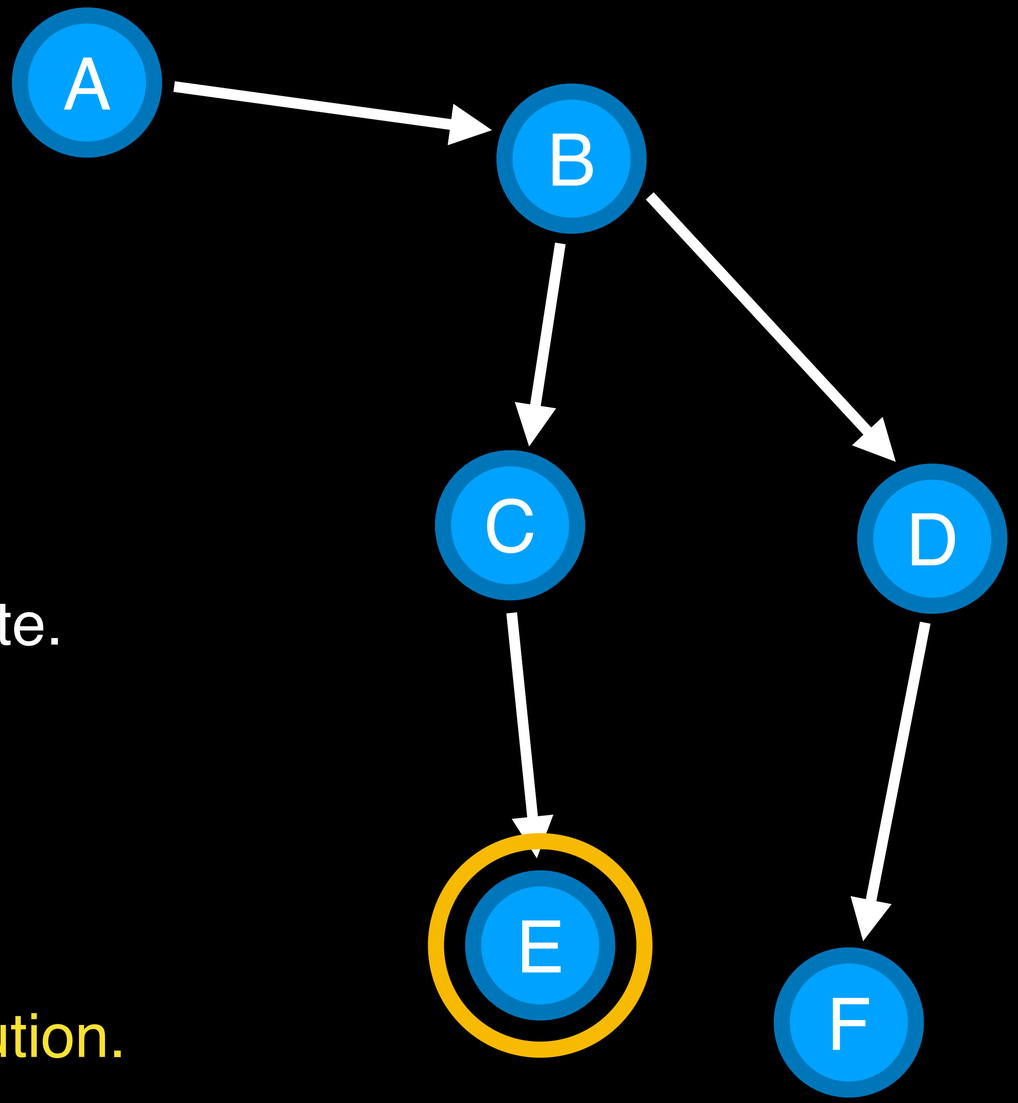

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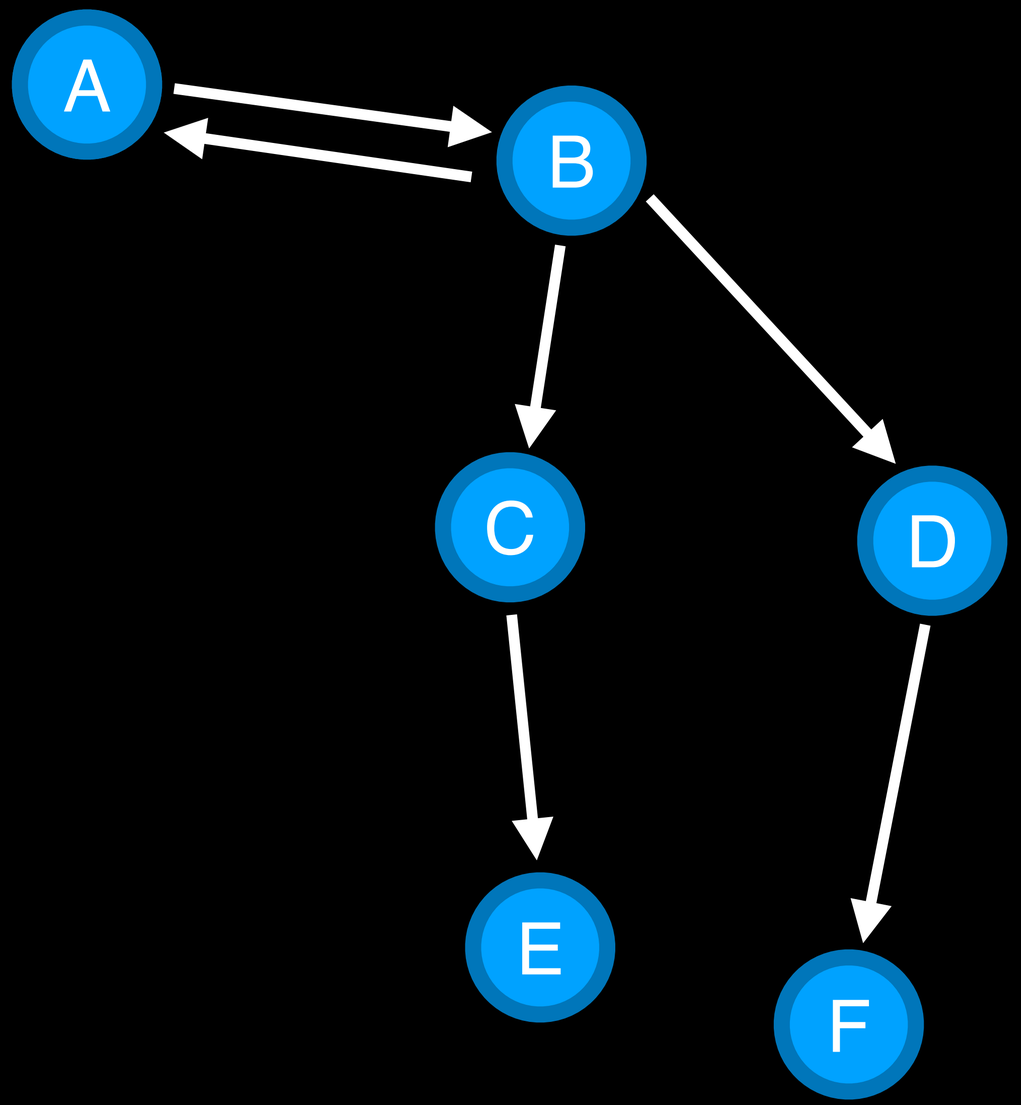
What could go wrong?

Find a path from A to E.

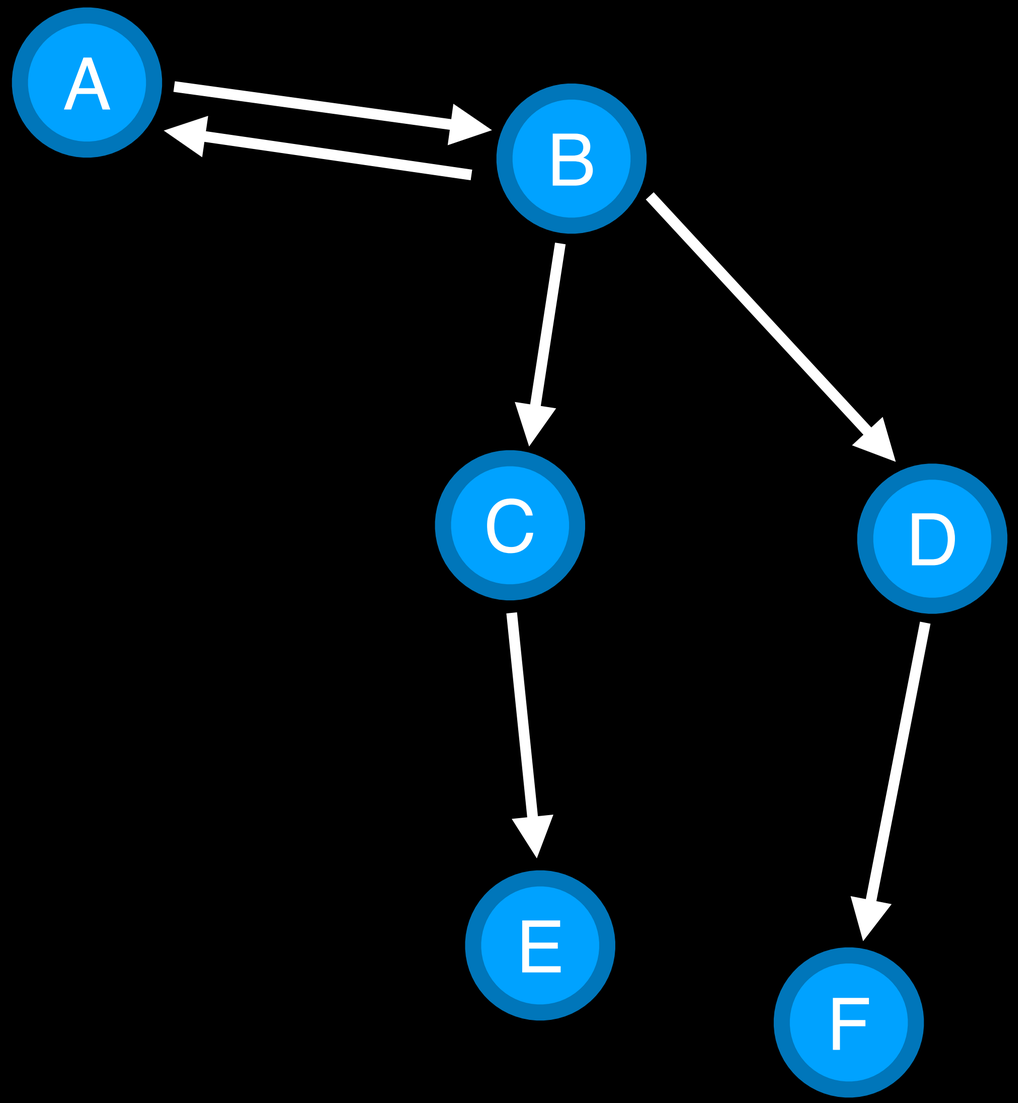
Frontier



Find a path from A to E.

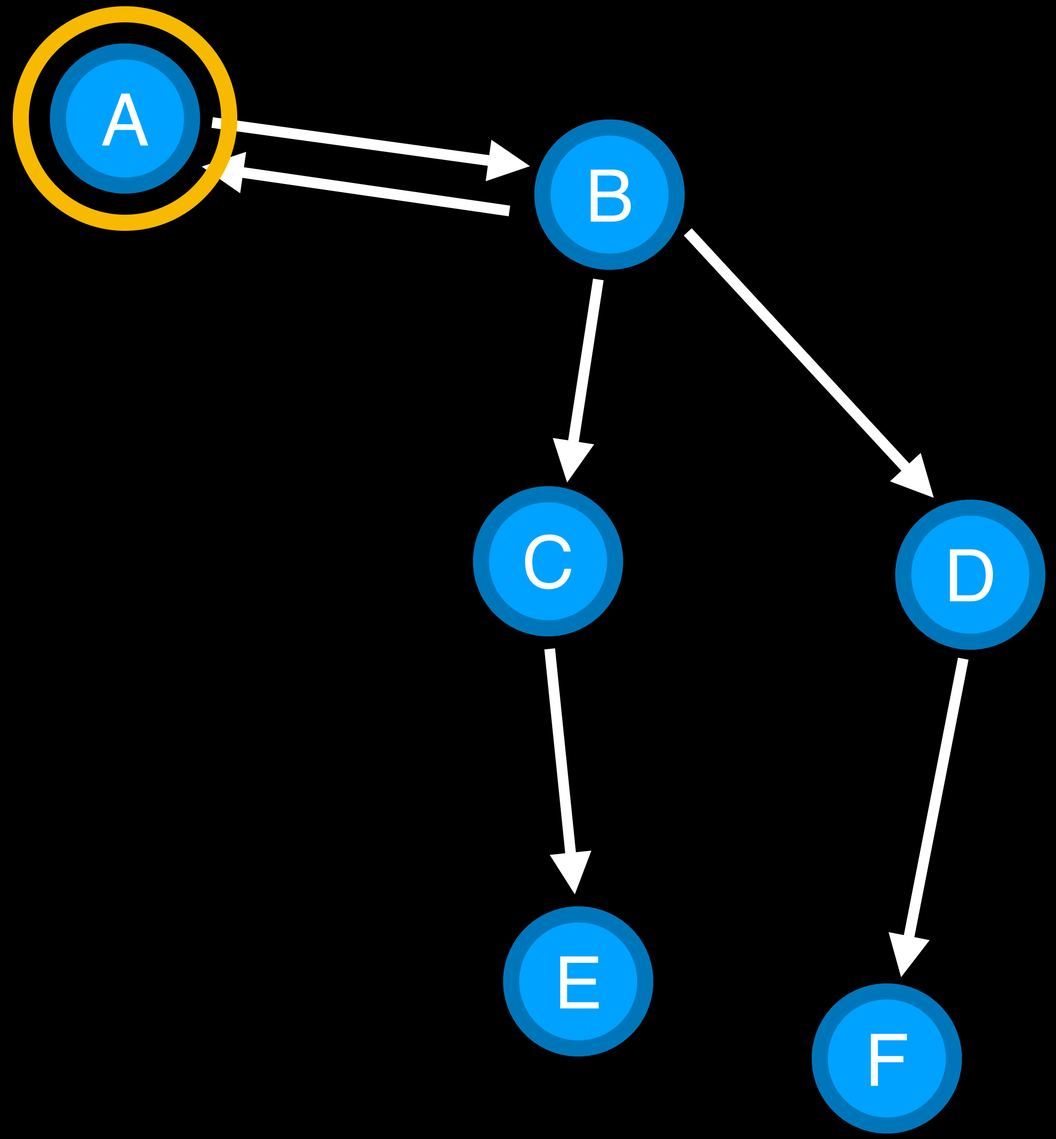
Frontier

A



Find a path from A to E.

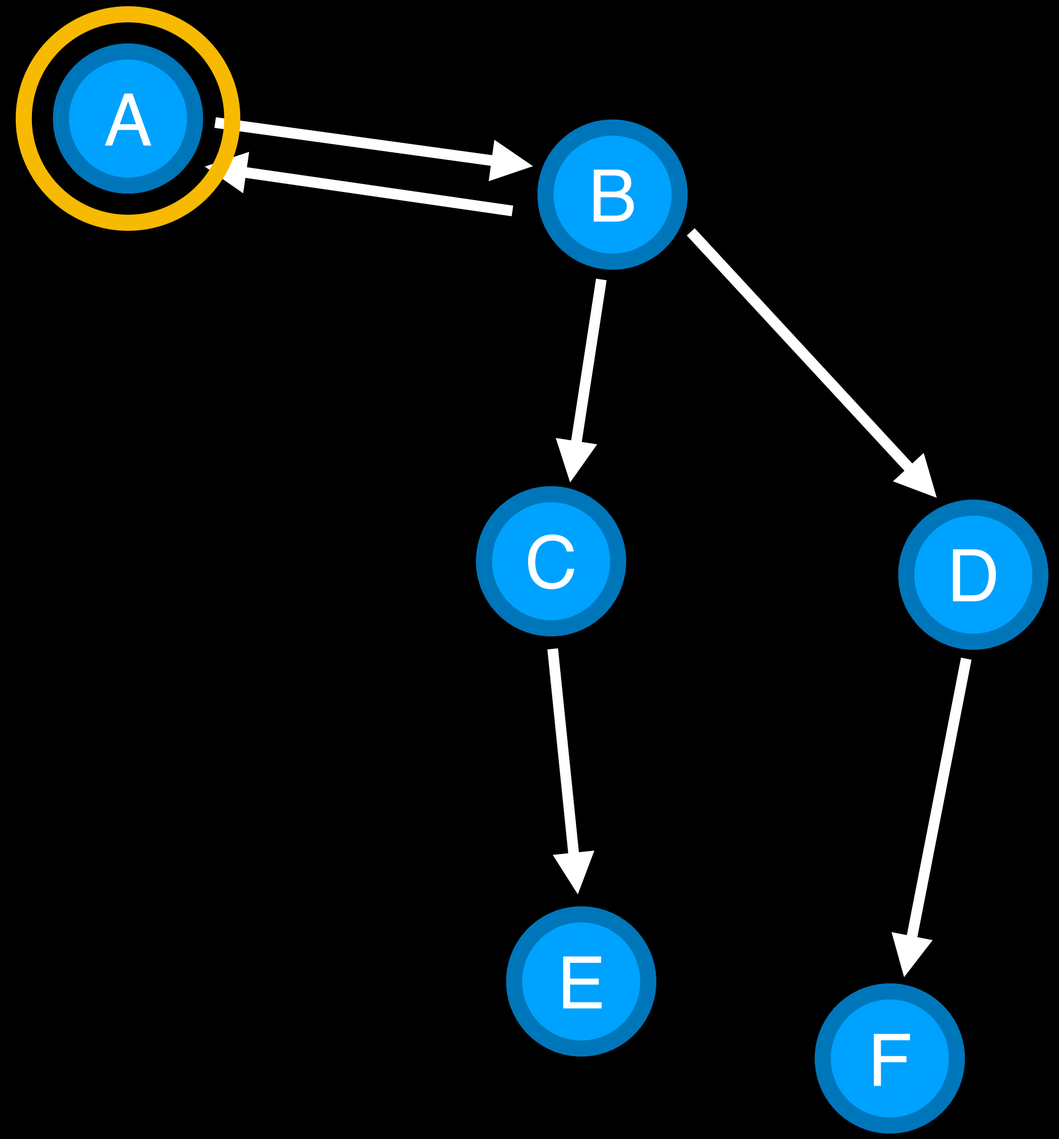
Frontier



Find a path from A to E.

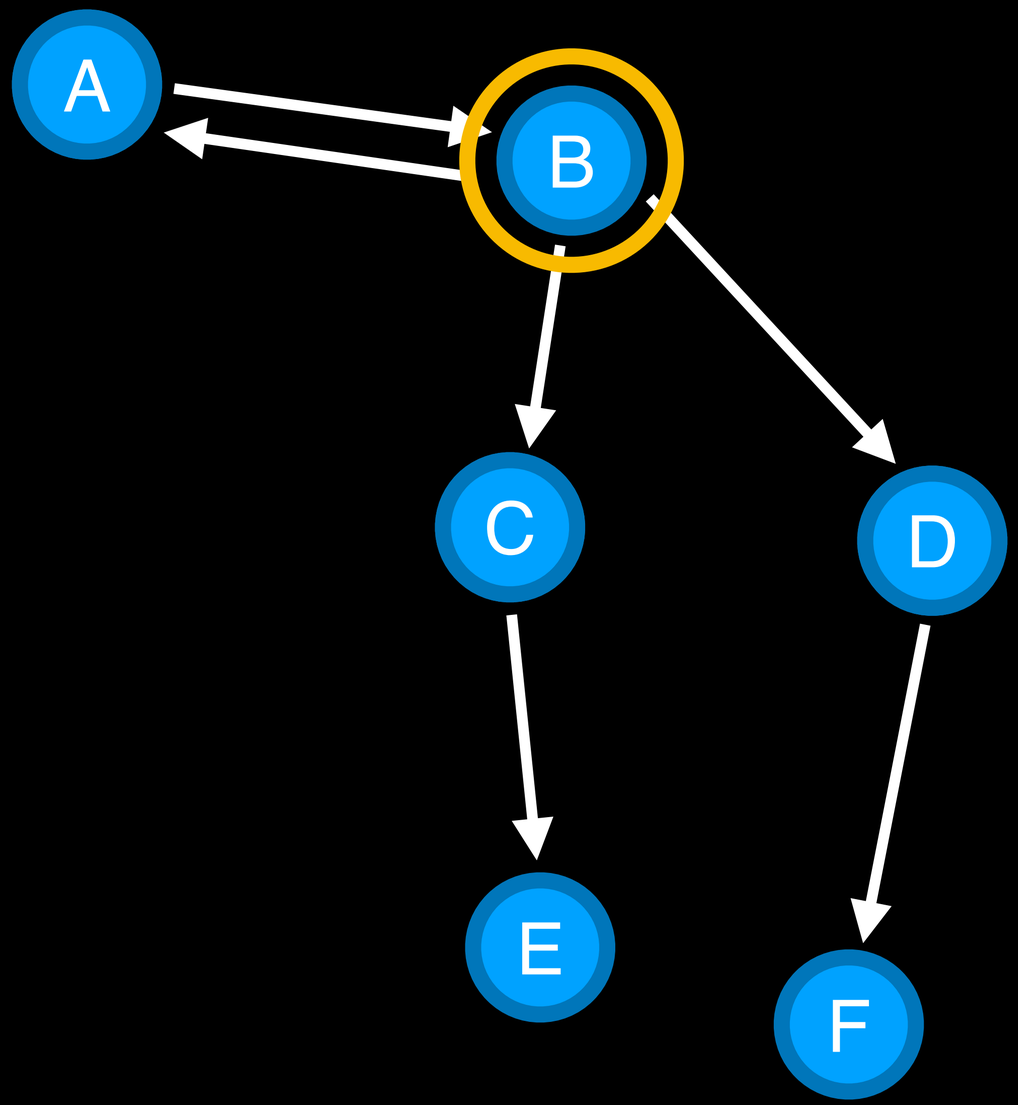
Frontier

B



Find a path from A to E.

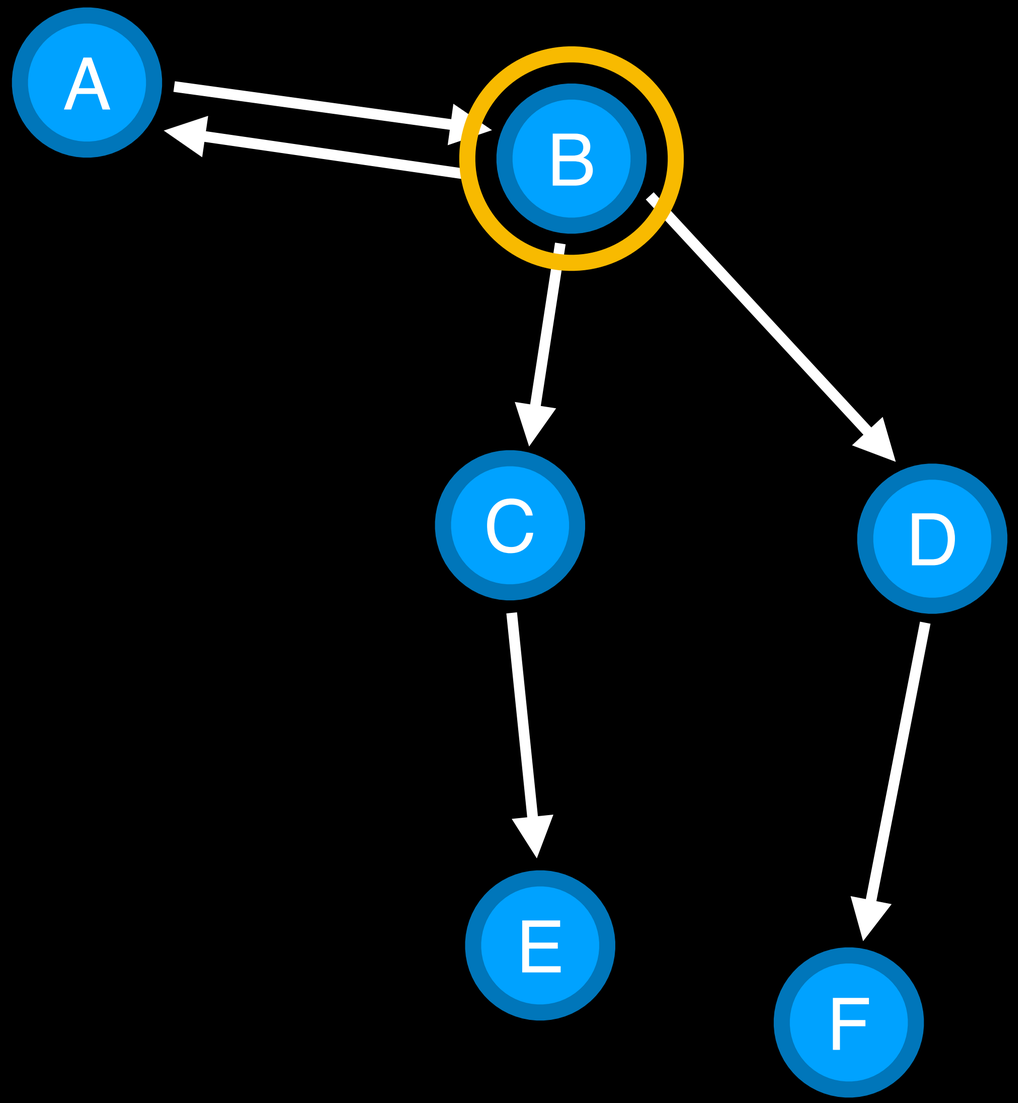
Frontier



Find a path from A to E.

Frontier

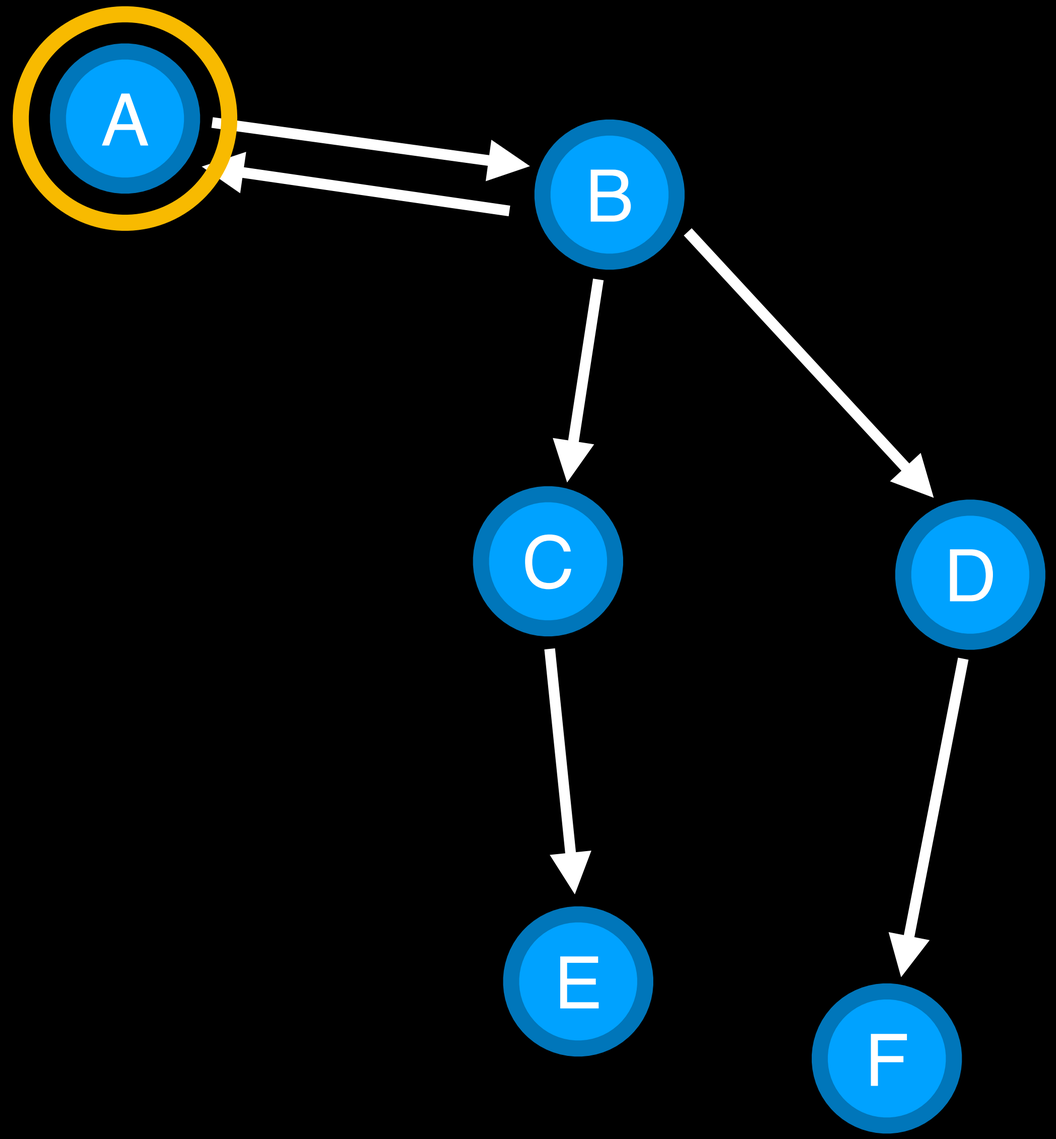
A C D



Find a path from A to E.

Frontier

C D



Revised Approach

- Start with a **frontier** that contains the initial state.
- Start with an empty **explored set**.
- Repeat:
    - If the frontier is empty, then no solution.
    - Remove a node from the frontier.
    - If node contains goal state, return the solution.
    - Add the node to the explored set.
    - **Expand** node, add resulting nodes to the frontier if they aren't already in the frontier or the explored set.

Revised Approach

- Start with a **frontier** that contains the initial state.
- Start with an empty **explored set**.
- Repeat:
  - If the frontier is empty, then no solution.
  - Remove a node from the frontier.
  - If node contains goal state, return the solution.
  - Add the node to the explored set.
  - **Expand** node, add resulting nodes to the frontier if they aren't already in the frontier or the explored set.

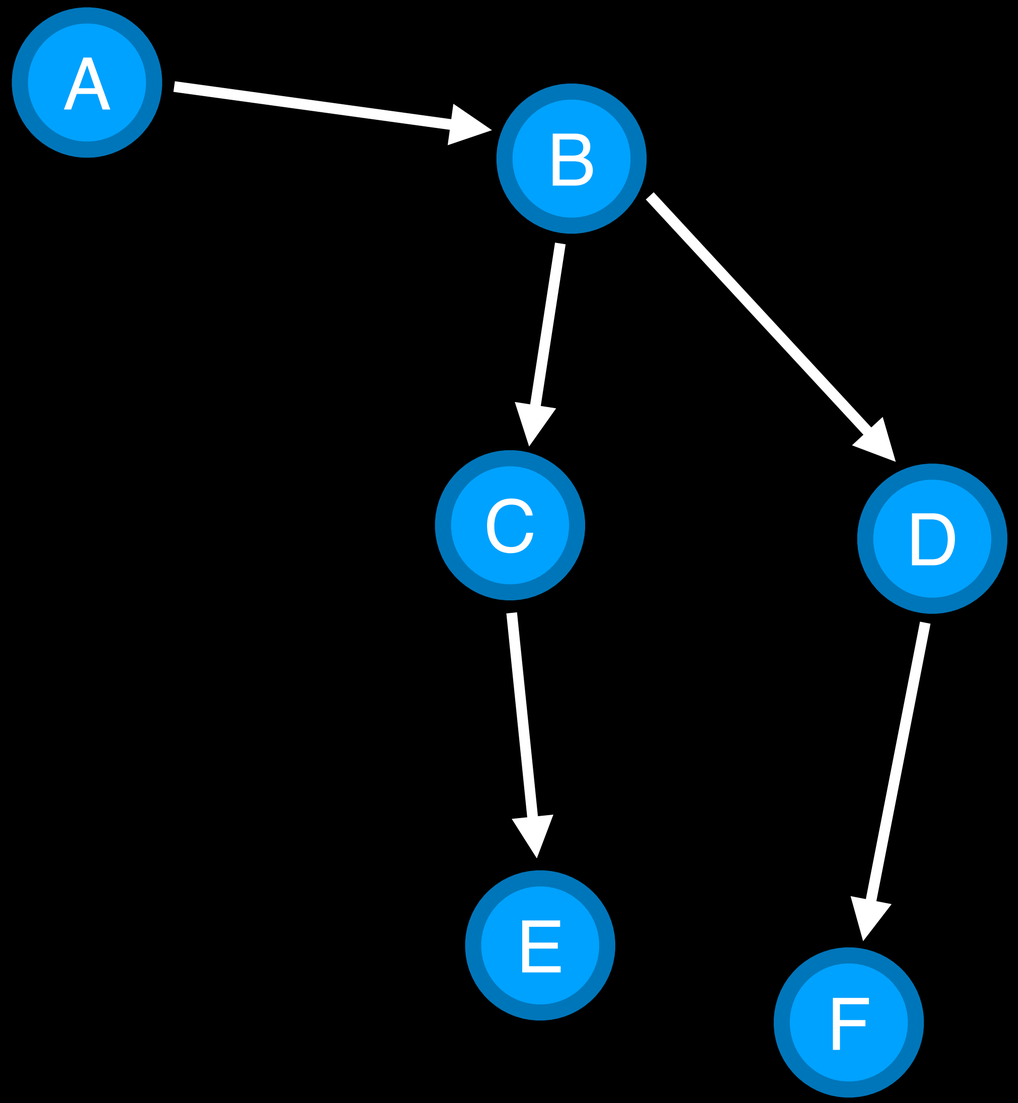
stack

last-in first-out data type

Find a path from A to E.

Frontier

Explored Set

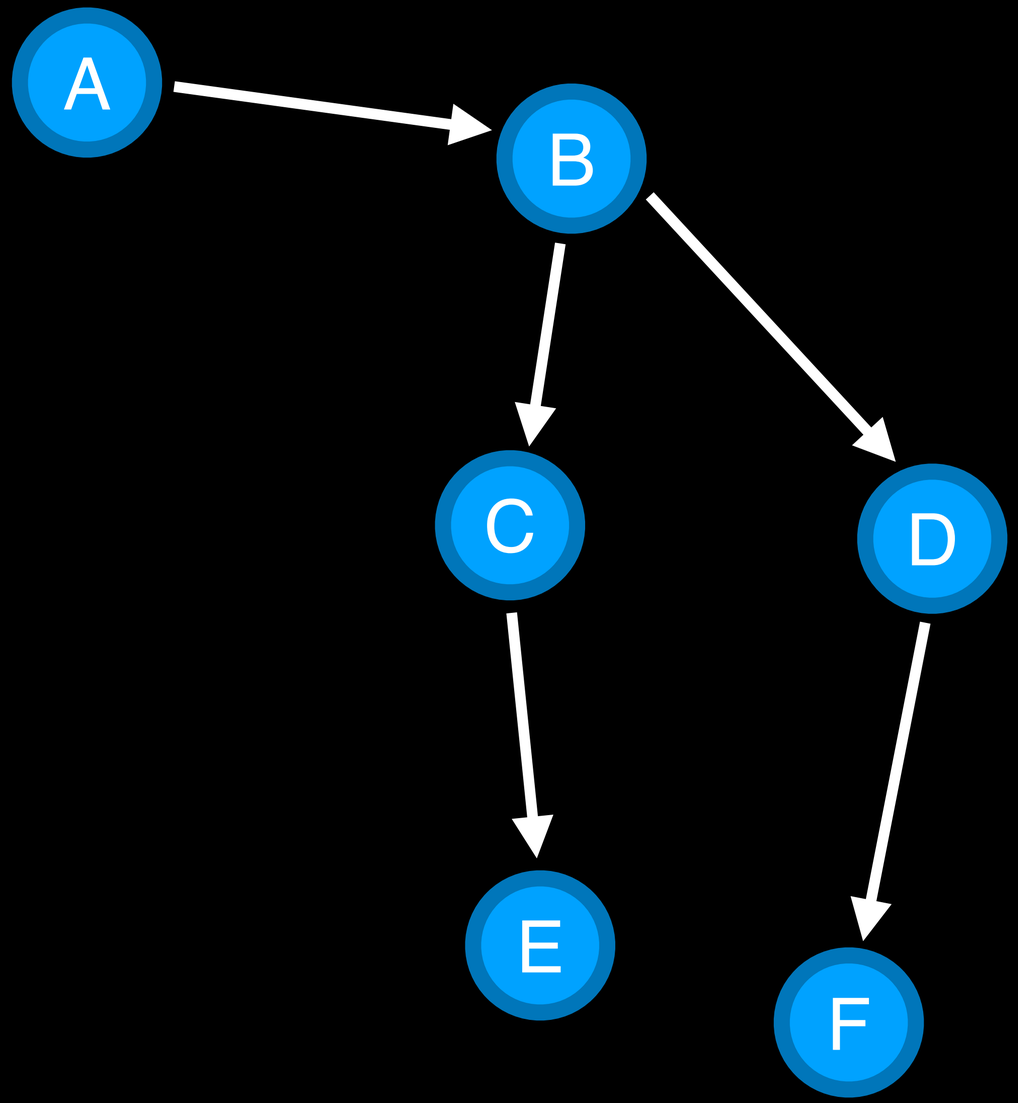


Find a path from A to E.

Frontier

A

Explored Set

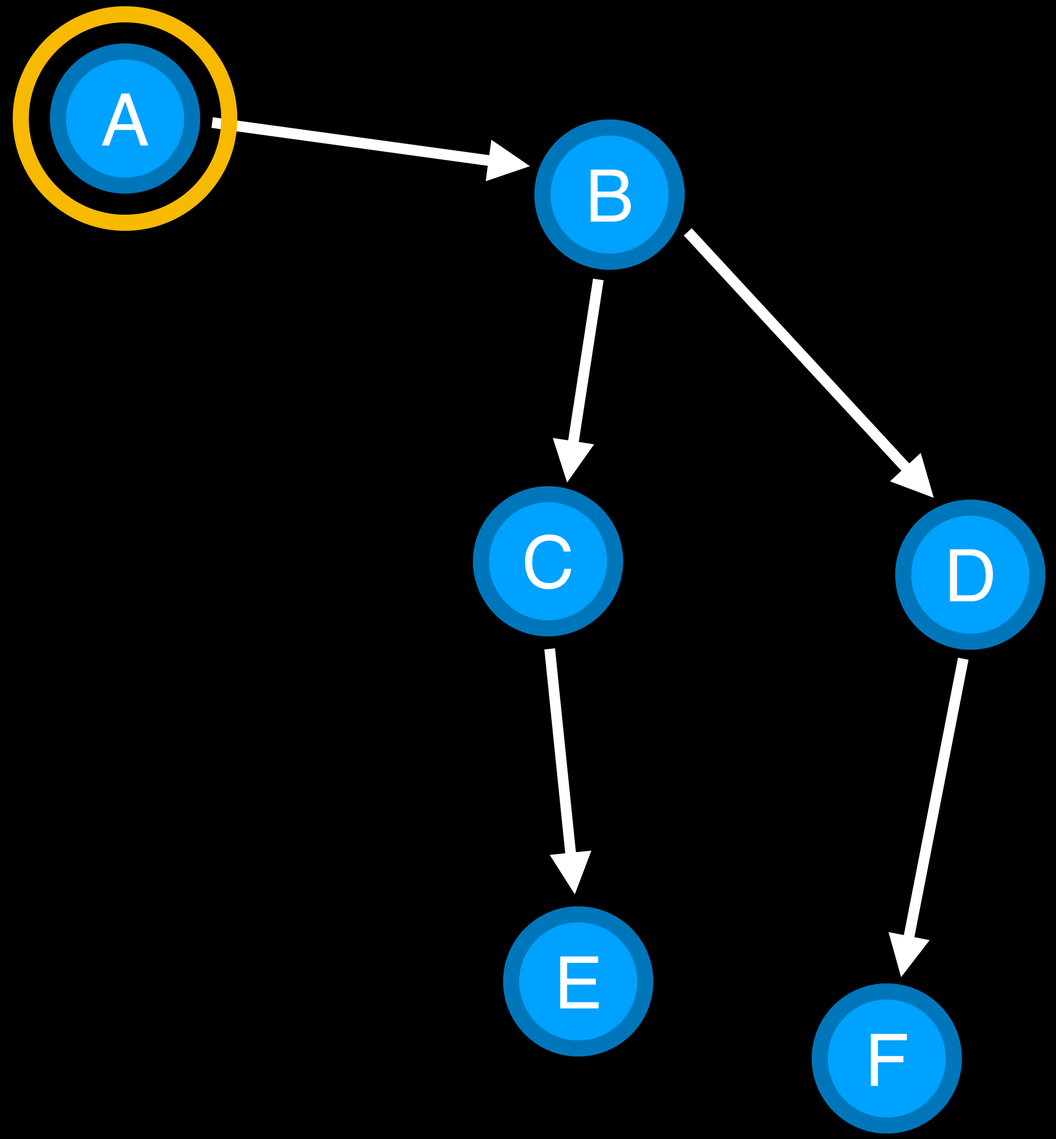


Find a path from A to E.

Frontier

Explored Set

A



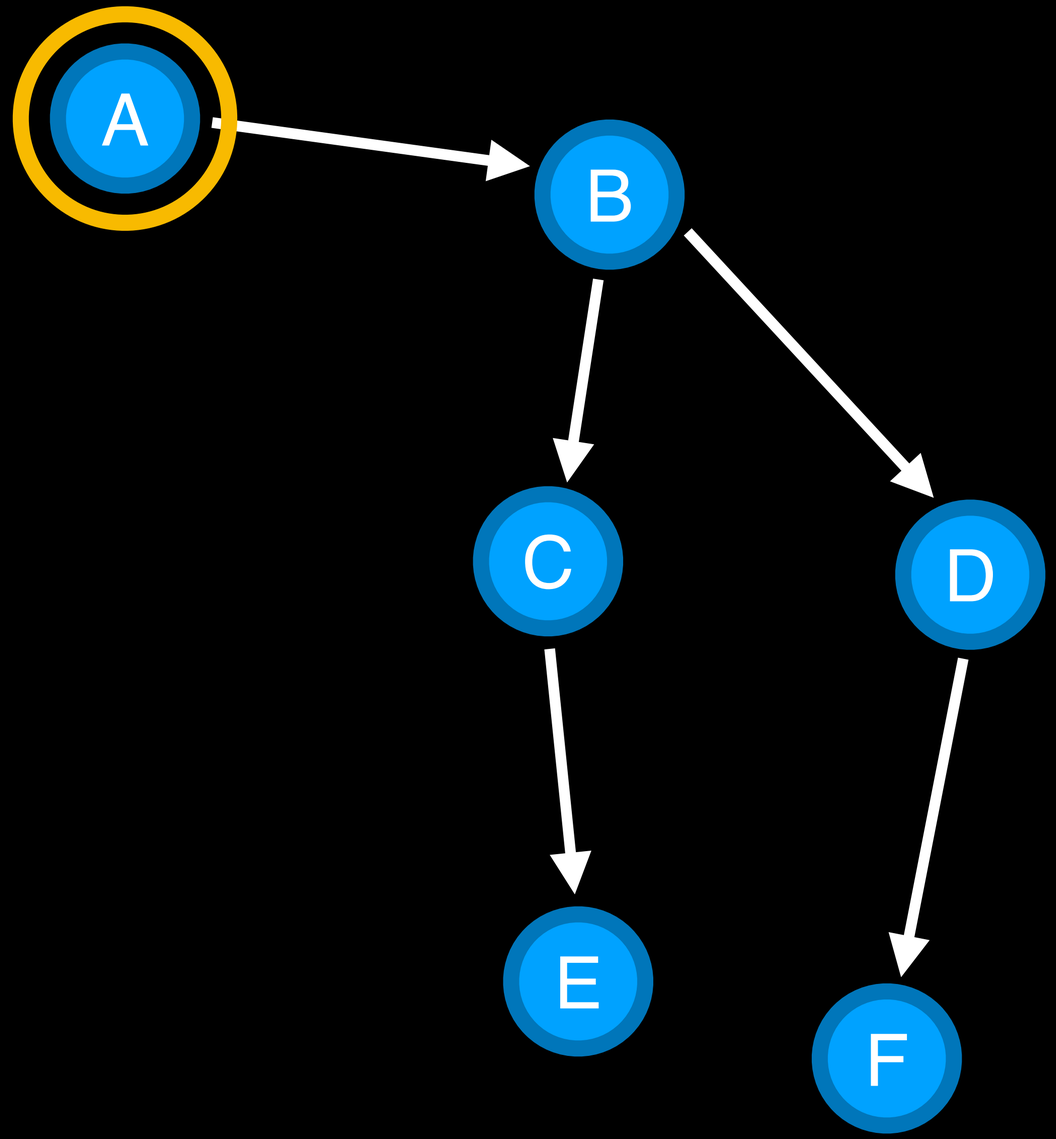
Find a path from A to E.

Frontier

B

Explored Set

A

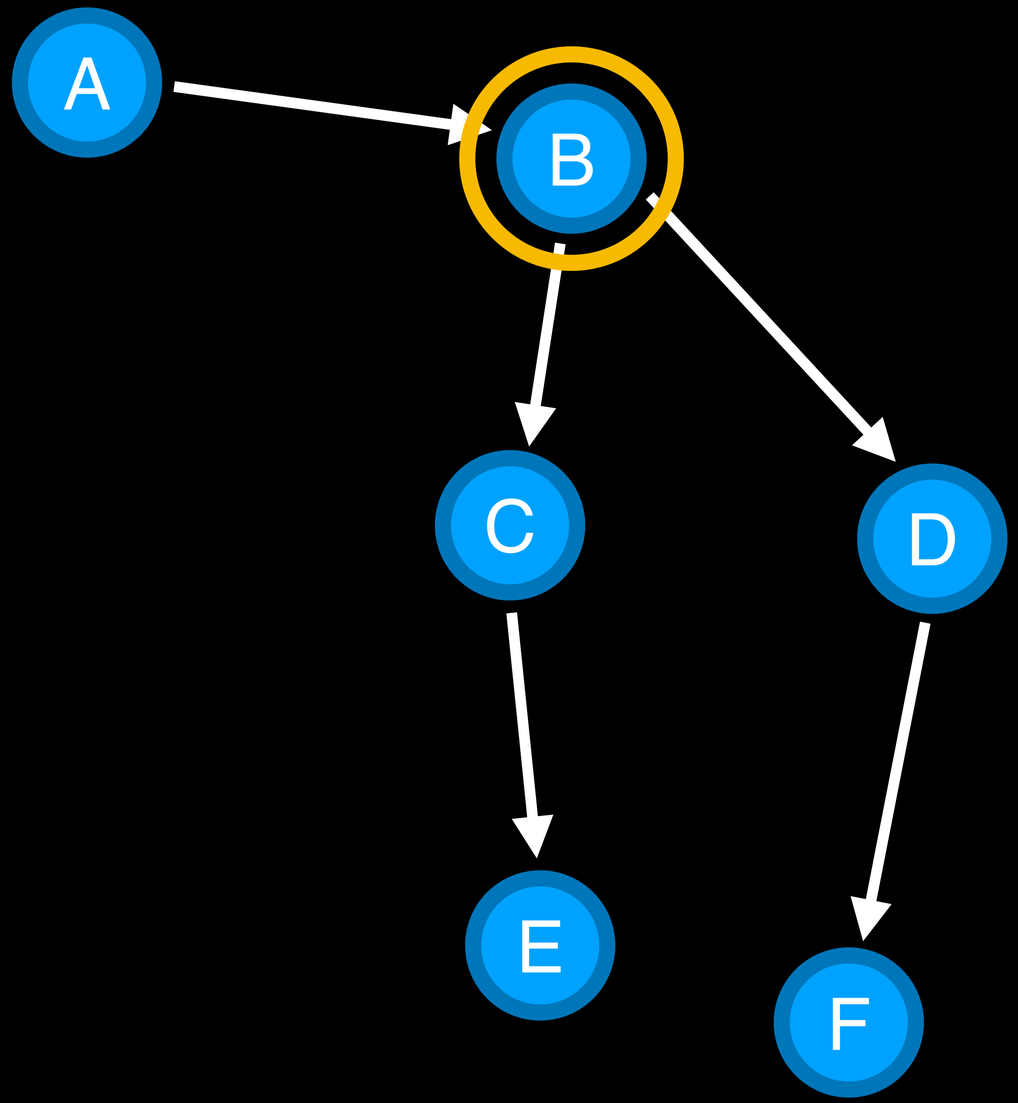


Find a path from A to E.

Frontier

Explored Set

A B



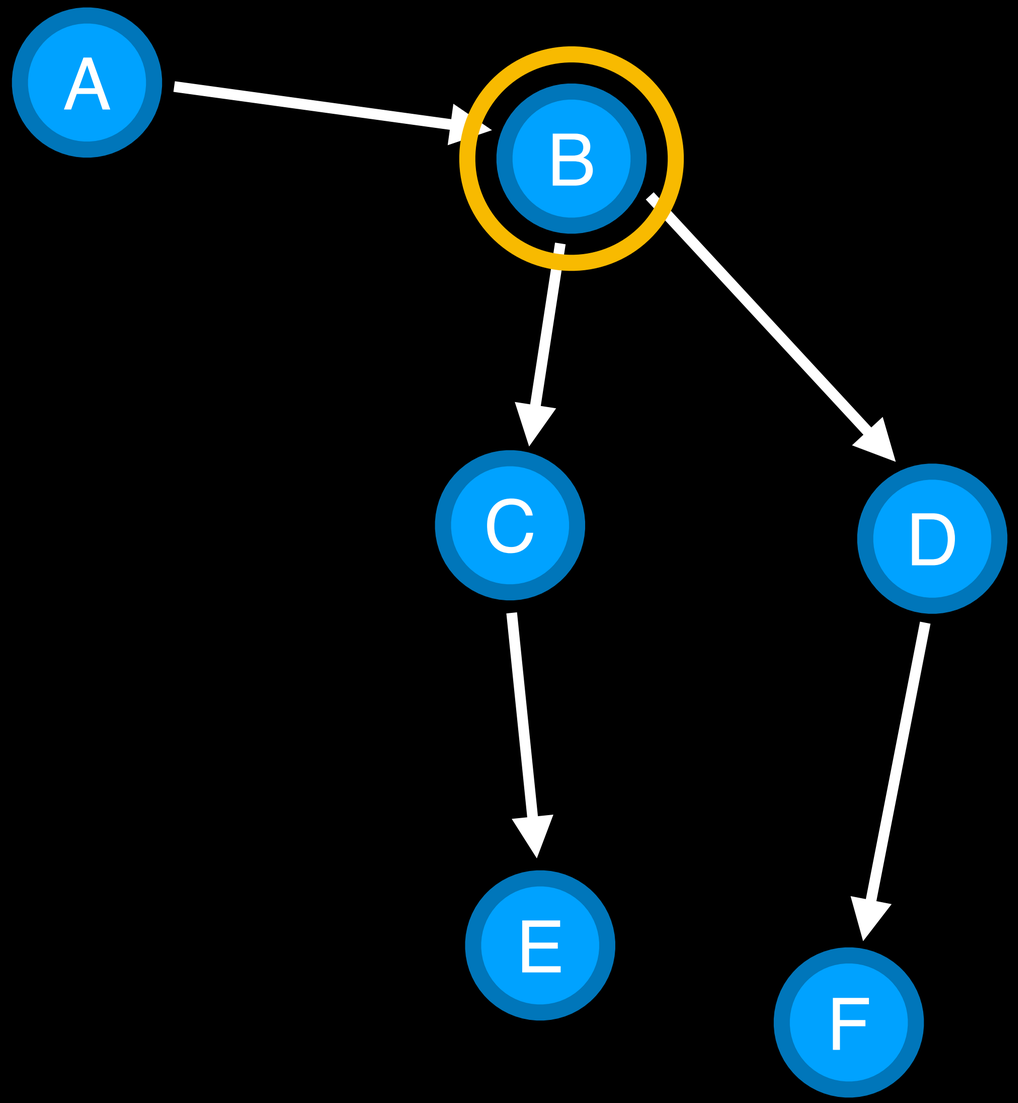
Find a path from A to E.

Frontier

C D

Explored Set

A B



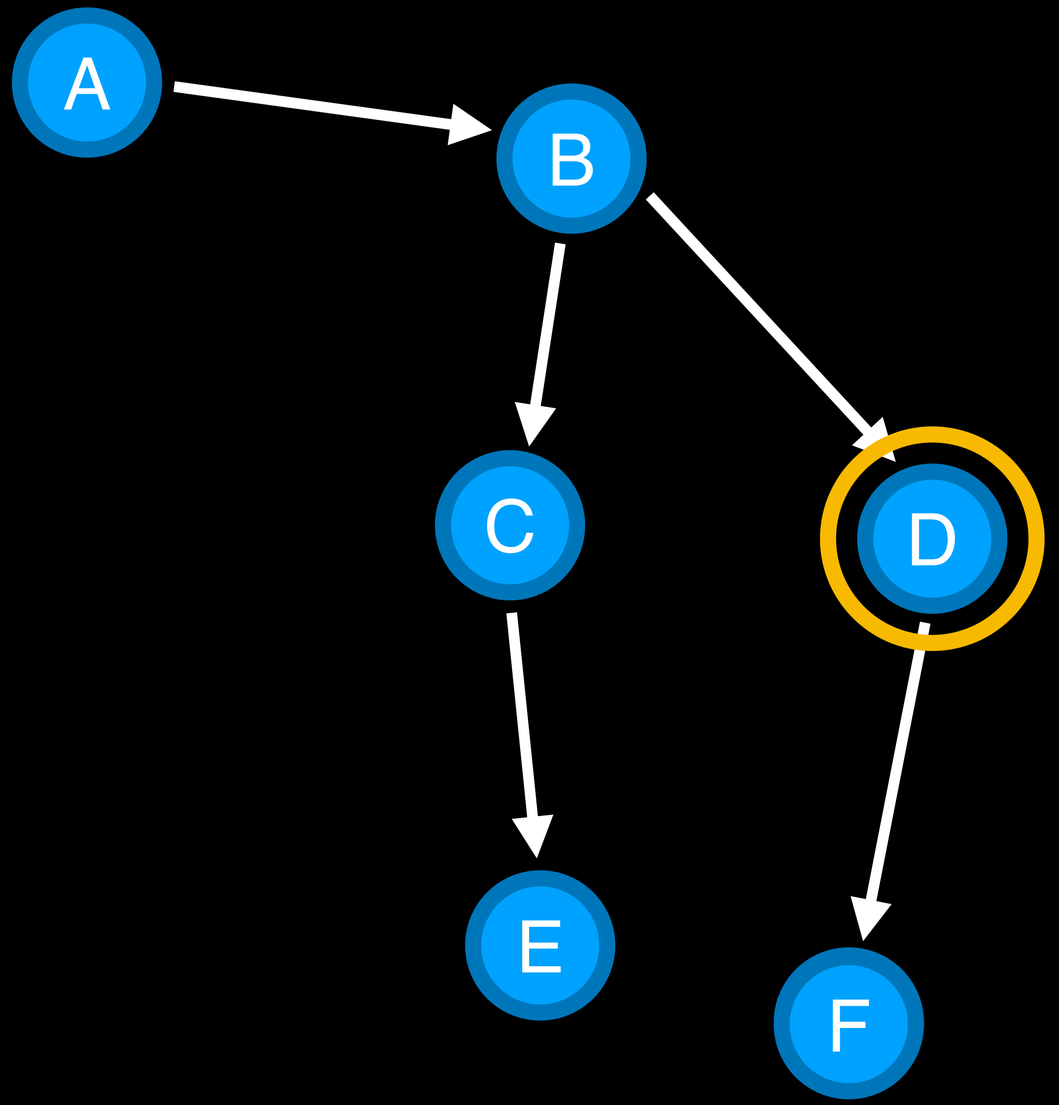
Find a path from A to E.

Frontier

C

Explored Set

A B D



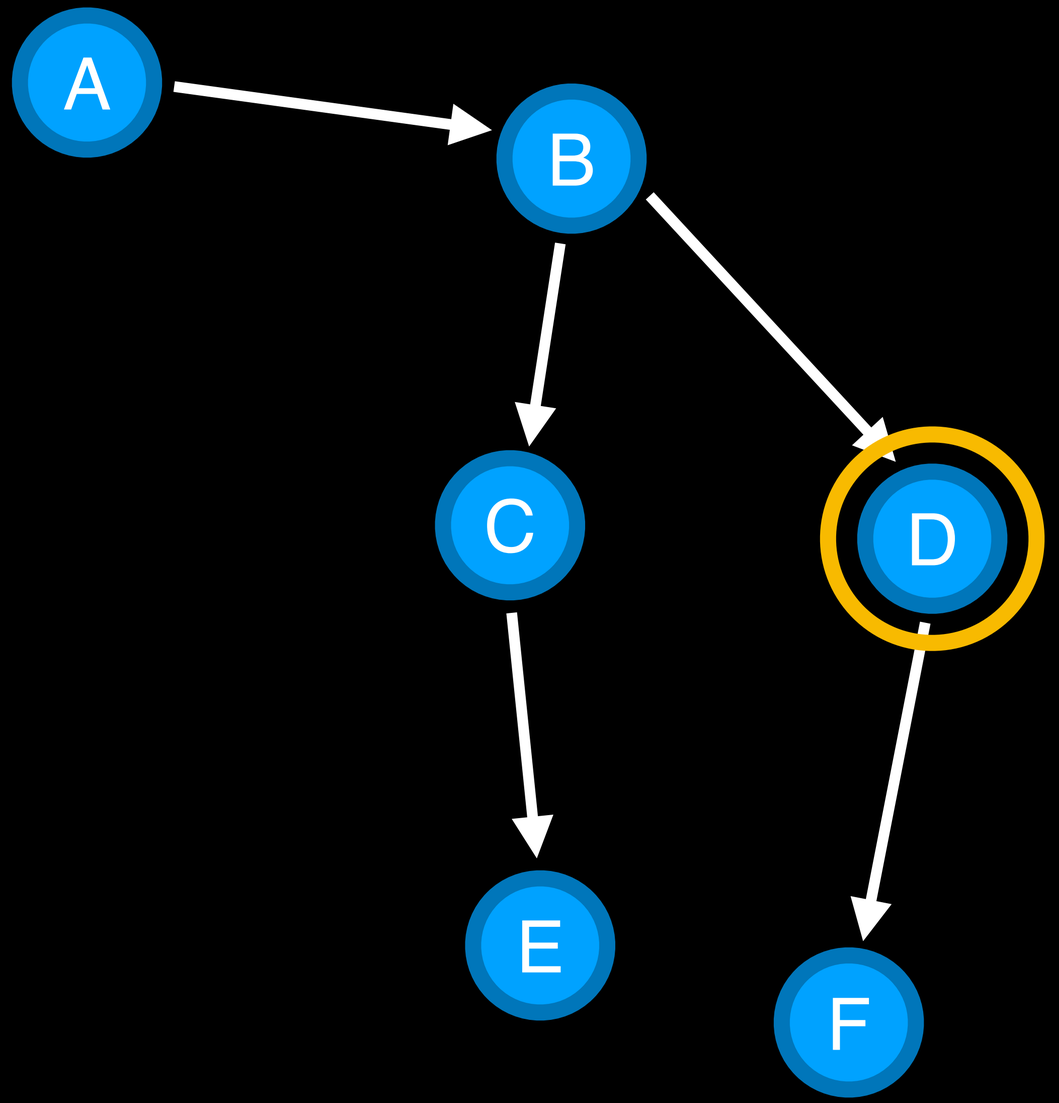
Find a path from A to E.

Frontier

C F

Explored Set

A B D



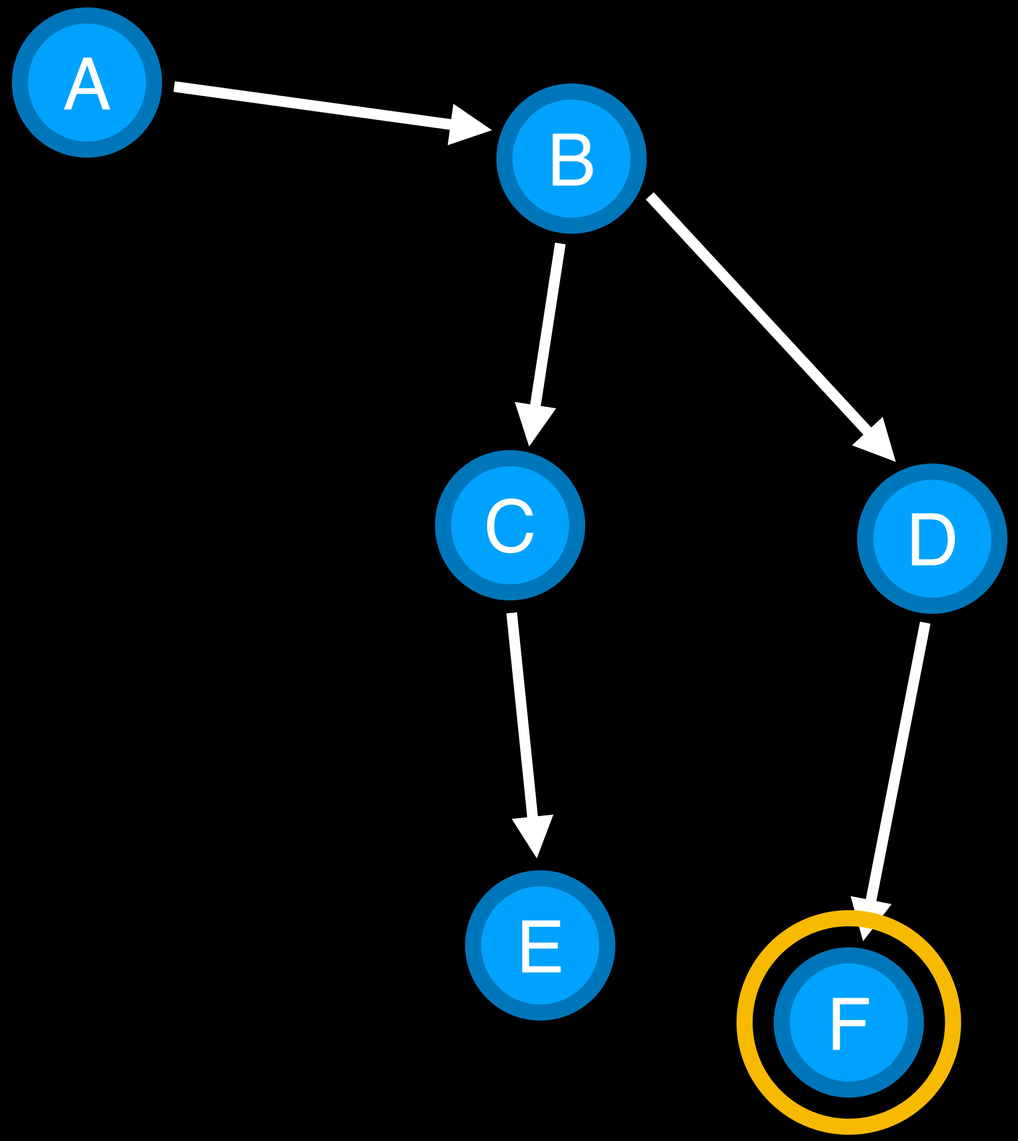
Find a path from A to E.

Frontier

C

Explored Set

A B D F

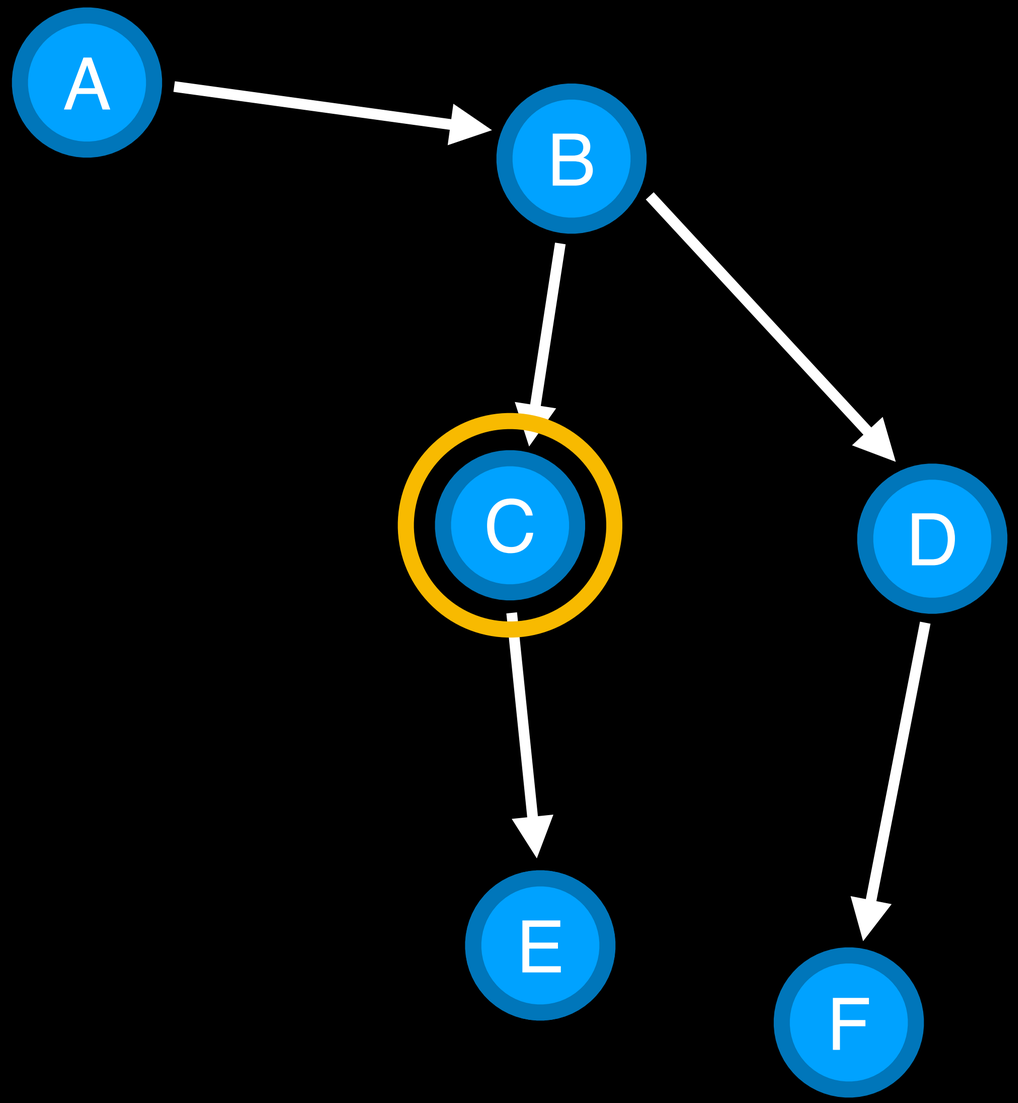


Find a path from A to E.

Frontier

Explored Set

A B D F C



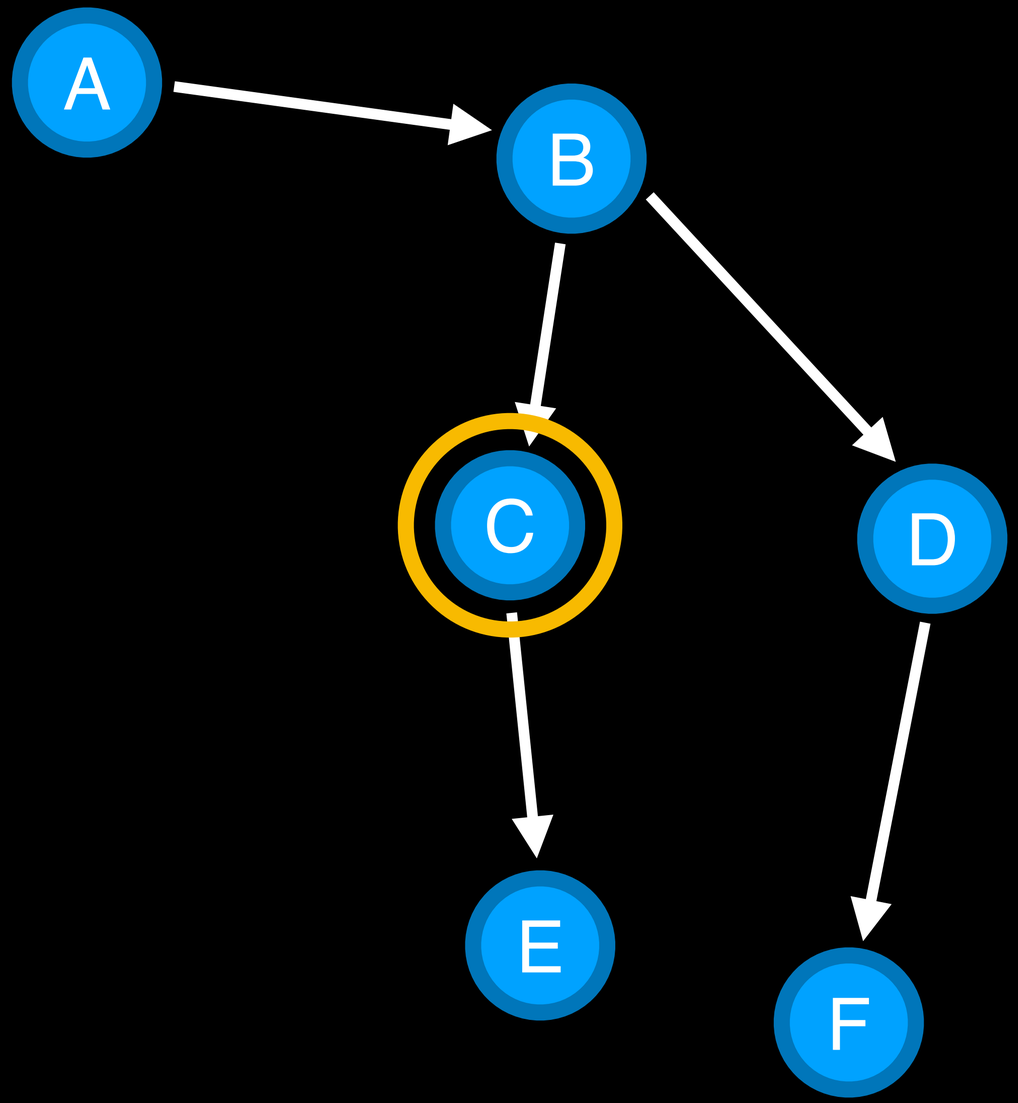
Find a path from A to E.

Frontier

E

Explored Set

A B D F C

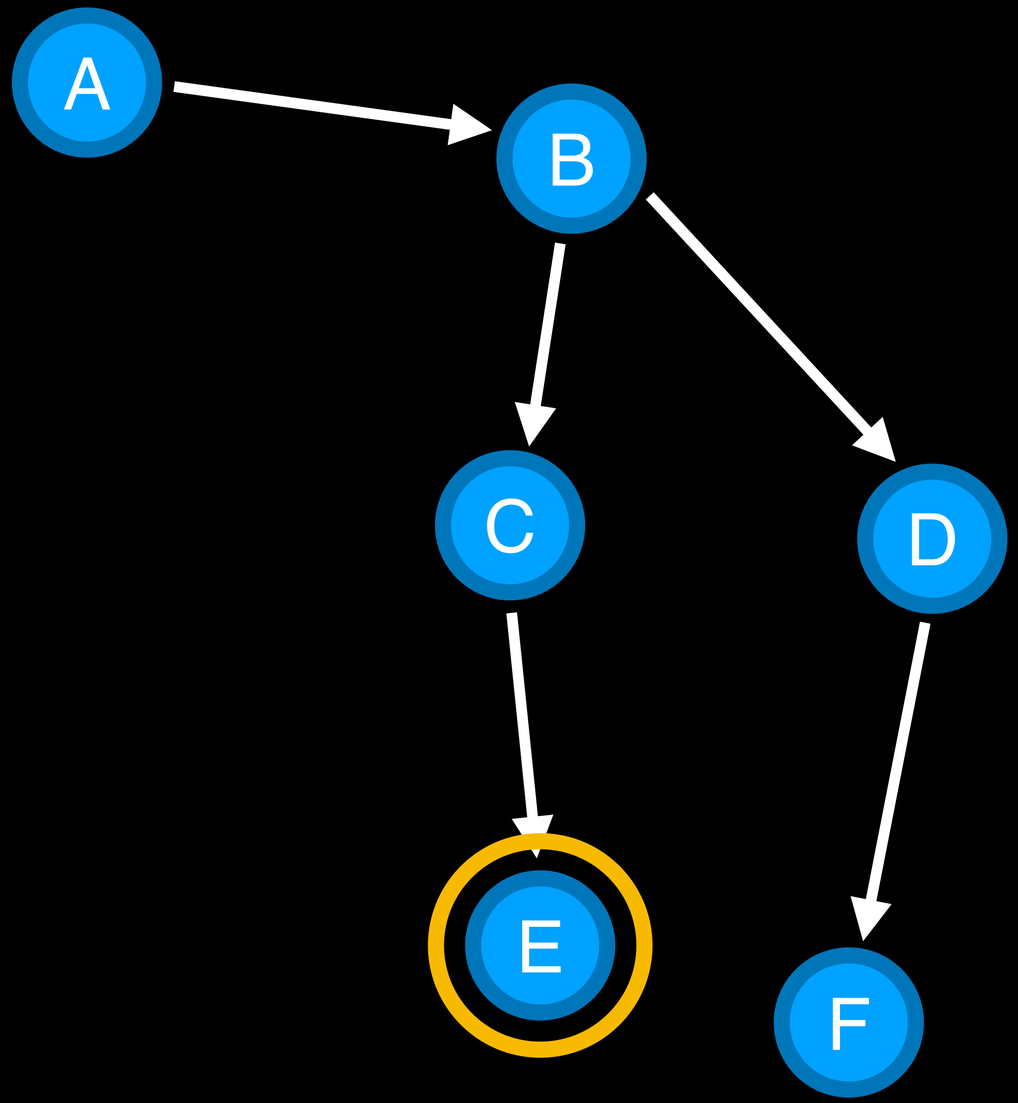


Find a path from A to E.

Frontier

Explored Set

A B D F C



Depth-First Search

depth-first search

search algorithm that always expands the deepest node in the frontier

Breadth-First Search

breadth-first search

search algorithm that always expands the shallowest node in the frontier

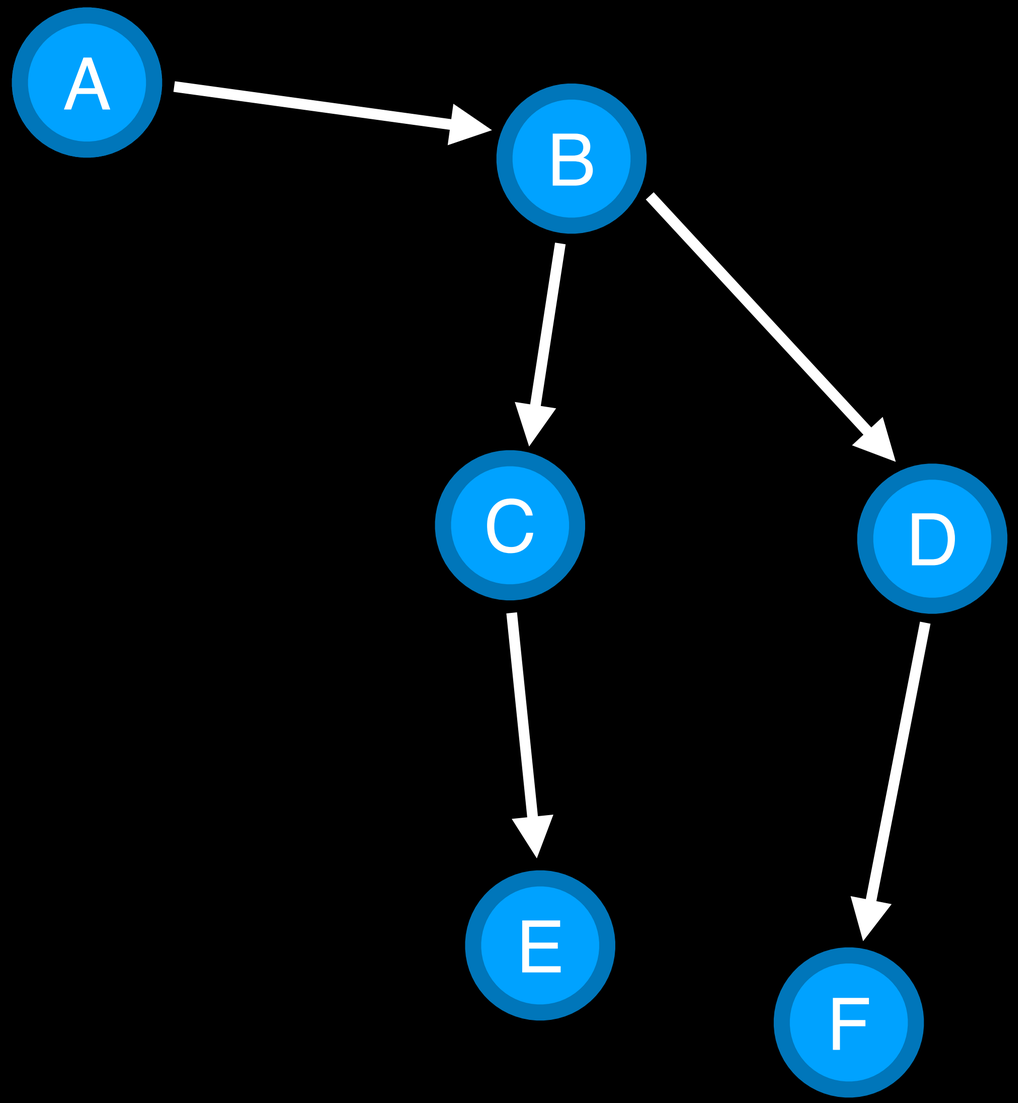
queue

first-in first-out data type

Find a path from A to E.

Frontier

Explored Set

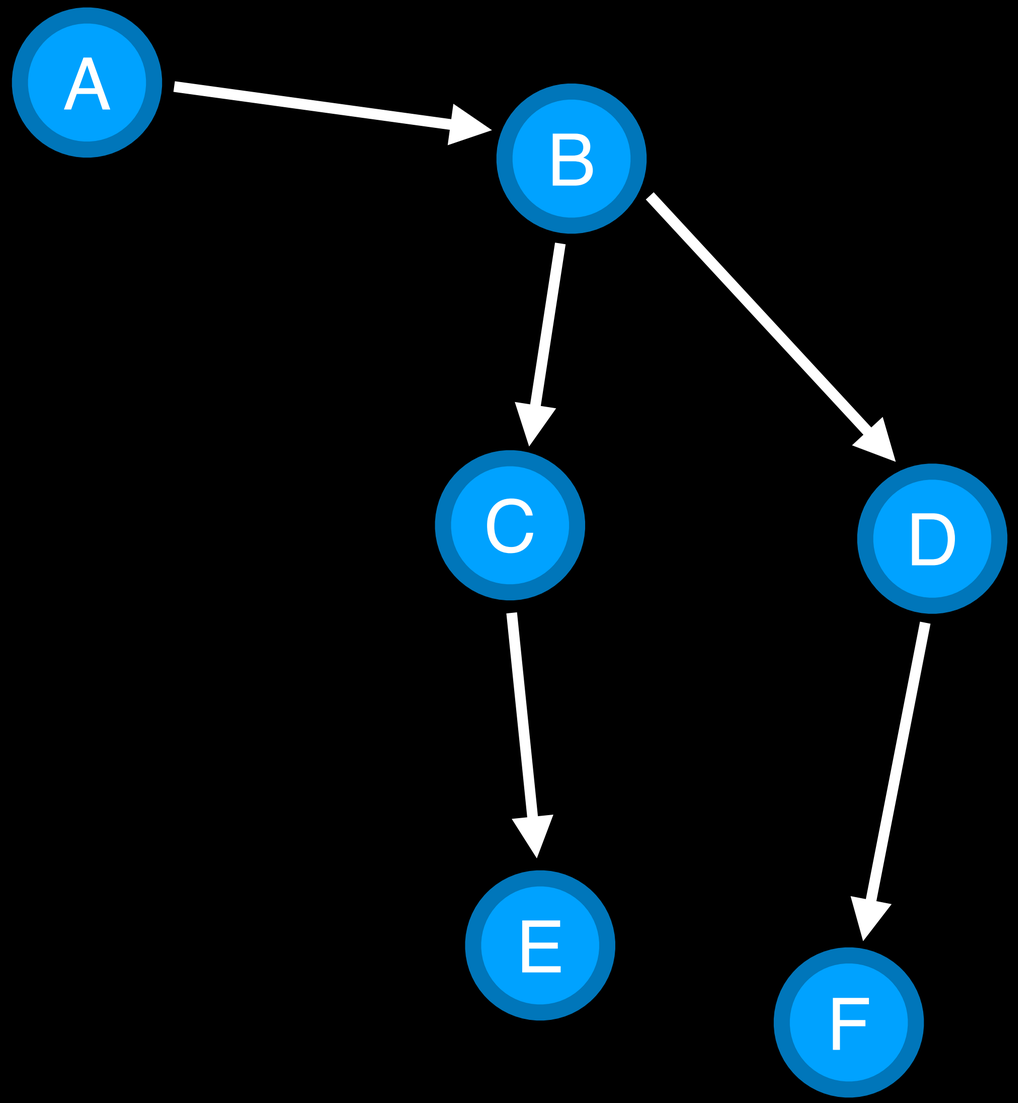


Find a path from A to E.

