

# Limit Order Book and High Frequency Trading<sup>1</sup>

Jing Guo

Strats Associate, Goldman Sachs

PhD in Financial Engineering, Columbia University

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<sup>1</sup>Reference: Avellaneda (2011), Maglaras (2015)

- 1 Top of Book Analysis
- 2 The Need for Speed in Placing Passive Orders
- 3 The Need for Speed in Placing Aggressive Orders

# Simple Limit Order Book

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ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid1	55	100.00	100.01	2,000	Offer1
Bid2	1,000	100.00	100.02	2950	Offer2
Bid3	3,100	99.99	100.02	600	Offer3
Bid4	200	99.99	100.03	300	Offer4
Bid5	5,000	99.98	100.04	1,000	Offer5

Table 1: Mockup of an Order Book for a Fictitious Ticker

Last Trade Price = ?    Last Trade Size = ?

Best Bid Price = 100.00    Best Bid Size = 1,055

Best Offer Price = 100.01    Best Offer Size = 2,000

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<sup>2</sup>Narang (2014)

# Simple Limit Order Book (Cont'd)

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid1	55	100.00	100.02	1,950	Offer2
Bid2	1,000	100.00	100.02	600	Offer3
Bid3	3,100	99.99	100.03	300	Offer4
Bid4	200	99.99	100.04	1,000	Offer5
Bid5	5,000	99.98			

Table 2: Mockup of an Order Book for a Fictitious Ticker after a 3,000-share Market Share to Buy

Last Trade Price = 100.01    Last Trade Size = 3000

Best Bid Price = 100.00    Best Bid Size = 55

Best Offer Price = 100.02    Best Offer Size = 1,950

# Simple Limit Order Book (Cont'd)

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid2	55	100.00	100.02	1,950	Offer2
Bid3	1,000	100.00	100.02	600	Offer3
Bid4	3,100	99.99	100.03	300	Offer4
Bid5	200	99.99	100.04	1,000	Offer5

Table 3: Mockup of an Order Book for a Fictitious Ticker after a 1,000-share Limit Order to Sell at \$100.00

Last Trade Price = 100.00    Last Trade Size = 1000

Best Bid Price = 100.00    Best Bid Size = 55

Best Offer Price = 100.02    Best Offer Size = 1,950

# Simple Limit Order Book (Cont'd)

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid2	55	100.00	100.02	1,950	Offer2
Bid3	1,000	100.00	100.02	600	Offer3
Bid4	3,100	99.99	100.02	1,000	Offer6
Bid5	200	99.99	100.03	300	Offer4
			100.04	1,000	Offer5

**Table 4:** Mockup of an Order Book for a Fictitious Ticker after a 1,000-share Limit Order Joins the Best Offer \$100.02

Last Trade Price = 100.00    Last Trade Size = 1000

Best Bid Price = 100.00    Best Bid Size = 55

Best Offer Price = 100.02    Best Offer Size = 1,950

# Simple Limit Order Book (Cont'd)

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid2	55	100.00	100.01	2,000	Offer7
Bid3	1,000	100.00	100.02	1,950	Offer2
Bid4	3,100	99.99	100.02	600	Offer3
Bid5	200	99.99	100.02	1,000	Offer6
			100.03	300	Offer4
			100.04	1,000	Offer5

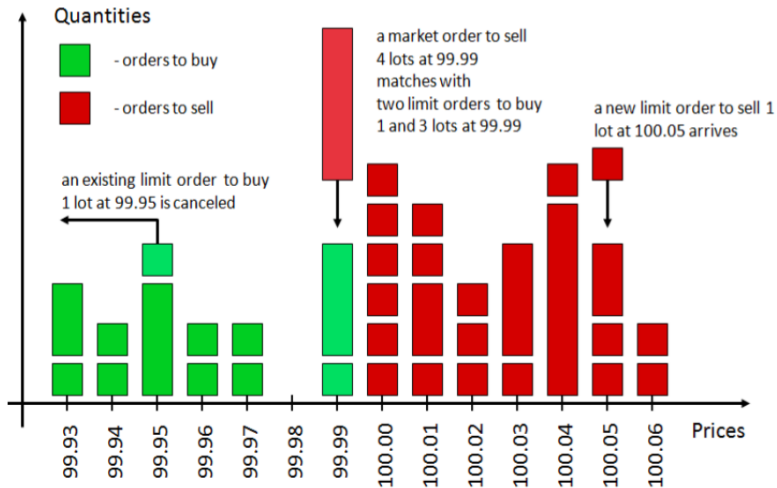
**Table 5:** Mockup of an Order Book for a Fictitious Ticker after a 2,000-share Limit Order Improves the Best Offer \$100.01

