NOTE: This Jupyter notebook is derived from the Jupyter notebook located at https://github.com/ageron/handson-ml as of 01/06/2019, as discussed in Hands-On Machine Learning. It is used here for educational purposes only.

Chapter 4 – Training Linear Models

Setup

[1]

```
from __future__ import division, print_function, unicode_literals

# Common imports
import numpy as np
import os

# to make this notebook's output stable across runs
np.random.seed(42)

# To plot pretty figures
%matplotlib inline
import matplotlib as mpl
import matplotlib.pyplot as plt
mpl.rc('axes', labelsize=14)
mpl.rc('xtick', labelsize=12)
mpl.rc('ytick', labelsize=12)

# Where to save the figures
PROJECT_ROOT_DIR = "../.."
CHAPTER_ID = "training_linear_models"


def save_fig(fig_id, tight_layout=True):
    path = os.path.join(PROJECT_ROOT_DIR, "images", CHAPTER_ID, fig_id
    print("Saving figure", fig_id)
    if tight_layout:
        plt.tight_layout()
    try:
        plt.savefig(path, format='png', dpi=300)
    except:
        plt.savefig(fig_id + ".png", format='png', dpi=300)
```

```python
import warnings
warnings.filterwarnings(action="ignore", message="^internal gelsd")
```

Linear regression using the Normal Equation

[2]

```python
from pathlib import Path
import pandas as pd
import numpy as np

github_p = "https://raw.githubusercontent.com/Finance-781/FinML/master/

my_file = Path("data/returns.csv") # Defines path
if my_file.is_file():              # See if file exists
    print("Local file found")
    df = pd.read_csv('data/returns.csv')
    rf = pd.read_csv('data/rf.csv')
else:
    print("Be patient: loading from github (2 minutes)")
    df = pd.read_csv(github_p+'data/returns.csv')
    rf = pd.read_csv(github_p+'data/rf.csv')

    print("Done")
df = df.set_index("date",drop=True)
df.head()
```

```
Local file found
```

	A	AAME	AAON	AAP	AAPL	
date						
20071231	-0.028813	-0.200000	0.049921	0.058120	0.087038	0.
20080131	-0.078389	0.135714	-0.101917	-0.058173	-0.316640	-0
20080229	-0.095983	-0.025157	-0.072472	-0.062605	-0.076389	0.
20080331	-0.025482	-0.012903	0.213204	0.016995	0.147816	0.
20080430	0.012739	0.555556	-0.097354	0.018502	0.212195	0.

5 rows × 1779 columns

[475]

```python
from sklearn.linear_model import LinearRegression
df["label"] = 1
reg = LinearRegression(fit_intercept=False).fit(df.drop(["label"],axis=

df_coeff = pd.DataFrame(df.drop(["label"],axis=1).columns)
df_coeff.columns = ["TICKER"]
df_coeff["coeff"] = reg.coef_
df_coeff["coeff_norm"] = (df_coeff["coeff"]-df_coeff["coeff"].min())/(d
df_coeff["weight"] = df_coeff["coeff_norm"]/df_coeff["coeff_norm"].sum(
print("Normal Regression Weights")
df_coeff[df_coeff["weight"]>0.001].head(5)  ### Stocks where portfolio
```

```
Normal Regression Weights
```

	TICKER	coeff	coeff_norm	weight
1128	NFLX	0.441748	1.0	0.001102

[476]

```python
from sklearn.linear_model import Lasso
df["label"] = 1
reg = Lasso(fit_intercept=False,alpha=0.0005, positive=True).fit(df.dro

df_coeff = pd.DataFrame(df.drop(["label"],axis=1).columns)
df_coeff.columns = ["TICKER"]
df_coeff["coeff"] = reg.coef_
df_coeff["coeff_norm"] = (df_coeff["coeff"]-df_coeff["coeff"].min())/(d
df_coeff["weight"] = df_coeff["coeff_norm"]/df_coeff["coeff_norm"].sum(
print("Lasso Regression Weights, alpha=0.0005")
df_coeff[df_coeff["weight"]>0.001].head(5) ### Stocks where portfolio w
```

```
Lasso Regression Weights, alpha=0.0005
```

	TICKER	coeff	coeff_norm	weight

	TICKER	coeff	coeff_norm	weight
10	ABMD	0.113090	0.032014	0.004746
62	ALGT	0.080879	0.022895	0.003394
111	ARL	0.157040	0.044455	0.006590
112	ARNA	0.142277	0.040276	0.005971
244	BVX	0.204516	0.057895	0.008583

[477]

```
from sklearn.linear_model import Ridge
df["label"] = 1
reg = Ridge(fit_intercept=False,alpha=1).fit(df.drop(["label"],axis=1),

df_coeff = pd.DataFrame(df.drop(["label"],axis=1).columns)
df_coeff.columns = ["TICKER"]
df_coeff["coeff"] = reg.coef_
df_coeff["coeff_norm"] = (df_coeff["coeff"]-df_coeff["coeff"].min())/(d
df_coeff["weight"] = df_coeff["coeff_norm"]/df_coeff["coeff_norm"].sum(
print("Ridge Regression Weights")
df_coeff[df_coeff["weight"]>0.001].head(5) ### Stocks where portfolio w
```

```
Ridge Regression Weights
```

	TICKER	coeff	coeff_norm	weight
1128	NFLX	0.396506	1.0	0.00109

[478]

```
### Each month, calculate the weight and the portfolios, choosing Lasso
### Use 5 years data and roll forward, monthtly basis
```

[479]

```
df.head()
```

	A	AAME	AAON	AAP	AAPL	
date						
20071231	-0.028813	-0.200000	0.049921	0.058120	0.087038	0.
20080131	-0.078389	0.135714	-0.101917	-0.058173	-0.316640	-0
20080229	-0.095983	-0.025157	-0.072472	-0.062605	-0.076389	0.
20080331	-0.025482	-0.012903	0.213204	0.016995	0.147816	0.
20080430	0.012739	0.555556	-0.097354	0.018502	0.212195	0.