Frontier

A

Explored Set

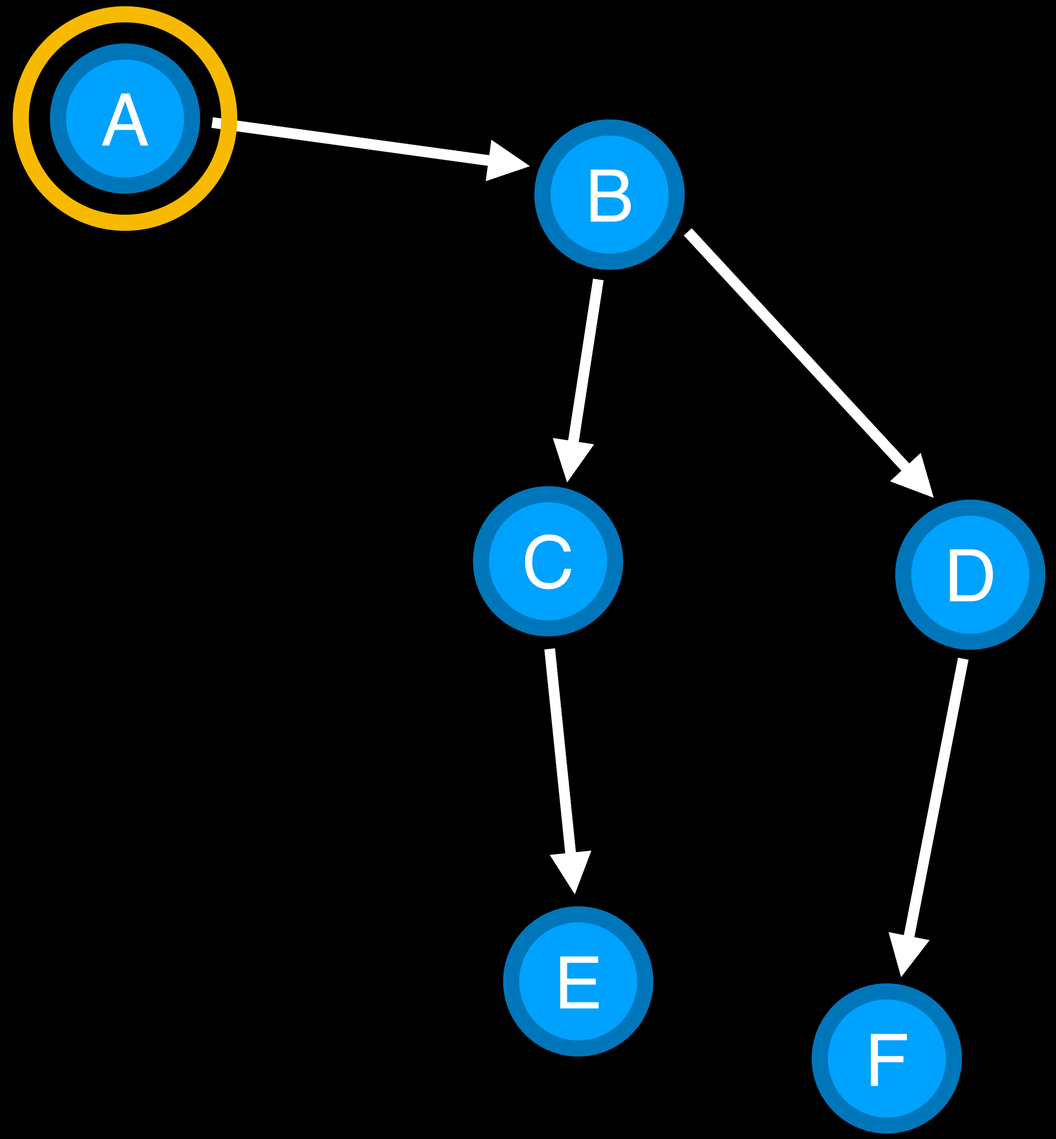


Find a path from A to E.

Frontier

Explored Set

A



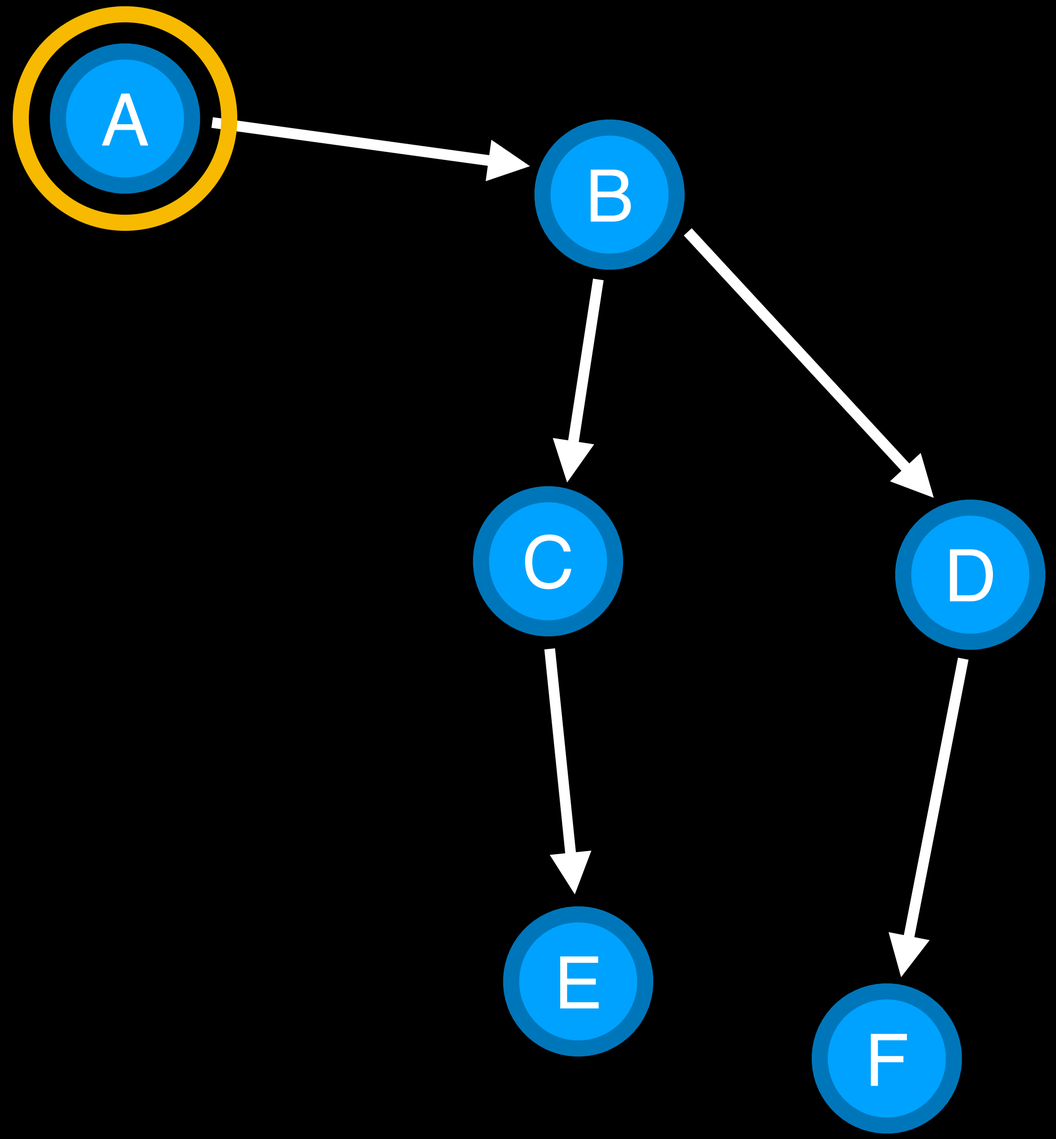
Find a path from A to E.

Frontier

B

Explored Set

A

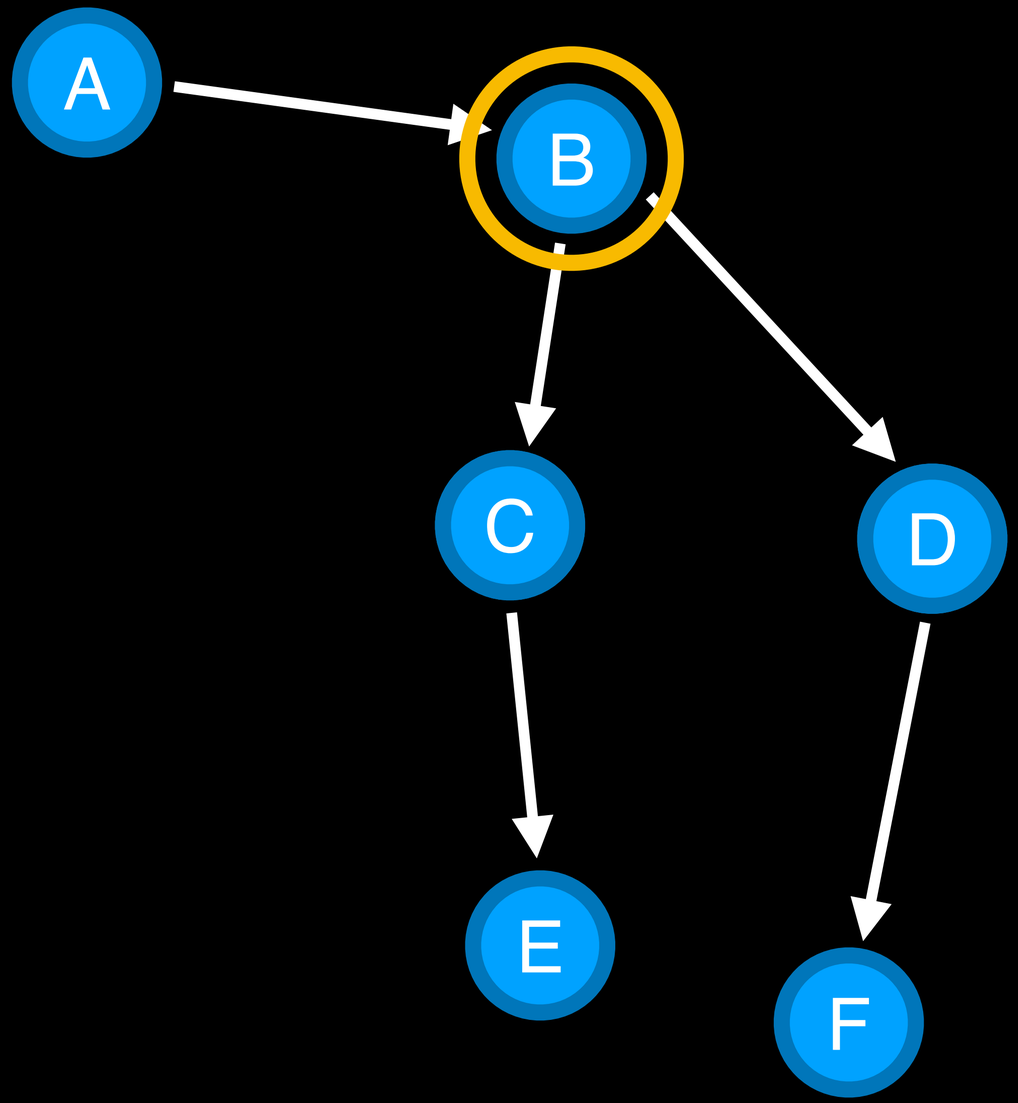


Find a path from A to E.

Frontier

Explored Set

A B



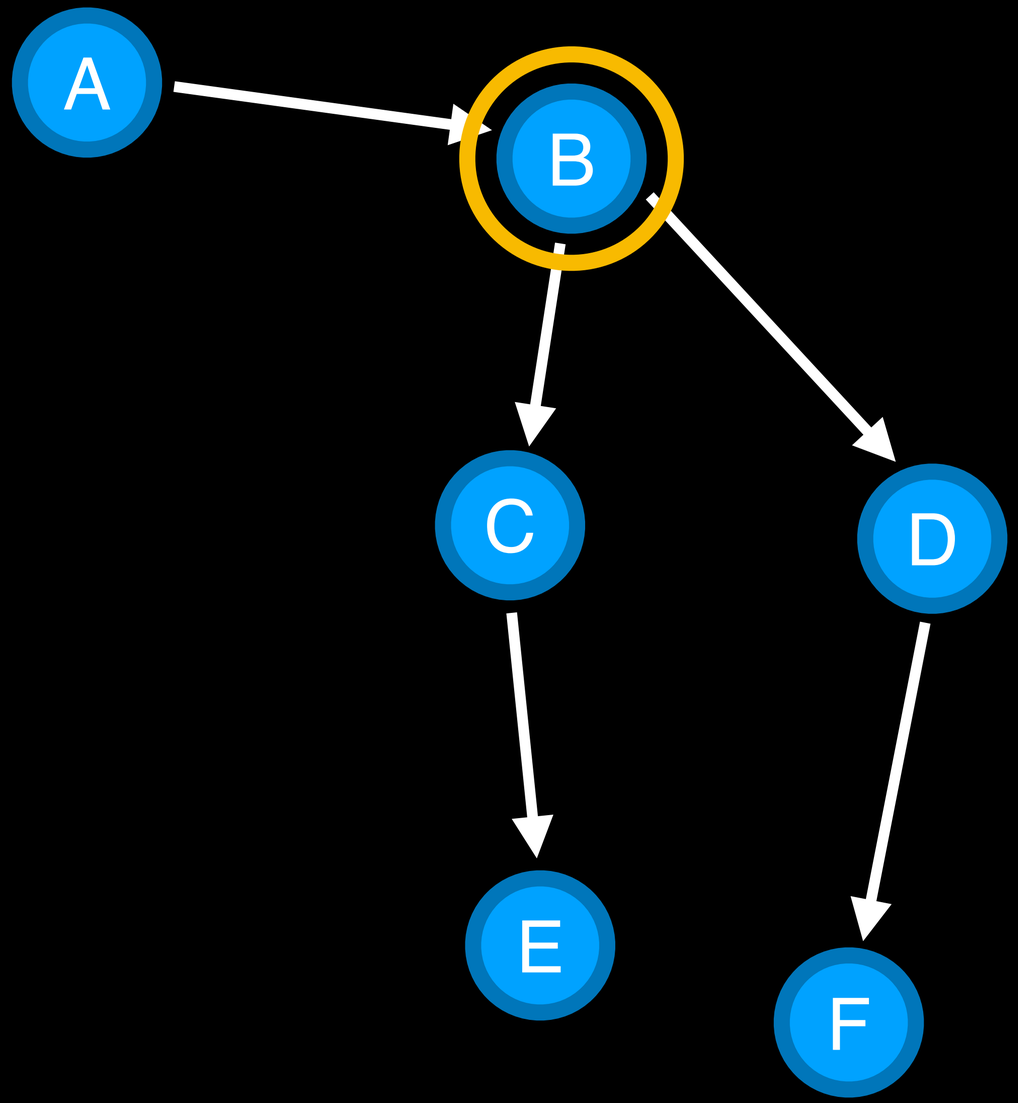
Find a path from A to E.

Frontier

C D

Explored Set

A B



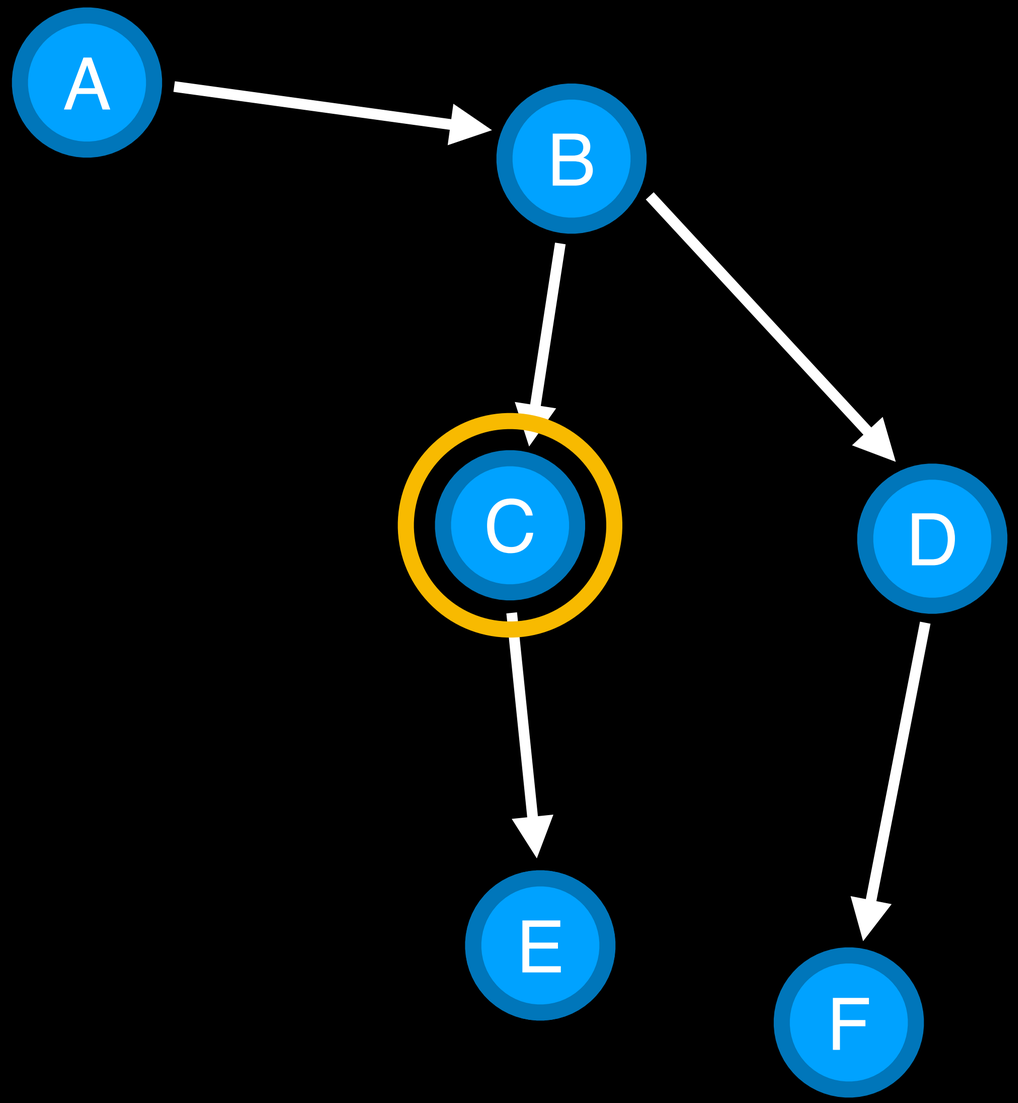
Find a path from A to E.

Frontier

D

Explored Set

A B C



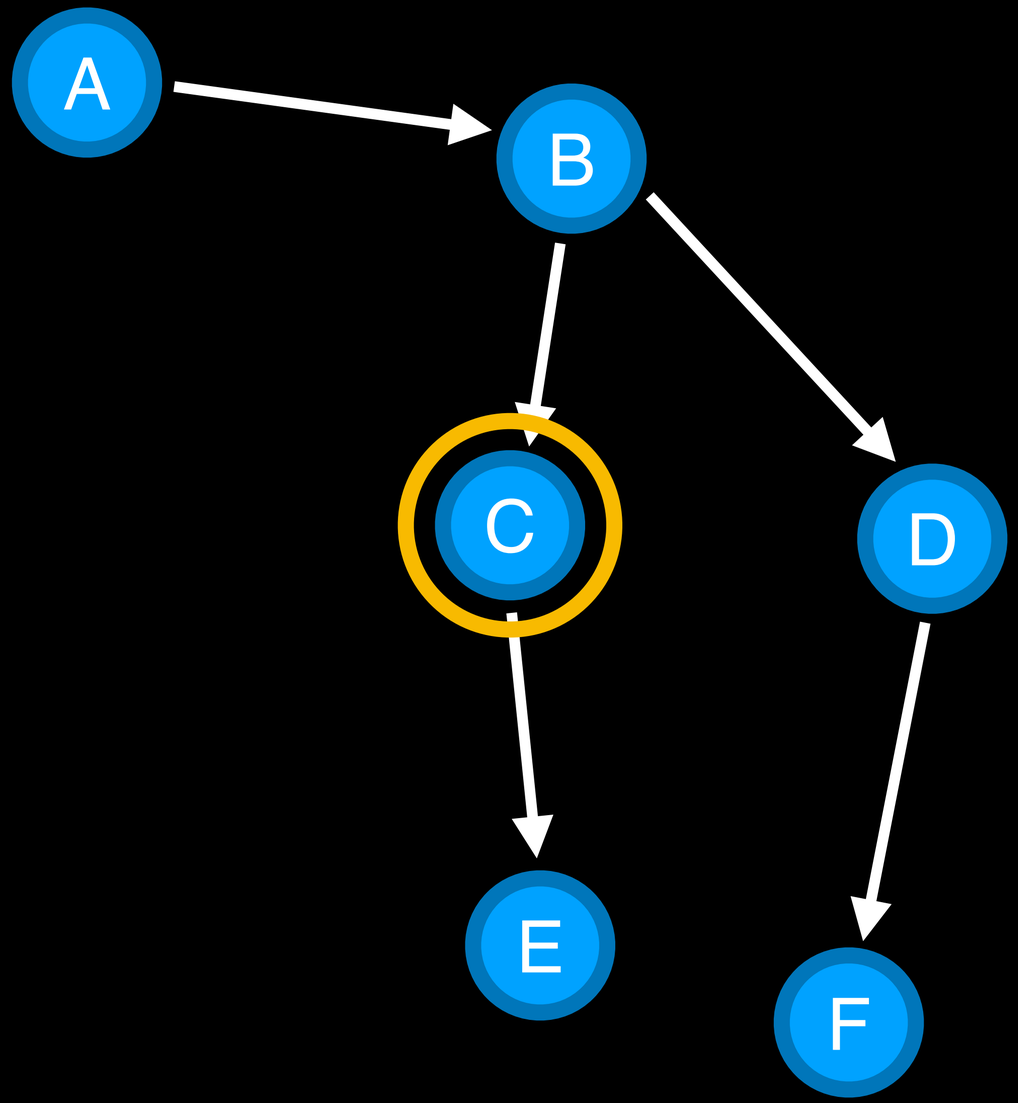
Find a path from A to E.

Frontier

D E

Explored Set

A B C



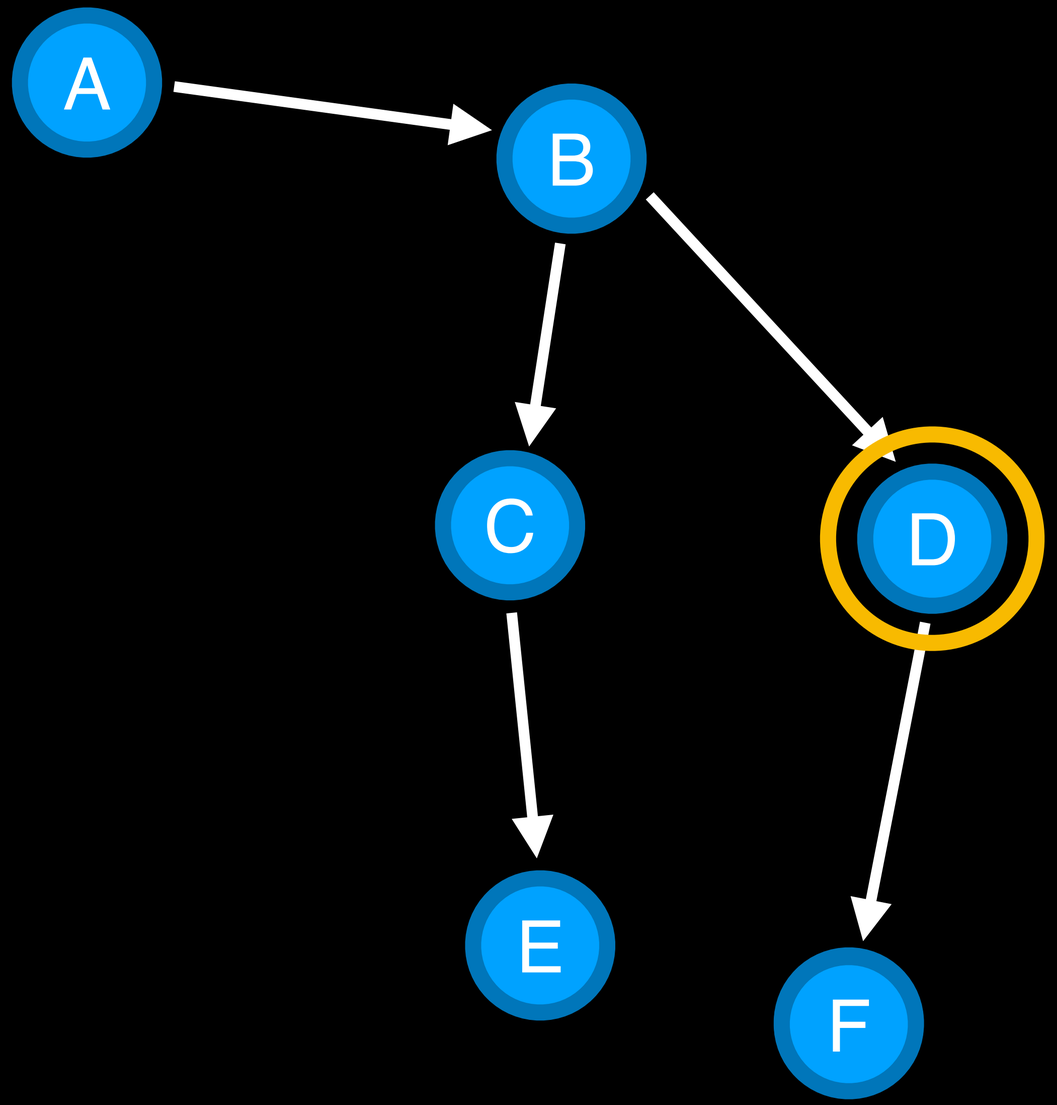
Find a path from A to E.

Frontier

E

Explored Set

A B C D



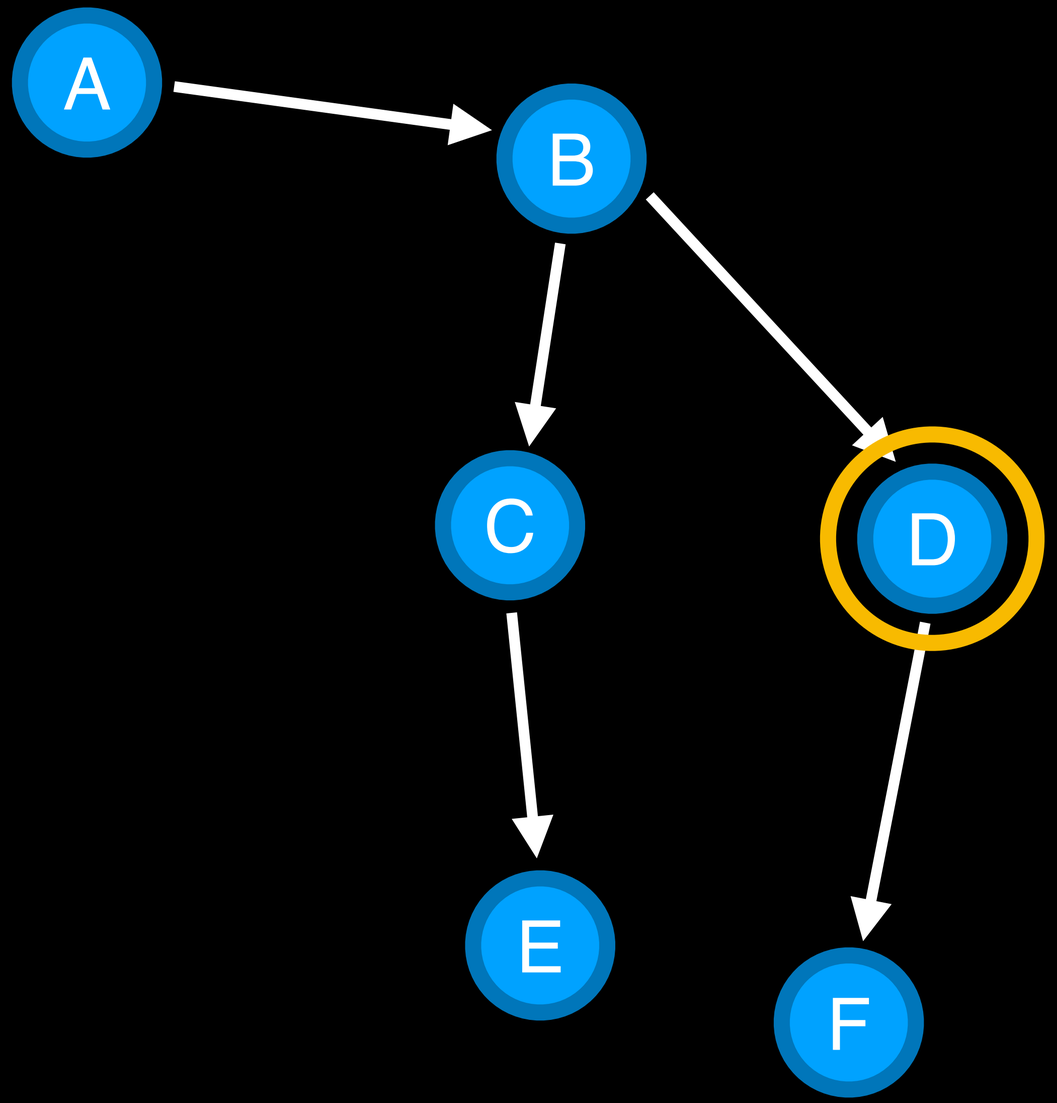
Find a path from A to E.

Frontier

E F

Explored Set

A B C D



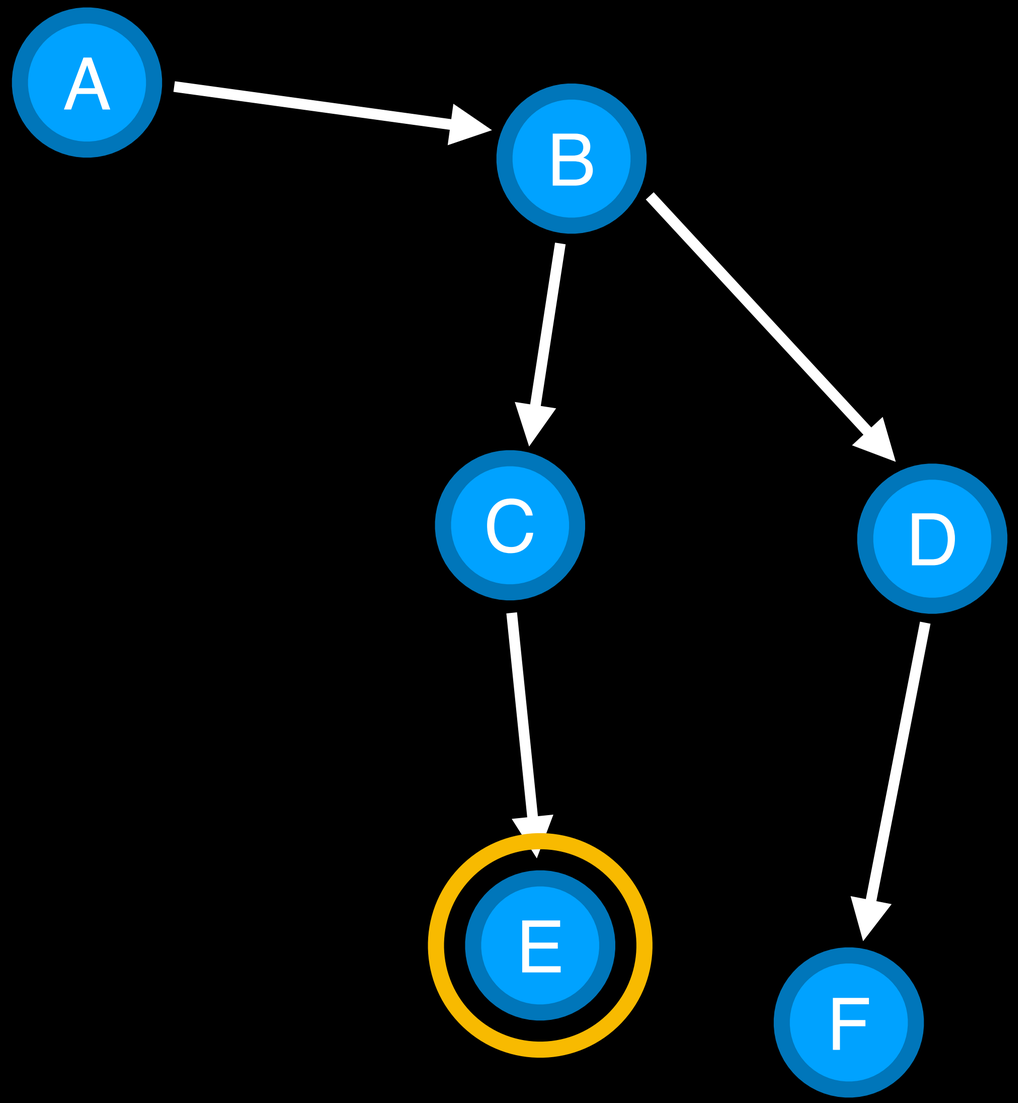
Find a path from A to E.

