CHAPTERS 4–5: NON-CLASSICAL AND ADVERSARIAL SEARCH

DIT410/TIN174, Artificial Intelligence

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REPETITION

UNINFORMED SEARCH (R&N 3.4)

Search problems, graphs, states, arcs, goal test, generic search algorithm, tree search, graph search, depth-first search, breadth-first search, uniform cost search, iterative deepending, bidirectional search, …

HEURISTIC SEARCH (R&N 3.5–3.6)

Greedy best-first search, A* search, heuristics, admissibility, consistency, dominating heuristics, …

LOCAL SEARCH (R&N 4.1)

Hill climbing / gradient descent, random moves, random restarts, beam search, simulated annealing, …

NON-CLASSICAL SEARCH

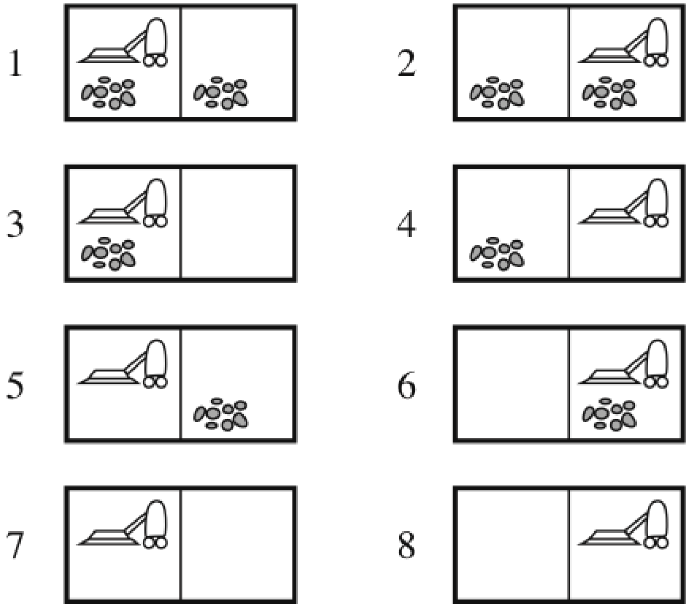
NONDETERMINISTIC SEARCH (R&N 4.3)

PARTIAL OBSERVATIONS (R&N 4.4)

NONDETERMINISTIC SEARCH (R&N 4.3)

- Contingency plan (strategy)
- ***And-or*** search trees
- And-or graph search algorithm

THE VACUUM CLEANER WORLD, AGAIN



The eight possible states of the vacuum world; states 7 and 8 are goal states.

There are three actions: *Left, Right, Suck*

AN ERRATIC VACUUM CLEANER

Assume that the *Suck* action works as follows:

- if the square is dirty, it is cleaned but sometimes also the adjacent square is
- if the square is clean, the vacuum cleaner sometimes deposists dirt

Now we need a more general *result* function:

- instead of returning a single state, it returns a set of possible outcome states
- e.g., $\text{Results}(\text{Suck}, 1) = \{5, 7\}$ and $\text{Results}(\text{Suck}, 5) = \{1, 5\}$

We also need to generalise the notion of a *solution*:

- instead of a single sequence (path) from the start to the goal, we need a *strategy* (or a *contingency plan*)
- i.e., we need **if-then-else** constructs
- this is a possible solution from state 1:
    - [*Suck*, if *State*=5 then [*Right*, *Suck*] else []]

HOW TO FIND CONTINGENCY PLANS

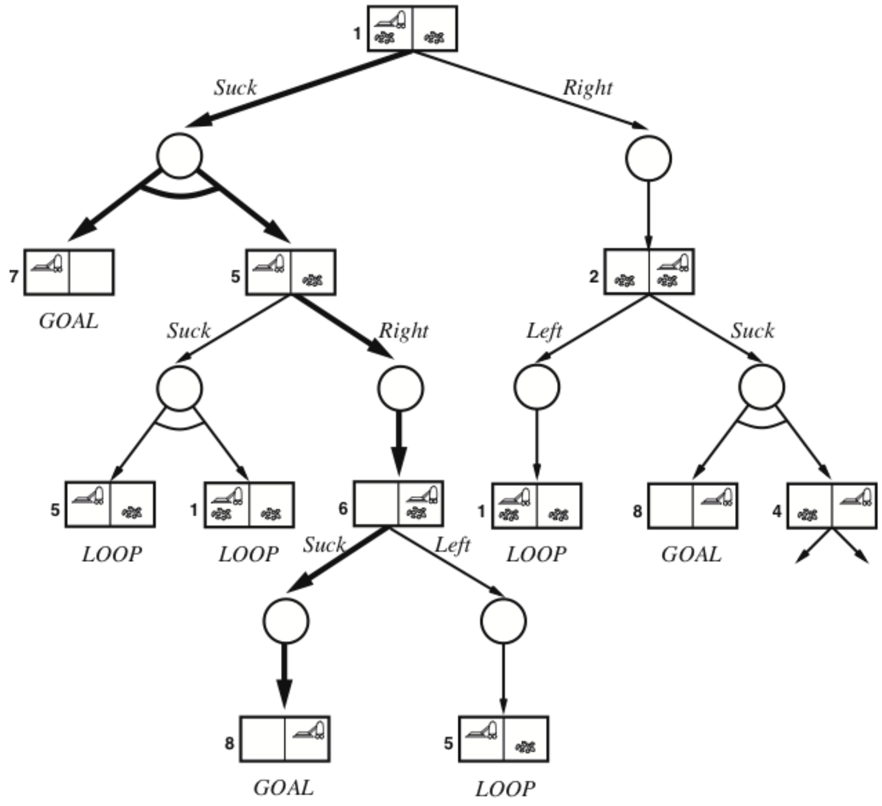
We need a new kind of nodes in the search tree:

- ***and** nodes*:
  these are used whenever an action is nondeterministic
- normal nodes are called ***or** nodes*:
  they are used when we have several possible actions in a state

A solution for an ***and-or*** search problem is a subtree that:

- has a goal node at every leaf
- specifies exactly one action at each of its ***or** node*
- includes every branch at each of its ***and** node*

A SOLUTION TO THE ERRATIC VACUUM CLEANER



The solution subtree is shown in bold, and corresponds to the plan:

[*Suck*, if *State*=5 then [*Right*, *Suck*] else []]

AN ALGORITHM FOR FINDING A CONTINGENCY PLAN

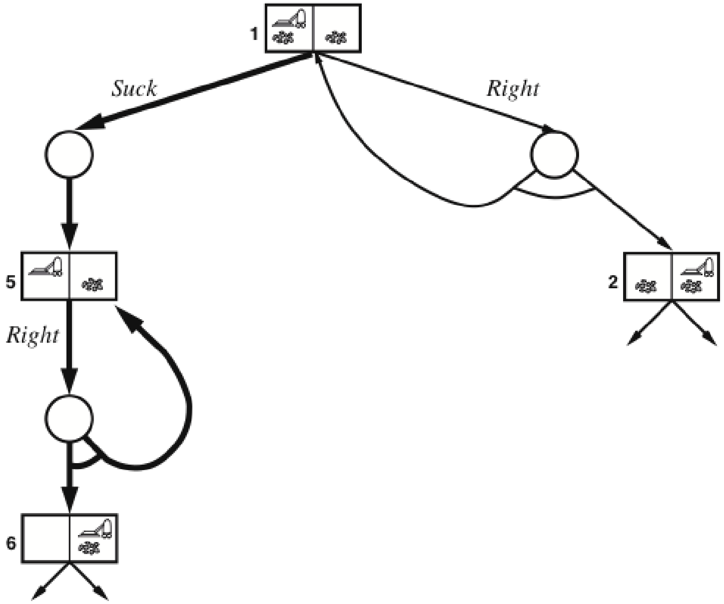
This algorithm does a depth-first search in the *and-or* tree,
so it is not guaranteed to find the best or shortest plan:

function AndOrGraphSearch(*problem*):
    return OrSearch(*problem*.InitialState, *problem*, [])

function OrSearch(*state*, *problem*, *path*):
    if *problem*.GoalTest(*state*) **then return** []
    if *state* is on *path* **then return** failure
    for each *action* in *problem*.Actions(*state*):
        plan := AndSearch(*problem*.Results(*state*, *action*), *problem*, [*state*] ++ *path*)
        if *plan* ≠ failure **then return** [*action*] ++ *plan*
    return failure

function AndSearch(*states*, *problem*, *path*):
    for each s_i in *states*:
        $plan_i$:= OrSearch(s_i, *problem*, *path*)
        if $plan_i$ = failure **then return** failure
    return [if s_1 then $plan_1$ else if s_2 then $plan_2$ else … if s_n then $plan_n$]

WHILE LOOPS IN CONTINGENCY PLANS



If the search graph contains cycles, **if-then-else** is not enough in a contingency plan:

- we need **while** loops instead

In the slippery vacuum world above, the cleaner don’t always move when told:

- the solution is a sub-graph (not a subtree), shown in bold above
- this solution translates to [*Suck*, while *State*=5 do *Right*, *Suck*]

PARTIAL OBSERVATIONS (R&N 4.4)

- Belief states: goal test, transitions, …
- Sensor-less (conformant) problems
- Partially observable problems

OBSERVABILITY VS DETERMINISM

A problem is *nondeterministic* if there are several possible outcomes of an action

- deterministic — nondeterministic (chance)

It is *partially observable* if the agent cannot tell exactly which state it is in

- fully observable (perfect info.) — partially observable (imperfect info.)

A problem can be either nondeterministic, or partially observable, or both:

	deterministic	chance
perfect information	**chess, checkers, go, othello**	**backgammon monopoly**
imperfect information	**battleships, blind tictactoe**	**bridge, poker, scrabble nuclear war**

BELIEF STATES

Instead of searching in a graph of states, we use *belief states*

- A belief state is a *set of states*

In a sensor-less (or conformant) problem, the agent has *no information at all*

- The initial belief state is the set of all problem states
  - e.g., for the vacuum world the initial state is {1,2,3,4,5,6,7,8}

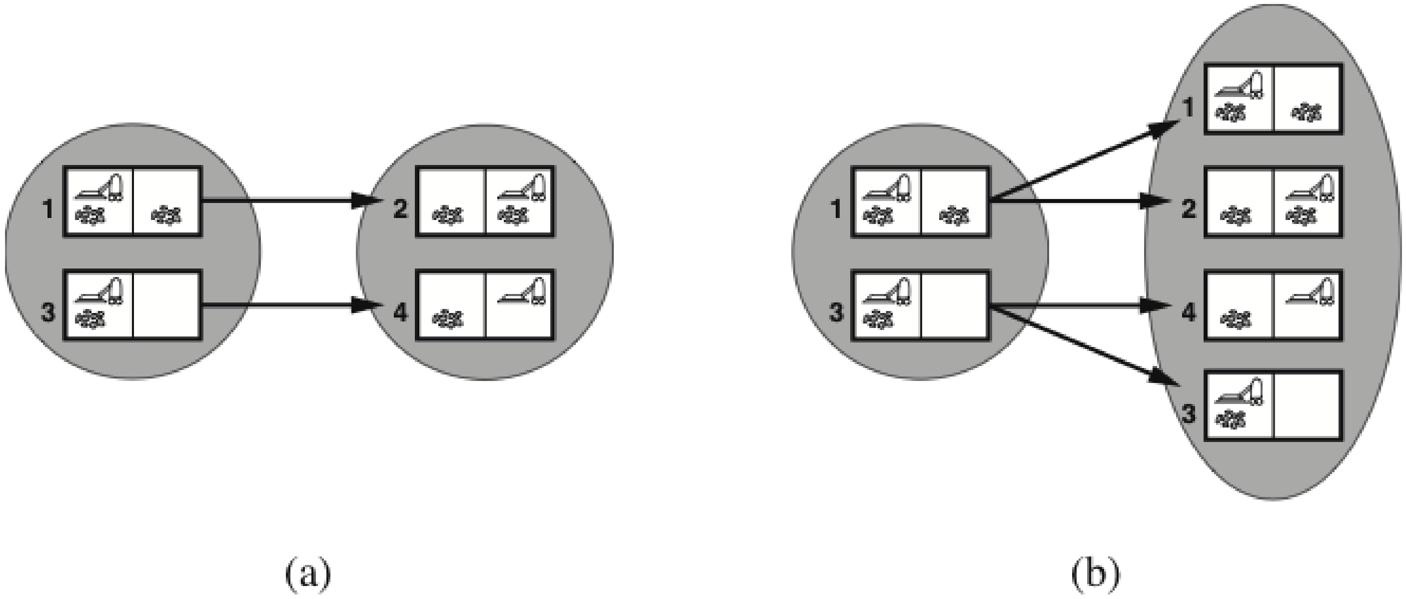
The goal test has to check that *all* members in the belief state is a goal

- e.g., for the vacuum world, the following are goal states: {7}, {8}, and {7,8}

The result of performing an action is the *union* of all possible results

- i.e., $\text{Predict}(b, a) = \{\text{Result}(s, a) \text{ for each } s \in b\}$
- if the problem is also nondeterministic:
  - $\text{Predict}(b, a) = \bigcup\{\text{Results}(s, a) \text{ for each } s \in b\}$

