CHAPTER 6: SEARCH PART IV, AND CONSTRAINT SATISFACTION PROBLEMS, PART II

DIT410/TIN174, Artificial Intelligence

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REPETITION OF SEARCH

CLASSICAL SEARCH (R&N 3.1–3.6)

Generic search algorithm, tree search, graph search, depth-first search, breadth-first search, uniform cost search, iterative deepending, bidirectional search, greedy best-first search, A* search, heuristics, admissibility, consistency, dominating heuristics, …

NON-CLASSICAL SEARCH (R&N 4.1, 4.3–4.4)

Hill climbing, random moves, random restarts, beam search, nondeterministic actions, contingency plan, and-or search trees, partial observations, belief states, sensor-less problems, …

ADVERSARIAL SEARCH (R&N 5.1–5.3)

Cooperative, competetive, zero-sum games, game trees, minimax, α-β pruning, …

MORE GAMES

IMPERFECT DECISIONS (R&N 5.4–5.4.2)

STOCHASTIC GAMES (R&N 5.5)

IMPERFECT DECISIONS (R&N 5.4–5.4.2)

- H-minimax algorithm
- evaluation function, cutoff test
- features, weighted linear function
- quiescence search, horizon effect

REPETITION: MINIMAX SEARCH FOR ZERO-SUM GAMES

Given two players called MAX and MIN:

- MAX wants to maximize the utility value,
- MIN wants to minimize the same value.

$\Rightarrow$ MAX should choose the alternative that maximizes assuming that MIN minimizes.

function Minimax(*state*):
  if TerminalTest(*state*) **then return** Utility(*state*)
  A := Actions(*state*)
  if *state* is a MAX node **then return** $\max_{a \in A}$ Minimax(Result(*state*, *a*))
  if *state* is a MIN node **then return** $\min_{a \in A}$ Minimax(Result(*state*, *a*))

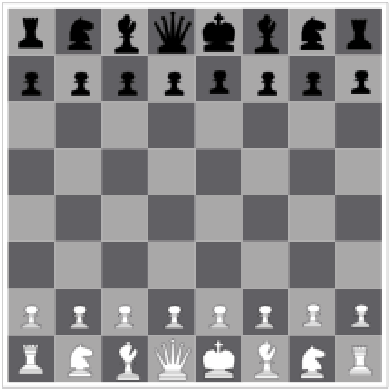
H-MINIMAX ALGORITHM

The *Heuristic* Minimax algorithm is similar to normal Minimax

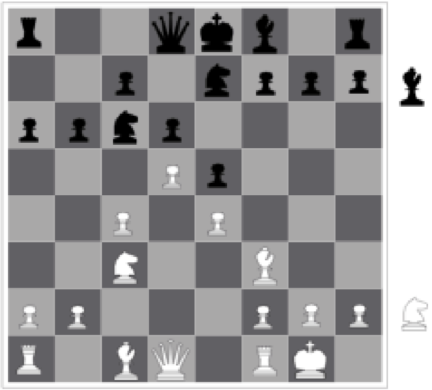
- it replaces **TerminalTest** and **Utility** with **CutoffTest** and **Eval**

function H-Minimax(*state*, *depth*):
    if CutoffTest(*state*, *depth*) **then return** Eval(*state*)
    A := Actions(*state*)
    if *state* is a MAX node **then return** $\max_{a \in A}$ H-Minimax(Result(*state*, *a*), *depth*+1)
    if *state* is a MIN node **then return** $\min_{a \in A}$ H-Minimax(Result(*state*, *a*), *depth*+1)

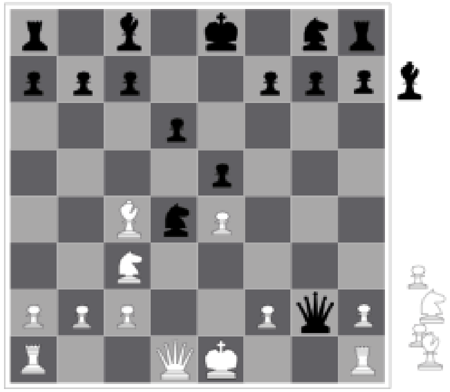
CHESS POSITIONS: HOW TO EVALUATE



(a) White to move
Fairly even



(b) Black to move
White slightly better



(c) White to move
Black winning

(d) Black to move
White about to lose

WEIGHTED LINEAR EVALUATION FUNCTIONS

A very common evaluation function is to use a weighted sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \cdots + w_n f_n(s) = \sum_{i=1}^{n} w_i f_i(s)$$

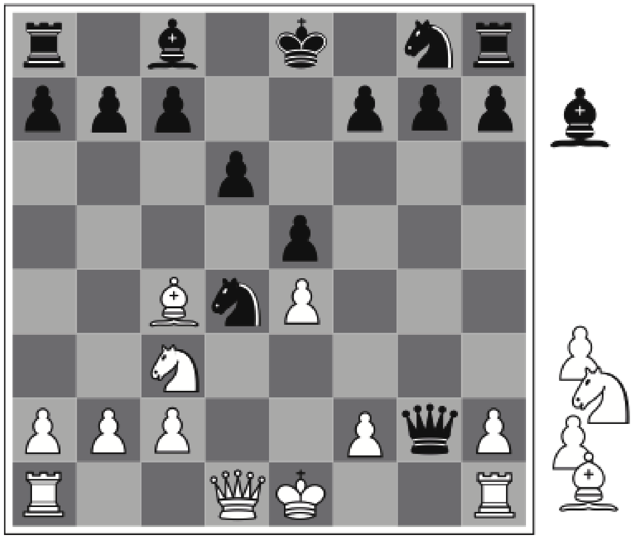
This relies on a strong assumption: all features are *independent of each other*