Frontier

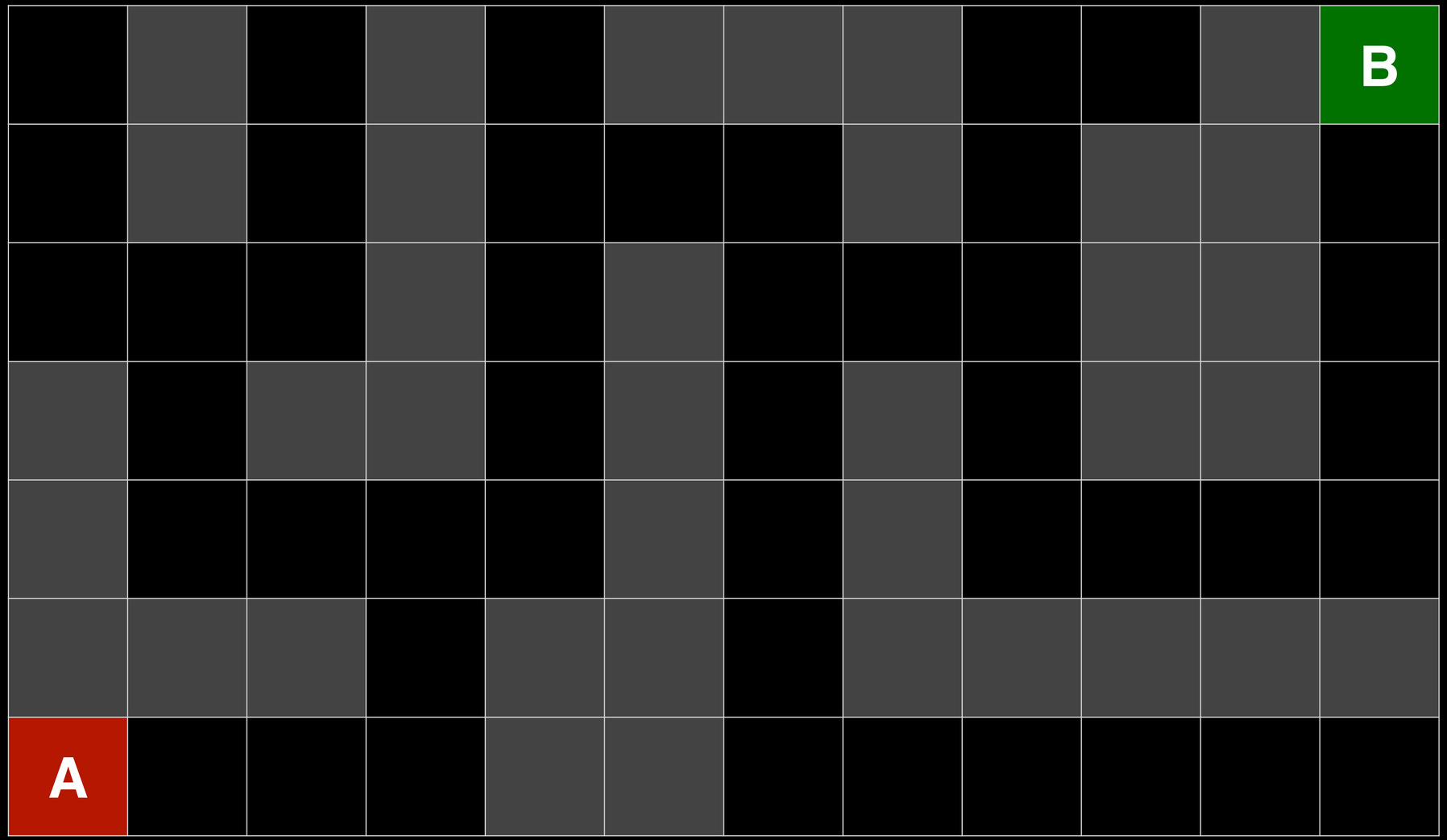
F

Explored Set

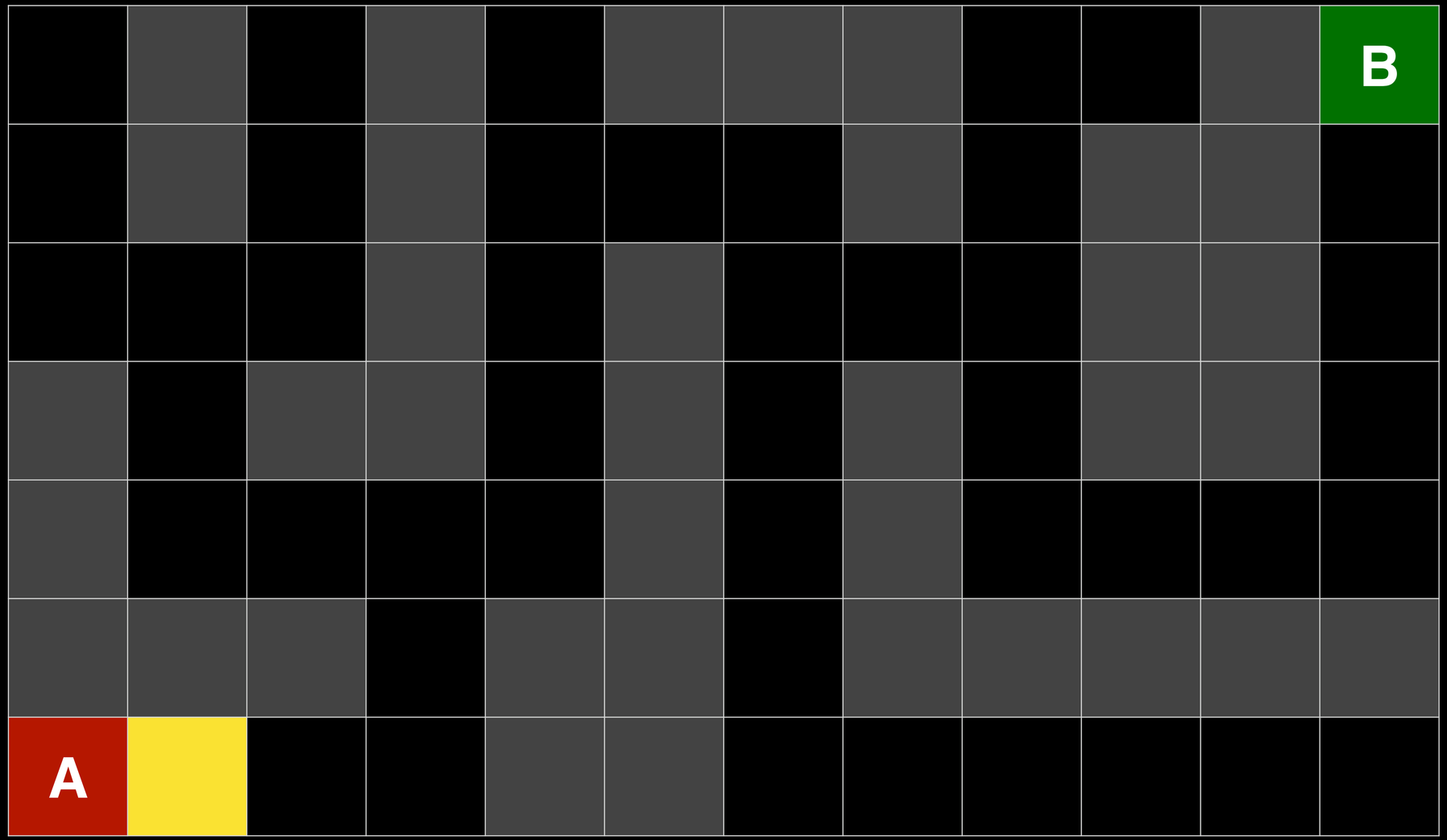
A B C D



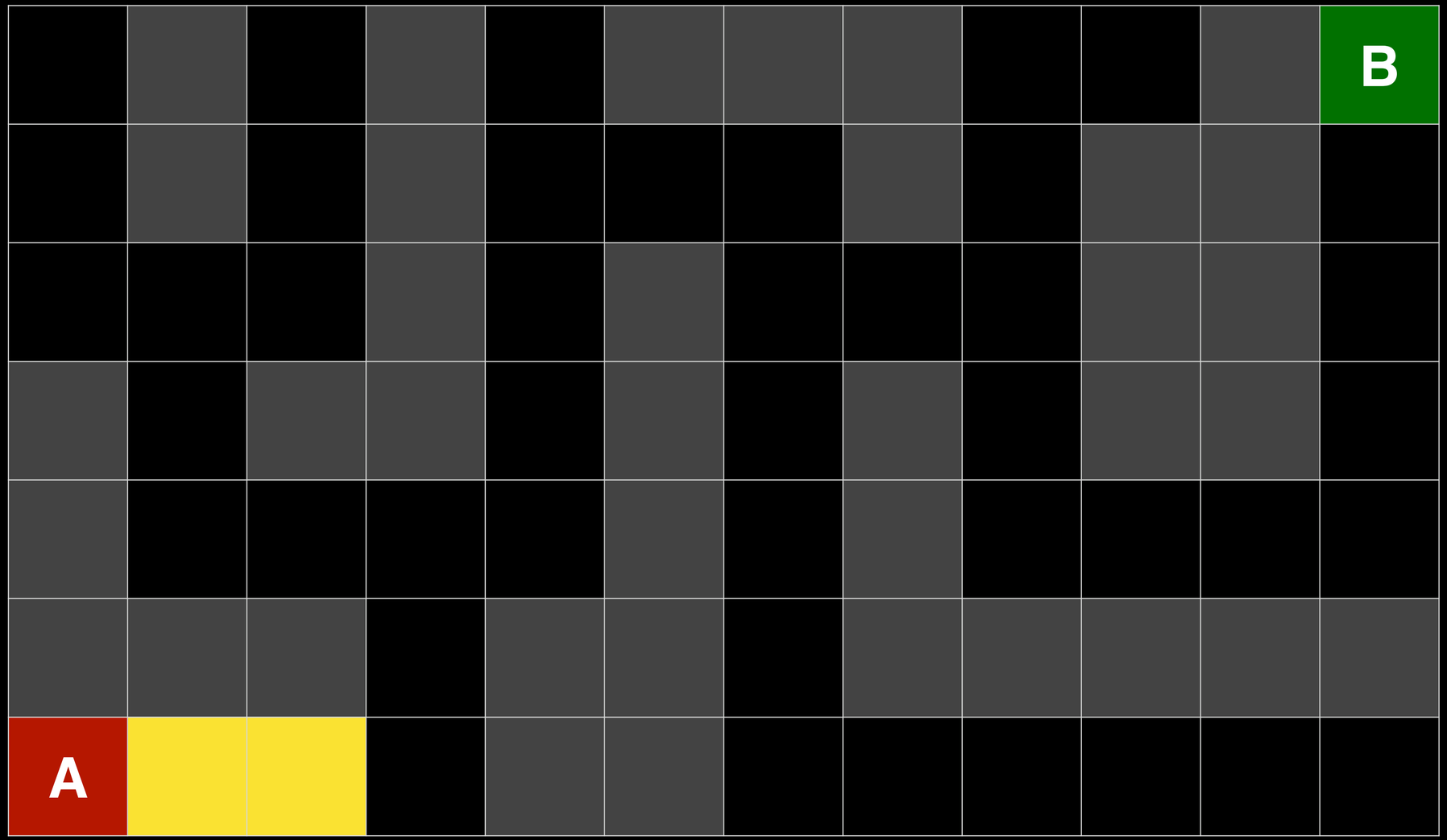
Depth-First Search



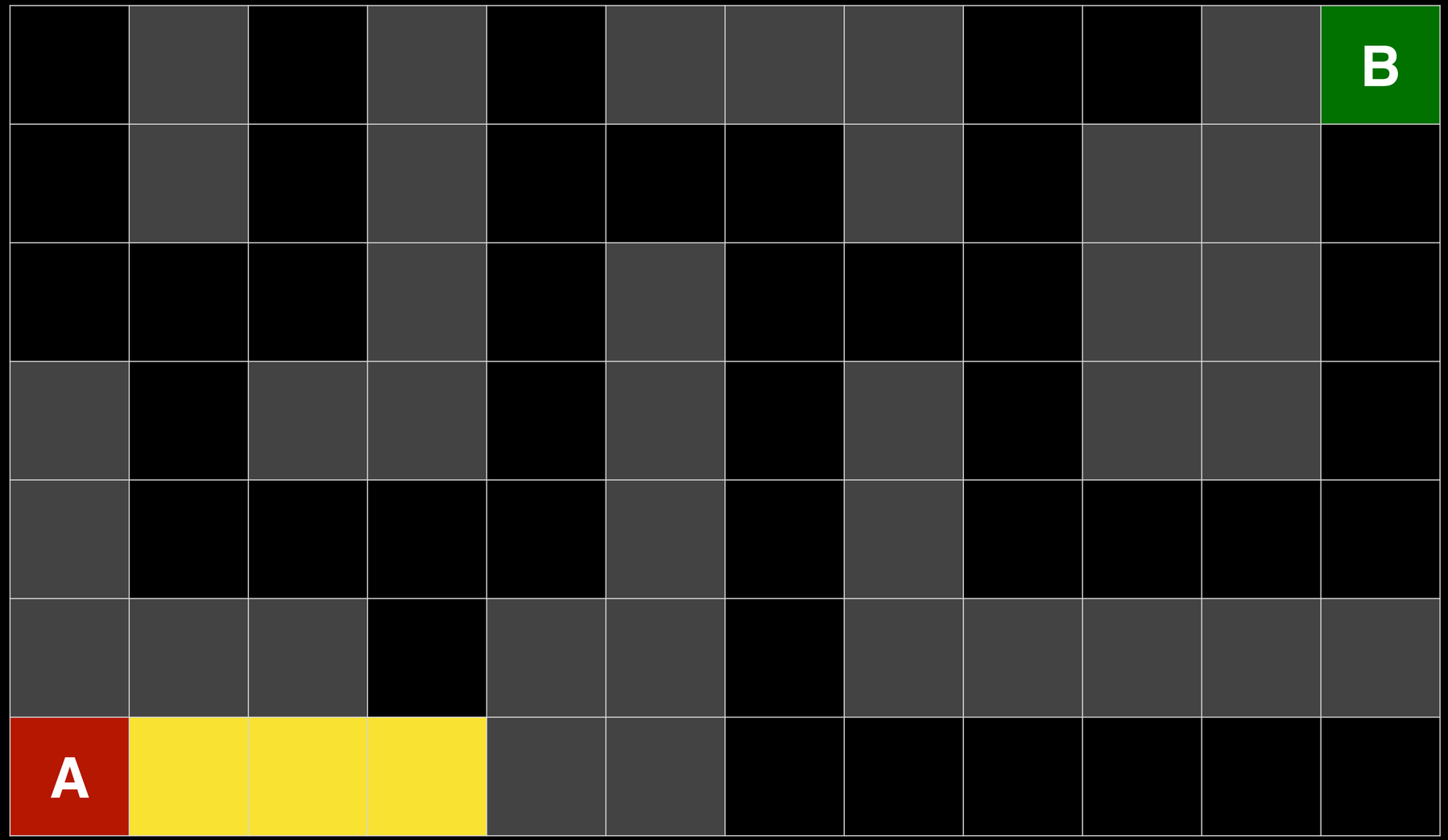
Depth-First Search



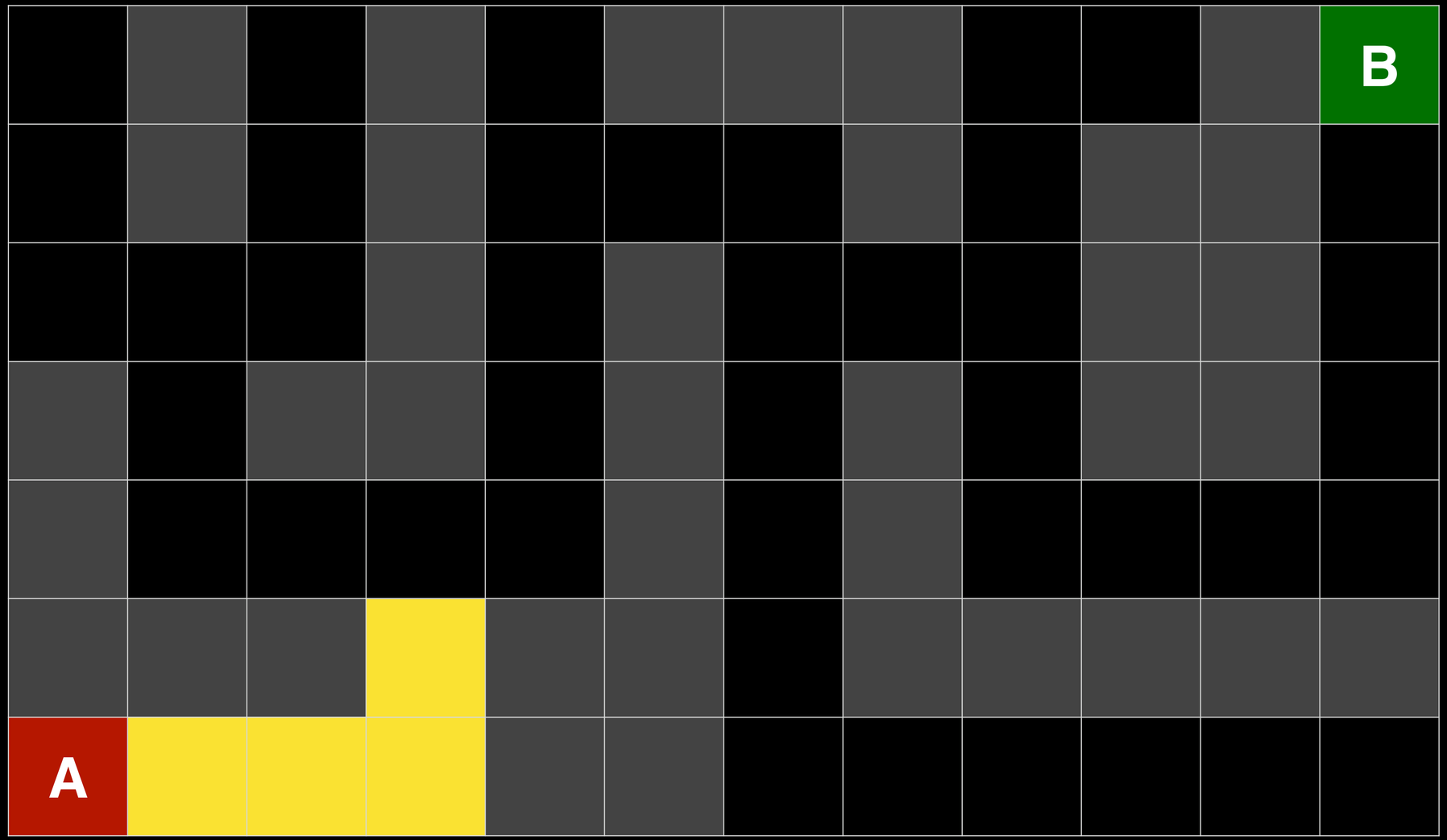
Depth-First Search



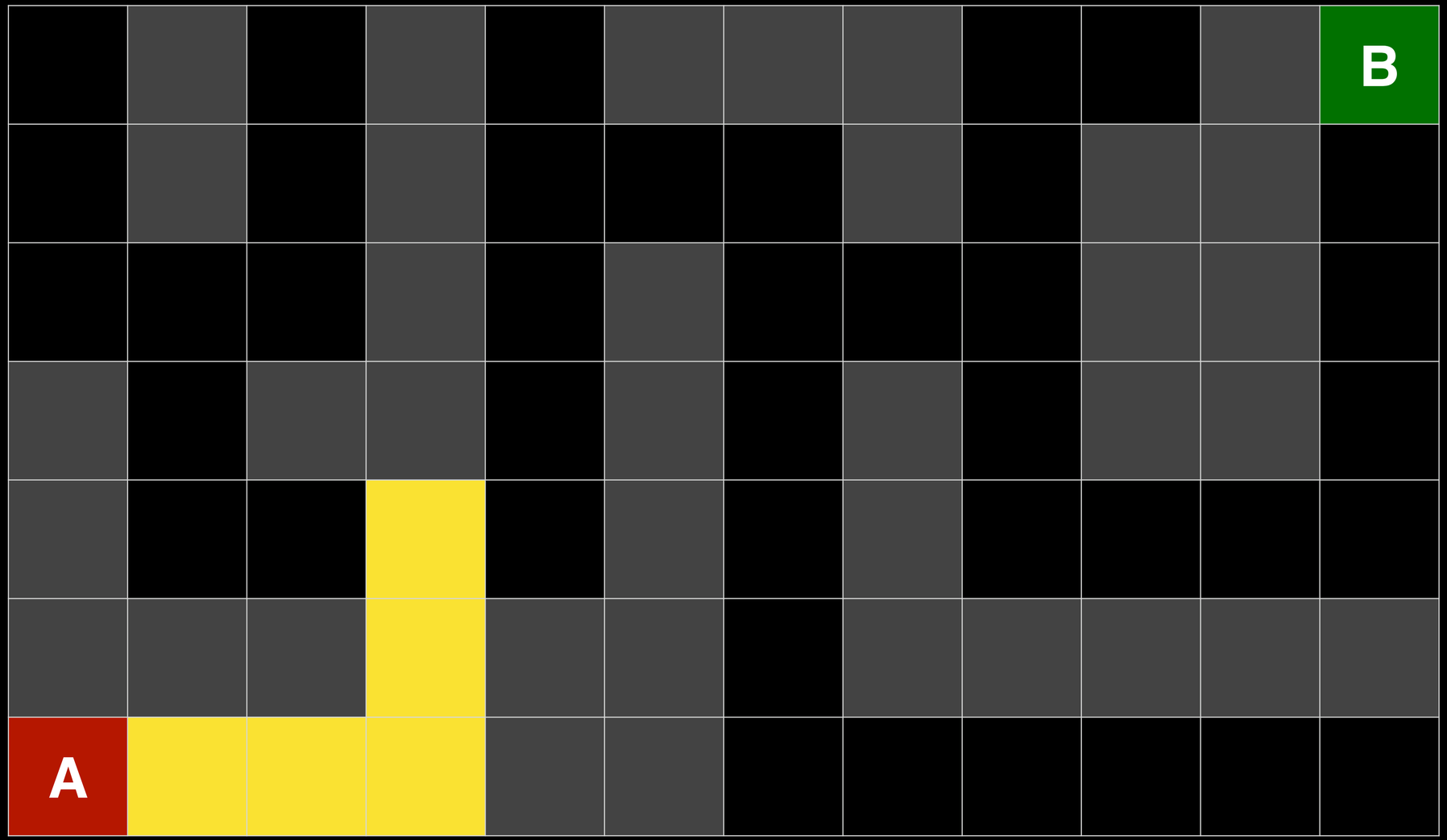
Depth-First Search



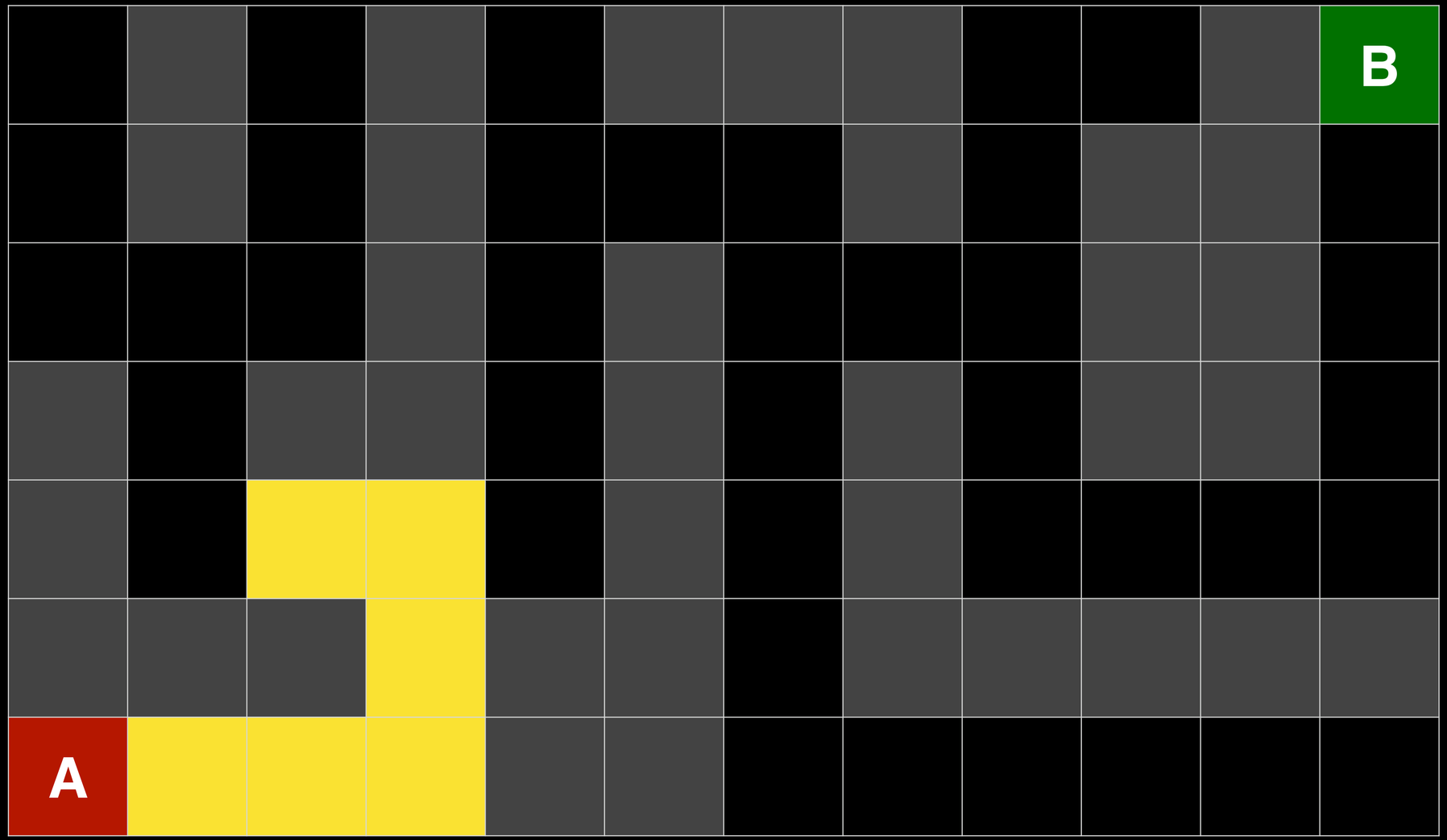
Depth-First Search



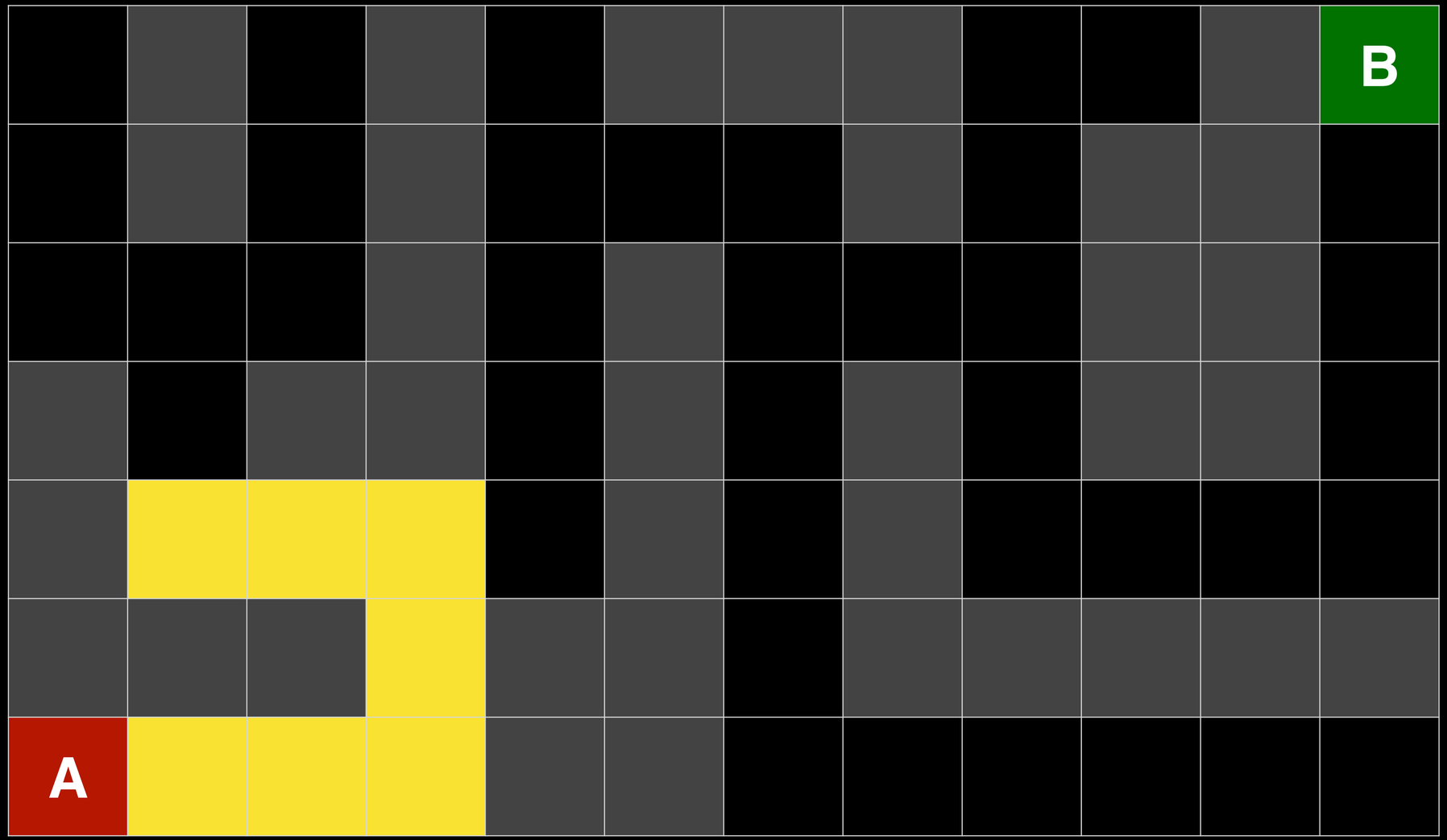
Depth-First Search



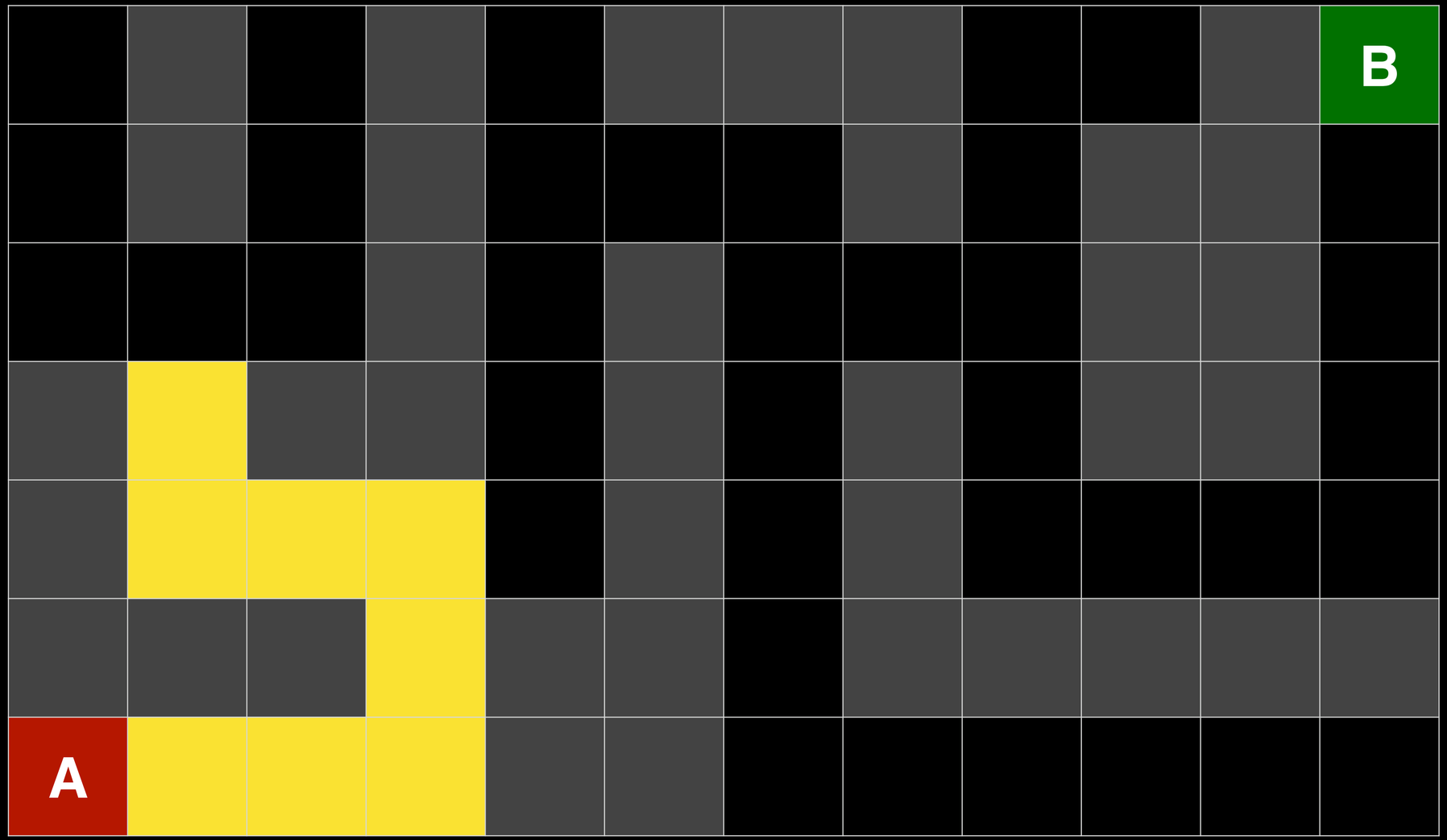
Depth-First Search



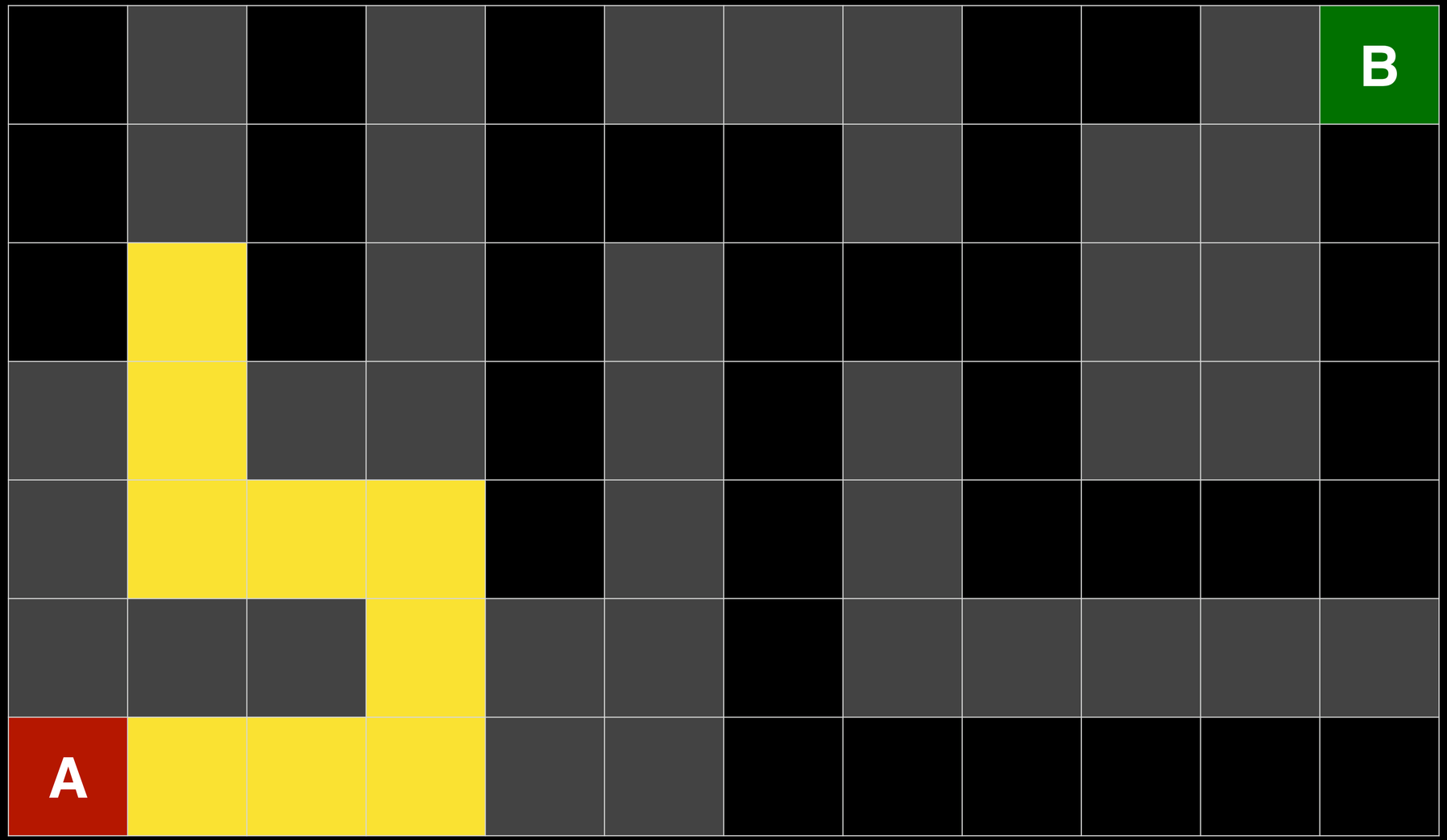
Depth-First Search



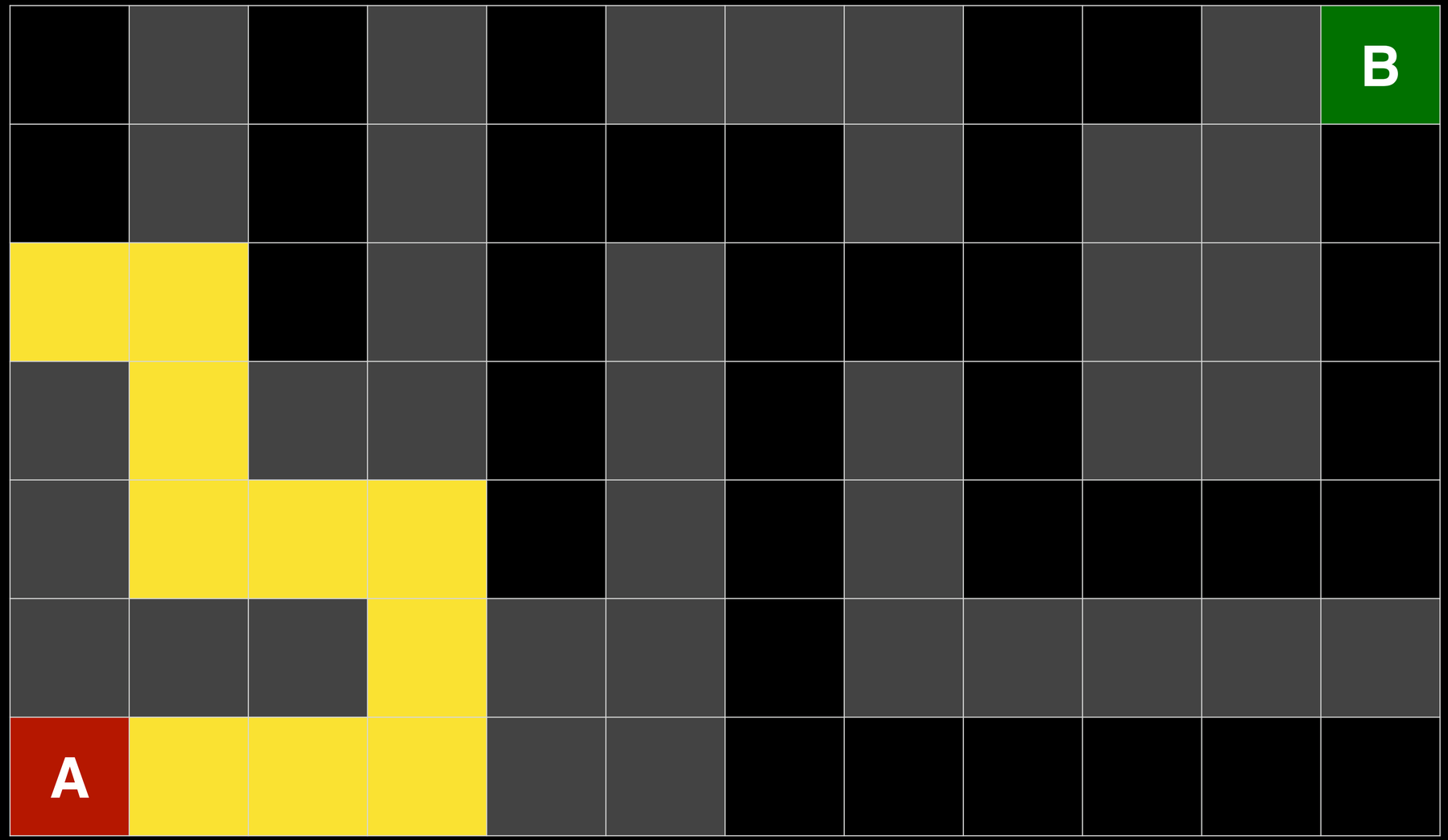
Depth-First Search



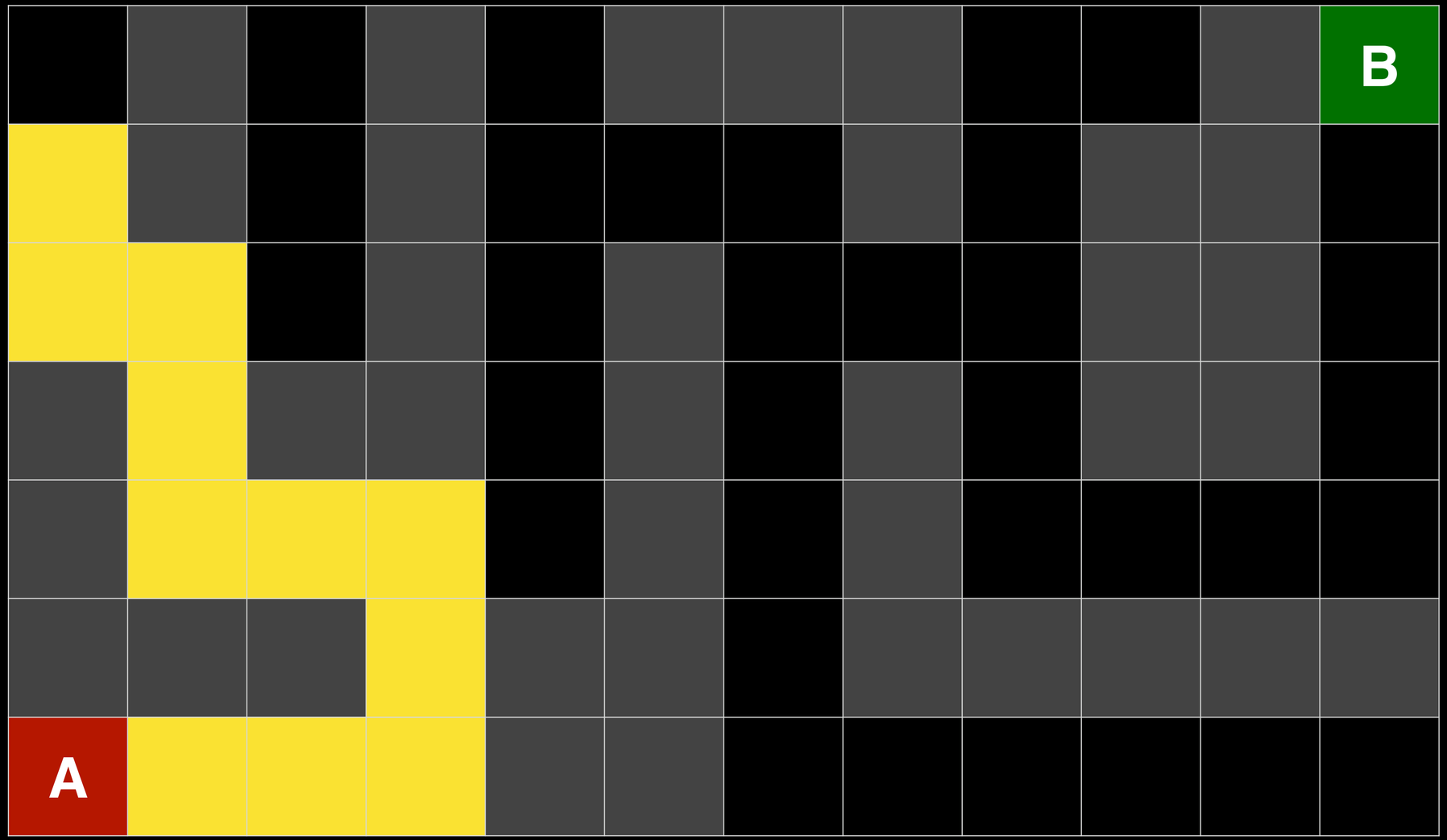
Depth-First Search



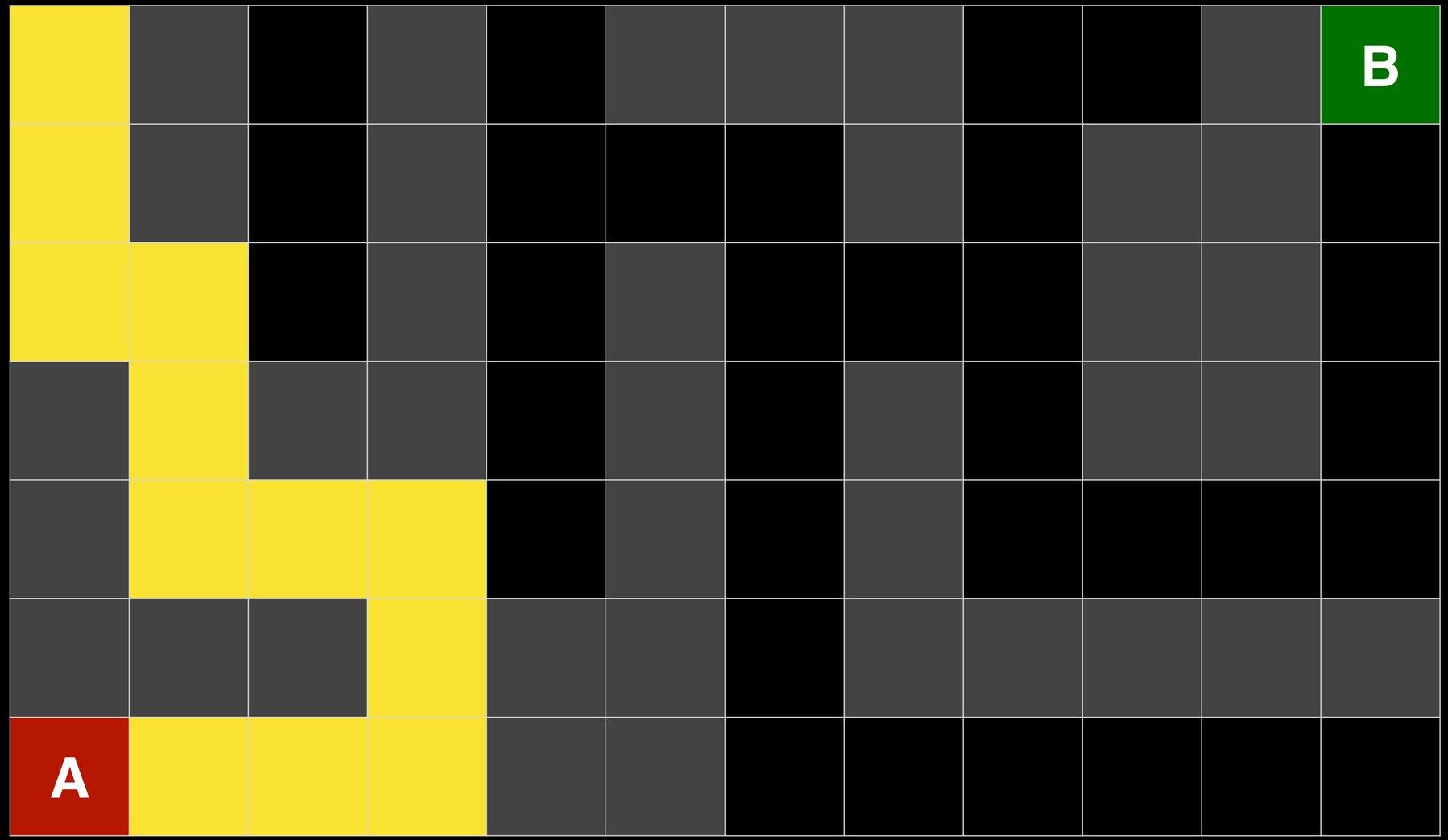
Depth-First Search



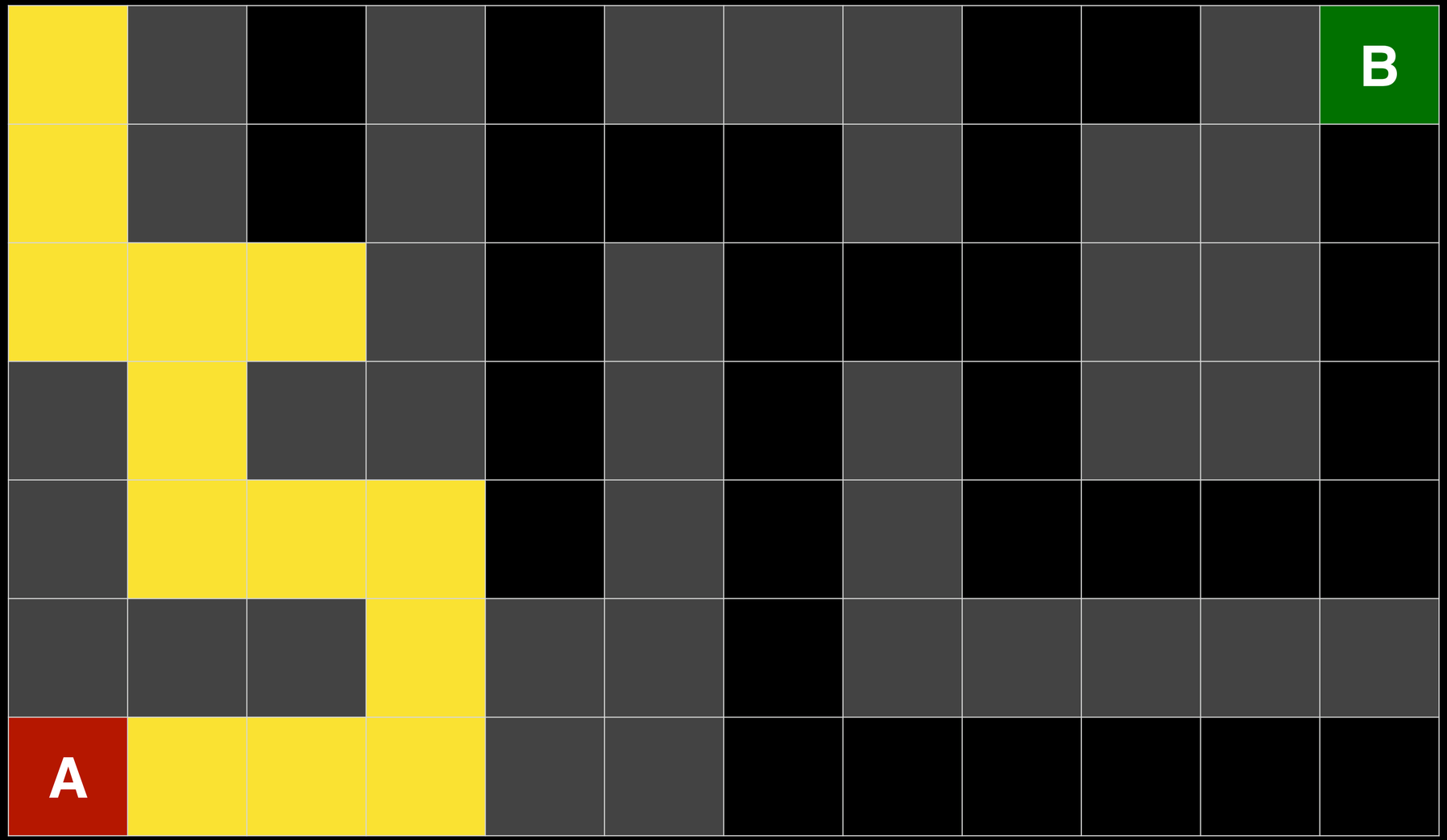
Depth-First Search



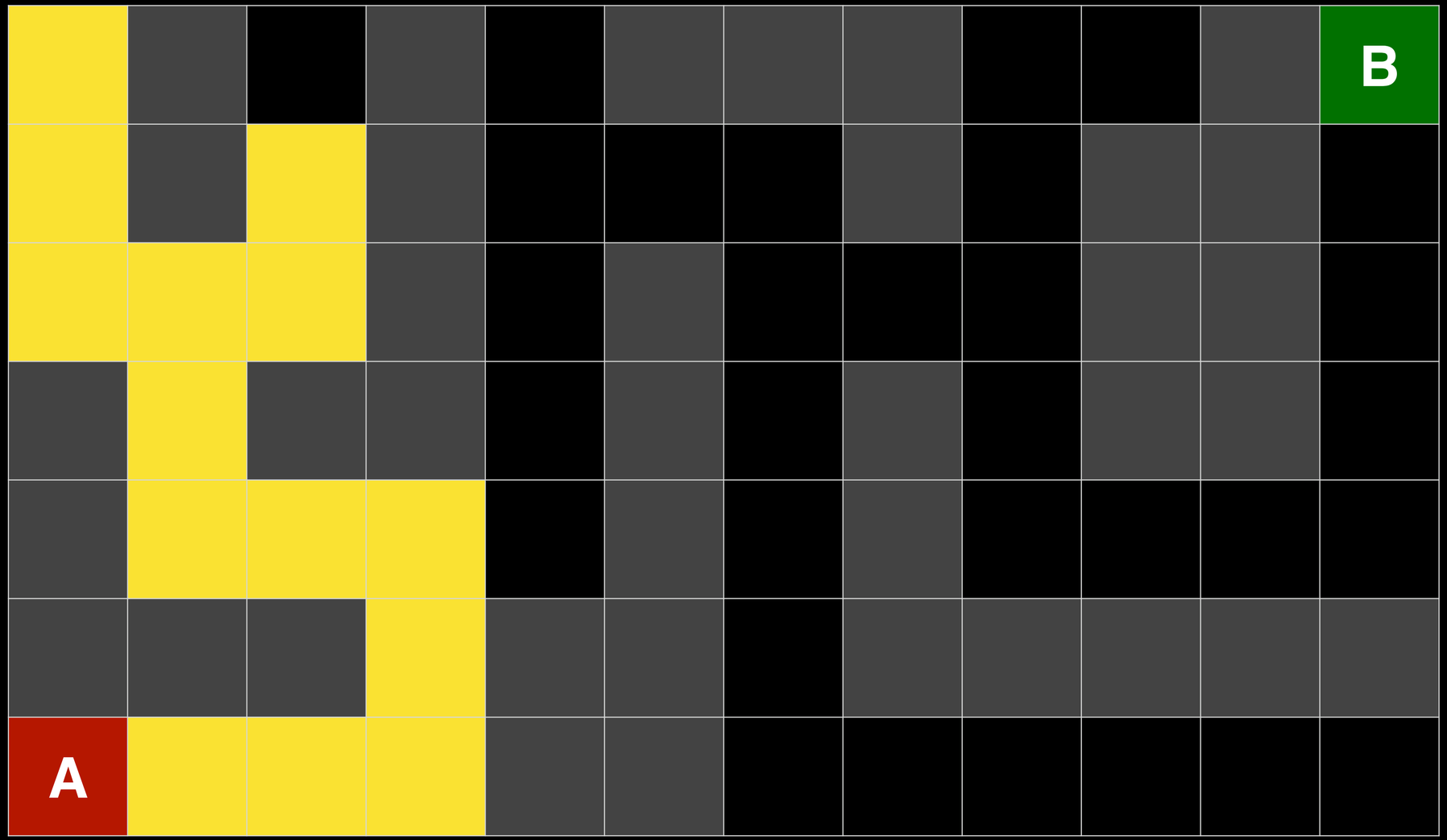
Depth-First Search



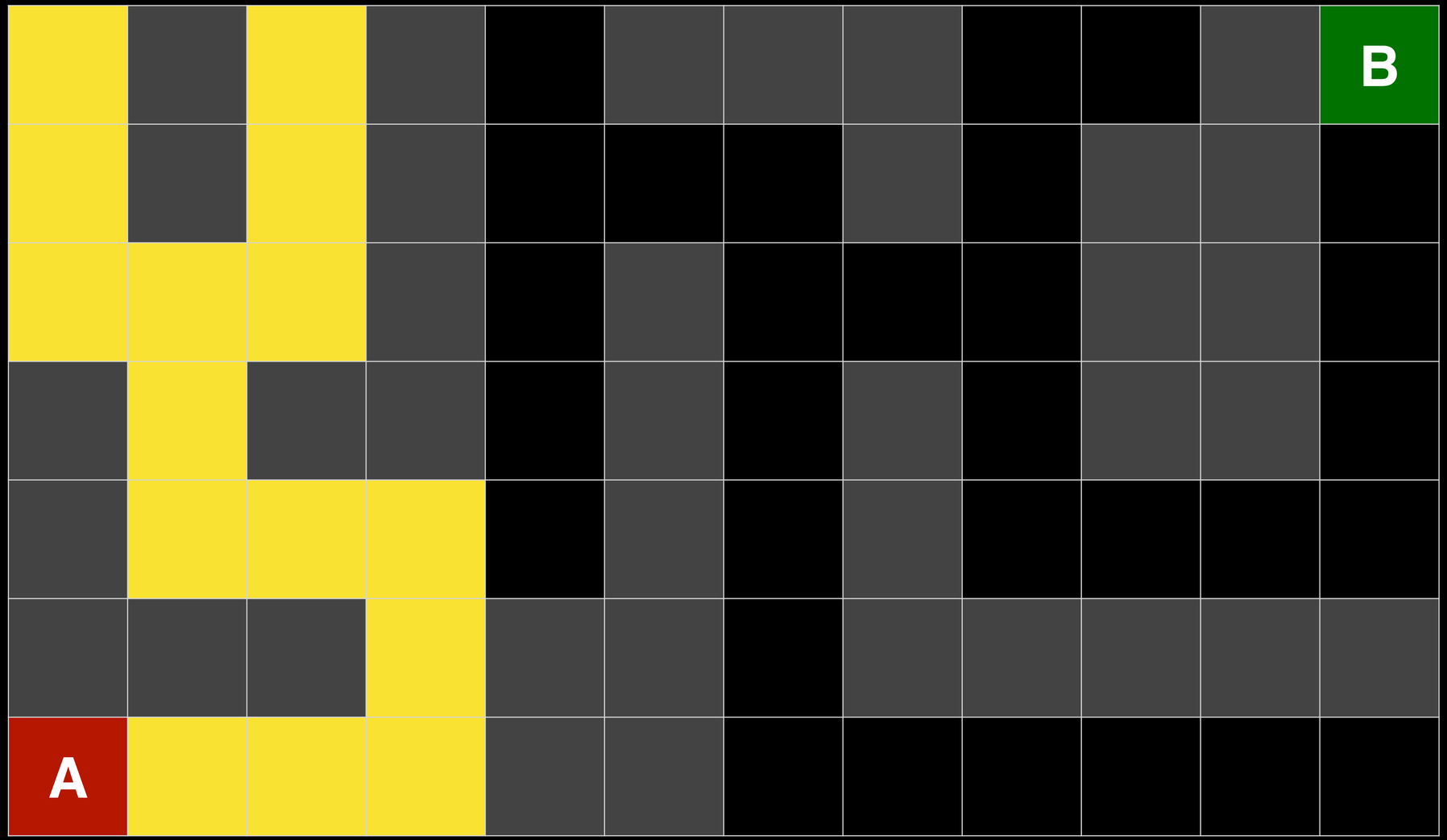
Depth-First Search



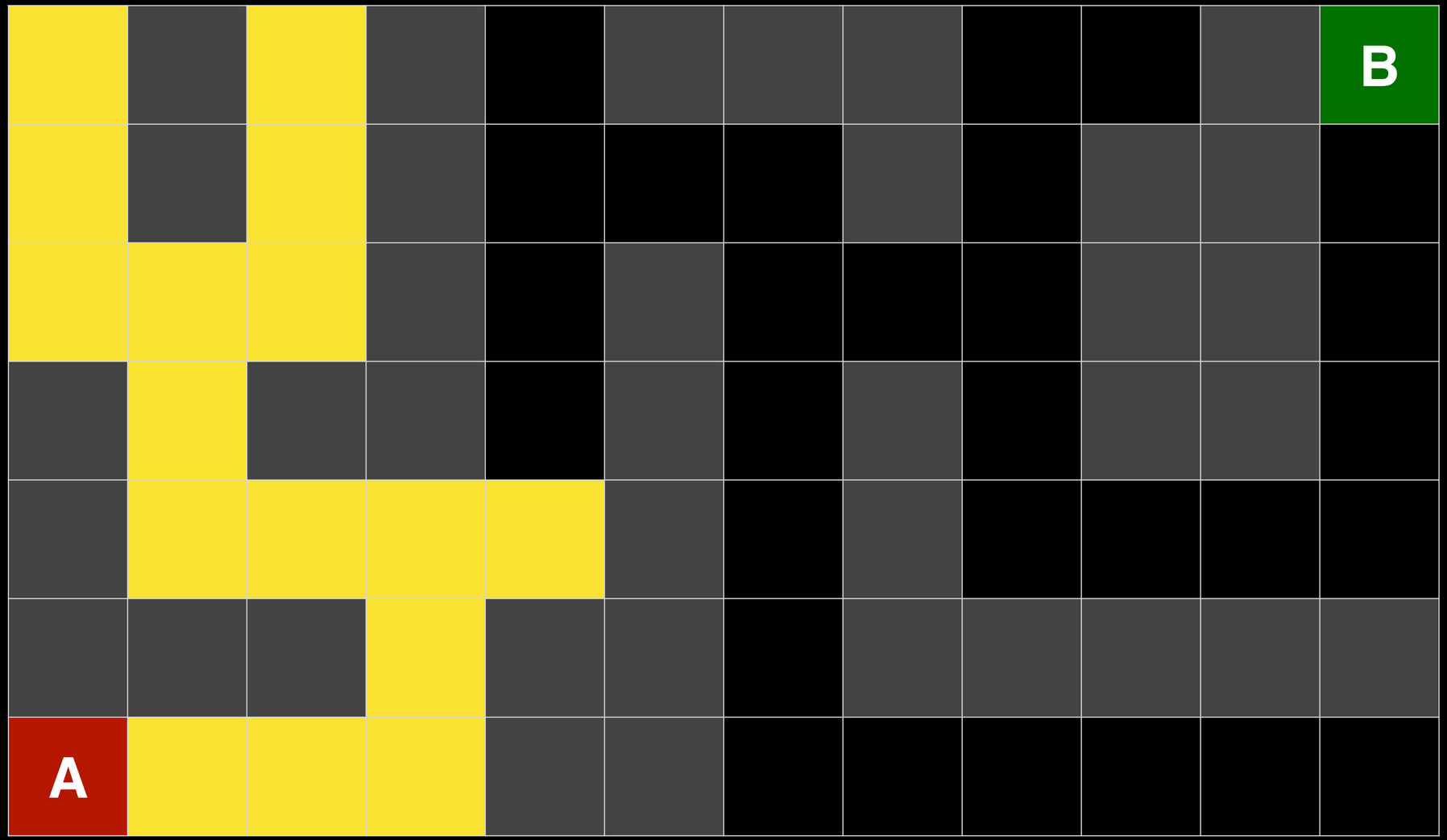
Depth-First Search



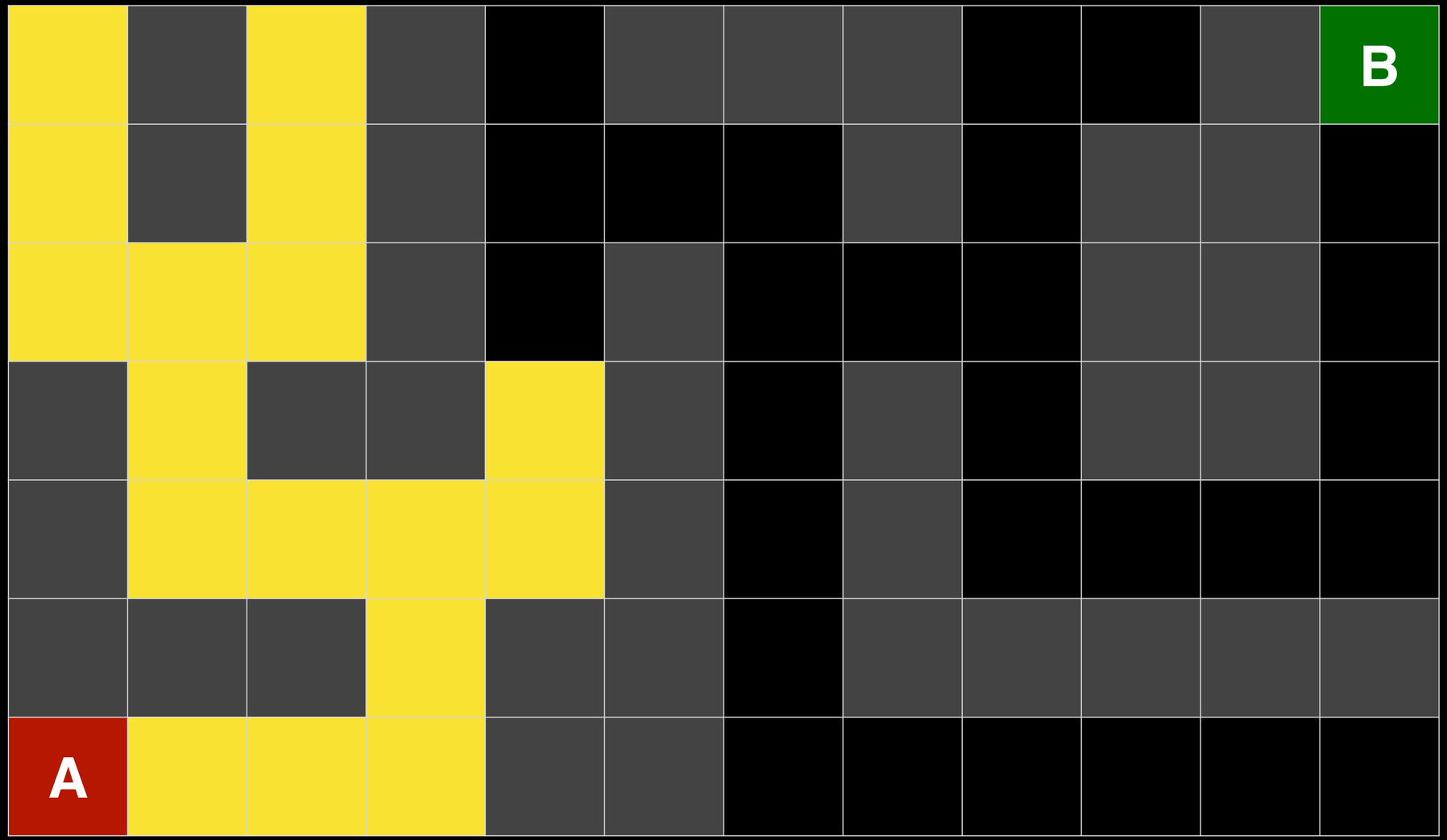
Depth-First Search



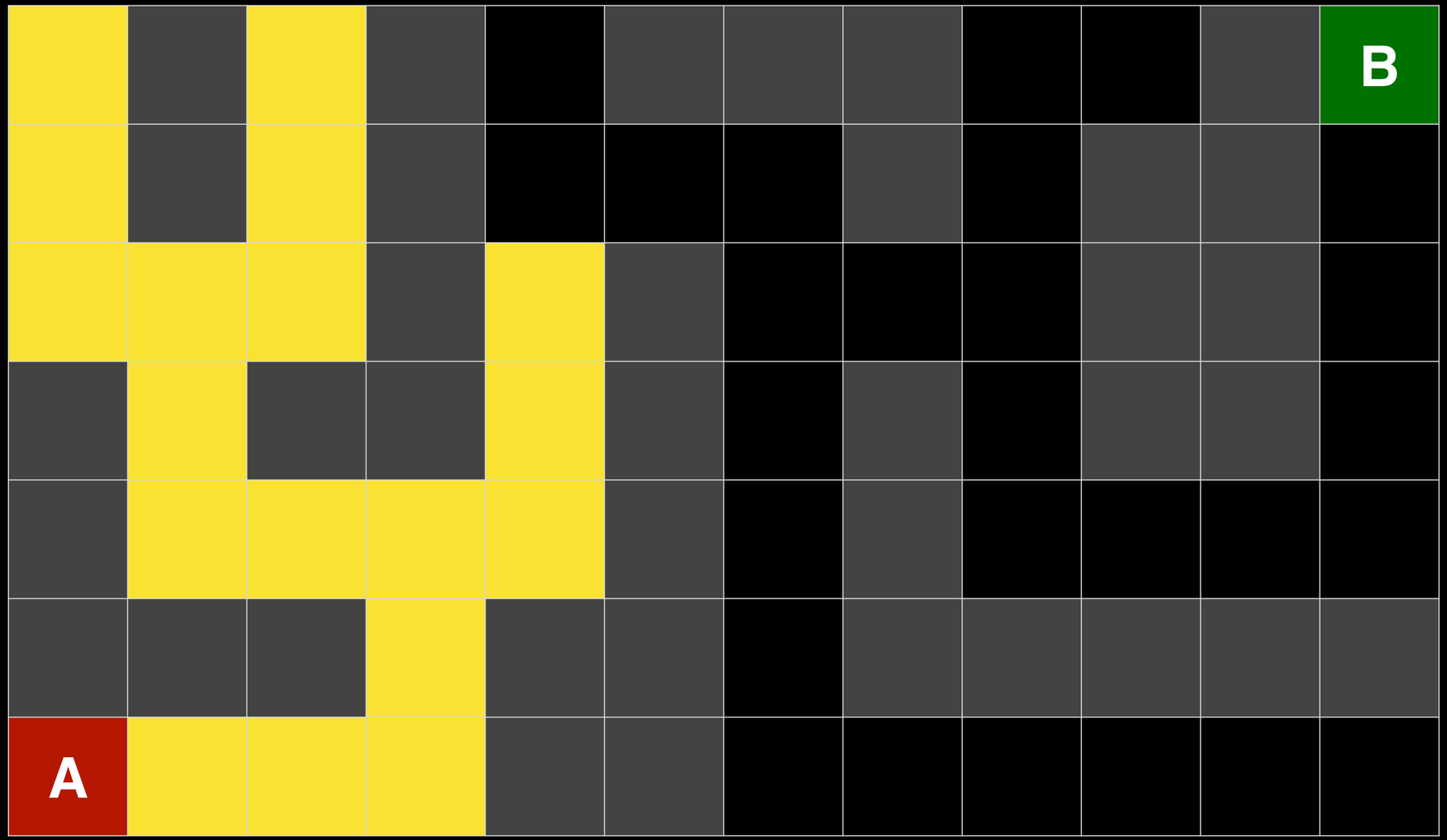
Depth-First Search



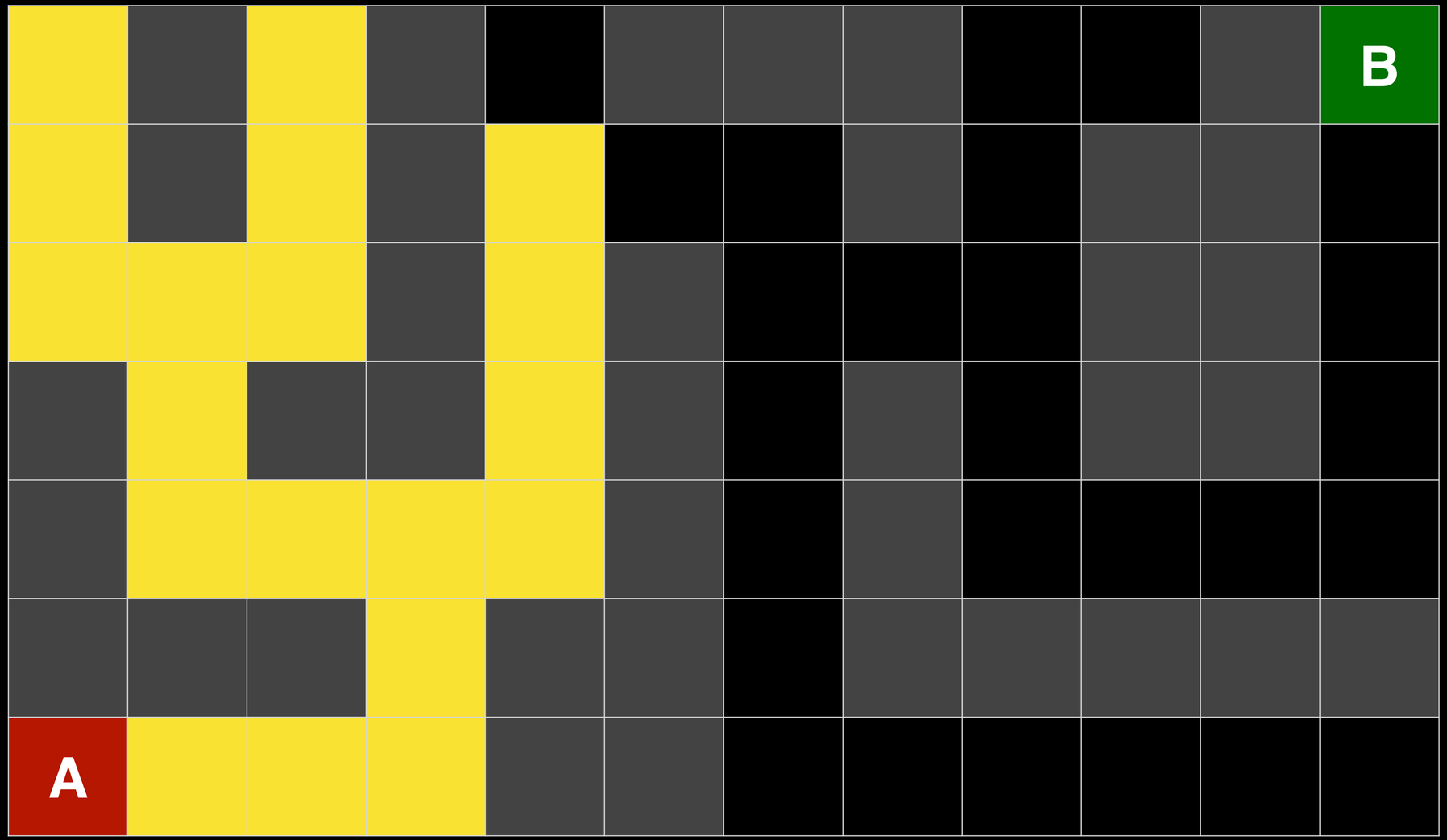
Depth-First Search



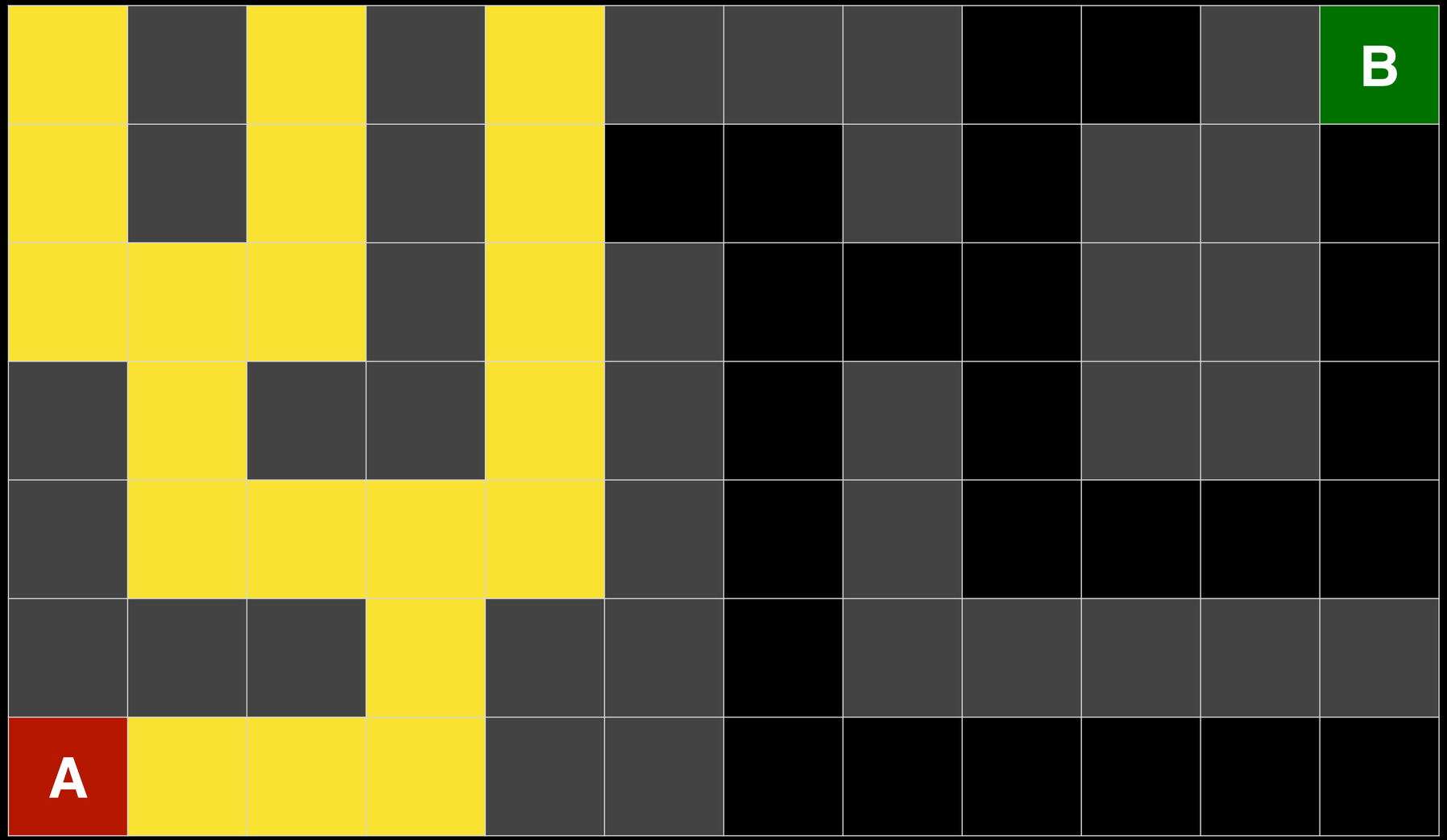
Depth-First Search



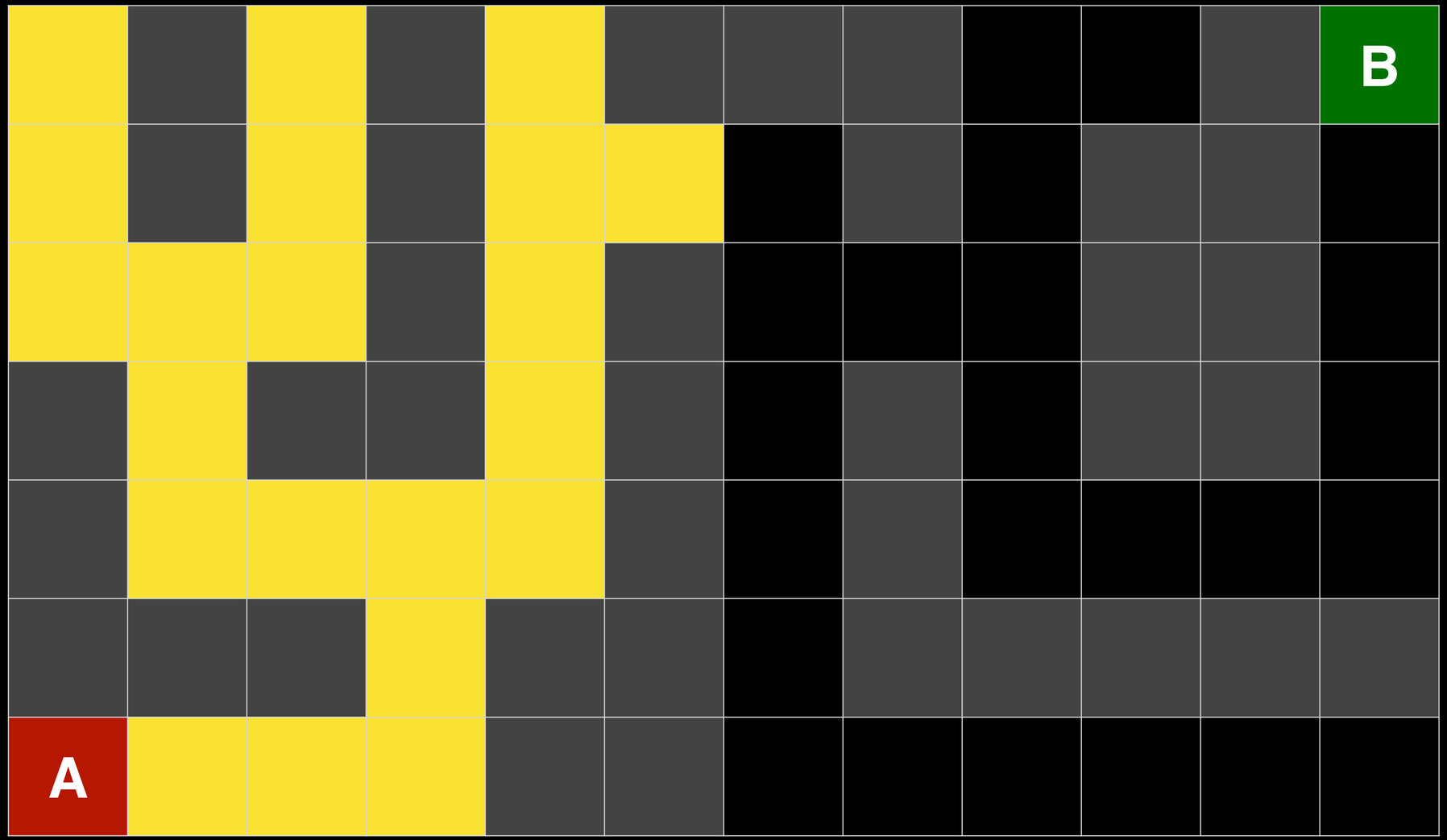
Depth-First Search



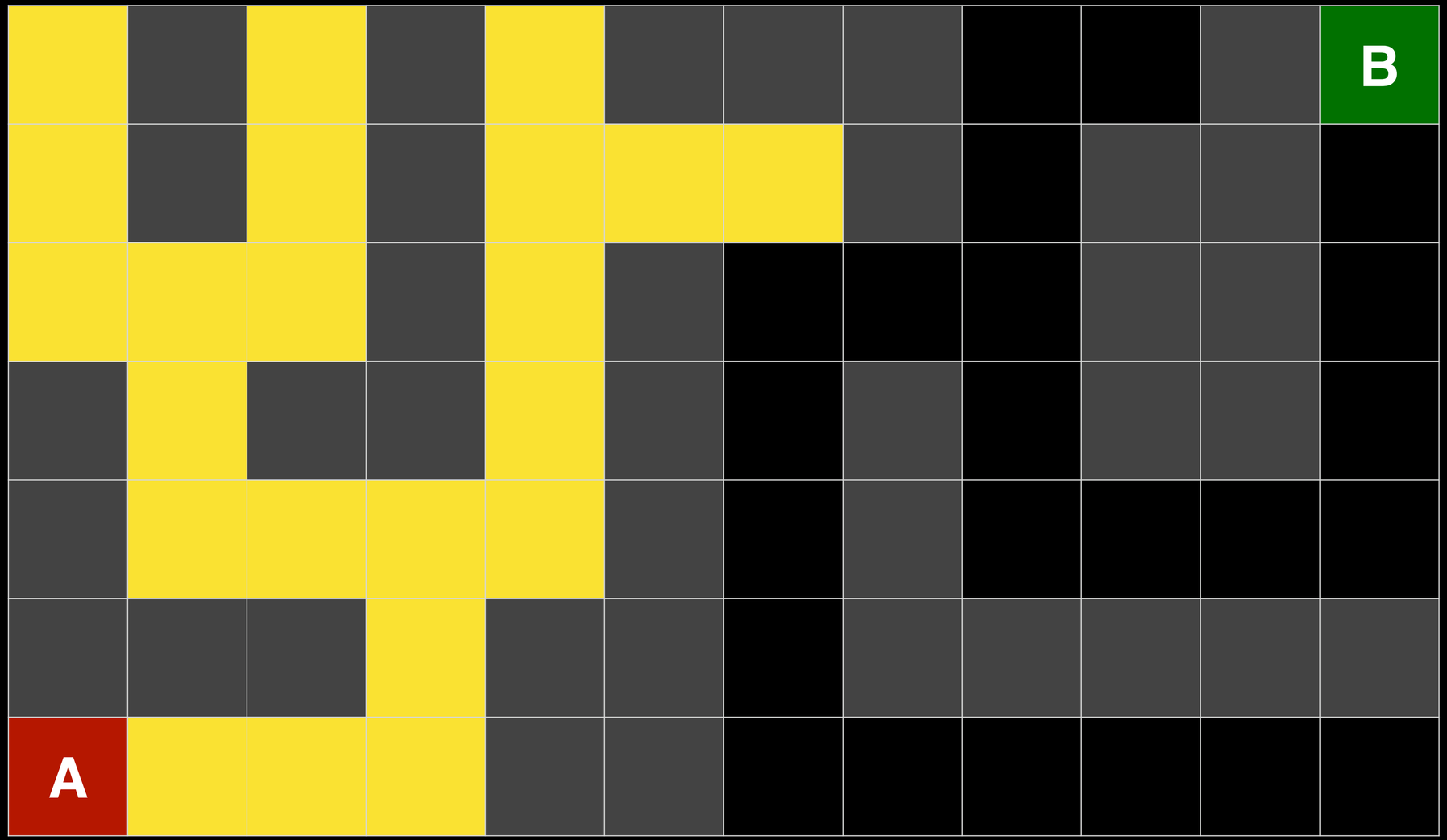
Depth-First Search



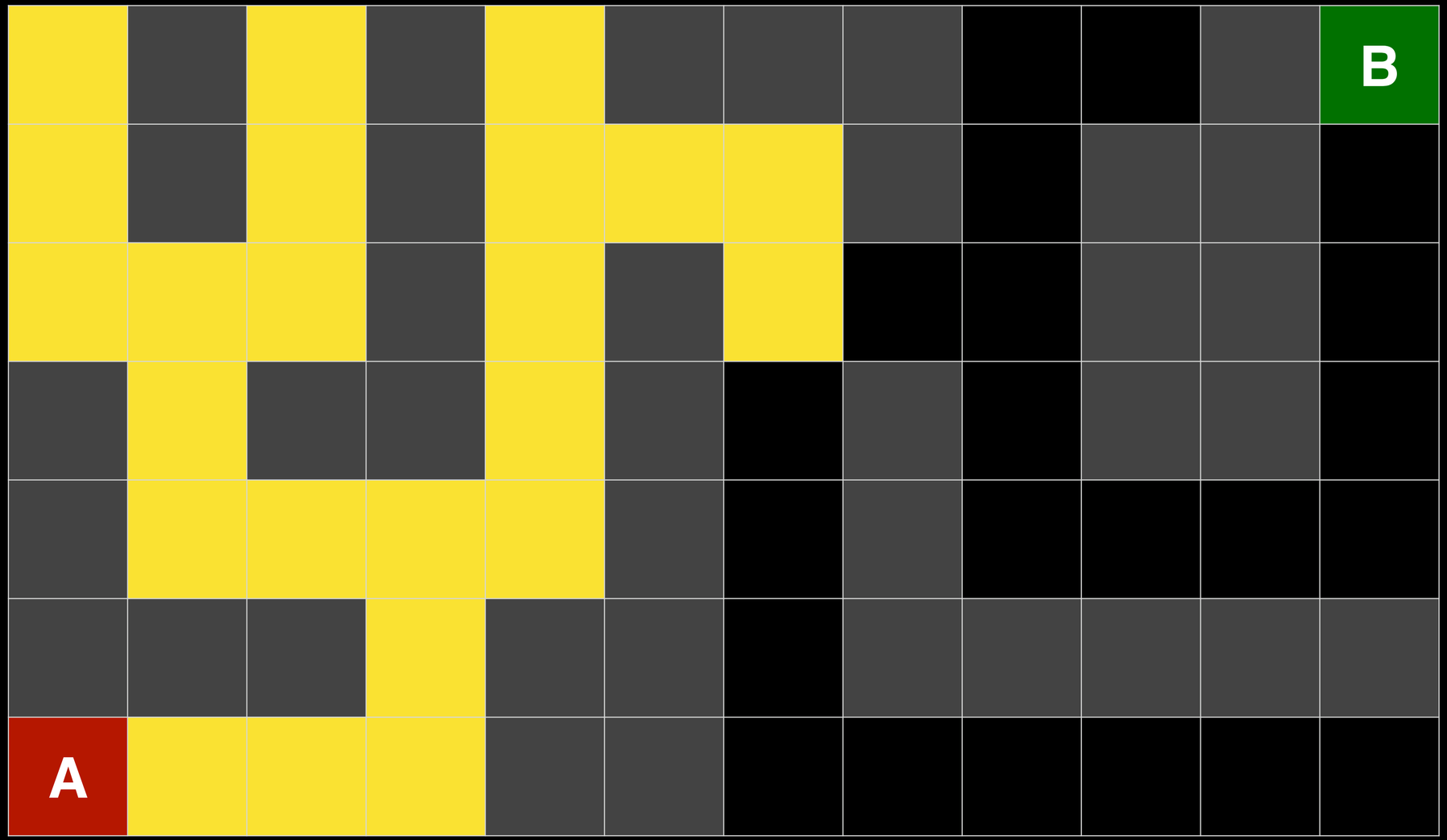
Depth-First Search



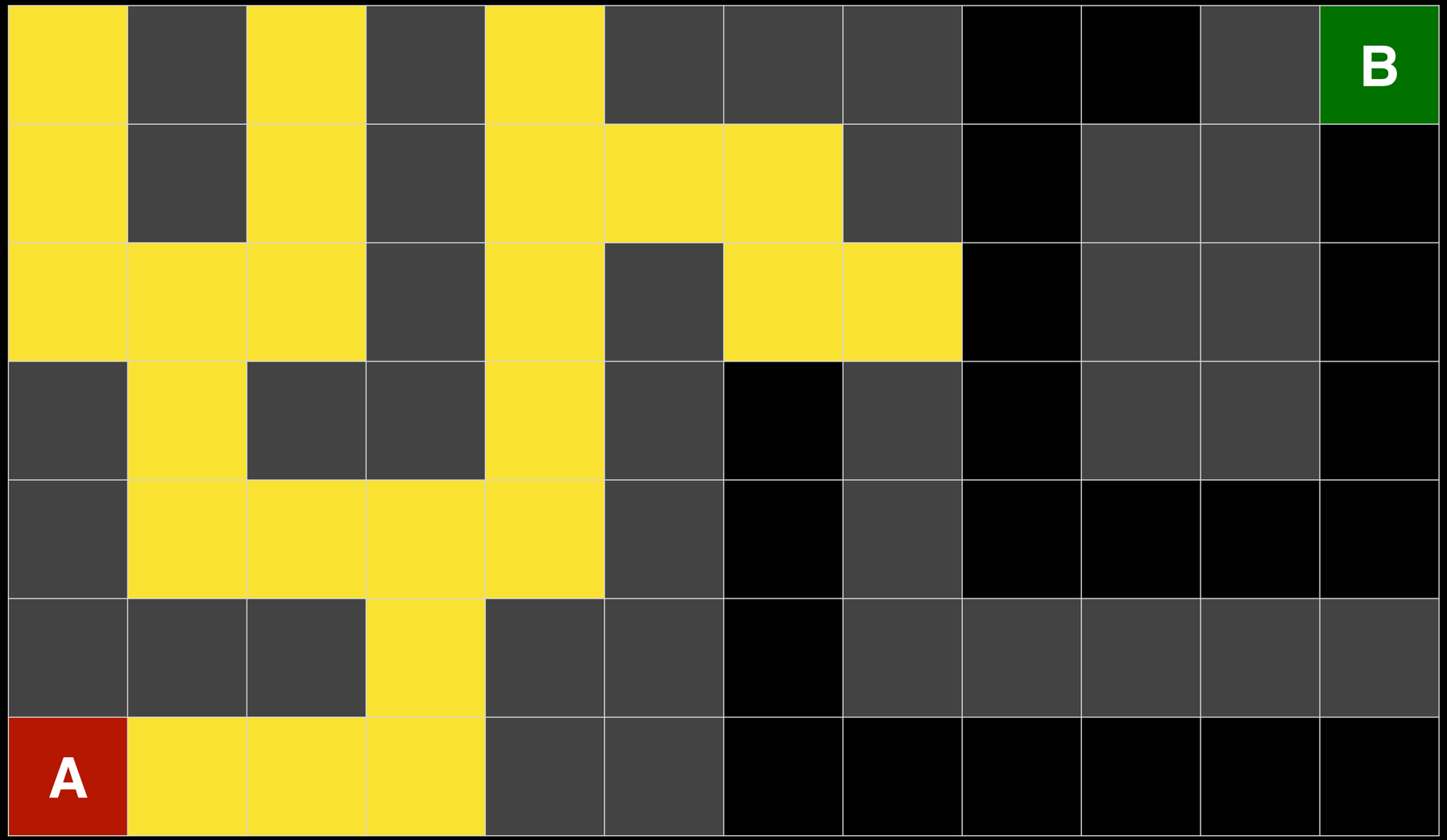
Depth-First Search



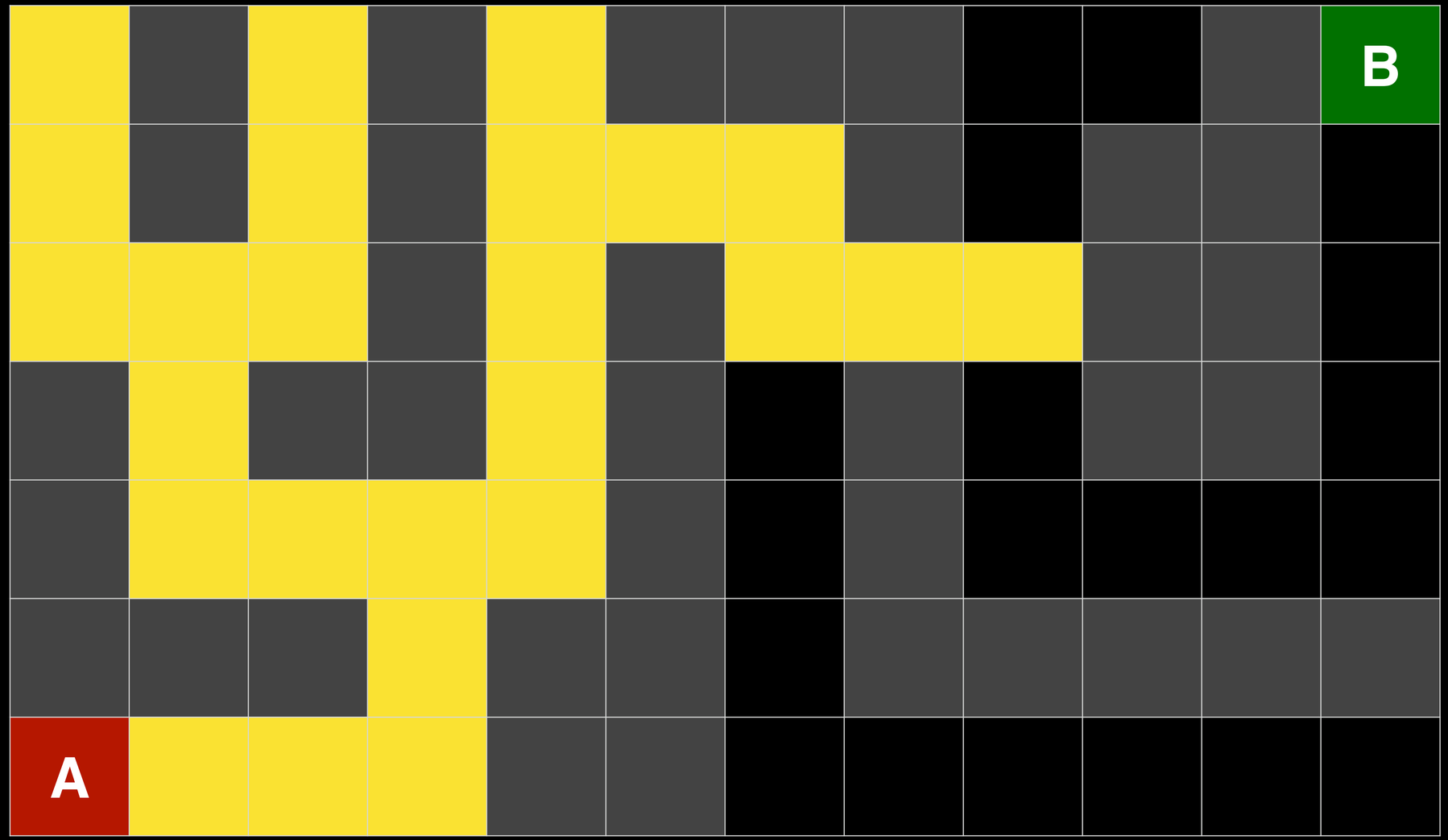
Depth-First Search



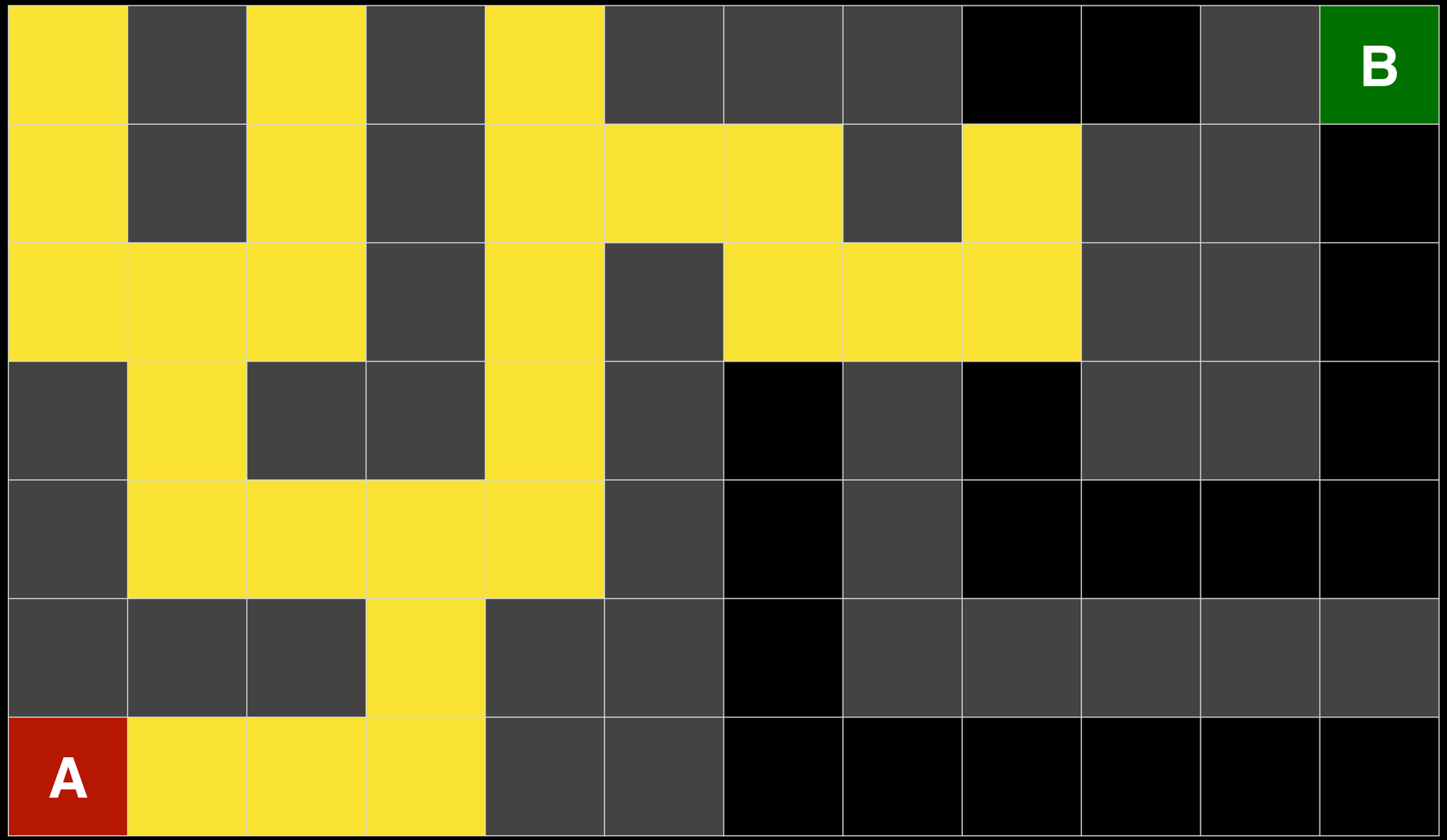
Depth-First Search



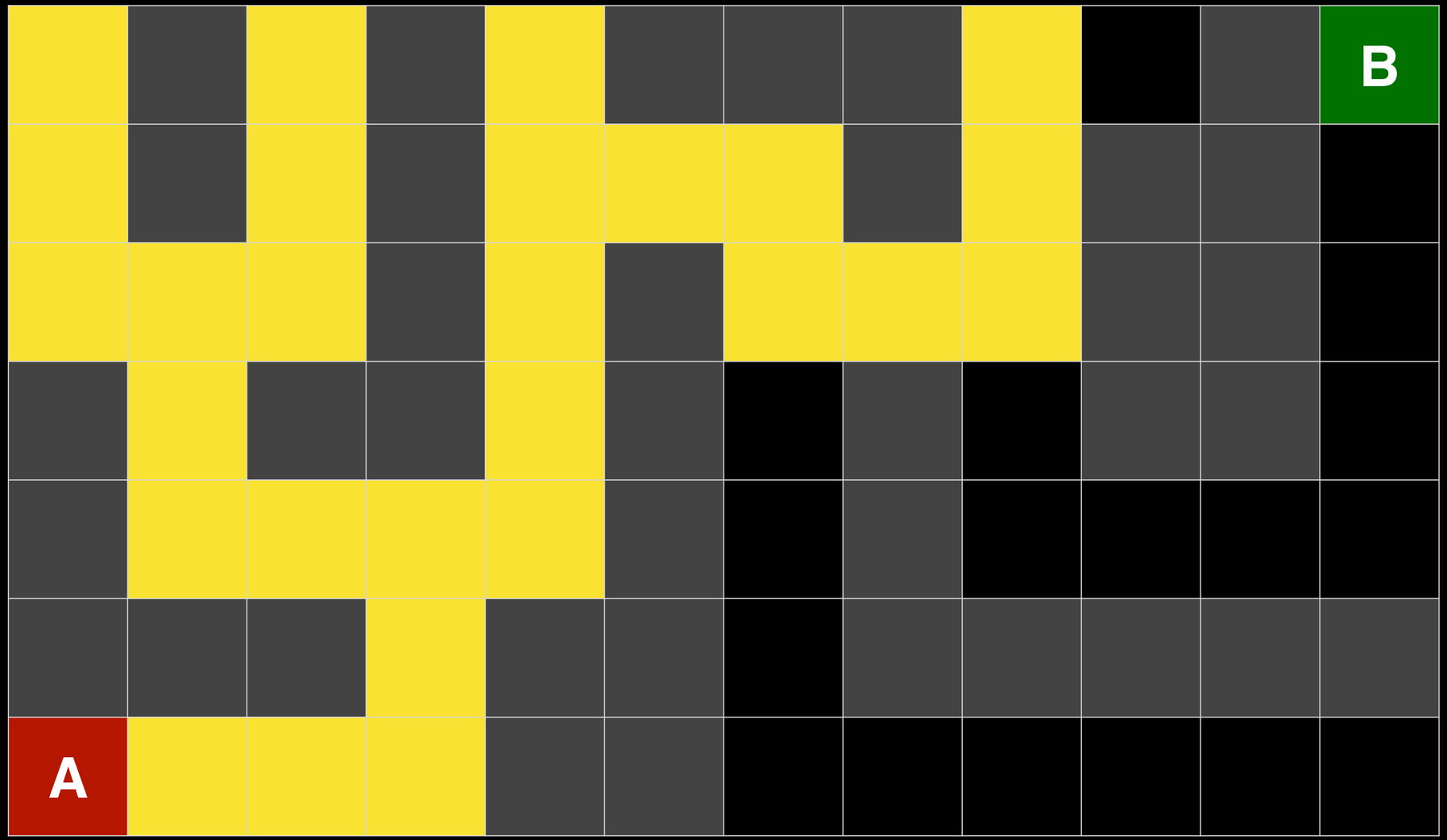
Depth-First Search



Depth-First Search



Depth-First Search



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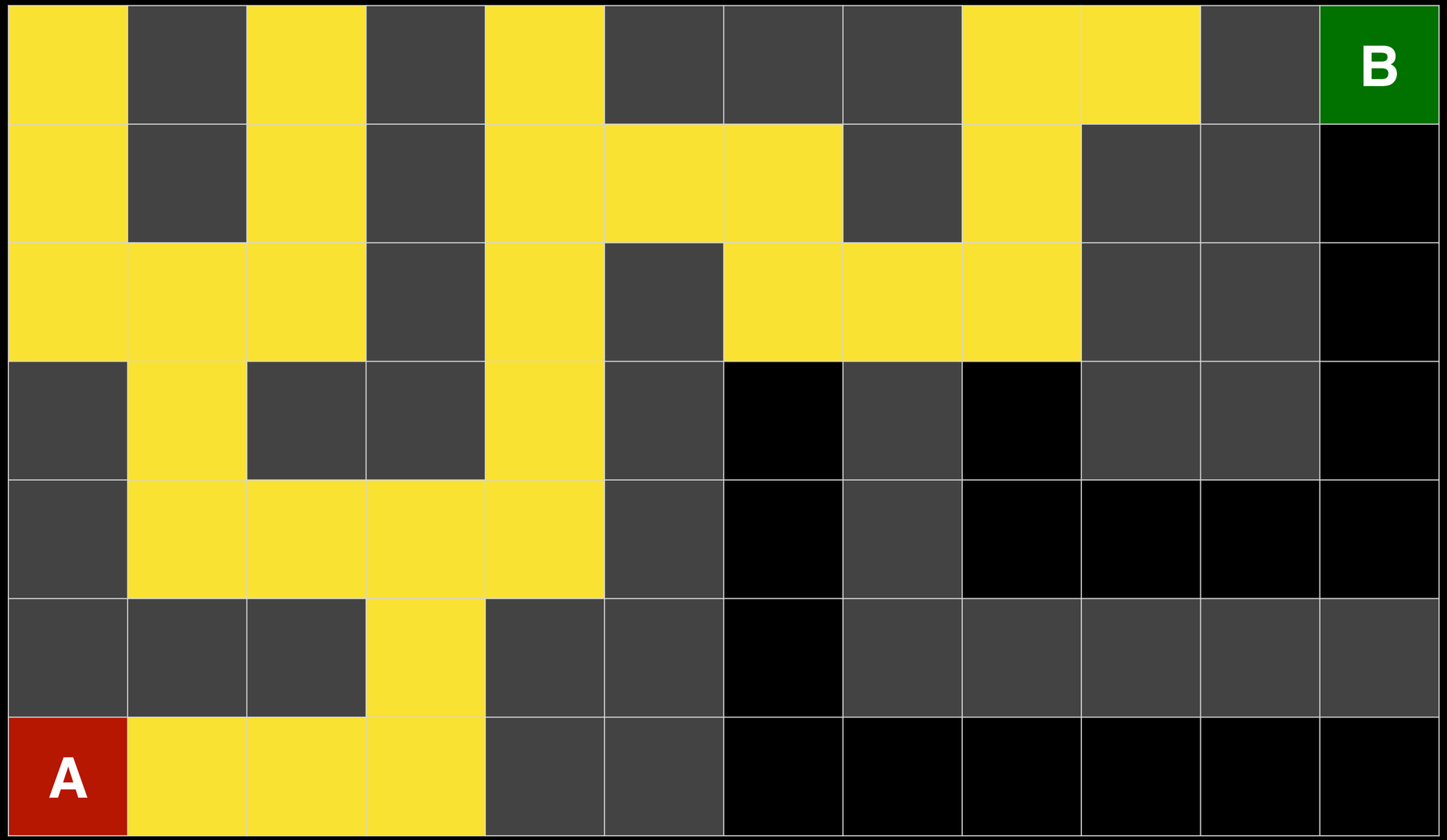

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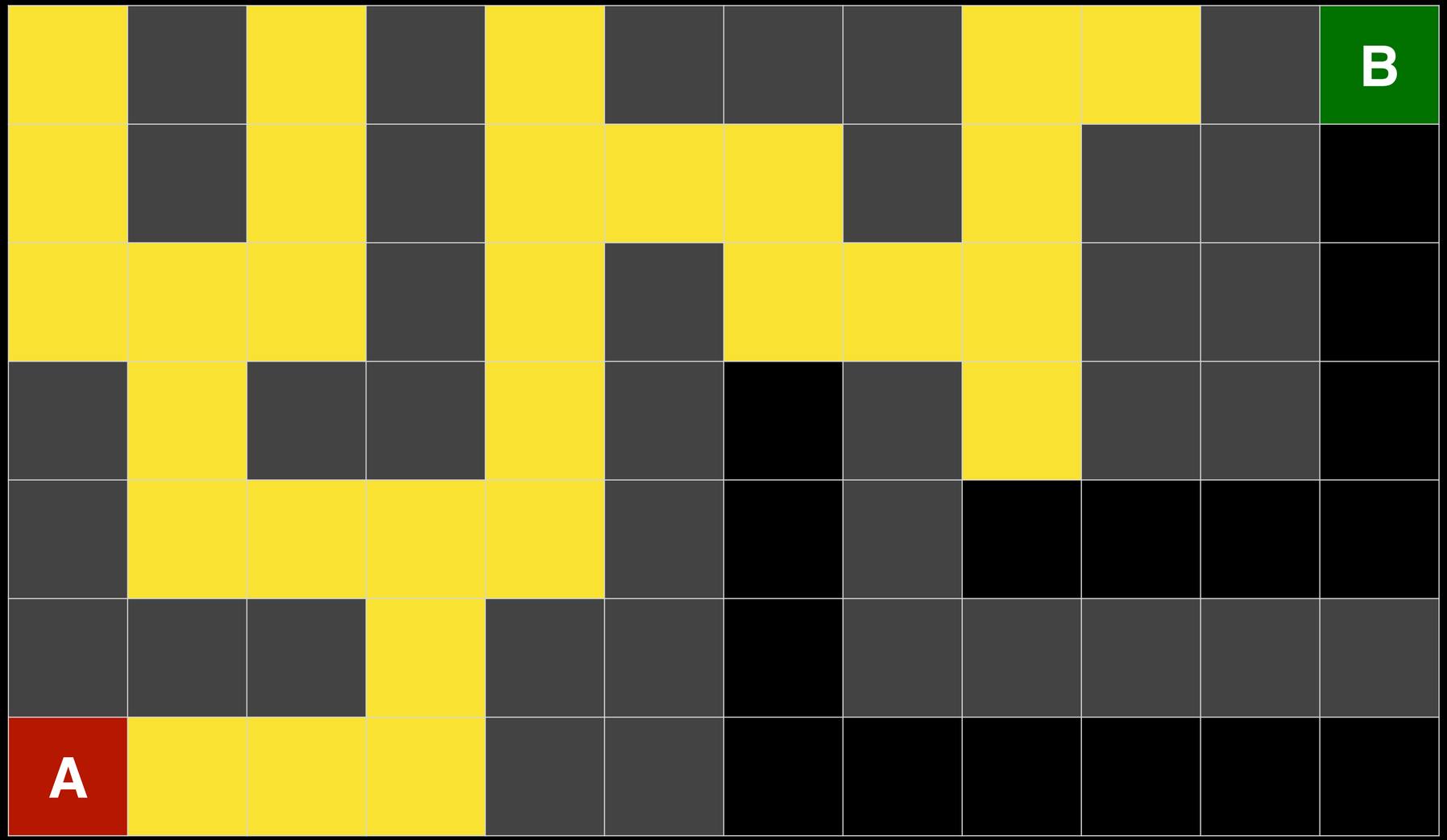

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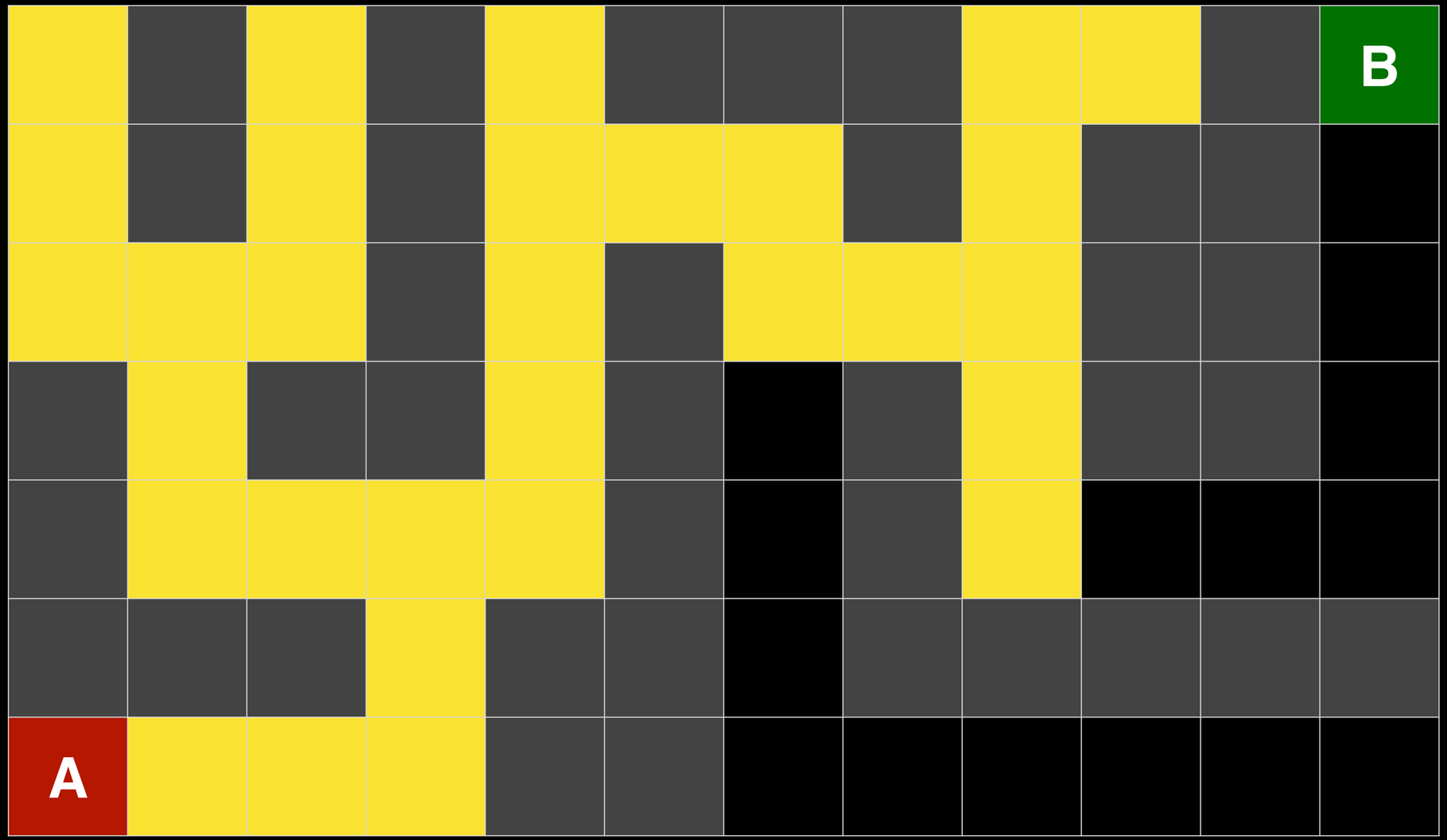

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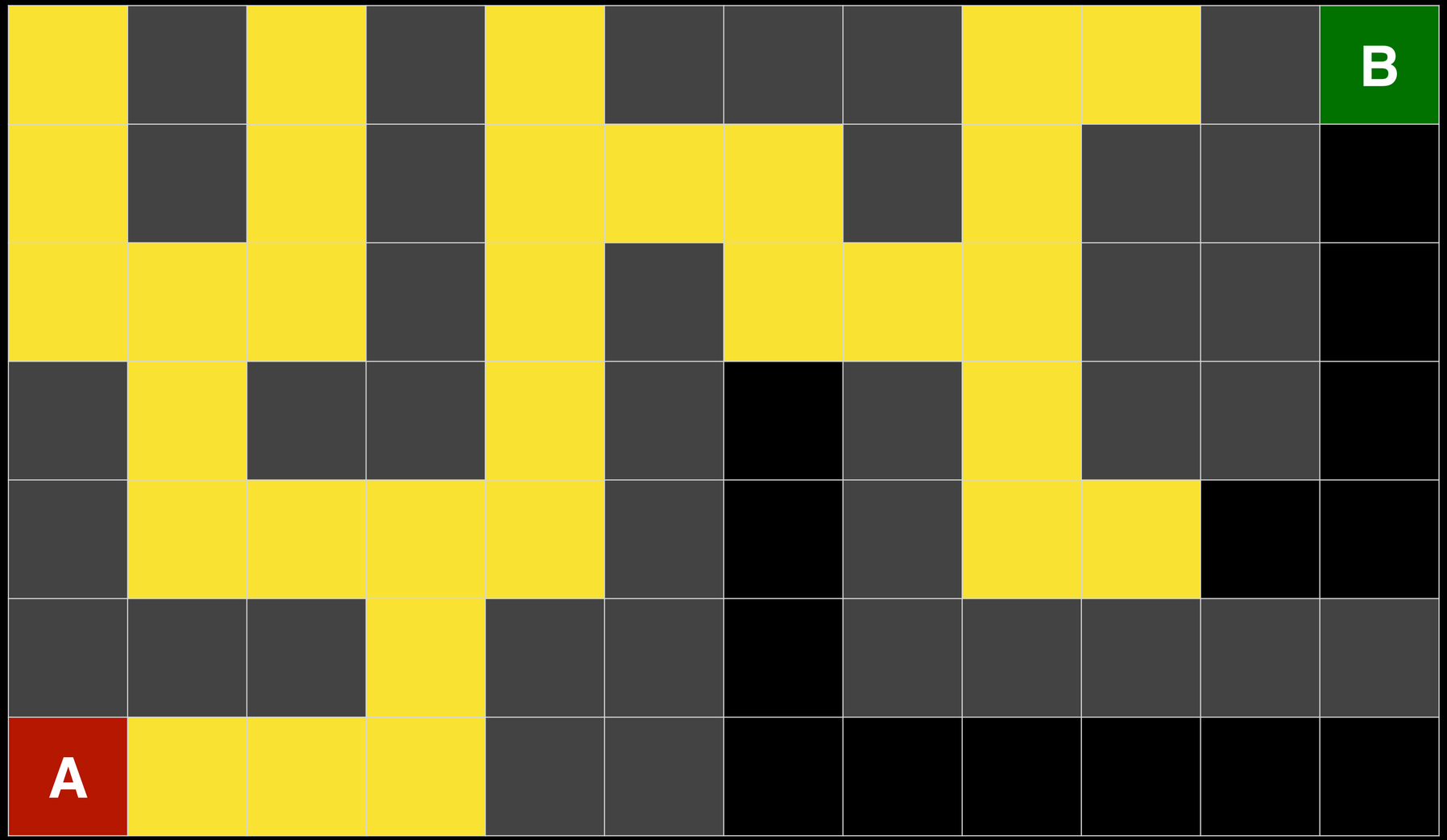

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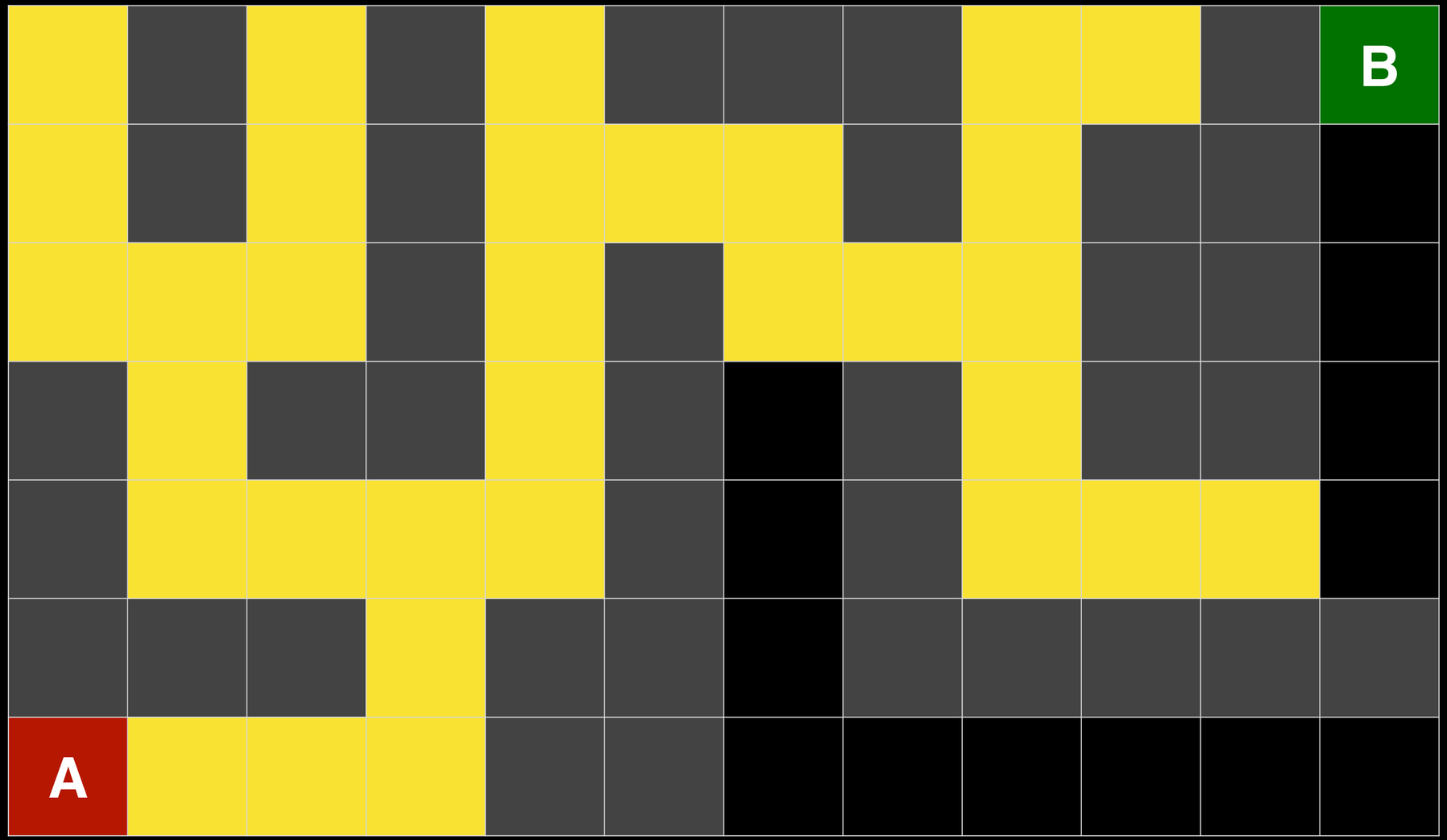

