

# *Chapter 10: Multiagent systems*

*DIT410/TIN172 Artificial Intelligence*

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# Outline

- 1 *Multiple agents (10.1)*
- 2 *Games (10.2)*
  - Normal form of games (10.2.1)
  - Extensive form of games (10.2.2)
- 3 *Zero-sum games (10.3)*

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# Multiple agents

Let's consider multiple agents, where:

- the agents select actions autonomously
- each agent has its own information state
  - ▶ they can have different information (even conflicting)
- the outcome depends on the actions of all agents
- each agent has its own utility function (that depends on the total outcome)

## Types of agents

There are two extremes of multiagent systems:

*Cooperative:* The agents share the same utility function

*Example:* Automatic trucks in a warehouse

*Competitive:* When one agent wins all other agents lose

A common special case is when  $\sum_a u_a(o) = 0$  for any outcome  $o$ . This is called a *zero-sum game*.

*Example:* Most board games

Most multiagent systems are between these two extremes.

*Example:* Long-distance bike races are usually both *cooperative* (the bikers usually form clusters where they take turns in leading the group), and *competitive* (only one of them can win in the end).

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## Normal-form games

This is the most basic representation of a game. It consists of:

- a finite set of agents,  $I = \{1, \dots, n\}$
- a set of actions  $A_i$  for each agent  $i \in I$ 
  - ▶ an **action profile**  $\sigma = \langle a_1, \dots, a_n \rangle$ , means that every agent  $i \in I$  carries out action  $a_i$
- a utility function  $u(\sigma, i)$  which gives the expected utility for agent  $i$  when all agents follow the action profile  $\sigma$



# Rock-papers-scissors

Here's a *payoff matrix* of the game “rock-paper-scissors”:

	<i>rock</i>	<i>paper</i>	<i>scissors</i>
<i>rock</i>	0, 0	-1, 1	1, -1
<i>paper</i>	1, -1	0, 0	-1, 1
<i>scissors</i>	-1, 1	1, -1	0, 0

*Note:* This is a zero-sum game.

## *Prisoner's dilemma*

Two criminals are arrested for a crime, and each of them is given the following opportunities:

- betray the other one by saying that (s)he did it
- remain silent

If both betray each other they will serve 2 years in prison, but if both remain silent they will only serve 1 year.

However, if one betrays and the other says nothing, the silent person will serve 3 years while the other one will be set free.

## *Prisoner's dilemma, payoff matrix*

Here is the payoff matrix for the prisoner's dilemma:

	<i>B stays silent</i>	<i>B betrays A</i>
<i>A stays silent</i>	-1, -1	-3, 0
<i>A betrays B</i>	0, -3	-2, -2

Assuming that both criminals are independent and rational agents, what will be their actions?

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Assuming that both criminals are independent and rational agents, what will be their actions?

- if *B* stays silent, then it's better for *A* to betray
- if *B* betrays, then it's better for *A* to betray

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- if *B* stays silent, then it's better for *A* to betray
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So, both will get the 2 year sentence since both are rational agents and will come to the same decision.

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## Extensive form

The normal form of a game can only represent games that are momentaneous, i.e., that have no representation of time.

- With the *extensive form* of a game we can represent the unfolding of a game through time.
- For now we assume that the game is fully observable.

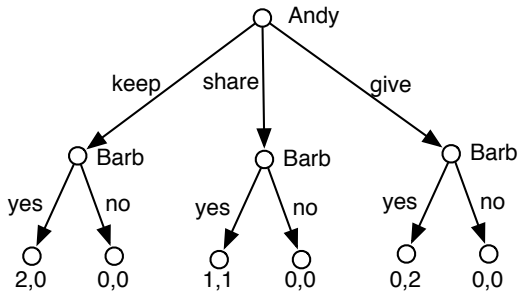
A *perfect information game* in extensive form is also known as a *game tree*.



## Extensive form for perfect information games

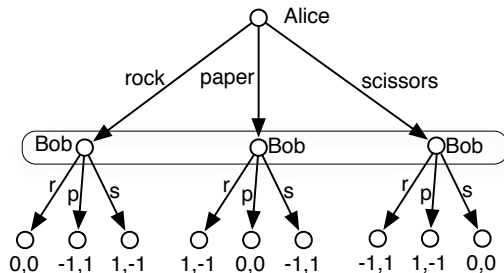
A *perfect information game* in extensive form is a finite tree where the nodes are states and the arcs correspond to actions by the agents, in particular:

- Each internal node is labeled with an agent.
- Each outgoing arc corresponds to an action for its agent.
- The leaves represent final outcomes and are labeled with utilities.



## Imperfect information games

Rock-paper-scissors is not a perfect information game:



In this example, Bob cannot know which of the nodes he is in, since he doesn't know which action Alice will perform.

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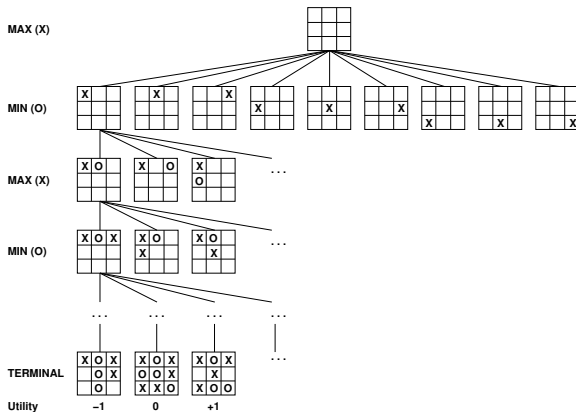
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## Perfect information games

- Perfect information games are solvable in a manner similar to fully observable single-agent systems. We can either do it backward using dynamic programming or forward using search.
- If two agents are competing so that a positive reward for one is a negative reward for the other agent, we have a two-agent *zero-sum game*.
- The value of a game zero-sum game can be characterized by a single number that one agent is trying to maximize and the other agent is trying to minimize.
- This leads to a *minimax* strategy:
  - ▶ A node is either a MAX node (if it is controlled by the maximising agent),
  - ▶ or is a MIN node (if it is controlled by the minimising agent).