- which is usually not true, so the best programs for chess (and other games) also use nonlinear feature combinations

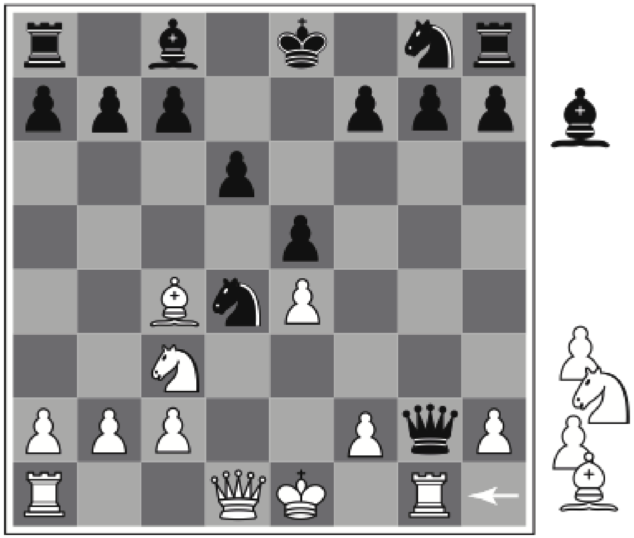
The weights can be calculated using machine learning algorithms, but a human still has to come up with the features.

- using recent advances in deep machine learning, the computer can learn the features too

EVALUATION FUNCTIONS



(a) White to move



(b) White to move

A naive weighted sum of features will not see the difference between these two states.

PROBLEMS WITH CUTOFF TESTS

Too simplistic cutoff tests and evaluation functions can be problematic:

- e.g., if the cutoff is only based on the current depth
- then it might cut off the search in unfortunate positions (such as (b) on the previous slide)

We want more sophisticated cutoff tests:

- only cut off search in *quiescent* positions
- i.e., in positions that are “stable”, unlikely to exhibit wild swings in value
- non-quiescent positions should be expanded further

Another problem is the *horizon effect*:

- if a bad position is unavoidable (e.g., loss of a piece), but the system can delay it from happening, it might push the bad position “over the horizon”
- in the end, the resulting delayed position might be even worse

DETERMINISTIC GAMES IN PRACTICE

Chess:

- DeepBlue (IBM) beats world champion Garry Kasparov, 1997.
- Modern chess programs: Houdini, Critter, Stockfish.

Checkers/Othello/Reversi:

- Logistello beats the world champion in Othello/Reversi, 1997.
- Chinook plays checkers perfectly, 2007. It uses an endgame database defining perfect play for all 8-piece positions on the board, (a total of 443,748,401,247 positions).

Go:

- AlphaGo (Google DeepMind) beats one of the world's best players, Lee Sedol by 4–1, in April 2016.
- Modern programs: MoGo, Zen, GNU Go, AlphaGo.

GAMES OF IMPERFECT INFORMATION

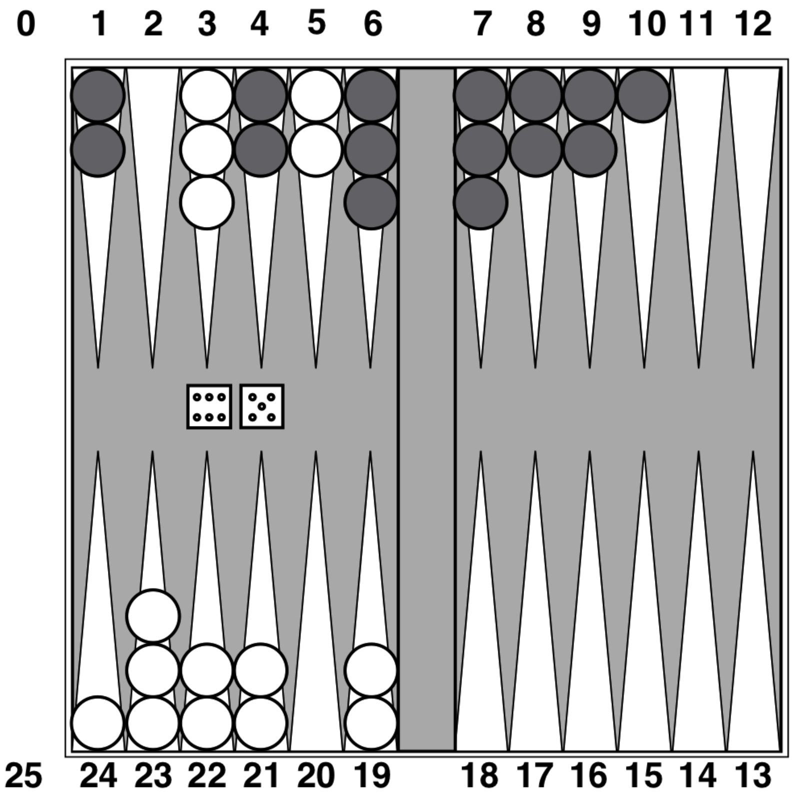
Imperfect information games

- e.g., card games, where the opponent's initial cards are unknown
- typically we can calculate a probability for each possible deal
- seems just like having one big dice roll at the beginning of the game
- main idea: compute the minimax value of each action in each deal, then choose the action with highest expected value over all deals

STOCHASTIC GAMES (R&N 5.5)