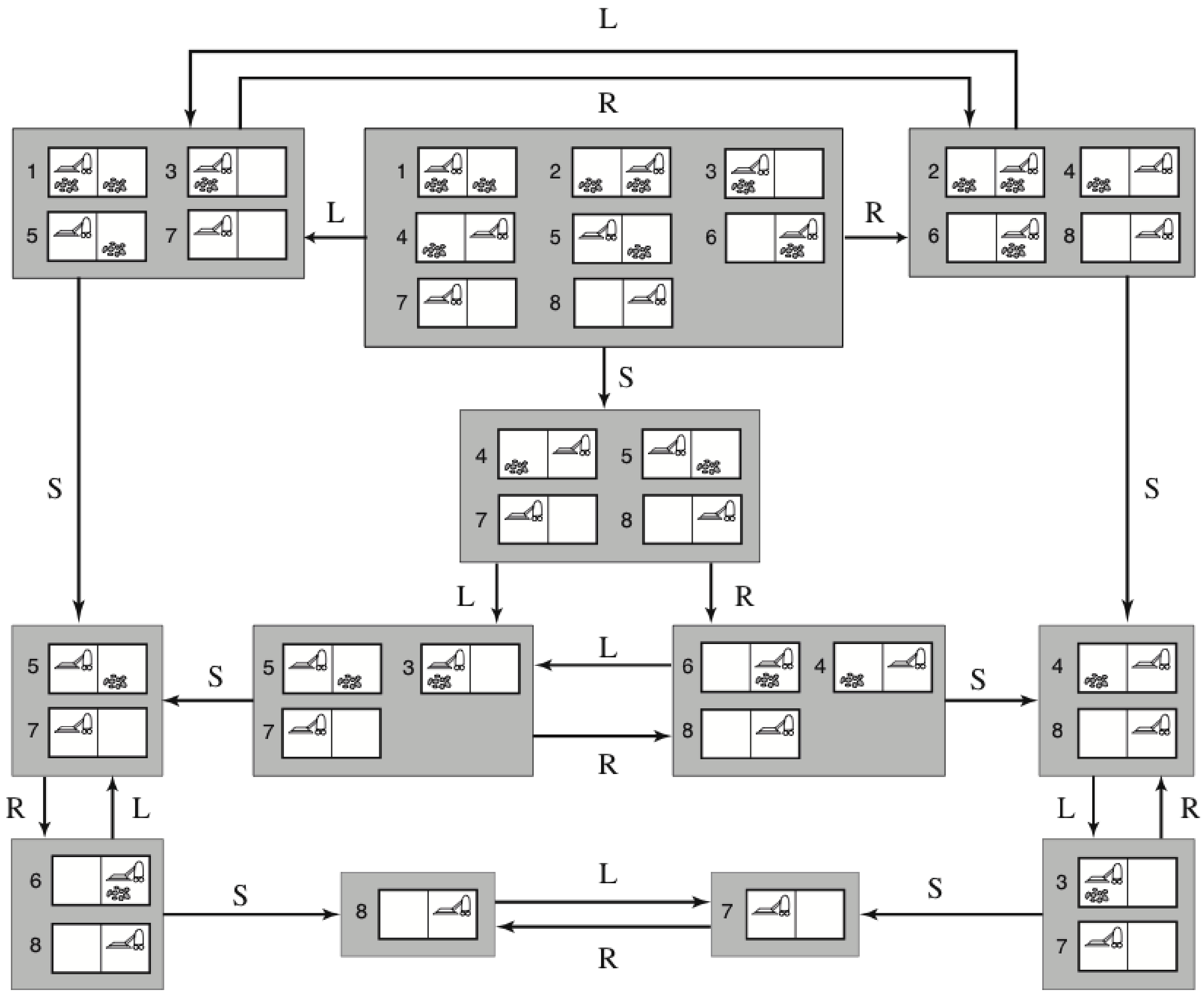
PREDICTING BELIEF STATES IN THE VACUUM WORLD



(a) Predicting the next belief state for the sensorless vacuum world with a deterministic action, *Right*.

(b) Prediction for the same belief state and action in the nondeterministic slippery version of the sensorless vacuum world.

THE DETERMINISTIC SENSORLESS VACUUM WORLD



PARTIAL OBSERVATIONS: STATE TRANSITIONS

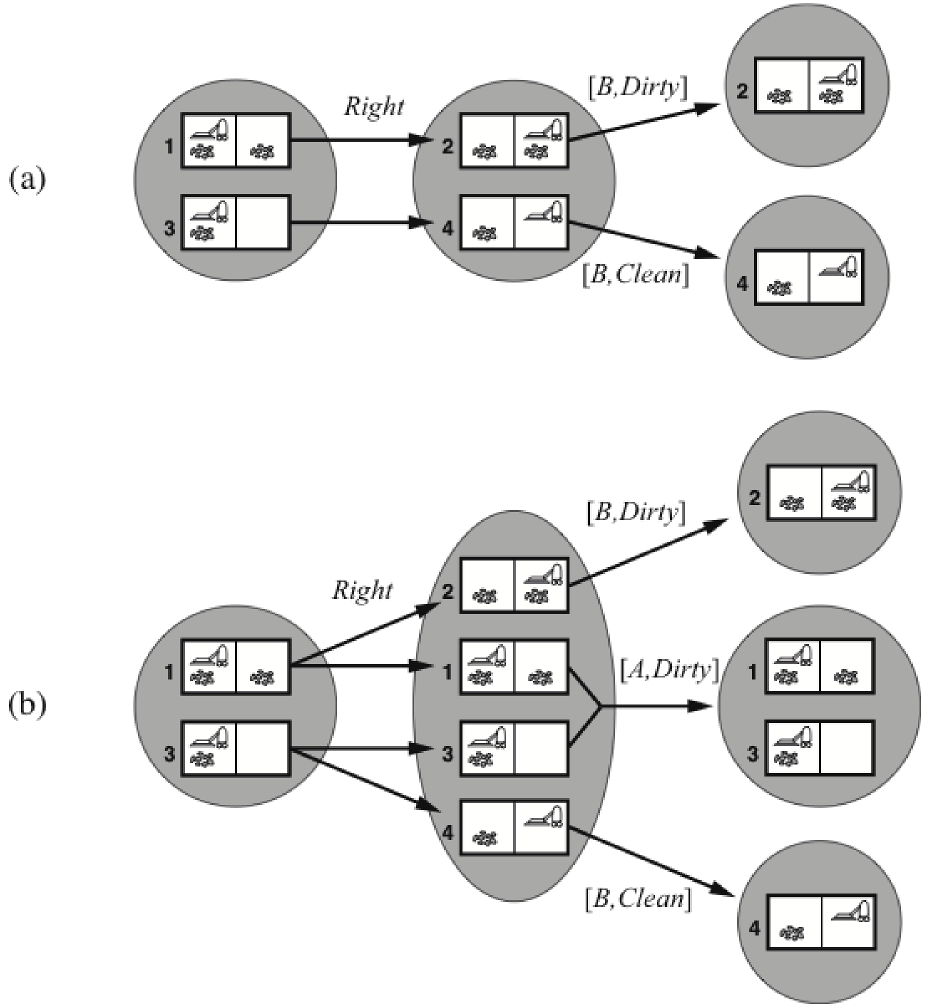
With partial observations, we can think of belief state transitions in three stages:

- **Prediction**, the same as for sensorless problems:
  - $b' = \text{Predict}(b, a) = \{\text{Result}(s, a) \text{ for each } s \in b\}$
- **Observation prediction**, determines the percepts that can be observed:
  - $\text{PossiblePercepts}(b') = \{\text{Percept}(s) \text{ for each } s \in b'\}$
- **Update**, filters the predicted states according to the percepts:
  - $\text{Update}(b', o) = \{s \text{ for each } s \in b' \text{ such that } o = \text{Percept}(s)\}$

Belief state transitions:

- $\text{Results}(b, a) = \{\text{Update}(b', o) \text{ for each } o \in \text{PossiblePercepts}(b')\}$
  where $b' = \text{Predict}(b, a)$

TRANSITIONS IN PARTIALLY OBSERVABLE VACUUM WORLDS

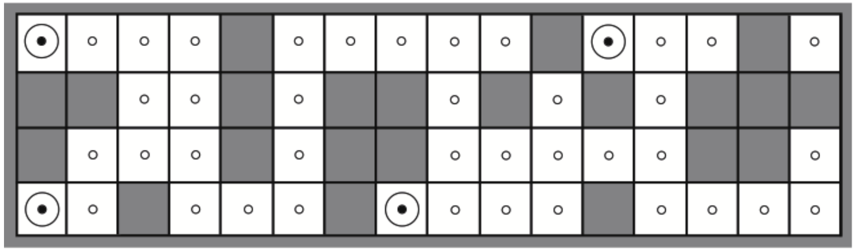


The percepts return the current position and the dirtyness of that square.

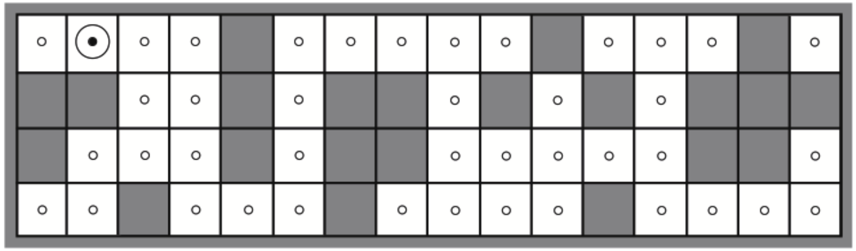
(a) The deterministic world: *Right* always succeeds.

