

Chapter 5 - Financial Forwards and Futures

Tyler J. Brough

February 1, 2017

Section 5.1 Alternative Ways to Buy a Stock

Introduction

- ▶ Financial futures and forwards
 - ▶ On stocks and indexes
 - ▶ On currencies
 - ▶ On interest rates
- ▶ How are they used?
- ▶ How are they priced?
- ▶ How are they hedged?

Alternative Ways to Buy a Stock

- ▶ Four different payment and receipt timing combinations
 - ▶ Outright purchase: ordinary transaction
 - ▶ Fully leveraged purchase: investor borrows the full amount
 - ▶ Prepaid forward contract: pay today, receive the share later
 - ▶ Forward contract: agree on price now, pay/receive later
- ▶ Payments, receipts, and their timing

TABLE 5.1

Four different ways to buy a share of stock that has price S_0 at time 0. At time 0 you agree to a price, which is paid either today or at time T . The shares are received either at 0 or T . The interest rate is r .

Description	Pay at Time	Receive Security at Time	Payment
Outright purchase	0	0	S_0 at time 0
Fully leveraged purchase	T	0	$S_0 e^{rT}$ at time T
Prepaid forward contract	0	T	?
Forward contract	T	T	$? \times e^{rT}$

Pricing Prepaid Forwards

- ▶ If we can price the *prepaid* forward (F^P), then we can calculate the price for a forward contract

$$F = \text{Future Value of } F^P$$

- ▶ Three possible methods to price prepaid forwards
 - ▶ Pricing by analogy
 - ▶ Pricing by discounted cash flows
 - ▶ Pricing by arbitrage
- ▶ For now, assume that there are no dividends

Pricing Prepaid Forwards (cont'd)

- ▶ Pricing by analogy
 - ▶ In the absence of dividends, the timing of delivery is irrelevant
 - ▶ Price of the prepaid forward contract same as current stock price
 - ▶ $F^P = S_0$ (where the asset is bought at $t = 0$, delivered at $t = T$)
- ▶ Pricing by discounted present value (α : risk-adjusted discount rate)
 - ▶ If expected $t = T$ stock price at $t = 0$ is $E_0(S_T)$, then $F^P = E_0(S_T)e^{-\alpha T}$
 - ▶ Since $t = 0$ expected value of price at $t = T$ is $E_0(S_T) = S_0e^{\alpha T}$
 - ▶ Combining the two, $F_{0,T}^P = S_0e^{\alpha T} = S_0$

Pricing Prepaid Forwards (cont'd)

- Pricing by arbitrage
 - **Arbitrage**: a situation in which one can generate positive cash flow by simultaneously buying and selling related assets, with no net investment and with no risk. Free money!
 - If at time $t = 0$, the prepaid forward price somehow exceeded the stock price, i.e., $F_{0,T}^P > S_0$, an arbitrageur could do the following

TABLE 5.2

Cash flows and transactions to undertake arbitrage when the prepaid forward price, $F_{0,T}^P$, exceeds the stock price, S_0 .

Transaction	Cash Flows	
	Time 0	Time T (expiration)
Buy stock @ S_0	$-S_0$	$+S_T$
Sell prepaid forward @ $F_{0,T}^P$	$+F_{0,T}^P$	$-S_T$
Total	$F_{0,T}^P - S_0$	0

- The price mechanism will ensure that these sort of arbitrage opportunities cannot persist, at equilibrium we can expect:

$$F_{0,T}^P = S_0$$

Pricing Prepaid Forwards (cont'd)

- ▶ What if there are dividends? Is $F_{0,T}^P = S_0$ still valid?
 - ▶ No, because the holder of the forward will not receive dividends that will be paid to the holder of the stock, $F_{0,T}^P > S_0$
 - ▶ $F_{0,T}^P = S_0 - \text{PV}(\text{all dividends paid from } t = 0 \text{ to } t = T)$
- ▶ For discrete dividends D_{t_i} at times $t_i, i = 1, \dots, n$
 - ▶ The prepaid forward price: $F_{0,T}^P = S_0 - \sum_{i=1}^n \text{PV}_0(D_{t_i})$
 - ▶ For continuous dividends with an annualized yield δ , the prepaid forward price is $F_{0,T}^P = S_0 e^{-\delta T}$

Pricing Prepaid Forwards (cont'd)

- ▶ Example 5.1

- ▶ XYZ stock costs \$100 today and is expected to pay a quarterly dividend of \$1.25. If the risk-free rate is 10% compounded continuously, how much does a 1-year prepaid forward cost?
- ▶ $F_{0,1}^P = \$100 - \sum_{i=1}^4 \$1.25e^{-0.025i} = \$95.30$

Pricing Prepaid Forwards (cont'd)

- ▶ Example 5.2

- ▶ The index is \$125 and the dividend yield is 3% continuously compounded. How much does a 1-year prepaid forward cost?
- ▶ $F_{0,1}^P = \$125e^{-0.03} = \121.31