Last Trade Price = 100.00    Last Trade Size = 1000

Best Bid Price = 100.00    Best Bid Size = 55

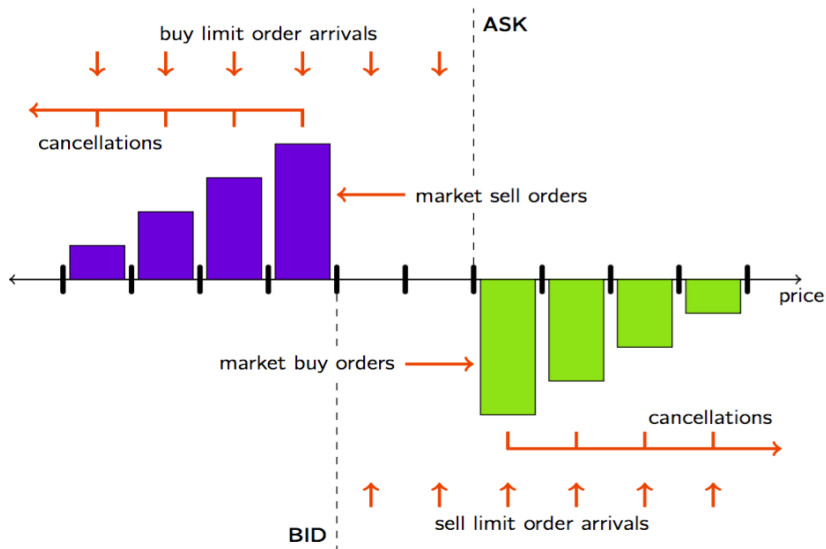
Best Offer Price = 100.01    Best Offer Size = 2,000

# LOB Schematic

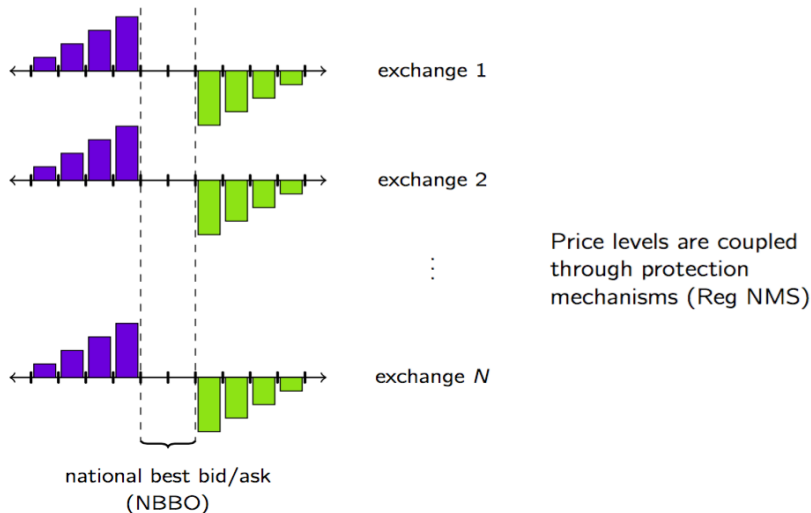




# The Limit Order Book (LOB)



# Multiple Limit Order Books



- 1 Top of Book Analysis
- 2 The Need for Speed in Placing Passive Orders
- 3 The Need for Speed in Placing Aggressive Orders

# The Need for Speed in Placing *Passive* Orders<sup>3</sup>

- Passive Orders are susceptible to adverse selection.
  - Slower orders are more likely to be traded on toxic orders.
- According to an internal research by Tradeworx, the average return of passive order on the most liquid stocks was  $\approx -0.2$  cents per share (year 2010).
- Buying the bid and selling the offer is a *money-losing* proposition in the absence of liquidity provision rebates.

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<sup>3</sup>Narang (2014)

# Top of Order Book

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid4	1,000	100.00	100.01	2,000	Offer1
Bid5	6,000	100.00	100.02	2,950	Offer2
Bid1	3,100	99.99	100.02	600	Offer3
Bid2	200	99.99	100.03	300	Offer4
Bid3	5,000	99.98	100.04	1,000	Offer5

Table 6: Mockup of an Order Book for a Fictitious Ticker with 1,000-share Bid

- Imagine that you place an order to buy 1,000 shares of XYZ at \$100.00 (Bid6).
- Further imagine that there are a large number of shares bid just after Bid4 at the same price (Bid5).

