Market Impact Model and Microstructure[1]

Jing Guo

Strats Associate, Goldman Sachs

PhD in Financial Engineering, Columbia University

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[1] Reference: Maglaras (2015), Rama Cont and Stoikov (2014), C. Maglaras (n.d.)

Outline

1. Execution in LOB and Market Impact

2. Example of Market/Price Impact Model

Algo Trading Systems: Typically Decomposed into 3 Steps

- **Trade scheduling (macro-trader):** splits parent order into $\sim$ 5 min slices (**Lecture 2**)
  - Relevant time-scale: minutes-hours
  - Schedule follows user selected strategy (VWAP, POV, IS, ...)
  - Reflects urgency, alpha, risk/return tradeoff
  - Schedule updated during execution to reflect price/liquidity/...
- **Optimal execution of a slice (micro-trader):** further divides slice into child orders (**Lecture 3**)
  - Relevant time-scale: secondsminutes
  - Strategy optimizes pricing and placing of orders in the LOB
  - Execution adjusts to speed of LOB dynamics, price momentum, ...
- **Order routing:** decides where to send each child order (**Lecture 4**)
  - Relevant time-scale: $\sim 1 - 50$ ms
  - Optimizes fee/rebate tradeoff, liquidity/price, latency, etc

Stylized Optimal Execution in a LOB

- **Objective**: How to buy C shares within time T at the lowest price.
- **Controls**:
  - How much, when, at what prices to trade?
  - Trade with *limit orders* or *market orders*?
  - Trade with *block trades* or *continuously submitted trades*? When to submit
- Notes:
  - T is of same order of magnitude as the queueing delays ($\approx 1 \sim 5$ mins).
  - Microstrcture of LOB impacts execution and resulting costs.

Structure of Optimal Policy

- Market orders:
  - Trade with a small block trade $t = 0^+$ (do not move price)
  - Trade continuously and slowly without moving price in $(0, T)$.
  - Market orders do not push the price until T.
- Limit orders: since price will not move (up or down) in $(0, T)$,
  - post maximal limit order quantity that could execute taking into account queue dynamics and queue position.
- Block trade: submit at T, if needed, to complete target quantity C.

Practical Consideration

- Avoid *clean up*[2] trades, especially if this is a slice of a longer trade.
- Often times micro-trader does not have to complete C by T.
- Account of multiple exchanges in deciding how much and where to post.
- Do not post all limit order quantity in one block to
  - Avoid information leakage.
  - Avoid adverse selection (block trade fills most of the order $t \approx 0$).
  - Spread limit orders (accounting for queueing) to "trade uniformly over $[0, T]$".

[2] Purchase/sale of all the remaining supply of stock, or the last piece of a block, in a trade-leaving a net zero position.

Essential Building Block: Market Impact Model

- Optimizing the trade schedule, i.e., how to split a large trade over smaller waves to be executed over time, requires a cost function for:
  - Immediate costs due to current trading decisions (e.g., next 3 min)
  - Impact of current decisions on future prices (and future trades)
- Key considerations:
  - Transient costs: impact of current trading decisions on price
  - Decay of transient costs: instantaneous? impact decays over time?
  - Permanent costs: is there a permanent cost (information content)?
  - Time-scales: interpretation of transient, decay, permanent
- Calibration
  - How to model? functional forms? (depends on relevant time-scale)
  - What data is needed
  - Stock segmentation

What Causes Market Impact

- Short-term: institutional trading shifts buy/sell short-term imbalance.
  - Market makers detect imbalance and move price to source more liquidity.
  - Institutional trading "stimulates" other trading strategies.
    - e.g., price momentum or volume sensitive.
  - What happens when our order ends? It depends...
- Information: institutional trading carries information about our beliefs about price.
  - How does this get incorporated into price?
  - What happens after trade ends?

Market Impact Modeling

- Relevant time scales of *market impact*:
  - Permanent
  - Instantaneous (recover quickly)
  - Transient (decay over time)
    - Short-term costs
  - Intraday characteristics of volume, spread, tick size, volatility.
  - With respect to participation rate.
  - With respect to microstructure variables.
- Tactical trading decisions.

Drivers of Short-Term Execution Cost

- Percentage of order traded via limit orders:
  - Queue length (bid side)
  - Trading volume (bid side)
  - Cancellation behavior
  - Short-term alpha signal
- Percentage of order traded via market orders at top-of-book:
  - Queue length (ask side)
  - Trading volume (ask side)
  - Arrival rate of limit orders at top-of-book (ask side)
  - Quote resiliency
  - Short-term alpha signal
- Percentage of order traded via market orders at higher (when buying) price levels:
  - Tick size
  - Depth of book

Proprietary Trade Data

realized trade stats: 5min slices for 2013/7-2013/9, $> 1,800$ securities traded

	JUL 2013	AUG 2013	SEP 2013
Sample Size			
5min Slices	27,760	30,054	29,226
Parent Orders	3,396	3,607	3,882
Distinct Securities	988	896	885
Characteristics			
Average Daily Volume (shares)	3,014,000	2,595,000	2,509,000
Size of 5min Slices (shares)	1,294	1,043	849
Average Queue Length	10,280	21,730	17,750
Realized Participation Rate	9.60%	9.40%	8.39%
Price ($)	46.80	38.16	41.41
Spread ($)	0.031	0.025	0.025
Daily Volatility	2.23%	1.90%	1.94%
Implementation Shortfall (bps)	3.04	3.09	3.48