- chance nodes
- expected value
- expecti-minimax algorithm

STOCHASTIC GAME EXAMPLE: BACKGAMMON

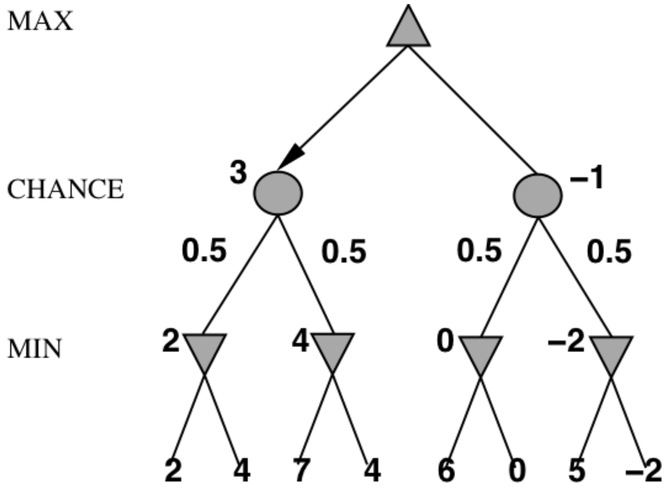


STOCHASTIC GAMES IN GENERAL

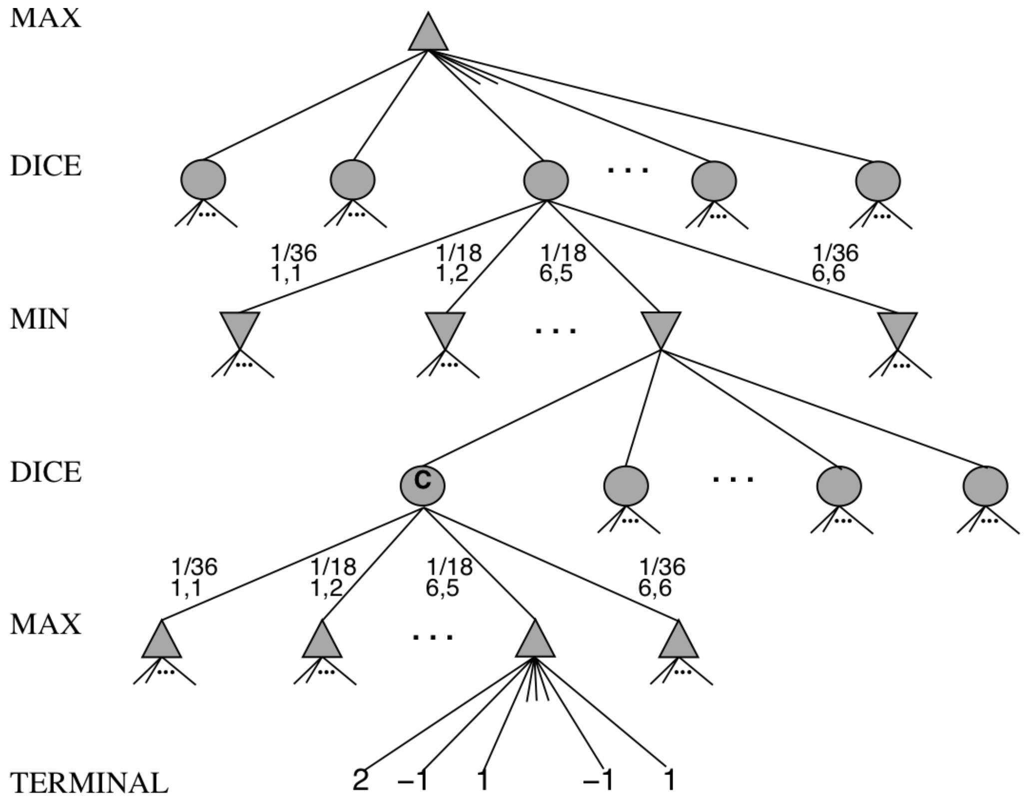
In stochastic games, chance is introduced by dice, card-shuffling, etc.

- We introduce *chance nodes* to the game tree.
- We can't calculate a definite minimax value,
  instead we calculate the *expected value* of a position.
- The expected value is the average of all possible outcomes.

A very simple example with coin-flipping and arbitrary values:



BACKGAMMON GAME TREE



ALGORITHM FOR STOCHASTIC GAMES

The ExpectiMinimax algorithm gives perfect play;
it's just like Minimax, except we must also handle chance nodes:

function ExpectiMinimax(*state*):
  if TerminalTest(*state*) **then return** Utility(*state*)
  A := Actions(*state*)
  if *state* is a MAX node **then return** $\max_{a \in A}$ Minimax(*state*, a)
  if *state* is a MAX node **then return** $\min_{a \in A}$ Minimax(*state*, a)
  if *state* is a chance node **then return** $\sum_{a \in A} P(a)$ Minimax(*state*, a)

where $P(a)$ is the probability that action a occurs.

STOCHASTIC GAMES IN PRACTICE

Dice rolls increase the branching factor *b*:

- there are 21 possible rolls with 2 dice

Backgammon has ≈20 legal moves:

- depth $4 \Rightarrow 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$ nodes

As depth increases, the probability of reaching a given node shrinks:

- value of lookahead is diminished
- α-β pruning is much less effective

TDGammon (1995) used depth-2 search + very good Eval:

- the evaluation function was learned by self-play
- world-champion level

REPETITION OF CSP

CONSTRAINT SATISFACTION PROBLEMS (R&N 6.1)

Variables, domains, constraints (unary, binary, n-ary), constraint graph

CSP AS A SEARCH PROBLEM (R&N 6.3–6.3.2)

Backtracking search, heuristics (minimum remaining values, degree, least constraining value), forward checking, maintaining arc-consistency (MAC)

CONSTRAINT PROGAGATION (R&N 6.2–6.2.2)

Consistency (node, arc, path, k, …), global constratints, the AC-3 algorithm

CSP: CONSTRAINT SATISFACTION PROBLEMS (R&N 6.1)