## Top of Order Book (Cont'd)

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid5	6,000	100.00	100.01	2,000	Offer1
Bid1	3,100	99.99	100.02	2,950	Offer2
Bid2	200	99.99	100.02	600	Offer3
Bid3	5,000	99.98	100.03	300	Offer4
			100.04	1,000	Offer5

**Table 7:** Mockup of an Order Book for a Fictitious Ticker after a 1,000-share Aggressive Sell Order

- You were at the front of the order book queue.
- The best price is still \$100.00.
- You bought at the best price.

## Top of Order Book (Cont'd)

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid1	3,100	99.99	100.00	2,000	Offer6
Bid2	200	99.99	100.01	2,000	Offer1
Bid3	5,000	99.98	100.02	2,950	Offer2
			100.02	600	Offer3
			100.03	300	Offer4
			100.04	1,000	Offer5

**Table 8:** Mockup of an Order Book for a Fictitious Ticker after All \$100.00 Bid Shares are Removed

- Imagine your order is the *last* one in the book at \$100.00.
- The best bid price becomes \$99.99 when finally the order is executed.
- The price you received immediately became the *subsequent* best offer.

# Need for Speed<sup>4</sup>

- The impact of queue placement was examined empirically by Tradeworx as approximately a 1.7 cents per share difference in profitability of being first versus being last at a given price.
- Reaching the top tier of speed costs a great deal of money.
- The need for speed among passive orders is a function of:
  - Adverse selection metrics
  - Volume of shares traded
  - The cost of building and maintaining top-tier speed

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<sup>4</sup>Narang (2014)



# Outline

- 1 Top of Book Analysis
- 2 The Need for Speed in Placing Passive Orders
- 3 The Need for Speed in Placing Aggressive Orders

# The Need for Speed in Placing *Aggressive Limit* Orders

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid1	3,100	99.99	100.01	2,000	Offer1
Bid2	200	99.99	100.02	2,950	Offer2
Bid3	5,000	99.98	100.02	600	Offer3
			100.03	300	Offer4
			100.04	1,000	Offer5

Table 9: Mockup of an Order Book for a Fictitious Ticker

- The top of order book is shown as above.
- Imagine two trades each want to buy 2,000 shares at \$100.01 and they both enter limit orders.
- Only the first order will interact with the resting \$100.01 offer for \$2,000 shares.

## Placing Aggressive Limit Orders (Cont'd)

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid4	2,000	100.01	100.02	2,950	Offer2
Bid1	3,100	99.99	100.02	600	Offer3
Bid2	200	99.99	100.03	300	Offer4
Bid3	5,000	99.98	100.04	1,000	Offer5

Table 10: Mockup of an Order Book for a Fictitious Ticker

- The new best bid belongs to the second (slower) order.
- One of three scenarios may apply to this order:
  - It will be filled at \$100.01 but subject to adverse selection.
  - He cancels and replaces the order with a higher-priced bid.
  - It will end up not being filled at all.

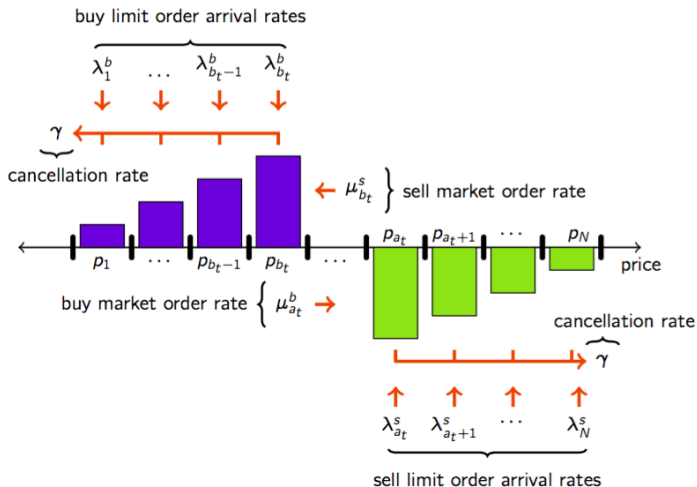
# Need for Speed in Placing *Aggressive Market Orders*<sup>5</sup>

- A slowly transmitted market order suffers from adverse selection.
  - It is less likely that there are other buyers behind us.
  - We will most likely end up with a worse fill.
- Market orders also have *slippage issues*.
- The greater the accuracy of the forecasts (from the alpha model), the larger our concern over slippage.
  - A more accurate forecast is more likely to move in the direction we expect.

