Stochastic Finance (FIN 519)
Homework Solutions

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2018-19 Module 3 (Spring 2019)

1. **HW 1-1 SCFA** Exercise 1.1

2. **HW 1-2 SCFA** Exercise 1.3

3. **HW 2-1** In class, we derived the moments of the standard normal distribution:

$$E(Z^{2n}) = (2n-1)(2n-3)\cdot 3\cdot 1 \quad \text{for} \quad Z \sim N(0,1).$$

We can derive the same result using the moment generating function. First, derive the moment generating function,

$$M_X(t) = E(\exp(tX)) = \exp(\mu t + \frac{1}{2}\sigma^2 t^2) \quad \text{for} \quad X \sim N(\mu, \sigma^2).$$

Then, using the Taylor expansion of $M_X(t)$, derive the moment of Z. (After this problem, you can understand **SCFA Exercise 3.4** better. The solution is already provided.)

4. **HW 2-2** Martingale page in (WIKIPEDIA) gives the following example of Martingale. Prove (or disprove) the statement.

> Suppose each amoeba either splits into two amoebas, with probability p, or eventually dies, with probability $q = 1-p$. Let X_n be the number of amoebas surviving in the n-th generation (in particular $X_n = 0$ if the population has become extinct by that time). Let r be the probability of eventual extinction. (Finding r as a function of p is an instructive exercise. Hint: The probability that the descendants of an amoeba eventually die out is equal to the probability that either of its immediate offspring dies out, given that the original amoeba has split.) Then
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> $$\{\, r^{X_n} : n = 1, 2, 3, \dots \}$$
>
> is a martingale with respect to $\{X_n : n = 1, 2, 3, \dots\}$.

See **2017-18 Midterm Exam Problem 1** to derive the extinction probability r as a function of p (and q).