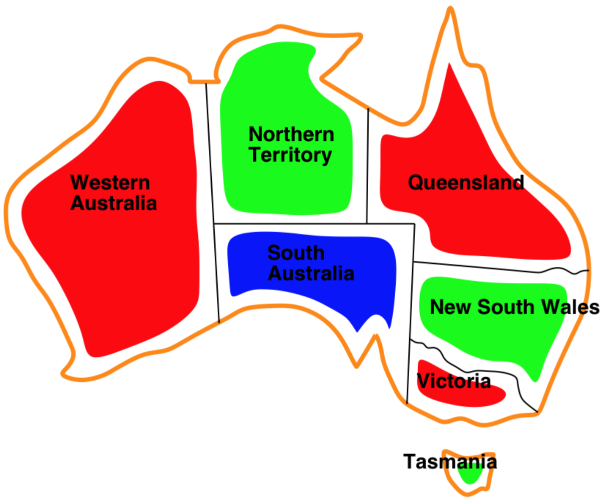
CSP is a specific kind of search problem:

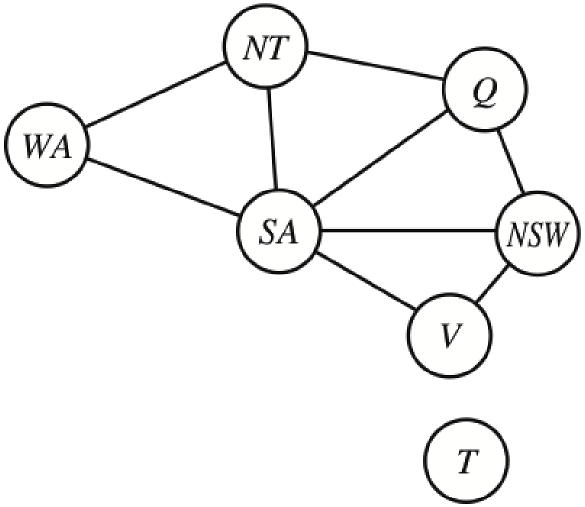
- the *state* is defined by *variables* X_i, each taking values from the domain D_i
- the *goal test* is a set of *constraints*:
  - each constraint specifies allowed values for a subset of variables
  - all constraints must be satisfied

Differences to general search problems:

- the path to a goal isn't important, only the solution is.
- there are no predefined starting state
- often these problems are huge, with thousands of variables,
  so systematically searching the space is infeasible

EXAMPLE: MAP COLOURING (BINARY CSP)

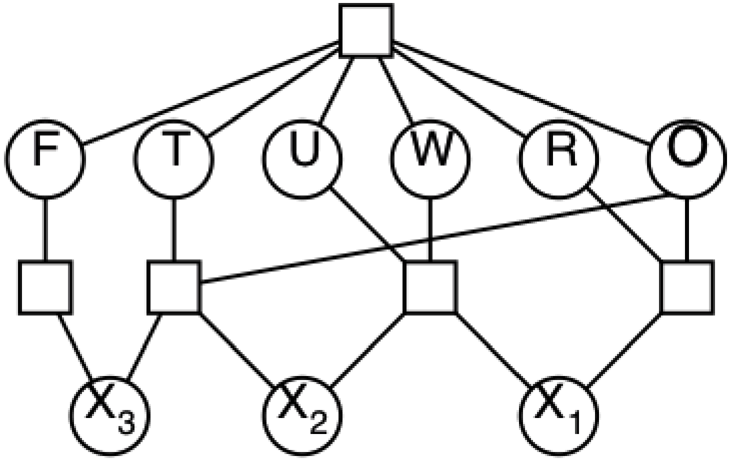




Variables:	*WA*, *NT*, *Q*, *NSW*, *V*, *SA*, *T*
Domains:	D_i = {red, green, blue}
Constraints:	*SA≠WA, SA≠NT, SA≠Q, SA≠NSW, SA≠V, WA≠NT, NT≠Q, Q≠NSW, NSW≠V*
Constraint graph:	Every variable is a node, every binary constraint is an arc.

EXAMPLE: CRYPTARITHMETIC PUZZLE (HIGHER-ORDER CSP)

```
    T W O
+   T W O
---------
  F O U R
```



Variables:	*F, T, U, W, R, O,* X_1, X_2, X_3
Domains:	D_i = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
Constraints:	*Alldiff*(*F,T,U,W,R,O*), *O*+*O*=*R*+10·X_1, etc.
Constraint graph:	This is not a binary CSP! The graph is a *constraint hypergraph*.

CSP AS A SEARCH PROBLEM (R&N 6.3–6.3.2)

- backtracking search
- select variable: minimum remaining values, degree heuristic
- order domain values: least constraining value
- inference: forward checking and arc consistency

ALGORITHM FOR BACKTRACKING SEARCH

At each depth level, decide on one single variable to assign:

- this gives branching factor $b = d$, so there are d^n leaves

Depth-first search with single-variable assignments is called *backtracking search*:

function BacktrackingSearch(*csp*):
    return Backtrack(*csp*, { })

function Backtrack(*csp*, *assignment*):
    if *assignment* is complete **then return** *assignment*
    var := SelectUnassignedVariable(*csp*, *assignment*)
    for each *value* **in** OrderDomainValues(*csp*, *var*, *assignment*):
        if *value* is consistent with *assignment*:
            inferences := Inference(*csp*, *var*, *value*)
            if *inferences* ≠ failure:
                result := Backtrack(*csp*, *assignment* ∪ {*var*=*value*} ∪ *inferences*)
                if *result* ≠ failure **then return** *result*
    return failure