Outline

Market Impact Model: Rama Cont and Stoikov (2014)

- Rama Cont and Stoikov (2014) proposed a regression model for price change due to order imbalance:

$$\Delta P_i = \alpha_i + \beta_i OFI_i + \varepsilon_i,$$

where:

- ΔP_i is the difference of the first and last bid-ask mid price in interval $[T_{i-1}, T_i]$,
- Order flow imbalance $OFI_i := \sum_{t \in [T_{i-1}, T_i]} e_t$,
- where

$$\begin{aligned} e_n = & q_n^b \mathbb{1}_{\{P_n^b \geq P_{n-1}^b\}} - q_{n-1}^b \mathbb{1}_{\{P_n^b \leq P_{n-1}^b\}} - q_n^s \mathbb{1}_{\{P_n^s \geq P_{n-1}^s\}} \\ & + q_{n-1}^s \mathbb{1}_{\{P_n^s \geq P_{n-1}^s\}}, \end{aligned}$$

where $P_n^b(P_n^s)$ is n-th bid (ask) price, and $q_n^b(q_n^s)$ is n-th bid (ask) volume.

Market Impact Model: Rama Cont and Stoikov (2014) (Cont'd)

Ticker	Order flow imbalance			
	R^2	$t(\hat{\beta}_i)$	$\{\beta_i \neq 0\}$	F
AMD	64%	11.10	100%	382
APOL	63%	10.74	96%	396
AXP	69%	14.12	100%	449
AZO	47%	7.02	99%	179
BAC	79%	19.08	100%	774
BDX	63%	10.77	100%	362
BK	74%	15.56	100%	610
BSX	58%	7.55	88%	338
BTU	72%	14.75	100%	527
CAT	71%	14.80	100%	498
CB	64%	12.61	100%	378
CCL	70%	14.16	100%	478
CINF	70%	11.66	99%	552
CME	35%	5.46	96%	112
COH	69%	13.13	100%	457
COP	68%	12.79	100%	450
CVH	65%	11.74	99%	418
DNR	69%	13.78	99%	471
DVN	65%	12.11	100%	414

[3] [4]

[3] Features are constructed in every $T_i - T_{t-1} = \Delta T =$10 seconds.

[4] Regression is renewed in every 30 minutes.

Market Impact Model: C. Maglaras (n.d.)

- C. Maglaras (n.d.) proposed a linear regression for *implementation shortfall* (IS, **execution cost**):

$$IS = \beta_0 + \beta_1 \cdot s^* + \beta_2 \cdot (R^L s^*) + \beta_3 \cdot (R^M \delta^*) + \beta_4 \cdot \delta^*,$$

  where:
  - $s^* := \frac{s}{p}$ is bid-ask spread relative to stock price,
  - $\delta^* := \frac{\delta}{p}$ is tick size relative to stock price,
  - R^L is price adjustment due to limit order,
  - R^M is price adjustment due to market order.

In-sample Regressions (ADV $\geq$ 300,000 shares; POV $\in$ (1%, 30%))

Monthly linear regression results for microstructure market impact model

	JUL 2013	AUG 2013	SEP 2013
(intercept)			
coefficient	-0.6888***	-0.6941***	-0.5832**
std. error	0.1232	0.1140	0.1076
spread (bps): s^*			
coefficient	0.3187***	0.3905***	0.3950***
std. error	0.0069	0.0077	0.0070
limit order: $R^L s^*$			
coefficient	-0.3027***	-0.3415***	-0.3658***
std. error	0.0107	0.0100	0.0099
add. tick to pay: $R^M \sigma^*$			
coefficients	0.0991***	0.1480***	0.1486***
std. error	0.0234	0.0225	0.0348
tick size: σ^*			
coefficients	2.3238***	1.8508***	2.4290***
std. error	0.1098	0.0997	0.0996
R-squared	9.91%	10.62%	13.48%

[5] Significance: *** p<0.001, ** p<0.01, * p<0.05

[5] Daily volatility σ^* is used a proxy of effective tick size δ^*.

- Cross-Validation
  - C. Maglaras (n.d.) "micro" model:

$$IS = \beta_0 + \beta_1 \cdot s^* + \beta_2 \cdot (R^L s^*) + \beta_3 \cdot (R^M \delta^*) + \beta_4 \cdot \delta^*.$$

  - Benchmark "macro" model

$$IS = \beta_0 + \beta_1 \cdot (\text{Percent of Market Vol.})^\alpha \sigma^* + \beta_2 \cdot \sigma^*.$$

  - Out-of-sample R^2: C. Maglaras (n.d.) model 11% VS. benchmark models 3%.

	Our Model	Linear	Square Root
avg. out-of-sample R^2	11.03%	3.11%	3.12%
relative improvement	0.00%	255%	254%

References I

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