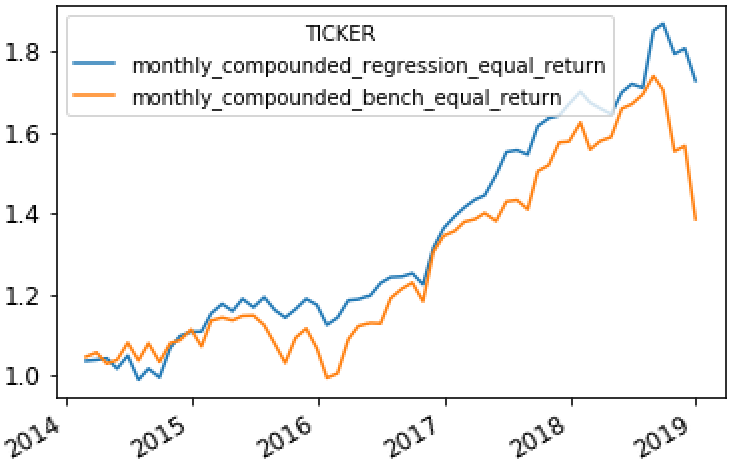
[480]

```
from sklearn.linear_model import Lasso

years_lookback = 5
reg_periods = df.shape[0]-(12*years_lookback)


reg_type="Lasso"
#reg_type = "Normal"
```

```
print(reg_type)

for n in range(len(df)-reg_periods-1):
    df_reg = df.iloc[n:reg_periods+n,:].copy()
    weight_date = df.iloc[reg_periods+n+1].copy().name
    df_reg["label"] = 1
    if reg_type == "Lasso":
        reg = Lasso(fit_intercept=False,alpha=0.0005, positive=True,ran
    elif reg_type == "Normal":
        reg = LinearRegression(fit_intercept=False).fit(df.drop(["label
    else:
        print("please set reg = to Lasso or Normal")
        break

    df_coeff = pd.DataFrame(df_reg.drop(["label"],axis=1).columns)
    df_coeff.columns = ["TICKER"]
    df_coeff["coeff"] = reg.coef_
    df_coeff["coeff_norm"] = (df_coeff["coeff"]-df_coeff["coeff"].min()
    df_coeff[weight_date] = df_coeff["coeff_norm"]/df_coeff["coeff_norm


    df_coeff = df_coeff.set_index("TICKER")

    df_coeff = df_coeff[[weight_date]].T
    df_coeff = df_coeff.replace(0., np.nan)

    if n ==0:
        df_all = df_coeff
    else:
        df_all = pd.concat((df_all,df_coeff),axis=0)

df_all.head()

df_ret = df.loc[df_all.index[0]:].drop("label",axis=1) # select returns

df_port_returns = df_all*df_ret.values

df_port_returns["monthly_returns"] = df_port_returns.sum(axis=1) # equa

df_port_returns["monthly_compounded_regression_equal_return"]= ((df_por


##bench
df_port_returns["monthly_returns_bench_equal"] = df_ret.mean(axis=1) #

df_port_returns["monthly_compounded_bench_equal_return"]= ((df_port_ret

df_port_returns.index = pd.to_datetime(df_port_returns.index, format='%

df_port_returns[["monthly_compounded_regression_equal_return","monthly_
```

Lasso

<matplotlib.axes._subplots.AxesSubplot at 0x1a69faa390>

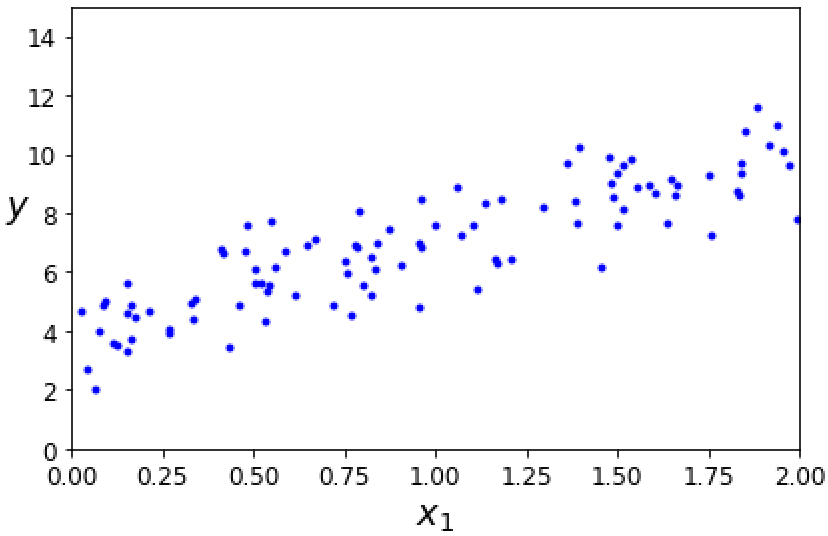


This part forward follows the original notebook

[402]
```
import numpy as np

X = 2 * np.random.rand(100, 1)
y = 4 + 3 * X + np.random.randn(100, 1)
```

[411]
```
plt.plot(X, y, "b.")
plt.xlabel("$x_1$", fontsize=18)
plt.ylabel("$y$", rotation=0, fontsize=18)
plt.axis([0, 2, 0, 15])
save_fig("generated_data_plot")
plt.show()
```

Saving figure generated_data_plot



[412]
```
X_b = np.c_[np.ones((100, 1)), X]  # add x0 = 1 to each instance
theta_best = np.linalg.inv(X_b.T.dot(X_b)).dot(X_b.T).dot(y)
```

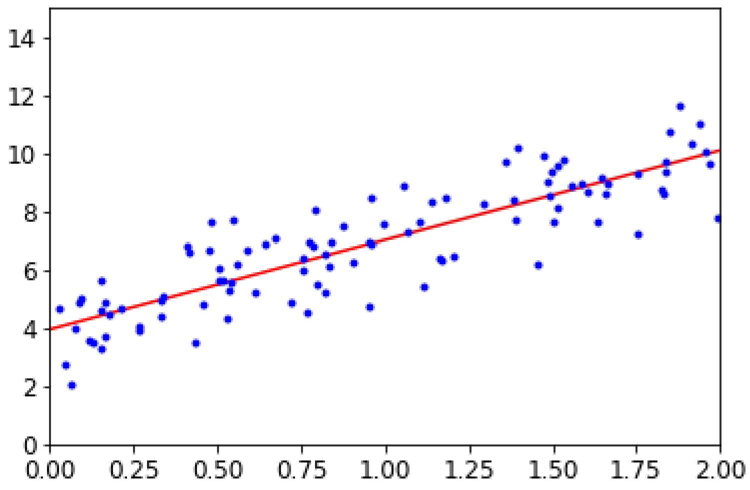
[413]
```
theta_best
```

```
array([[3.95574706],
       [3.073125  ]])
```

[414]
```
X_new = np.array([[0], [2]])
X_new_b = np.c_[np.ones((2, 1)), X_new]  # add x0 = 1 to each instance
y_predict = X_new_b.dot(theta_best)
y_predict
```

```
array([[ 3.95574706],
       [10.10199706]])
```

[415]
```
plt.plot(X_new, y_predict, "r-")
plt.plot(X, y, "b.")
plt.axis([0, 2, 0, 15])
plt.show()
```

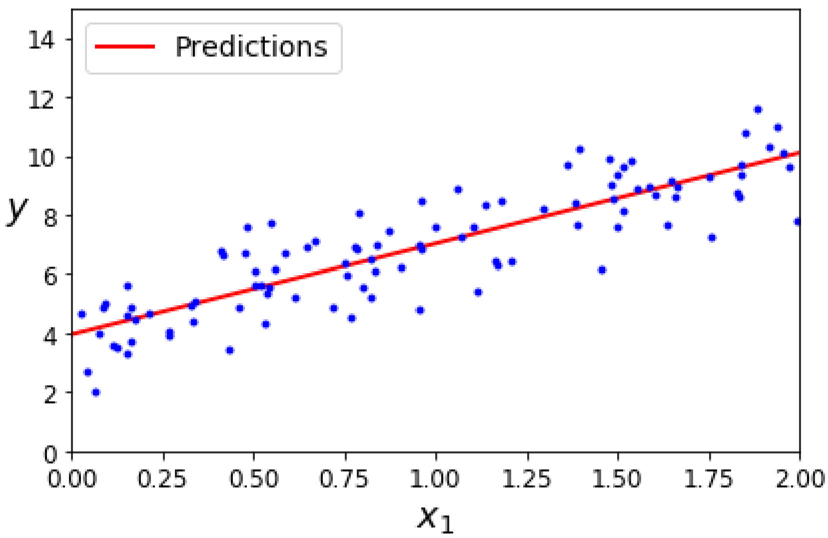


The figure in the book actually corresponds to the following code, with a legend and axis labels:

[416]

```
plt.plot(X_new, y_predict, "r-", linewidth=2, label="Predictions")
plt.plot(X, y, "b.")
plt.xlabel("$x_1$", fontsize=18)
plt.ylabel("$y$", rotation=0, fontsize=18)
plt.legend(loc="upper left", fontsize=14)
plt.axis([0, 2, 0, 15])
save_fig("linear_model_predictions")
plt.show()
```

```
Saving figure linear_model_predictions
```



[417]

```
from sklearn.linear_model import LinearRegression
lin_reg = LinearRegression()
lin_reg.fit(X, y)
lin_reg.intercept_, lin_reg.coef_
```

```
(array([3.95574706]), array([[3.073125]]))
```

[418]
```python
lin_reg.predict(X_new)
```

```
array([[ 3.95574706],
       [10.10199706]])
```

The `LinearRegression` class is based on the `scipy.linalg.lstsq()` function (the name stands for "least squares"), which you could call directly:

[419]
```python
theta_best_svd, residuals, rank, s = np.linalg.lstsq(X_b, y, rcond=1e-6
theta_best_svd
```

```
array([[3.95574706],
       [3.073125  ]])
```

This function computes $\mathbf{X}^+\mathbf{y}$, where $\mathbf{X}^+$ is the *pseudoinverse* of $\mathbf{X}$ (specifically the Moore-Penrose inverse). You can use `np.linalg.pinv()` to compute the pseudoinverse directly:

[420]
```python
np.linalg.pinv(X_b).dot(y)
```

```
array([[3.95574706],
       [3.073125  ]])
```

Note: the first releases of the book implied that the `LinearRegression` class was based on the Normal Equation. This was an error, my apologies: as explained above, it is based on the pseudoinverse, which ultimately relies on the SVD matrix decomposition of $\mathbf{X}$ (see chapter 8 for details about the SVD decomposition). Its time complexity is $O(n^2)$ and it works even when $m < n$ or when some features are linear combinations of other features (in these cases, $\mathbf{X}^T\mathbf{X}$ is not invertible so the Normal Equation fails), see issue #184 for more details. However, this does not change the rest of the description of the `LinearRegression` class, in particular, it is based on an analytical solution, it does not scale well with the number of features, it scales linearly with the number of instances, all the data must fit in memory, it does not require feature scaling and the order of the instances in the training set does not matter.

Linear regression using batch gradient descent

[421]
```
eta = 0.1
n_iterations = 1000
m = 100
theta = np.random.randn(2,1)

for iteration in range(n_iterations):
    gradients = 2/m * X_b.T.dot(X_b.dot(theta) - y)
    theta = theta - eta * gradients
```

[422]
```
theta
```

```
array([[3.95574706],
       [3.073125  ]])
```

[423]
```
X_new_b.dot(theta)
```

```
array([[ 3.95574706],
       [10.10199706]])
```

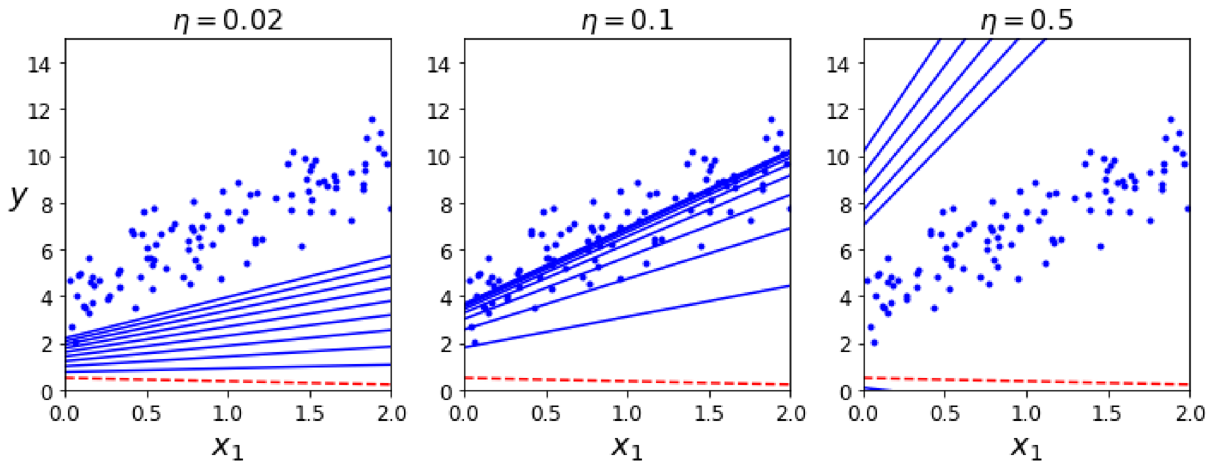
[424]
```
theta_path_bgd = []

def plot_gradient_descent(theta, eta, theta_path=None):
    m = len(X_b)
    plt.plot(X, y, "b.")
    n_iterations = 1000
    for iteration in range(n_iterations):
        if iteration < 10:
            y_predict = X_new_b.dot(theta)
            style = "b-" if iteration > 0 else "r--"
            plt.plot(X_new, y_predict, style)
        gradients = 2/m * X_b.T.dot(X_b.dot(theta) - y)
        theta = theta - eta * gradients
        if theta_path is not None:
            theta_path.append(theta)
```

```
    plt.axis([0, 2, 0, 15])
    plt.title(r"$\eta = {}$".format(eta), fontsize=16)
```

[425]

```
np.random.seed(42)
theta = np.random.randn(2,1)  # random initialization

plt.figure(figsize=(10,4))
plt.subplot(131); plot_gradient_descent(theta, eta=0.02)
plt.ylabel("$y$", rotation=0, fontsize=18)
plt.subplot(132); plot_gradient_descent(theta, eta=0.1, theta_path=thet
plt.subplot(133); plot_gradient_descent(theta, eta=0.5)

save_fig("gradient_descent_plot")
plt.show()
```

```
Saving figure gradient_descent_plot
```



Stochastic Gradient Descent

[426]