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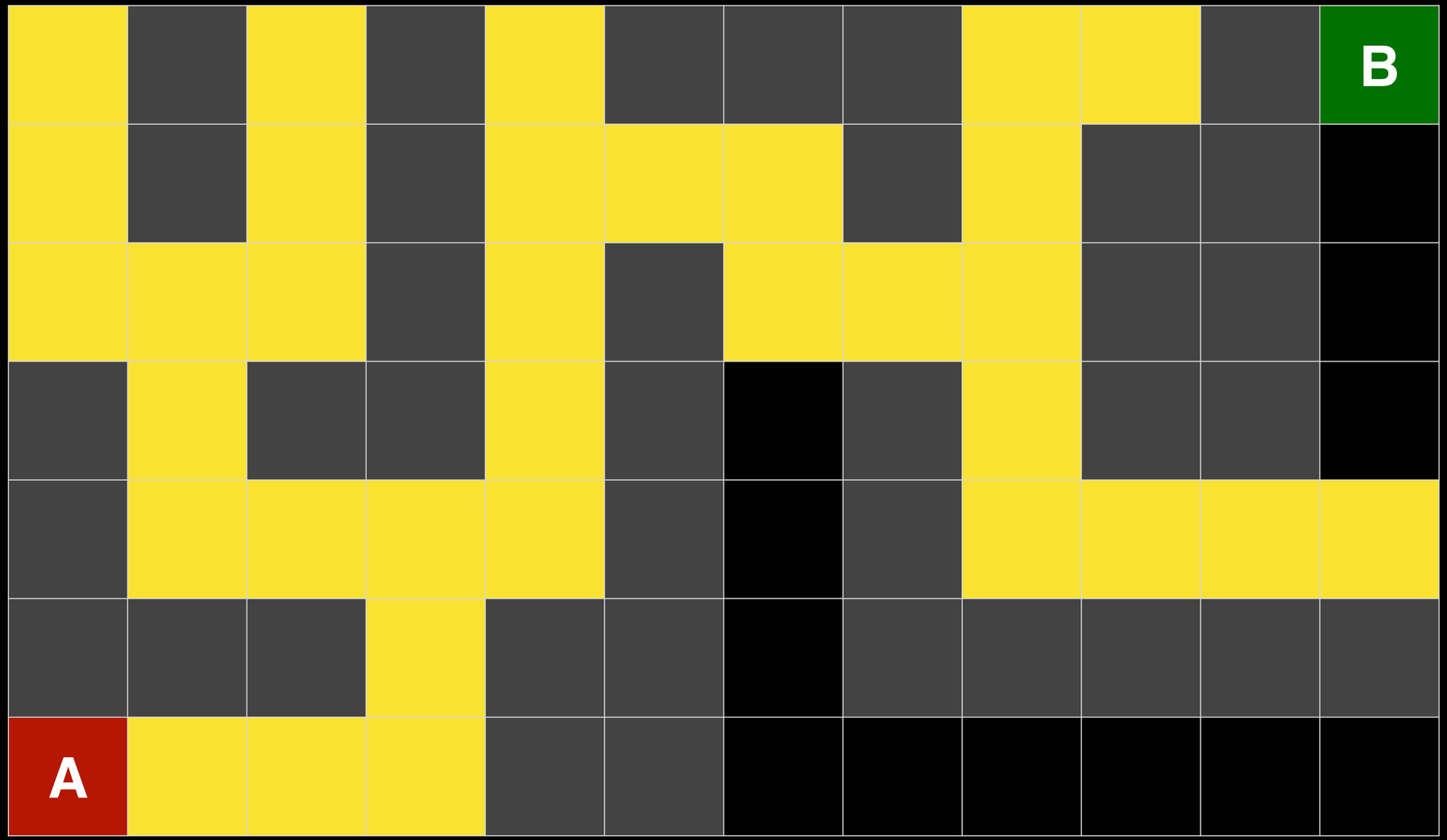

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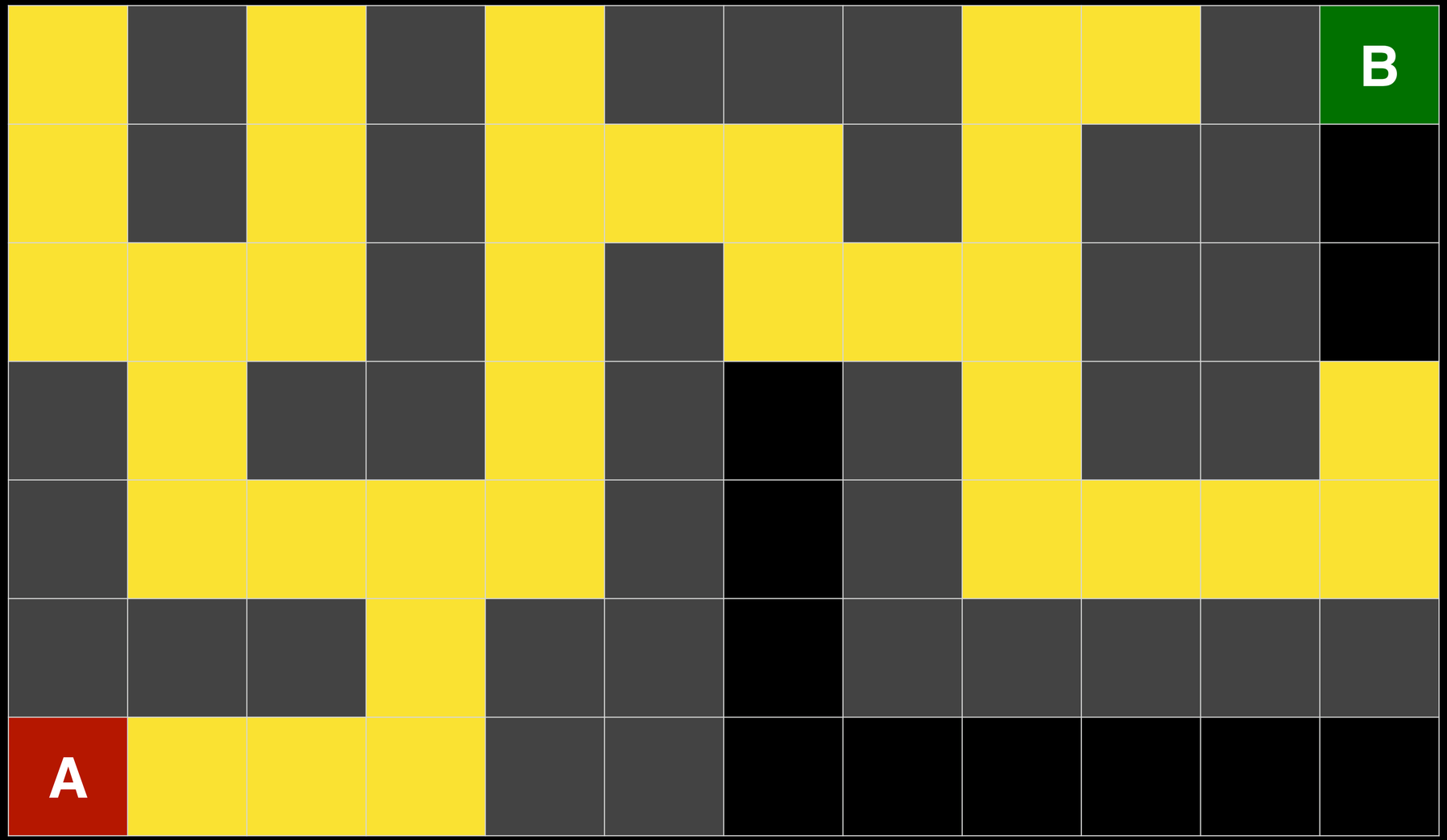

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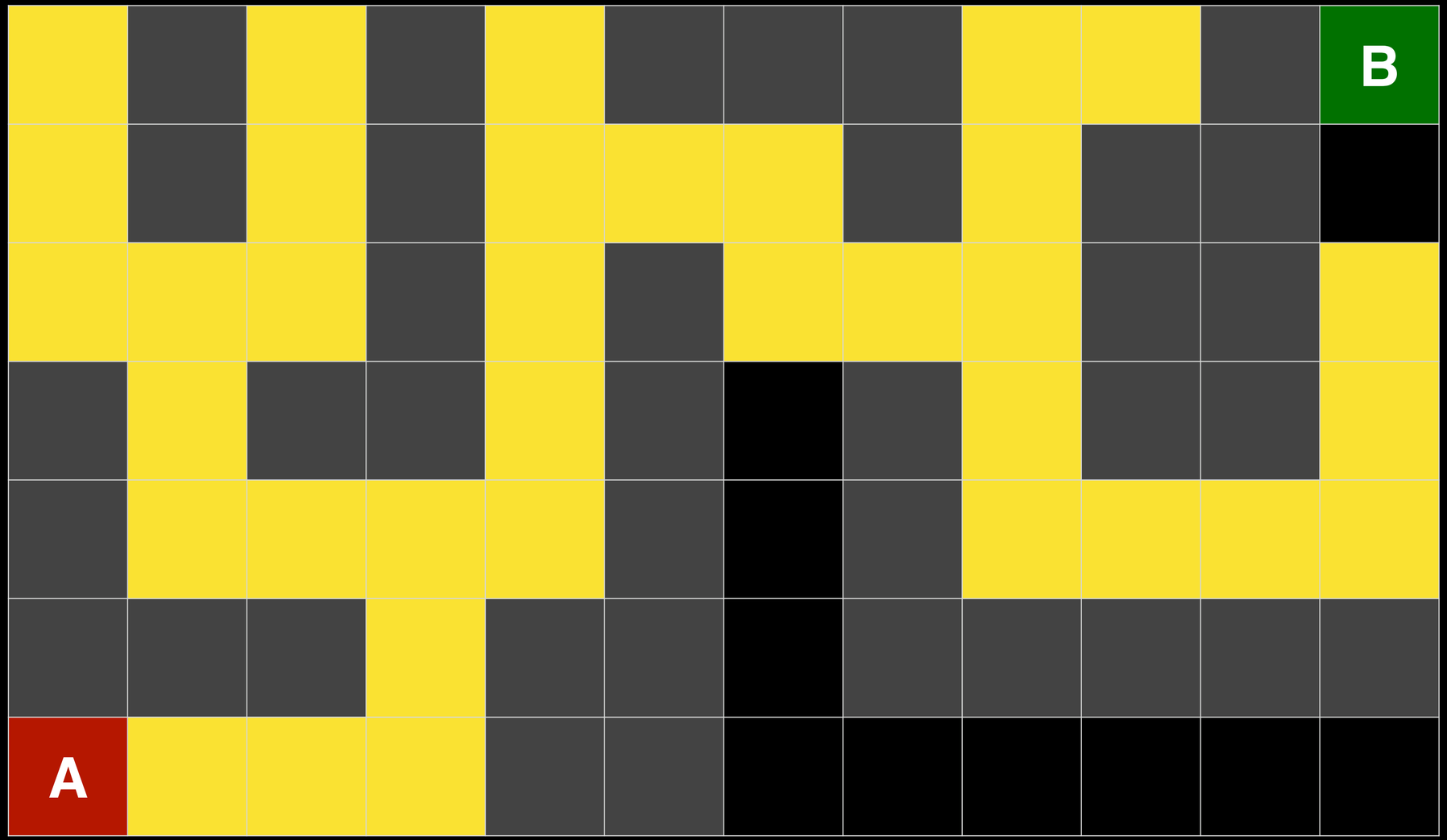

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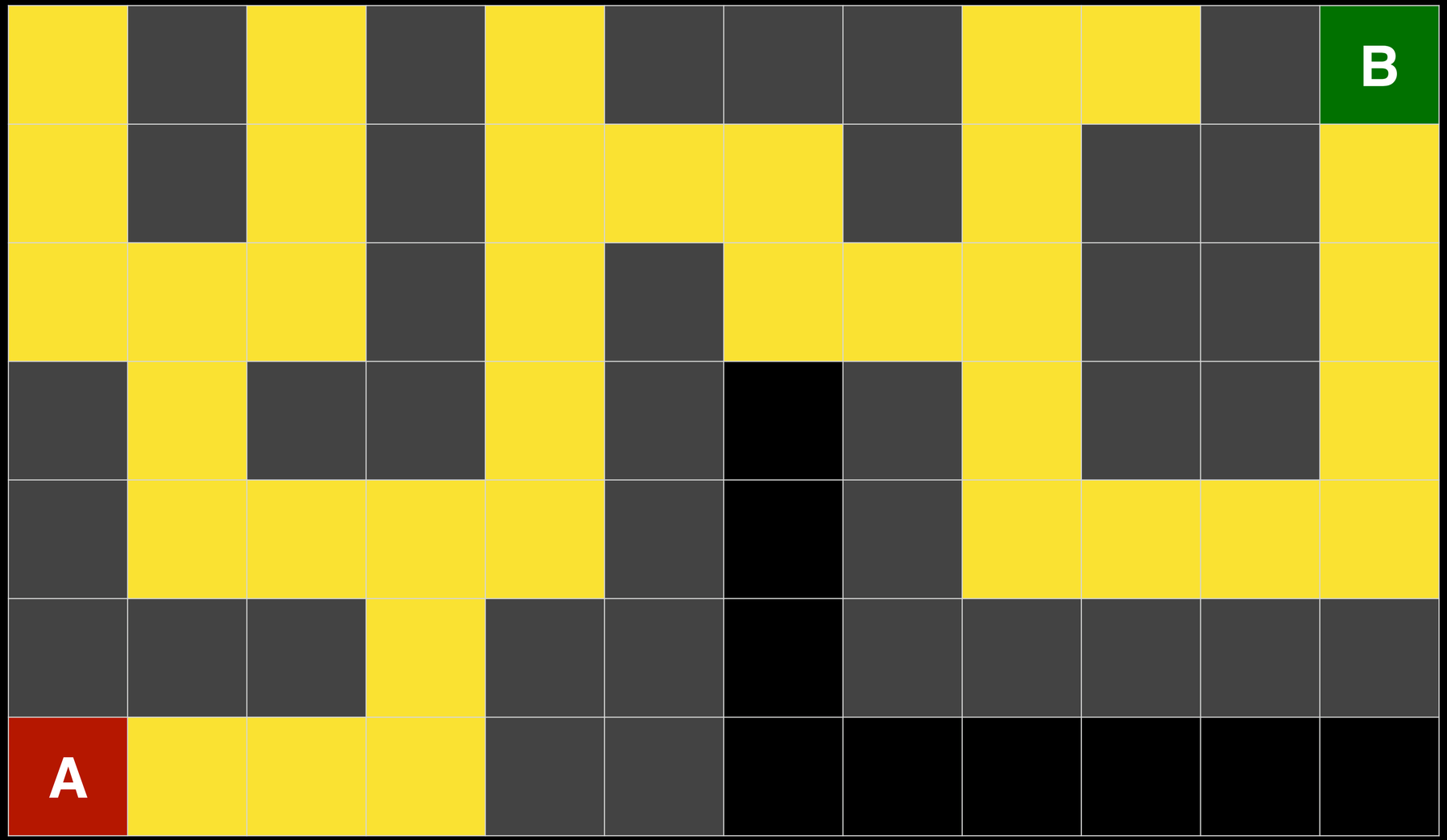

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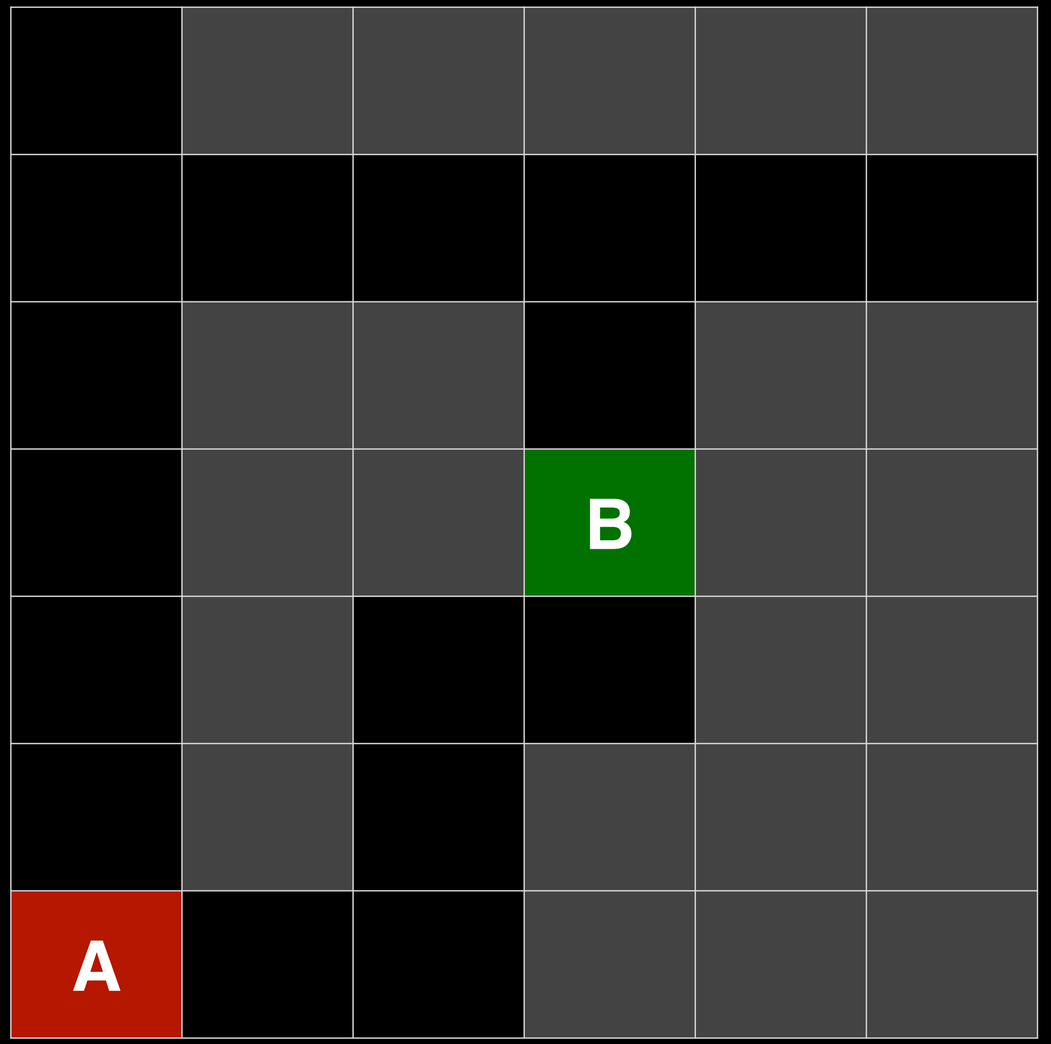

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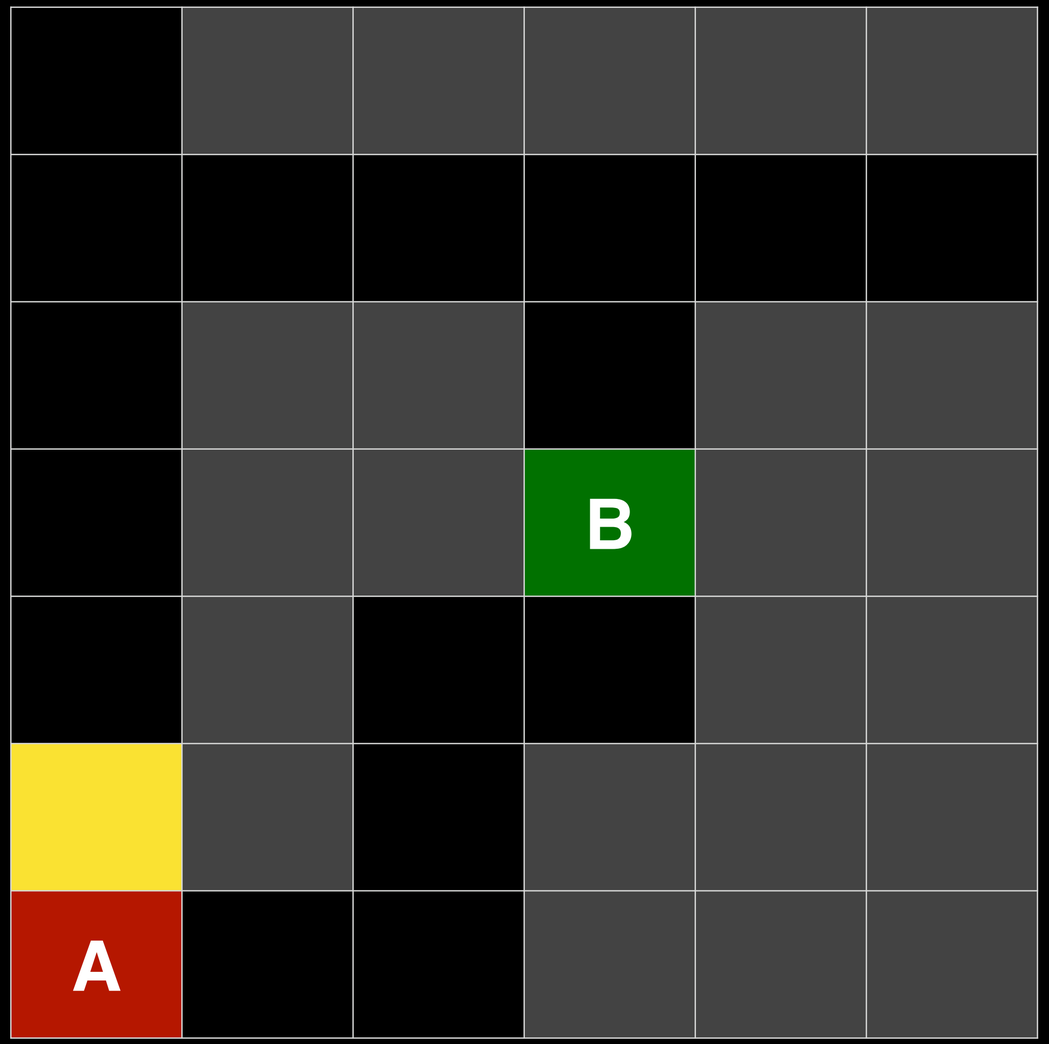

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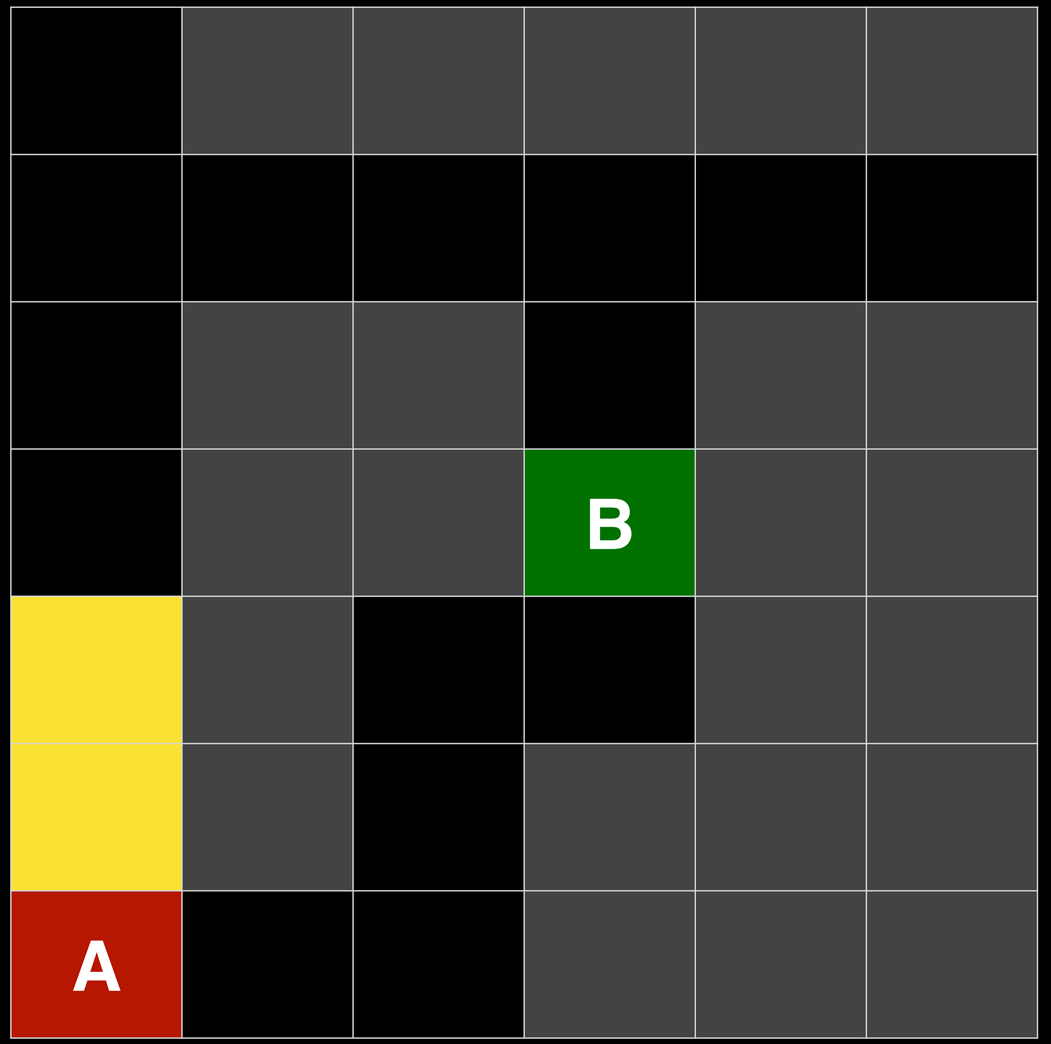

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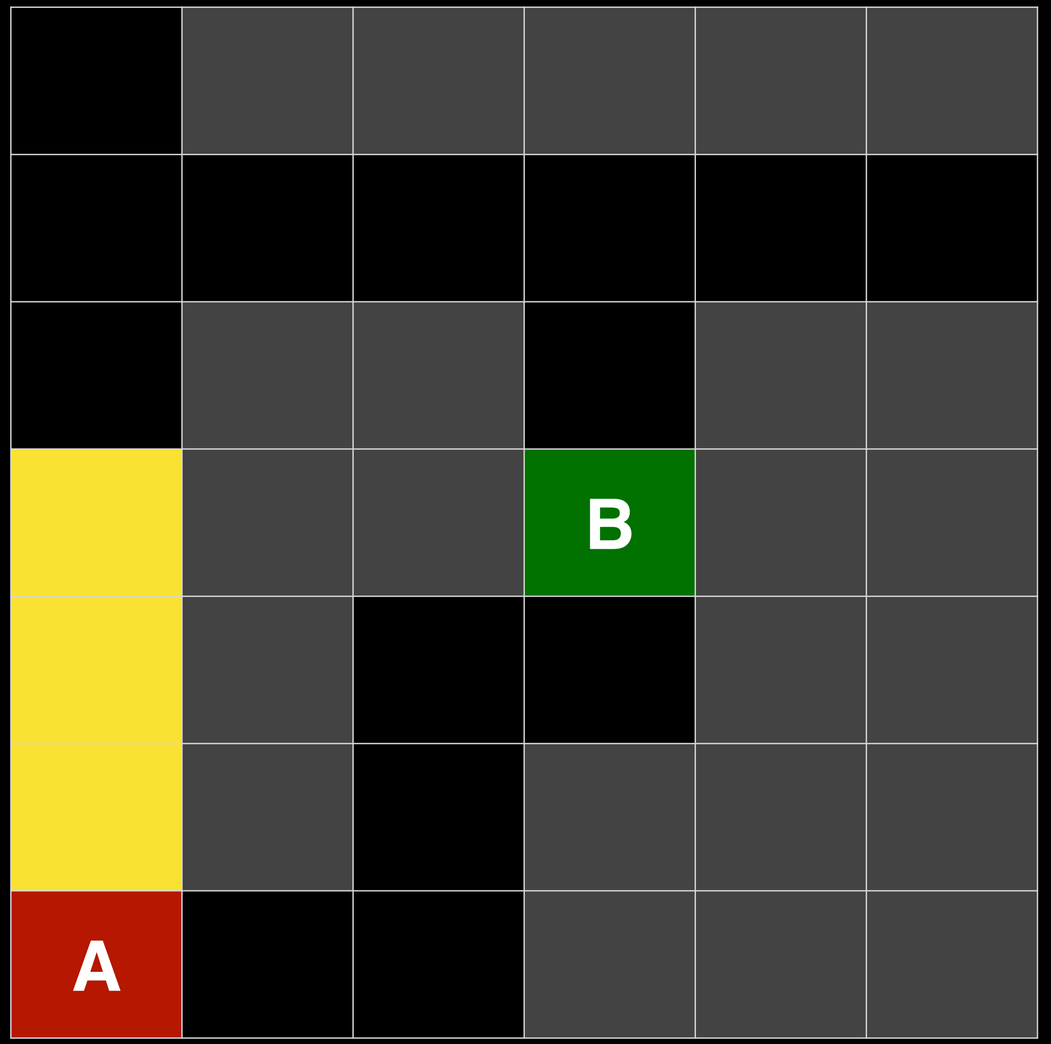

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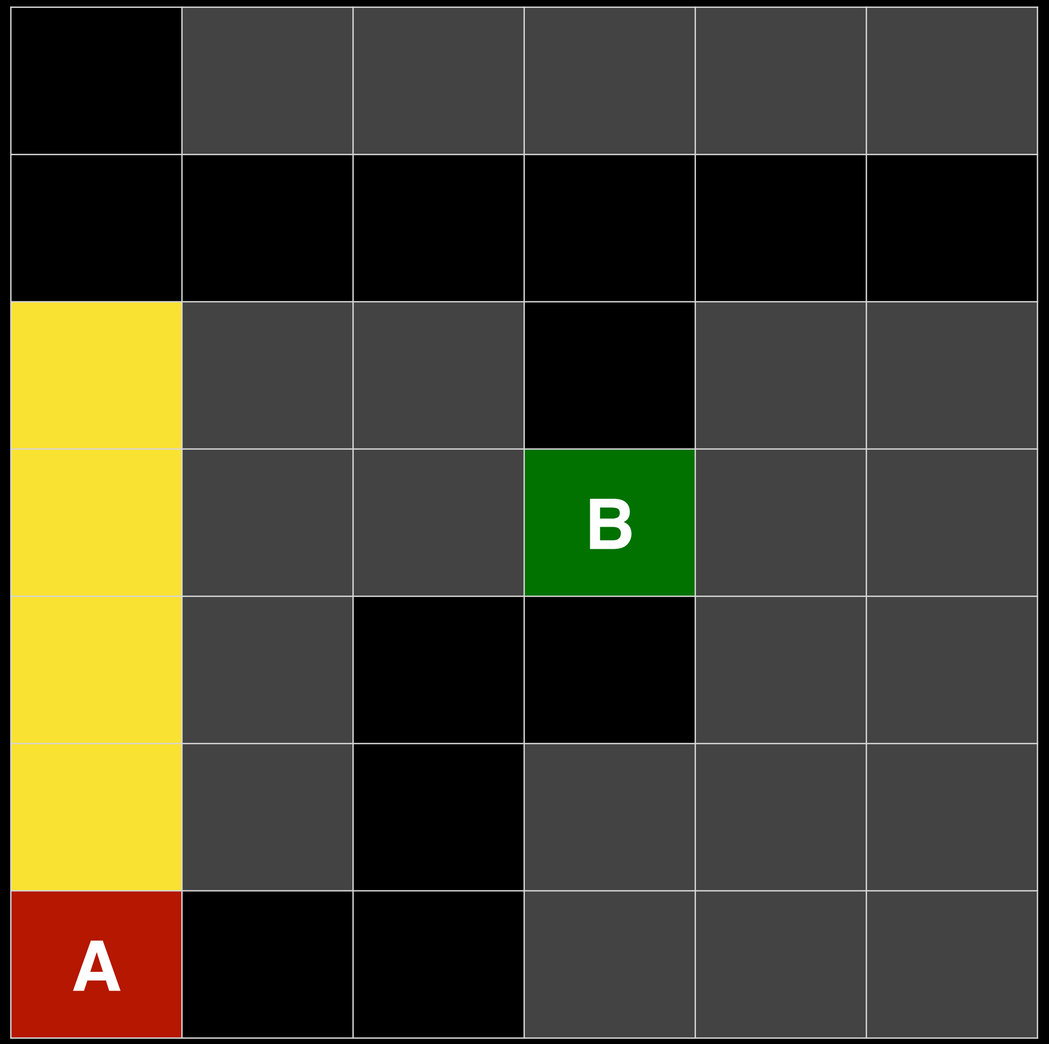

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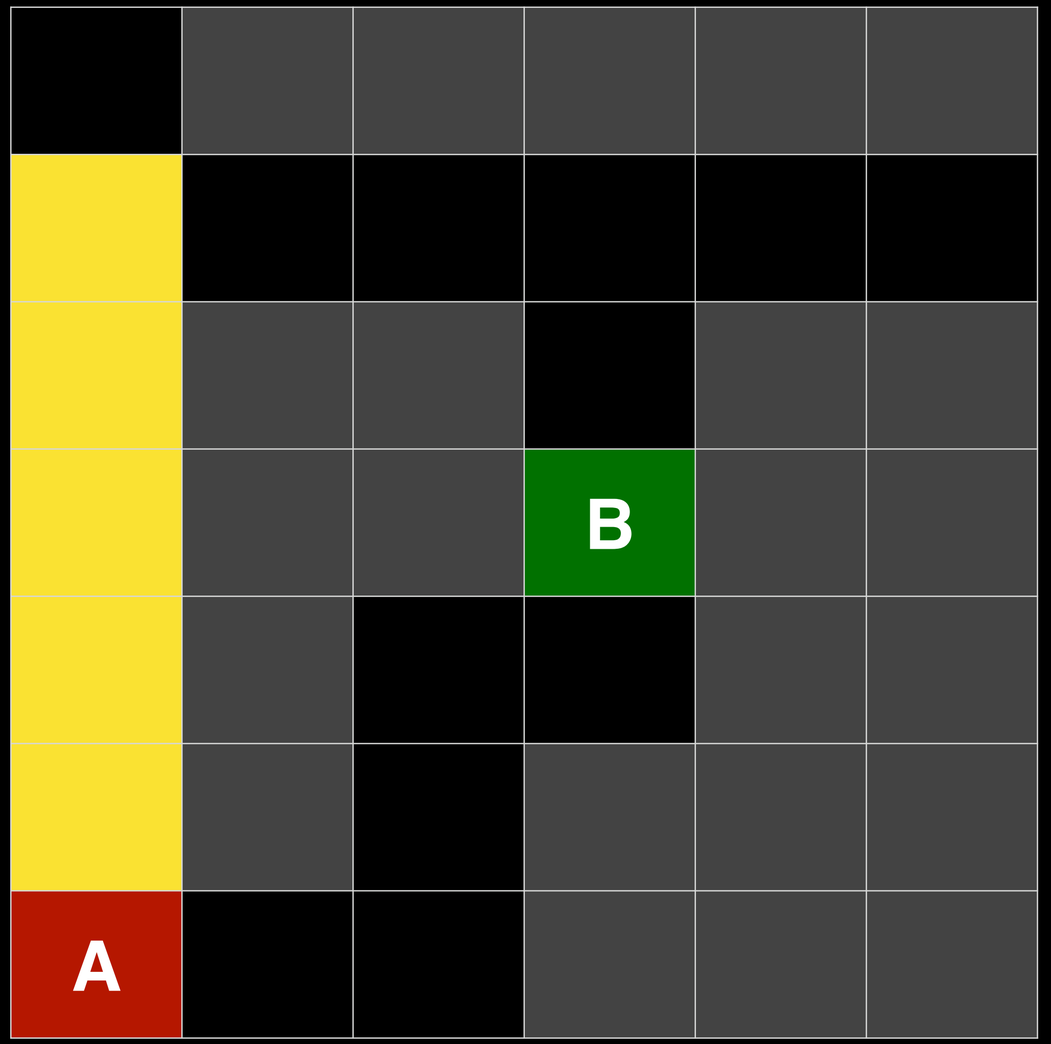

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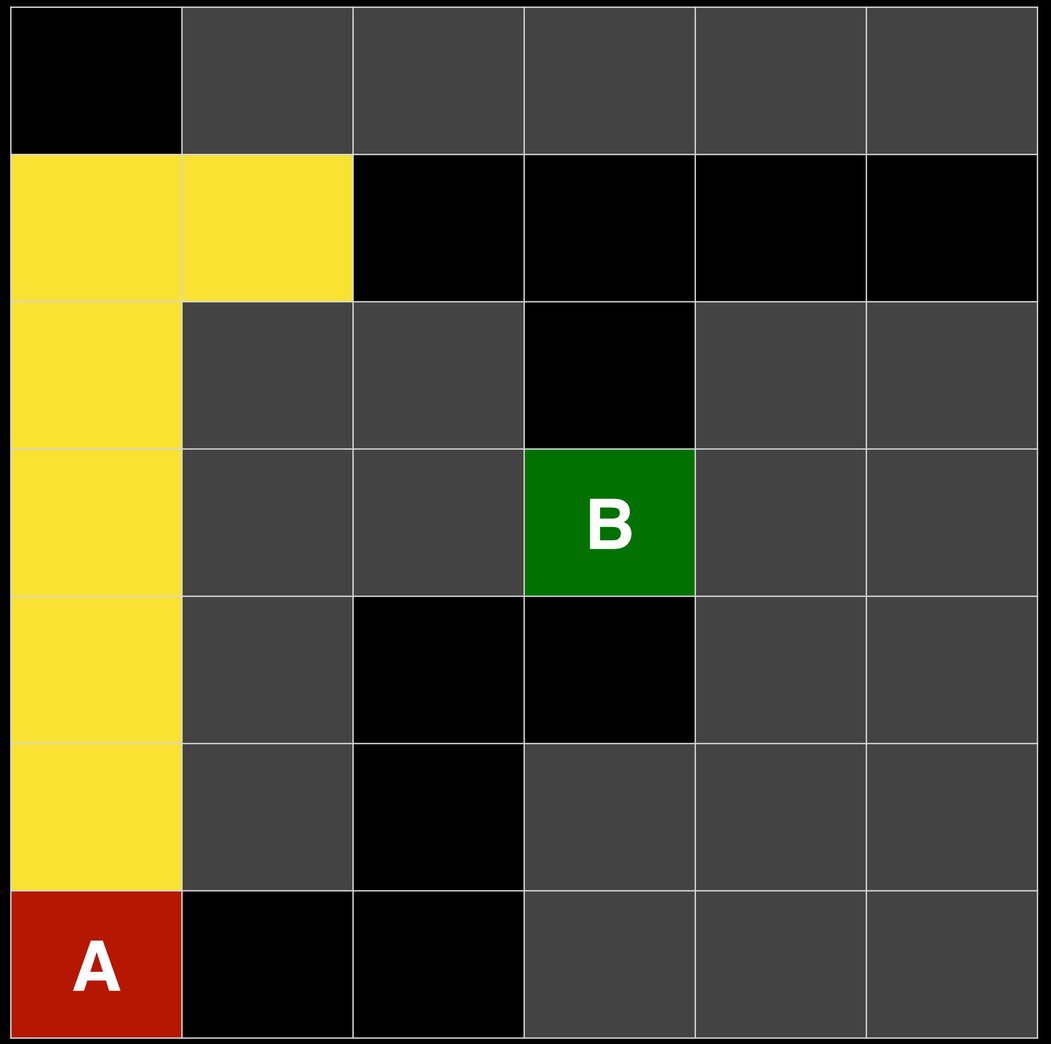

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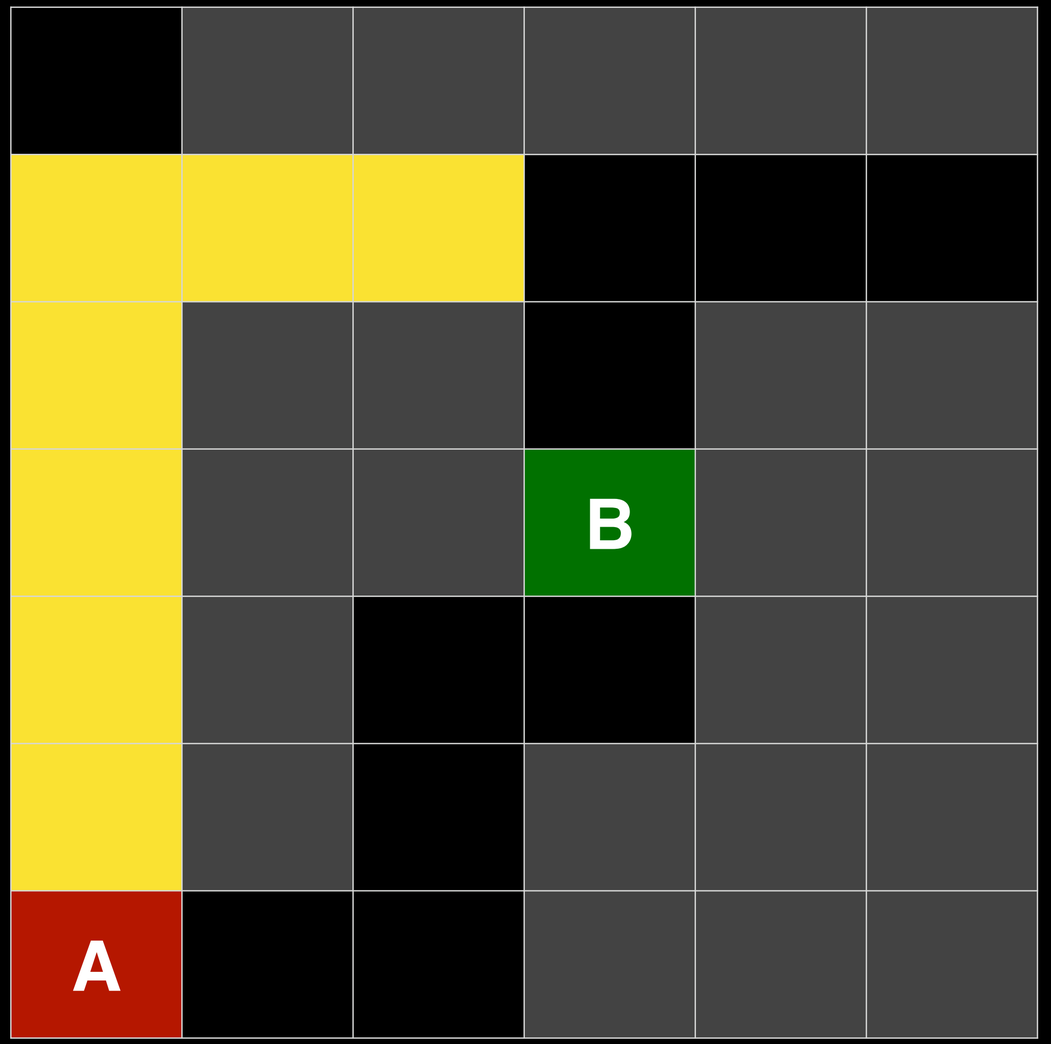

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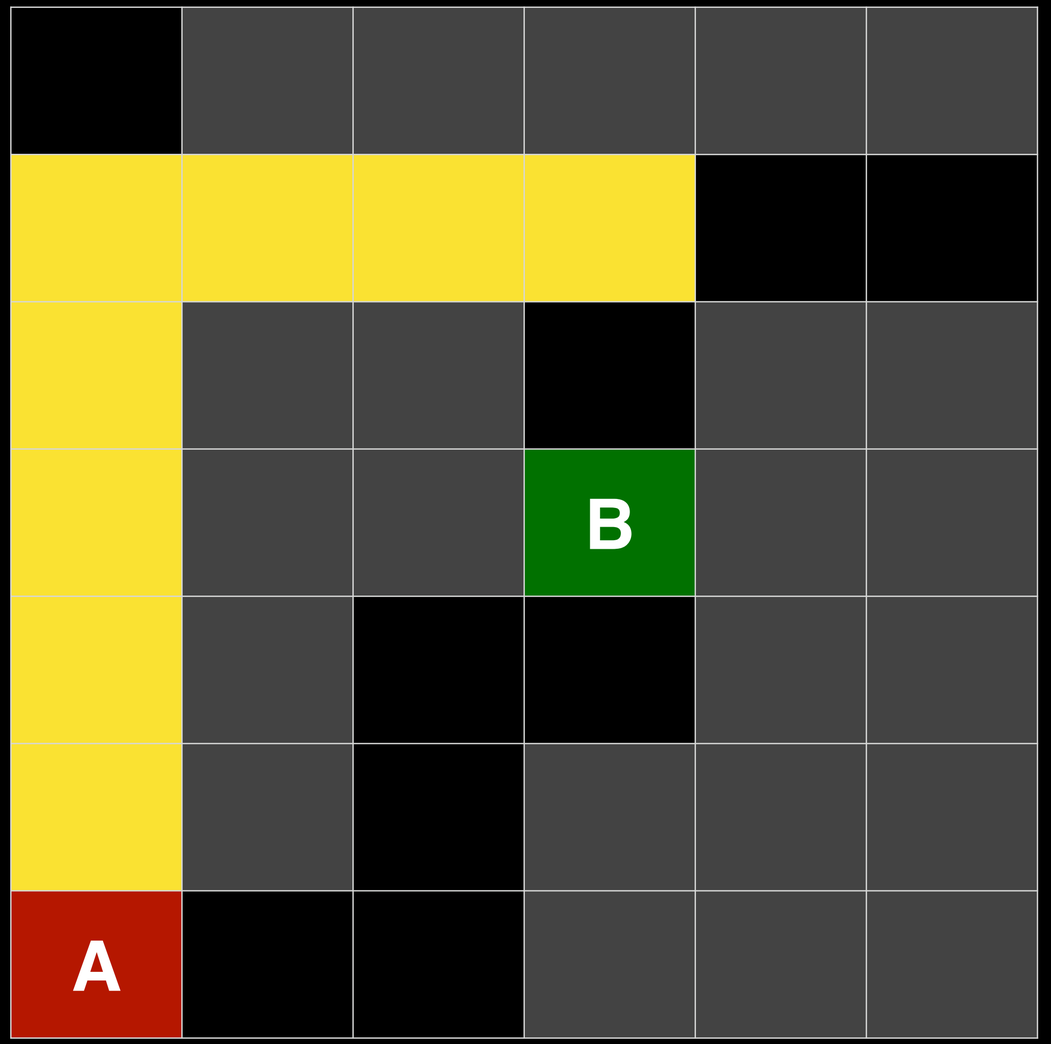

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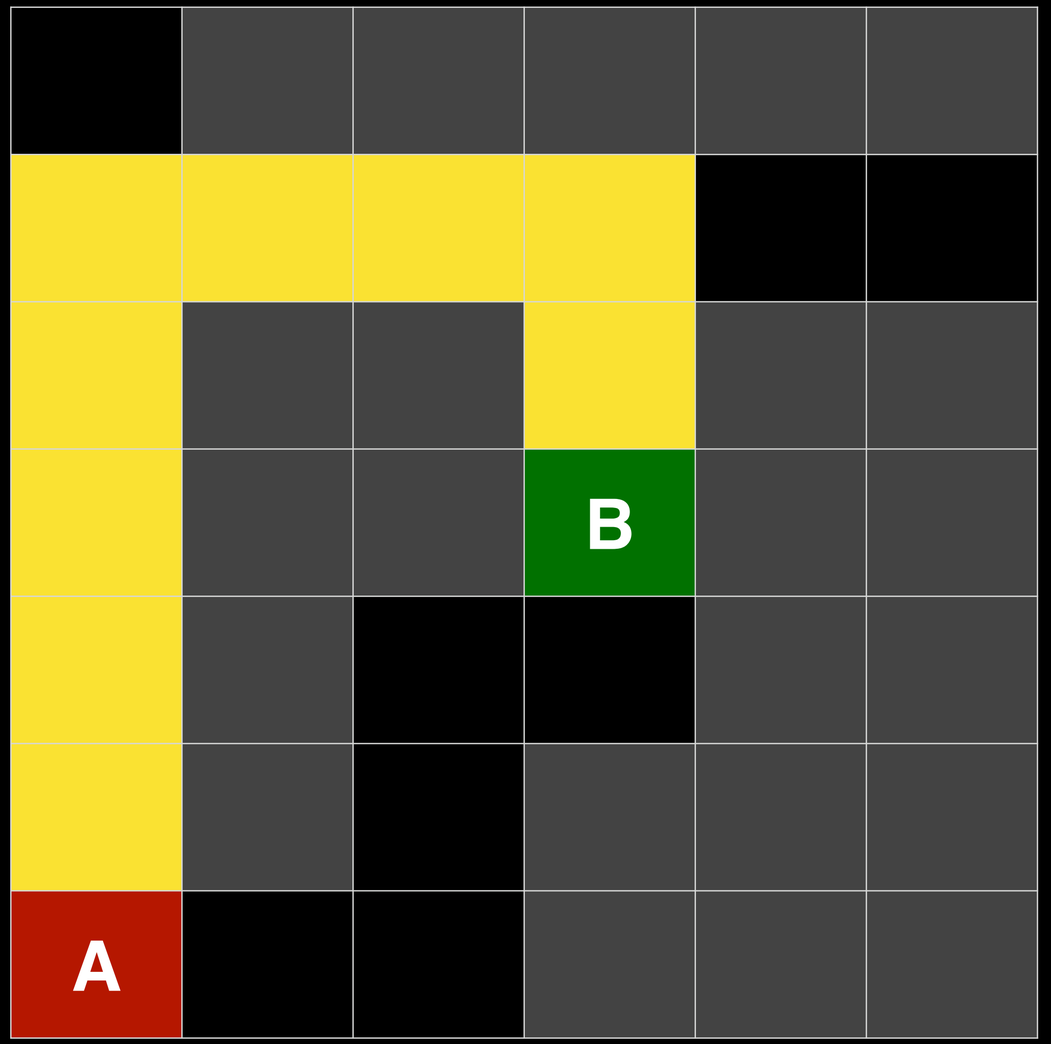

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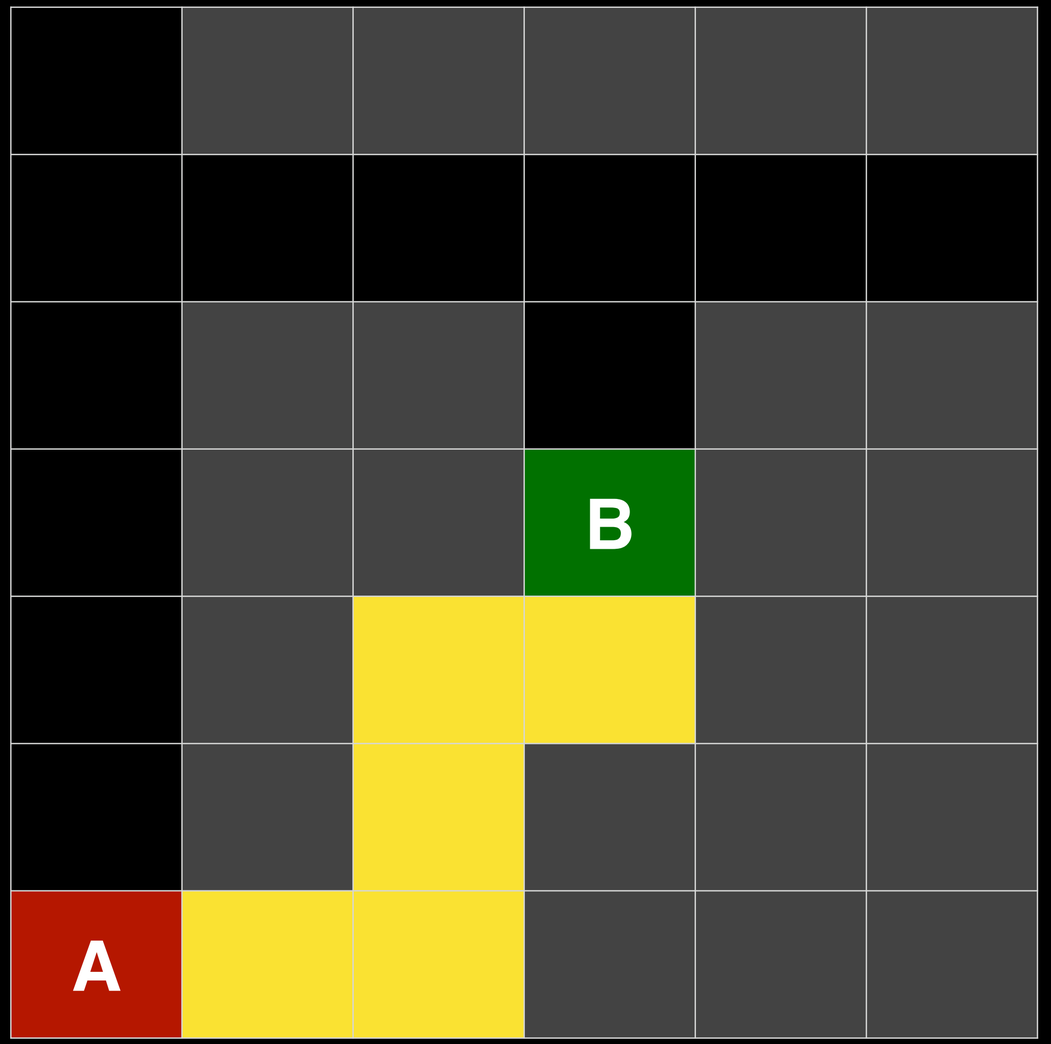

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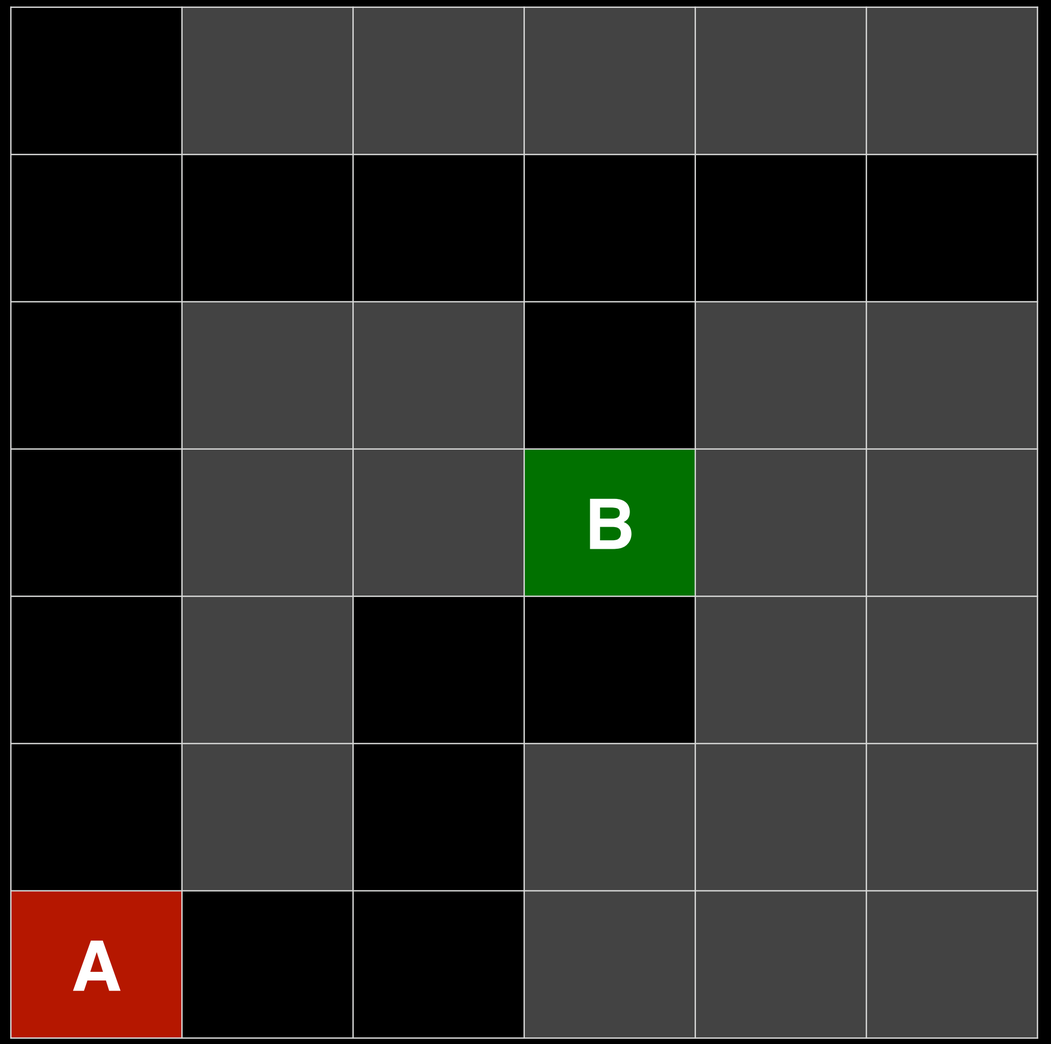

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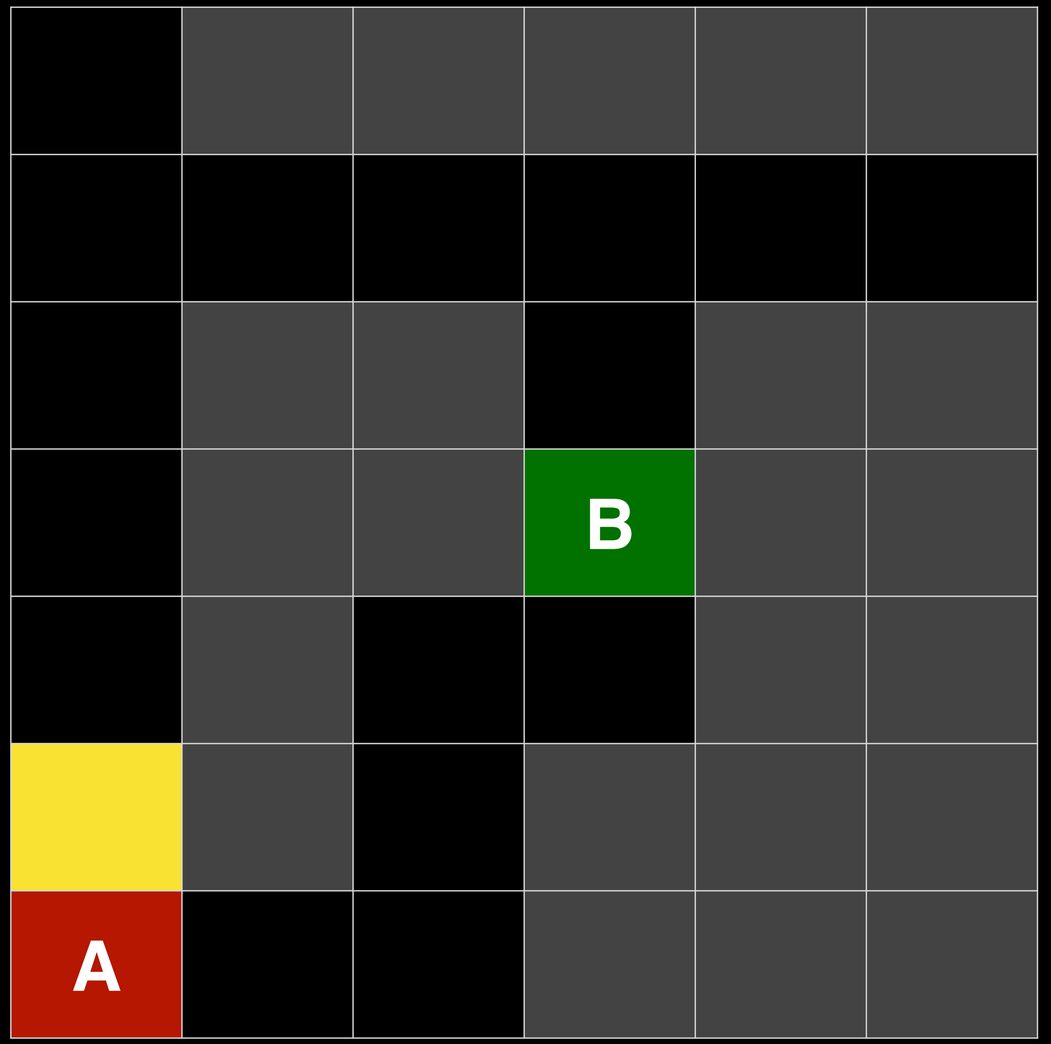

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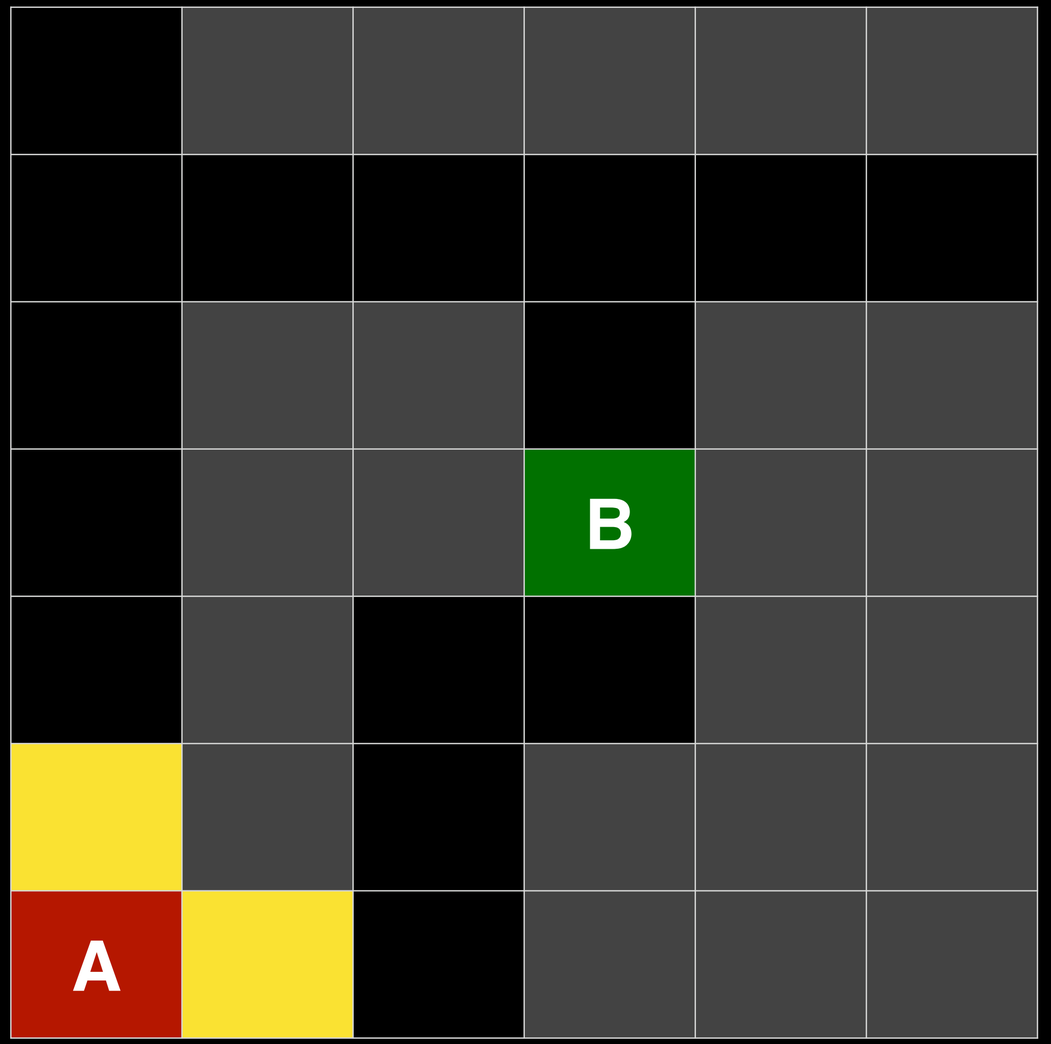

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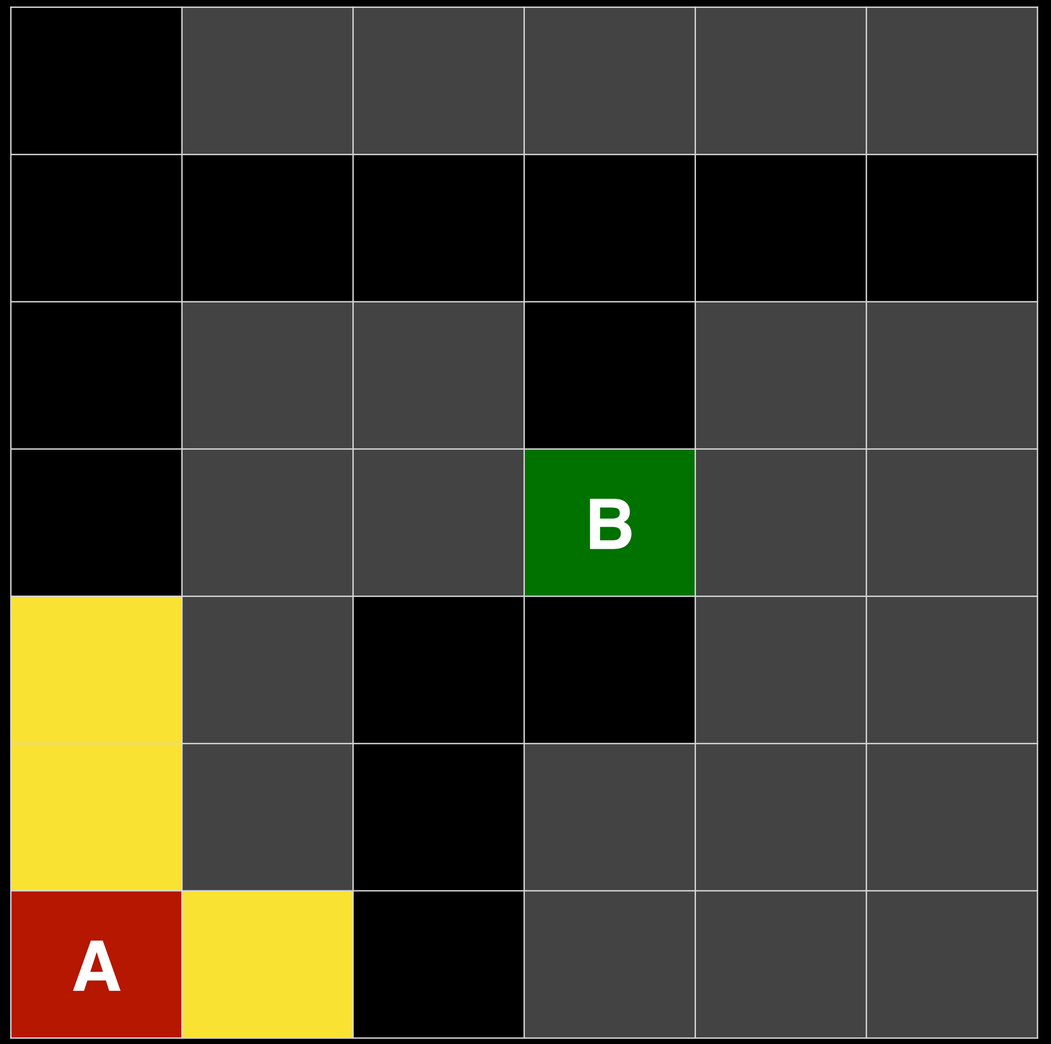

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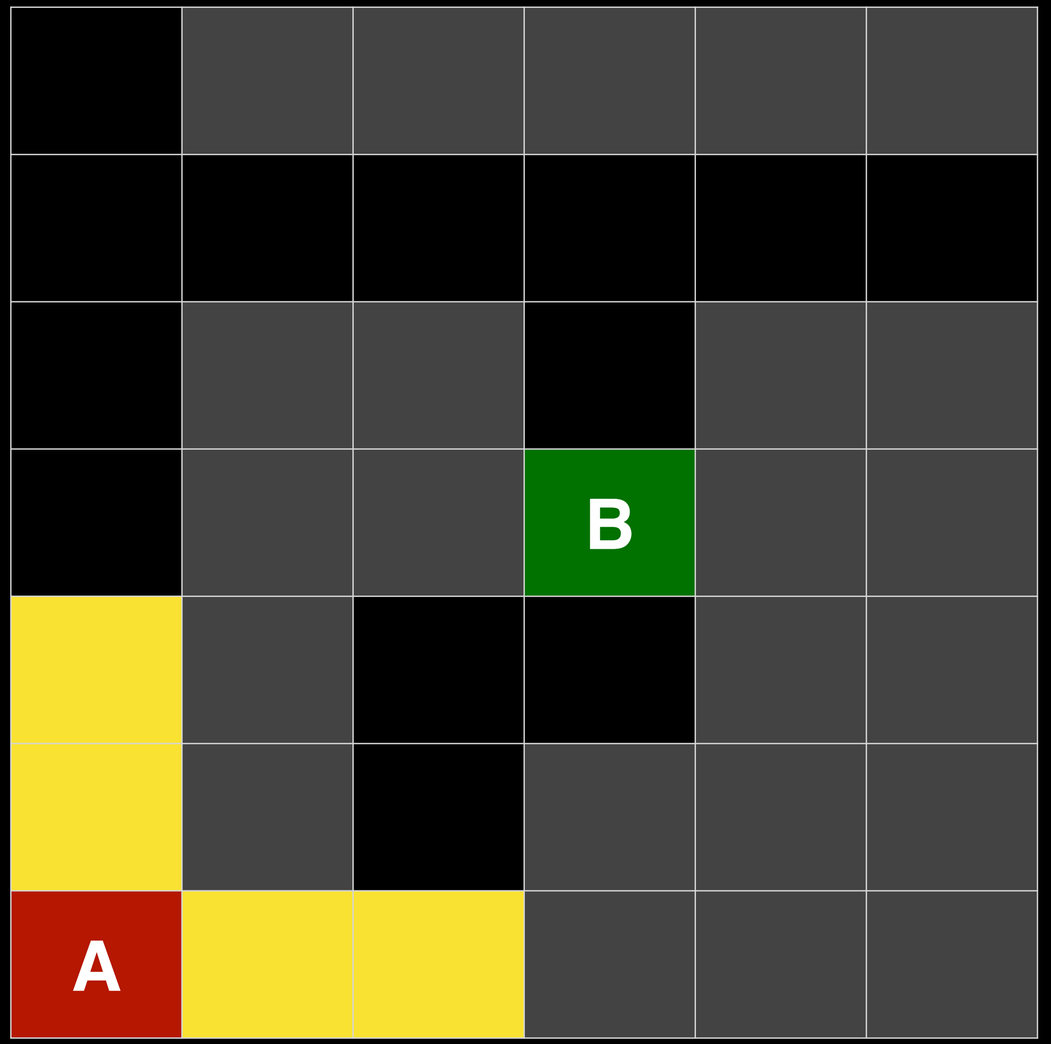

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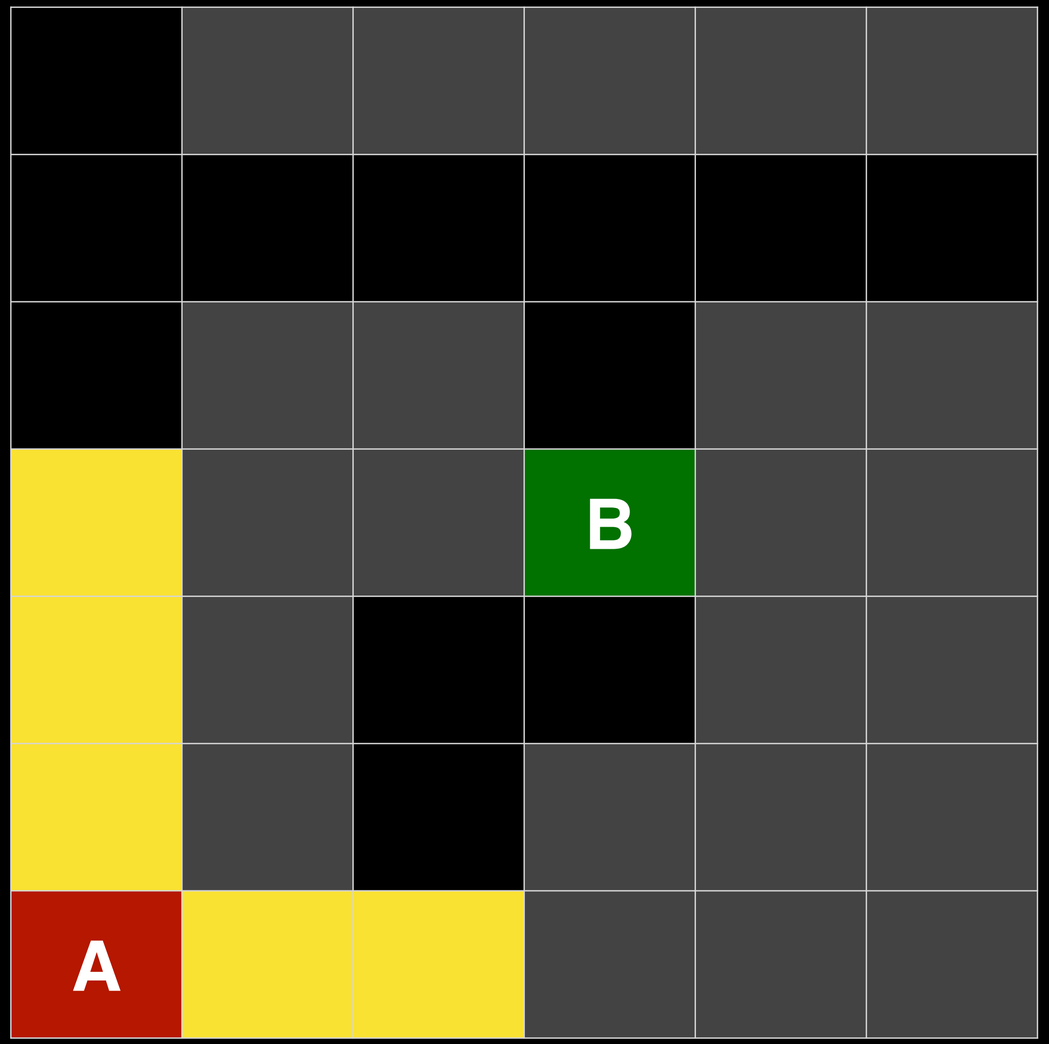

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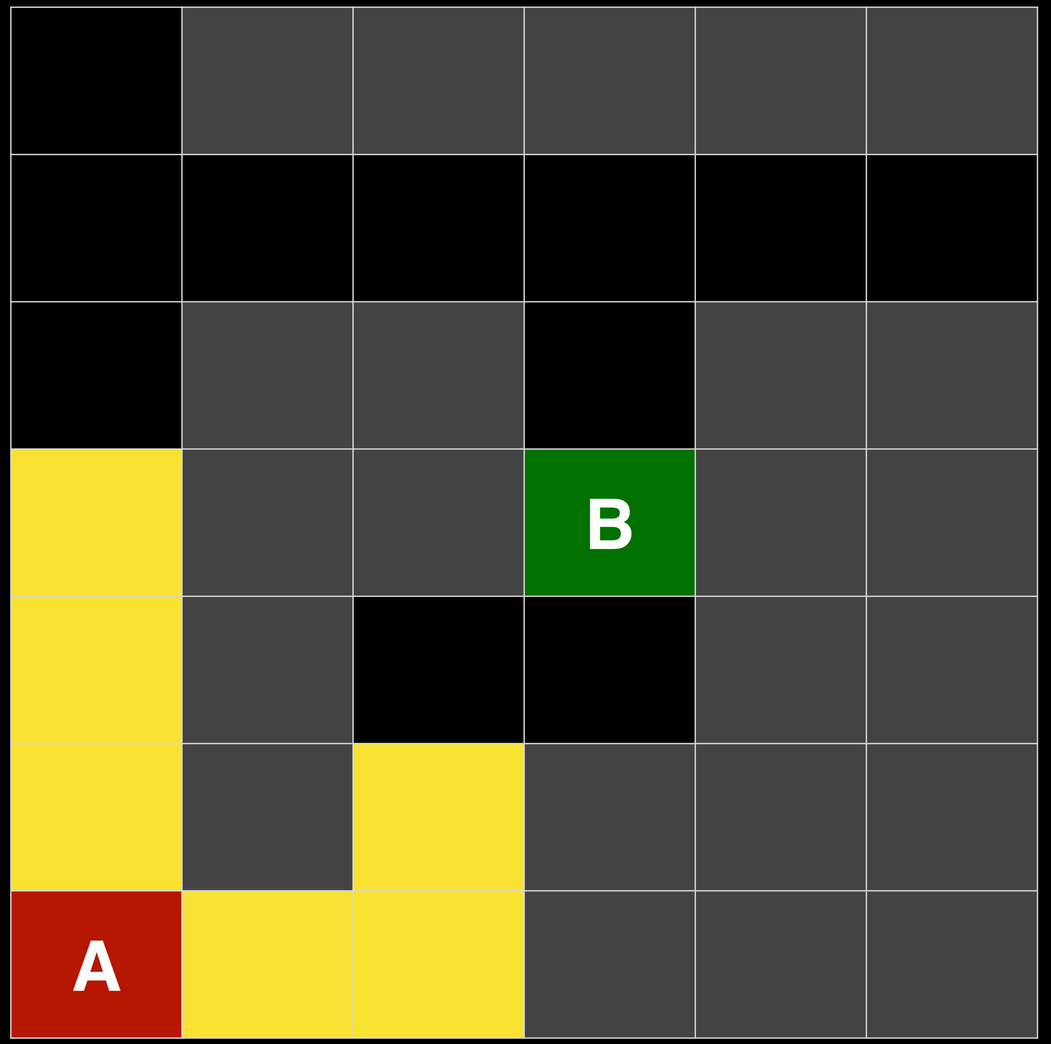

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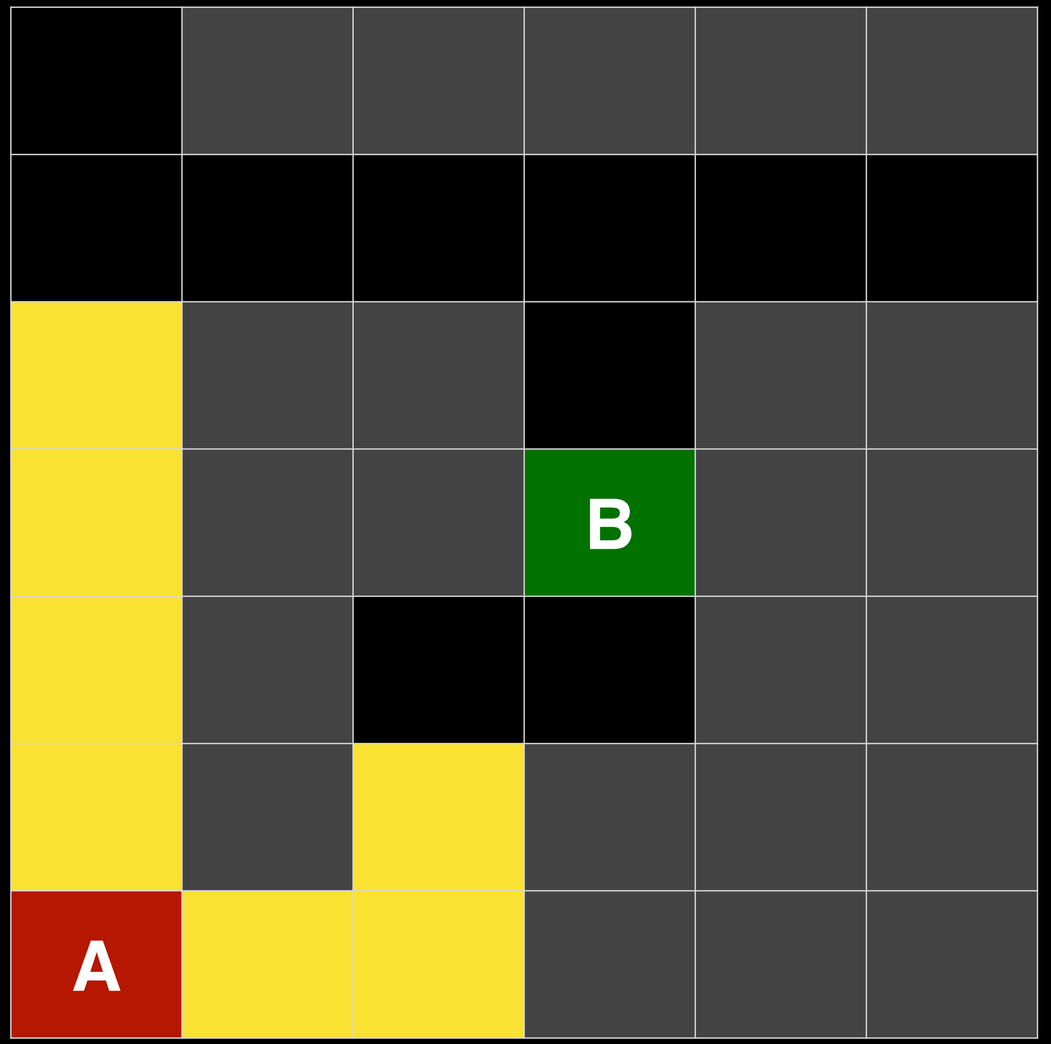