IMPROVING BACKTRACKING EFFICIENCY

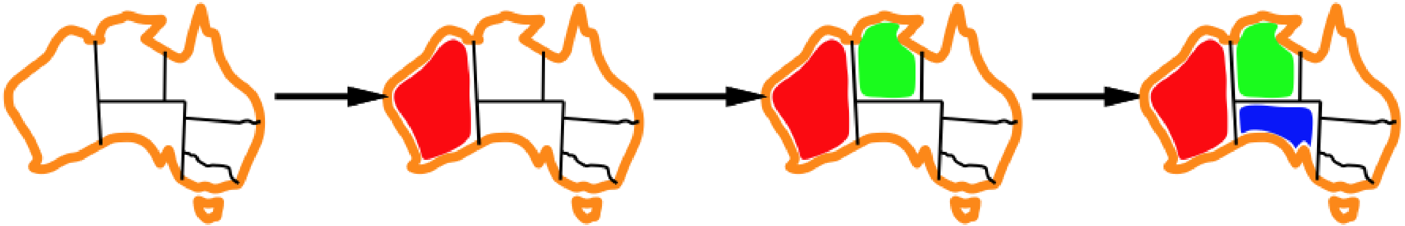
The general-purpose algorithm gives rise to several questions:

- Which variable should be assigned next?
  - SelectUnassignedVariable(*csp*, *assignment*)
- In what order should its values be tried?
  - OrderDomainValues(*csp*, *var*, *assignment*)
- What inferences should be performed at each step?
  - Inference(*csp*, *var*, *value*)
- Can the search avoid repeating failures?
  - Conflict-directed backjumping, constraint learning, no-good sets (R&N 6.3.3, not covered in this course)

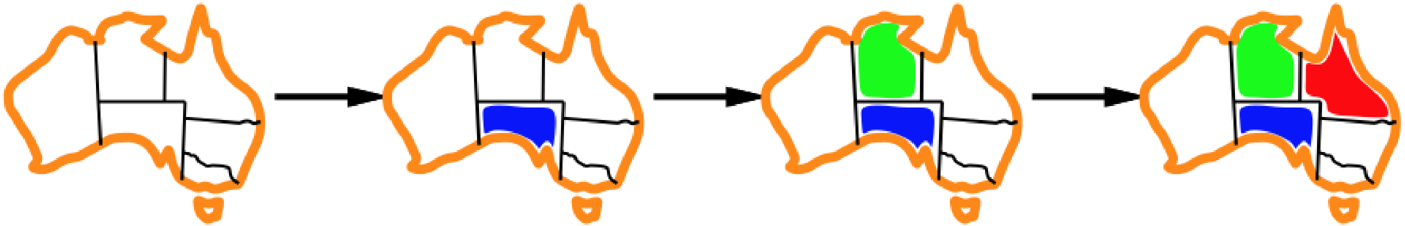
SELECTING UNASSIGNED VARIABLES

Heuristics for selecting the next unassigned variable:

- Minimum remaining values (MRV):
  $\Longrightarrow$ choose the variable with the fewest legal values



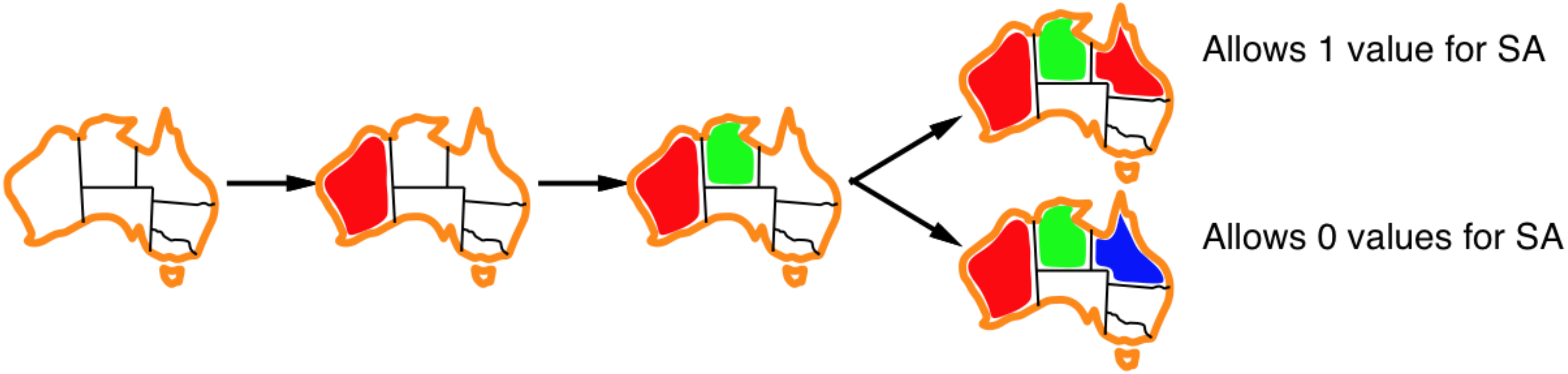
- Degree heuristic (if there are several MRV variables):
  $\Longrightarrow$ choose the variable with most constraints on remaining variables



ORDERING DOMAIN VALUES

Heuristics for ordering the values of a selected variable:

- Least constraining value:
  $\Longrightarrow$ prefer the value that rules out the fewest choices for the neighboring variables in the constraint graph