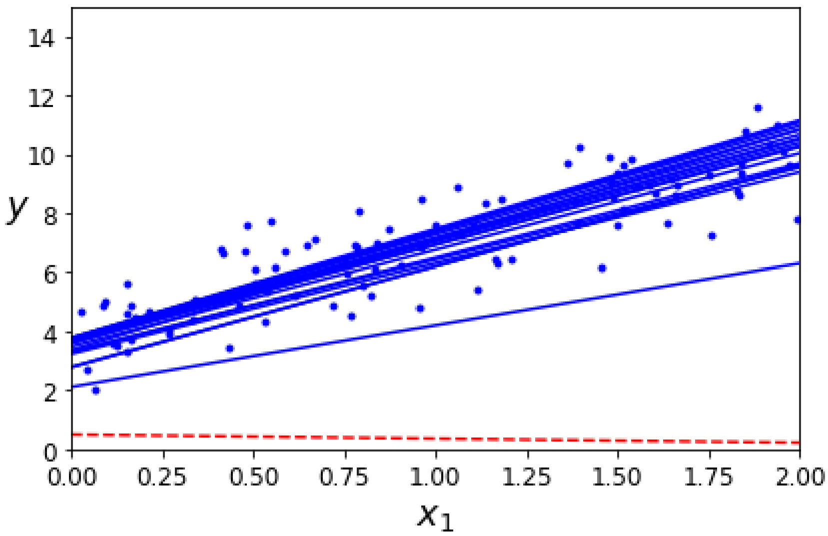
```
theta_path_sgd = []
m = len(X_b)
np.random.seed(42)
```

[427]

```
n_epochs = 50
t0, t1 = 5, 50  # learning schedule hyperparameters

def learning_schedule(t):
    return t0 / (t + t1)
```

```
for epoch in range(n_epochs):
    for i in range(m):
        if epoch == 0 and i < 20:                    # not shown in the
            y_predict = X_new_b.dot(theta)           # not shown
            style = "b-" if i > 0 else "r--"         # not shown
            plt.plot(X_new, y_predict, style)        # not shown
        random_index = np.random.randint(m)
        xi = X_b[random_index:random_index+1]
        yi = y[random_index:random_index+1]
        gradients = 2 * xi.T.dot(xi.dot(theta) - yi)
        eta = learning_schedule(epoch * m + i)
        theta = theta - eta * gradients
        theta_path_sgd.append(theta)                 # not shown

plt.plot(X, y, "b.")                                 # not shown
plt.xlabel("$x_1$", fontsize=18)                     # not shown
plt.ylabel("$y$", rotation=0, fontsize=18)           # not shown
plt.axis([0, 2, 0, 15])                              # not shown
save_fig("sgd_plot")                                 # not shown
plt.show()                                           # not shown
```

```
Saving figure sgd_plot
```



[428]
```
theta
```

```
array([[3.92793287],
       [3.06981492]])
```

[429]
```
from sklearn.linear_model import SGDRegressor
sgd_reg = SGDRegressor(max_iter=50, tol=-np.infty, penalty=None, eta0=0
sgd_reg.fit(X, y.ravel())
```

```
SGDRegressor(alpha=0.0001, average=False, epsilon=0.1, eta0=0.1,
       fit_intercept=True, l1_ratio=0.15, learning_rate='invscaling',
       loss='squared_loss', max_iter=50, n_iter=None, penalty=None,
       power_t=0.25, random_state=42, shuffle=True, tol=-inf, verbose=0,
       warm_start=False)
```

[430]

```
sgd_reg.intercept_, sgd_reg.coef_
```

```
(array([3.88184084]), array([2.96620713]))
```

Mini-batch gradient descent

[23]

```
theta_path_mgd = []

n_iterations = 50
minibatch_size = 20

np.random.seed(42)
theta = np.random.randn(2,1)  # random initialization

t0, t1 = 200, 1000
def learning_schedule(t):
    return t0 / (t + t1)

t = 0
for epoch in range(n_iterations):
    shuffled_indices = np.random.permutation(m)
    X_b_shuffled = X_b[shuffled_indices]
    y_shuffled = y[shuffled_indices]
    for i in range(0, m, minibatch_size):
        t += 1
        xi = X_b_shuffled[i:i+minibatch_size]
        yi = y_shuffled[i:i+minibatch_size]
        gradients = 2/minibatch_size * xi.T.dot(xi.dot(theta) - yi)
        eta = learning_schedule(t)
        theta = theta - eta * gradients
        theta_path_mgd.append(theta)
```

[24]

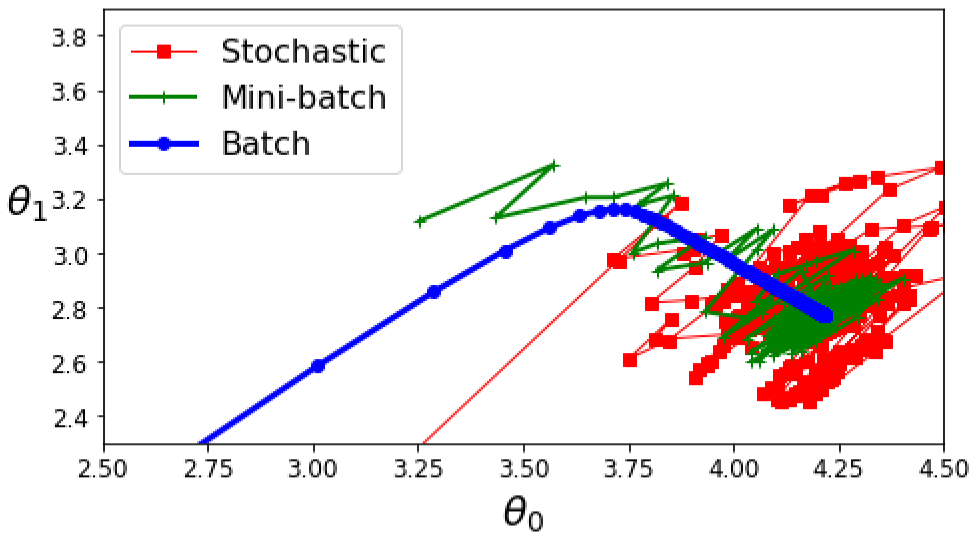
```
theta
```

```
array([[4.25214635],
       [2.7896408 ]])
```

```
[25]
theta_path_bgd = np.array(theta_path_bgd)
theta_path_sgd = np.array(theta_path_sgd)
theta_path_mgd = np.array(theta_path_mgd)
```

```
[26]
plt.figure(figsize=(7,4))
plt.plot(theta_path_sgd[:, 0], theta_path_sgd[:, 1], "r-s", linewidth=1
plt.plot(theta_path_mgd[:, 0], theta_path_mgd[:, 1], "g-+", linewidth=2
plt.plot(theta_path_bgd[:, 0], theta_path_bgd[:, 1], "b-o", linewidth=3
plt.legend(loc="upper left", fontsize=16)
plt.xlabel(r"$\theta_0$", fontsize=20)
plt.ylabel(r"$\theta_1$   ", fontsize=20, rotation=0)
plt.axis([2.5, 4.5, 2.3, 3.9])
save_fig("gradient_descent_paths_plot")
plt.show()
```

```
Saving figure gradient_descent_paths_plot
```



