Introduction to Limit Order Book[1]

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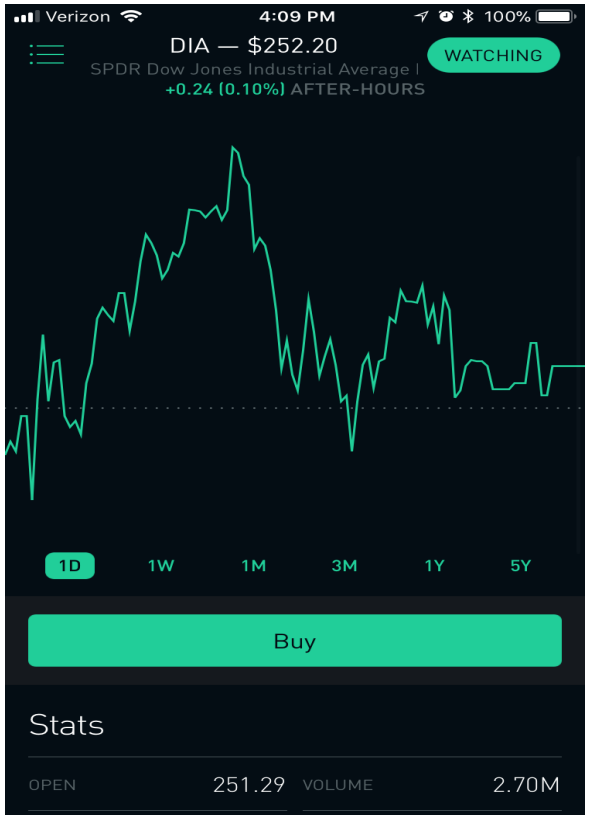
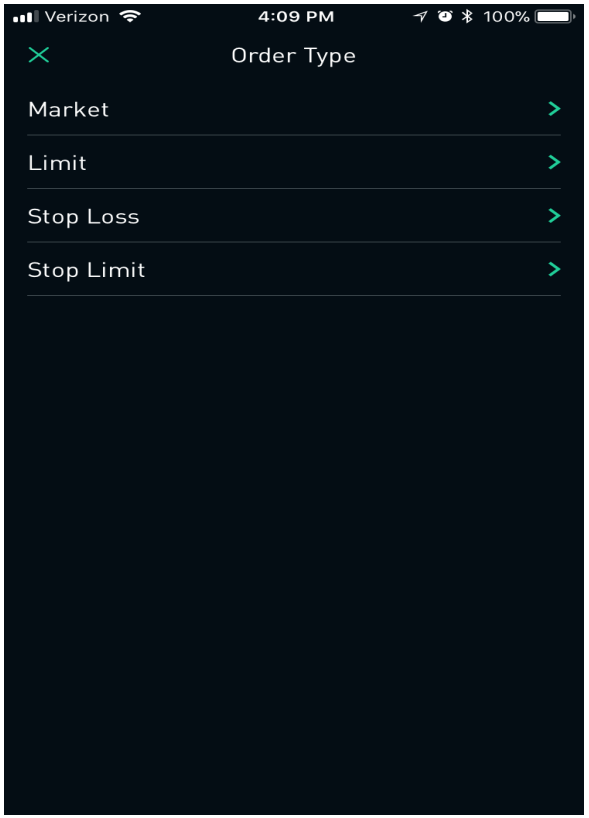
[1] Reference: Narang (2014)

Outline

Market Order Versus Limit Order

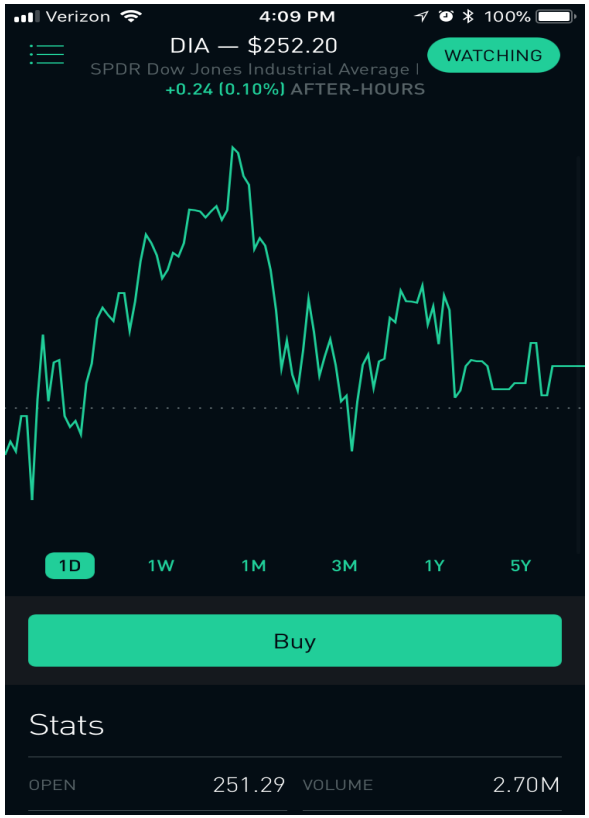
- A **market order** is an order to buy or sell *immediately* at the best available price.

