

# Market Impact Model and Microstructure<sup>1</sup>

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<sup>1</sup>Reference: Maglaras (2015), Rama Cont and Stoikov (2014), C. Maglaras (n.d.)

- 1 Execution in LOB and Market Impact
- 2 Example of Market/Price Impact Model

# Algo Trading Systems: Typically Decomposed into 3 Steps

- **Trade scheduling (macro-trader):** splits parent order into  $\sim 5$  min slices (**Lecture 2**)
  - Relevant time-scale: minutes-hours
  - Schedule follows user selected strategy (VWAP, POV, IS, ... )
  - Reflects urgency, alpha, risk/return tradeoff
  - Schedule updated during execution to reflect price/liquidity/...
- **Optimal execution of a slice (micro-trader):** further divides slice into child orders (**Lecture 3**)
  - Relevant time-scale: secondsminutes
  - Strategy optimizes pricing and placing of orders in the LOB
  - Execution adjusts to speed of LOB dynamics, price momentum, ...
- **Order routing:** decides where to send each child order (**Lecture 4**)
  - Relevant time-scale:  $\sim 1 - 50$  ms
  - Optimizes fee/rebate tradeoff, liquidity/price, latency, etc

# Stylized Optimal Execution in a LOB

- **Objective:** How to buy  $C$  shares within time  $T$  at the lowest price.
- **Controls:**
  - How much, when, at what prices to trade?
  - Trade with *limit orders* or *market orders*?
  - Trade with *block trades* or *continuously submitted trades*? When to submit
- **Notes:**
  - $T$  is of same order of magnitude as the queueing delays ( $\approx 1 \sim 5$  mins).
  - Microstructure of LOB impacts execution and resulting costs.

# Structure of Optimal Policy

- Market orders:
  - Trade with a small block trade  $t = 0^+$  (do not move price)
  - Trade continuously and slowly without moving price in  $(0, T)$ .
  - Market orders do not push the price until  $T$ .
- Limit orders: since price will not move (up or down) in  $(0, T)$ ,
  - post maximal limit order quantity that could execute taking into account queue dynamics and queue position.
- Block trade: submit at  $T$ , if needed, to complete target quantity  $C$ .

# Practical Consideration

- Avoid *clean up*<sup>2</sup> trades, especially if this is a slice of a longer trade.
- Often times micro-trader does not have to complete  $C$  by  $T$ .
- Account of multiple exchanges in deciding how much and where to post.
- Do not post all limit order quantity in one block to
  - Avoid information leakage.
  - Avoid adverse selection (block trade fills most of the order  $t \approx 0$ ).
  - Spread limit orders (accounting for queueing) to "trade uniformly over  $[0, T]$ ".

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<sup>2</sup>Purchase/sale of all the remaining supply of stock, or the last piece of a block, in a trade-leaving a net zero position.

# Essential Building Block: Market Impact Model

- Optimizing the trade schedule, i.e., how to split a large trade over smaller waves to be executed over time, requires a cost function for:
  - Immediate costs due to current trading decisions (e.g., next 3 min)
  - Impact of current decisions on future prices (and future trades)
- Key considerations:
  - Transient costs: impact of current trading decisions on price
  - Decay of transient costs: instantaneous? impact decays over time?
  - Permanent costs: is there a permanent cost (information content)?
  - Time-scales: interpretation of transient, decay, permanent
- Calibration
  - How to model? functional forms? (depends on relevant time-scale)
  - What data is needed
  - Stock segmentation

# What Causes Market Impact

- Short-term: institutional trading shifts buy/sell short-term imbalance.
  - Market makers detect imbalance and move price to source more liquidity.
  - Institutional trading "stimulates" other trading strategies.
    - e.g., price momentum or volume sensitive.
  - What happens when our order ends? It depends...
- Information: institutional trading carries information about our beliefs about price.
  - How does this get incorporated into price?
  - What happens after trade ends?



- Relevant time scales of *market impact*:
  - Permanent
  - Instantaneous (recover quickly)
  - Transient (decay over time)
    - Short-term costs
  - Intraday characteristics of volume, spread, tick size, volatility.
  - With respect to participation rate.
  - With respect to microstructure variables.
- Tactical trading decisions.

# Drivers of Short-Term Execution Cost

- Percentage of order traded via limit orders:
  - Queue length (bid side)
  - Trading volume (bid side)
  - Cancellation behavior
  - Short-term alpha signal
- Percentage of order traded via market orders at top-of-book:
  - Queue length (ask side)
  - Trading volume (ask side)
  - Arrival rate of limit orders at top-of-book (ask side)
  - Quote resiliency
  - Short-term alpha signal
- Percentage of order traded via market orders at higher (when buying) price levels:
  - Tick size
  - Depth of book

# Executed Algorithmic Orders

realized trade stats: 5min slices for 2013/7-2013/9, > 1,800 securities traded

	JUL 2013	AUG 2013	SEP 2013
<b>Sample Size</b>			
5min Slices	27,760	30,054	29,226
Parent Orders	3,396	3,607	3,882
Distinct Securities	988	896	885
<b>Characteristics</b>			
Average Daily Volume (shares)	3,014,000	2,595,000	2,509,000
Size of 5min Slices (shares)	1,294	1,043	849
Average Queue Length	10,280	21,730	17,750
Realized Participation Rate	9.60%	9.40%	8.39%
Price (\$)	46.80	38.16	41.41
Spread (\$)	0.031	0.025	0.025
Daily Volatility	2.23%	1.90%	1.94%
Implementation Shortfall (bps)	3.04	3.09	3.48

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<sup>3</sup>Only slices that correspond to VWAP, TWAP, and POV strategies are reported.

