Market Impact Model and Microstructure[1]

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[1] Reference: Maglaras (2015), Rama Cont and Stoikov (2014), C. Maglaras (n.d.)

Outline

Algo Trading Systems: Typically Decomposed into 3 Steps

- **Trade scheduling (macro-trader):** splits parent order into $\sim$ 5 min slices (**Lecture 2**)
  - Relevant time-scale: minutes-hours
  - Schedule follows user selected strategy (VWAP, POV, IS, ...)
  - Reflects urgency, alpha, risk/return tradeoff
  - Schedule updated during execution to reflect price/liquidity/...
- **Optimal execution of a slice (micro-trader):** further divides slice into child orders (**Lecture 3**)
  - Relevant time-scale: secondsminutes
  - Strategy optimizes pricing and placing of orders in the LOB
  - Execution adjusts to speed of LOB dynamics, price momentum, ...
- **Order routing:** decides where to send each child order (**Lecture 4**)
  - Relevant time-scale: $\sim 1 - 50$ ms
  - Optimizes fee/rebate tradeoff, liquidity/price, latency, etc

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  - Microstrcture of LOB impacts execution and resulting costs.

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- Block trade: submit at T, if needed, to complete target quantity C.

Practical Consideration

- Avoid *clean up*[2] trades, especially if this is a slice of a longer trade.

[2] Purchase/sale of all the remaining supply of stock, or the last piece of a block, in a trade-leaving a net zero position.

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- Do not post all limit order quantity in one block to
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  - Avoid adverse selection (block trade fills most of the order $t \approx 0$).
  - Spread limit orders (accounting for queueing) to "trade uniformly over $[0, T]$".

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- Percentage of order traded via market orders at higher (when buying) price levels:
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  - Depth of book

Executed Algorithmic Orders

realized trade stats: 5min slices for 2013/7-2013/9, $> 1,800$ securities traded

	JUL 2013	AUG 2013	SEP 2013
Sample Size			
5min Slices	27,760	30,054	29,226
Parent Orders	3,396	3,607	3,882
Distinct Securities	988	896	885
Characteristics			
Average Daily Volume (shares)	3,014,000	2,595,000	2,509,000
Size of 5min Slices (shares)	1,294	1,043	849
Average Queue Length	10,280	21,730	17,750
Realized Participation Rate	9.60%	9.40%	8.39%
Price ($)	46.80	38.16	41.41
Spread ($)	0.031	0.025	0.025
Daily Volatility	2.23%	1.90%	1.94%
Implementation Shortfall (bps)	3.04	3.09	3.48

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[3] Only slices that correspond to VWAP, TWAP, and POV strategies are reported.

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$$\begin{aligned} e_n =& q_n^b \mathbb{1}_{\{P_n^b \geq P_{n-1}^b\}} - q_{n-1}^b \mathbb{1}_{\{P_n^b \leq P_{n-1}^b\}} - q_n^s \mathbb{1}_{\{P_n^s \geq P_{n-1}^s\}} \\ &+ q_{n-1}^s \mathbb{1}_{\{P_n^s \geq P_{n-1}^s\}}, \end{aligned}$$

where $P_n^b(P_n^s)$ is n-th bid (ask) price, and $q_n^b(q_n^s)$ is n-th bid (ask) volume.

Market Impact Model: Rama Cont and Stoikov (2014) (Cont'd)

Ticker	Order flow imbalance			
	R^2	$t(\hat{\beta}_i)$	$\{\beta_i \neq 0\}$	F
AMD	64%	11.10	100%	382
APOL	63%	10.74	96%	396
AXP	69%	14.12	100%	449
AZO	47%	7.02	99%	179
BAC	79%	19.08	100%	774
BDX	63%	10.77	100%	362
BK	74%	15.56	100%	610
BSX	58%	7.55	88%	338
BTU	72%	14.75	100%	527
CAT	71%	14.80	100%	498
CB	64%	12.61	100%	378
CCL	70%	14.16	100%	478
CINF	70%	11.66	99%	552
CME	35%	5.46	96%	112
COH	69%	13.13	100%	457
COP	68%	12.79	100%	450
CVH	65%	11.74	99%	418
DNR	69%	13.78	99%	471
DVN	65%	12.11	100%	414

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[4] Features are constructed in every $T_t - T_{t-1} = \Delta T$ =10 seconds.