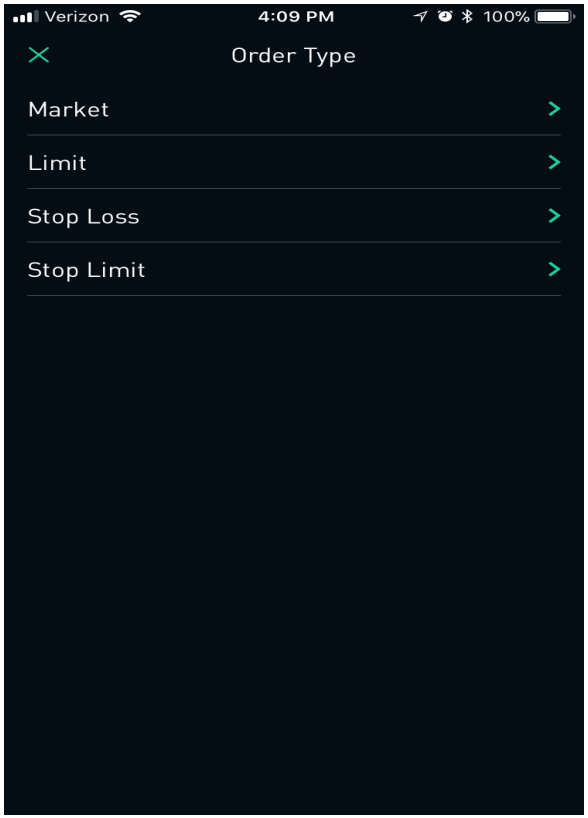




Market Order Versus Limit Order

- A **market order** is an order to buy or sell *immediately* at the best available price.
- A **limit order** sets the *maximum* or *minimum* price you are willing to buy or sell.





Simple Limit Order Book

2

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid1	55	100.00	100.01	2,000	Offer1
Bid2	1,000	100.00	100.02	2950	Offer2
Bid3	3,100	99.99	100.02	600	Offer3
Bid4	200	99.99	100.03	300	Offer4
Bid5	5,000	99.98	100.04	1,000	Offer5

Table 1: Mockup of an Order Book for a Fictitious Ticker

$$\text{Last Trade Price} = ? \quad \text{Last Trade Size} = ?$$
$$\text{Best Bid Price} = 100.00 \quad \text{Best Bid Size} = 1{,}055$$
$$\text{Best Offer Price} = 100.01 \quad \text{Best Offer Size} = 2{,}000$$

[2] Narang (2014)

LOB after 3,000-share Market Share to Buy

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid1	55	100.00	100.02	1,950	Offer2
Bid2	1,000	100.00	100.02	600	Offer3
Bid3	3,100	99.99	100.03	300	Offer4
Bid4	200	99.99	100.04	1,000	Offer5
Bid5	5,000	99.98			

Table 2: Mockup of an Order Book for a Fictitious Ticker after a 3,000-share Market Share to Buy

$$\text{Last Trade Price} = 100.01 \quad \text{Last Trade Size} = 3000$$
$$\text{Best Bid Price} = 100.00 \quad \text{Best Bid Size} = 55$$
$$\text{Best Offer Price} = 100.02 \quad \text{Best Offer Size} = 1{,}950$$

LOB after a 1,000-share Limit Order to Sell at $100.00

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid2	55	100.00	100.02	1,950	Offer2
Bid3	3,100	99.99	100.02	600	Offer3
Bid4	200	99.99	100.03	300	Offer4
Bid5	5000	99.8	100.04	1,000	Offer5