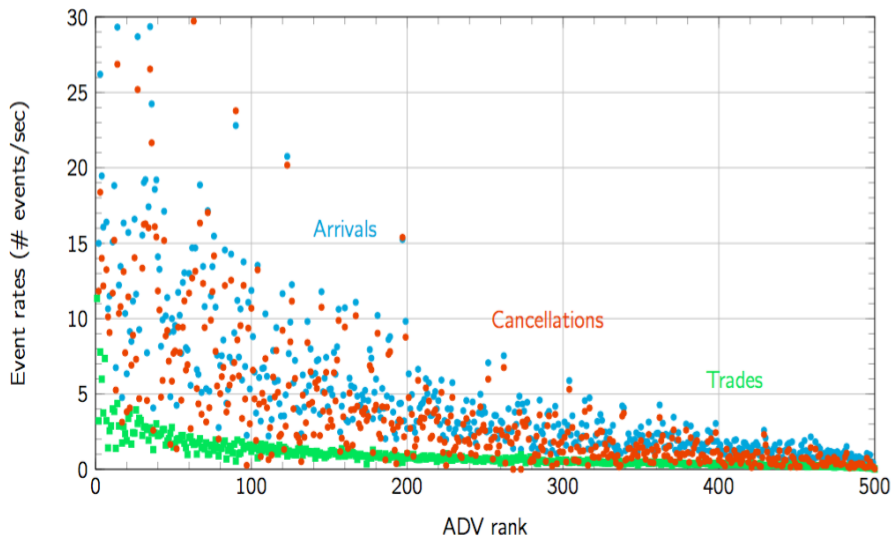
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<sup>5</sup>Narang (2014)