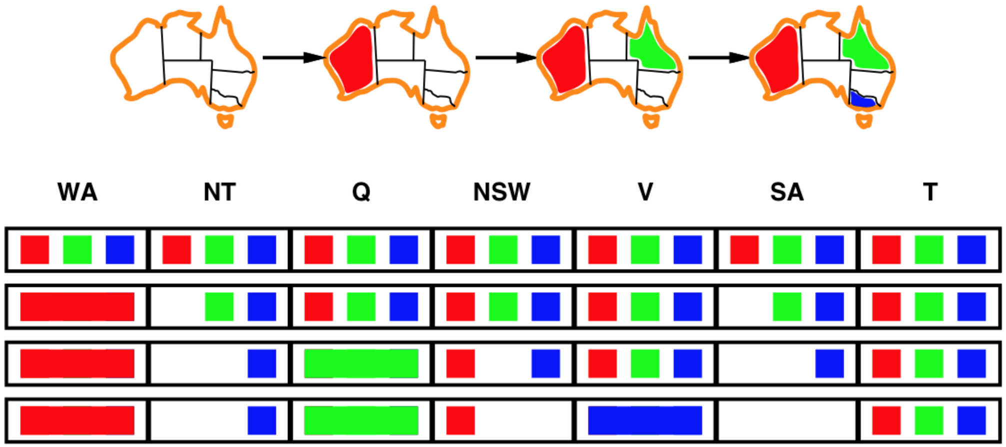
CONSTRAINT PROGAGATION (R&N 6.2–6.2.2)

- consistency (node, arc, path, k, …)
- global constratints
- the AC-3 algorithm
- maintaining arc consistency

INFERENCE: FORWARD CHECKING AND ARC CONSISTENCY

Forward checking is a simple form of inference:

- Keep track of remaining legal values for unassigned variables
- When a new variable is assigned, recalculate the legal values for its neighbors



Arc consistency: $X \rightarrow Y$ is ac iff for every x in X, there is some allowed y in Y

- since NT and SA cannot both be blue, the problem becomes arc inconsistent before forward checking notices
- arc consistency detects failure earlier than forward checking

ARC CONSISTENCY ALGORITHM, AC-3

Keep a set of arcs to be considered: pick one arc (X, Y) at the time and make it consistent (i.e., make X arc consistent to Y).

- Start with the set of all arcs $\{(X, Y), (Y, X), (X, Z), (Z, X), \ldots\}$.

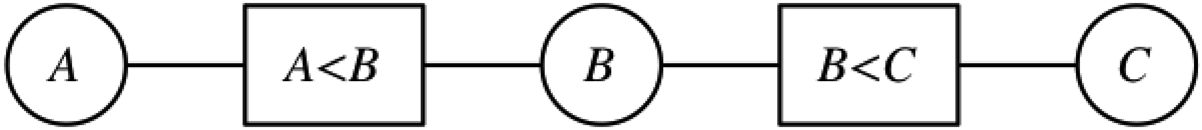
When an arc has been made arc consistent, does it ever need to be checked again?

- An arc (X, Y) needs to be revisited if the domain of Y is revised.

function AC-3(**inout** *csp*):
    initialise *queue* to all arcs in *csp*
    while *queue* is not empty:
        (X, Y) := RemoveOne(*queue*)
        if Revise(*csp*, X, Y):
            if $D_X = \emptyset$ **then return** failure
            for each Z **in** X.neighbors–{Y} **do** add (Z, X) to *queue*

function Revise(**inout** *csp*, X, Y):
    delete every x from D_X such that there is no value y in D_Y satisfying the constraint C_{XY}

AC-3 EXAMPLE



remove	D_A	D_B	D_C	add	queue
	1234	1234	1234		$A<B$, $B<C$, $C>B$, $B>A$
$A<B$	***123***	1234	1234		$B<C$, $C>B$, $B>A$
$B<C$	123	***123***	1234	$A<B$	$C>B$, $B>A$, ***A<B***
$C>B$	123	123	***234***		$B>A$, $A<B$
$B>A$	123	***23***	234	$C>B$	$A<B$, ***C>B***
$A<B$	***12***	23	234		$C>B$
$C>B$	12	23	***34***		$\emptyset$

COMBINING BACKTRACKING WITH AC-3

What if some domains have more than one element after AC?

We can resort to backtracking search:

- Select a variable and a value using some heuristics (e.g., minimum-remaining-values, degree-heuristic, least-constraining-value)
- Make the graph arc-consistent again
- Backtrack and try new values/variables, if AC fails
- Select a new variable/value, perform arc-consistency, etc.

Do we need to restart AC from scratch?

- no, only some arcs risk becoming inconsistent after a new assignment
- restart AC with the queue $\{(Y_i, X) | X \rightarrow Y_i\}$, i.e., only the arcs (Y_i, X) where Y_i are the neighbors of X
- this algorithm is called *Maintaining Arc Consistency* (MAC)

CONSISTENCY PROPERTIES

There are several kinds of consistency properties and algorithms:

- *Node consistency*: single variable, unary constraints (straightforward)
- *Arc consistency*: pairs of variables, binary constraints (AC-3 algorithm)
- *Path consistency*: triples of variables, binary constraints (PC-2 algorithm)
- *k-consistency*: k variables, k-ary constraints ($O(k)$ algorithms)
- Consistency for global constraints:
    - special-purpose algorithms for different constraints, e.g.:
    - *Alldiff*$(X_1, \ldots, X_m)$ is inconsistent if $m > |D_1 \cup \cdots \cup D_m|$
    - *Atmost*$(n, X_1, \ldots, X_m)$ is inconsistent if $n < \sum_i \min(D_i)$

MORE ABOUT CSP

LOCAL SEARCH FOR CSPS (R&N 6.4)

PROBLEM STRUCTURE (R&N 6.5)

LOCAL SEARCH FOR CSPS (R&N 6.4)

Given an assignment of a value to each variable:

- A conflict is an unsatisfied constraint.
- The goal is an assignment with zero conflicts.

Local search / Greedy descent algorithm:

- Start with a complete assignment.
- Repeat until a satisfying assignment is found:
    - select a variable to change
    - select a new value for that variable

MIN CONFLICTS ALGORITHM

Heuristic function to be minimized: the number of conflicts.

- this is the *min-conflicts* heuristics

Note: this does not always work!