Table 3: Mockup of an Order Book for a Fictitious Ticker after a 1,000-share Limit Order to Sell at $100.00

$$\text{Last Trade Price} = 100.00 \quad \text{Last Trade Size} = 1000$$
$$\text{Best Bid Price} = 100.00 \quad \text{Best Bid Size} = 55$$
$$\text{Best Offer Price} = 100.02 \quad \text{Best Offer Size} = 1{,}950$$

LOB after a 1,000-share Limit Order to Sell at $100.02

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid2	55	100.00	100.02	1,950	Offer2
Bid3	3,100	99.99	100.02	600	Offer3
Bid4	200	99.99	100.02	1,000	Offer6
Bid5	5000	99.98	100.03	300	Offer4
			100.04	1,000	Offer5

Table 4: Mockup of an Order Book for a Fictitious Ticker after a 1,000-share Limit Order Joins the Best Offer $100.02

$$\text{Last Trade Price} = 100.00 \quad \text{Last Trade Size} = 1000$$
$$\text{Best Bid Price} = 100.00 \quad \text{Best Bid Size} = 55$$
$$\text{Best Offer Price} = 100.02 \quad \text{Best Offer Size} = 1{,}950$$

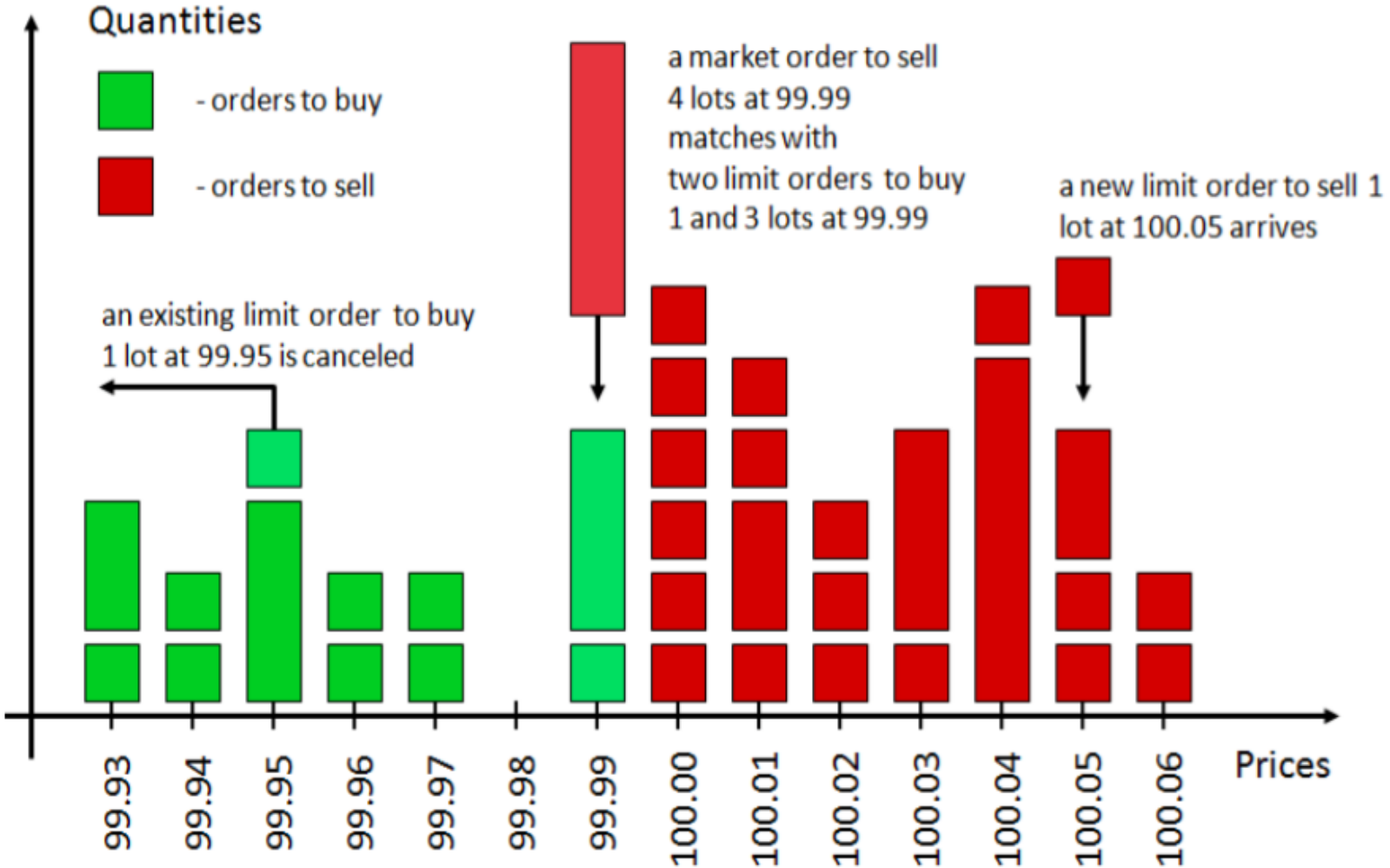
LOB after a 2,000-share Limit Order to sell at $100.01