Polynomial regression

```
[27]
import numpy as np
import numpy.random as rnd

np.random.seed(42)
```

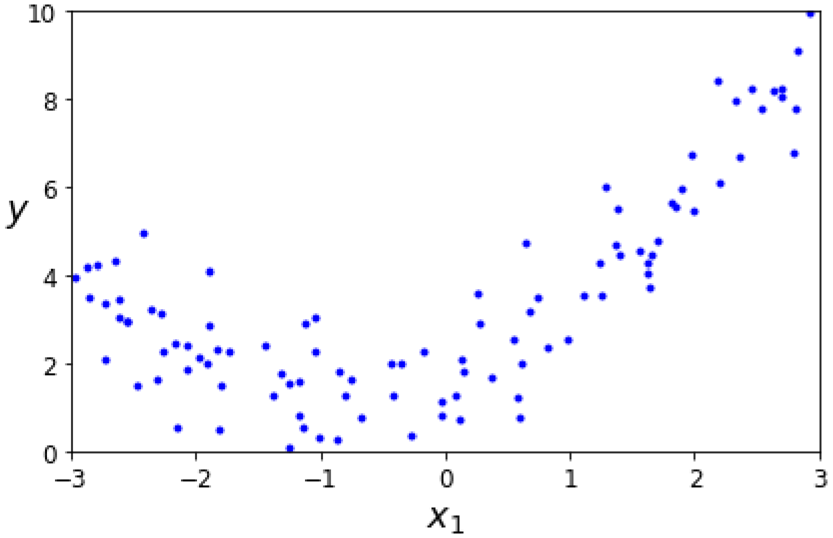
[28]

```
m = 100
X = 6 * np.random.rand(m, 1) - 3
y = 0.5 * X**2 + X + 2 + np.random.randn(m, 1)
```

[29]

```
plt.plot(X, y, "b.")
plt.xlabel("$x_1$", fontsize=18)
plt.ylabel("$y$", rotation=0, fontsize=18)
plt.axis([-3, 3, 0, 10])
save_fig("quadratic_data_plot")
plt.show()
```

```
Saving figure quadratic_data_plot
```



[30]

```
from sklearn.preprocessing import PolynomialFeatures
poly_features = PolynomialFeatures(degree=2, include_bias=False)
X_poly = poly_features.fit_transform(X)
X[0]
```

```
array([-0.75275929])
```

[31]

```
X_poly[0]
```

```
array([-0.75275929,  0.56664654])
```

[32]

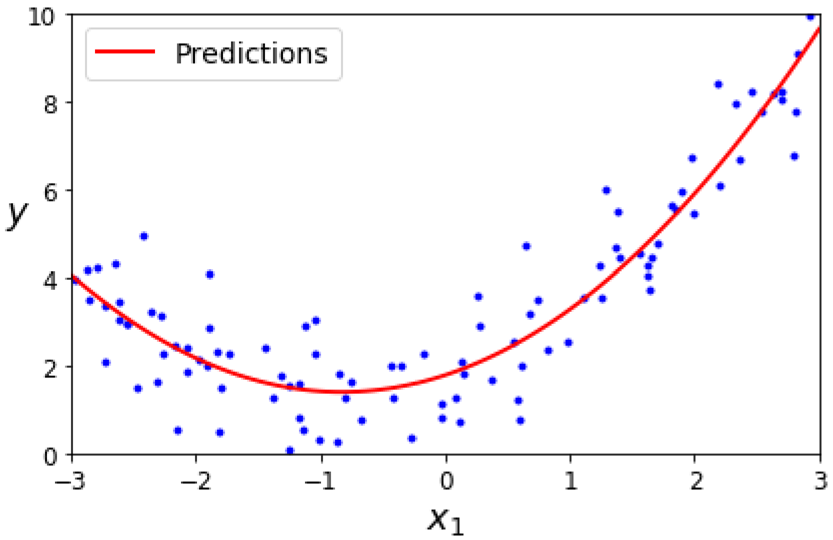
```
lin_reg = LinearRegression()
lin_reg.fit(X_poly, y)
lin_reg.intercept_, lin_reg.coef_
```

```
(array([1.78134581]), array([[0.93366893, 0.56456263]]))
```

[33]

```python
X_new=np.linspace(-3, 3, 100).reshape(100, 1)
X_new_poly = poly_features.transform(X_new)
y_new = lin_reg.predict(X_new_poly)
plt.plot(X, y, "b.")
plt.plot(X_new, y_new, "r-", linewidth=2, label="Predictions")
plt.xlabel("$x_1$", fontsize=18)
plt.ylabel("$y$", rotation=0, fontsize=18)
plt.legend(loc="upper left", fontsize=14)
plt.axis([-3, 3, 0, 10])
save_fig("quadratic_predictions_plot")
plt.show()
```

```
Saving figure quadratic_predictions_plot
```

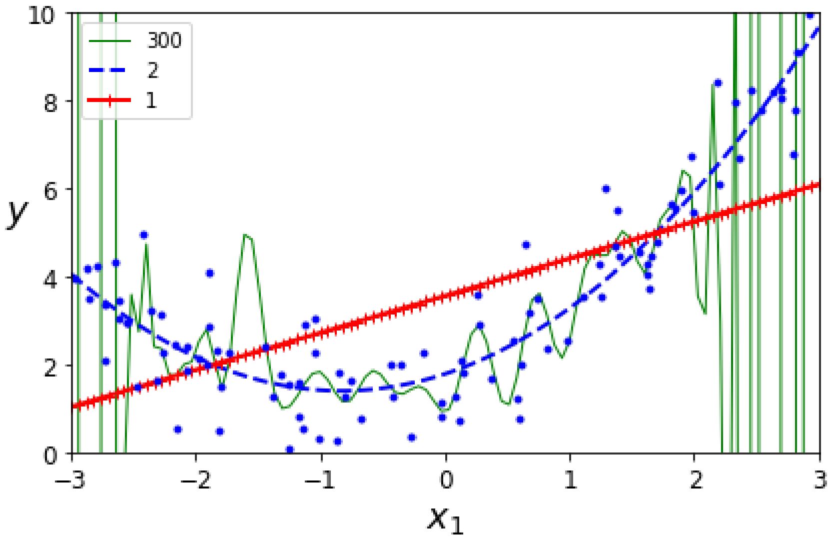


[34]

```python
from sklearn.preprocessing import StandardScaler
from sklearn.pipeline import Pipeline

for style, width, degree in (("g-", 1, 300), ("b--", 2, 2), ("r-+", 2,
    polybig_features = PolynomialFeatures(degree=degree, include_bias=F
    std_scaler = StandardScaler()
    lin_reg = LinearRegression()
    polynomial_regression = Pipeline([
            ("poly_features", polybig_features),
            ("std_scaler", std_scaler),
            ("lin_reg", lin_reg),
        ])
    polynomial_regression.fit(X, y)
    y_newbig = polynomial_regression.predict(X_new)
    plt.plot(X_new, y_newbig, style, label=str(degree), linewidth=width

plt.plot(X, y, "b.", linewidth=3)
```

```
plt.xlabel("$x_1$", fontsize=18)
plt.ylabel("$y$", rotation=0, fontsize=18)
plt.axis([-3, 3, 0, 10])
save_fig("high_degree_polynomials_plot")
plt.show()
```

```
Saving figure high_degree_polynomials_plot
```