Minimax search: tic-tac-toe

Here is an example of a minimax search tree for tic-tac-toe:

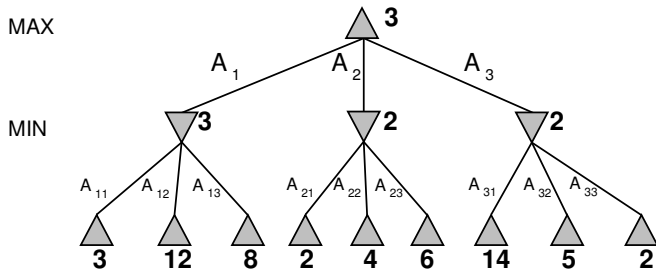


## $\alpha - \beta$ pruning

- Suppose, we reach a node  $t$  in the game tree which has leaves  $t_1, \dots, t_k$  corresponding to moves of player MIN.
- Let  $\alpha$  be the best value of a position on a path from the root node to  $t$ .
- Then, if any of the leaves evaluates to  $u(t_i) \leq \alpha$ , we can discard  $t$ , because any further evaluation will not improve the value of  $t$ .
- Analogously, define  $\beta$  values for evaluating response moves of MAX.

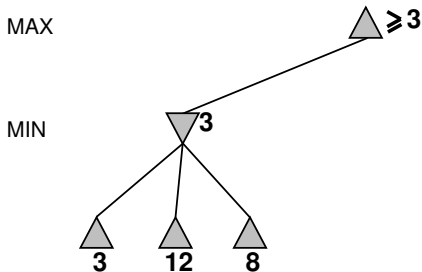
# Minimax example

Assume the following minimax search tree:



# Minimax example, with $\alpha - \beta$ pruning

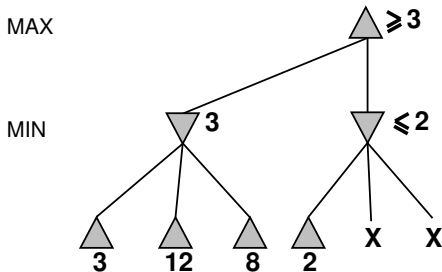
This is how  $\alpha - \beta$  pruning might work:





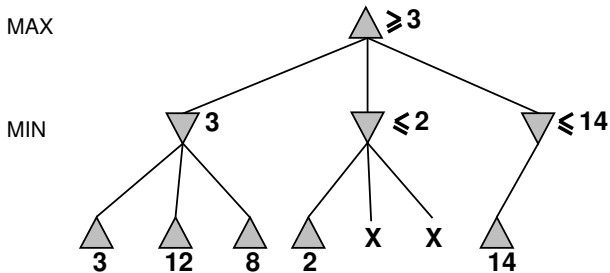
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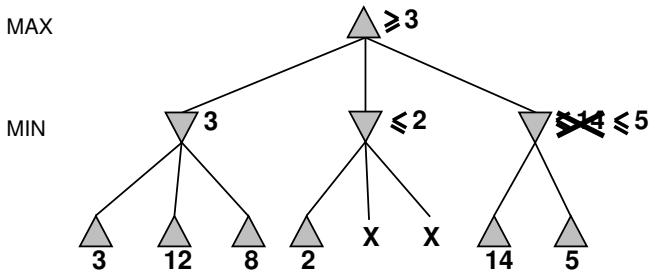
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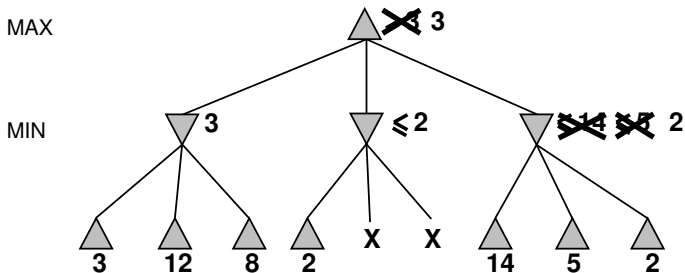
# Minimax example, with $\alpha - \beta$ pruning

This is how  $\alpha - \beta$  pruning might work:



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## *How efficient is $\alpha - \beta$ pruning?*

The amount of pruning provided by the  $\alpha - \beta$  algorithm depends on the ordering of the children of each node.

- It works best if a highest-valued child of a MAX node is selected first and if a lowest-valued child of a MIN node is returned first.
- In implementations of real games, much of the effort is made to try to ensure this outcome.

## Minimax and real games

Most real games are too big to carry out minimax search, even with  $\alpha$ - $\beta$  pruning.

- For these games, instead of only stopping at leaf nodes, it is possible to stop at any node.
- The value returned at the node where the algorithm stops is an estimate of the value for this node.
- The function used to estimate the value is an evaluation function.
- Much work goes into finding good evaluation functions.
- There is a trade-off between the amount of computation required to compute the evaluation function and the size of the search space that can be explored in any given time.