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid2	55	100.00	100.01	2,000	Offer7
Bid3	3,100	99.99	100.02	1,950	Offer2
Bid4	200	99.99	100.02	600	Offer3
Bid5	5000	99.98	100.02	1,000	Offer6
			100.03	300	Offer4
			100.04	1,000	Offer5

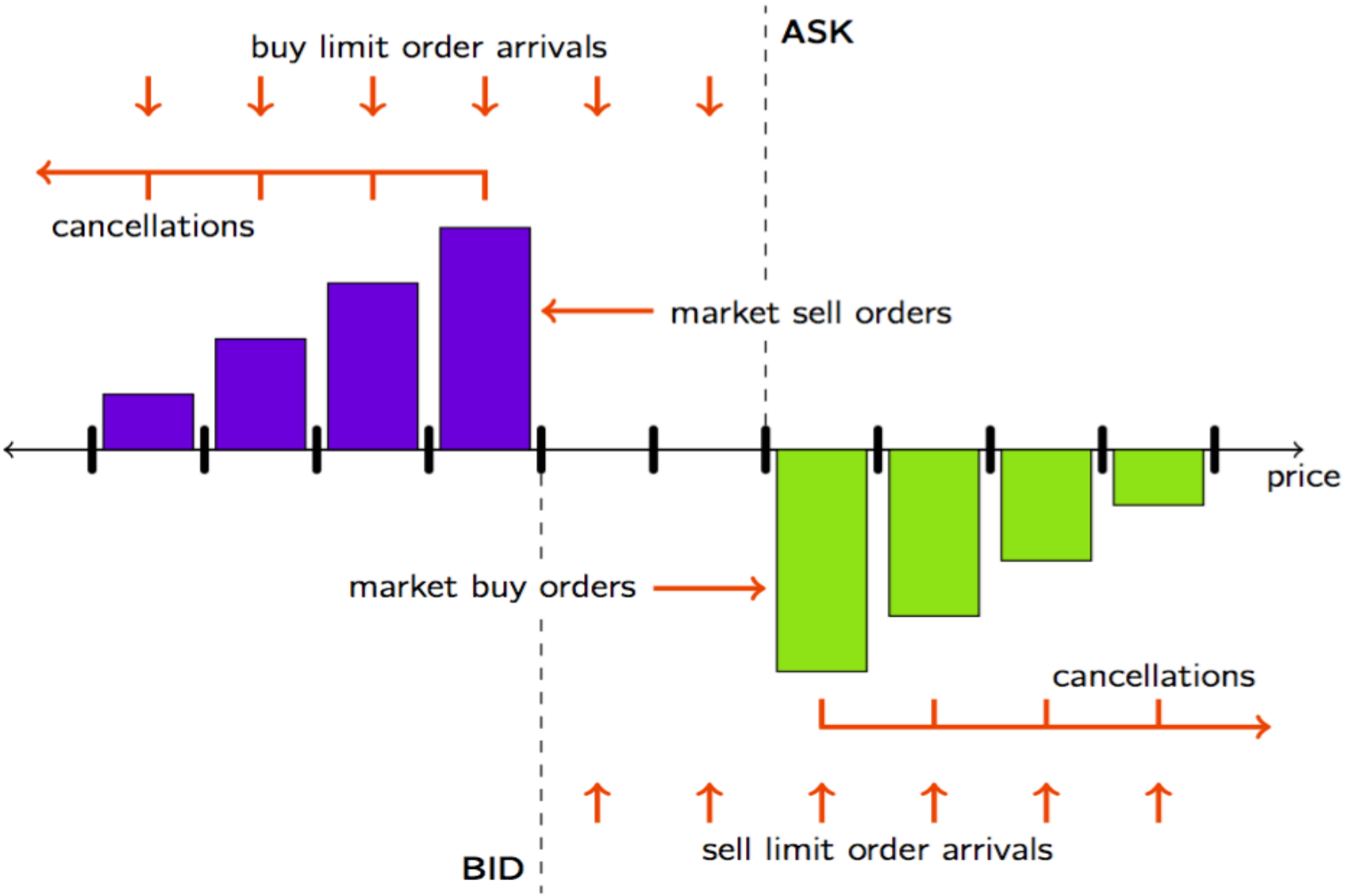
Table 5: Mockup of an Order Book for a Fictitious Ticker after a 2,000-share Limit Order Improves the Best Offer $100.01