(b) The slippery world: *Right* sometimes fails.

EXAMPLE: ROBOT LOCALISATION



(a) Possible locations of robot after $E_1 = NSW$



(b) Possible locations of robot After $E_1 = NSW, E_2 = NS$

The percepts return if there is a wall in each of the directions.

(a) Possible initial positions of the robot, after one observation.

(b) After moving right and a new observation, there is only one possible position left.

ADVERSARIAL SEARCH

TYPES OF GAMES (R&N 5.1)

MINIMAX SEARCH (R&N 5.2–5.3)

IMPERFECT DECISIONS (R&N 5.4–5.4.2)

STOCHASTIC GAMES (R&N 5.5)

TYPES OF GAMES (R&N 5.1)

- cooperative, competetive, zero-sum games
- game trees, ply/plies, utility functions

MULTIPLE AGENTS

Let’s consider problems with multiple agents, where:

- the agents select actions autonomously
- each agent has its own information state
    - they can have different information (even conflicting)
- the outcome depends on the actions of all agents
- each agent has its own utility function (that depends on the total outcome)

TYPES OF AGENTS

There are two extremes of multiagent systems:

- **Cooperative**: The agents share the same utility function
  - *Example*: Automatic trucks in a warehouse
- **Competetive**: When one agent wins all other agents lose
  - A common special case is when $\sum_a u_a(o) = 0$ for any outcome o. This is called a zero-sum game.
  - *Example*: Most board games

Many multiagent systems are between these two extremes.

- *Example*: Long-distance bike races are usually both cooperative (bikers usually form clusters where they take turns in leading a group), and competetive (only one of them can win in the end).

GAMES AS SEARCH PROBLEMS

The main difference to chapters 3–4:
now we have more than one agent that have different goals.

- All possible game sequences are represented in a game tree.
- The nodes are states of the game, e.g. board positions in chess.
- Initial state (root) and terminal nodes (leaves).
- States are connected if there is a legal move/ply.
  (a ply is a move by one player, i.e., one layer in the game tree)
- Utility function (payoff function). Terminal nodes have utility values $+x$ (player 1 wins), $-x$ (player 2 wins) and 0 (draw).

TYPES OF GAMES (AGAIN)

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information	battleships, blind tictactoe	bridge, poker, scrabble nuclear war

PERFECT INFORMATION GAMES: ZERO-SUM GAMES

Perfect information games are solvable in a manner similar to fully observable single-agent systems, e.g., using forward search.

If two agents are competing so that a positive reward for one is a negative reward for the other agent, we have a two-agent *zero-sum game*.

The value of a game zero-sum game can be characterized by a single number that one agent is trying to maximize and the other agent is trying to minimize.

This leads to a *minimax strategy*:

- A node is either a MAX node (if it is controlled by the maximising agent),
- or is a MIN node (if it is controlled by the minimising agent).

MINIMAX SEARCH (R&N 5.2–5.3)

- Minimax algorithm
- α-β pruning

MINIMAX SEARCH FOR ZERO-SUM GAMES

Given two players called MAX and MIN:

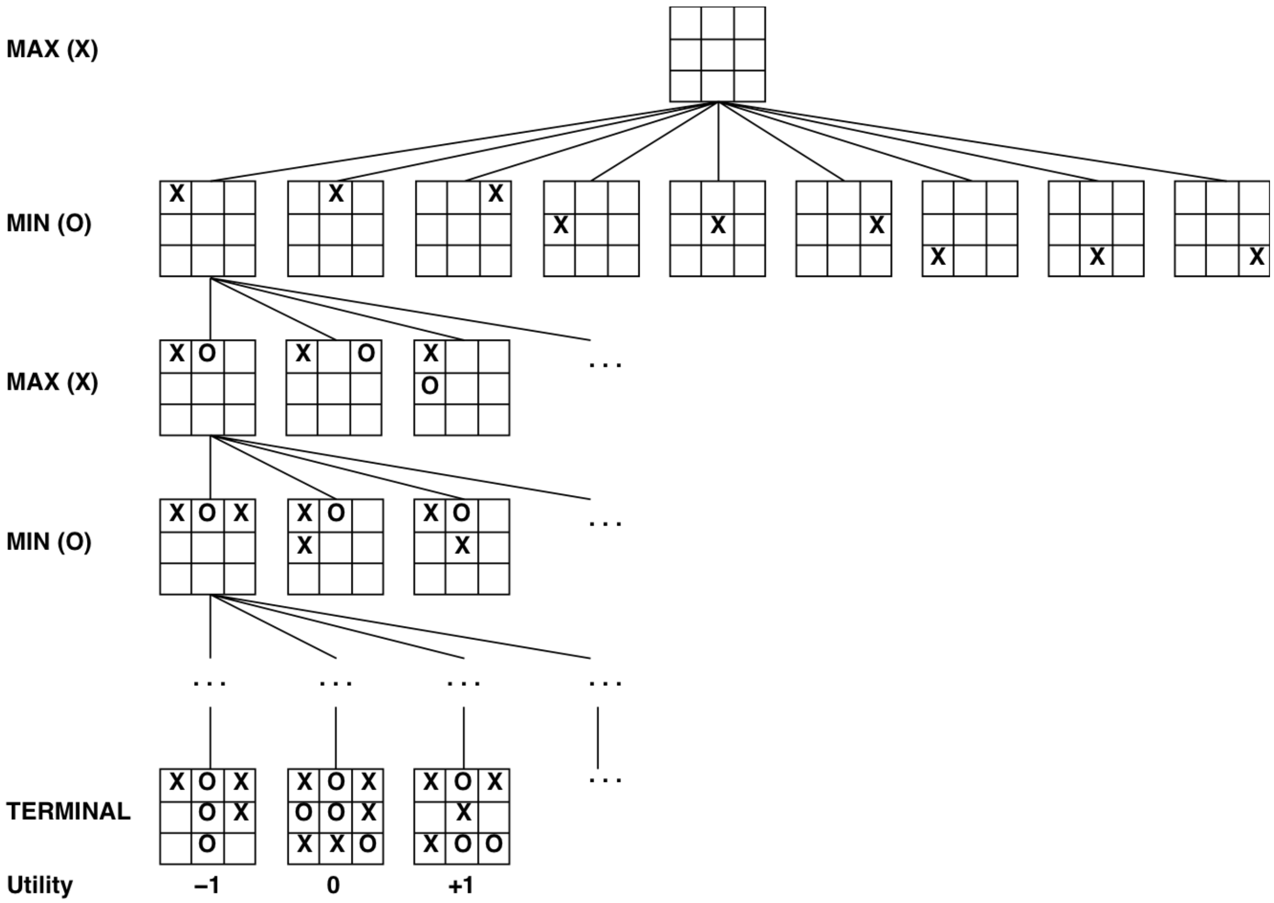
- MAX wants to maximize the utility value,
- MIN wants to minimize the same value.

$\Rightarrow$ MAX should choose the alternative that maximizes assuming that MIN minimizes.