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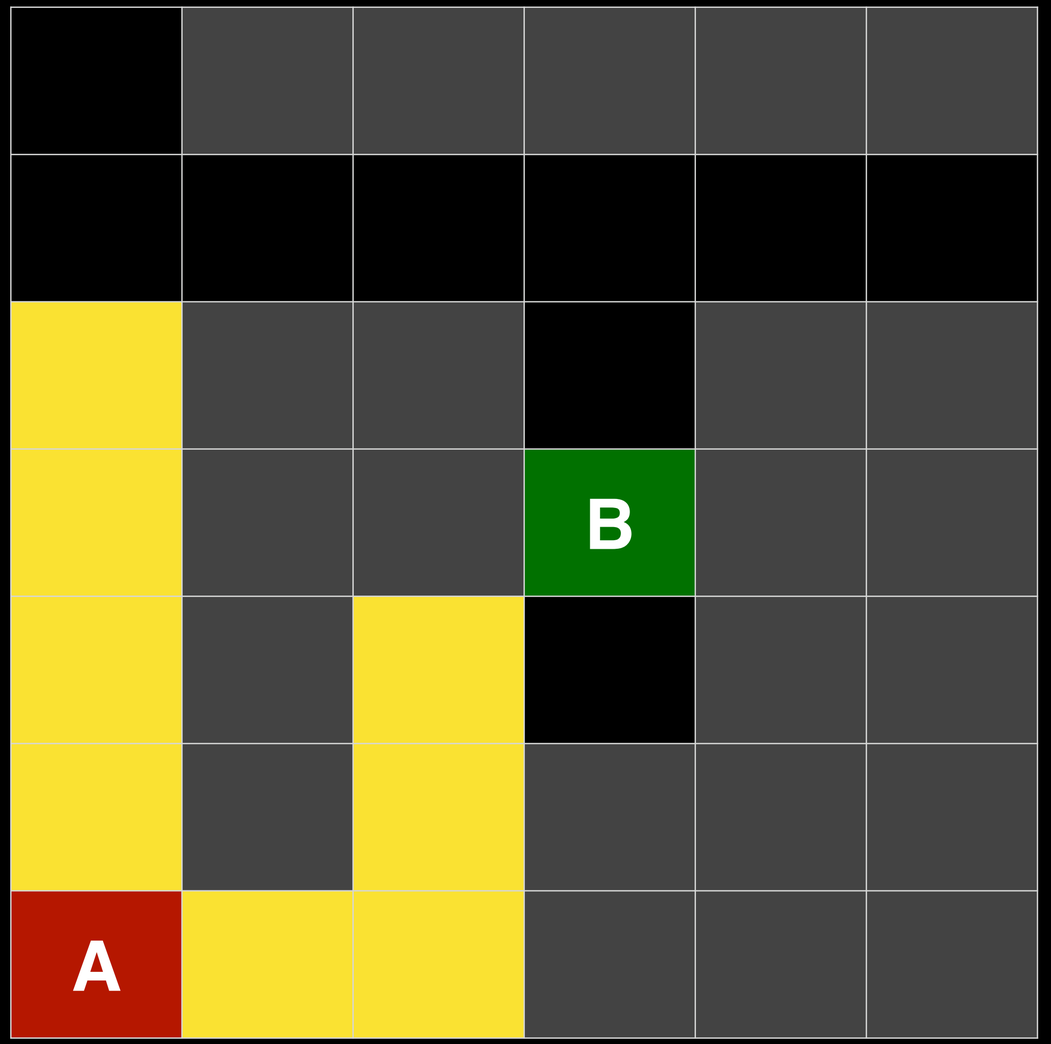

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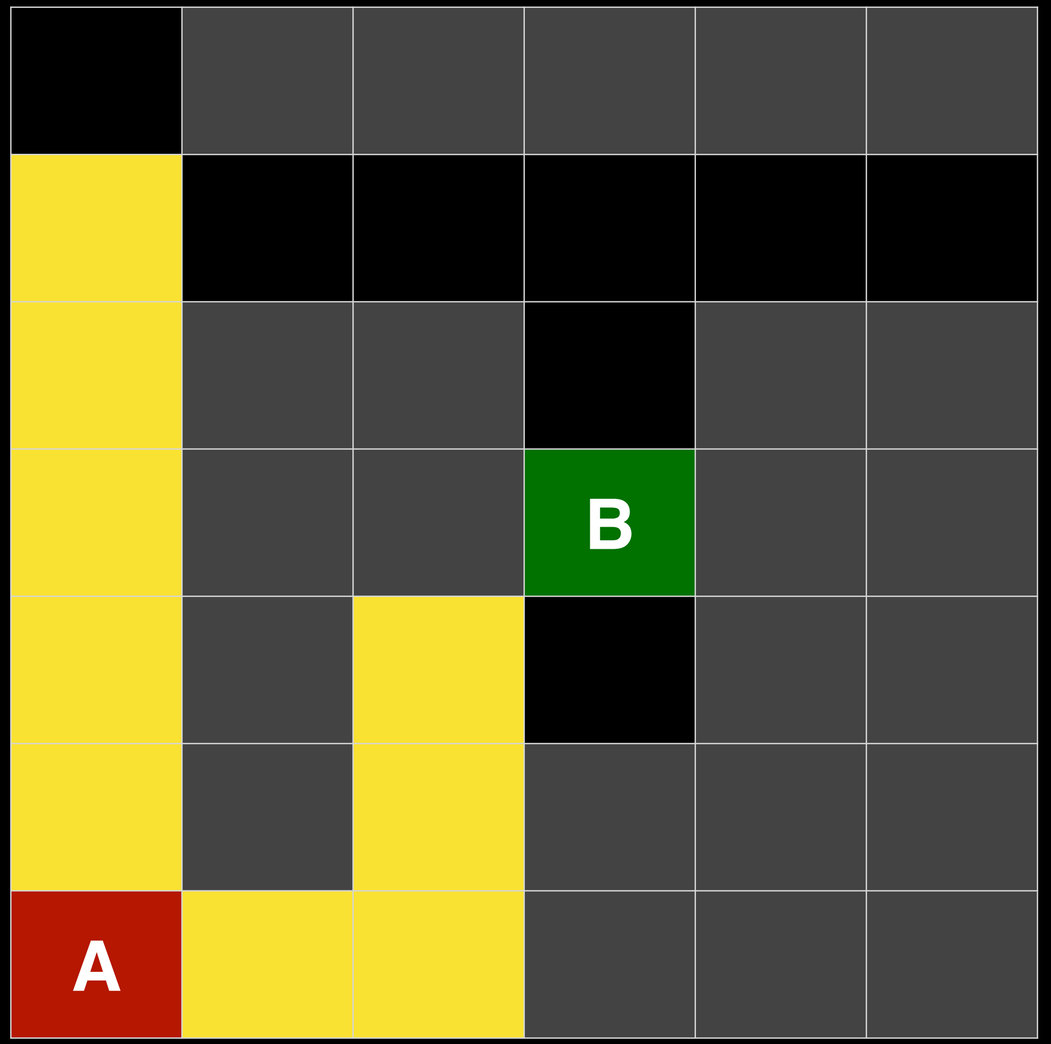

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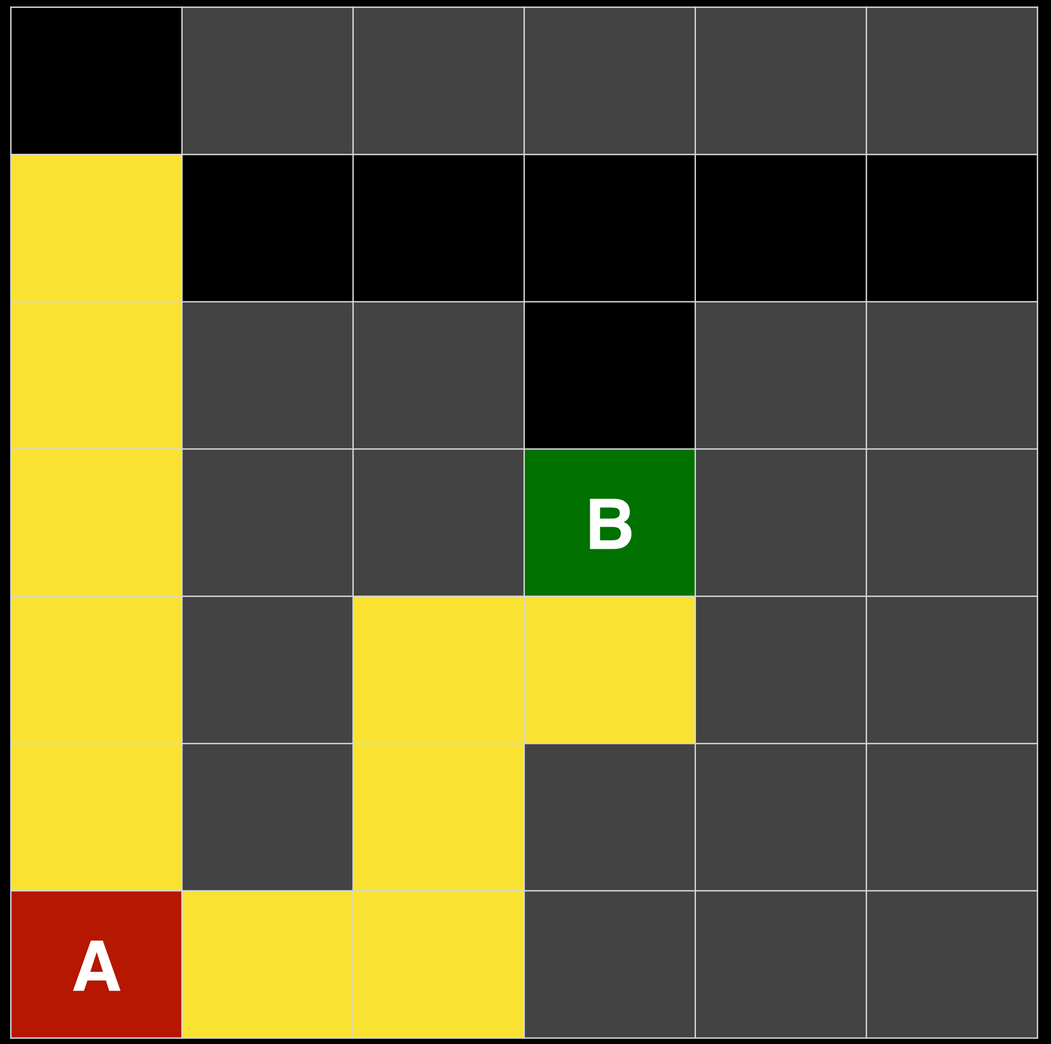

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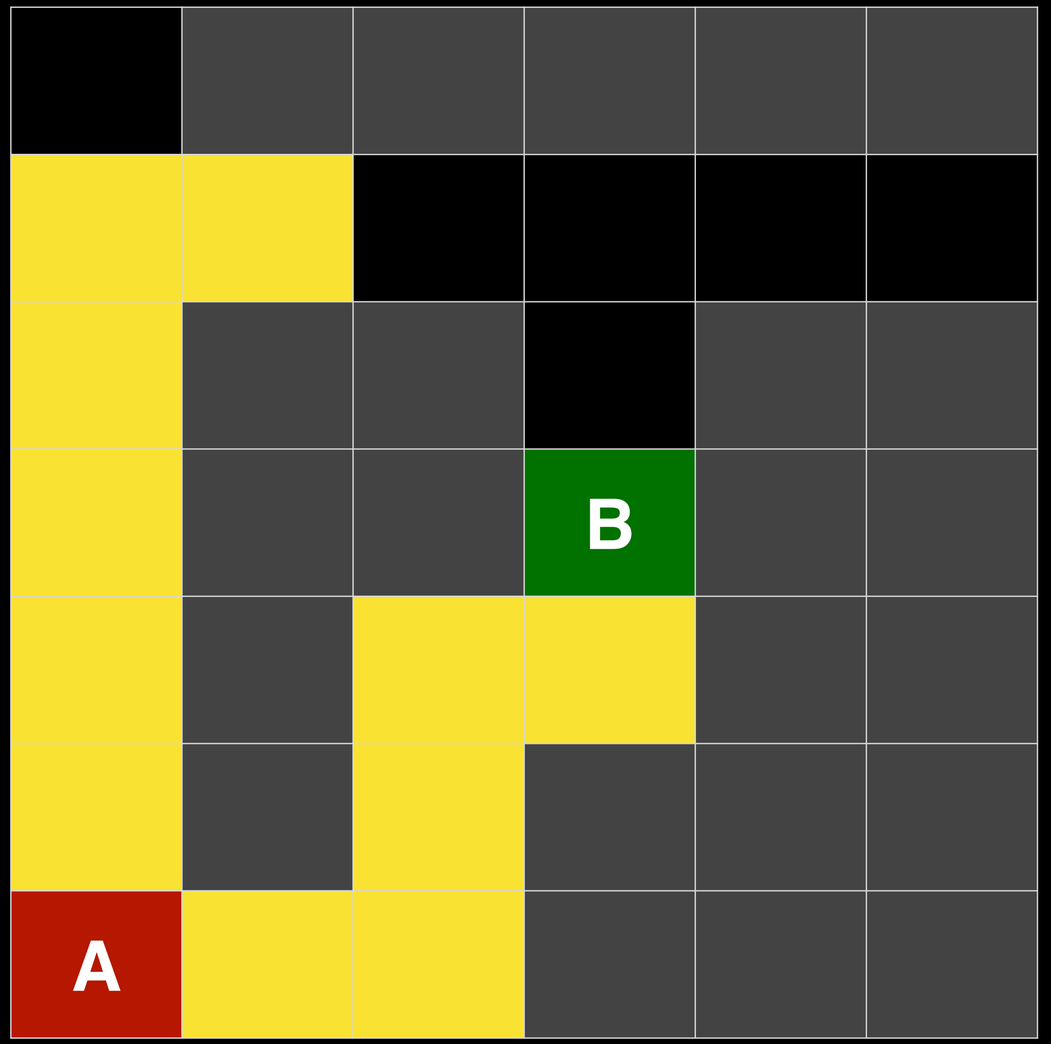

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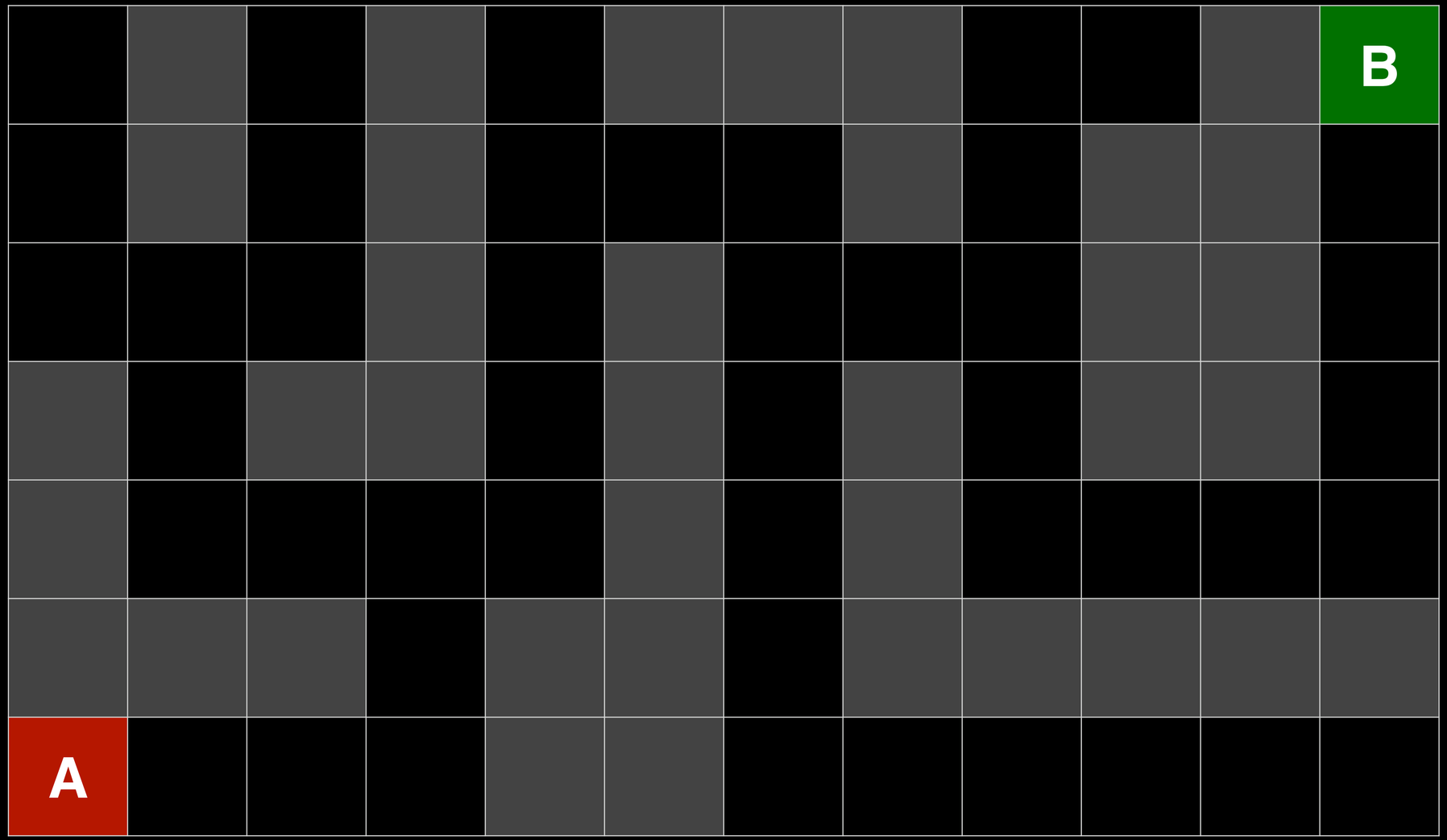

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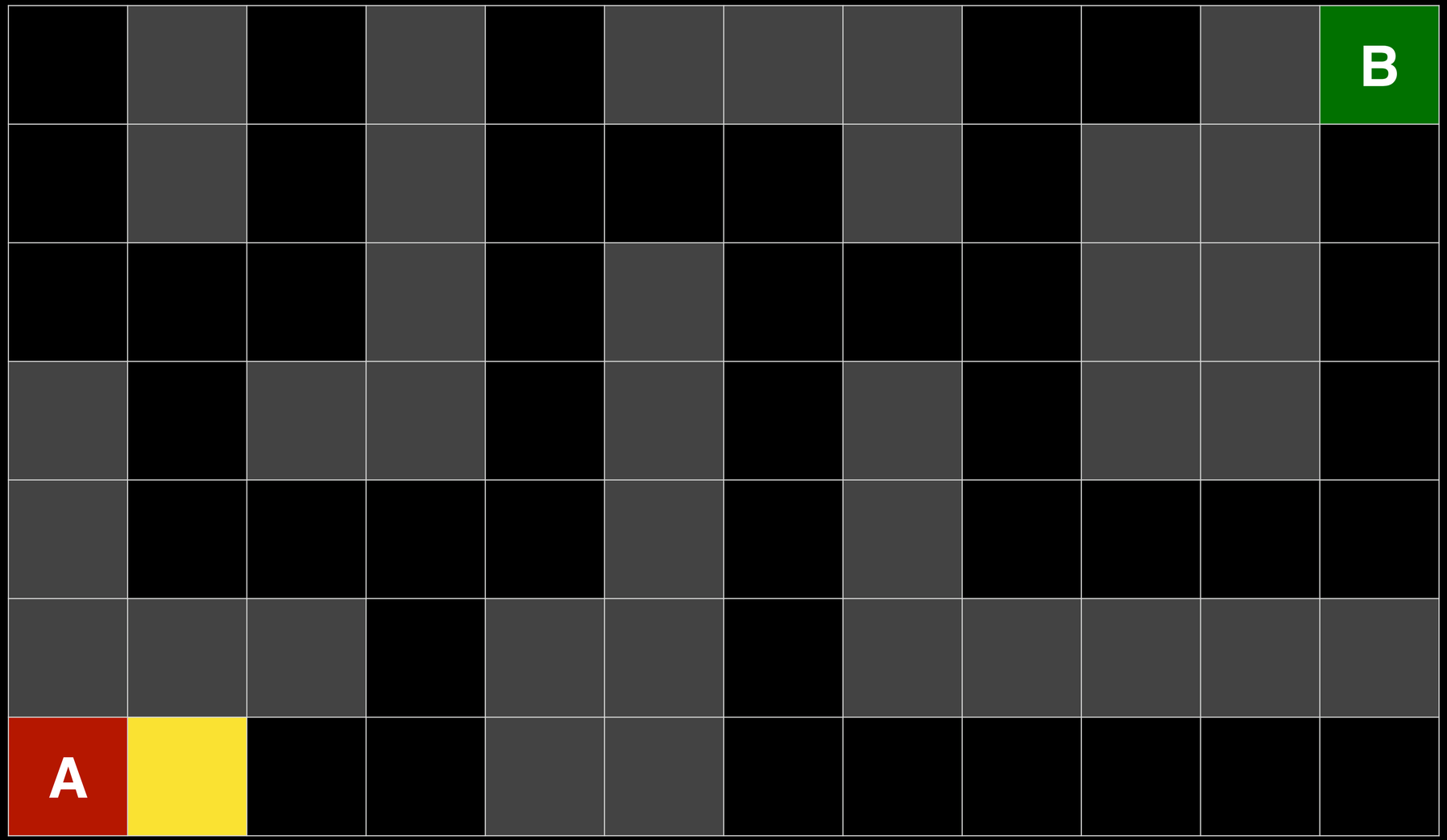

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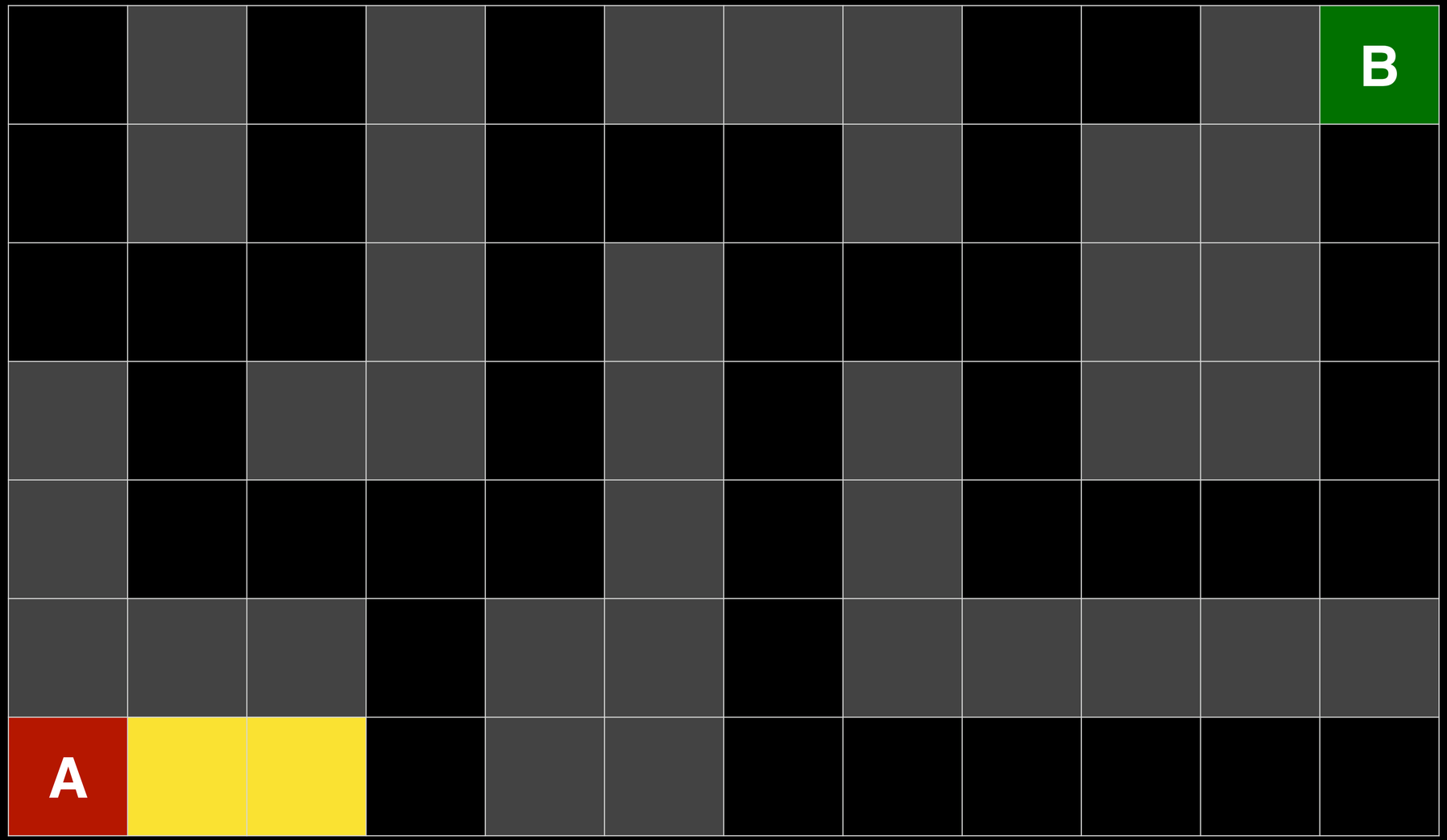

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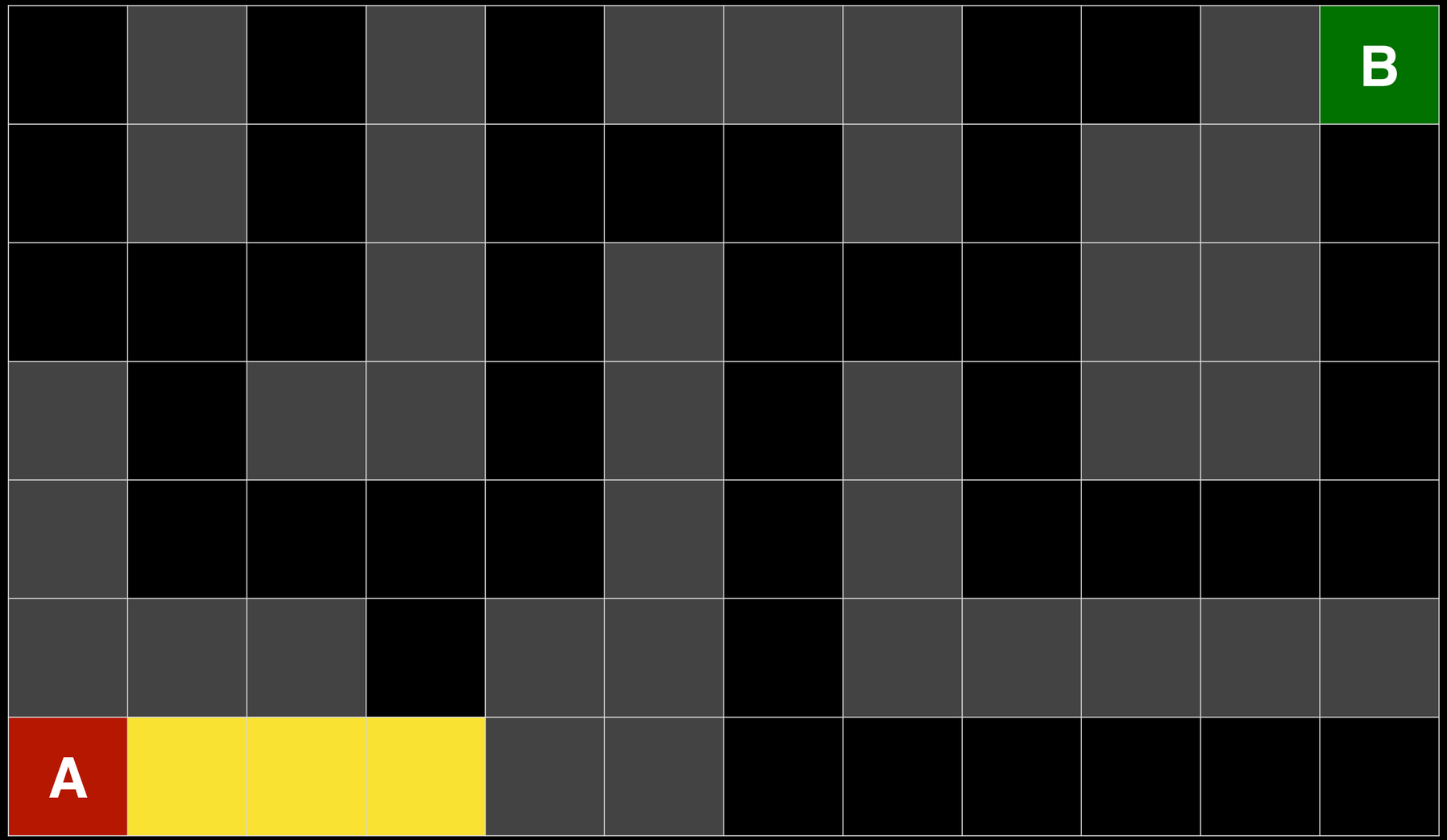

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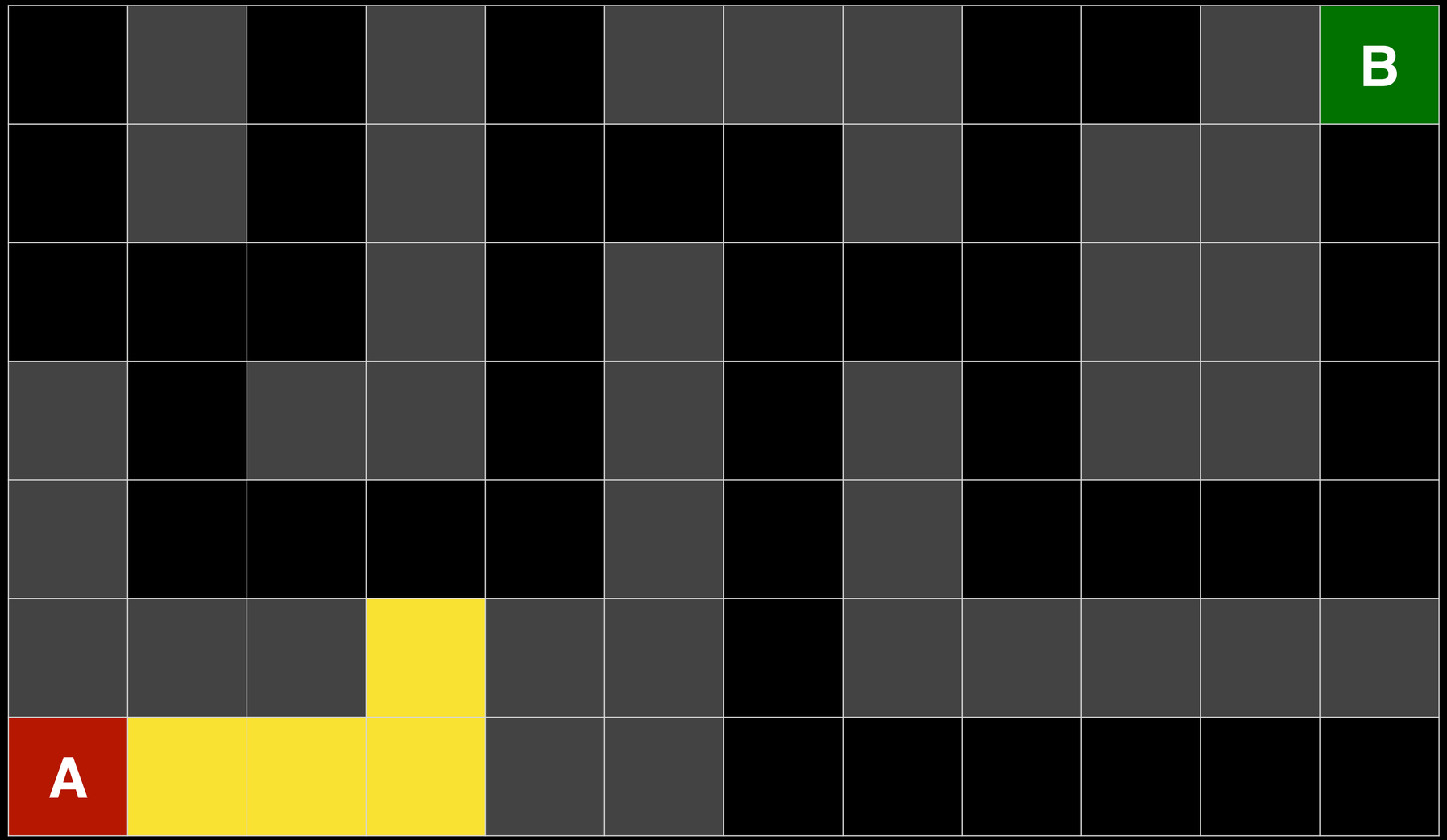

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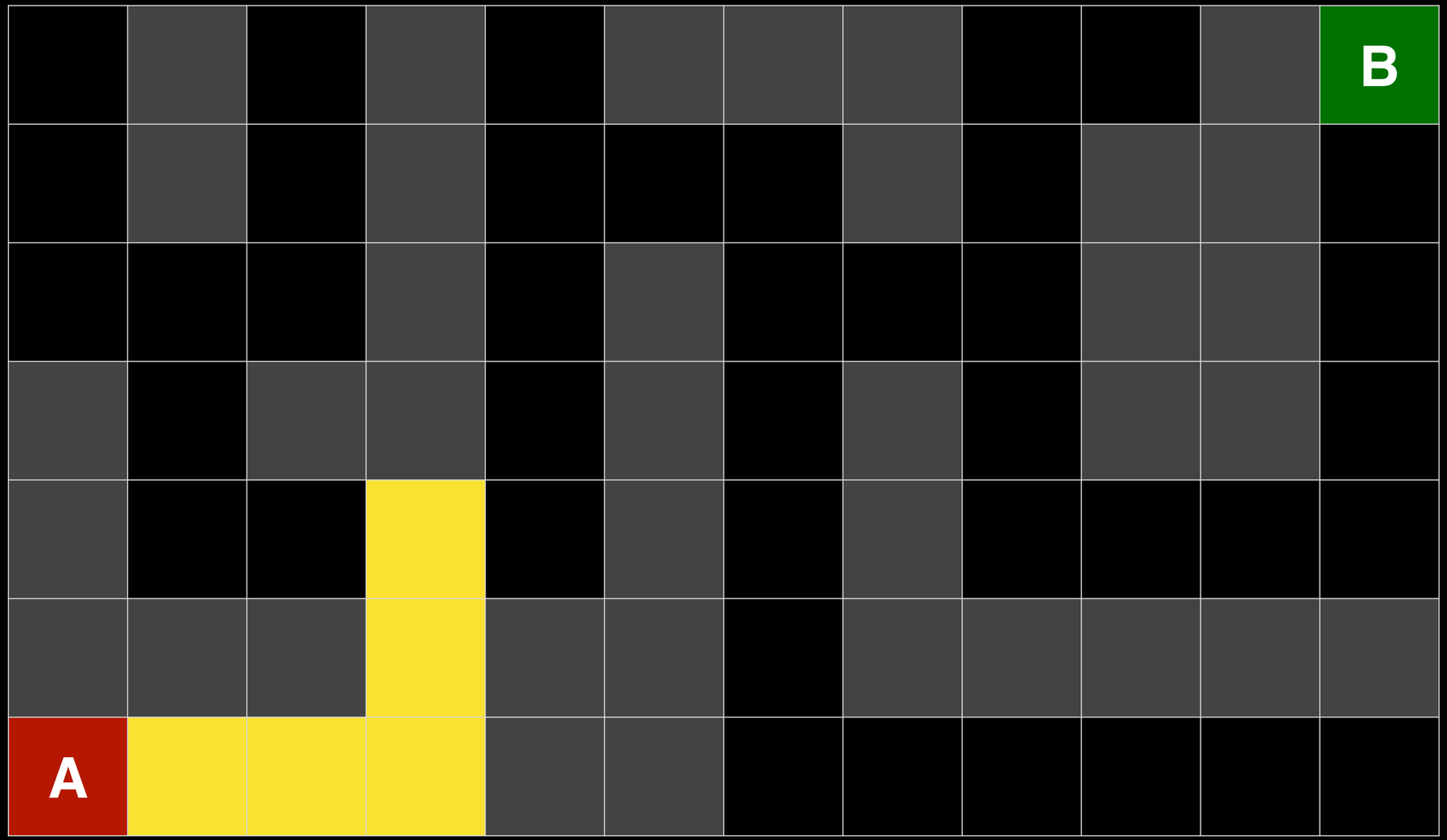

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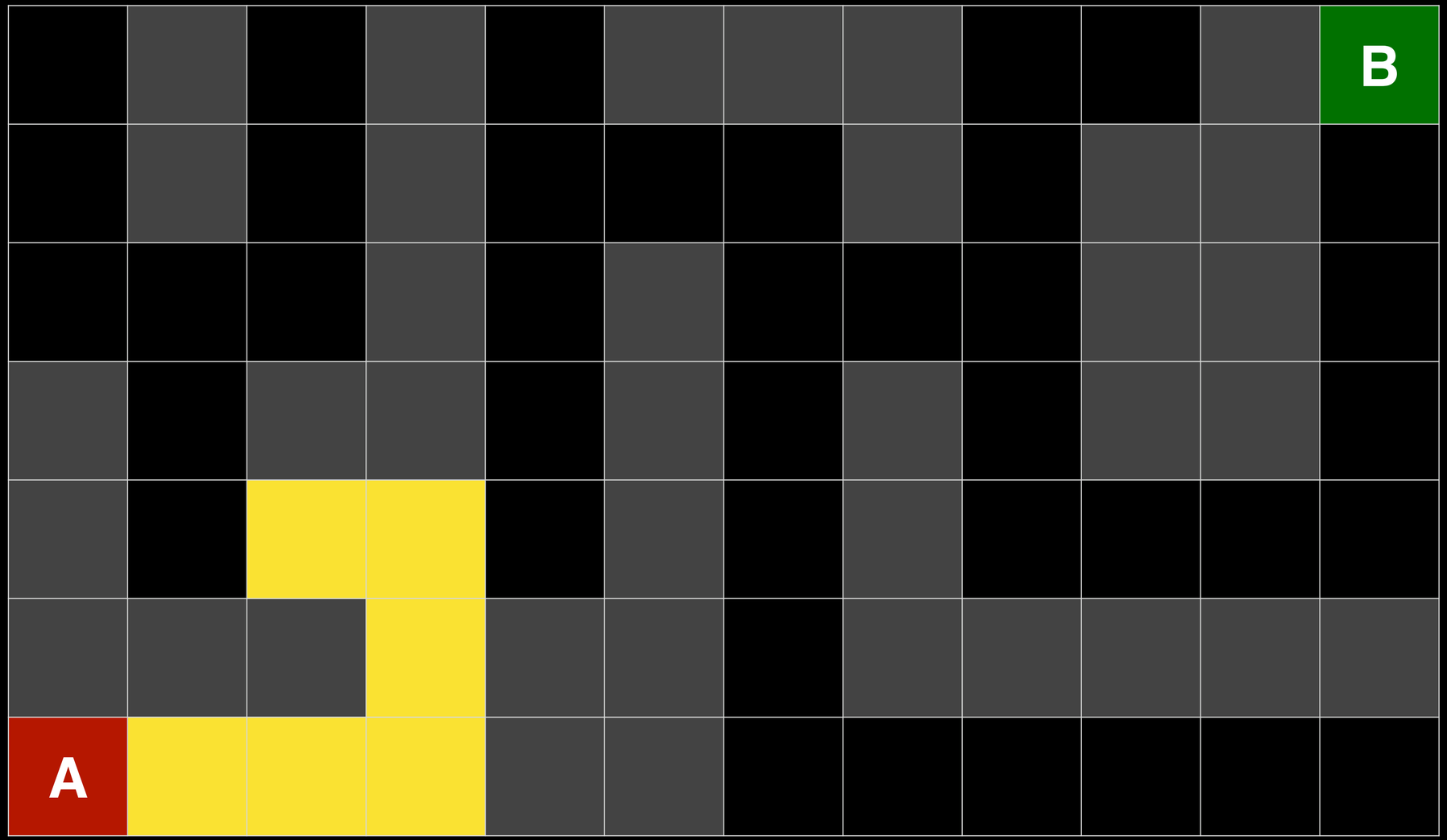

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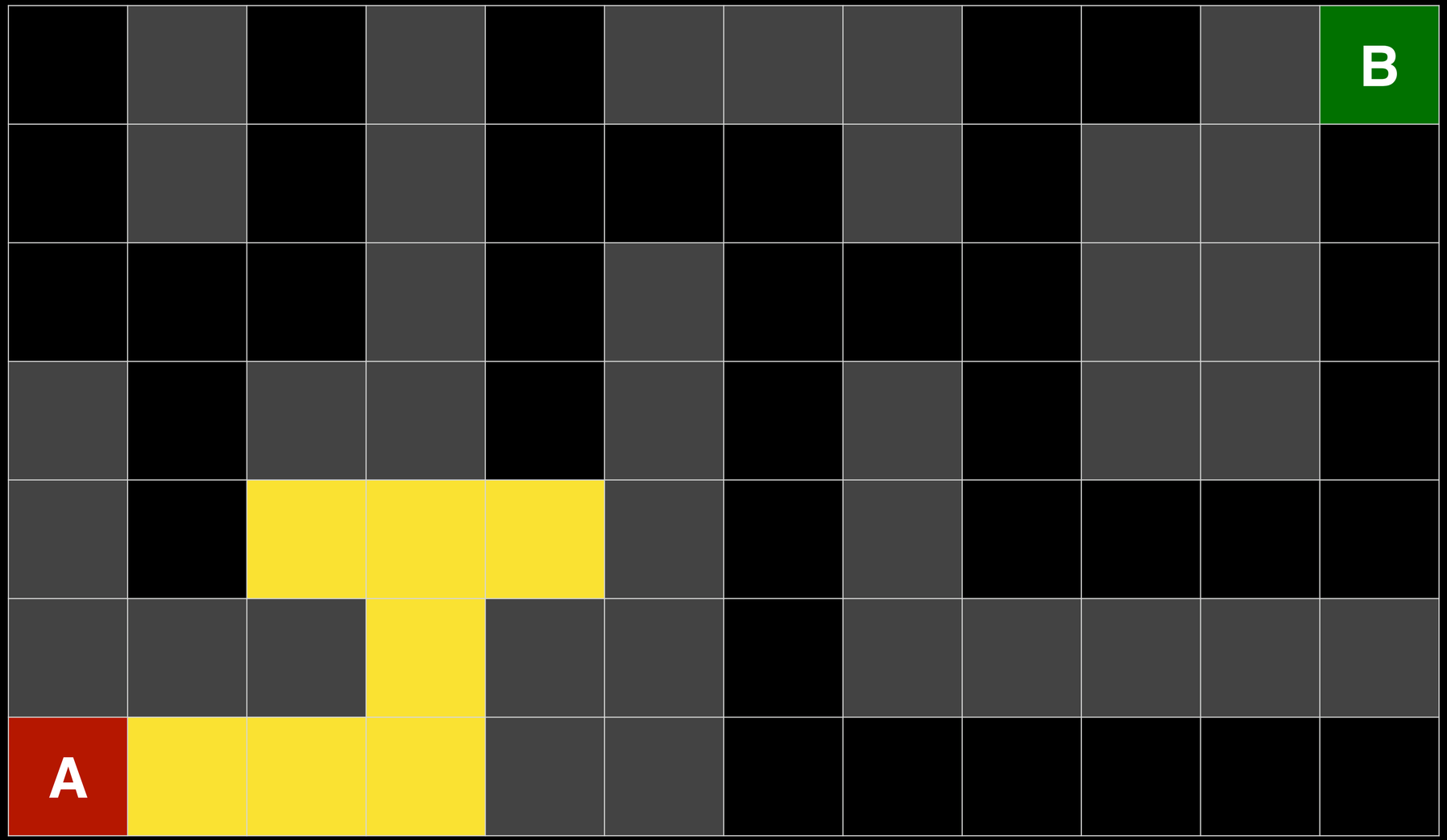

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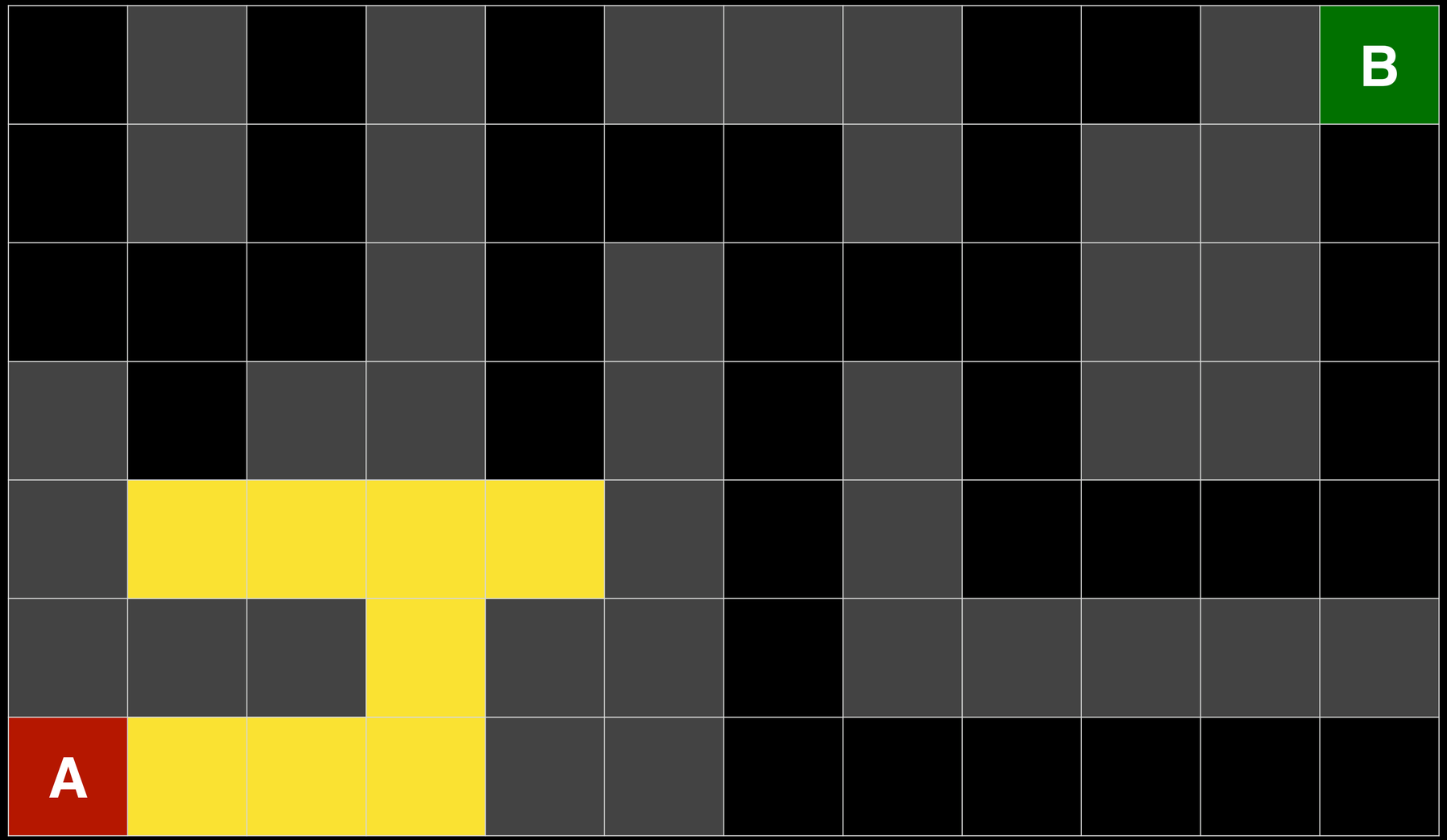

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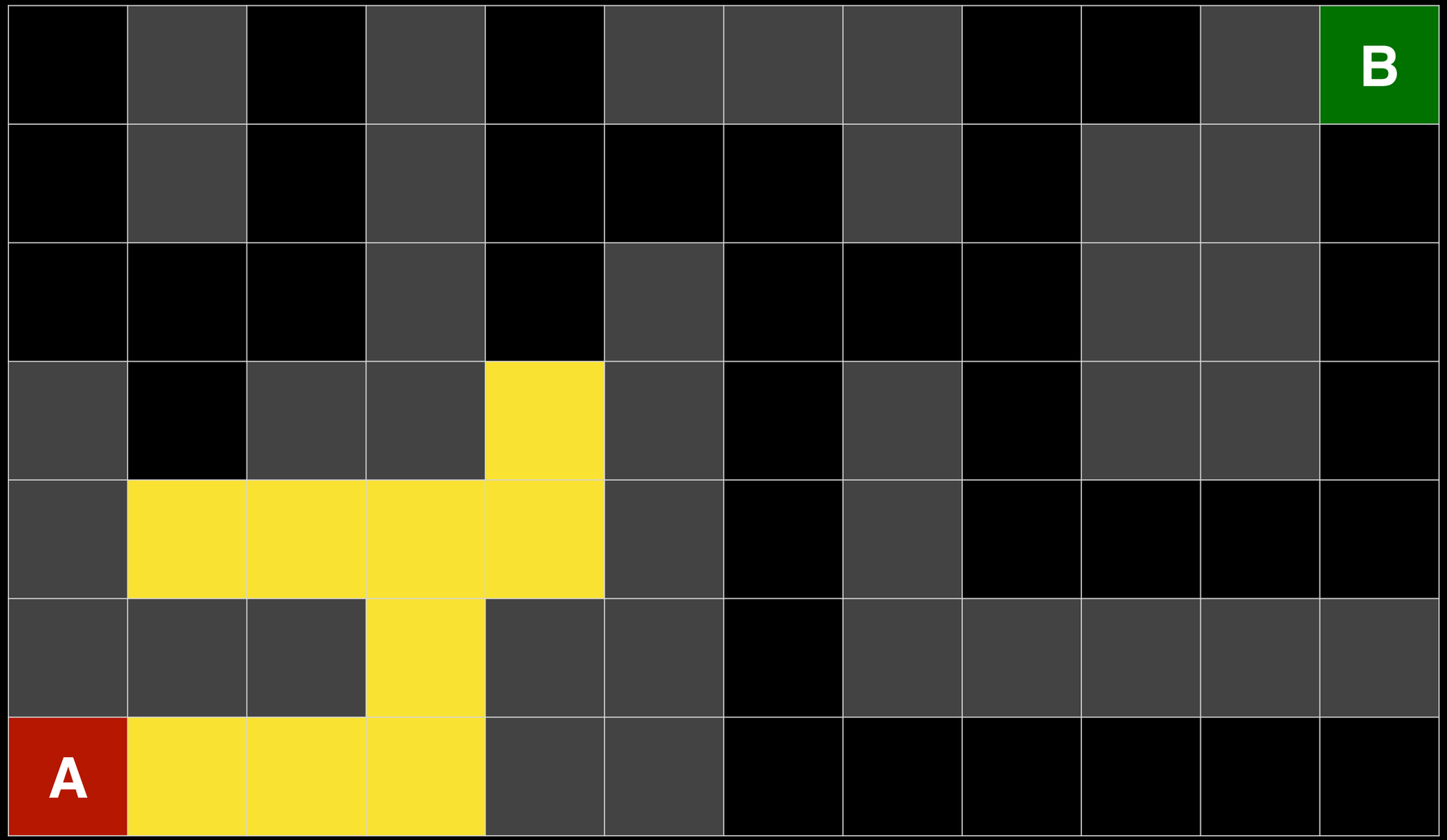

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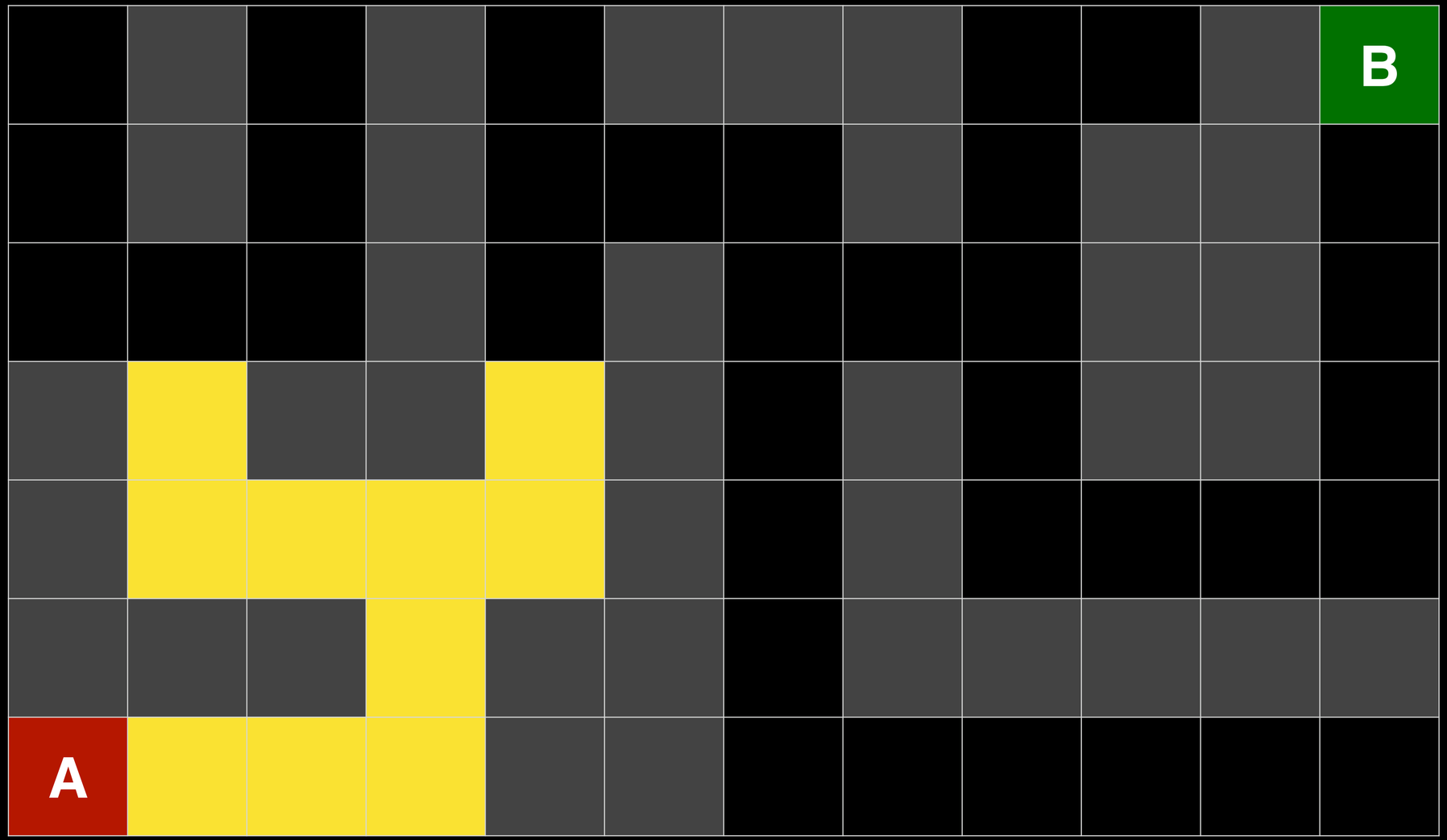

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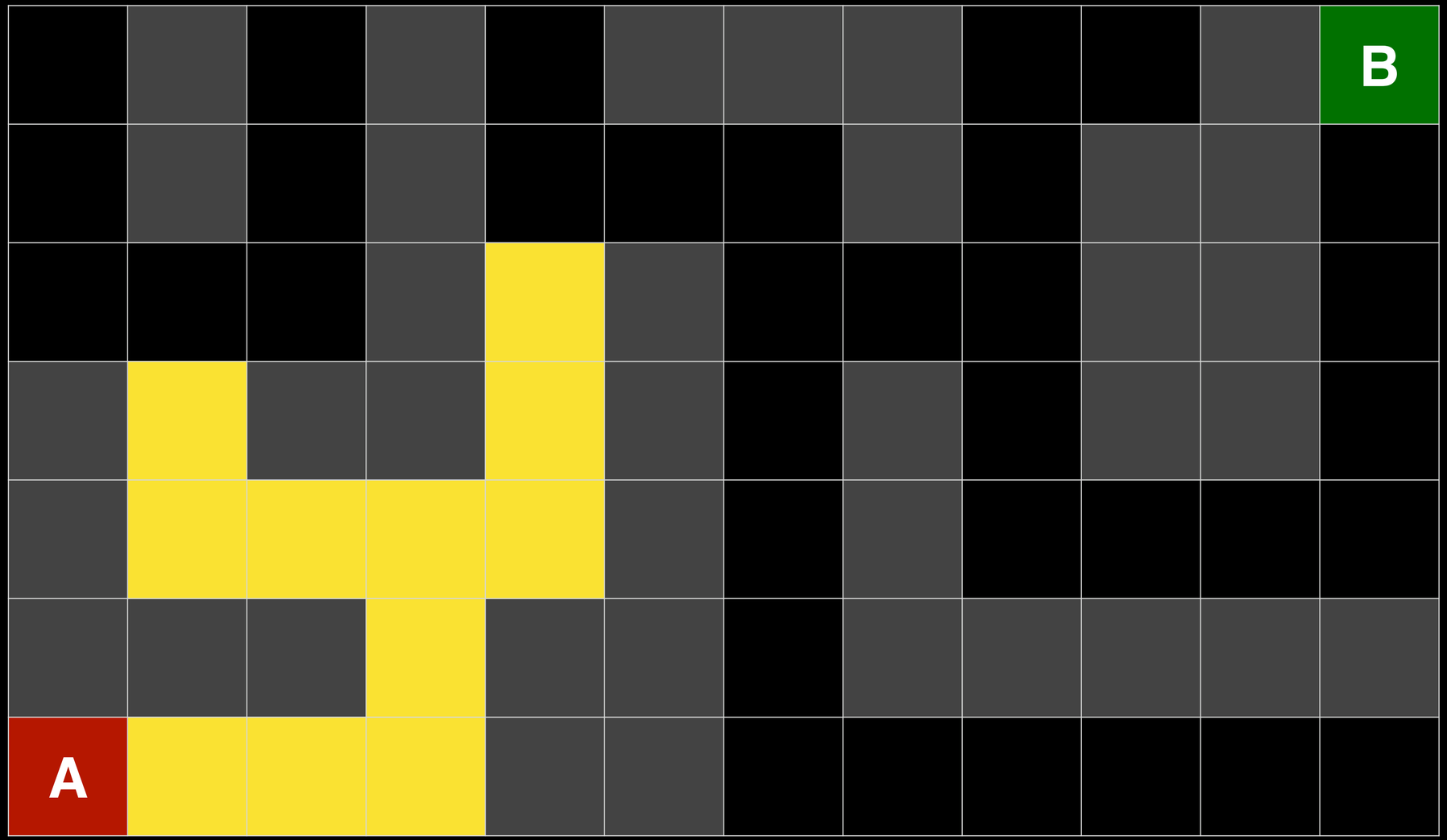

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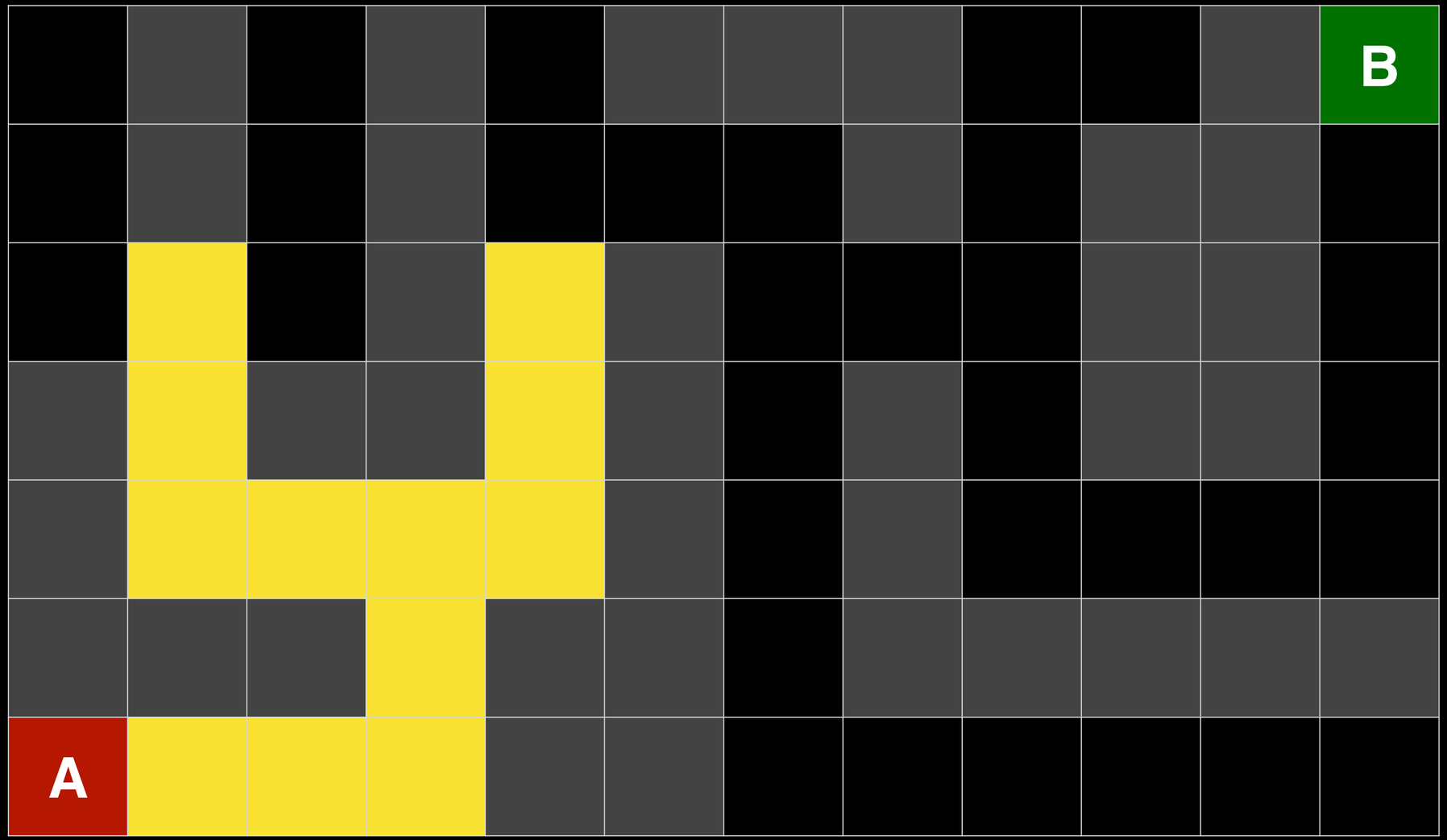

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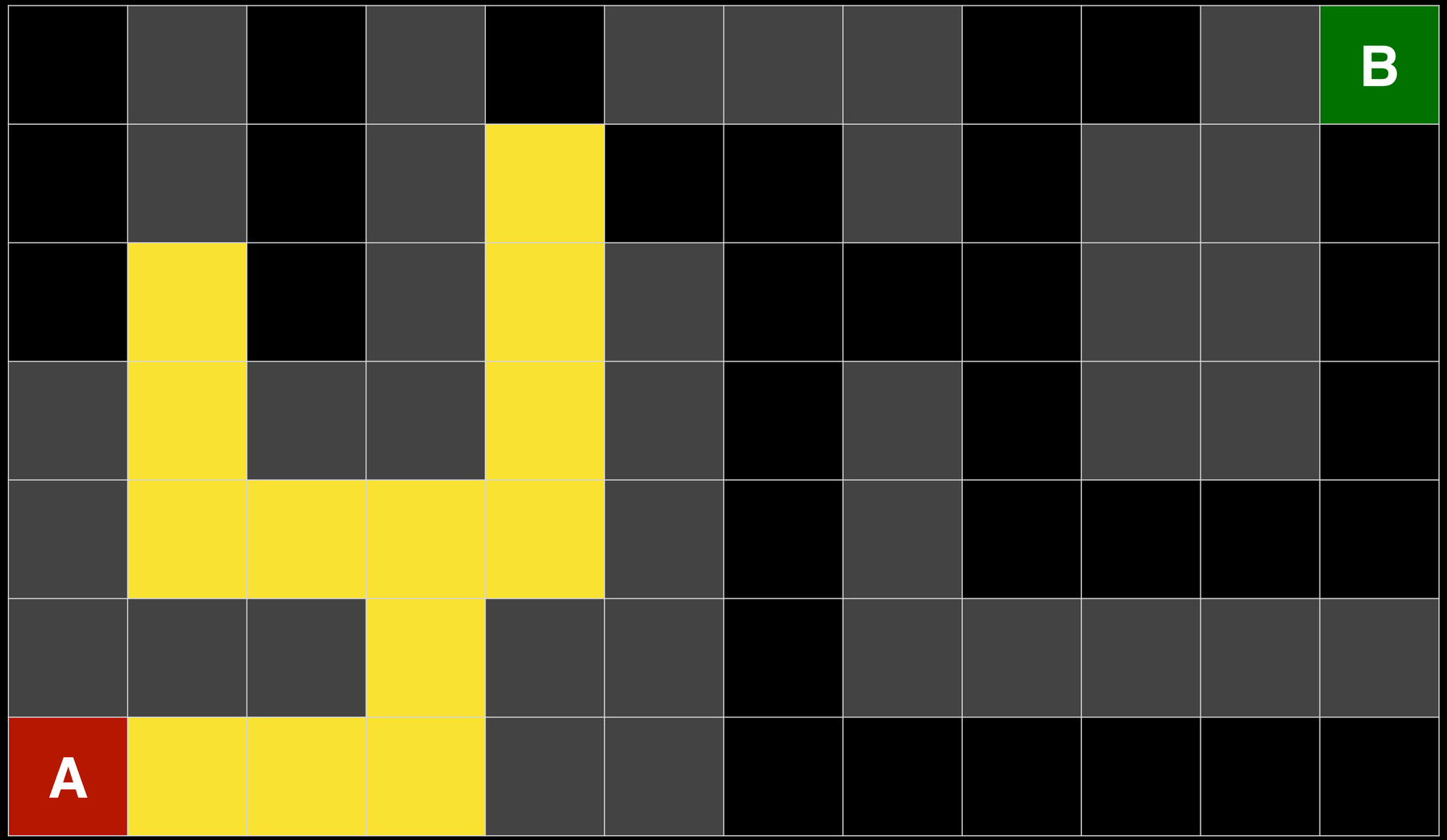

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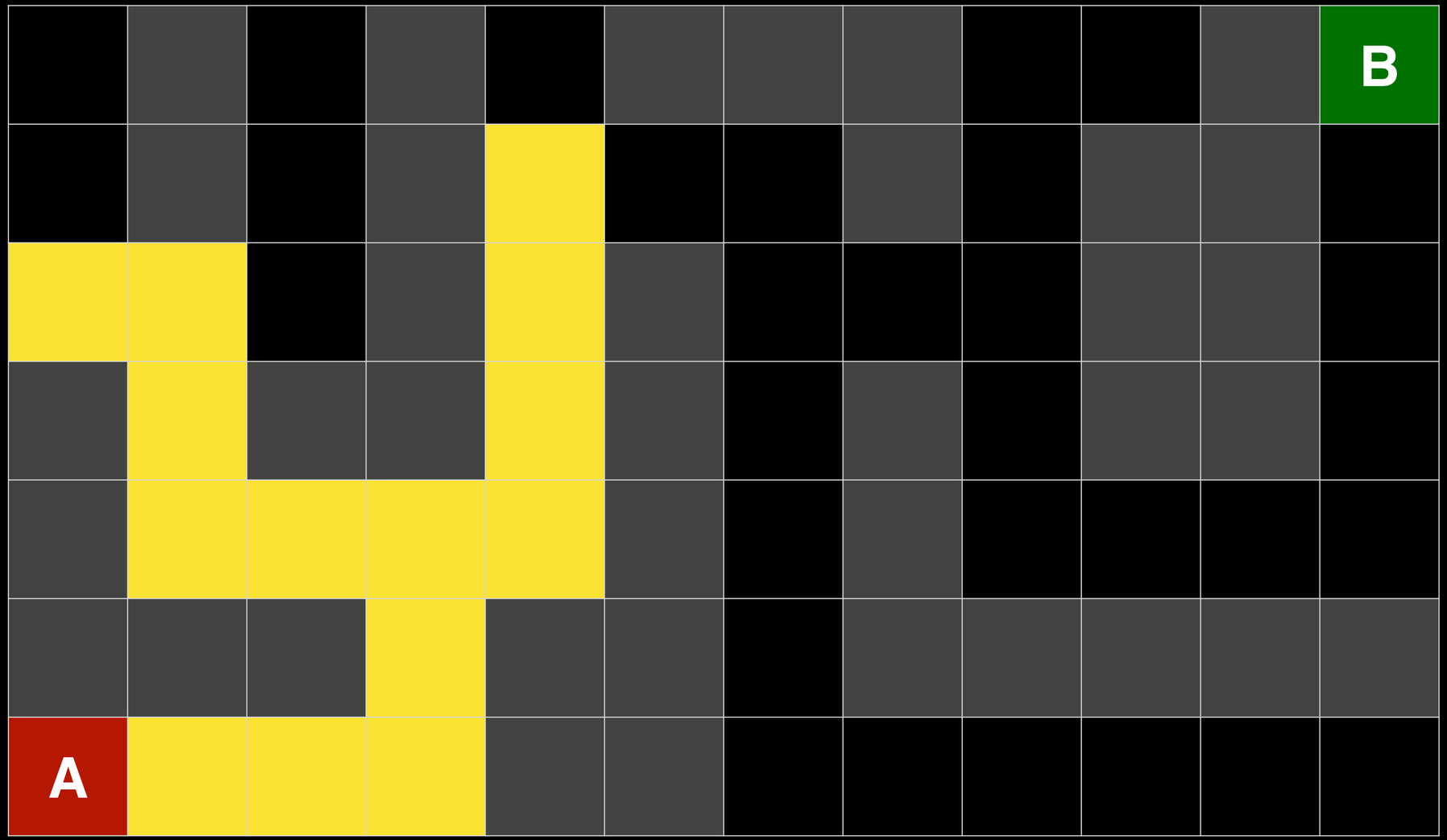

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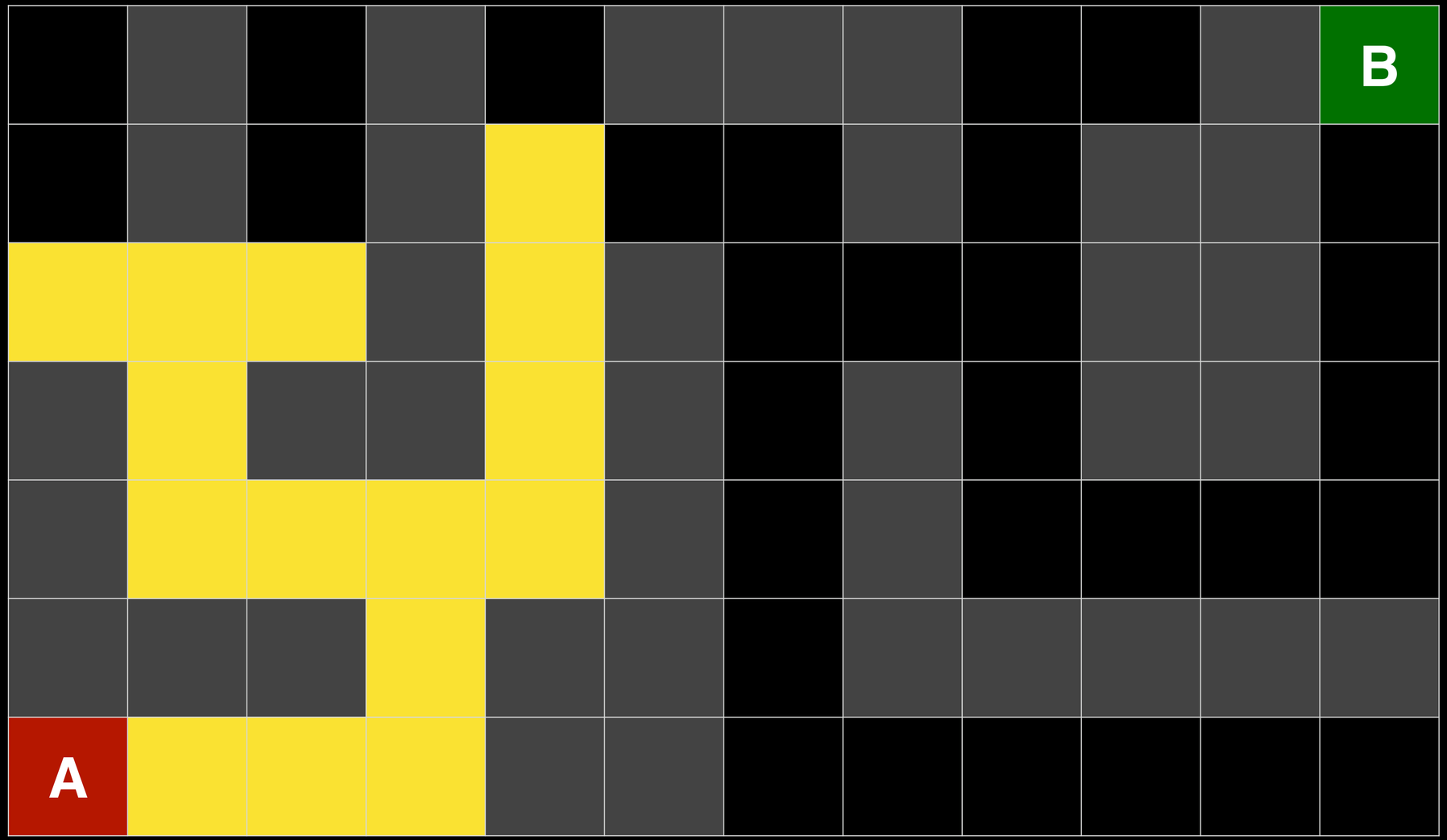

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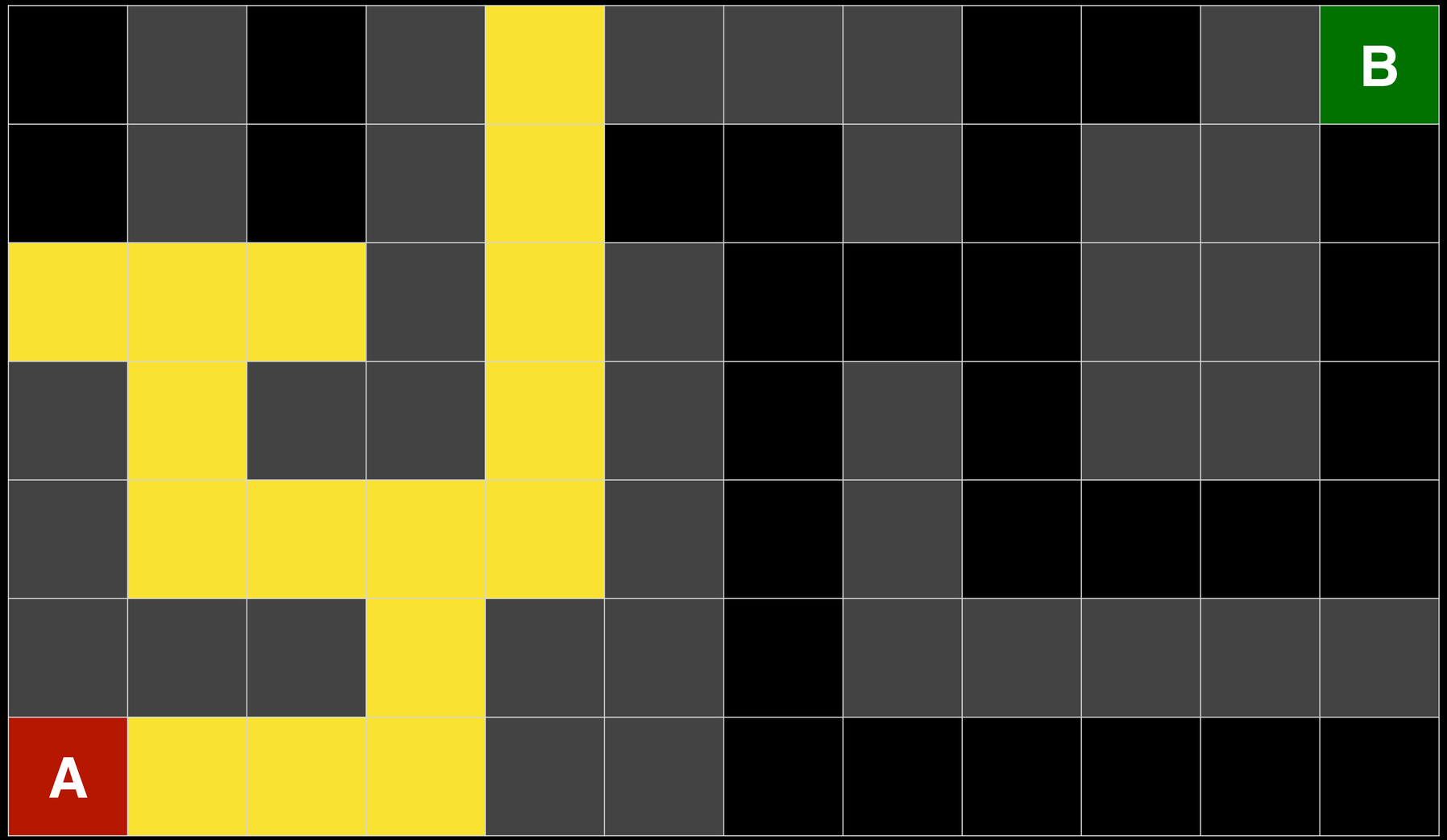