$$\text{Last Trade Price} = 100.00 \quad \text{Last Trade Size} = 1000$$
$$\text{Best Bid Price} = 100.00 \quad \text{Best Bid Size} = 55$$
$$\text{Best Offer Price} = 100.01 \quad \text{Best Offer Size} = 2{,}000$$

LOB Schematic



The Limit Order Book (LOB)



Reflection on LOB

- Best offers (bid/ask price/size) are often of greatest interest (why?).

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- Best offers (bid/ask price/size) are often of greatest interest (why?).
- If the best bid size is much larger than the best ask size, what does it mean?
- In general, if we have access to the whole order book info, it is high dimensional (2n*2). Can we reduce the dimension?

Outline

The Need for Speed in Placing *Passive* Orders[3]

- Passive Orders are susceptible to adverse selection.

[3] Narang (2014)

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  - Slower orders are more likely to be traded on toxic orders.

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The Need for Speed in Placing *Passive* Orders[3]

- Passive Orders are susceptible to adverse selection.
  - Slower orders are more likely to be traded on toxic orders.
- According to an internal research by Tradeworx, the average return of passive order on the most liquid stocks was ≈ -0.2 cents per share (year 2010).

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The Need for Speed in Placing *Passive* Orders[3]

- Passive Orders are susceptible to adverse selection.
  - Slower orders are more likely to be traded on toxic orders.
- According to an internal research by Tradeworx, the average return of passive order on the most liquid stocks was ≈ -0.2 cents per share (year 2010).
- Buying the bid and selling the offer is a *money-losing* proposition in the absence of liquidity provision rebates.

[3] Narang (2014)

Top of Order Book (Case I)

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid4	1,000	100.00	100.01	2,000	Offer1
Bid5	6,000	100.00	100.02	2,950	Offer2
Bid1	3,100	99.99	100.02	600	Offer3
Bid2	200	99.99	100.03	300	Offer4
Bid3	5,000	99.98	100.04	1,000	Offer5

Table 6: Mockup of an Order Book for a Fictitious Ticker with 1,000-share Bid

- Imagine that you place an order to buy 1,000 shares of XYZ at \$100.00 (Bid4).
- Further imagine that there are a large number of shares bid just after Bid4 at the same price (Bid5).

Top of LOB after a 1,000-share Market Order to Sell

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid5	6,000	100.00	100.01	2,000	Offer1
Bid1	3,100	99.99	100.02	2,950	Offer2
Bid2	200	99.99	100.02	600	Offer3
Bid3	5,000	99.98	100.03	300	Offer4
			100.04	1,000	Offer5

Table 7: Mockup of an Order Book for a Fictitious Ticker after a 1,000-share Aggressive Sell Order

- You were at the front of the order book queue.
- The best price is still \$100.00.
- You bought at the best price.

Top of Order Book (Case II)

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid1	3,100	99.99	100.00	2,000	Offer6
Bid2	200	99.99	100.01	2,000	Offer1
Bid3	5,000	99.98	100.02	2,950	Offer2
			100.02	600	Offer3
			100.03	300	Offer4
			100.04	1,000	Offer5

Table 8: Mockup of an Order Book for a Fictitious Ticker after All $100.00 Bid Shares are Removed

- Now imagine your order is the *last* one in the book at $100.00.
- The best bid price becomes $99.99 when finally the order is executed.
- The price you received immediately became the *subsequent* best offer.