- 1 Execution in LOB and Market Impact
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# Market Impact Model: Rama Cont and Stoikov (2014)

- Rama Cont and Stoikov (2014) proposed a regression model for price change due to order imbalance:

$$\Delta P_i = \alpha_i + \beta_i OFI_i + \varepsilon_i,$$

where:

- $\Delta P_i$  is the difference of the first and last bid-ask mid price in interval  $[T_{i-1}, T_i]$ ,
- Order flow imbalance  $OFI_i := \sum_{t \in [T_{i-1}, T_i]} e_t$ ,
- where

$$\begin{aligned} e_n = & q_n^b \mathbb{1}_{\{P_n^b \geq P_{n-1}^b\}} - q_{n-1}^b \mathbb{1}_{\{P_n^b \leq P_{n-1}^b\}} - q_n^s \mathbb{1}_{\{P_n^s \geq P_{n-1}^s\}} \\ & + q_{n-1}^s \mathbb{1}_{\{P_n^s \geq P_{n-1}^s\}}, \end{aligned}$$

where  $P_n^b(P_n^s)$  is  $n$ -th bid (ask) price, and  $q_n^b(q_n^s)$  is  $n$ -th bid (ask) volume.

# Market Impact Model: Rama Cont and Stoikov (2014)

## (Cont'd)

Ticker	Order flow imbalance			
	$R^2$	$t(\hat{\beta}_i)$	$\{\beta_i \neq 0\}$	$F$
AMD	64%	11.10	100%	382
APOL	63%	10.74	96%	396
AXP	69%	14.12	100%	449
AZO	47%	7.02	99%	179
BAC	79%	19.08	100%	774
BDX	63%	10.77	100%	362
BK	74%	15.56	100%	610
BSX	58%	7.55	88%	338
BTU	72%	14.75	100%	527
CAT	71%	14.80	100%	498
CB	64%	12.61	100%	378
CCL	70%	14.16	100%	478
CINF	70%	11.66	99%	552
CME	35%	5.46	96%	112
COH	69%	13.13	100%	457
COP	68%	12.79	100%	450
CVH	65%	11.74	99%	418
DNR	69%	13.78	99%	471
DVN	65%	12.11	100%	414

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<sup>4</sup>Features are constructed in every  $T_i - T_{t-1} = \Delta T = 10$  seconds.

<sup>5</sup>Regression is renewed in every 30 minutes.

# Market Impact Model: C. Maglaras (n.d.)

- C. Maglaras (n.d.) proposed a linear regression for *implementation shortfall* (IS, **execution cost**):

$$IS = \beta_0 + \beta_1 \cdot s^* + \beta_2 \cdot (R^L s^*) + \beta_3 \cdot (R^M \delta^*) + \beta_4 \cdot \delta^*,$$

where:

- $s^* := \frac{s}{p}$  is bid-ask spread relative to stock price,
- $\delta^* := \frac{\delta}{p}$  is tick size<sup>6</sup> relative to stock price,
- $R^L \leq 1$  is price adjustment due to limit order (limit orders may reduce spread),
- $R^M$  is price adjustment due to market order.

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<sup>6</sup>A tick size is the minimum price movement of a trading instrument. It is estimated by daily volatility in this paper.

## Monthly linear regression results for microstructure market impact model

	JUL 2013	AUG 2013	SEP 2013
(intercept)			
coefficient	-0.6888***	-0.6941***	-0.5832**
std. error	0.1232	0.1140	0.1076
spread (bps): $s^*$			
coefficient	0.3187***	0.3905***	0.3950***
std. error	0.0069	0.0077	0.0070
limit order: $R^L s^*$			
coefficient	-0.3027***	-0.3415***	-0.3658***
std. error	0.0107	0.0100	0.0099
add. tick to pay: $R^M \sigma^*$			
coefficients	0.0991***	0.1480***	0.1486***
std. error	0.0234	0.0225	0.0348
tick size: $\sigma^*$			
coefficients	2.3238***	1.8508***	2.4290***
std. error	0.1098	0.0997	0.0996
R-squared	9.91%	10.62%	13.48%

Significance: \*\*\*  $p < 0.001$ , \*\*  $p < 0.01$ , \*  $p < 0.05$

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<sup>7</sup>Daily volatility  $\sigma^*$  is used a proxy of effective tick size  $\delta^*$ .



- Cross-Validation

- C. Maglaras (n.d.) "micro" model:

$$IS = \beta_0 + \beta_1 \cdot s^* + \beta_2 \cdot (R^L s^*) + \beta_3 \cdot (R^M \delta^*) + \beta_4 \cdot \delta^*.$$

- Benchmark "macro" model

$$IS = \beta_0 + \beta_1 \cdot (\text{Percent of Market Vol.})^\alpha \sigma^* + \beta_2 \cdot \sigma^*.$$

- Out-of-sample  $R^2$ : C. Maglaras (n.d.) model 11% VS. benchmark models 3%.

	Our Model	Linear	Square Root
avg. out-of-sample $R^2$	11.03%	3.11%	3.12%
relative improvement	0.00%	255%	254%

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- Rama Cont, A. K. and Stoikov, S. (2014). The price impact of order book events, *J. Finan. Econometrics* **12**(1): 47–88.