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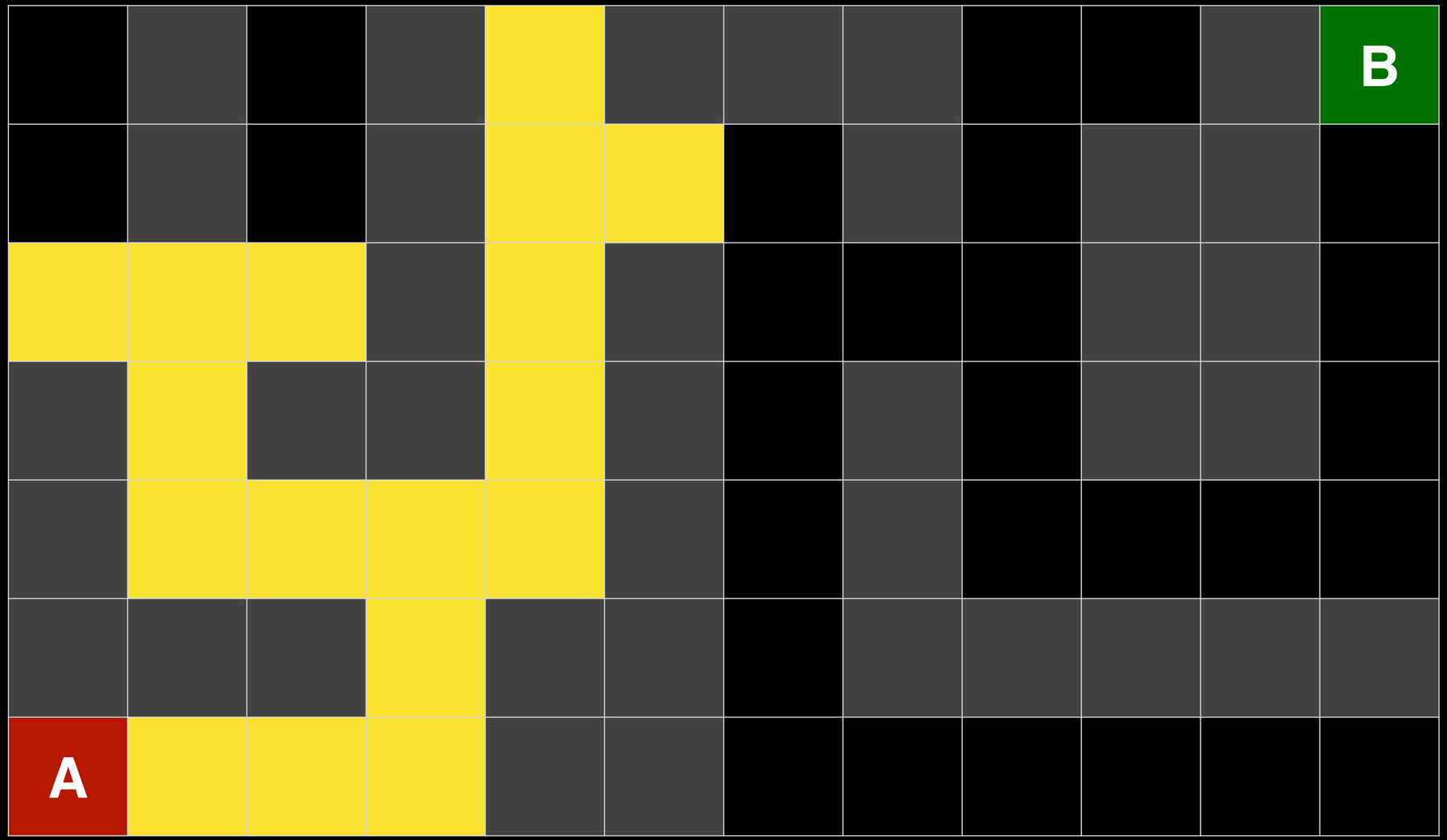

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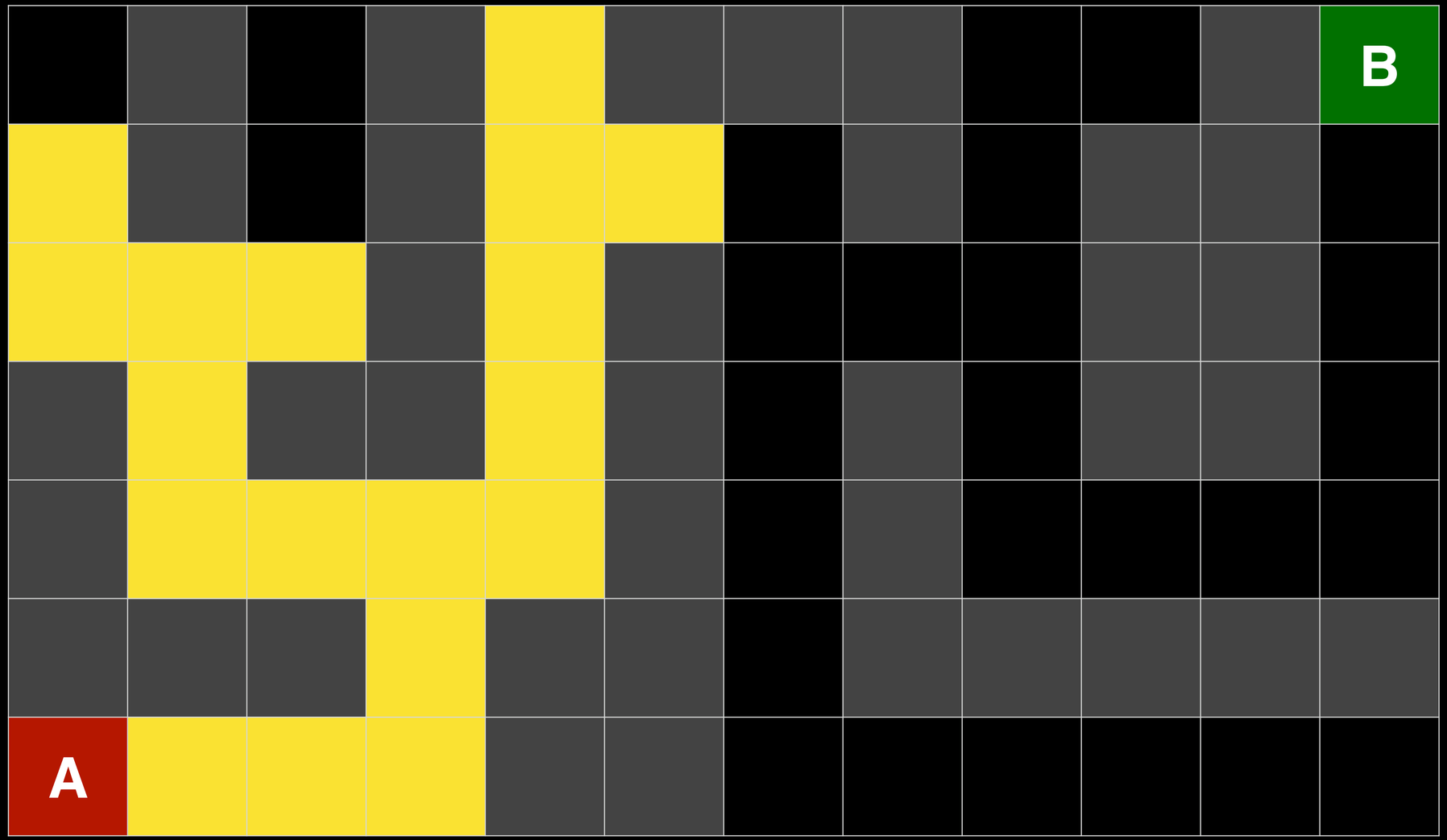

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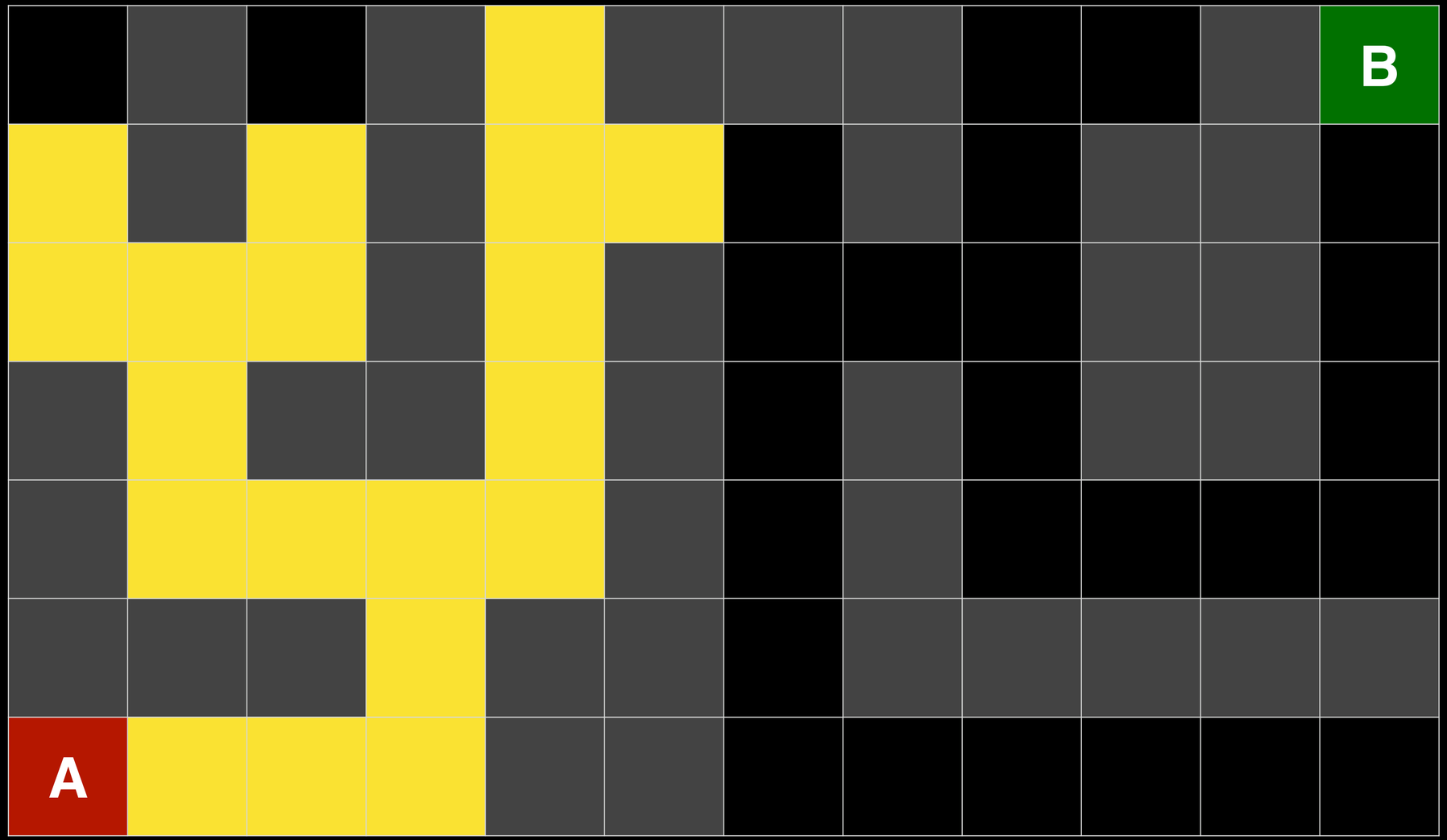

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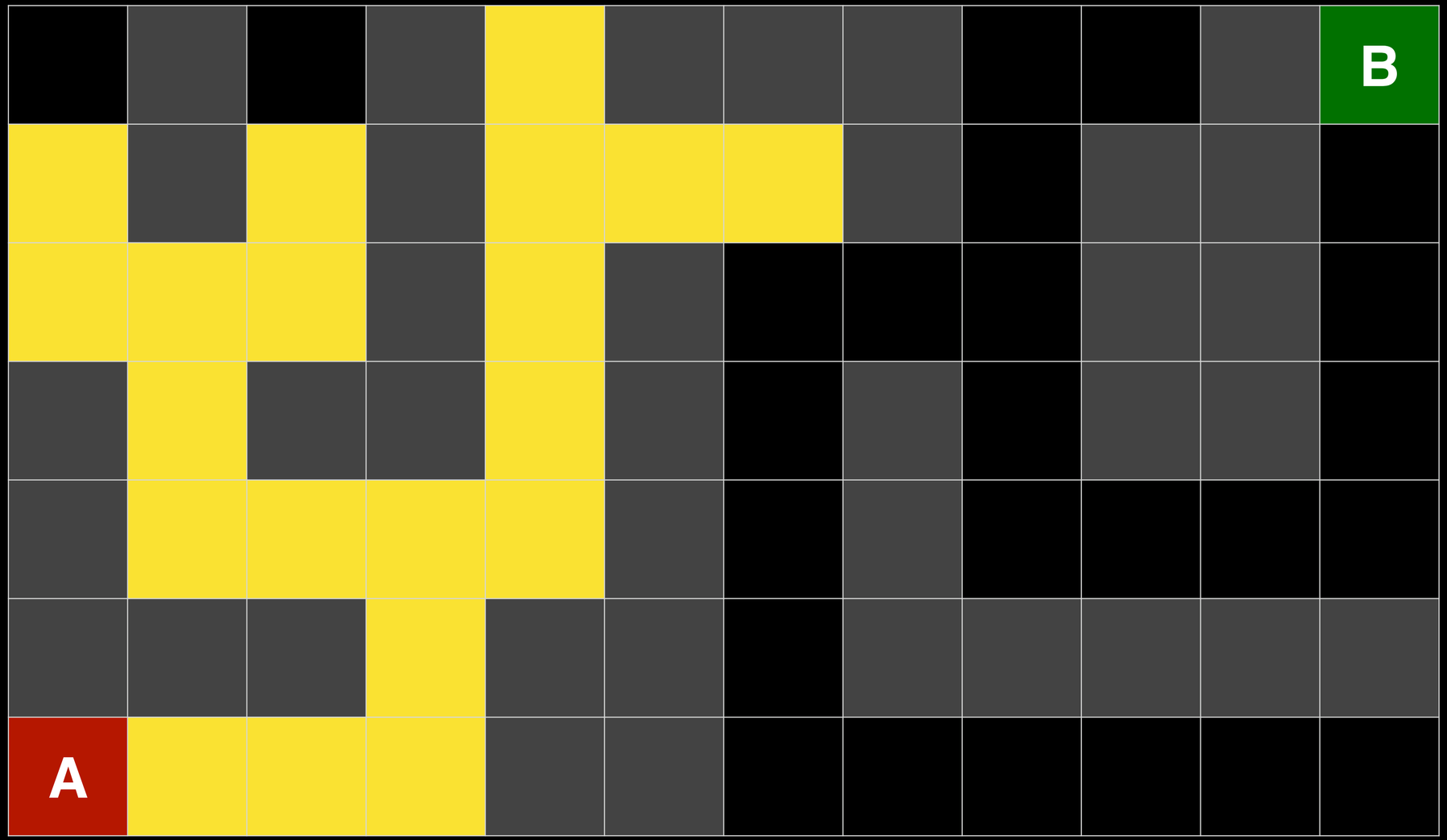

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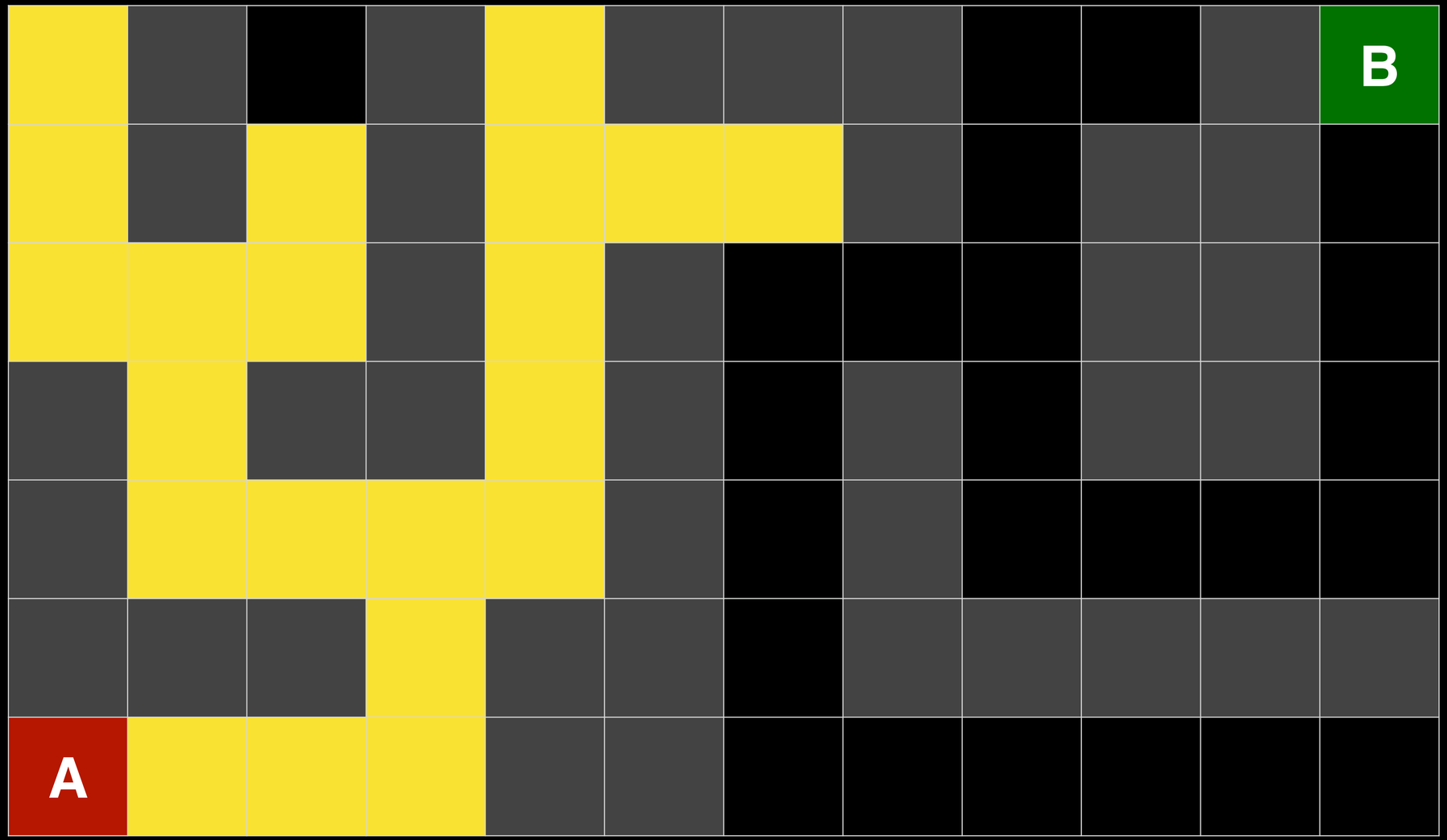

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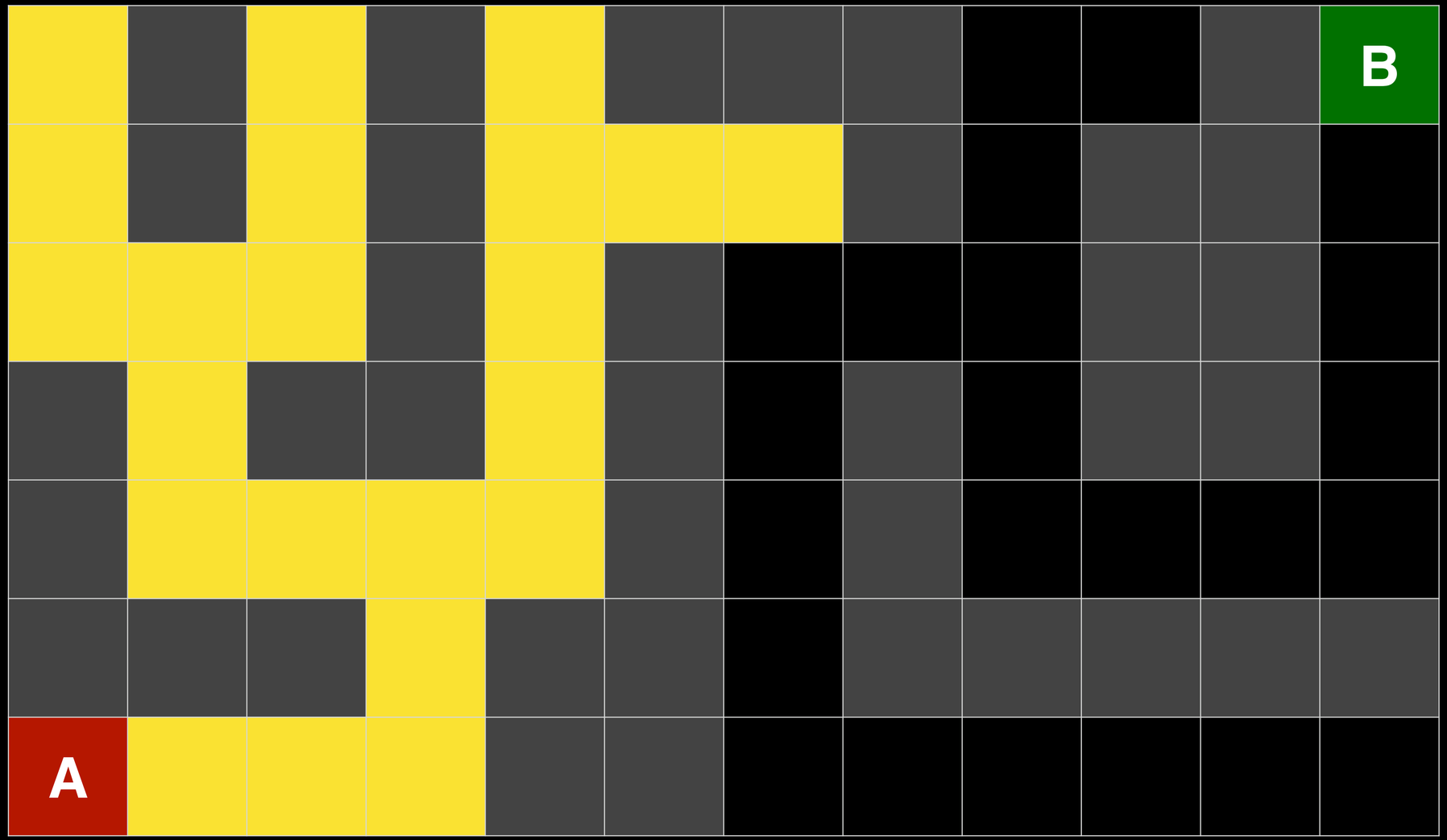

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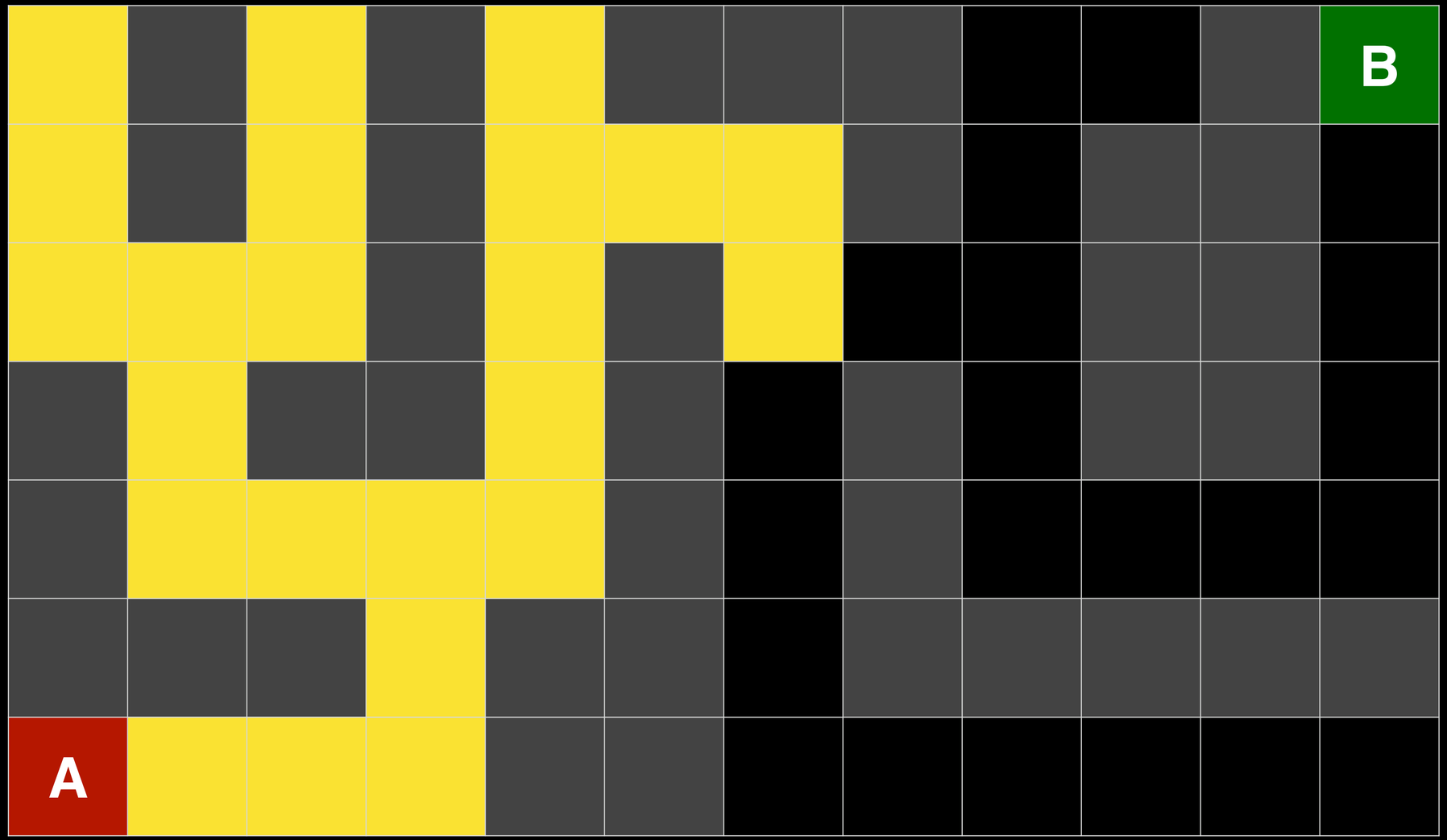

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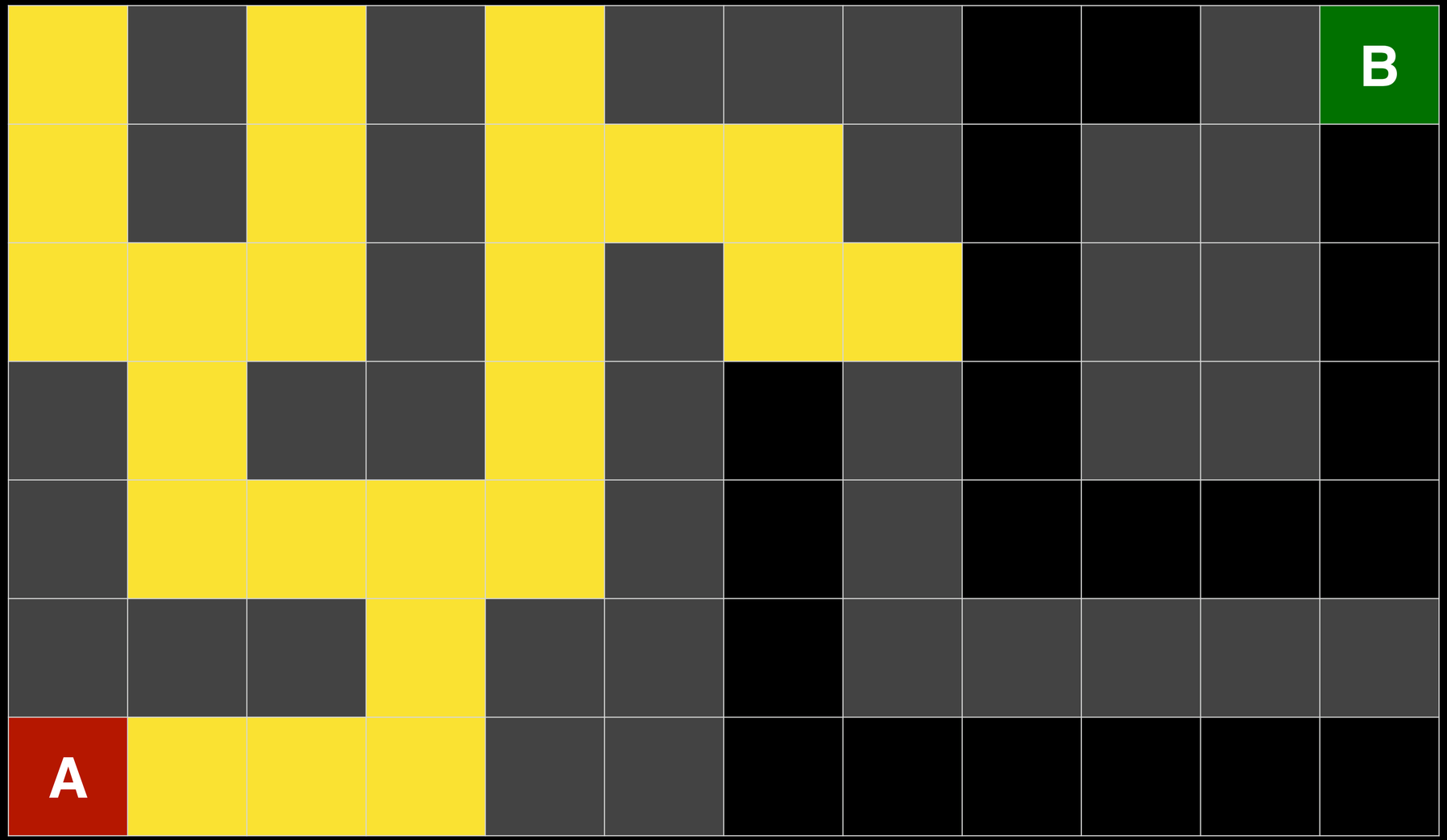

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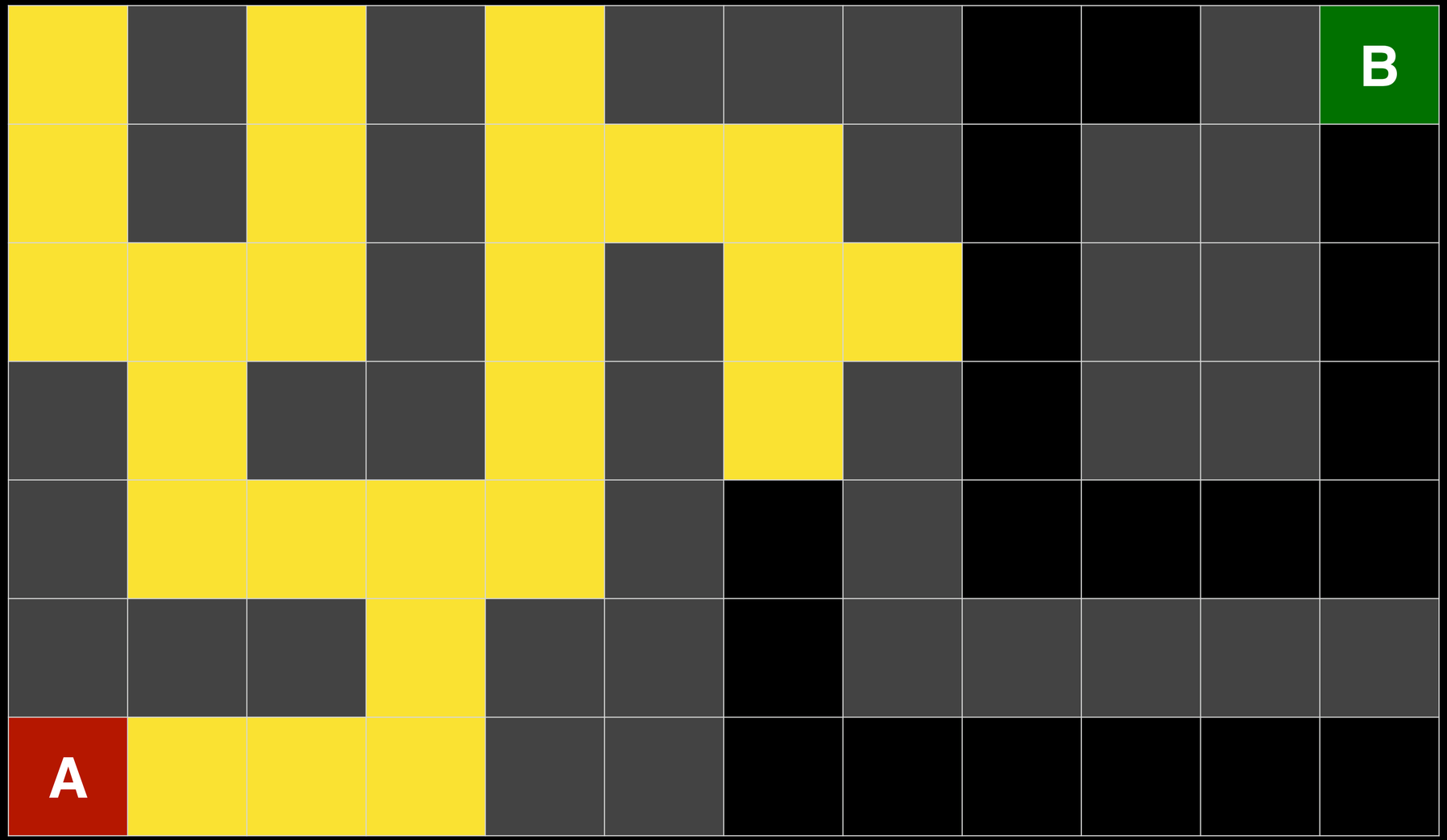

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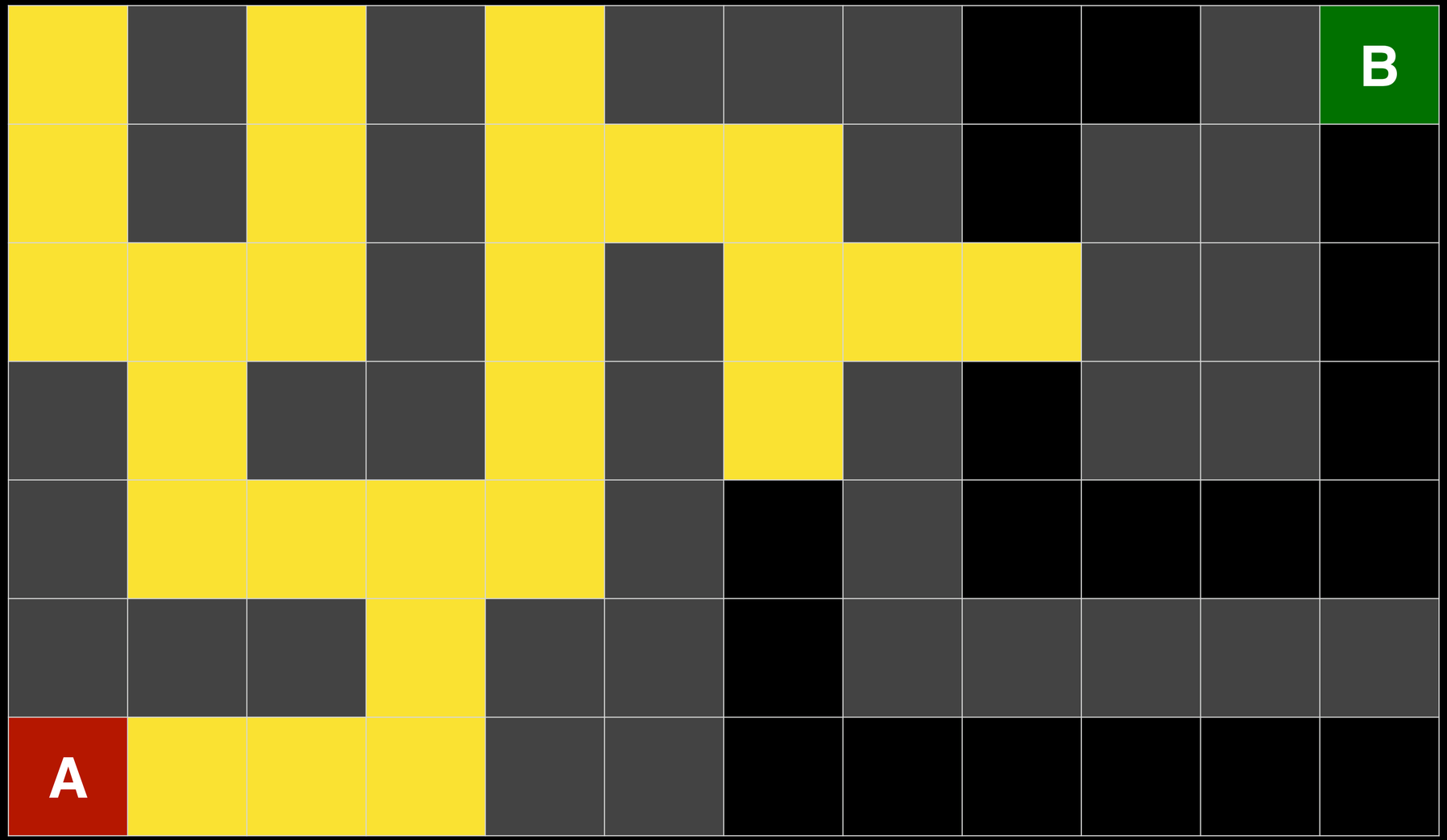

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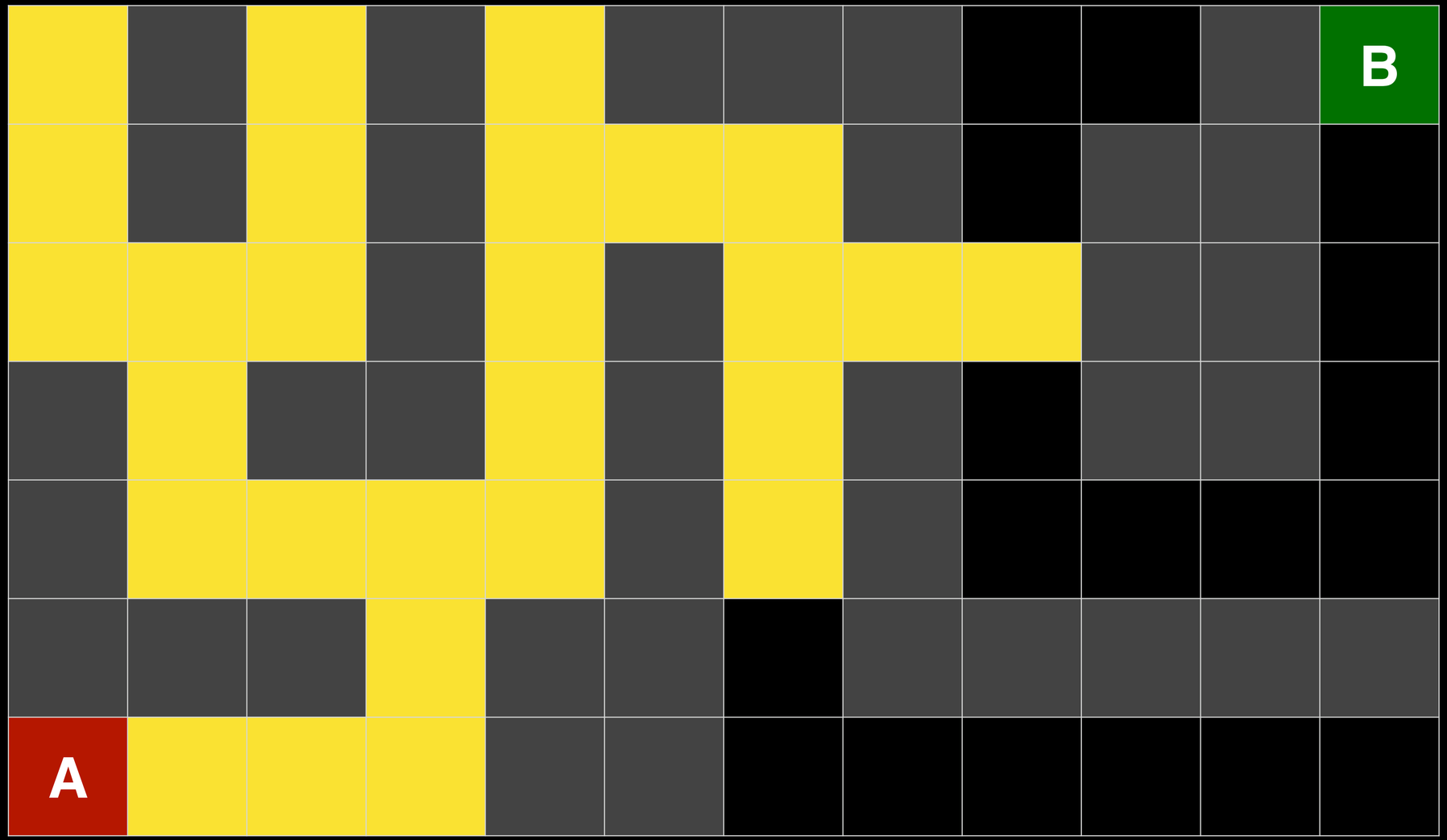

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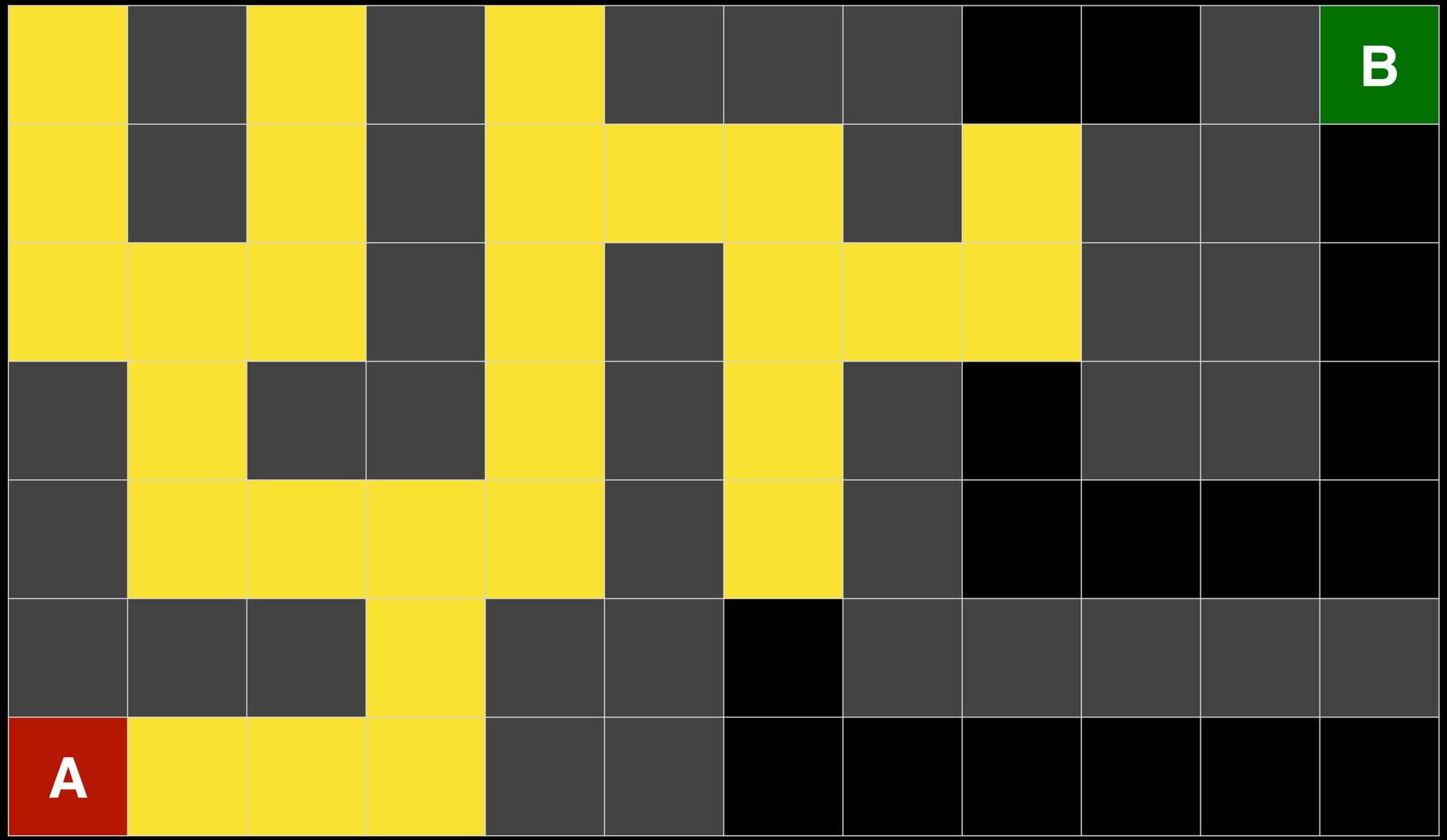

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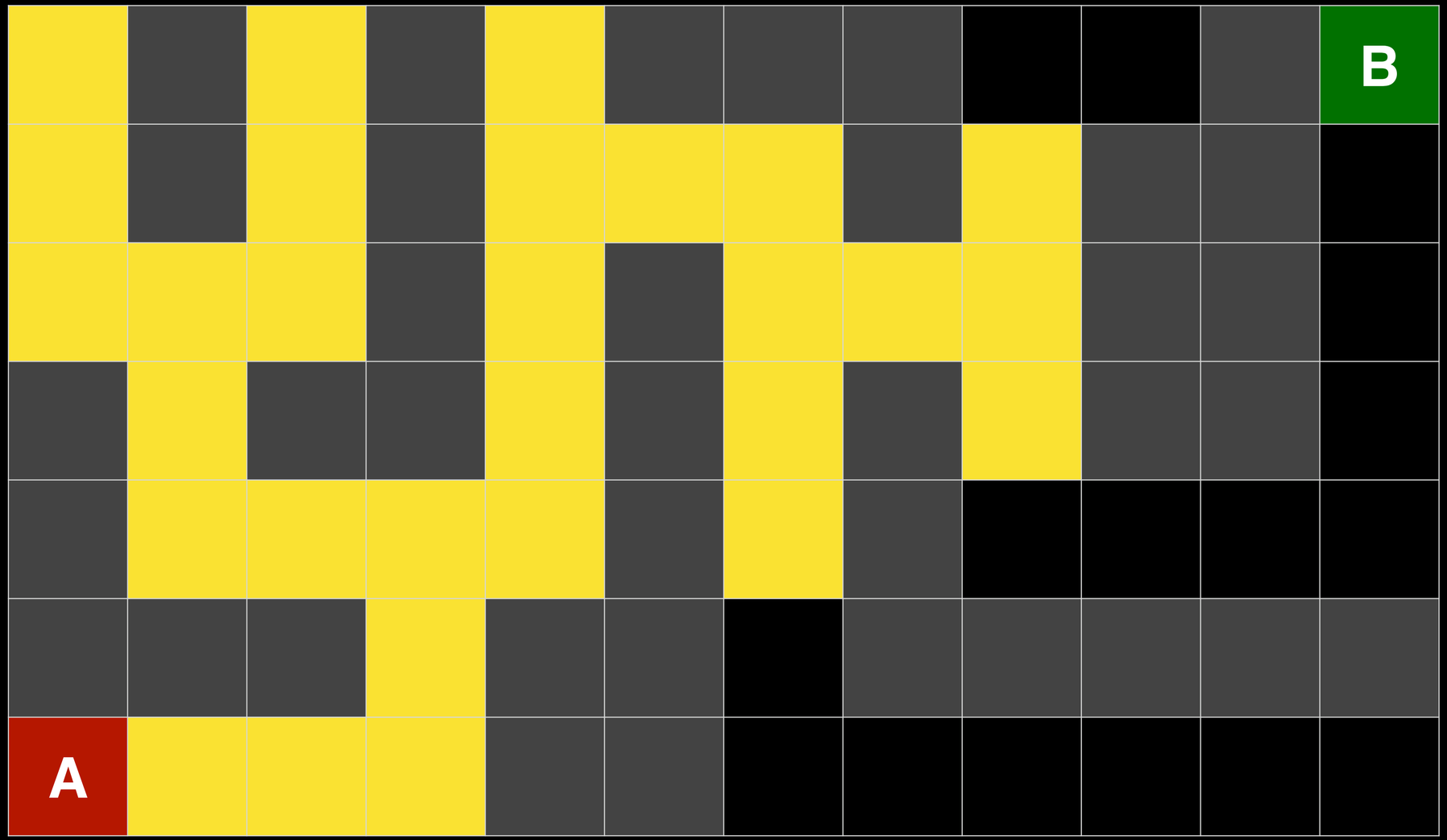

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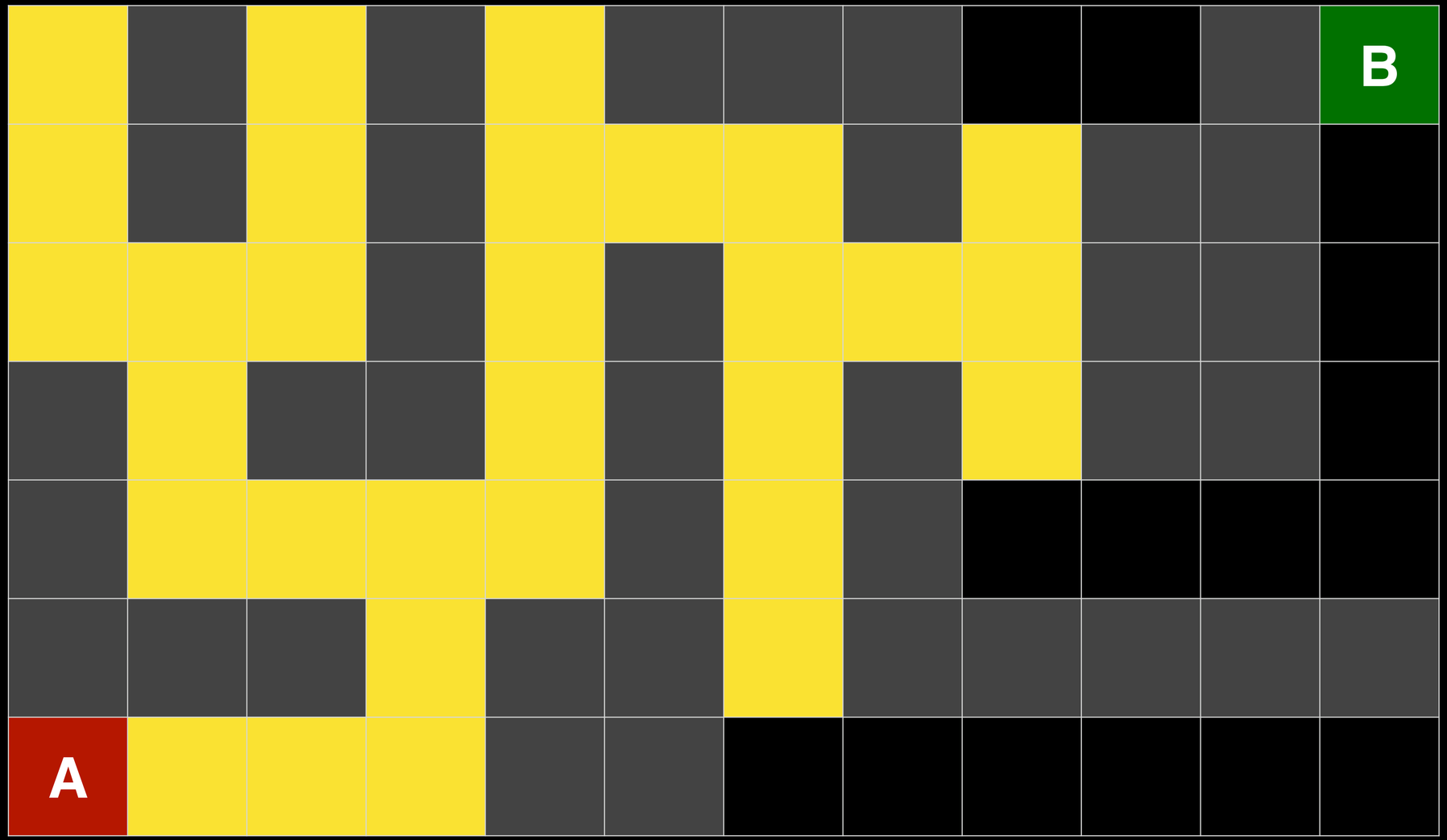

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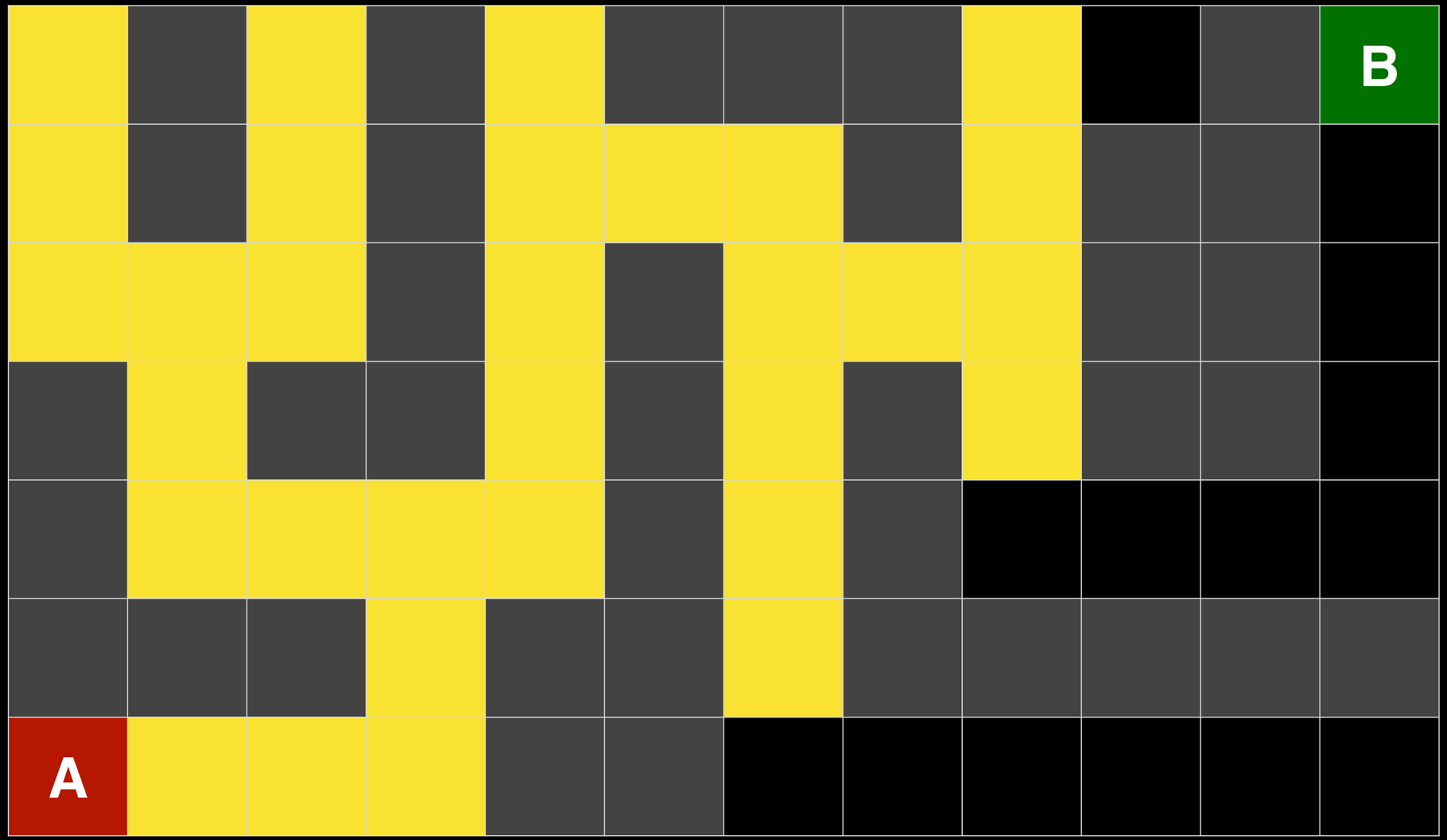

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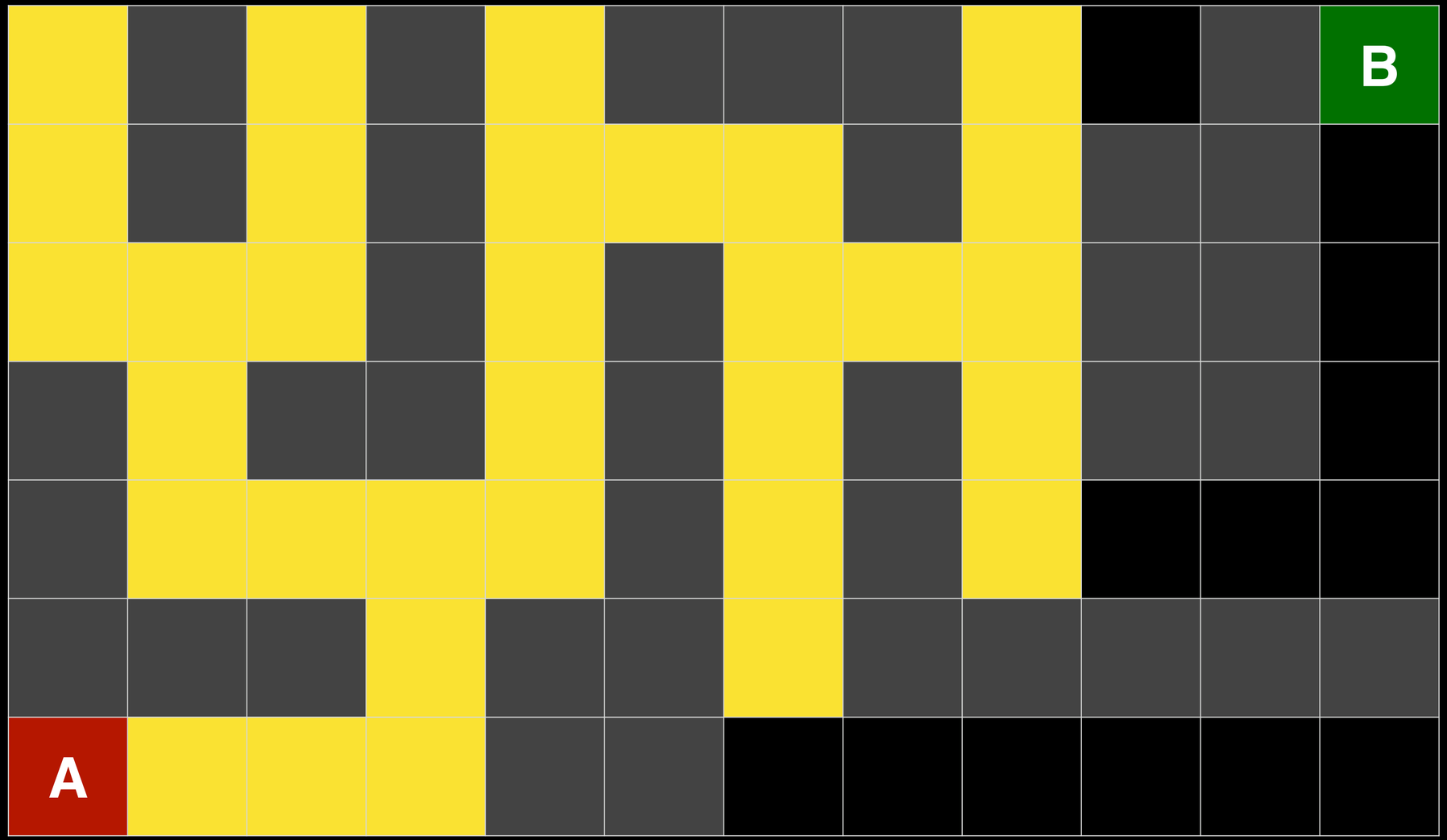

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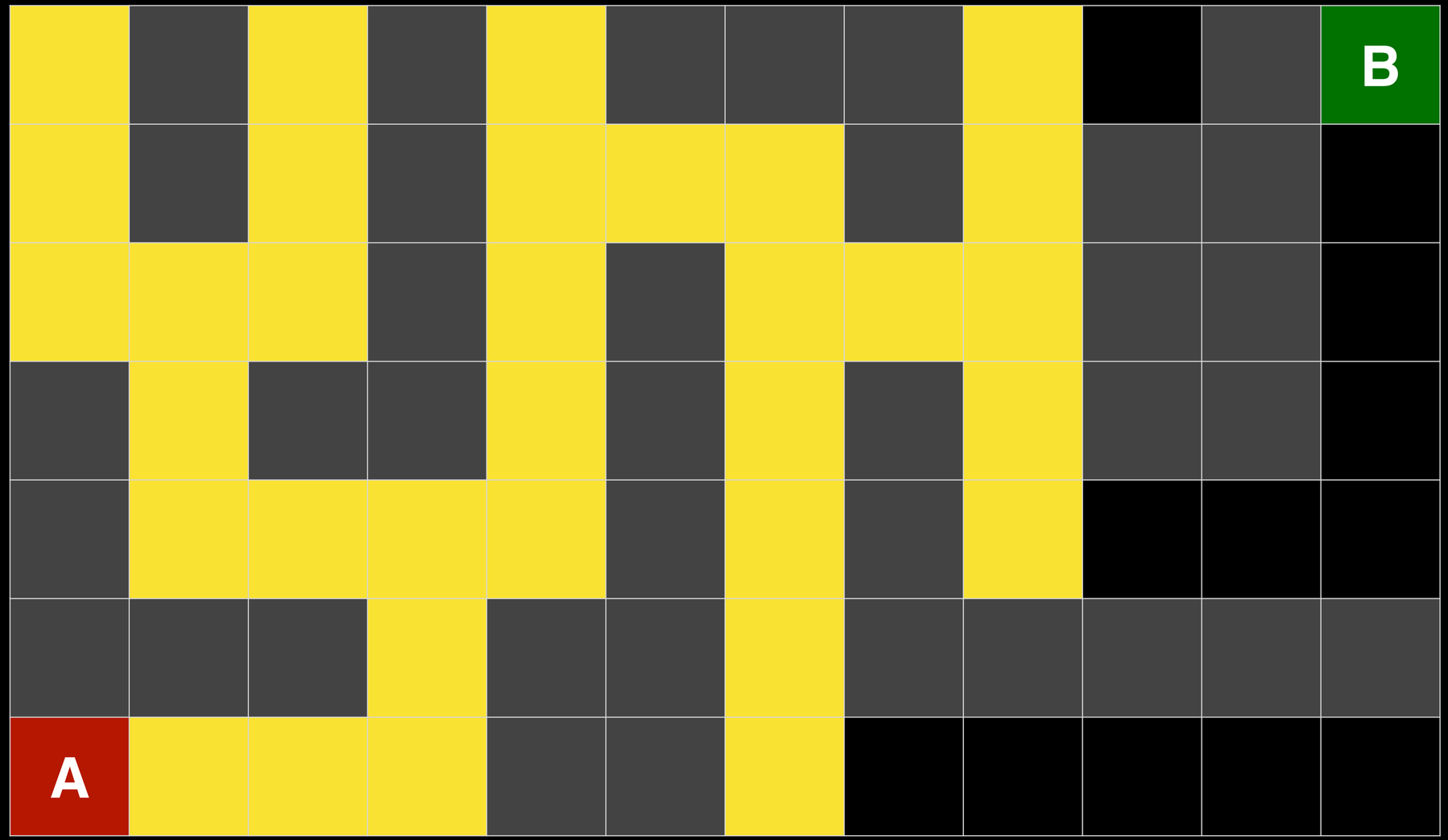

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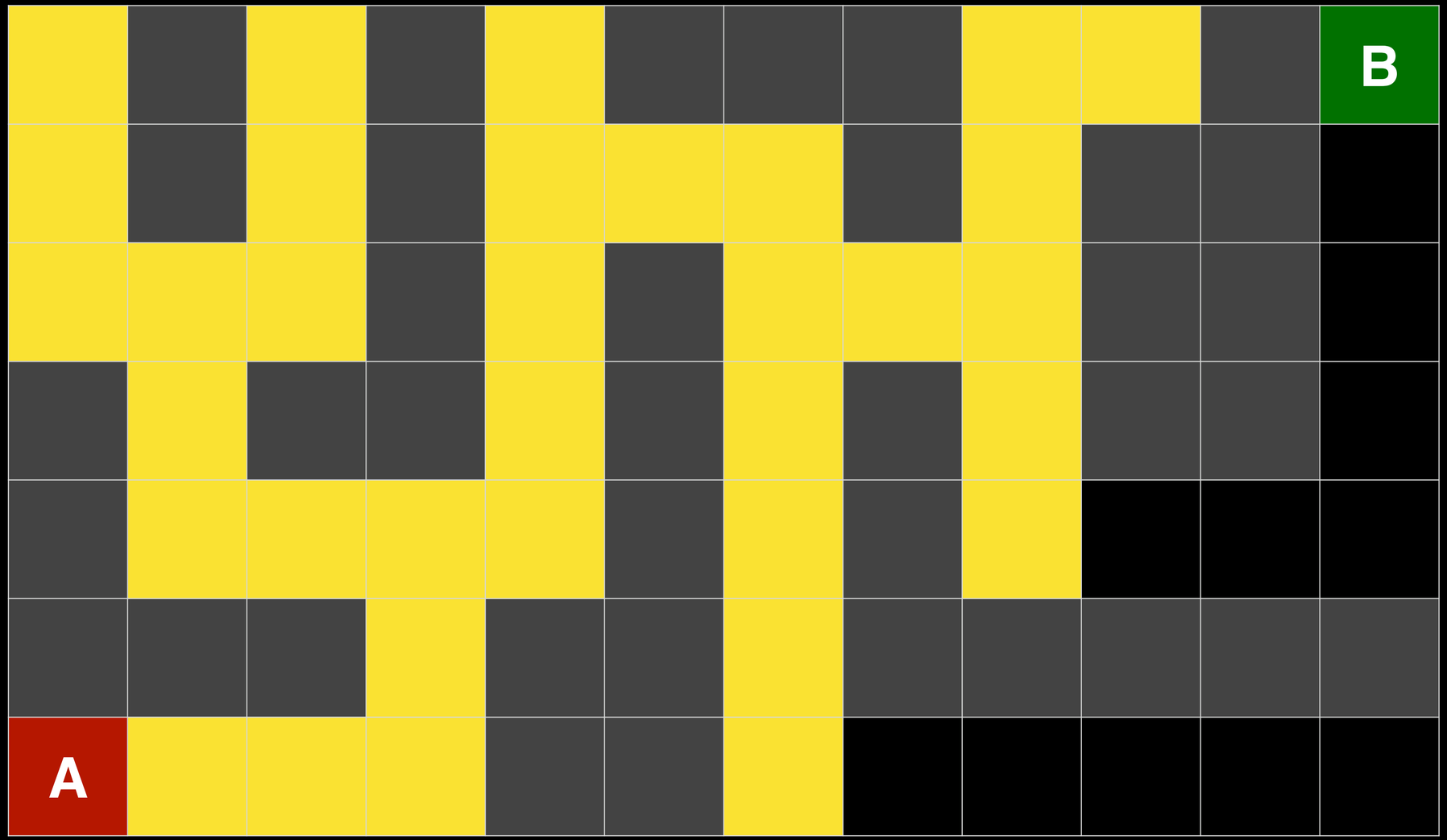

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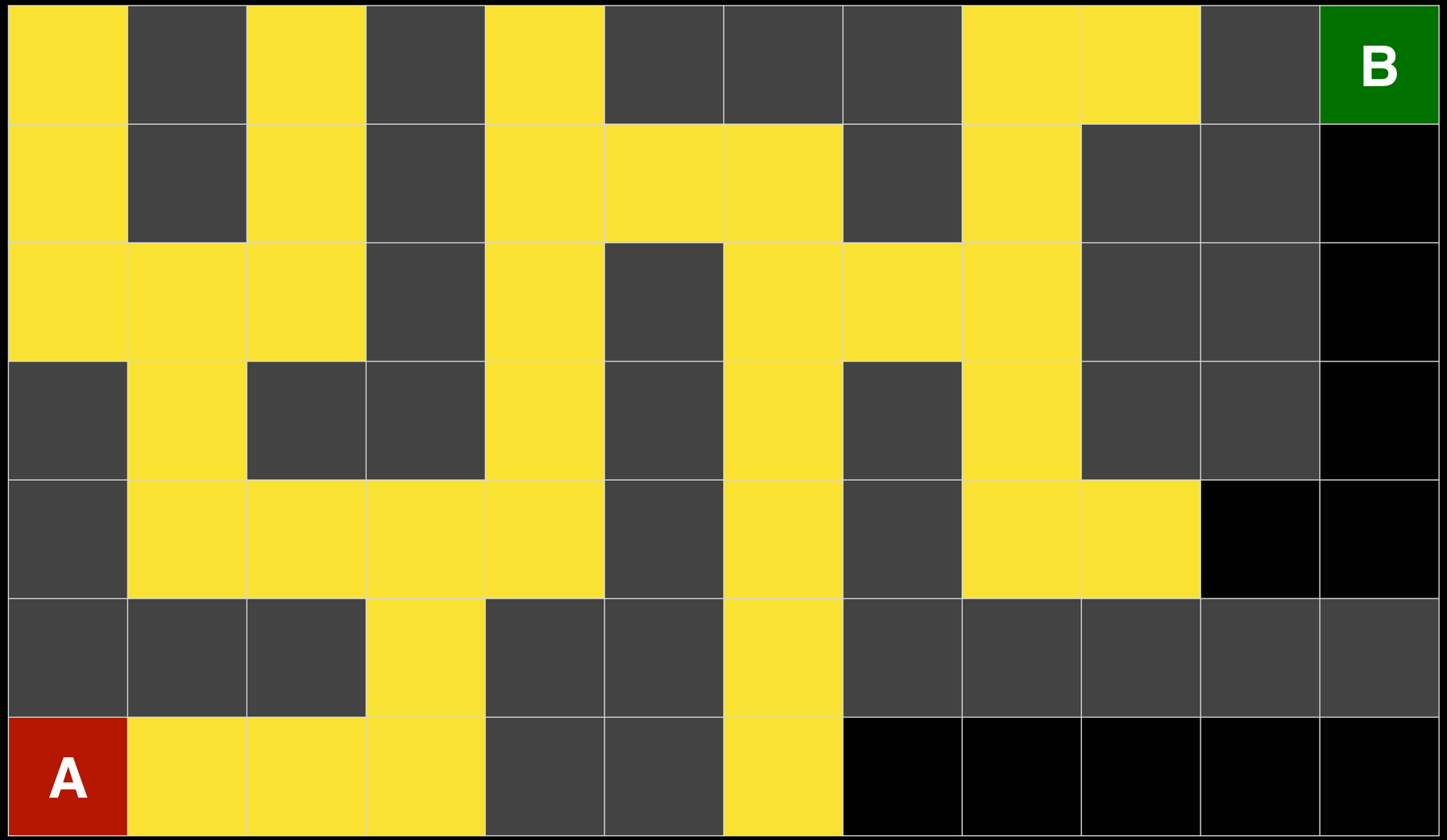

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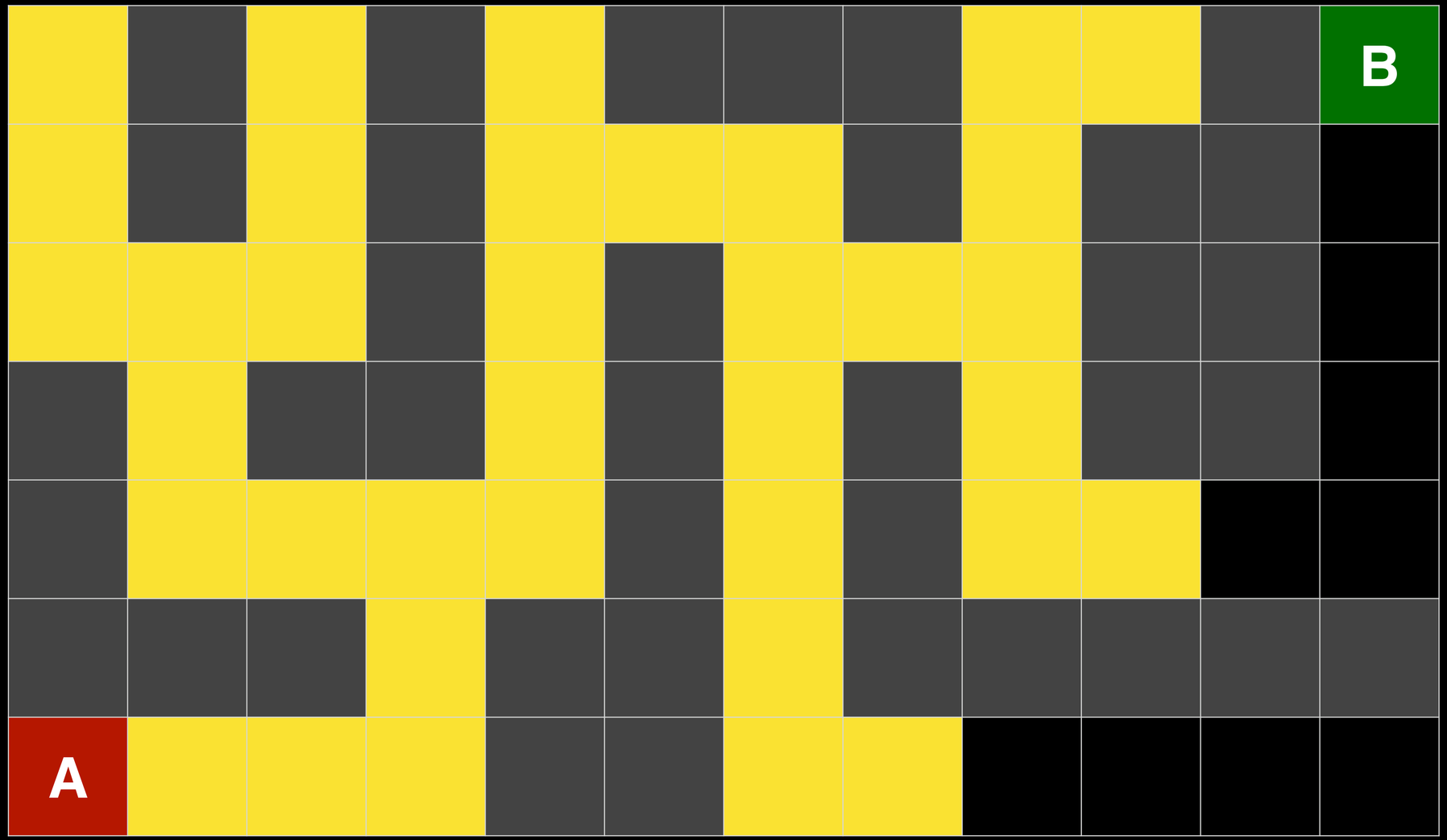

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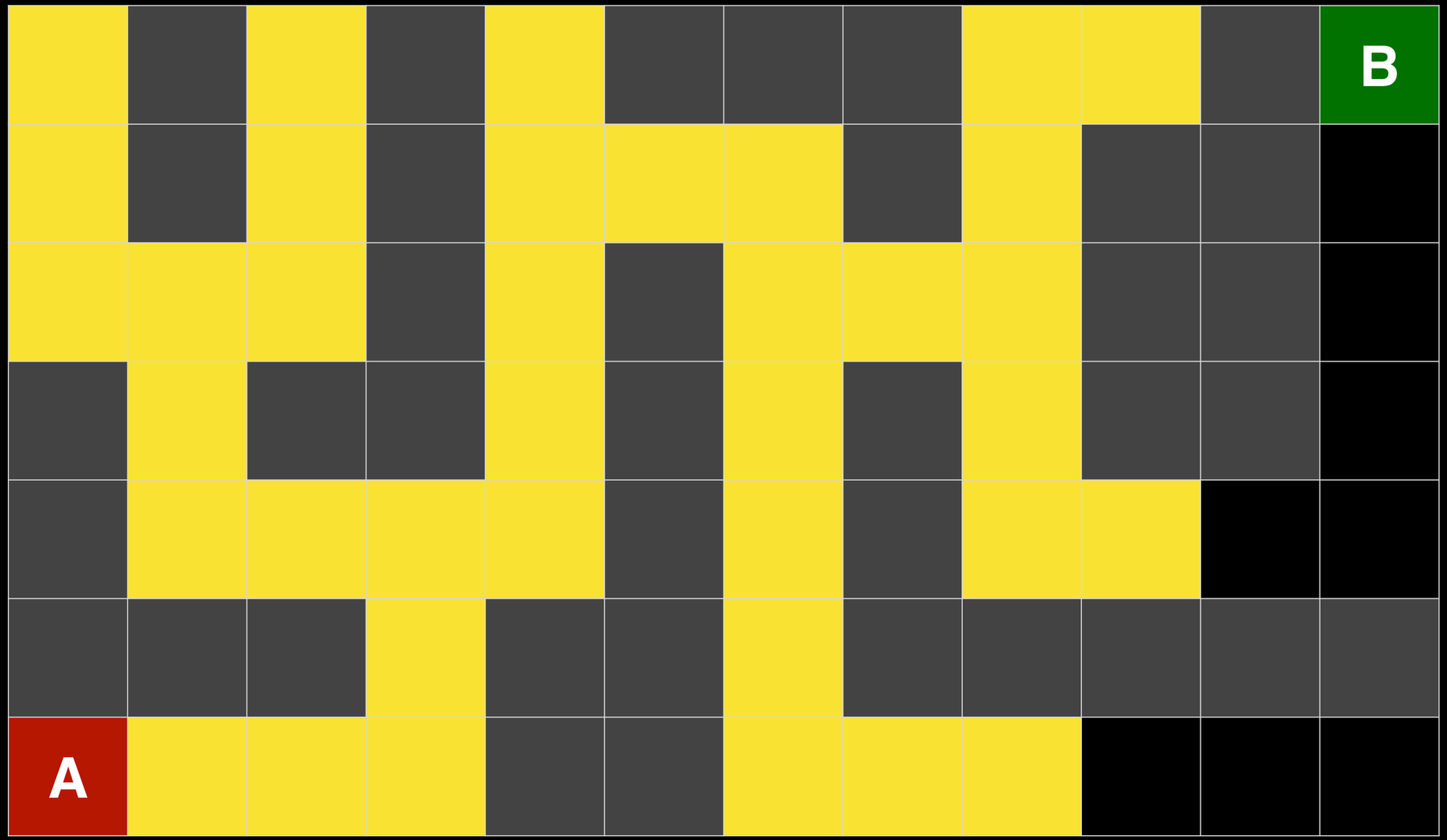

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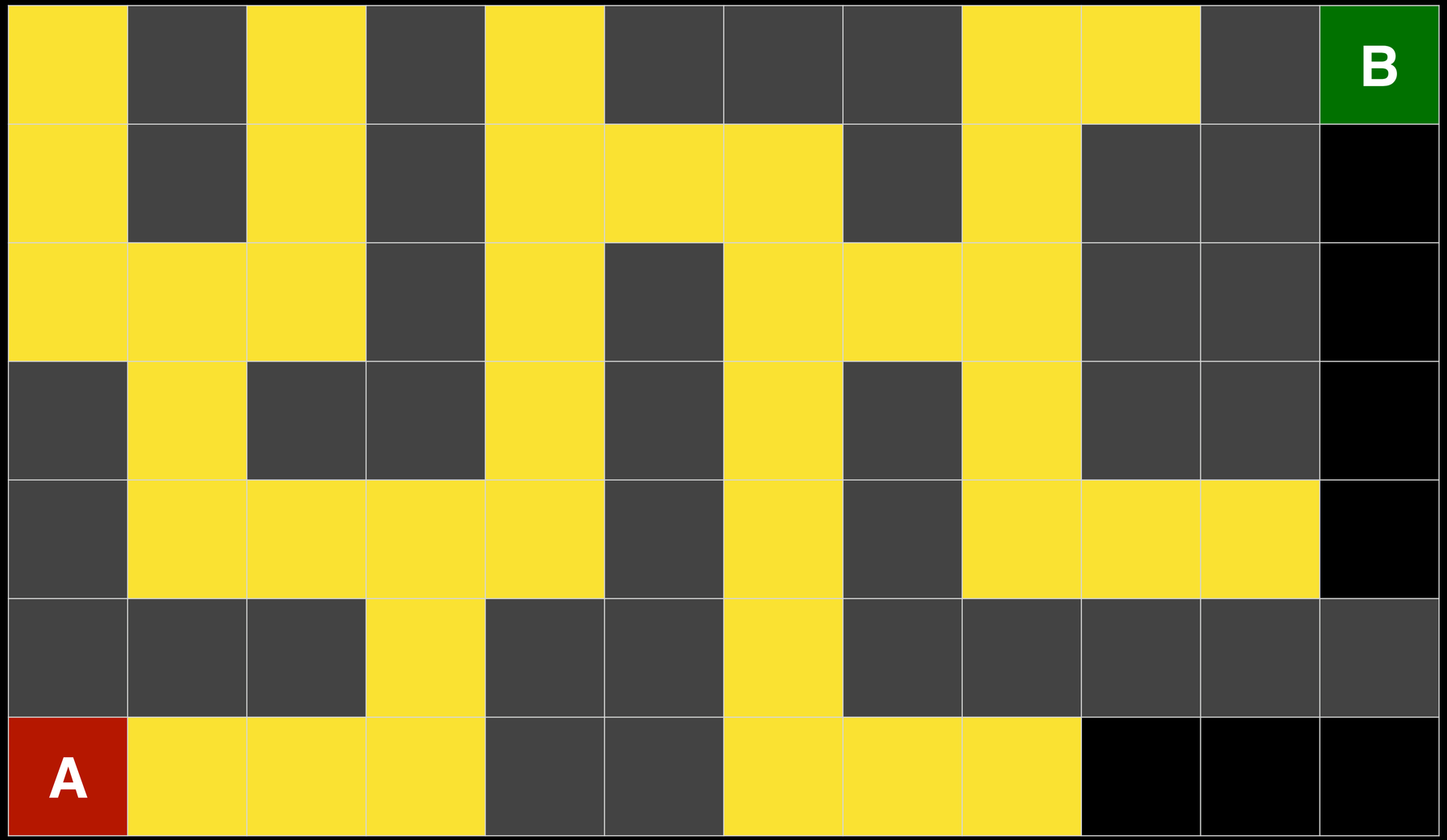