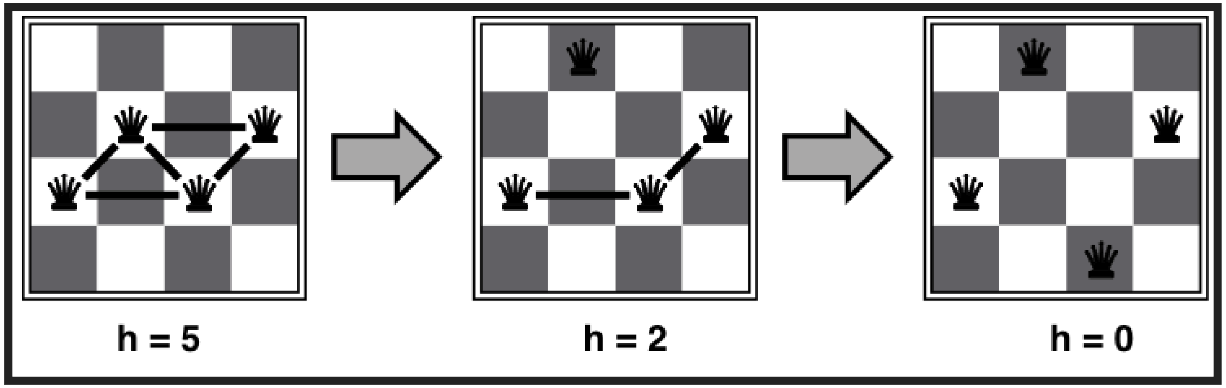
- it can get stuck in a *local minimum*

```
function MinConflicts(csp, max_steps)
    current := an initial complete assignment for csp
    repeat max_steps times:
        if current is a solution for csp then return current
        var := a randomly chosen conflicted variable from csp
        value := the value v for var that minimises Conflicts(var, v, current, csp)
        current[var] = value
    return failure
```

EXAMPLE: n-QUEENS (REVISITED)

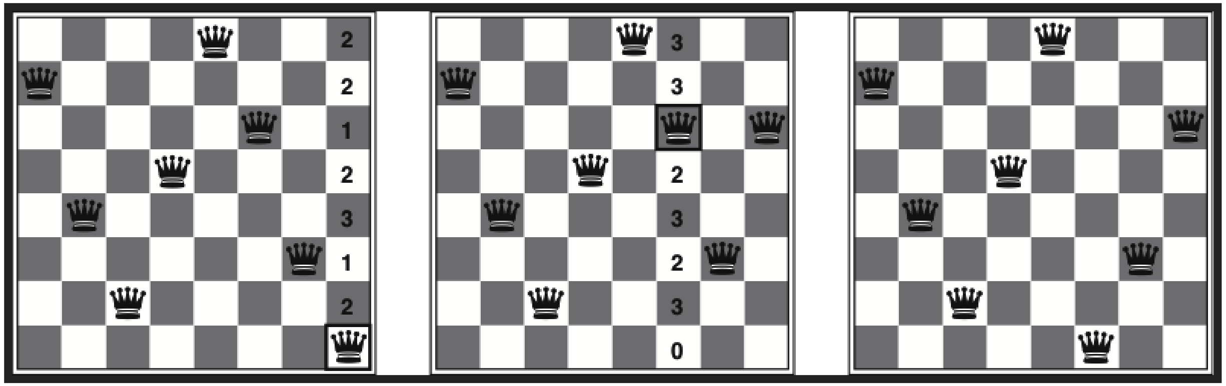
Do you remember this example?

- Put n queens on an $n \times n$ board, in separate columns
- Conflicts = unsatisfied constraints = n:o of threatened queens
- Move a queen to reduce the number of conflicts
    - repeat until we cannot move any queen anymore
    - then we are at a local maximum — hopefully it is global too



EASY AND HARD PROBLEMS

Two-step solution using min-conflicts for an 8-queens problem:



The runtime of min-conflicts on the *n-queens problem* is *independent of problem size*!

- it solves even the *million*-queens problem ≈50 steps

Why is *n*-queens easy for local search?

- because solutions are *densely distributed* throughout the state space!

VARIANTS OF GREEDY DESCENT

To choose a variable to change and a new value for it:

- Find a variable-value pair that minimizes the number of conflicts.
- Select a variable that participates in the most conflicts.
  Select a value that minimizes the number of conflicts.
- Select a variable that appears in any conflict.
  Select a value that minimizes the number of conflicts.
- Select a variable at random.
  Select a value that minimizes the number of conflicts.
- Select a variable and value at random;
  accept this change if it doesn't increase the number of conflicts.

All local search techniques from section 4.1 can be applied to CSPs, e.g.:

- random walk, random restarts, simulated annealing, beam search, …

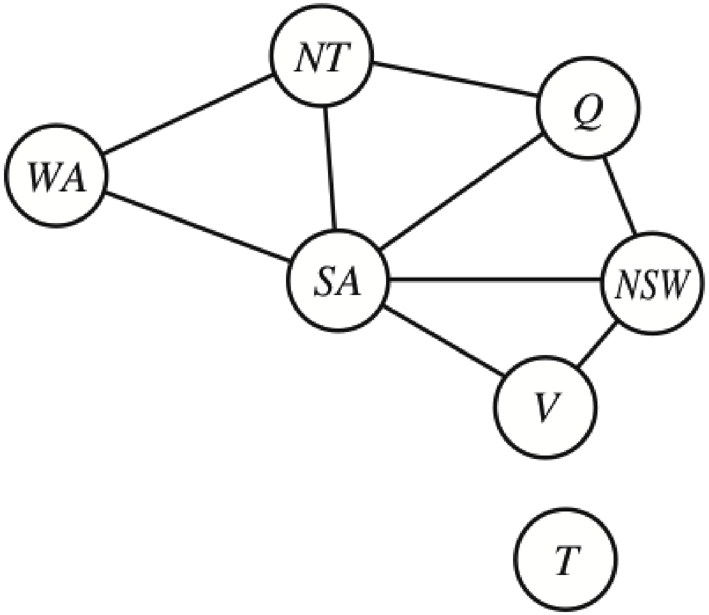
PROBLEM STRUCTURE (R&N 6.5)

- independent subproblems, connected components
- tree-structured CSP, topological sort
- converting to tree-structured CSP, cycle cutset, tree decomposition

INDEPENDENT SUBPROBLEMS

Tasmania is an *independent subproblem*:

- there are efficient algorithms for finding *connected components* in a graph



Suppose that each subproblem has c variables out of n total. The cost of the worst-case solution is $n/c \cdot d^c$, which is linear in n.