Minimax gives perfect play for deterministic, perfect-information games:

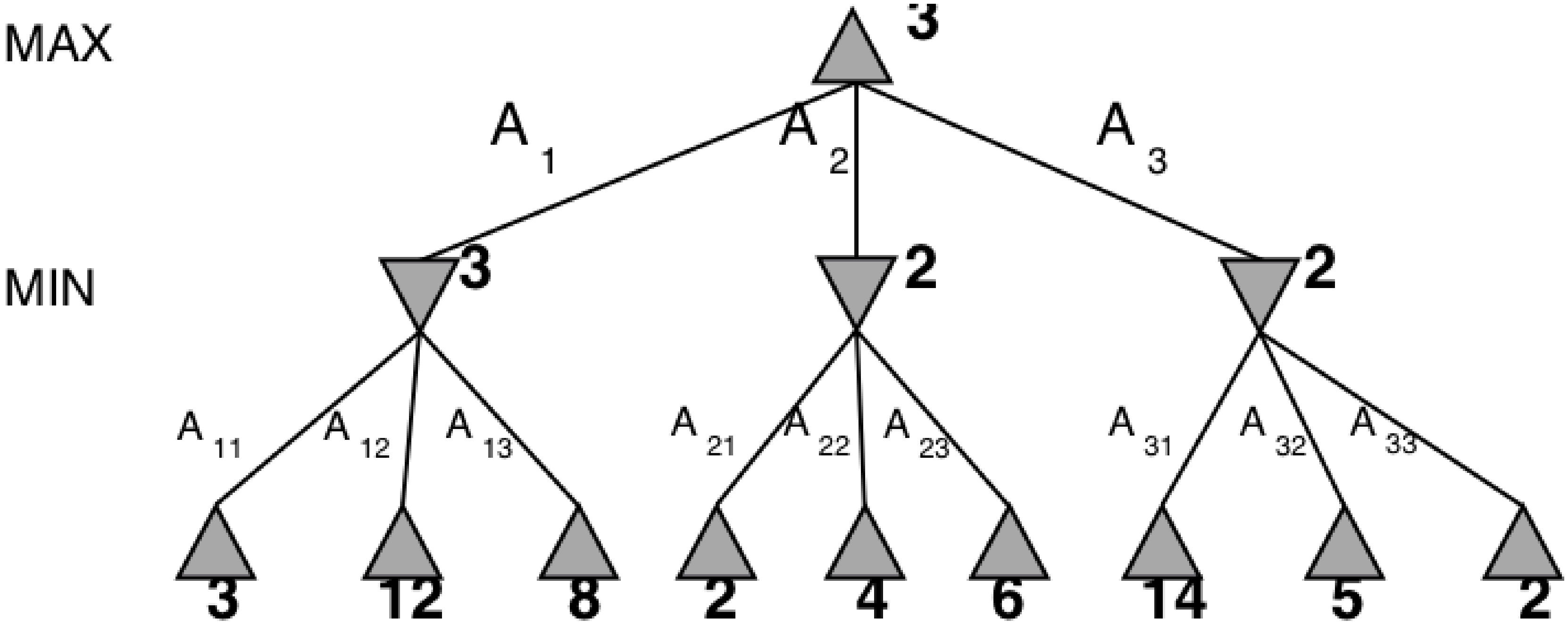
function Minimax(*state*):
  if TerminalTest(*state*) **then return** Utility(*state*)
  A := Actions(*state*)
  if *state* is a MAX node **then return** $\max_{a \in A}$ Minimax(Result(*state*, a))
  if *state* is a MIN node **then return** $\min_{a \in A}$ Minimax(Result(*state*, a))

MINIMAX SEARCH: TIC-TAC-TOE



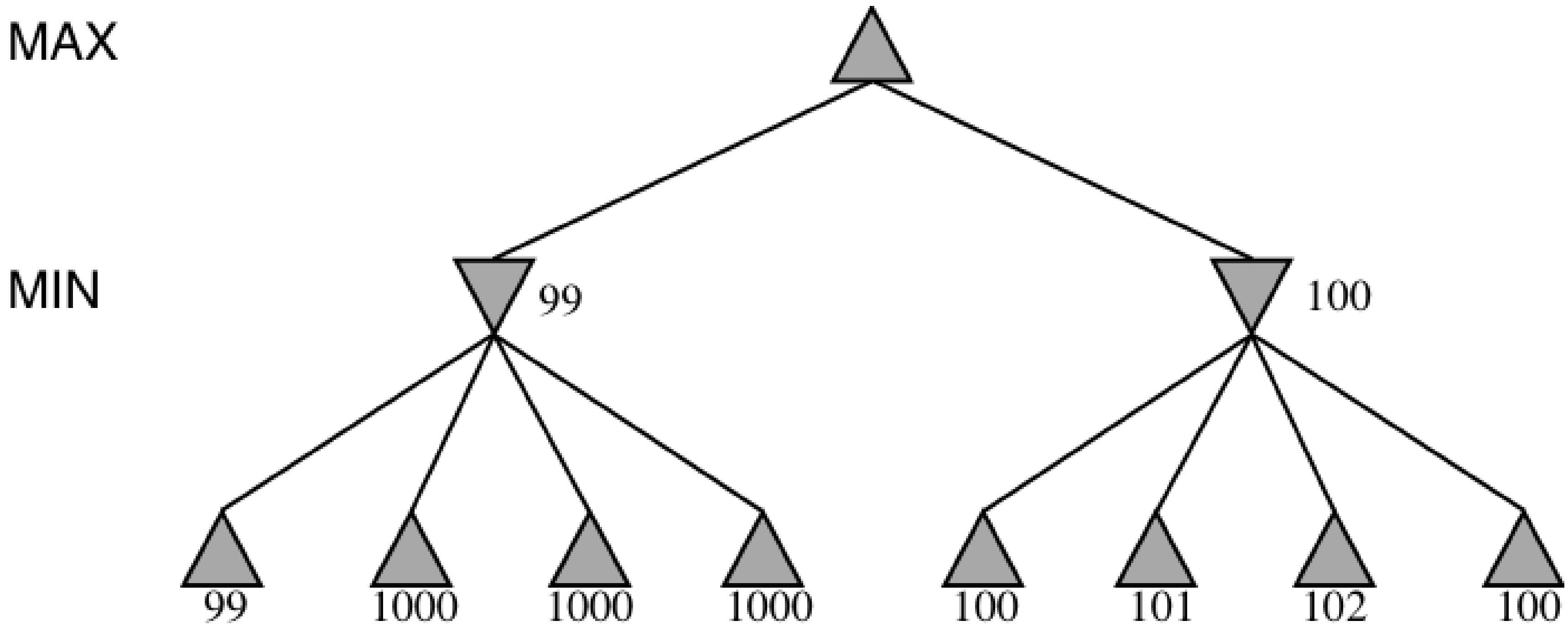
MINIMAX EXAMPLE

The Minimax algorithm gives perfect play for deterministic, perfect-information games.



CAN MINIMAX BE WRONG?