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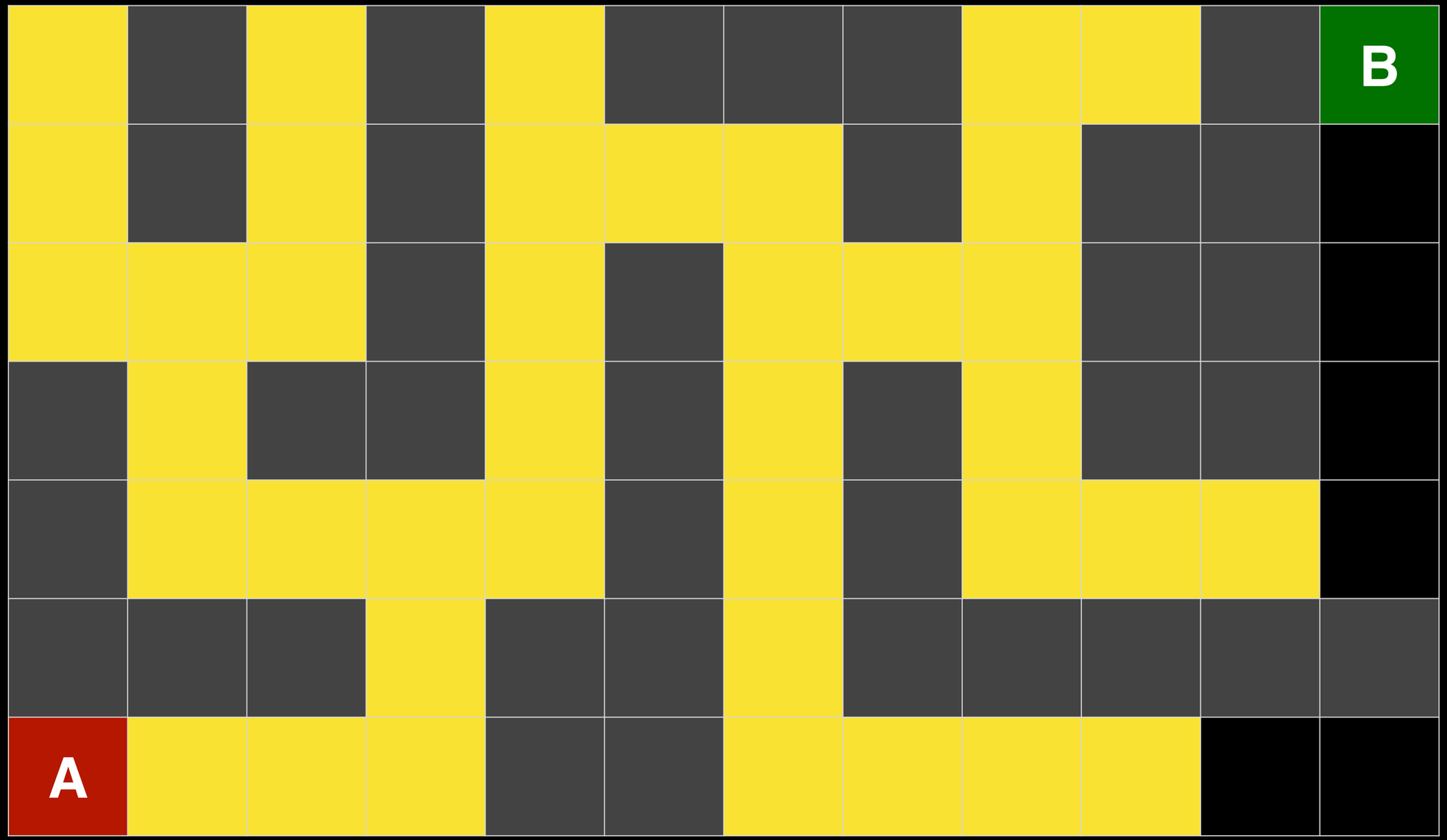

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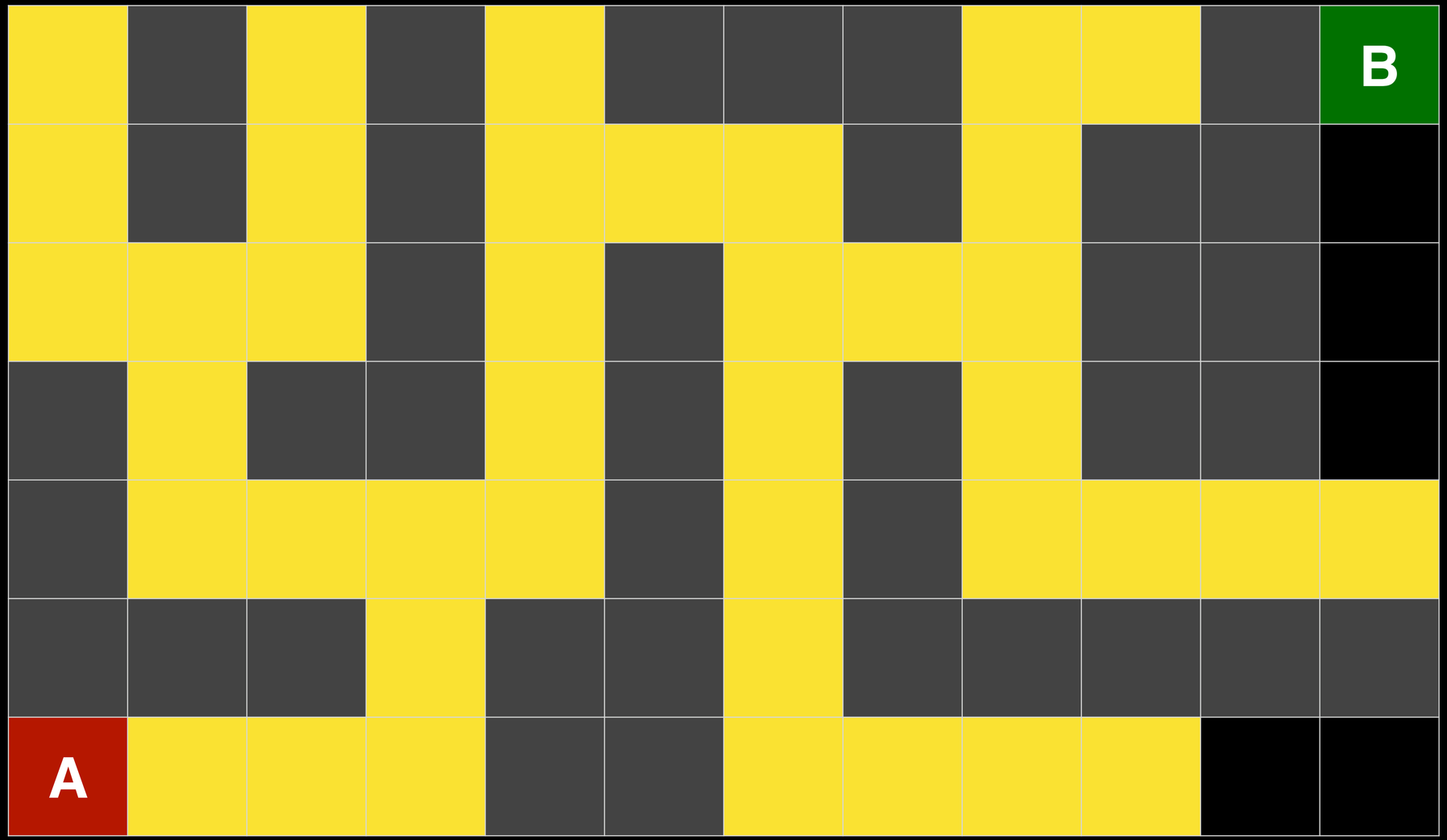

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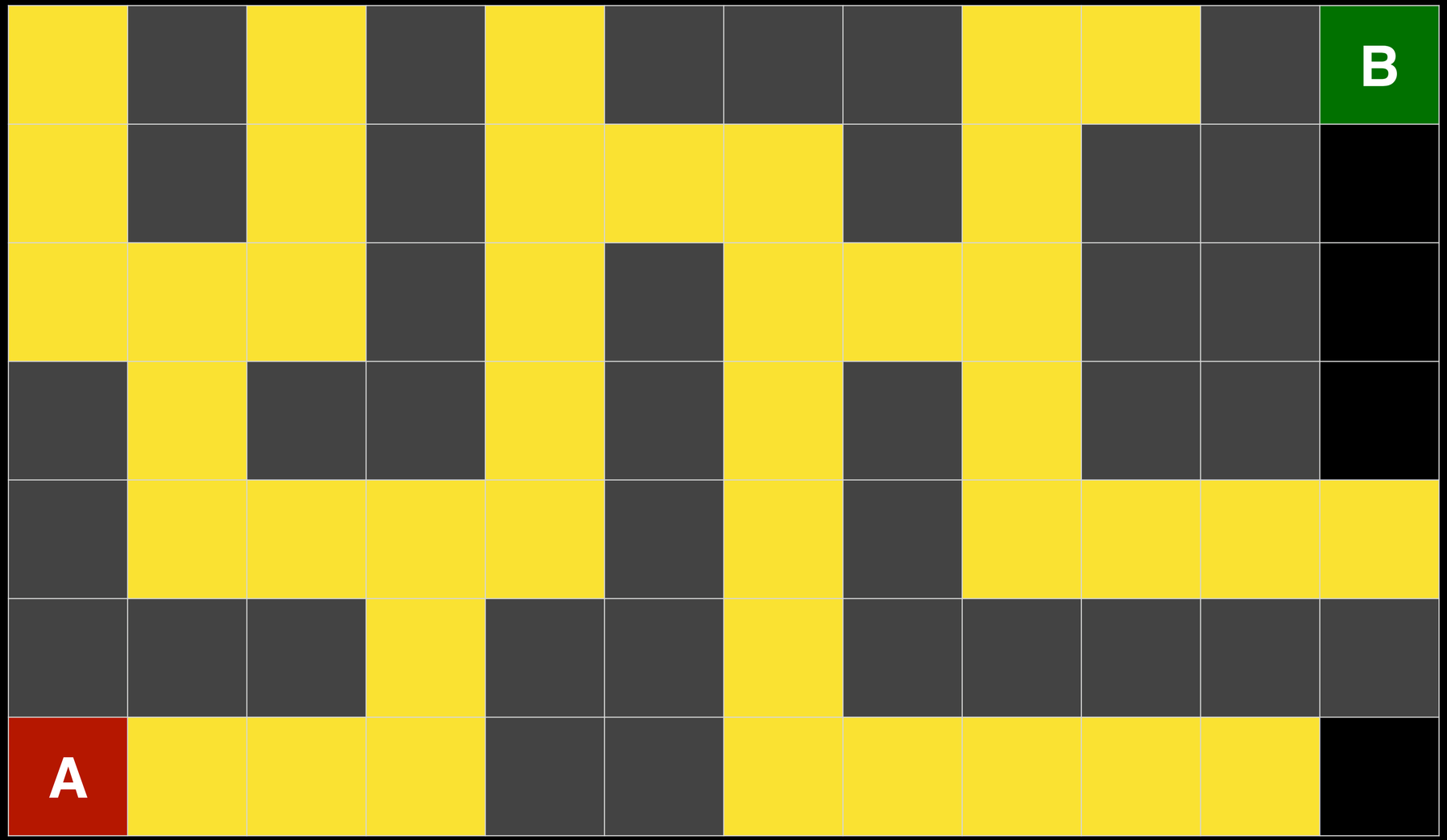

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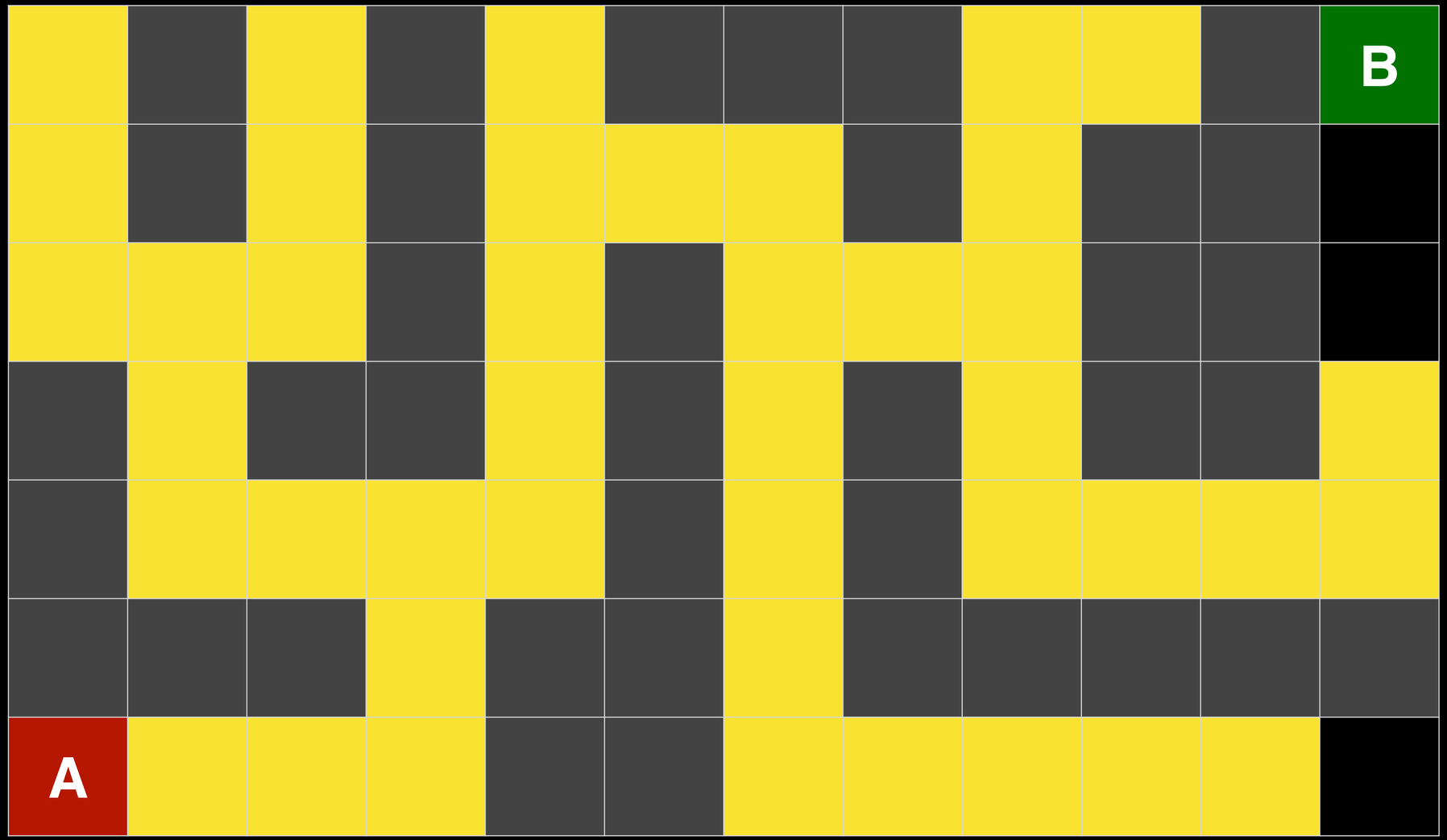

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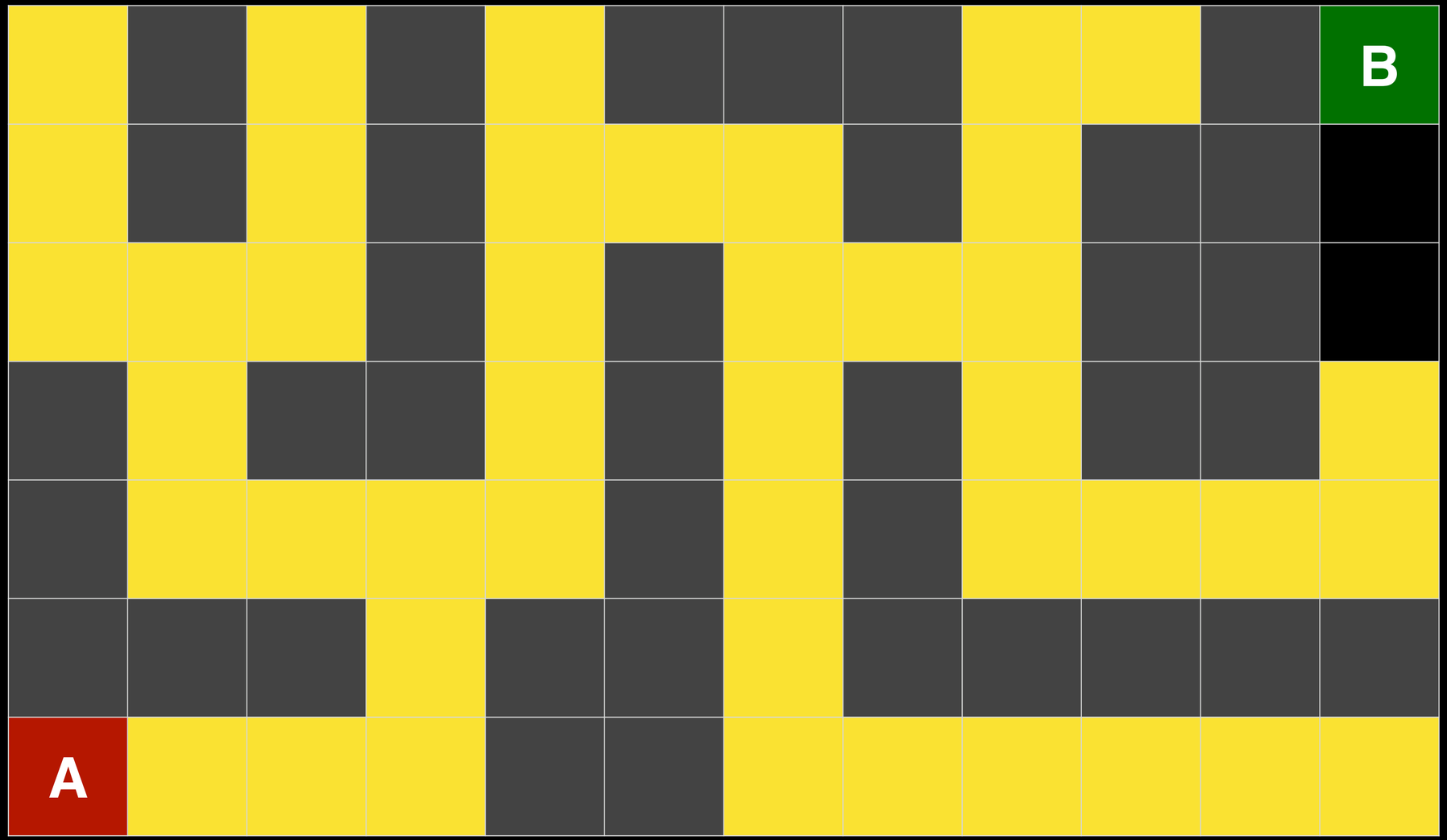

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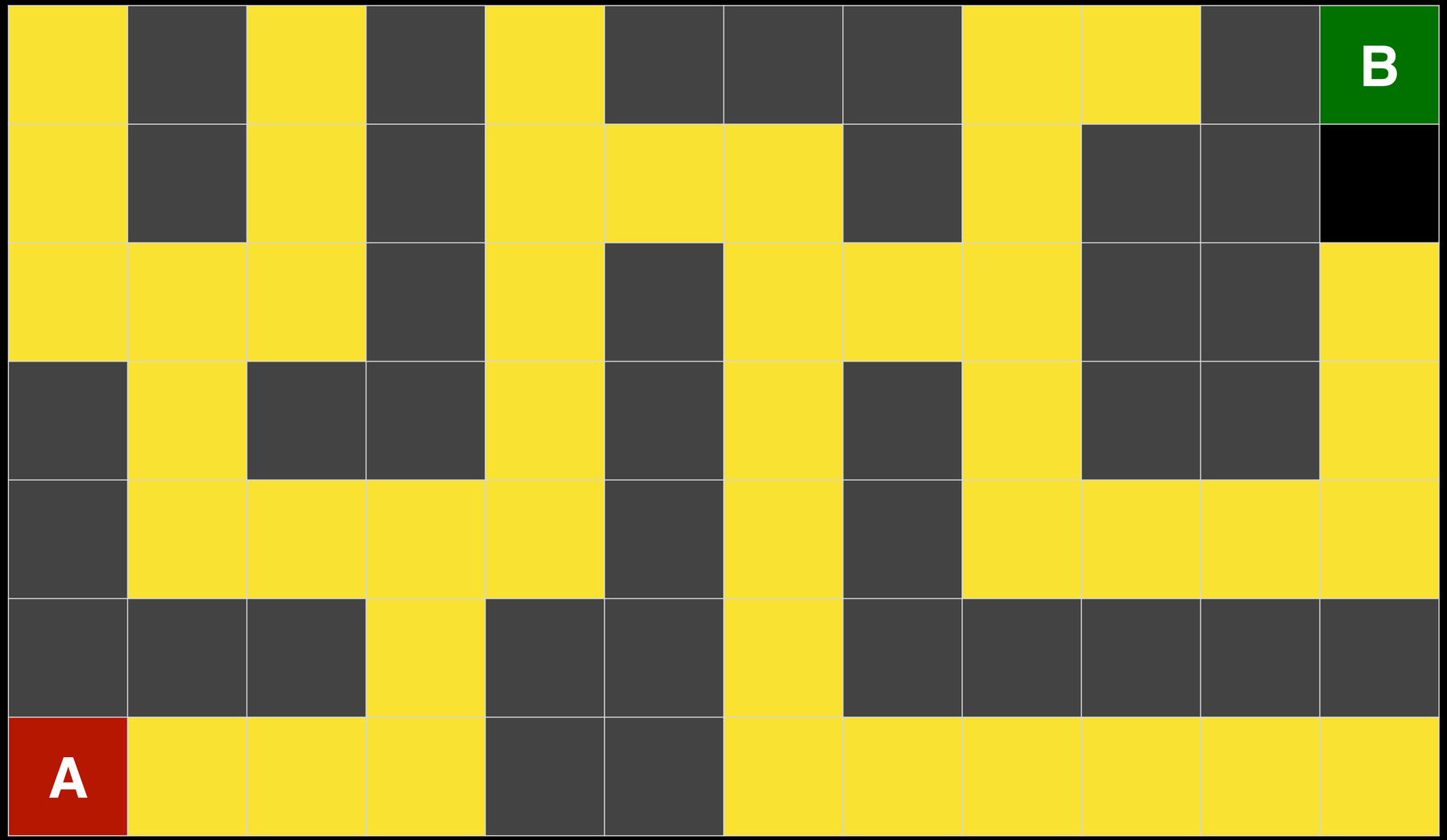

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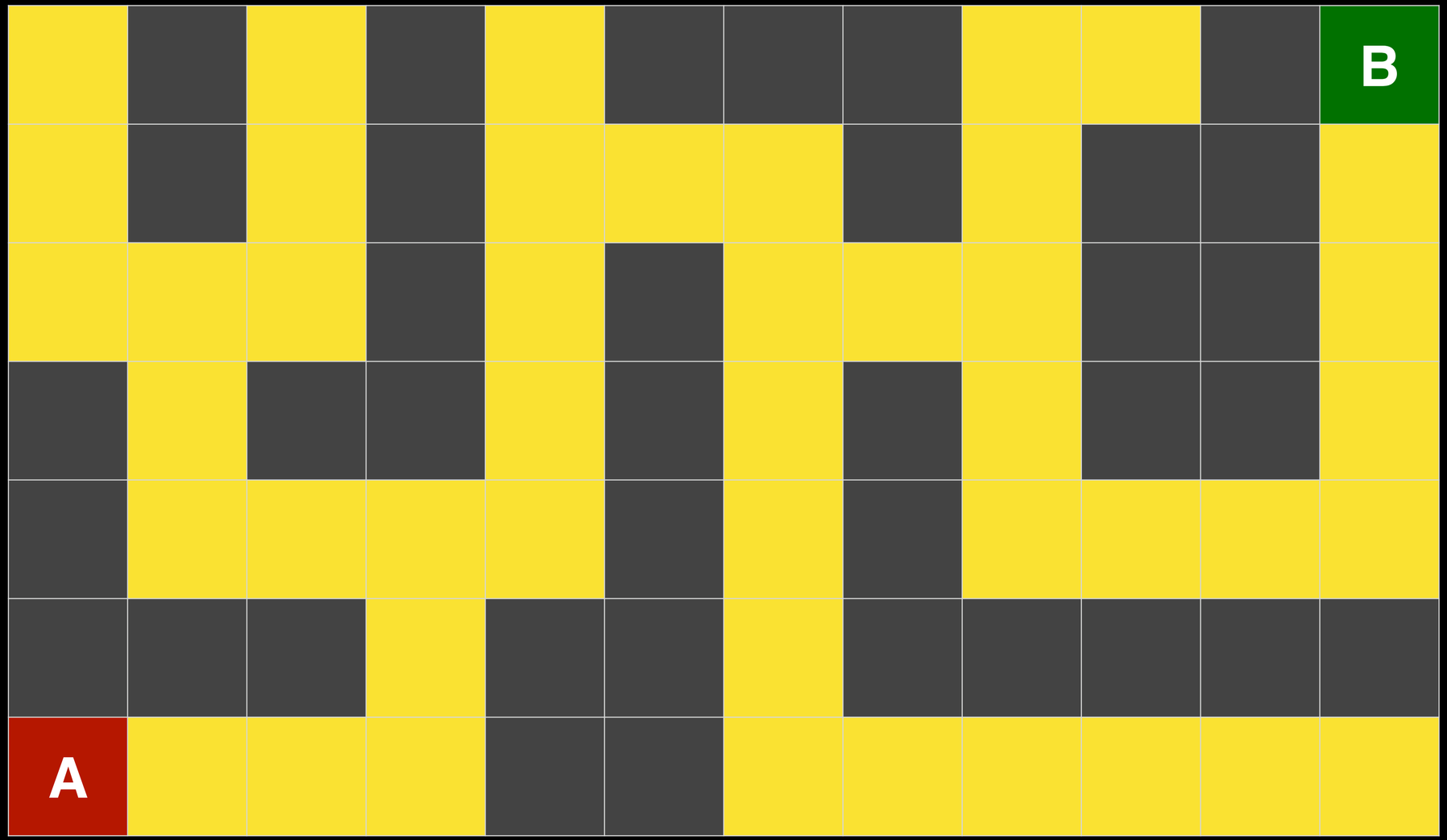

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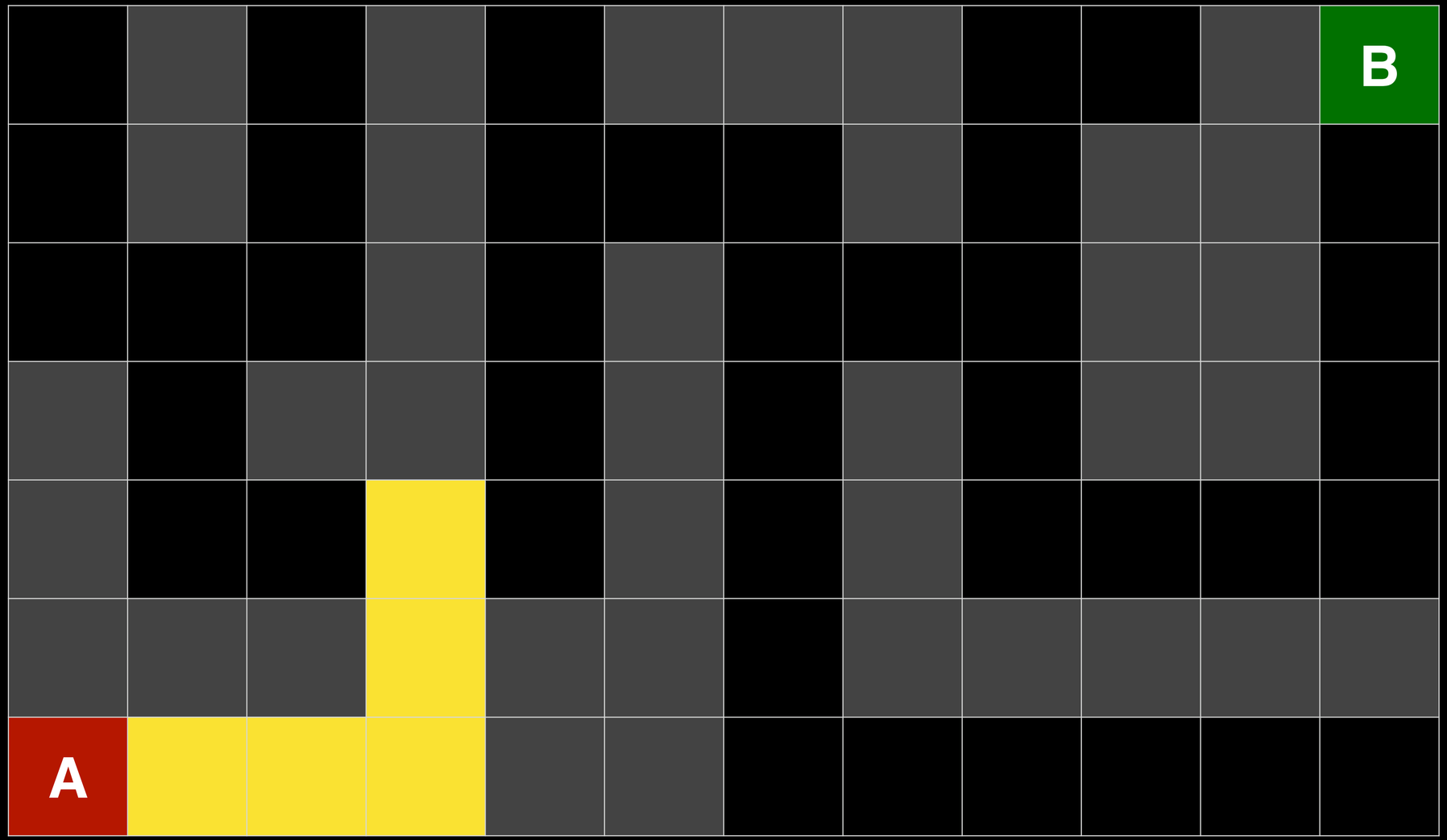


uninformed search

search strategy that uses no problem-specific knowledge

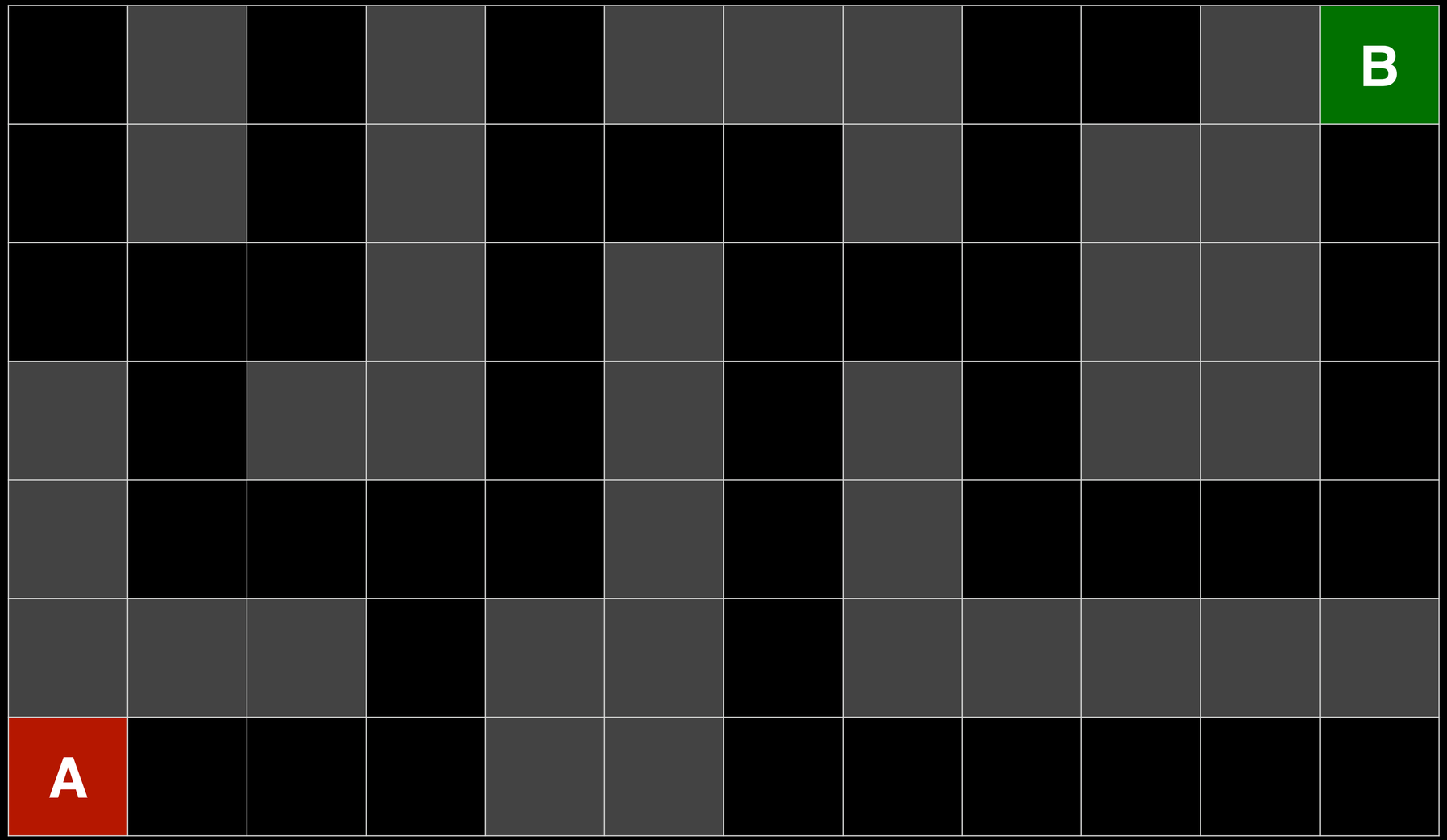
informed search

search strategy that uses problem-specific knowledge to find solutions more efficiently

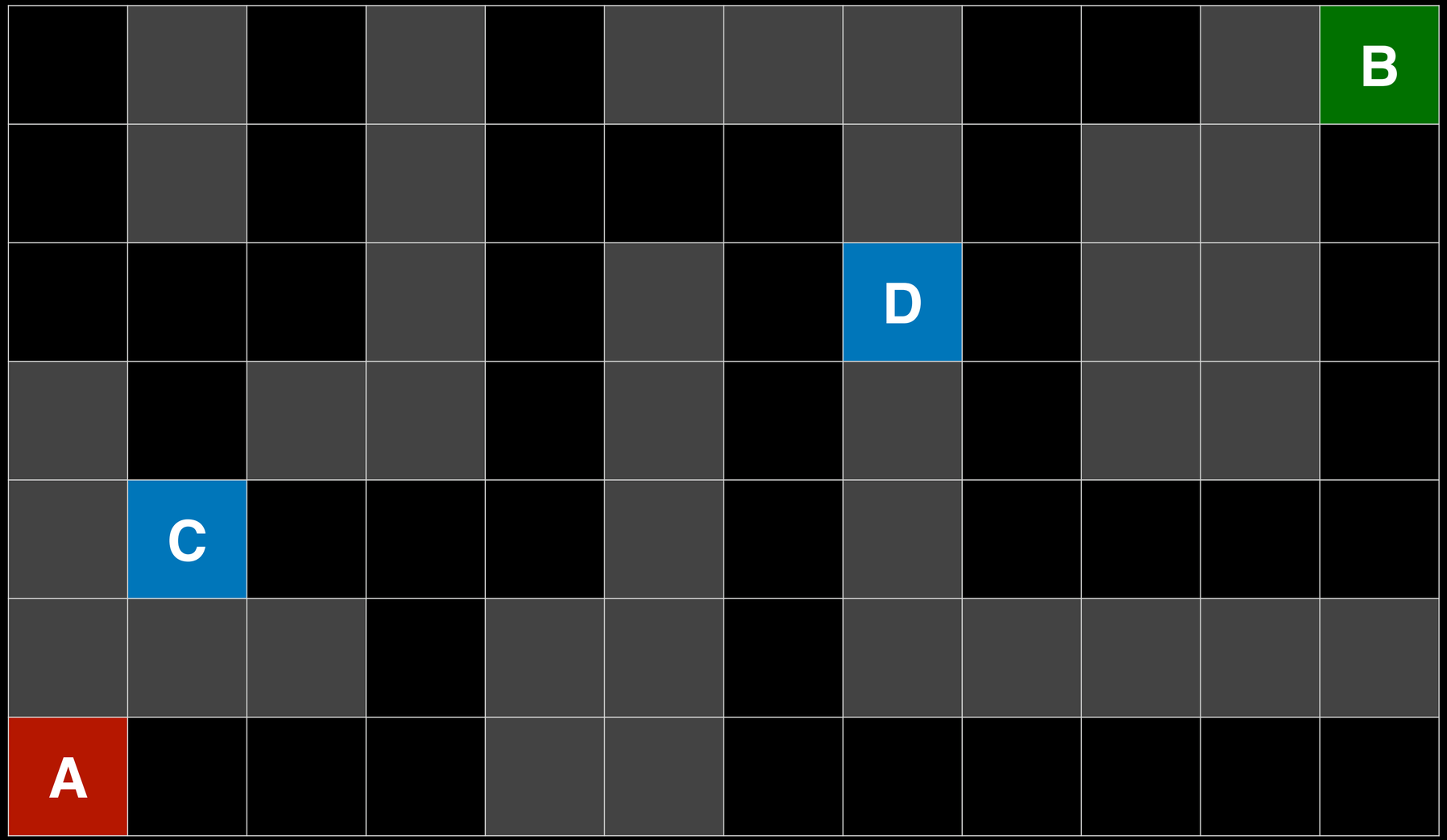
greedy best-first search

search algorithm that expands the node that is closest to the goal, as estimated by a heuristic function $h(n)$

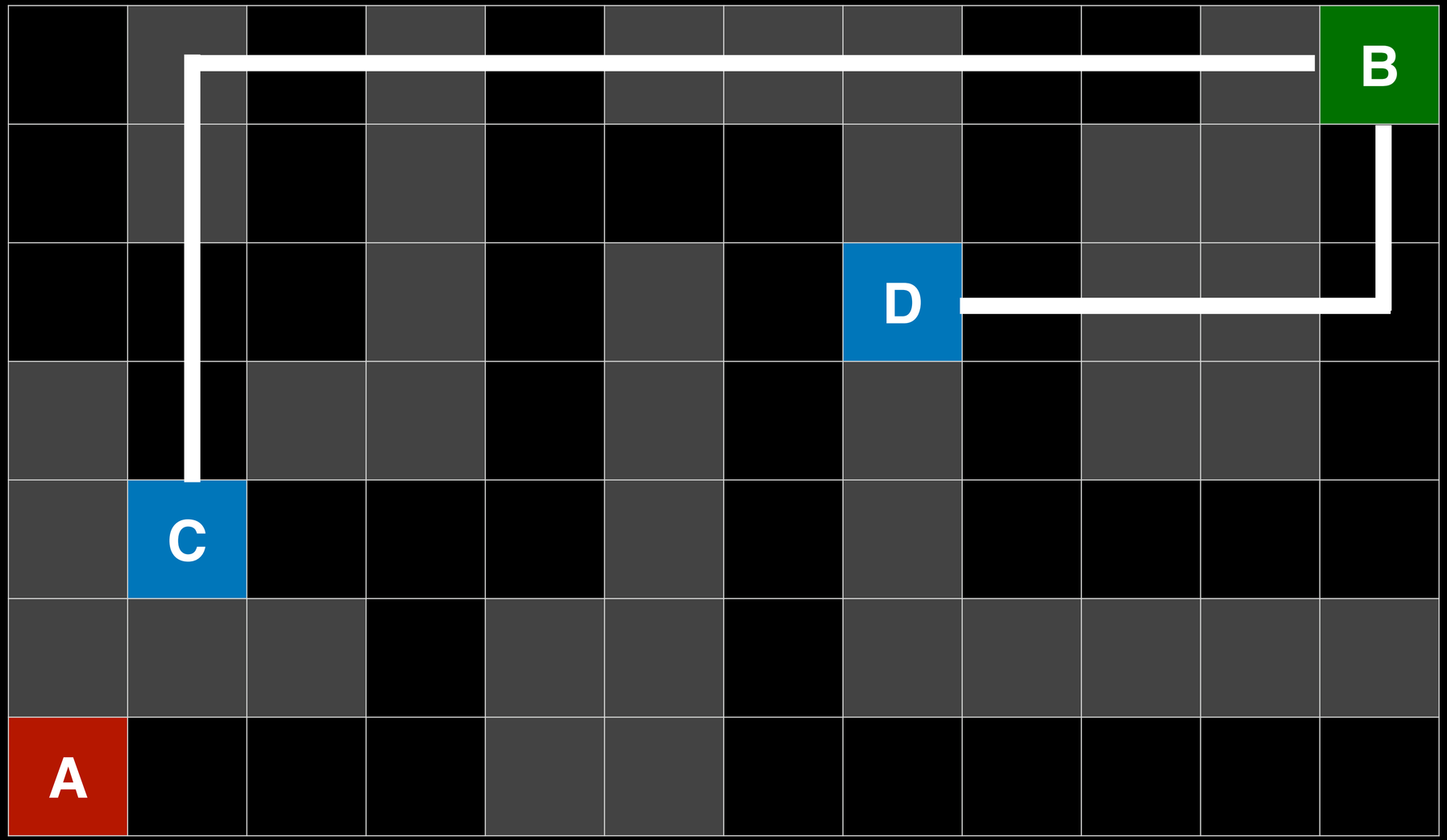
Heuristic function?



Heuristic function?



Heuristic function? Manhattan distance.



Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
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	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
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Greedy Best-First Search

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13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
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A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

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	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
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Greedy Best-First Search

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	14	13	12	11		9		7	6	5	4
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Greedy Best-First Search

11		9		7				3	2		**B**
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	14	13	12	11		9		7	6	5	4
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Greedy Best-First Search

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	14	13	12	11		9		7	6	5	4
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Greedy Best-First Search

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13	12	11		9		7	6	5			2
	13			10		8		6			3
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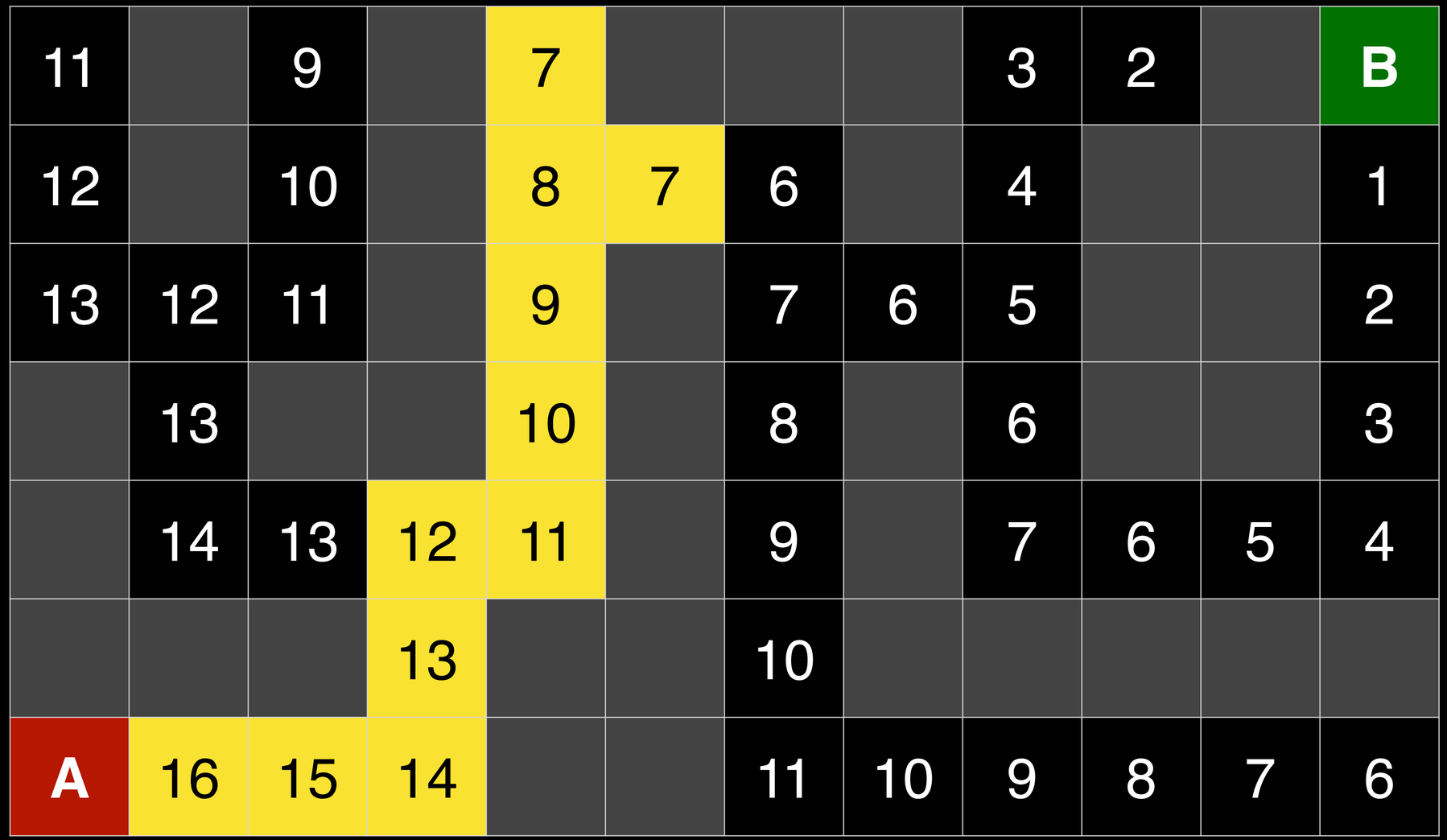
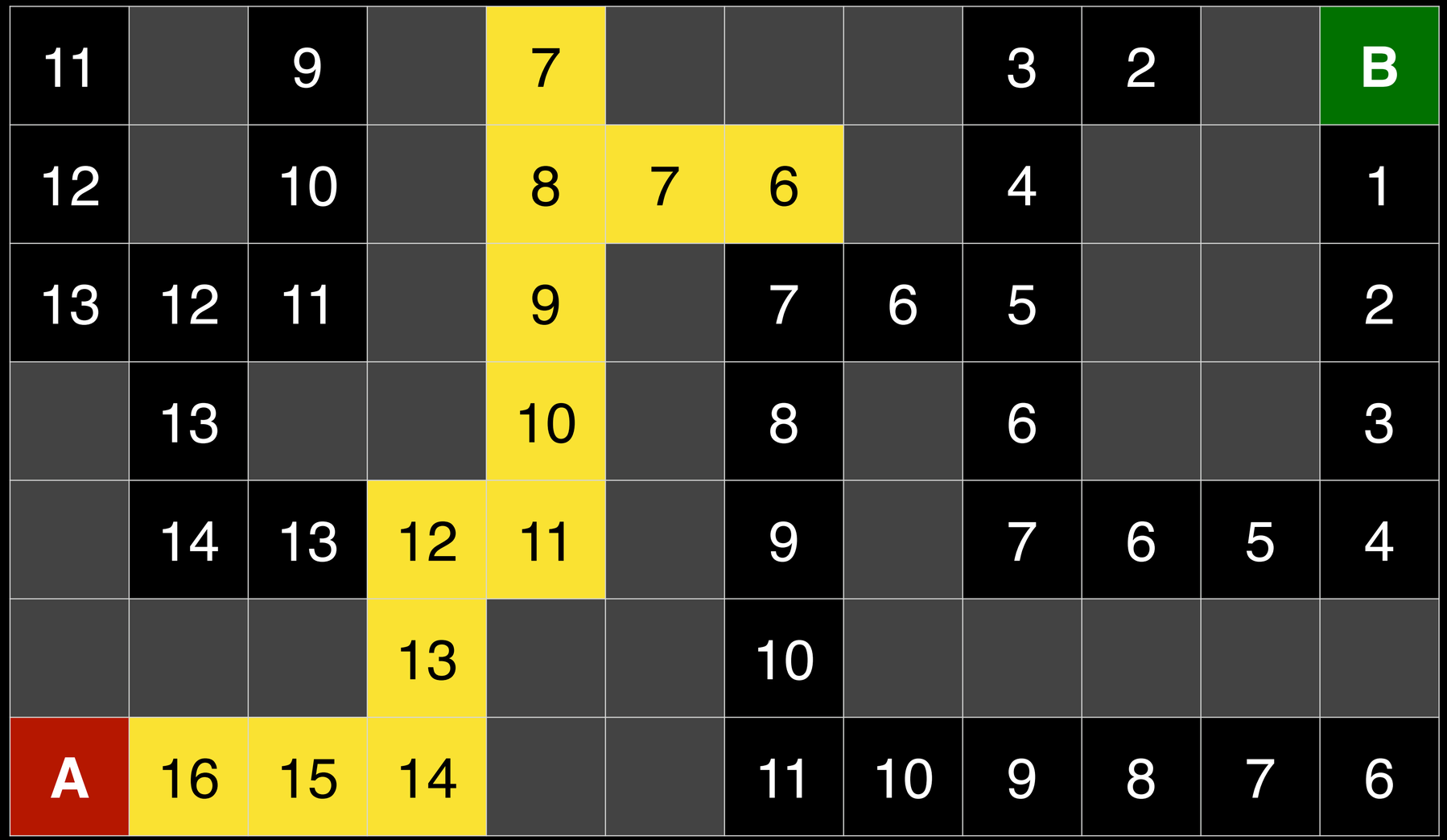
Greedy Best-First Search



Greedy Best-First Search



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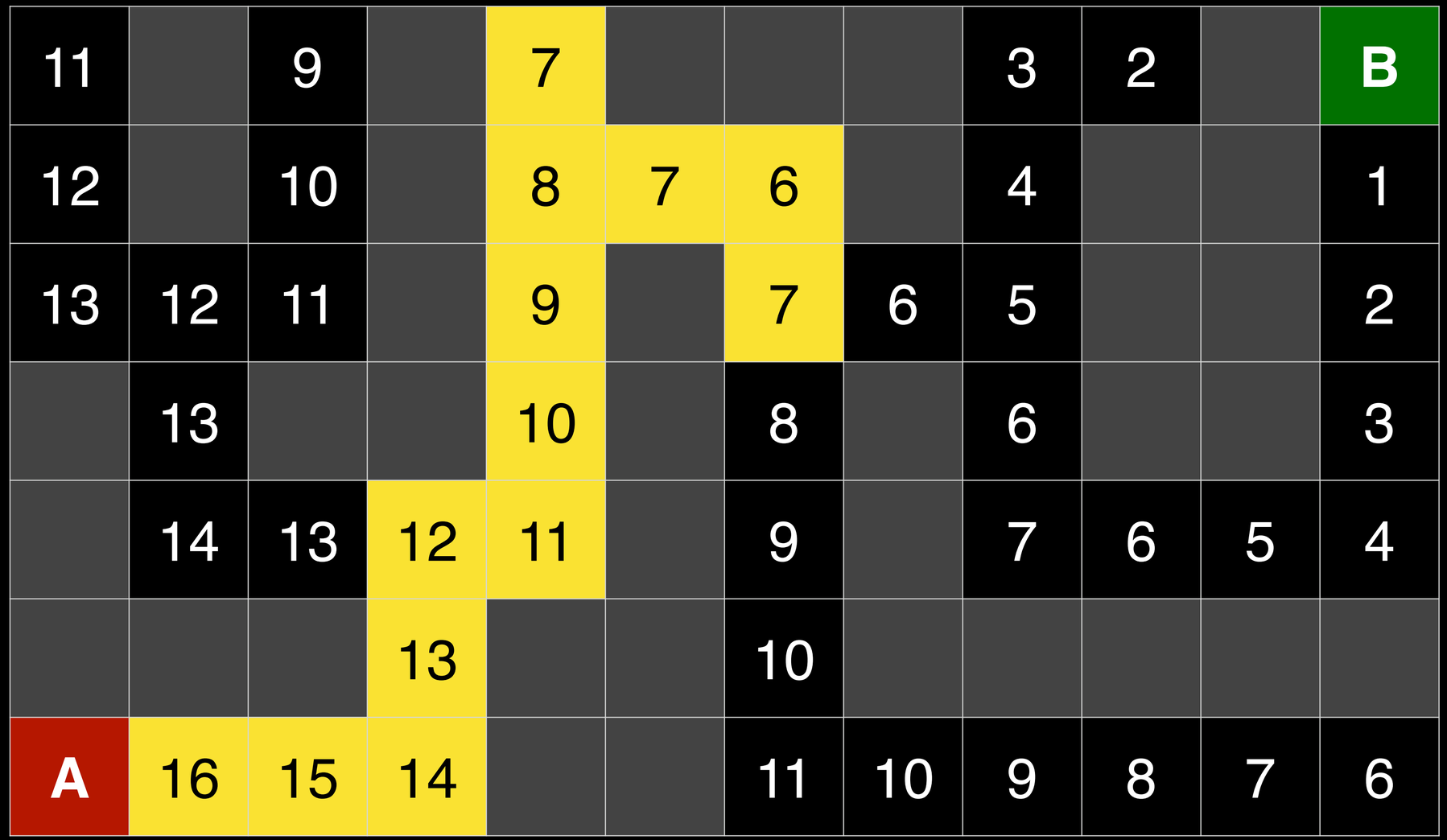

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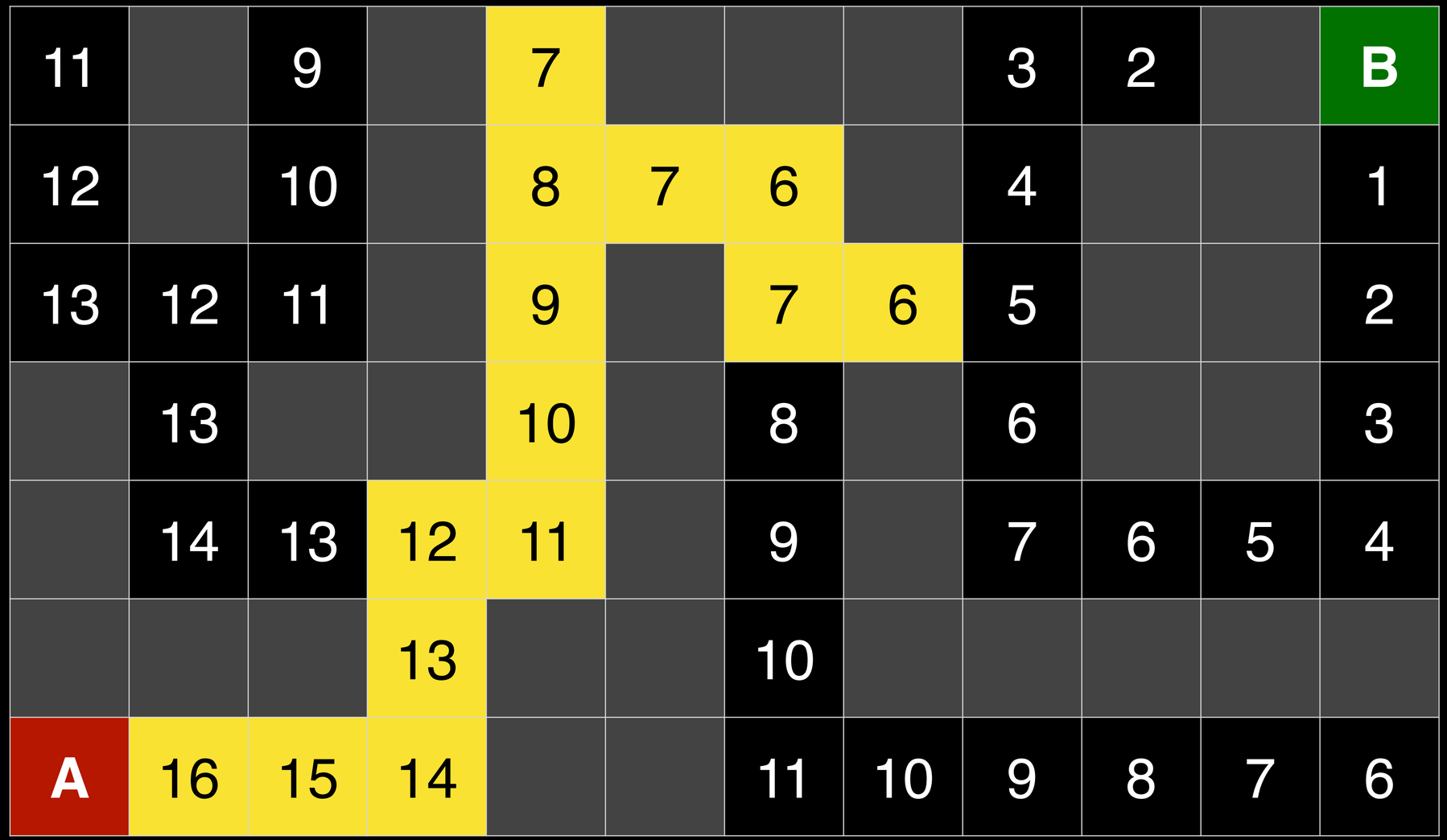
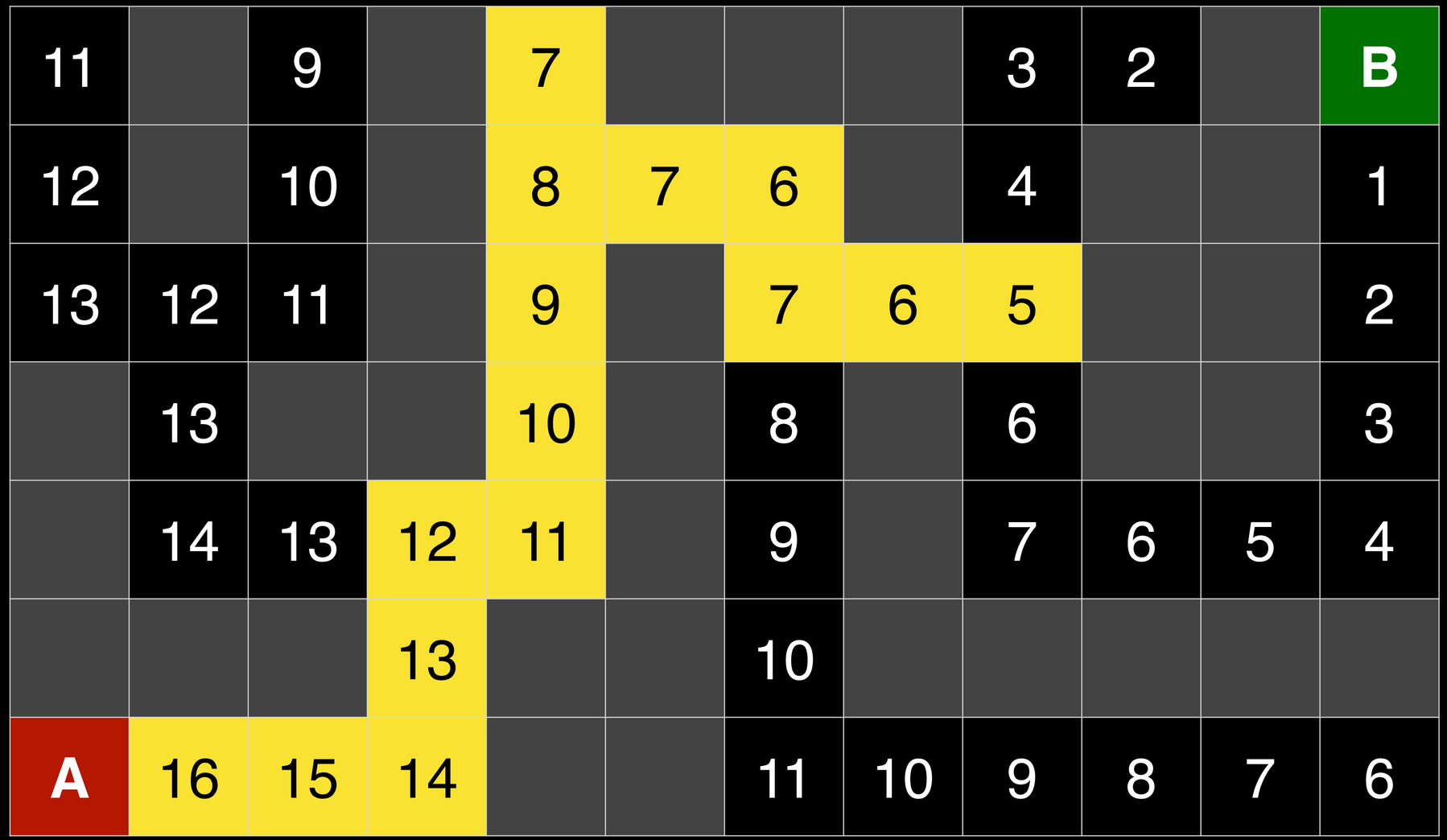
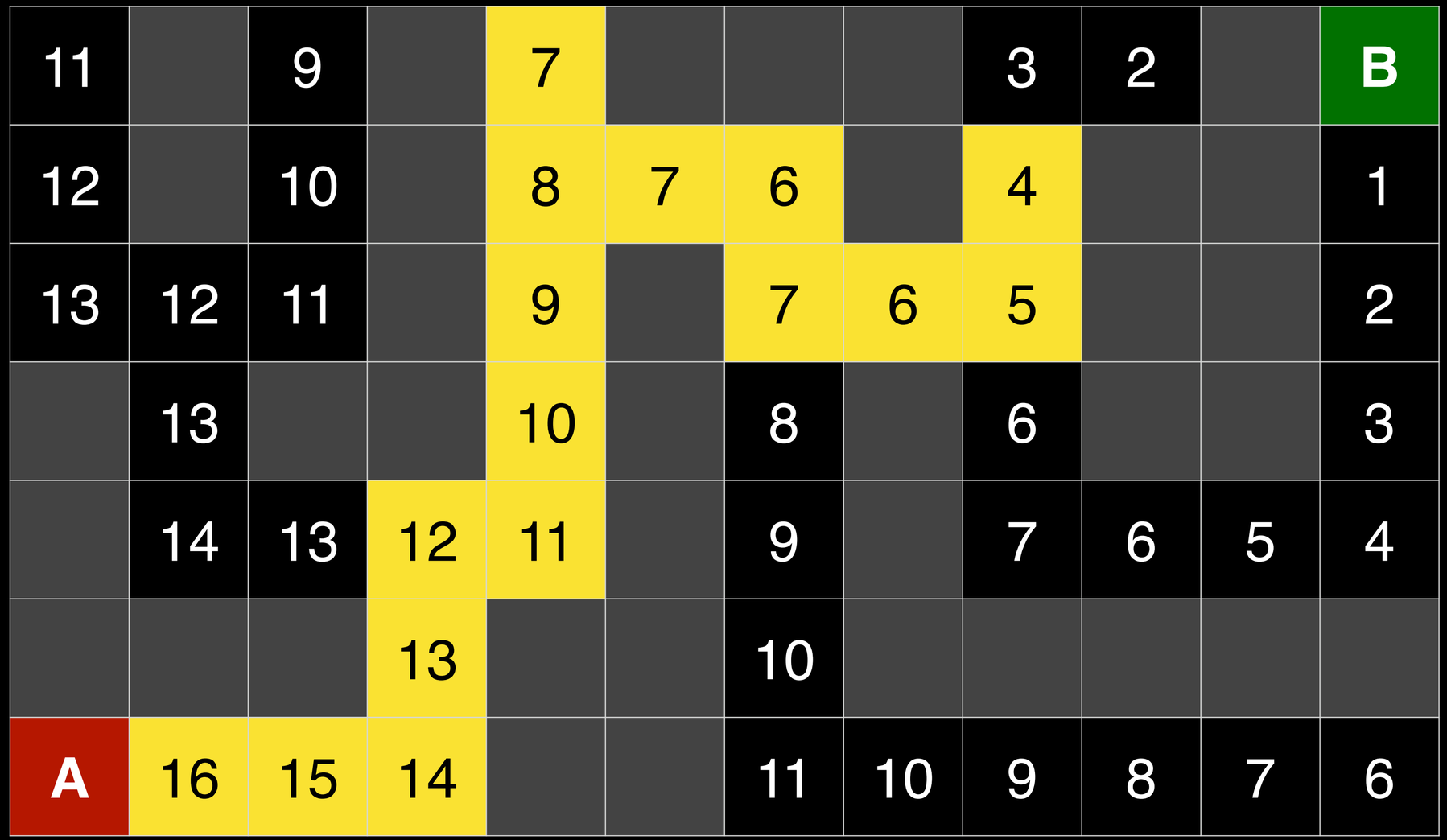


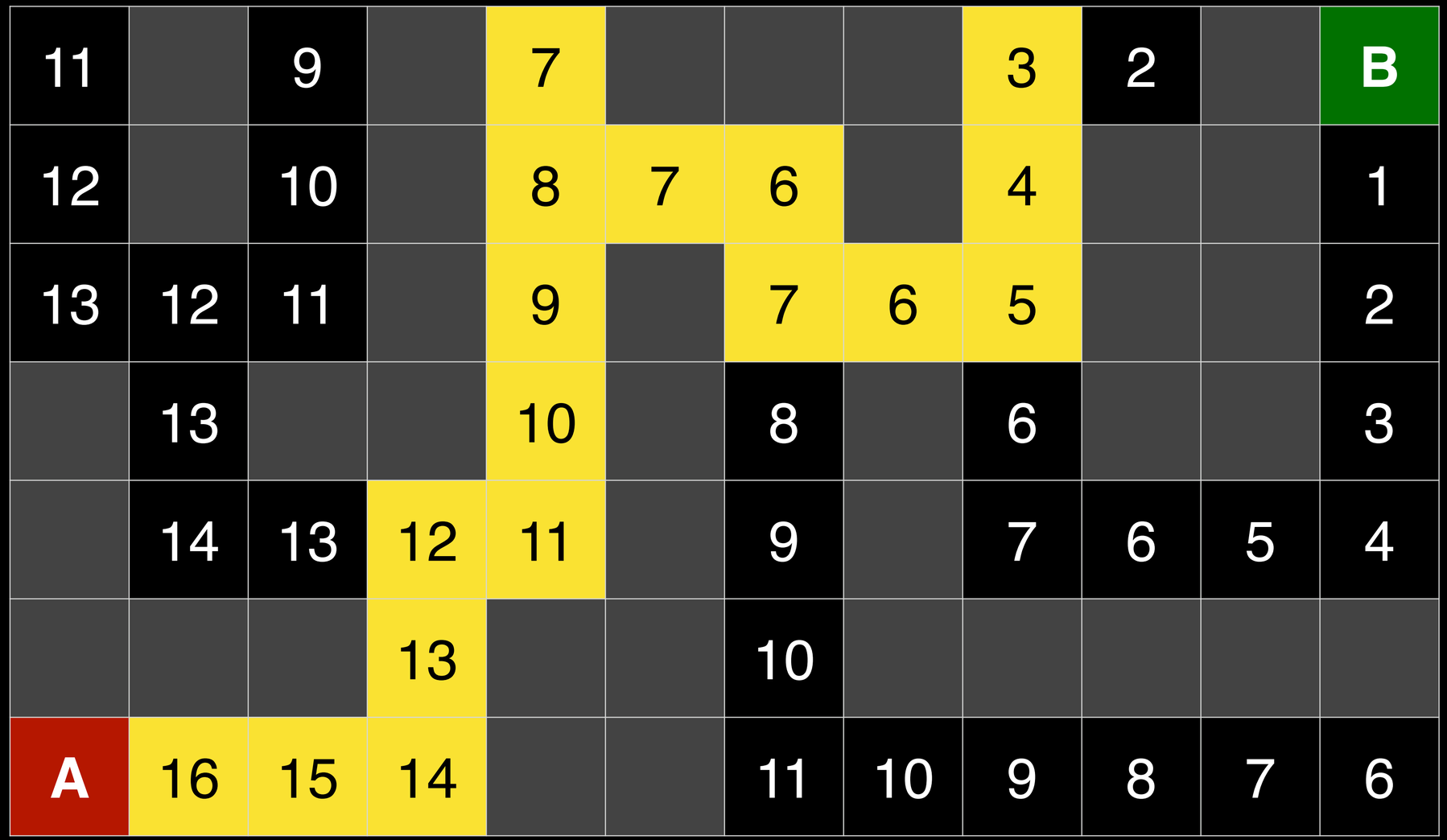
Greedy Best-First Search



Greedy Best-First Search



Greedy Best-First Search



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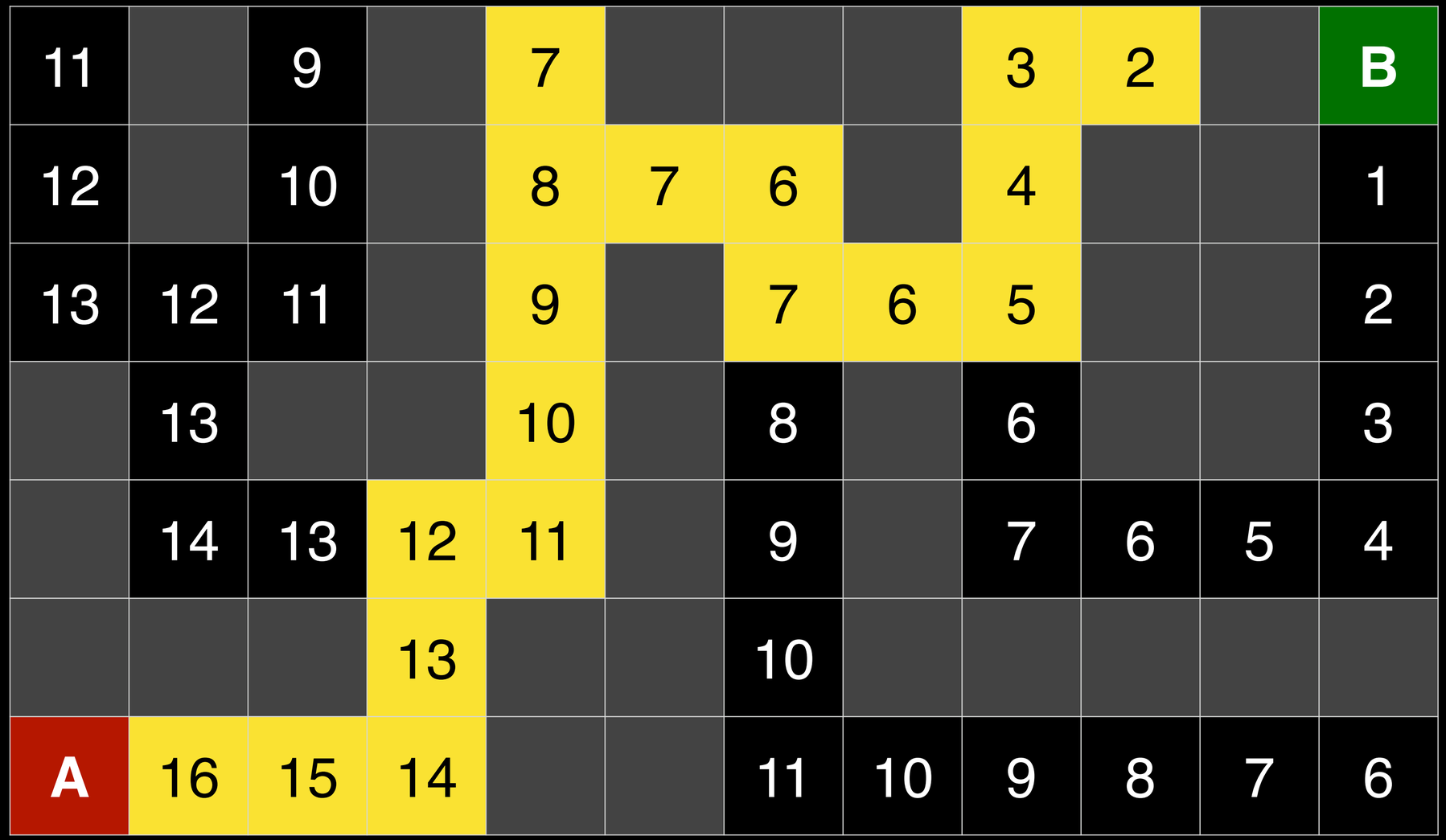

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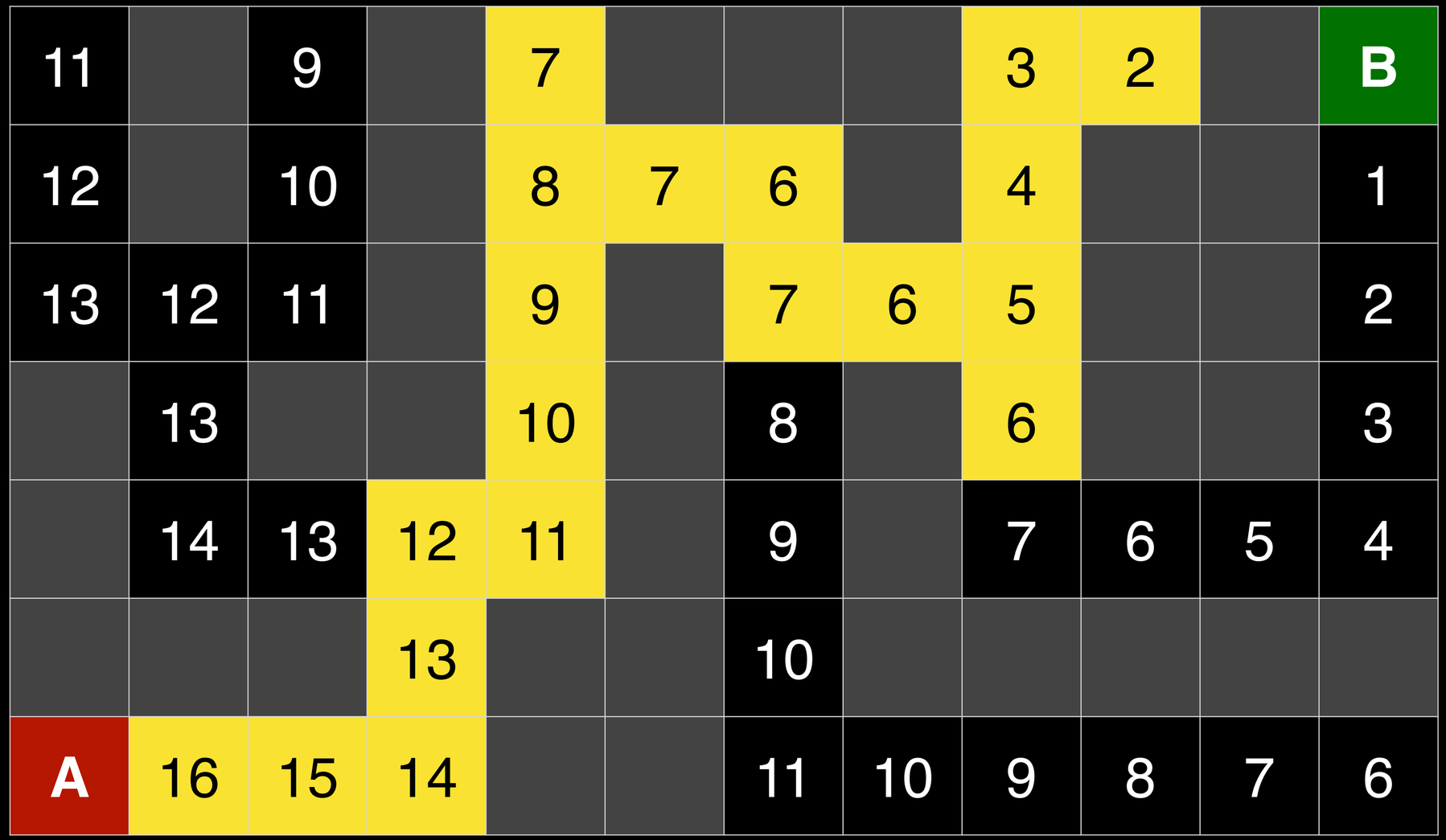

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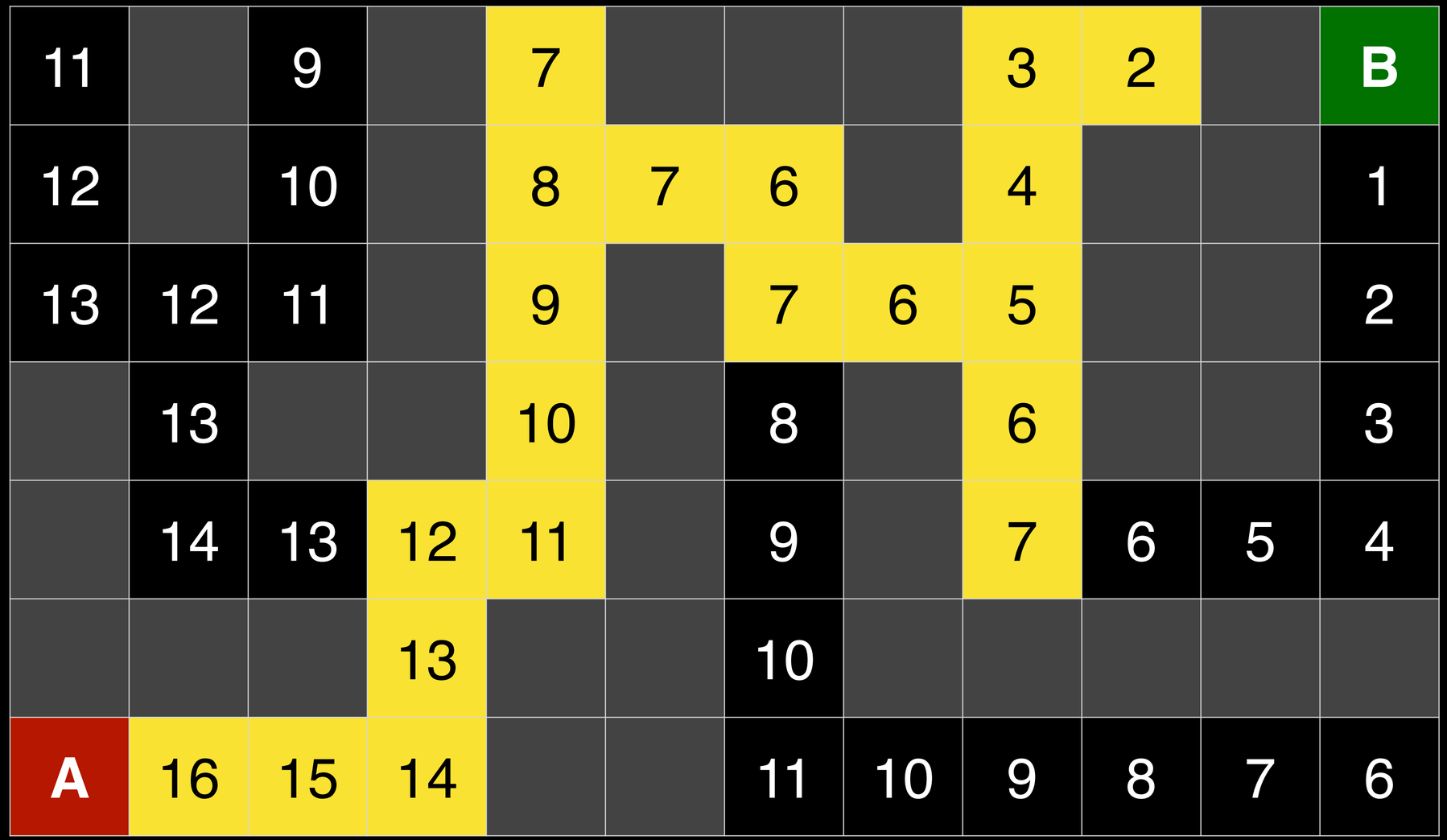