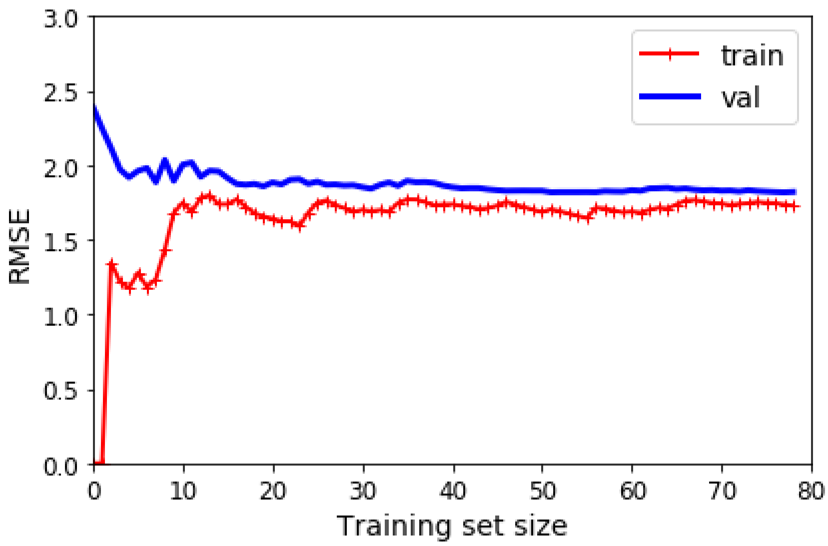
[35]

```
from sklearn.metrics import mean_squared_error
from sklearn.model_selection import train_test_split

def plot_learning_curves(model, X, y):
    X_train, X_val, y_train, y_val = train_test_split(X, y, test_size=0
    train_errors, val_errors = [], []
    for m in range(1, len(X_train)):
        model.fit(X_train[:m], y_train[:m])
        y_train_predict = model.predict(X_train[:m])
        y_val_predict = model.predict(X_val)
        train_errors.append(mean_squared_error(y_train[:m], y_train_pre
        val_errors.append(mean_squared_error(y_val, y_val_predict))

    plt.plot(np.sqrt(train_errors), "r-+", linewidth=2, label="train")
    plt.plot(np.sqrt(val_errors), "b-", linewidth=3, label="val")
    plt.legend(loc="upper right", fontsize=14)   # not shown in the boo
    plt.xlabel("Training set size", fontsize=14) # not shown
    plt.ylabel("RMSE", fontsize=14)              # not shown
```

[36]

```
lin_reg = LinearRegression()
plot_learning_curves(lin_reg, X, y)
plt.axis([0, 80, 0, 3])                         # not shown in the book
save_fig("underfitting_learning_curves_plot")   # not shown
plt.show()                                      # not shown
```

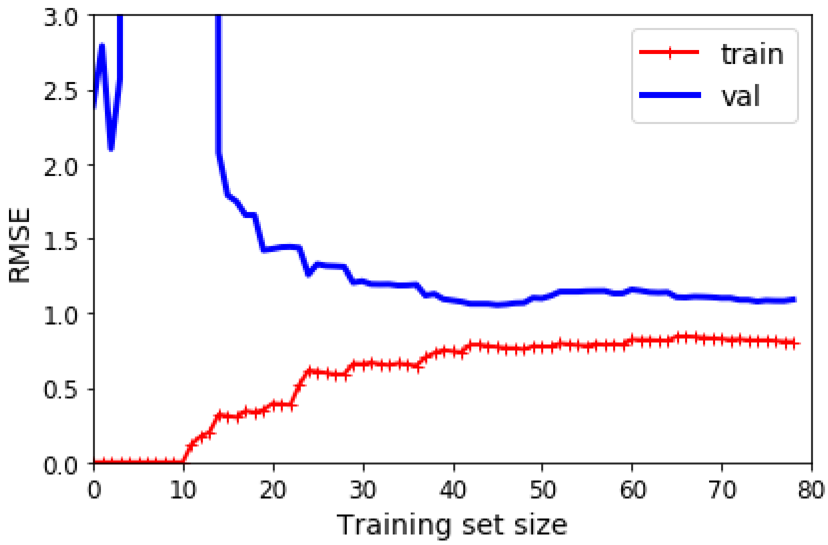
Saving figure underfitting_learning_curves_plot



[37]

```
from sklearn.pipeline import Pipeline

polynomial_regression = Pipeline([
        ("poly_features", PolynomialFeatures(degree=10, include_bias=Fa
        ("lin_reg", LinearRegression()),
    ])

plot_learning_curves(polynomial_regression, X, y)
plt.axis([0, 80, 0, 3])           # not shown
save_fig("learning_curves_plot")  # not shown
plt.show()                        # not shown
```