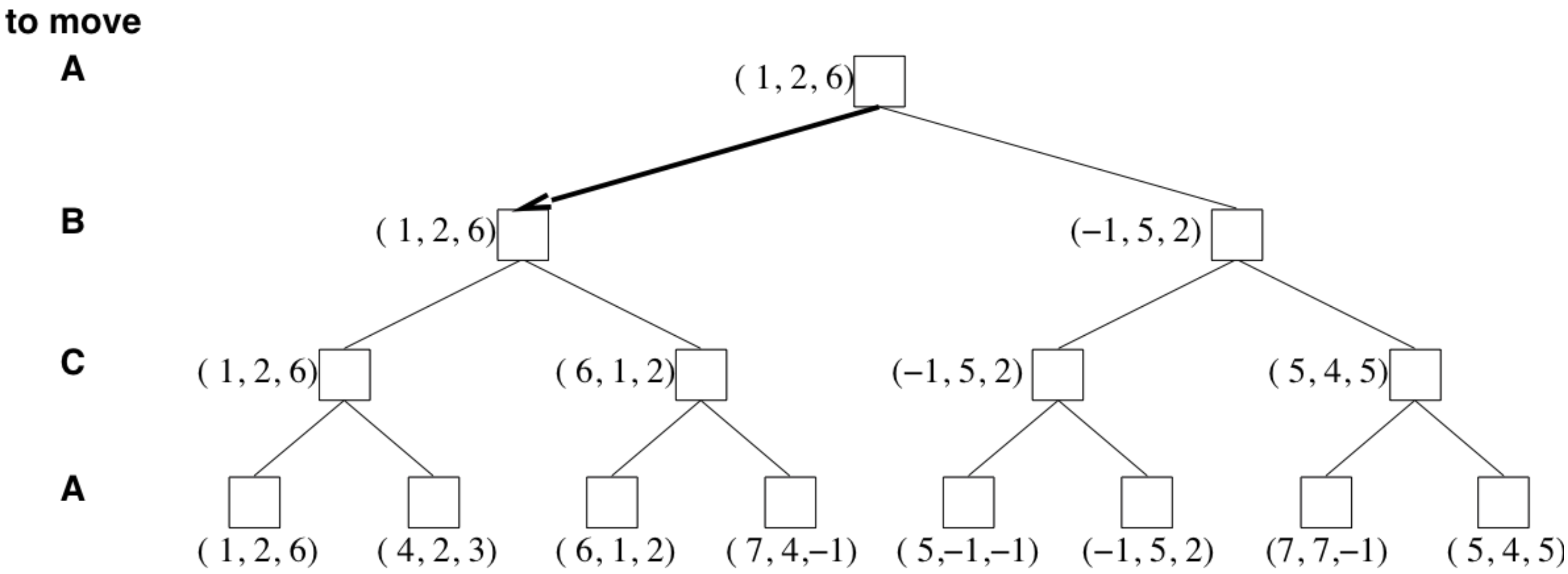
Minimax gives perfect play, but is that always the best strategy?



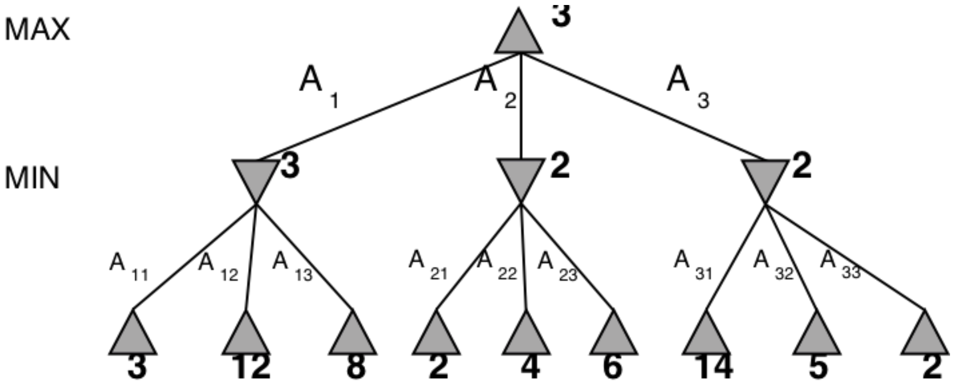
Perfect play assumes that the opponent is also a perfect player!

3-PLAYER MINIMAX

Minimax can also be used on multiplayer games



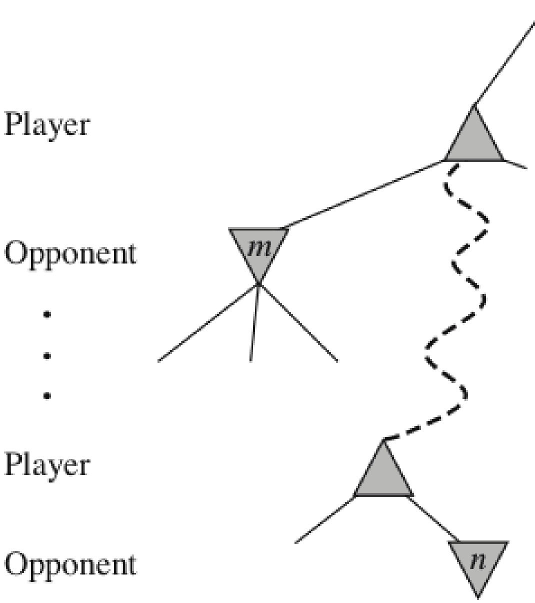
α–β PRUNING



$$
\begin{aligned}
\text{Minimax}(\textit{root}) &= \max(\min(3, 12, 8), \min(2, x, y), \min(14, 5, 2)) \\
&= \max(3, \min(2, x, y), 2) \\
&= \max(3, z, 2) \quad \text{where } z \leq 2 \\
&= 3
\end{aligned}
$$

I.e., we don't need to know the values of x and y!

α–β PRUNING, GENERAL IDEA

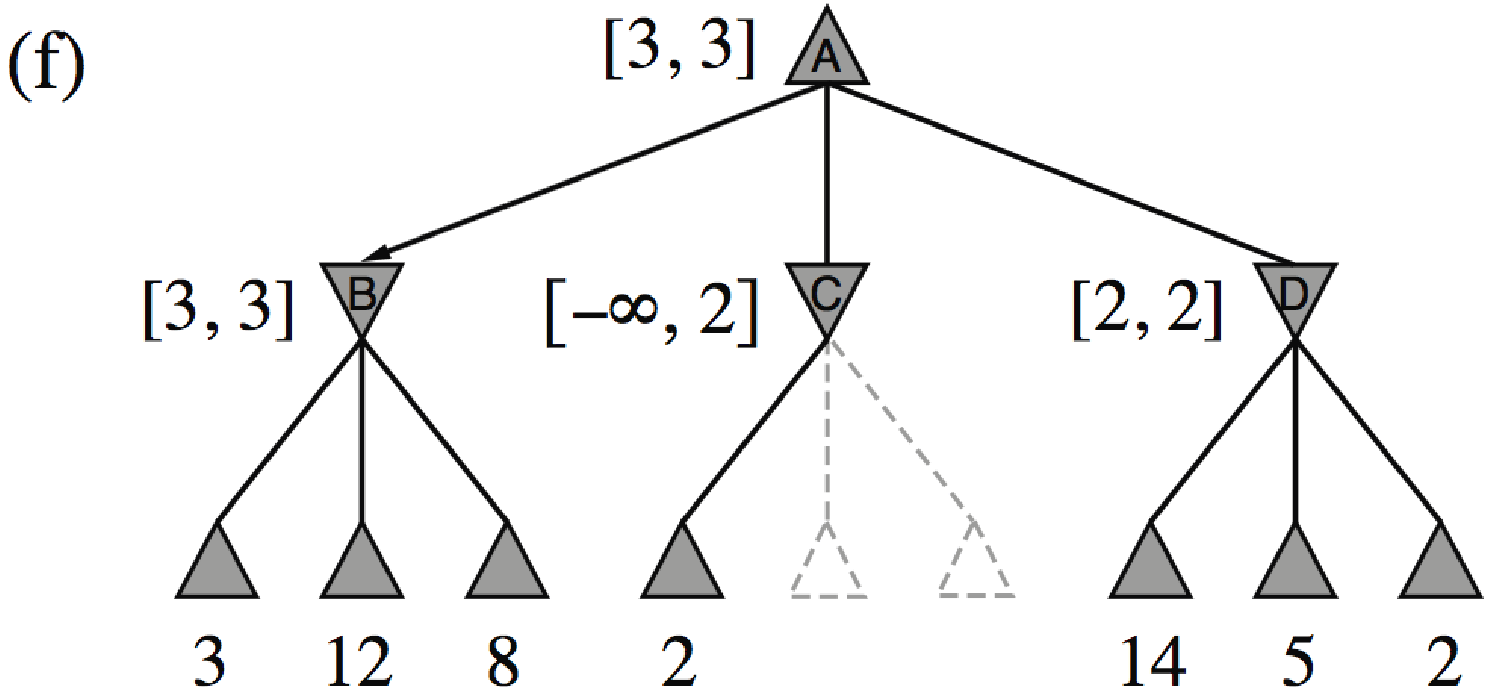


The general idea of α-β pruning is this:

- if m is better than n for Player, we don't want to pursue n
- so, once we know enough about n we can prune it
- sometimes it's enough to examine just one of n's descendants

α-β pruning keeps track of the possible range of values for every node it visits;
the parent range is updated when the child has been visited.

MINIMAX EXAMPLE, WITH α–β PRUNING



THE α−β ALGORITHM

function AlphaBetaSearch(*state*):
    v := MaxValue(*state*, $-\infty$, $+\infty$))
    return the *action* in Actions(*state*) that has value v

function MaxValue(*state*, α, β):
    if TerminalTest(*state*) **then return** Utility(*state*)
    $v := -\infty$
    for each *action* in Actions(*state*):
        v := max(v, MinValue(Result(*state*, *action*), α, β))
        if $v \geq \beta$ **then return** v
        α := max(α, v)
    return v

function MinValue(*state*, α, β):
    same as MaxValue but reverse the roles of α/β and *min*/*max* and $-\infty/+\infty$

HOW EFFICIENT IS $\alpha-\beta$ PRUNING?

The amount of pruning provided by the α-β algorithm depends on the ordering of the children of each node.

- It works best if a highest-valued child of a MAX node is selected first and if a lowest-valued child of a MIN node is returned first.
- In real games, much of the effort is made to optimise the search order.
- With a “perfect ordering”, the time complexity becomes $O(b^{m/2})$
    - this doubles the solvable search depth
    - however, $35^{80/2}$ (for chess) or $250^{160/2}$ (for go) is still impossible…

MINIMAX AND REAL GAMES

Most real games are too big to carry out minimax search, even with α-β pruning.

- For these games, instead of stopping at leaf nodes, we have to use a cutoff test to decide when to stop.
- The value returned at the node where the algorithm stops is an estimate of the value for this node.
- The function used to estimate the value is an evaluation function.
- Much work goes into finding good evaluation functions.
- There is a trade-off between the amount of computation required to compute the evaluation function and the size of the search space that can be explored in any given time.

IMPERFECT DECISIONS (R&N 5.4–5.4.2)

Note: this will be presented Tuesday 25th April instead!

- H-minimax algorithm
- Evaluation function, cutoff test
- features, weighted linear function
- quiescence search, horizon effect

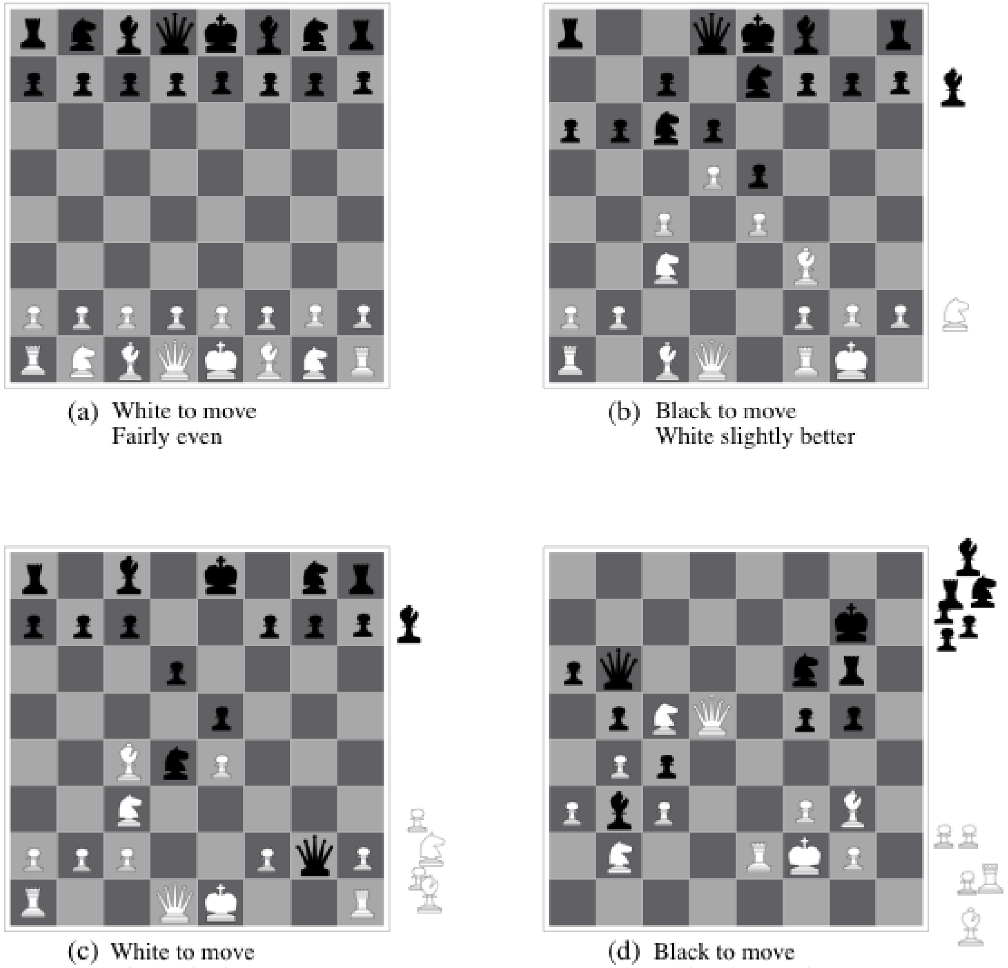
H-MINIMAX ALGORITHM

The *Heuristic* Minimax algorithm is similar to normal Minimax

- it replaces **TerminalTest** and **Utility** with **CutoffTest** and **Eval**

function H-Minimax(*state*, *depth*):
    if CutoffTest(*state*, *depth*) **then return** Eval(*state*)
    A := Actions(*state*)
    if *state* is a MAX node **then return** $\max_{a \in A}$ H-Minimax(Result(*state*, *a*), *depth*+1)
    if *state* is a MIN node **then return** $\min_{a \in A}$ H-Minimax(Result(*state*, *a*), *depth*+1)