Need for Speed[4]

- The impact of queue placement was examined empirically by Tradeworx as approximately a 1.7 cents per share difference in profitability of being first versus being last at a given price.

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- Reaching the top tier of speed costs a great deal of money.

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- Reaching the top tier of speed costs a great deal of money.
- The need for speed among passive orders is a function of:

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- Reaching the top tier of speed costs a great deal of money.
- The need for speed among passive orders is a function of:
  - Adverse selection metrics
  - Volume of shares traded

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Need for Speed[4]

- The impact of queue placement was examined empirically by Tradeworx as approximately a 1.7 cents per share difference in profitability of being first versus being last at a given price.
- Reaching the top tier of speed costs a great deal of money.
- The need for speed among passive orders is a function of:
  - Adverse selection metrics
  - Volume of shares traded
  - The cost of building and maintaining top-tier speed

[4] Narang (2014)

Outline

The Need for Speed in Placing *Aggressive Limit* Orders

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid1	3,100	99.99	100.01	2,000	Offer1
Bid2	200	99.99	100.02	2,950	Offer2
Bid3	5,000	99.98	100.02	600	Offer3
			100.03	300	Offer4
			100.04	1,000	Offer5

Table 9: Mockup of an Order Book for a Fictitious Ticker

- The top of order book is shown as above.
- Imagine two trades each want to buy 2,000 shares at \$100.01 and they both enter limit orders.
- Only the first order will interact with the resting \$100.01 offer for \$2,000 shares.

Placing Aggressive Limit Orders (Cont'd)

ID	Bid Size	Bid Price	Ask Price	Ask Size	ID
Bid4	2,000	100.01	100.02	2,950	Offer2
Bid1	3,100	99.99	100.02	600	Offer3
Bid2	200	99.99	100.03	300	Offer4
Bid3	5,000	99.98	100.04	1,000	Offer5

Table 10: Mockup of an Order Book for a Fictitious Ticker

- The new best bid belongs to the second (slower) order.
- One of three scenarios may apply to this order:
  - It will be filled at $100.01 but subject to adverse selection.
  - He cancels and replaces the order with a higher-priced bid.
  - It will end up not being filled at all.

Need for Speed in Placing *Aggressive Market* Orders[5]

- A slowly transmitted market order suffers from adverse selection.

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- A slowly transmitted market order suffers from adverse selection.
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- Market orders also have *slippage issues*.

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- The greater the accuracy of the forecasts (from the alpha model), the larger our concern over slippage.

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  - It is less likely that there are other buyers behind us.
  - We will most likely end up with a worse fill.
- Market orders also have *slippage issues*.
- The greater the accuracy of the forecasts (from the alpha model), the larger our concern over slippage.
  - A more accurate forecast is more likely to move in the direction we expect.

[5] Narang (2014)

Reflection on HFT

- It is more important to be fast than smart.