Saving figure learning_curves_plot

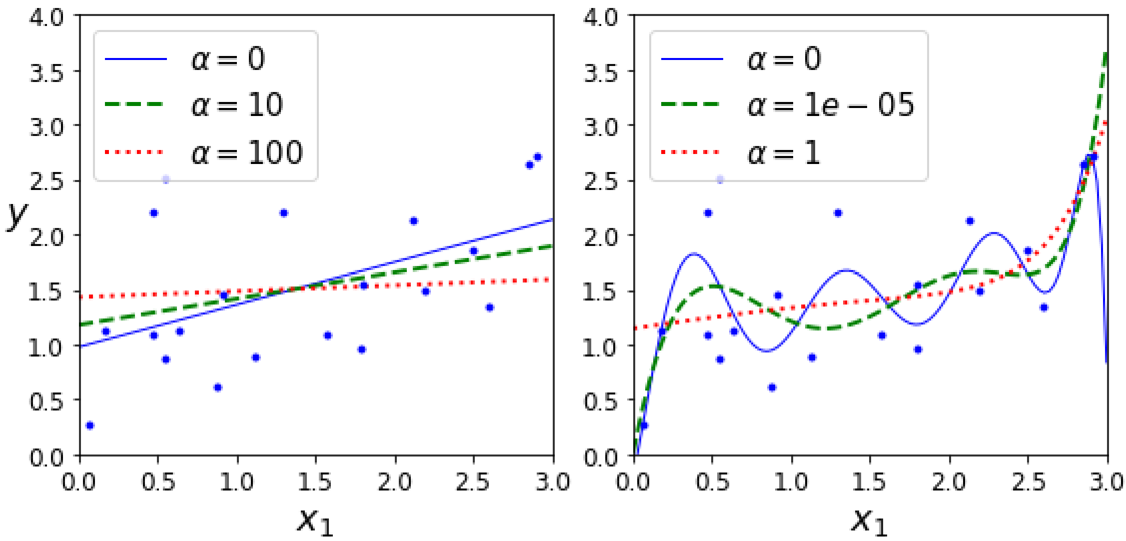


Regularized models

[38]

```python
from sklearn.linear_model import Ridge

np.random.seed(42)
m = 20
X = 3 * np.random.rand(m, 1)
y = 1 + 0.5 * X + np.random.randn(m, 1) / 1.5
X_new = np.linspace(0, 3, 100).reshape(100, 1)

def plot_model(model_class, polynomial, alphas, **model_kargs):
    for alpha, style in zip(alphas, ("b-", "g--", "r:")):
        model = model_class(alpha, **model_kargs) if alpha > 0 else Lin
        if polynomial:
            model = Pipeline([
                    ("poly_features", PolynomialFeatures(degree=10, inc
                    ("std_scaler", StandardScaler()),
                    ("regul_reg", model),
                ])
        model.fit(X, y)
        y_new_regul = model.predict(X_new)
        lw = 2 if alpha > 0 else 1
        plt.plot(X_new, y_new_regul, style, linewidth=lw, label=r"$\alp
    plt.plot(X, y, "b.", linewidth=3)
    plt.legend(loc="upper left", fontsize=15)
    plt.xlabel("$x_1$", fontsize=18)
    plt.axis([0, 3, 0, 4])

plt.figure(figsize=(8,4))
plt.subplot(121)
plot_model(Ridge, polynomial=False, alphas=(0, 10, 100), random_state=4
plt.ylabel("$y$", rotation=0, fontsize=18)
plt.subplot(122)
plot_model(Ridge, polynomial=True, alphas=(0, 10**-5, 1), random_state=

save_fig("ridge_regression_plot")
plt.show()
```

```
Saving figure ridge_regression_plot
```



[39]

```
from sklearn.linear_model import Ridge
ridge_reg = Ridge(alpha=1, solver="cholesky", random_state=42)
ridge_reg.fit(X, y)
ridge_reg.predict([[1.5]])
```

```
array([[1.55071465]])
```

[40]

```
sgd_reg = SGDRegressor(max_iter=50, tol=-np.infty, penalty="l2", random
sgd_reg.fit(X, y.ravel())
sgd_reg.predict([[1.5]])
```

```
array([1.49905184])
```

[41]

```
ridge_reg = Ridge(alpha=1, solver="sag", random_state=42)
ridge_reg.fit(X, y)
ridge_reg.predict([[1.5]])
```

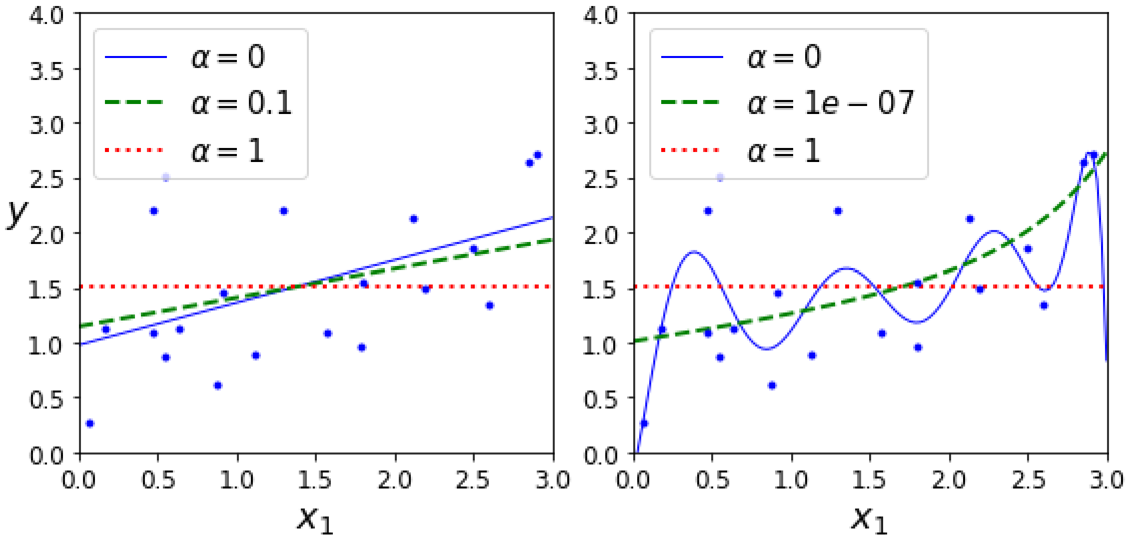
```
array([[1.5507201]])
```

[42]

```
from sklearn.linear_model import Lasso

plt.figure(figsize=(8,4))
plt.subplot(121)
plot_model(Lasso, polynomial=False, alphas=(0, 0.1, 1), random_state=42
plt.ylabel("$y$", rotation=0, fontsize=18)
```

```
plt.subplot(122)
plot_model(Lasso, polynomial=True, alphas=(0, 10**-7, 1), tol=1, random

save_fig("lasso_regression_plot")
plt.show()
```

Saving figure lasso_regression_plot



[43]

```
from sklearn.linear_model import Lasso
lasso_reg = Lasso(alpha=0.1)
lasso_reg.fit(X, y)
lasso_reg.predict([[1.5]])
```

array([1.53788174])

[44]

```
from sklearn.linear_model import ElasticNet
elastic_net = ElasticNet(alpha=0.1, l1_ratio=0.5, random_state=42)
elastic_net.fit(X, y)
elastic_net.predict([[1.5]])
```

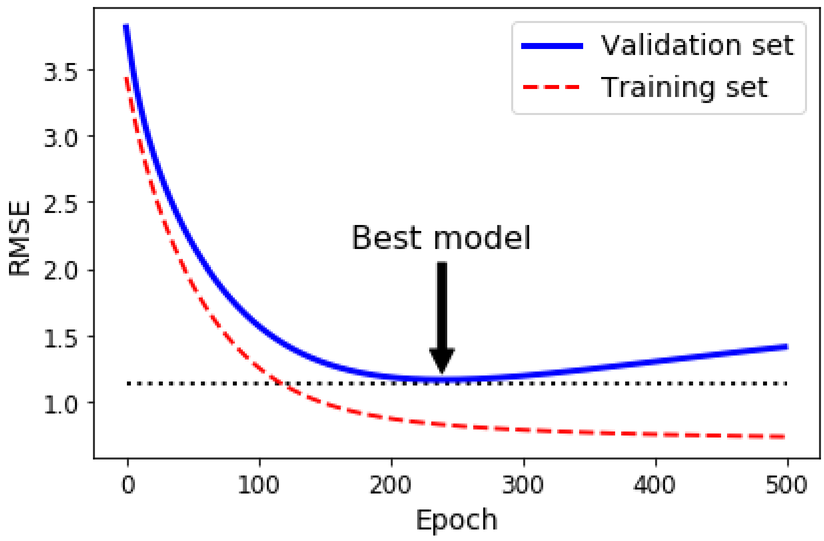
array([1.54333232])