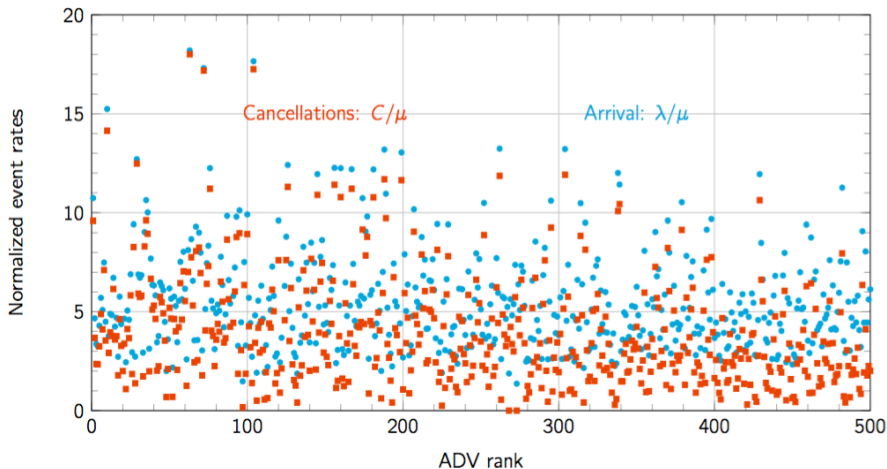
# LOB: event driven (short-term) view



# Event rates (top of book)

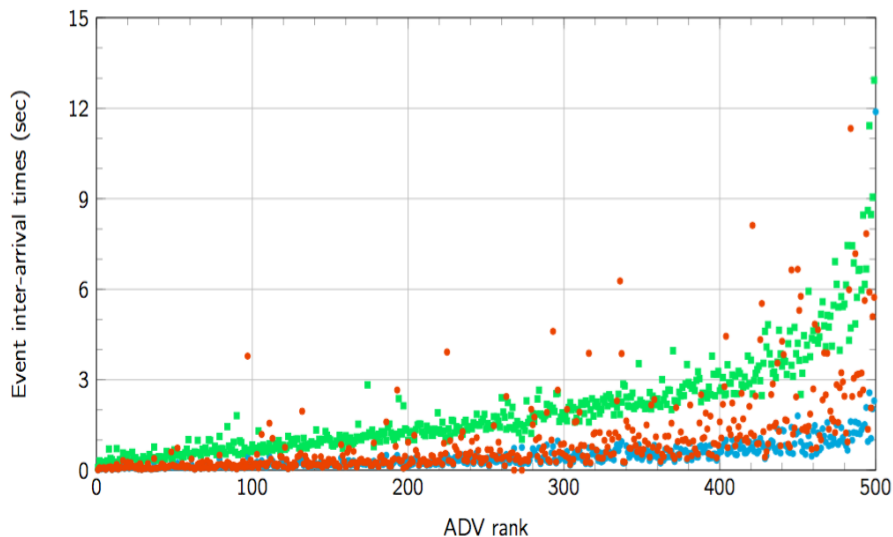


# Normalized event rates (top of book)



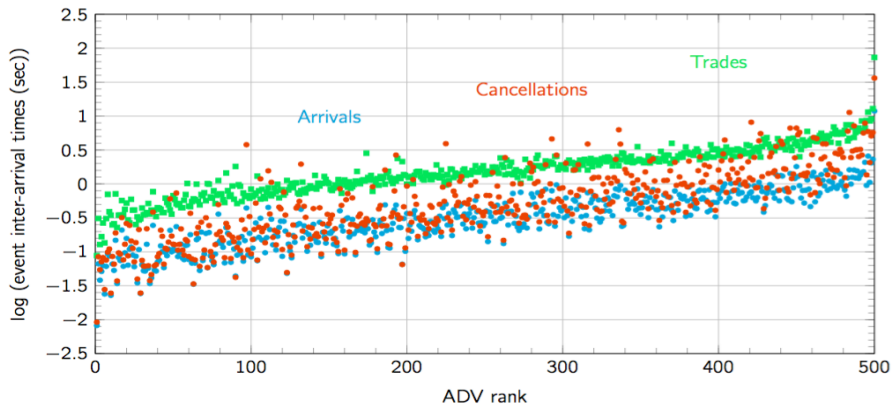
- cancellation volume (at top of book) trade volume
- arrival volume (limit orders at top of book) traded volume

# Interarrival times (top of book)





# Interarrival times (log scale) (top of book)



- liquid stocks: # trades, # cancellations, # large limit order arrivals
- # trades  $\approx$  1 order of magnitude less frequent than cancels or order arrivals

# Interarrival times (log scale) (top of book)

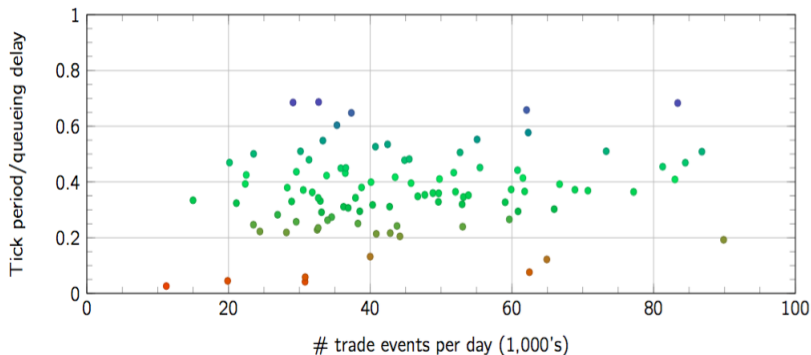


Figure 1: Tick period VS queueing delay: ratio against # trade events. (liquid names)

- tick period = avg time between changes in the mid-price
- tick period is on same (or smaller) order magnitude as queueing delay

# Some tactical questions in optimizing trading decisions

- given state of LOB, how long will it take for a limit order to trade?
  - e.g., VWAP schedule wants 1,000 shares over next 3 min, when do we post limit orders?
- which exchange to post limit order taking into account orders already posted there, and the way that market orders will prioritize over different exchanges due to rebate/fee differences
- what is the dependency on event dynamics on the bid-side and ask-side of the book
- how can we keep track of our queue position?
  - how do adverse selection costs depend on queue position:
  - queue ahead of my order
  - queue behind my order
  - queue on opposite side of the book
- market orders, cancellations, etc.,

- Avellaneda, M. (2011). Algorithmic and high-frequency trading: an overview.  
<https://www.math.nyu.edu/faculty/avellane/QuantCongressUSA2011AlgoTrading>
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<https://www0.gsb.columbia.edu/faculty/cmaglaras/papers/IC-Lectures-2015.pdf>.
- Narang, R. K. (2014). *Inside the Black Box: The Simple Truth About Quantitative Trading*, second edn, Brilliance Audio.