E.g., $n = 80, d = 2, c = 20$:

- 2^{80} = 4 billion years at 10 million nodes/sec

If we divide it into 4 equal-size subproblems:

- $4 \cdot 2^{20}$ =0.4 seconds at 10 million nodes/sec

Note: this only has a real effect if the subproblems are (roughly) equal size!

TREE-STRUCTURED CSP

A constraint graph is a tree when any two variables are connected by only one path.

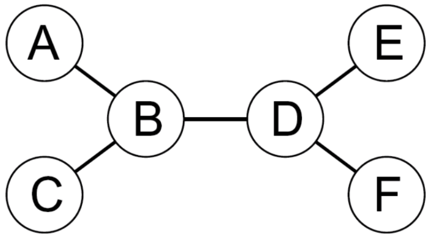
- then any variable can act as root in the tree
- tree-structured CSP can be solved in *linear time*, in the number of variables!

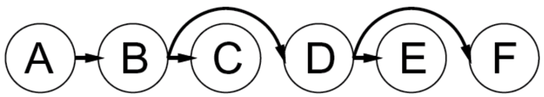
CSP is directed arc-consistent if:

- there is an orderning of variables $X_1, X_2, \ldots, X_n$ such that
- every X_i is arc-consistent with each X_j for all $j > i$

To solve a tree-structured CSP:

- first pick a variable to be the root of the tree
- then find a *topological sort* of the variables (with the root first)
- finally, make each arc consistent, in reverse topological order





SOLVING TREE-STRUCTURED CSP

```
function TreeCSPSolver(csp)
    n := number of variables in csp
    root := any variable in csp
    X_1 … X_n := TopologicalSort(csp, root)
    for j := n, n–1, …, 2:
        MakeArcConsistent(Parent(X_j), X_j)
        if it could not be made consistent then return failure
    assignment := an empty assignment
    for i := 1, 2, …, n:
        assignment[X_i] := any consistent value from D_i
    return assignment
```

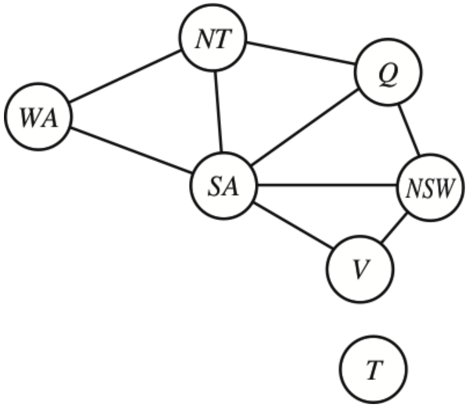
What is the runtime?

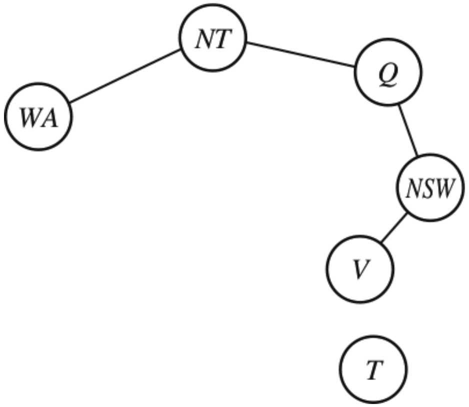
- to make an arc consistent, we must compare up to d^2 domain value pairs
- there are $n-1$ arcs, so the total runtime is $O(nd^2)$

CONVERTING TO TREE-STRUCTURED CSP

Most CSPs are *not* tree-structured, but sometimes we can reduce a problem to a tree

- one approach is to assign values to some variables, so that the remaining variables form a tree





If we assign a colour to South Australia, then the remaining variables form a tree

- a (worse) alternative is to assign values to {*NT,Q,V*}

Why is {*NT,Q,V*} a worse alternative?

- because then we have to try 3×3×3 different assignments, and for each of them solve the remaining tree-CSP

SOLVING ALMOST-TREE-STRUCTURED CSP

function SolveByReducingToTreeCSP(*csp*):
  S := a cycle cutset of variables, such that *csp*–S becomes a tree
  for each assignment for S that satisfies all constraints on S:
    remove any inconsistent values from neighboring variables of S
    solve the remaining tree-CSP (i.e., *csp*–S)
    if there is a solution **then return** it together with the assignment for S
  return failure

The set of variables that we have to assign is called a *cycle cutset*

- for Australia, {*SA*} is a cycle cutset and {*NT,Q,V*} is also a cycle cutset
- finding the smallest cycle cutset is NP-hard,
  but there are efficient approximation algorithms

TREE DECOMPOSITION

Another approach for reducing to a tree-CSP is *tree decomposition*:

- divide the original CSP into a set of connected subproblems, such that the connections form a *tree-structured graph*
- solve each subproblem independently
- since the decomposition is a tree, we can solve the main problem using directed arc consistency (the TreeCSPSolver algorithm)