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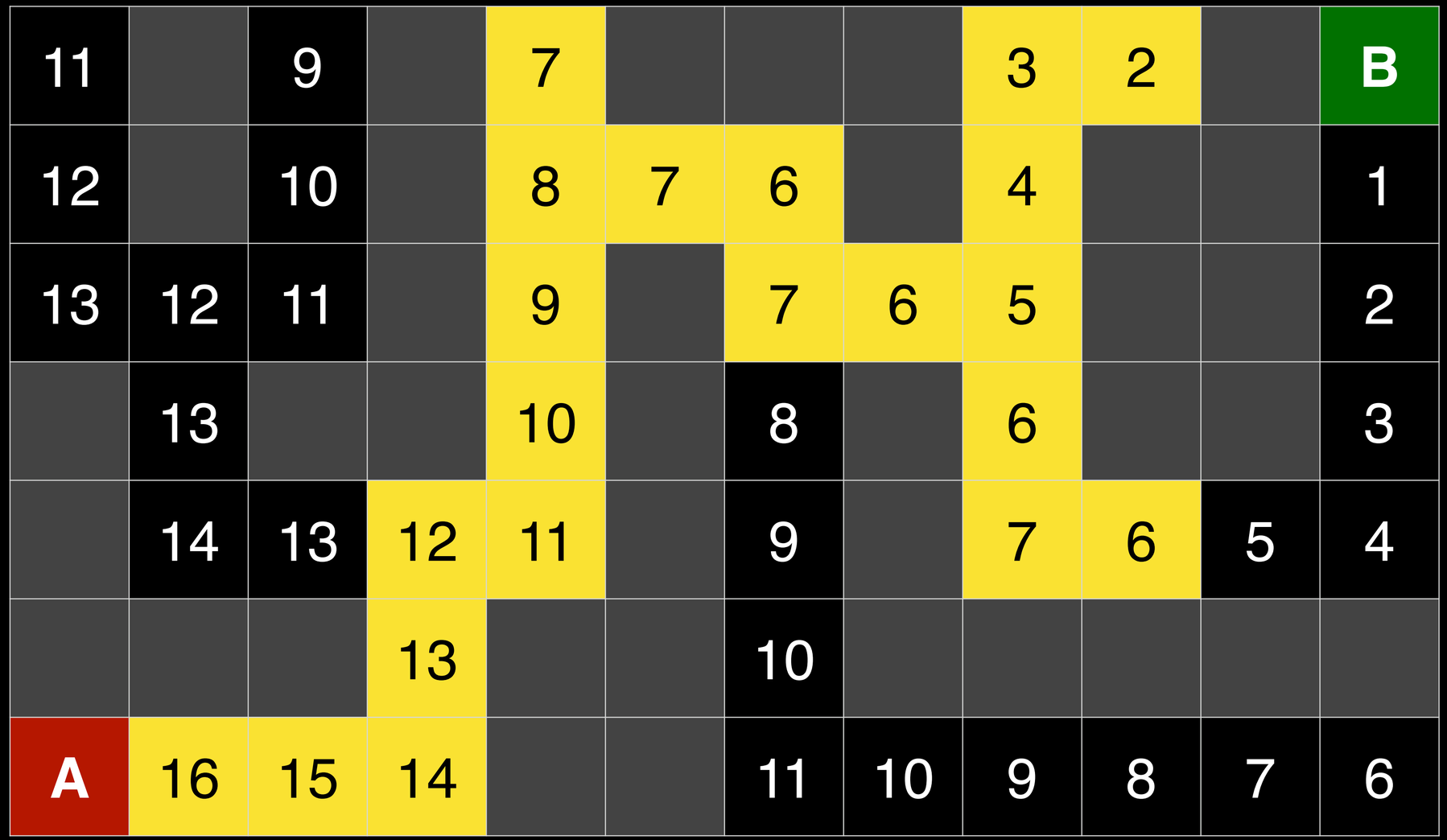

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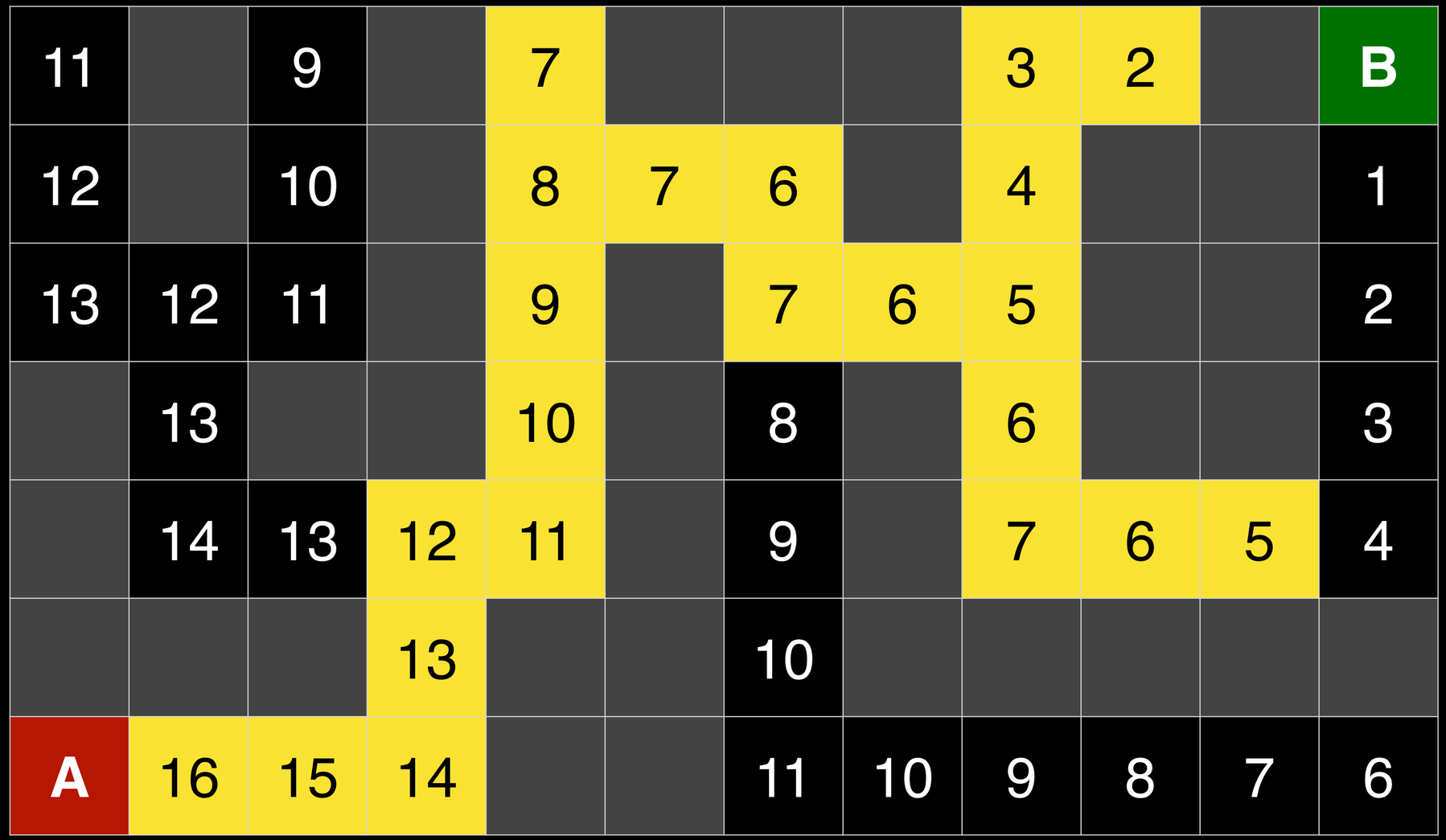

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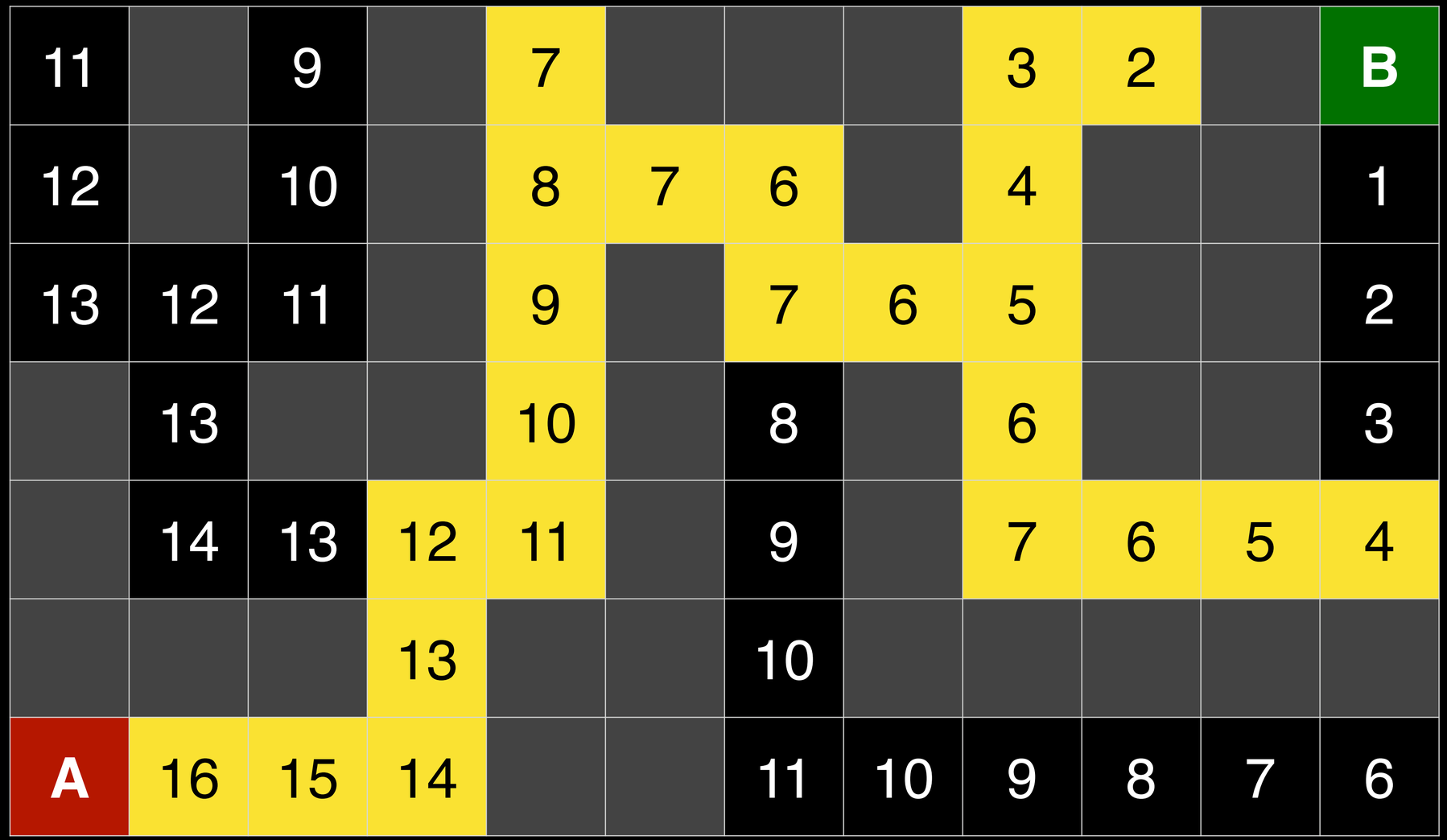

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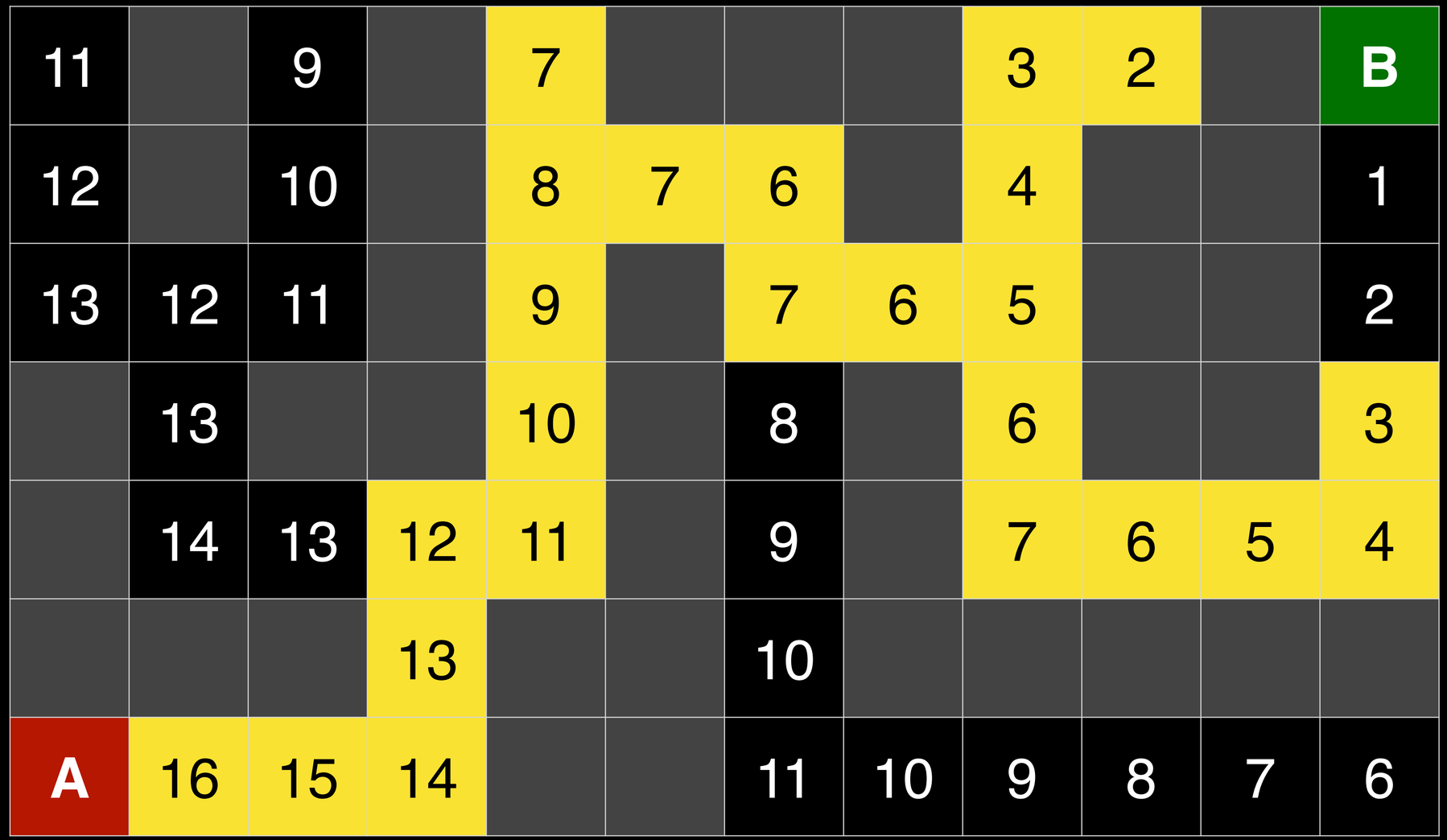
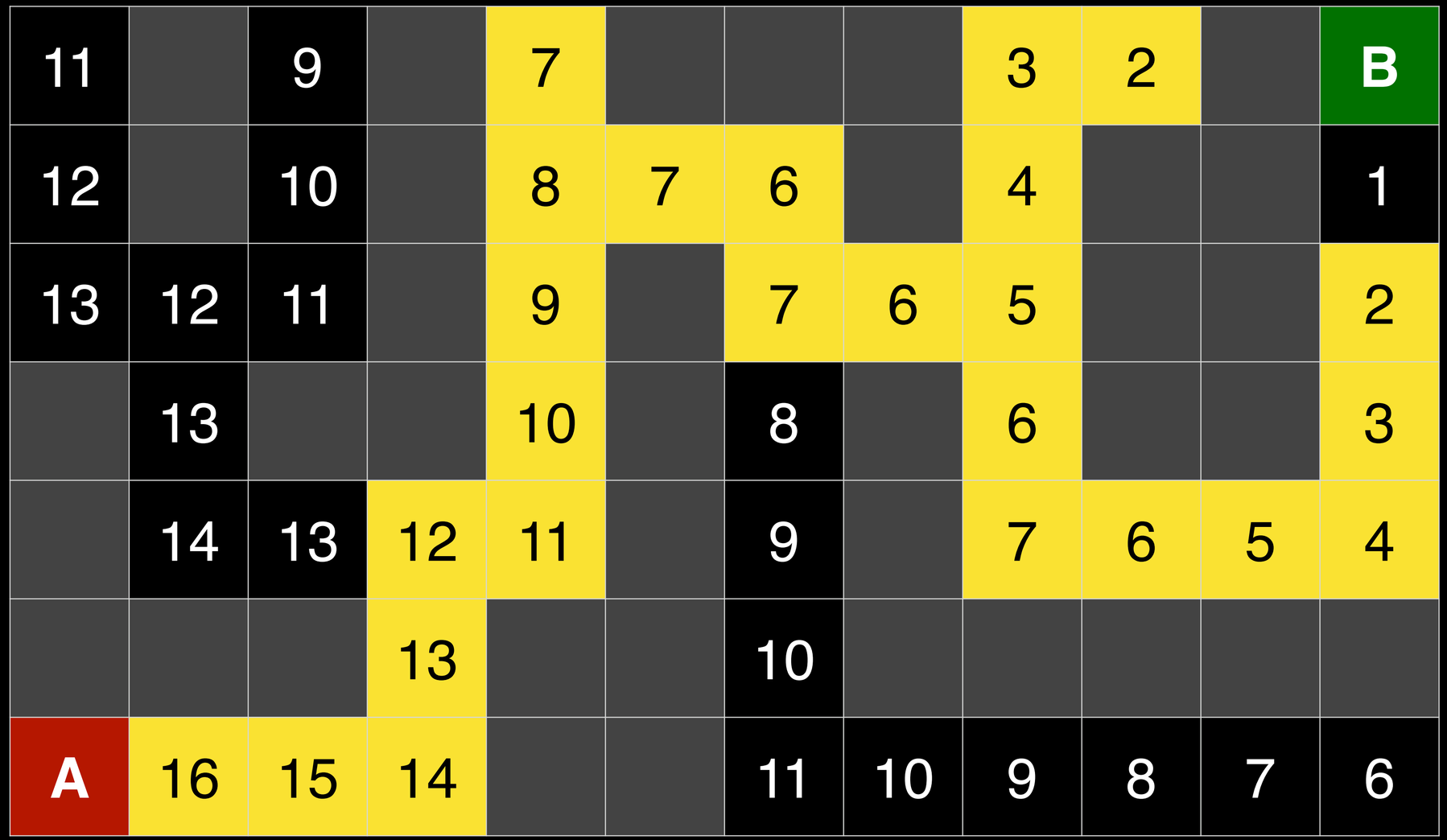
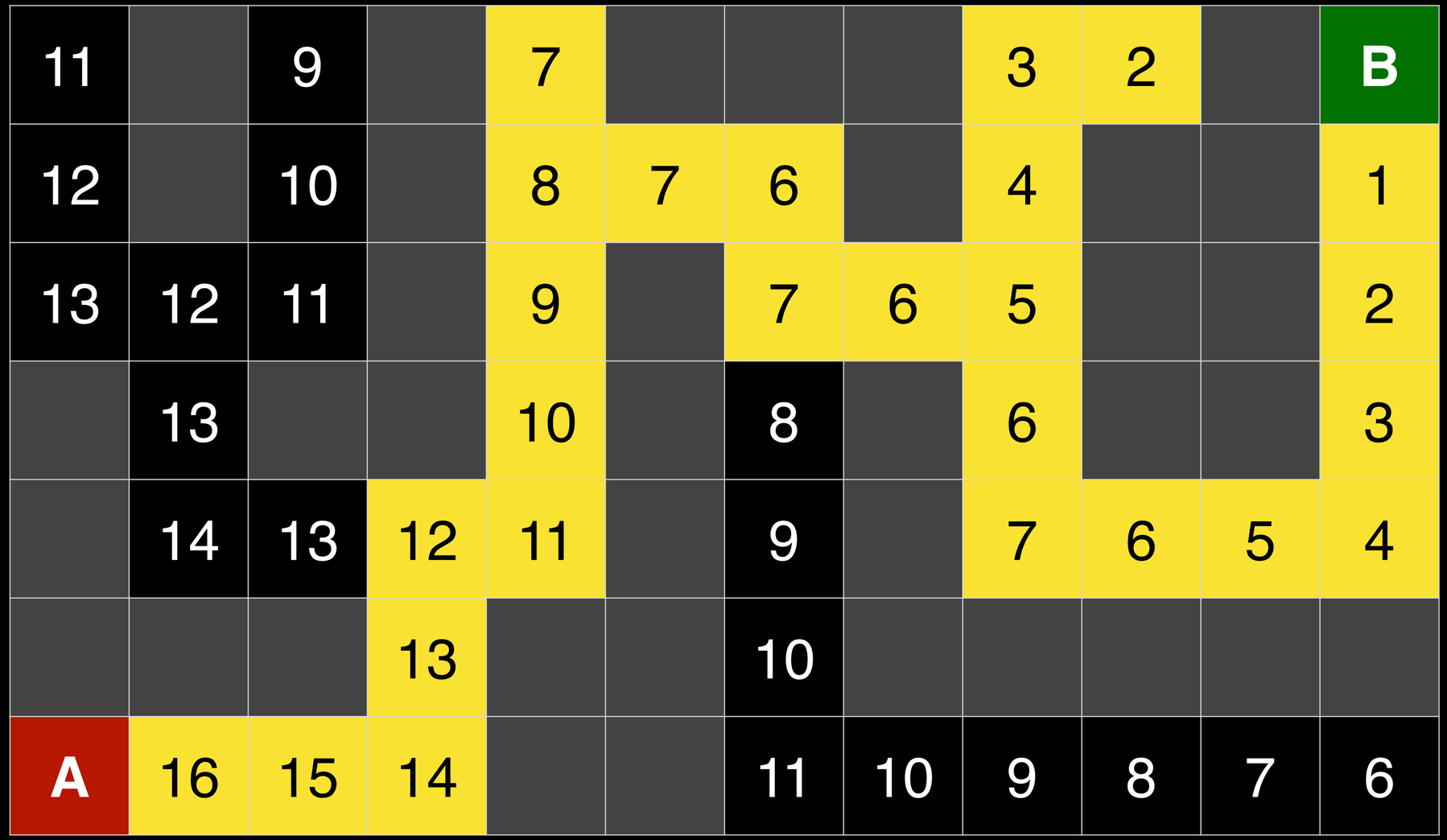


Greedy Best-First Search



Greedy Best-First Search



Greedy Best-First Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

Greedy Best-First Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

Greedy Best-First Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

A* search

search algorithm that expands node with lowest value of $g(n) + h(n)$

$g(n)$ = cost to reach node
$h(n)$ = estimated cost to goal

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	1+16	15	14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	1+16	2+15	14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	9	8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	14	6+13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	9	8	7	6	5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	8	7	6	5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	7	6	5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	6	5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	16+5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	16+5	17+4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	16+5	17+4	18+3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	16+5	17+4	18+3	19+2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

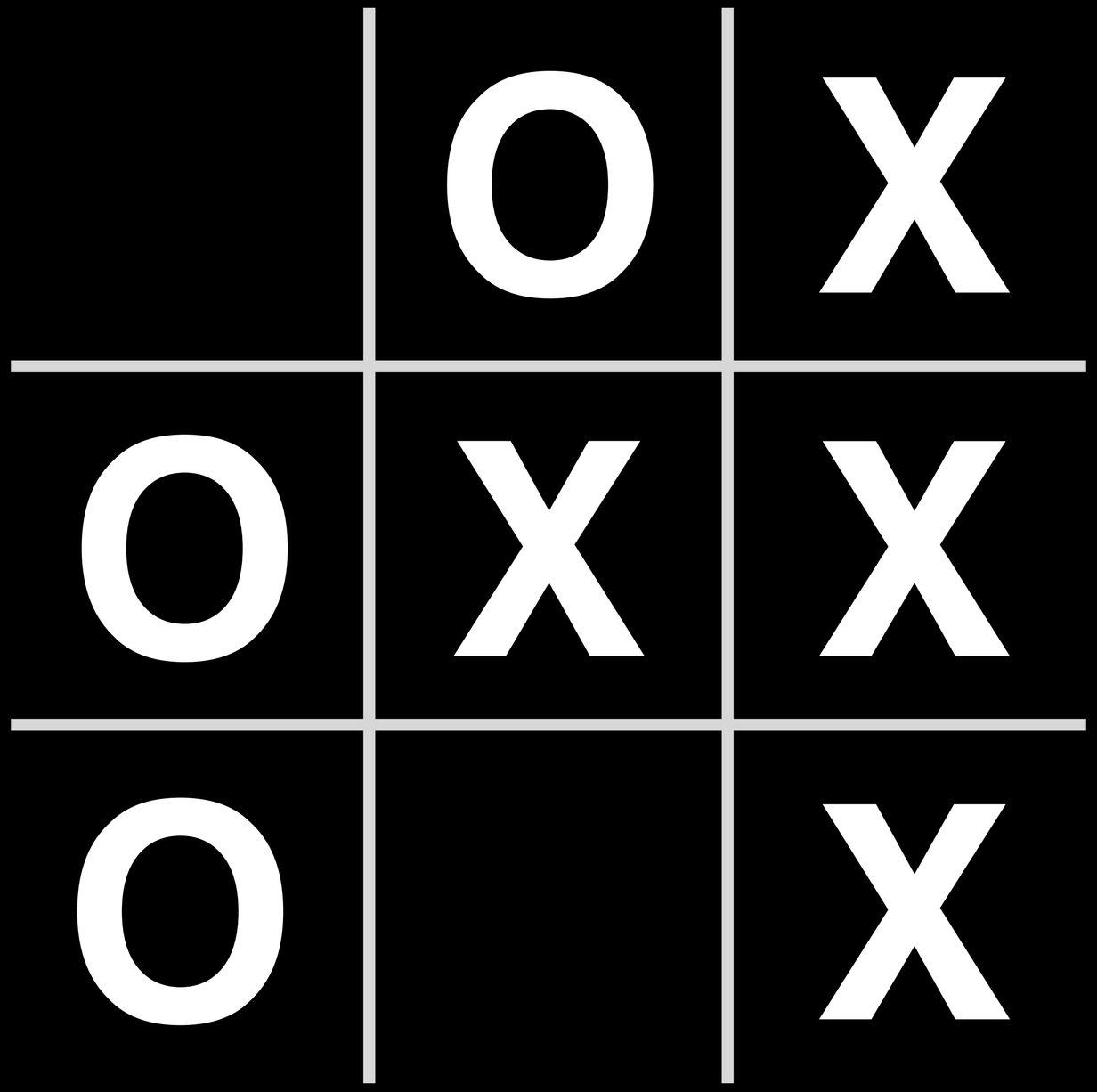
	11+10	12+9	13+8	14+7	15+6	16+5	17+4	18+3	19+2	20+1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

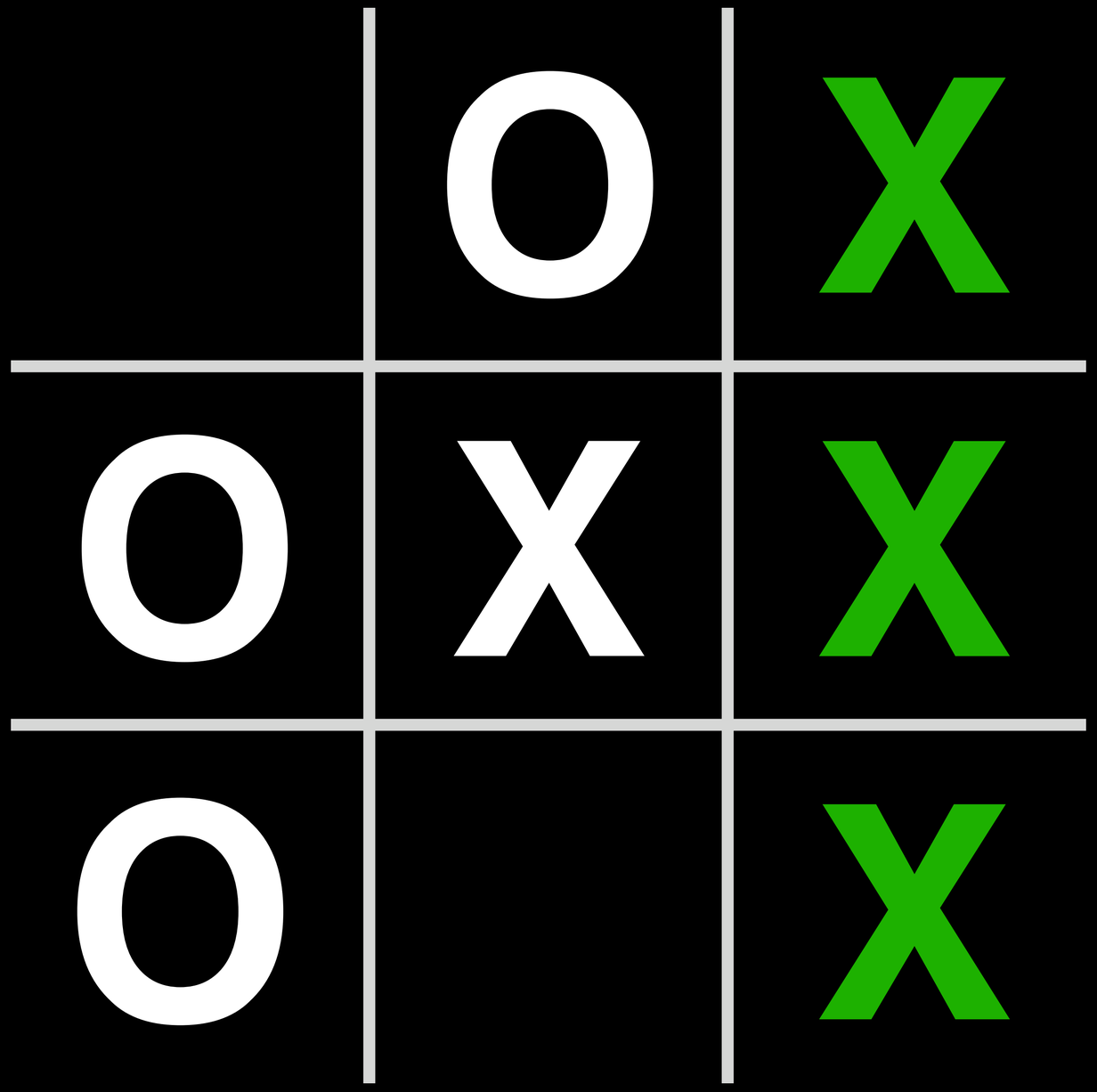
A* search

optimal if

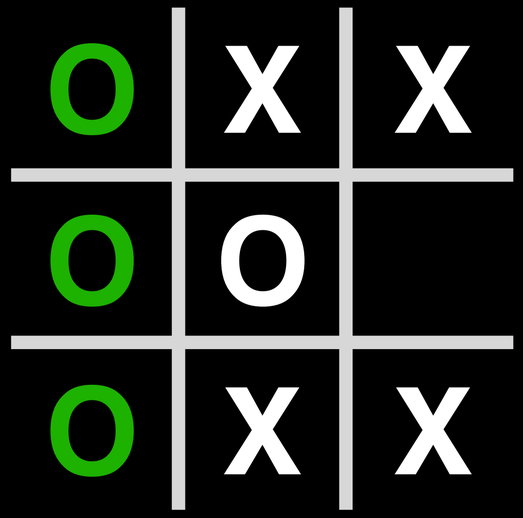
- $h(n)$ is admissible (never overestimates the true cost), and
- $h(n)$ is consistent (for every node n and successor n' with step cost c, $h(n) \leq h(n') + c$)

Adversarial Search

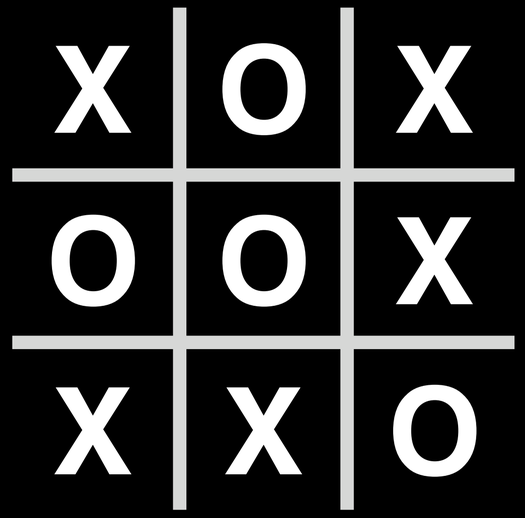

O X
O X X
O X


O
X
O
X
X
O
X

Minimax



-1



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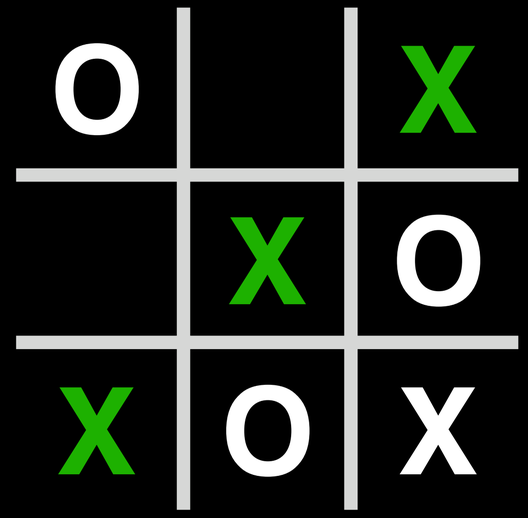


1

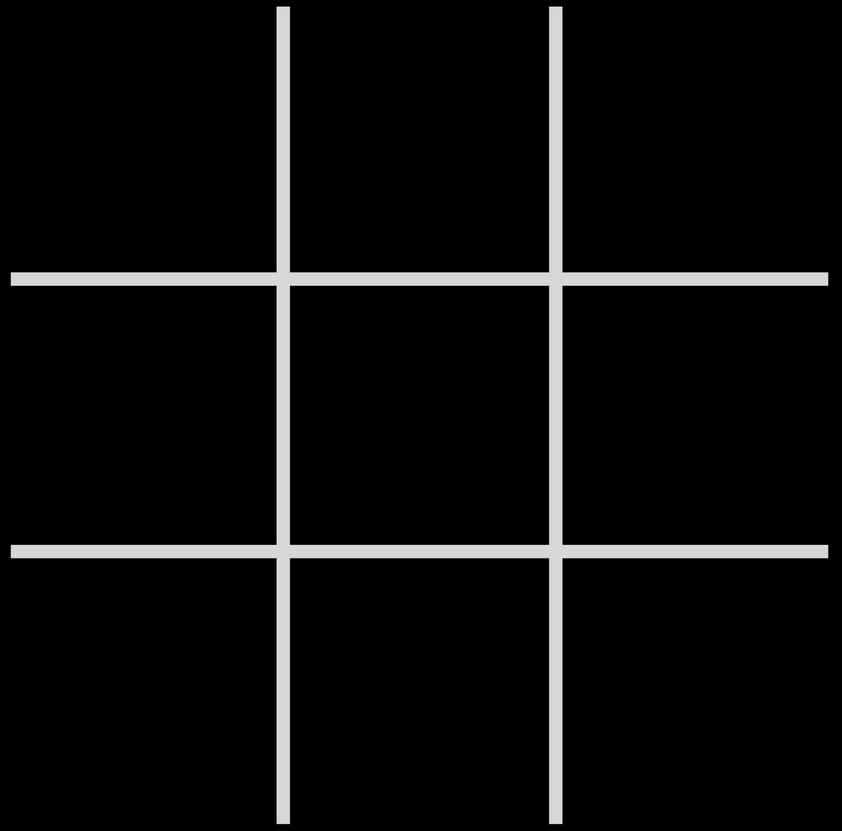
Minimax

- MAX (X) aims to maximize score.
- MIN (O) aims to minimize score.

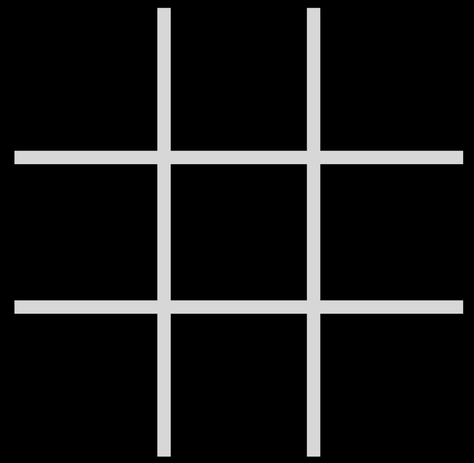
Game

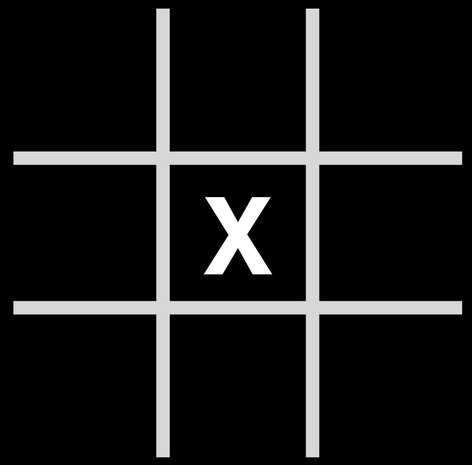
- S_0 : initial state
- PLAYER(s) : returns which player to move in state s
- ACTIONS(s) : returns legal moves in state s
- RESULT(s, a) : returns state after action a taken in state s
- TERMINAL(s) : checks if state s is a terminal state
- UTILITY(s) : final numerical value for terminal state s

Initial State



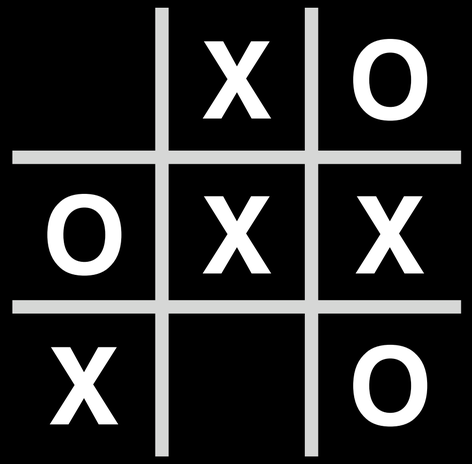
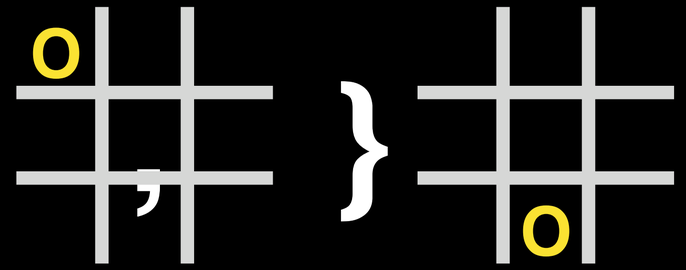
PLAYER(*s*)

PLAYER() = **X**

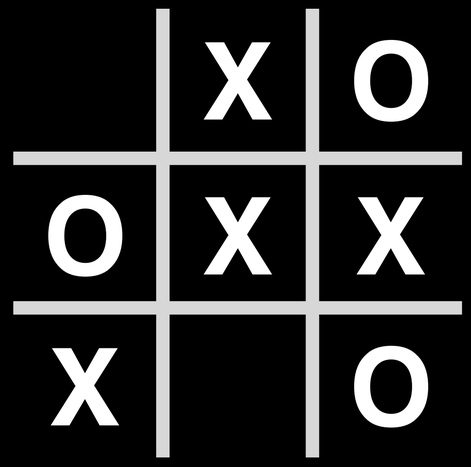
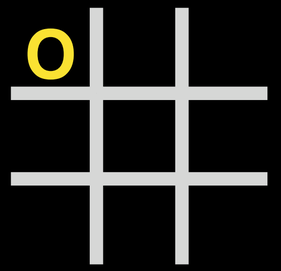
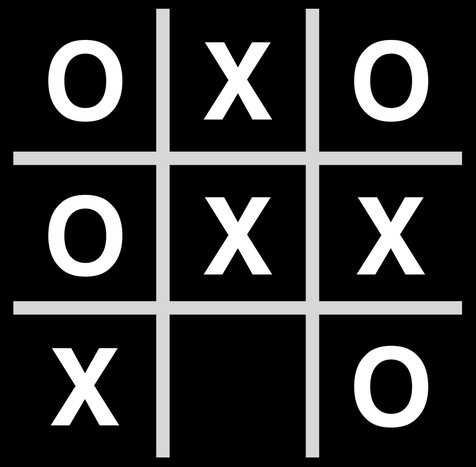
PLAYER(

) = **O**

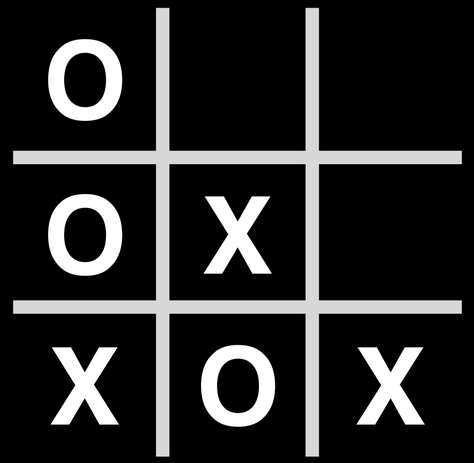
ACTIONS(*s*)

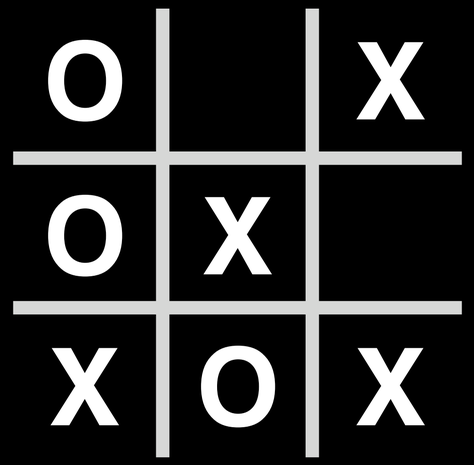
ACTIONS() = {  }

RESULT(*s*, *a*)

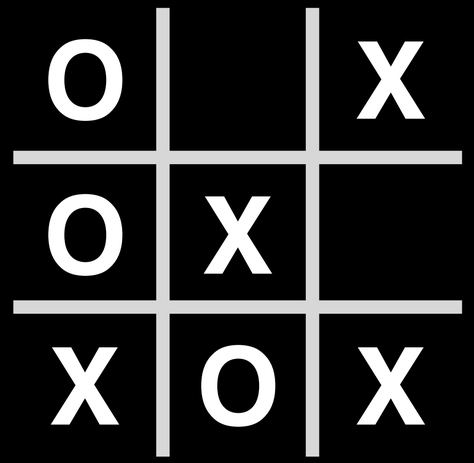
RESULT( , ) = 

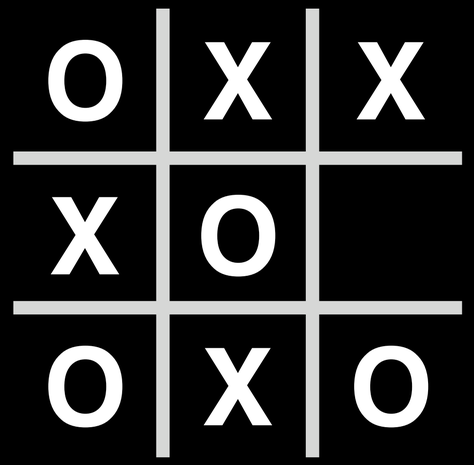
TERMINAL(*s*)

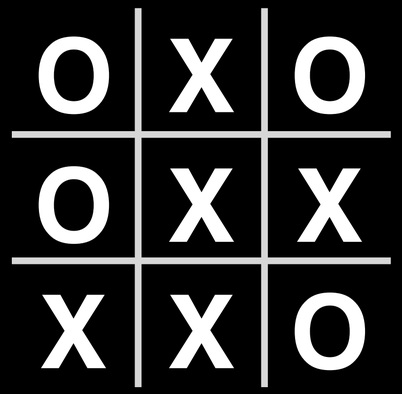
TERMINAL() = false

TERMINAL() = **true**

Utility(s)

Utility() = 1

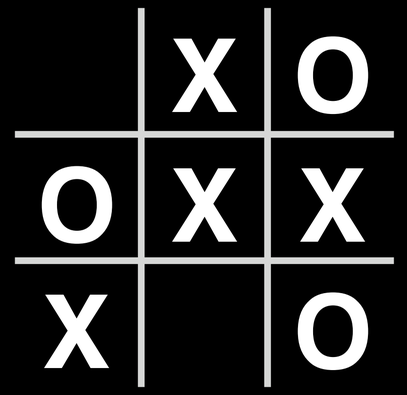
Utility() = -1

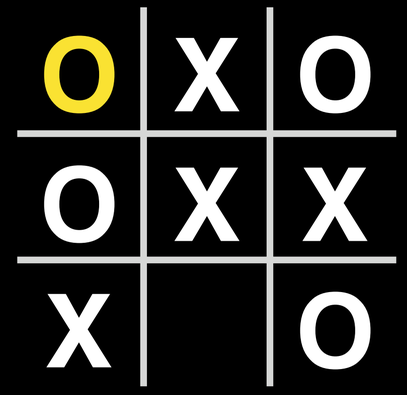


VALUE: 1

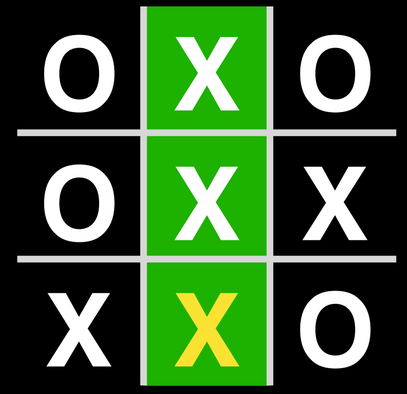
PLAYER(s) = O

MIN-VALUE: 0

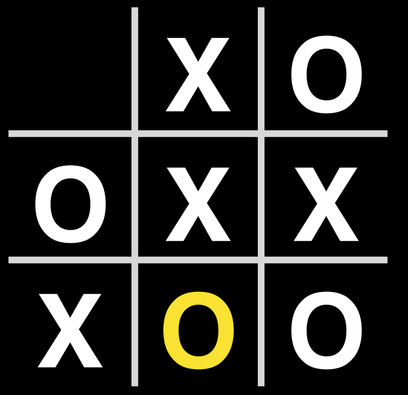




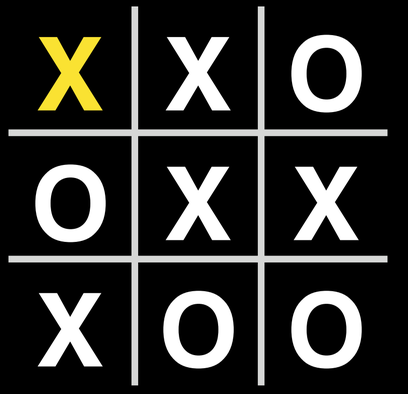
MAX-VALUE: 1



VALUE: 1



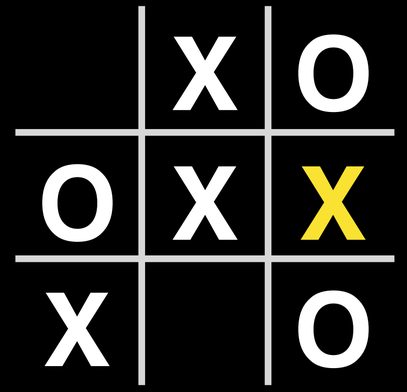
MAX-VALUE: 0

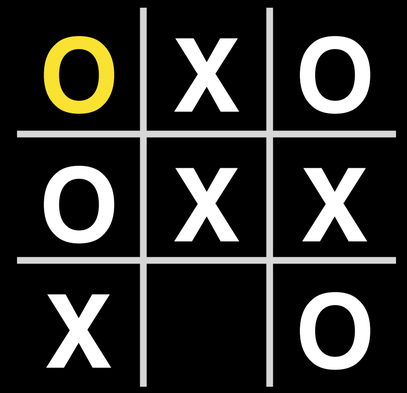


VALUE: 0

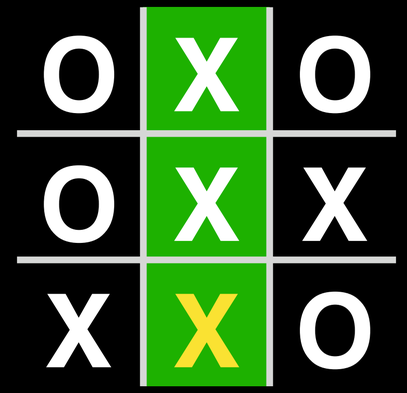
PLAYER(s) = O

MIN-VALUE: 0



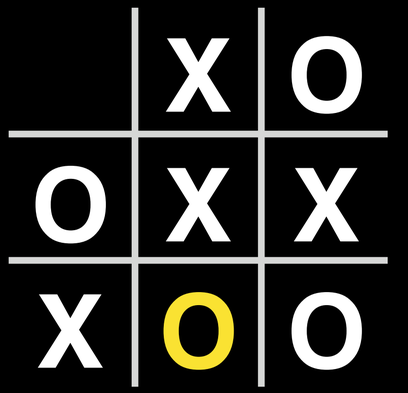


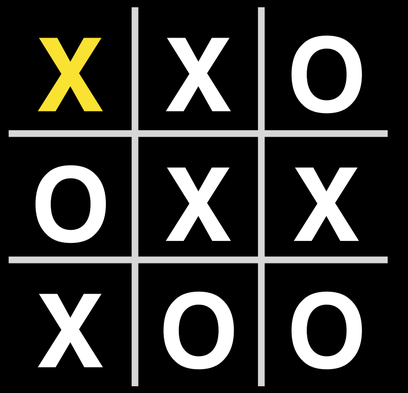
MAX-VALUE: 1



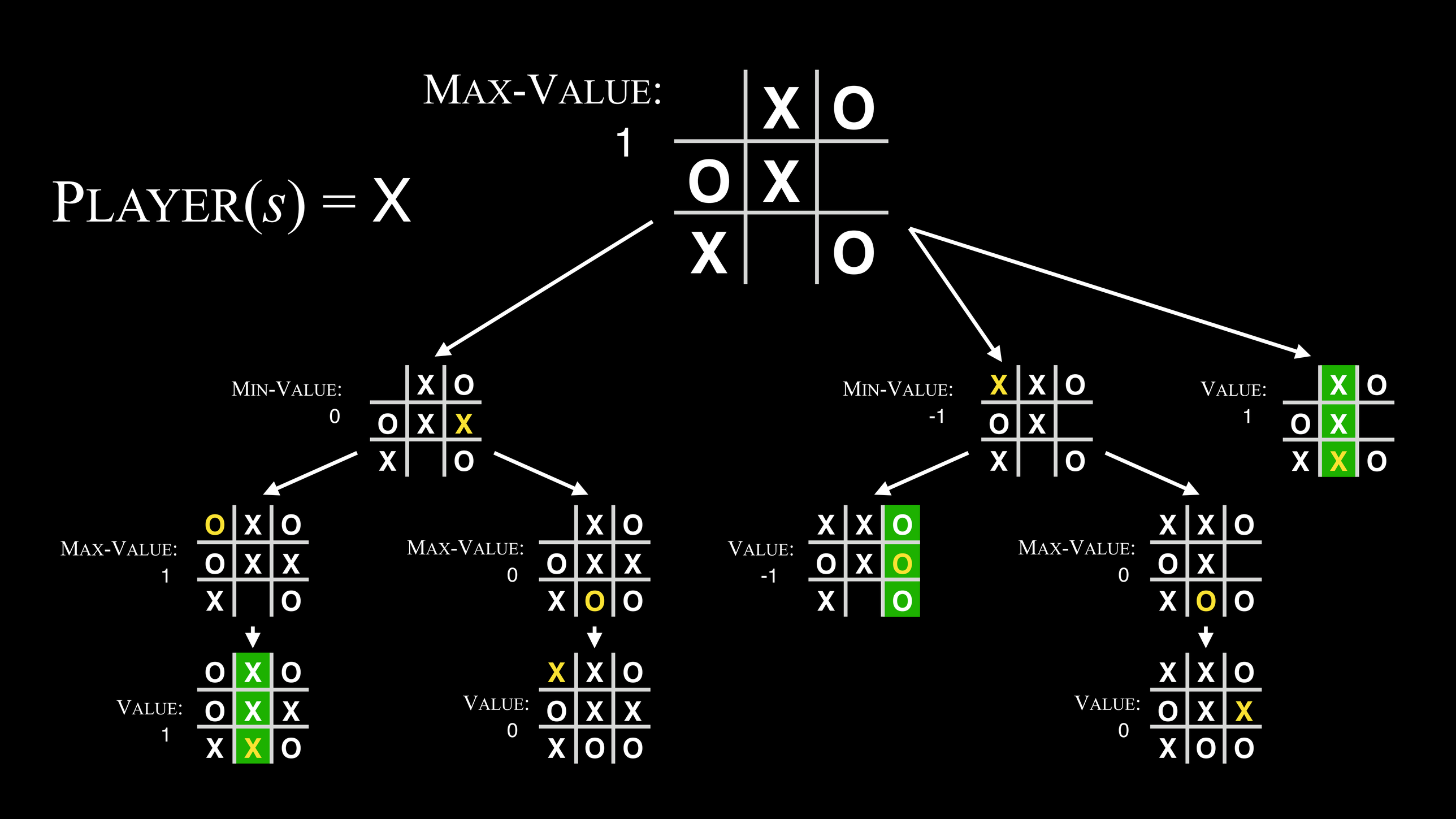
VALUE: 1

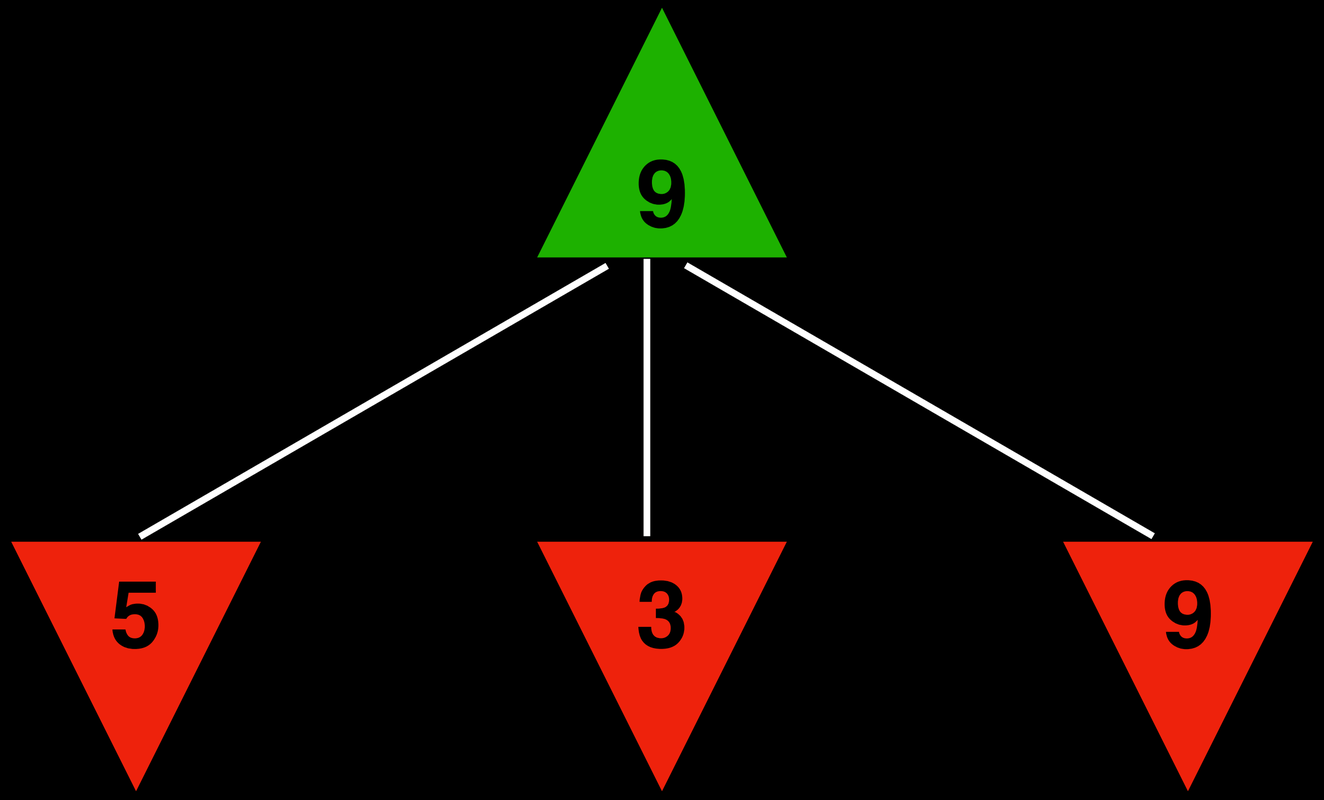
MAX-VALUE: 0

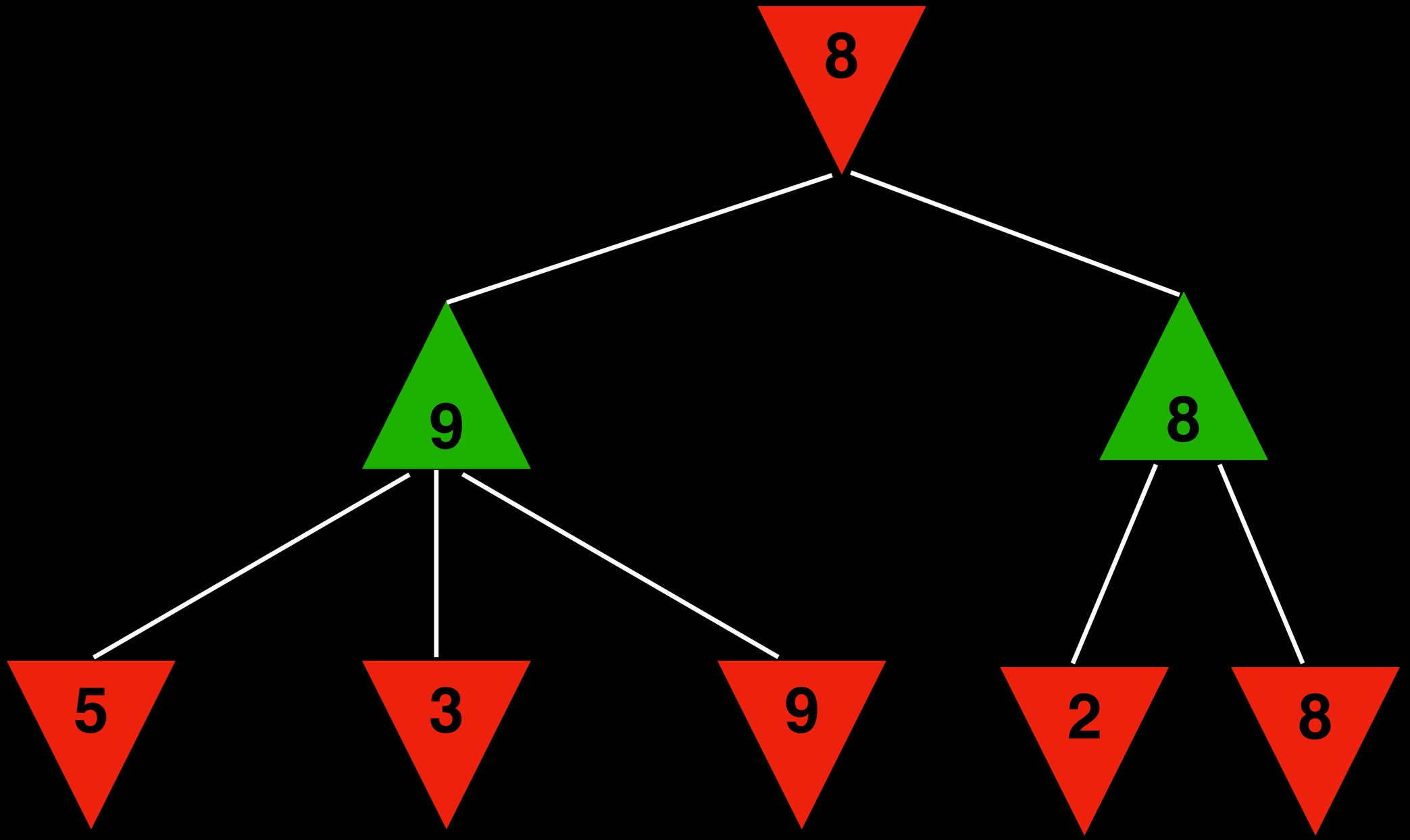




VALUE: 0


PLAYER(s) = X
MAX-VALUE:
1
MIN-VALUE:
0
MIN-VALUE:
-1
VALUE:
1
MAX-VALUE:
1
MAX-VALUE:
0
VALUE:
-1
MAX-VALUE:
0
VALUE:
1
VALUE:
0
VALUE:
0


9
5
3
9


8
9
8
5
3
9
2
8

Minimax

- Given a state s:
  - MAX picks action a in ACTIONS(s) that produces highest value of MIN-VALUE(RESULT(s, a))
  - MIN picks action a in ACTIONS(s) that produces smallest value of MAX-VALUE(RESULT(s, a))

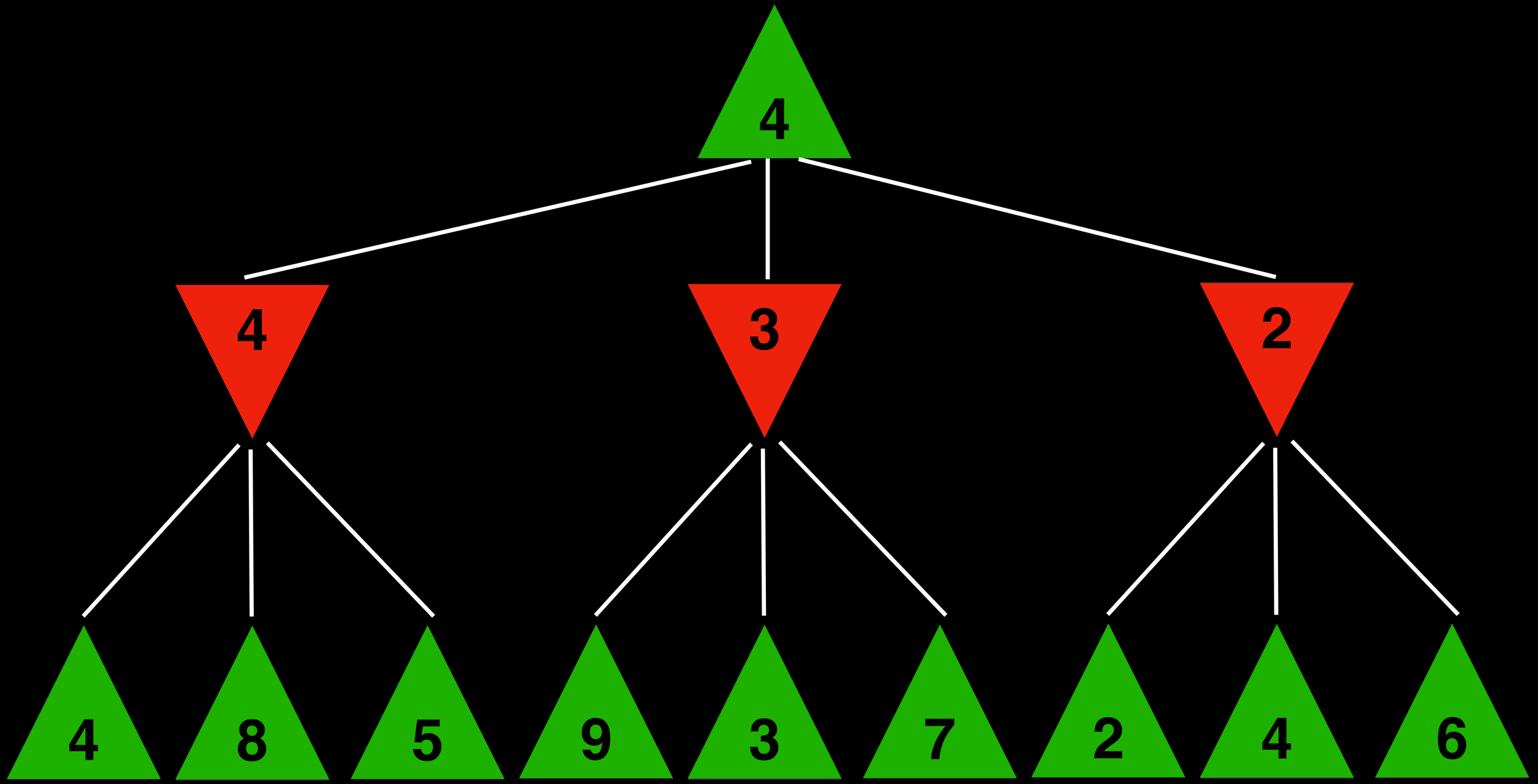
Minimax

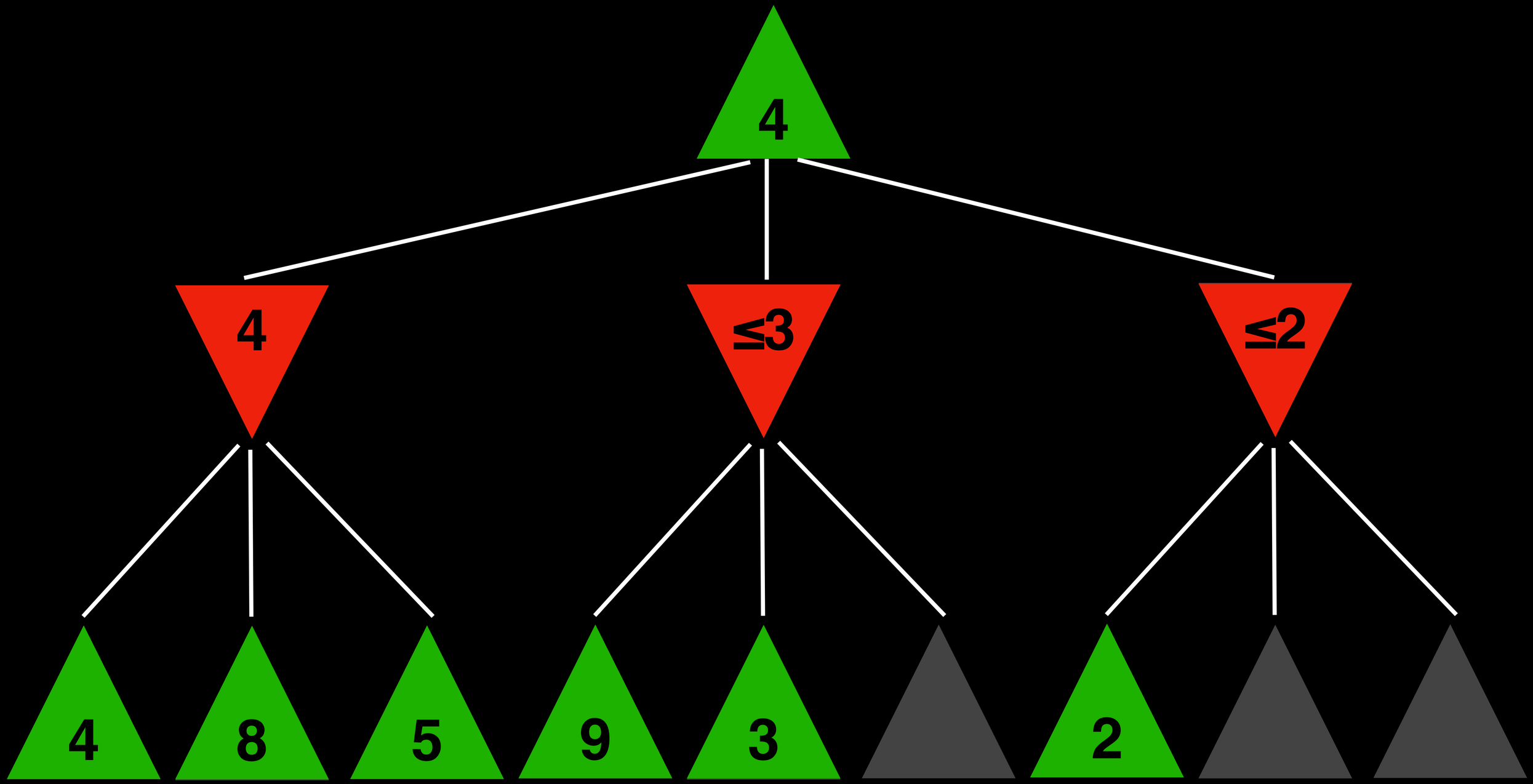
```
function MAX-VALUE(state):
    if TERMINAL(state):
        return UTILITY(state)
    v = -∞
    for action in ACTIONS(state):
        v = MAX(v, MIN-VALUE(RESULT(state, action)))
    return v
```

Minimax

```
function MIN-VALUE(state):
    if TERMINAL(state):
        return UTILITY(state)
    v = ∞
    for action in ACTIONS(state):
        v = MIN(v, MAX-VALUE(RESULT(state, action)))
    return v
```

Optimizations


4
4
3
2
4
8
5
9
3
7
2
4
6


4
4
≤3
≤2
4
8
5
9
3
2

Alpha-Beta Pruning

255,168

total possible Tic-Tac-Toe games

288,000,000,000

total possible chess games
after four moves each

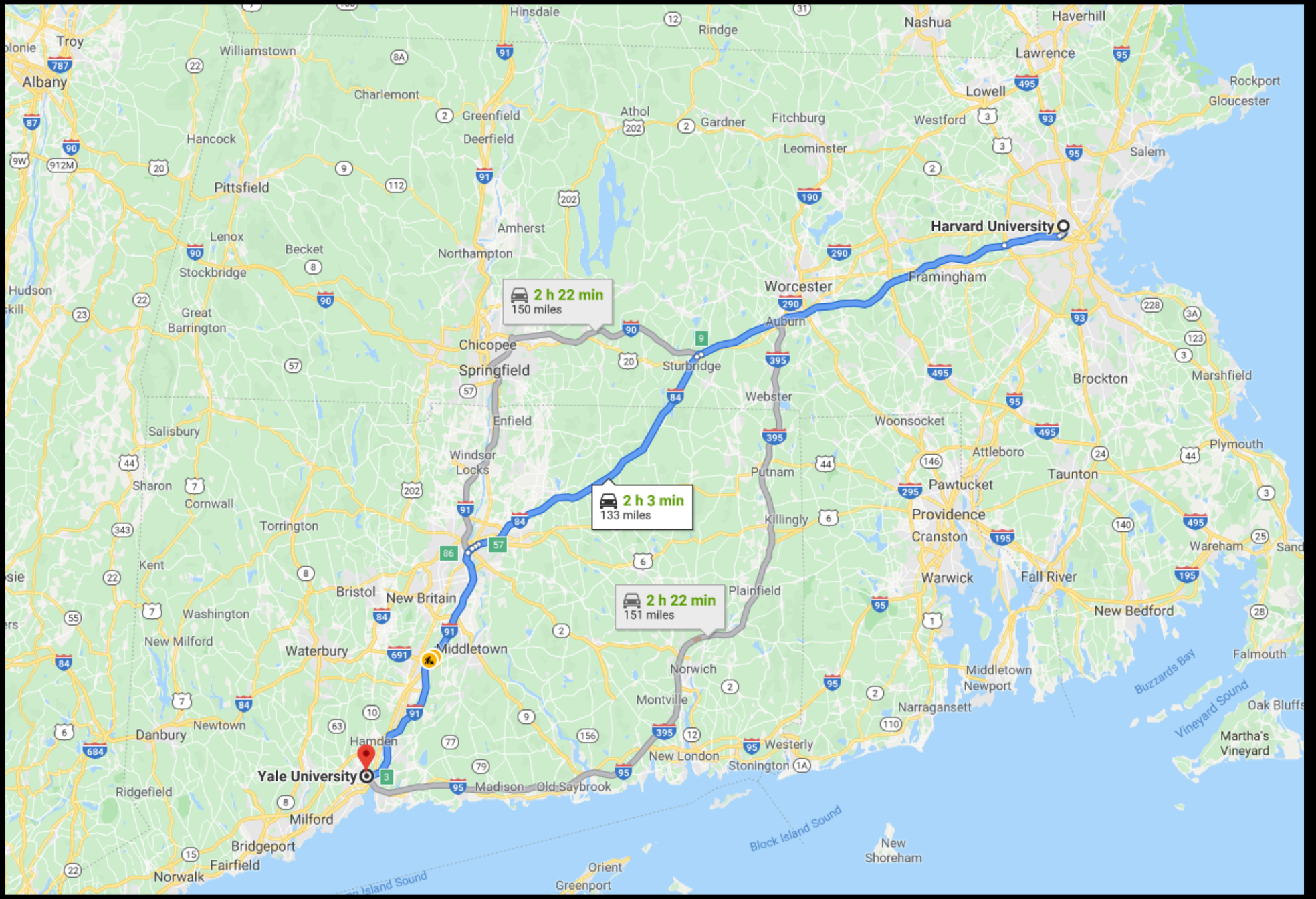
$$10^{29000}$$

total possible chess games
(lower bound)

Depth-Limited Minimax

evaluation function

function that estimates the expected utility of the game from a given state

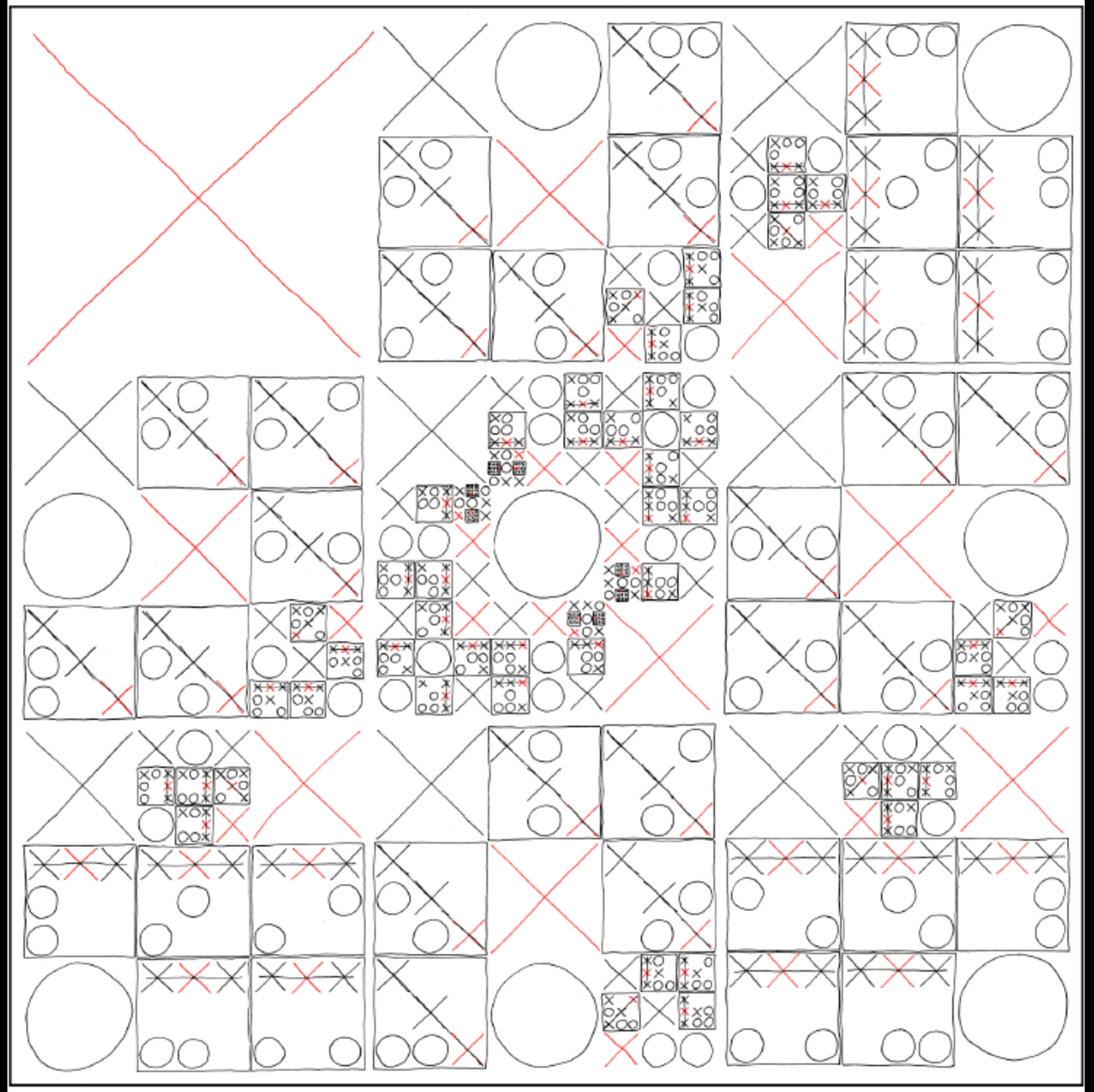


Harvard University
Yale University
2 h 22 min
150 miles
2 h 3 min
133 miles
2 h 22 min
151 miles
Hinsdale
Rindge
Nashua
Haverhill
Lawrence
Lowell
Rockport
Gloucester
Troy
Albany
Williamstown
Charlemont
Greenfield
Deerfield
Athol
Gardner
Fitchburg
Westford
Leominster
Salem
Hancock
Pittsfield
Amherst
Northampton
Lenox
Becket
Stockbridge
Great Barrington
Hudson
Worcester
Framingham
Auburn
Chicopee
Springfield
Sturbridge
Webster
Brockton
Marshfield
Enfield
Salisbury
Windsor Locks
Woonsocket
Attleboro
Plymouth
Sharon
Cornwall
Putnam
Pawtucket
Taunton
Torrington
Killingly
Providence
Cranston
Wareham
Kent
Bristol
New Britain
Plainfield
Warwick
Fall River
Washington
New Bedford
New Milford
Waterbury
Middletown
Norwich
Middletown
Newport
Falmouth
Buzzards Bay
Vineyard Sound
Oak Bluffs
Montville
Narragansett
Danbury
Newtown
Hamden
Westerly
New London
Stonington
Martha's Vineyard
Ridgefield
Madison
Old Saybrook
Milford
Block Island Sound
Bridgeport
Fairfield
Norwalk
Orient
Greenport
New Shoreham

COMPLETE MAP OF OPTIMAL TIC-TAC-TOE MOVES

YOUR MOVE IS GIVEN BY THE POSITION OF THE LARGEST RED SYMBOL ON THE GRID. WHEN YOUR OPPONENT PICKS A MOVE, ZOOM IN ON THE REGION OF THE GRID WHERE THEY WENT. REPEAT.

MAP FOR X:



https://xkcd.com/832/

Search

Introduction to
Artificial Intelligence
with Python