Section 5.3 Forward Contracts on Stock

Pricing Forwards on Stock

- ▶ Forward price is the future value of the *prepaid* forward price
 - ▶ No dividends
 - ▶ $F_{0,T} = FV(F_{0,T}^P) = FV(S_0) = S_0 e^{rT}$
 - ▶ Continuous dividends

$$F_{0,T} = S_0 e^{(r-\delta)T}$$

Pricing Forwards on Stock (cont'd)

- ▶ Forward premium
 - ▶ The difference between current forward price and stock price
 - ▶ Can be used to infer the current stock price from forward price
 - ▶ Definition:
 - ▶ Forward premium: $F_{0,T}/S_0$
 - ▶ Annualized forward premium $= (1/T) \ln(F_{0,T}/S_0)$

Creating a Synthetic Forward

- ▶ One can offset the risk of a forward by creating a *synthetic* forward to offset a position in the actual forward contract
- ▶ How can one do this? (assume continuous dividends at rate δ)
 - ▶ Recall the long forward payoff at expiration = $S_T - F_{0,T}$
 - ▶ Borrow and purchase shares as follows

TABLE 5.3

Demonstration that borrowing $S_0e^{-\delta T}$ to buy $e^{-\delta T}$ shares of the index replicates the payoff to a forward contract, $S_T - F_{0,T}$.

Transaction	Cash Flows	
	Time 0	Time T (expiration)
Buy $e^{-\delta T}$ units of the index	$-S_0e^{-\delta T}$	$+S_T$
Borrow $S_0e^{-\delta T}$	$+S_0e^{-\delta T}$	$-S_0e^{(r-\delta)T}$
Total	0	$S_T - S_0e^{(r-\delta)T}$

- ▶ Note that the total payoff at expiration is same as forward premium

Creating a Synthetic Forward (cont'd)

- ▶ The idea of creating synthetic forward leads to following
 - ▶ Forward = Stock - zero-coupon bond
 - ▶ Stock = Forward - zero-coupon bond
 - ▶ Zero-coupon bond = Stock - forward
- ▶ Cash-and-Carry arbitrage: Buy the index, short the forward

TABLE 5.6

Transactions and cash flows for a cash-and-carry: A market-maker is short a forward contract and long a synthetic forward contract.

Transaction	Cash Flows	
	Time 0	Time T (expiration)
Buy tailed position in stock, paying $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$+S_T$
Borrow $S_0 e^{-\delta T}$	$+S_0 e^{-\delta T}$	$-S_0 e^{(r-\delta)T}$
Short forward	0	$F_{0,T} - S_T$
Total	0	$F_{0,T} - S_0 e^{(r-\delta)T}$

Creating a Synthetic Forward (cont'd)

- ▶ Cash-and-carry arbitrage with transaction costs
 - ▶ Trading fees, bid-ask spreads, different borrowing/lending rates, the price effect of trading in large quantities, make arbitrage harder
 - ▶ Suppose
 - ▶ Bid-ask spreads: for stock $S^b < S^a$, and for forward $F^b < F^a$
 - ▶ Cost k of transacting forward
 - ▶ Interest rate for borrowing and lending are $r^b < r^l$
 - ▶ No dividends and no time T transaction costs for simplicity
 - ▶ Arbitrage possible if
 - ▶ $F^b > F^+ = (S_0^a + 2k)e^{r^b T}$
 - ▶ $F^a < F^- = (S_0^b - 2k)e^{r^l T}$

Other Issues in Forward Pricing

- ▶ Does the forward price predict the future price?
 - ▶ According to the formula $F_{0,T} = S_0 e^{-(r-\delta)T}$ the forward price conveys no additional information beyond what S_0 , r , and δ provides
 - ▶ Moreover, the forward price underestimates the future stock price
- ▶ Forward pricing formula and cost of carry

$$\text{Forward Price} = \text{Spot Price} + \underbrace{\text{Interest to carry the asset} - \text{asset lease rate}}_{\text{Cost of carry, } (r-\delta)S}$$

Section 5.4 Futures Contracts

Futures Contracts

- ▶ Exchange-traded “forward contracts”
- ▶ Typically features of futures contracts
 - ▶ Standardized, with specified delivery dates, locations, procedures
 - ▶ A clearinghouse
 - ▶ Matches buy and sell orders
 - ▶ Keeps track of members' obligations and payments
 - ▶ After matching the trades, becomes counterparty
- ▶ Differences from forward contracts
 - ▶ Settled daily through the mark-to-market process → low credit risk
 - ▶ Highly liquid → easier to offset an existing position
 - ▶ Highly standardized structure → harder to customize

Section 5.5 Uses of Index Futures

Section 5.6 Currency Contracts

Section 5.7 Eurodollar Futures