Reflection on HFT

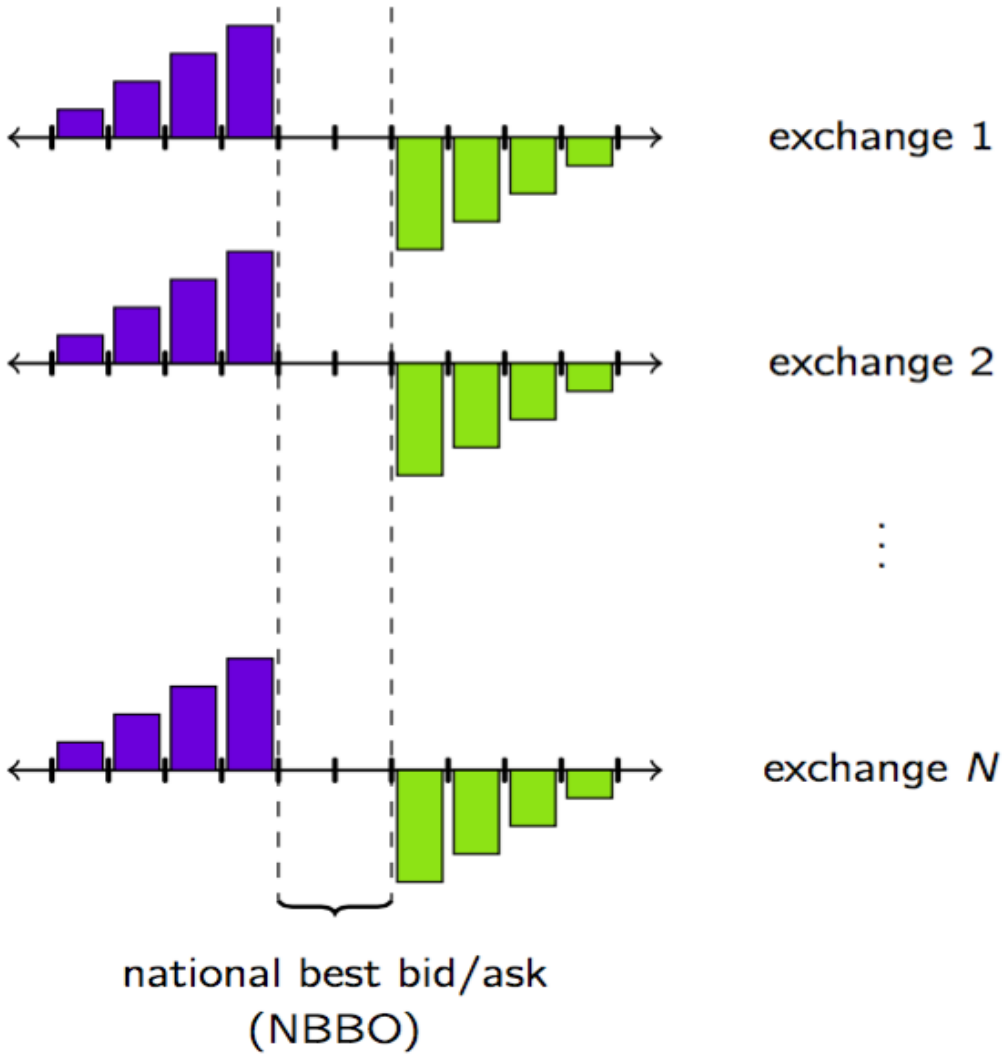
- It is more important to be fast than smart.
- It is more engineering than math.

Reflection on HFT

- It is more important to be fast than smart.
- It is more engineering than math.
- Profitability of HFT business.

Outline

Multiple Limit Order Books



Algorithmic trading and SOR (Wiki)

Preferred Venue		**Venue 1**		**Venue 2**	
Buy	Sell	Buy	Sell	Buy	Sell
	100@21.5		200@21.5		300@21.6

- An SOR Buy Day order for 1000@21.5 comes;
- Aggressive child to grab opportunity on preferable venue created: Buy *Immediate Or Cancel* (IOC) 100@21.5;
- Aggressive child to grab opportunity on Venue 1 created: Buy IOC 200@21.5;
- The remaining part placed passive to the Preferred venue:

Algorithmic trading and SOR (Wiki) (Cont'd)

Preferred Venue		**Venue 1**		**Venue 2**	
Buy	Sell	Buy	Sell	Buy	Sell
700@21.5					300@21.6

- New liquidity on Venue 2 appears: Sell 150@21.4:

Preferred Venue		**Venue 1**		**Venue 2**	
Buy	Sell	Buy	Sell	Buy	Sell
700@21.5					150@21.4
					300@21.6

Algorithmic trading and SOR (Wiki) (Cont'd)

- The algo "sweeps" from Preferred venue to grab the opportunity on Venue 2: Buy 150@21.4 IOC.

Preferred Venue		**Venue 1**		**Venue 2**	
Buy	Sell	Buy	Sell	Buy	Sell
550@21.5					300@21.6

- New liquidity on Venue 1 appears: Sell 600@21.5:

Preferred Venue		**Venue 1**		**Venue 2**	
Buy	Sell	Buy	Sell	Buy	Sell
550@21.5			600@21.5		300@21.6

Algorithmic trading and SOR (Wiki) (Cont'd)

- The algo "sweeps" from the Preferred venue to grab the opportunity on Venue 1: Buy 550@21.5 IOC.
- The trade happens, the algo terminates because all the intended shares were executed:

Preferred Venue		**Venue 1**		**Venue 2**	
Buy	Sell	Buy	Sell	Buy	Sell
			50@21.5		300@21.6

References I

Narang, R. K. (2014). *Inside the Black Box: The Simple Truth About Quantitative Trading*, second edn, Brilliance Audio.