[45]

```
np.random.seed(42)
m = 100
X = 6 * np.random.rand(m, 1) - 3
y = 2 + X + 0.5 * X**2 + np.random.randn(m, 1)

X_train, X_val, y_train, y_val = train_test_split(X[:50], y[:50].ravel(
```

```
        ("poly_features", PolynomialFeatures(degree=90, include_bias=Fa
        ("std_scaler", StandardScaler()),
    ])

X_train_poly_scaled = poly_scaler.fit_transform(X_train)
X_val_poly_scaled = poly_scaler.transform(X_val)

sgd_reg = SGDRegressor(max_iter=1,
                       tol=-np.infty,
                       penalty=None,
                       eta0=0.0005,
                       warm_start=True,
                       learning_rate="constant",
                       random_state=42)

n_epochs = 500
train_errors, val_errors = [], []
for epoch in range(n_epochs):
    sgd_reg.fit(X_train_poly_scaled, y_train)
    y_train_predict = sgd_reg.predict(X_train_poly_scaled)
    y_val_predict = sgd_reg.predict(X_val_poly_scaled)
    train_errors.append(mean_squared_error(y_train, y_train_predict))
    val_errors.append(mean_squared_error(y_val, y_val_predict))

best_epoch = np.argmin(val_errors)
best_val_rmse = np.sqrt(val_errors[best_epoch])

plt.annotate('Best model',
             xy=(best_epoch, best_val_rmse),
             xytext=(best_epoch, best_val_rmse + 1),
             ha="center",
             arrowprops=dict(facecolor='black', shrink=0.05),
             fontsize=16,
            )

best_val_rmse -= 0.03  # just to make the graph look better
plt.plot([0, n_epochs], [best_val_rmse, best_val_rmse], "k:", linewidth
plt.plot(np.sqrt(val_errors), "b-", linewidth=3, label="Validation set"
plt.plot(np.sqrt(train_errors), "r--", linewidth=2, label="Training set
plt.legend(loc="upper right", fontsize=14)
plt.xlabel("Epoch", fontsize=14)
plt.ylabel("RMSE", fontsize=14)
save_fig("early_stopping_plot")
plt.show()
```

```
Saving figure early_stopping_plot
```



[46]
```
from sklearn.base import clone
sgd_reg = SGDRegressor(max_iter=1, tol=-np.infty, warm_start=True, pena
                       learning_rate="constant", eta0=0.0005, random_st

minimum_val_error = float("inf")
best_epoch = None
best_model = None
for epoch in range(1000):
    sgd_reg.fit(X_train_poly_scaled, y_train)  # continues where it lef
    y_val_predict = sgd_reg.predict(X_val_poly_scaled)
    val_error = mean_squared_error(y_val, y_val_predict)
    if val_error < minimum_val_error:
        minimum_val_error = val_error
        best_epoch = epoch
        best_model = clone(sgd_reg)
```

[47]
```
best_epoch, best_model
```

```
(239,
 SGDRegressor(alpha=0.0001, average=False, early_stopping=False,
epsilon=0.1,
        eta0=0.0005, fit_intercept=True, l1_ratio=0.15,
        learning_rate='constant', loss='squared_loss', max_iter=1,
        n_iter=None, n_iter_no_change=5, penalty=None, power_t=0.25,
        random_state=42, shuffle=True, tol=-inf,
validation_fraction=0.1,
        verbose=0, warm_start=True))
```

[49]
```
t1a, t1b, t2a, t2b = -1, 3, -1.5, 1.5

# ignoring bias term
```

```python
t1s = np.linspace(t1a, t1b, 500)
t2s = np.linspace(t2a, t2b, 500)
t1, t2 = np.meshgrid(t1s, t2s)
T = np.c_[t1.ravel(), t2.ravel()]
Xr = np.array([[-1, 1], [-0.3, -1], [1, 0.1]])
yr = 2 * Xr[:, :1] + 0.5 * Xr[:, 1:]

J = (1/len(Xr) * np.sum((T.dot(Xr.T) - yr.T)**2, axis=1)).reshape(t1.sh

N1 = np.linalg.norm(T, ord=1, axis=1).reshape(t1.shape)
N2 = np.linalg.norm(T, ord=2, axis=1).reshape(t1.shape)

t_min_idx = np.unravel_index(np.argmin(J), J.shape)
t1_min, t2_min = t1[t_min_idx], t2[t_min_idx]

t_init = np.array([[0.25], [-1]])
```

[50]

```python
def bgd_path(theta, X, y, l1, l2, core = 1, eta = 0.1, n_iterations = 5
    path = [theta]
    for iteration in range(n_iterations):
        gradients = core * 2/len(X) * X.T.dot(X.dot(theta) - y) + l1 *

        theta = theta - eta * gradients
        path.append(theta)
    return np.array(path)

plt.figure(figsize=(12, 8))
for i, N, l1, l2, title in ((0, N1, 0.5, 0, "Lasso"), (1, N2, 0,  0.1,
    JR = J + l1 * N1 + l2 * N2**2

    tr_min_idx = np.unravel_index(np.argmin(JR), JR.shape)
    t1r_min, t2r_min = t1[tr_min_idx], t2[tr_min_idx]

    levelsJ=(np.exp(np.linspace(0, 1, 20)) - 1) * (np.max(J) - np.min(J
    levelsJR=(np.exp(np.linspace(0, 1, 20)) - 1) * (np.max(JR) - np.min
    levelsN=np.linspace(0, np.max(N), 10)

    path_J = bgd_path(t_init, Xr, yr, l1=0, l2=0)
    path_JR = bgd_path(t_init, Xr, yr, l1, l2)
    path_N = bgd_path(t_init, Xr, yr, np.sign(l1)/3, np.sign(l2), core=

    plt.subplot(221 + i * 2)
    plt.grid(True)
    plt.axhline(y=0, color='k')
    plt.axvline(x=0, color='k')
    plt.contourf(t1, t2, J, levels=levelsJ, alpha=0.9)
    plt.contour(t1, t2, N, levels=levelsN)
    plt.plot(path_J[:, 0], path_J[:, 1], "w-o")
    plt.plot(path_N[:, 0], path_N[:, 1], "y-^")
```

```python
        plt.title(r"$\ell_{}$ penalty".format(i + 1), fontsize=16)
        plt.axis([t1a, t1b, t2a, t2b])
        if i == 1:
            plt.xlabel(r"$\theta_1$", fontsize=20)
        plt.ylabel(r"$\theta_2$", fontsize=20, rotation=0)

        plt.subplot(222 + i * 2)
        plt.grid(True)
        plt.axhline(y=0, color='k')
        plt.axvline(x=0, color='k')
        plt.contourf(t1, t2, JR, levels=levelsJR, alpha=0.9)
        plt.plot(path_JR[:, 0], path_JR[:, 1], "w-o")
        plt.plot(t1r_min, t2r_min, "rs")
        plt.title(title, fontsize=16)
        plt.axis([t1a, t1b, t2a, t2b])
        if i == 1:
            plt.xlabel(r"$\theta_1$", fontsize=20)

save_fig("lasso_vs_ridge_plot")
plt.show()
```

```
Saving figure lasso_vs_ridge_plot
```