[5] Regression is renewed in every 30 minutes.

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- $R^L \leq 1$ is price adjustment due to limit order (limit orders may reduce spread),

[6] A tick size is the minimum price movement of a trading instrument. It is estimated by daily volatility in this paper.

Market Impact Model: C. Maglaras (n.d.)

- C. Maglaras (n.d.) proposed a linear regression for *implementation shortfall* (IS, **execution cost**):

$$IS = \beta_0 + \beta_1 \cdot s^* + \beta_2 \cdot (R^L s^*) + \beta_3 \cdot (R^M \delta^*) + \beta_4 \cdot \delta^*,$$

where:

- $s^* := \frac{s}{p}$ is bid-ask spread relative to stock price,
- $\delta^* := \frac{\delta}{p}$ is tick size[6] relative to stock price,
- $R^L \leq 1$ is price adjustment due to limit order (limit orders may reduce spread),
- R^M is price adjustment due to market order.

[6] A tick size is the minimum price movement of a trading instrument. It is estimated by daily volatility in this paper.

In-sample Regressions (ADV $\geq$ 300,000 shares; POV $\in$ (1%, 30%))

Monthly linear regression results for microstructure market impact model

	JUL 2013	AUG 2013	SEP 2013
(intercept)			
coefficient	-0.6888***	-0.6941***	-0.5832**
std. error	0.1232	0.1140	0.1076
spread (bps): s^*			
coefficient	0.3187***	0.3905***	0.3950***
std. error	0.0069	0.0077	0.0070
limit order: $R^L s^*$			
coefficient	-0.3027***	-0.3415***	-0.3658***
std. error	0.0107	0.0100	0.0099
add. tick to pay: $R^M \sigma^*$			
coefficients	0.0991***	0.1480***	0.1486***
std. error	0.0234	0.0225	0.0348
tick size: σ^*			
coefficients	2.3238***	1.8508***	2.4290***
std. error	0.1098	0.0997	0.0996
R-squared	9.91%	10.62%	13.48%

Significance: *** p<0.001, ** p<0.01, * p<0.05

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[7] Daily volatility σ^* is used a proxy of effective tick size δ^*.

Cross-Validation

- Cross-Validation

Cross-Validation

- Cross-Validation
  - C. Maglaras (n.d.) "micro" model:

$$IS = \beta_0 + \beta_1 \cdot s^* + \beta_2 \cdot (R^L s^*) + \beta_3 \cdot (R^M \delta^*) + \beta_4 \cdot \delta^*.$$

Cross-Validation

- Cross-Validation
  - C. Maglaras (n.d.) "micro" model:

$$IS = \beta_0 + \beta_1 \cdot s^* + \beta_2 \cdot (R^L s^*) + \beta_3 \cdot (R^M \delta^*) + \beta_4 \cdot \delta^*.$$

  - Benchmark "macro" model

$$IS = \beta_0 + \beta_1 \cdot (\text{Percent of Market Vol.})^\alpha \sigma^* + \beta_2 \cdot \sigma^*.$$

Cross-Validation

- Cross-Validation
  - C. Maglaras (n.d.) "micro" model:

$$IS = \beta_0 + \beta_1 \cdot s^* + \beta_2 \cdot (R^L s^*) + \beta_3 \cdot (R^M \delta^*) + \beta_4 \cdot \delta^*.$$

  - Benchmark "macro" model

$$IS = \beta_0 + \beta_1 \cdot (\text{Percent of Market Vol.})^\alpha \sigma^* + \beta_2 \cdot \sigma^*.$$

  - Out-of-sample R^2: C. Maglaras (n.d.) model 11% VS. benchmark models 3%.

	Our Model	Linear	Square Root
avg. out-of-sample R^2	11.03%	3.11%	3.12%
relative improvement	0.00%	255%	254%

References I

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