CHESS POSITIONS: HOW TO EVALUATE



(a) White to move
Fairly even

(b) Black to move
White slightly better

(c) White to move
Black winning

(d) Black to move
White about to lose

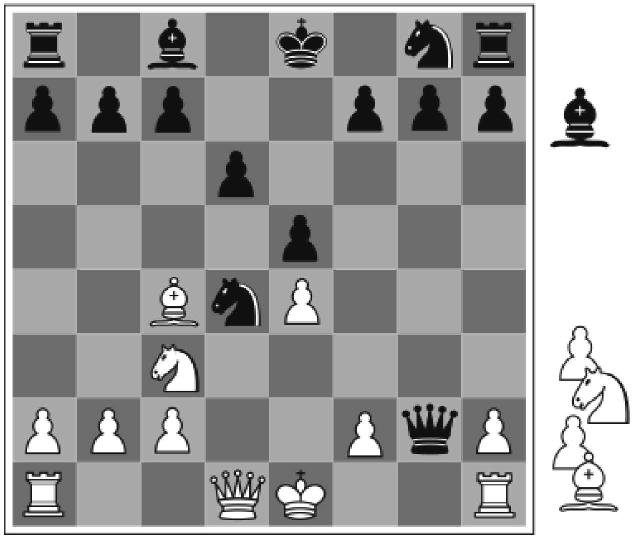
WEIGHTED LINEAR EVALUATION FUNCTIONS

A very common evaluation function is to use a weighted sum of features:

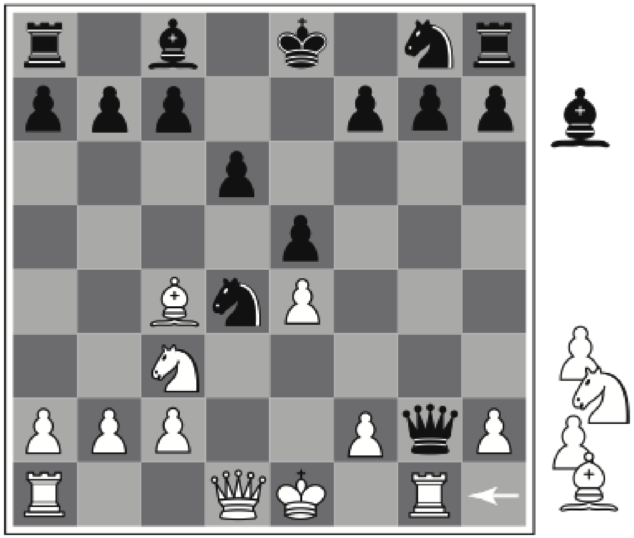
$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \cdots + w_n f_n(s) = \sum_{i=1}^{n} w_i f_i(s)$$

- This relies on a strong assumption: all features are *independent of each other*
    - which is usually not true, so the best programs for chess (and other games) also use nonlinear feature combinations
-
- The weights can be calculated using machine learning algorithms, but a human still has to come up with the features.
    - using recent advances in deep machine learning, the computer can learn the features too

EVALUATION FUNCTIONS



(a) White to move



(b) White to move

A naive weighted sum of features will not see the difference between these two states.

PROBLEMS WITH CUTOFF TESTS

Too simplistic cutoff tests and evaluation functions can be problematic:

- e.g., if the cutoff is only based on the current depth
- then it might cut off the search in unfortunate positions (such as (b) on the previous slide)

We want more sophisticated cutoff tests:

- only cut off search in *quiescent* positions
- i.e., in positions that are “stable”, unlikely to exhibit wild swings in value
- non-quiescent positions should be expanded further

Another problem is the *horizon effect*:

- if a bad position is unavoidable (e.g., loss of a piece), but the system can delay it from happening, it might push the bad position “over the horizon”
- in the end, the resulting delayed position might be even worse

DETERMINISTIC GAMES IN PRACTICE

Chess:

- DeepBlue (IBM) beats world champion Garry Kasparov, 1997.
- Modern chess programs: Houdini, Critter, Stockfish.

Checkers/Othello/Reversi:

- Logistello beats the world champion in Othello/Reversi, 1997.
- Chinook plays checkers perfectly, 2007. It uses an endgame database defining perfect play for all 8-piece positions on the board, (a total of 443,748,401,247 positions).

Go:

- AlphaGo (Google DeepMind) beats one of the world's best players, Lee Sedol by 4–1, in April 2016.
- Modern programs: MoGo, Zen, GNU Go, AlphaGo.

GAMES OF IMPERFECT INFORMATION

Imperfect information games

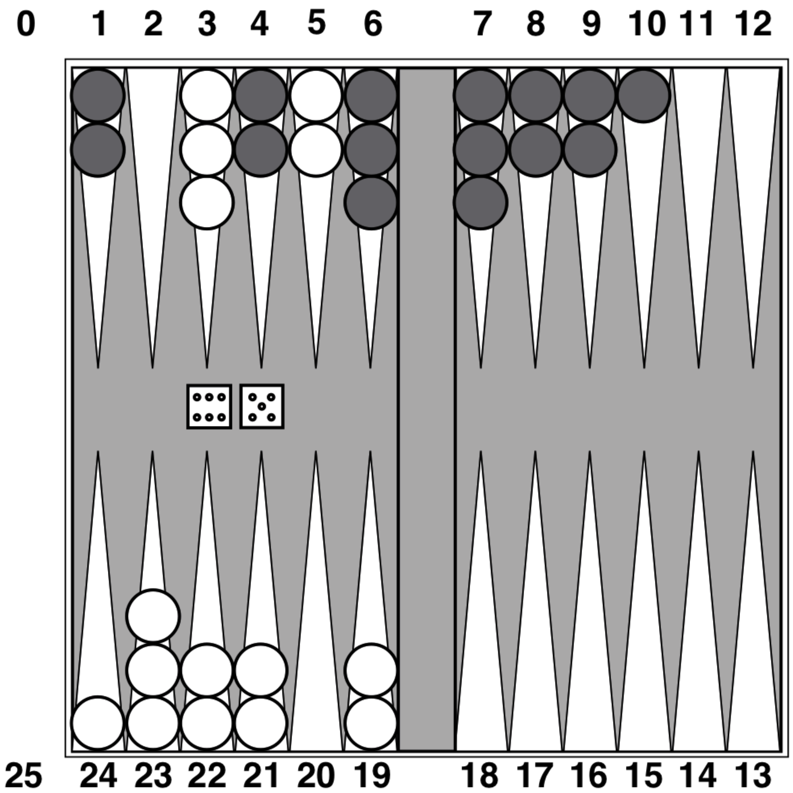
- e.g., card games, where the opponent's initial cards are unknown
- typically we can calculate a probability for each possible deal
- seems just like having one big dice roll at the beginning of the game
- main idea: compute the minimax value of each action in each deal, then choose the action with highest expected value over all deals

STOCHASTIC GAMES (R&N 5.5)

Note: this will be presented Tuesday 25th April instead!

- chance nodes
- expected value
- expecti-minimax algorithm

STOCHASTIC GAME EXAMPLE: BACKGAMMON

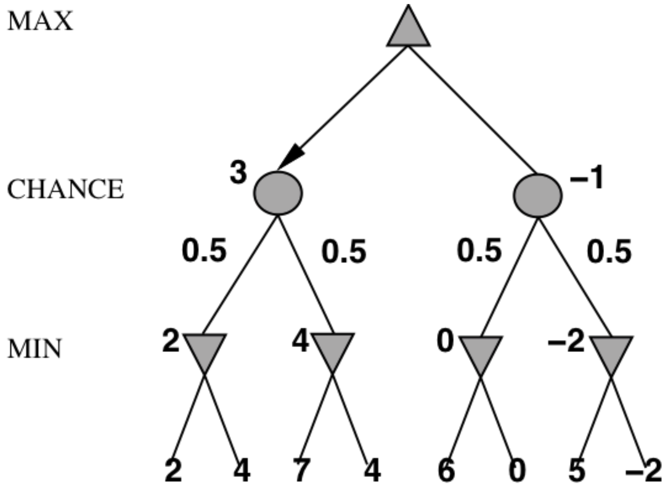


STOCHASTIC GAMES IN GENERAL

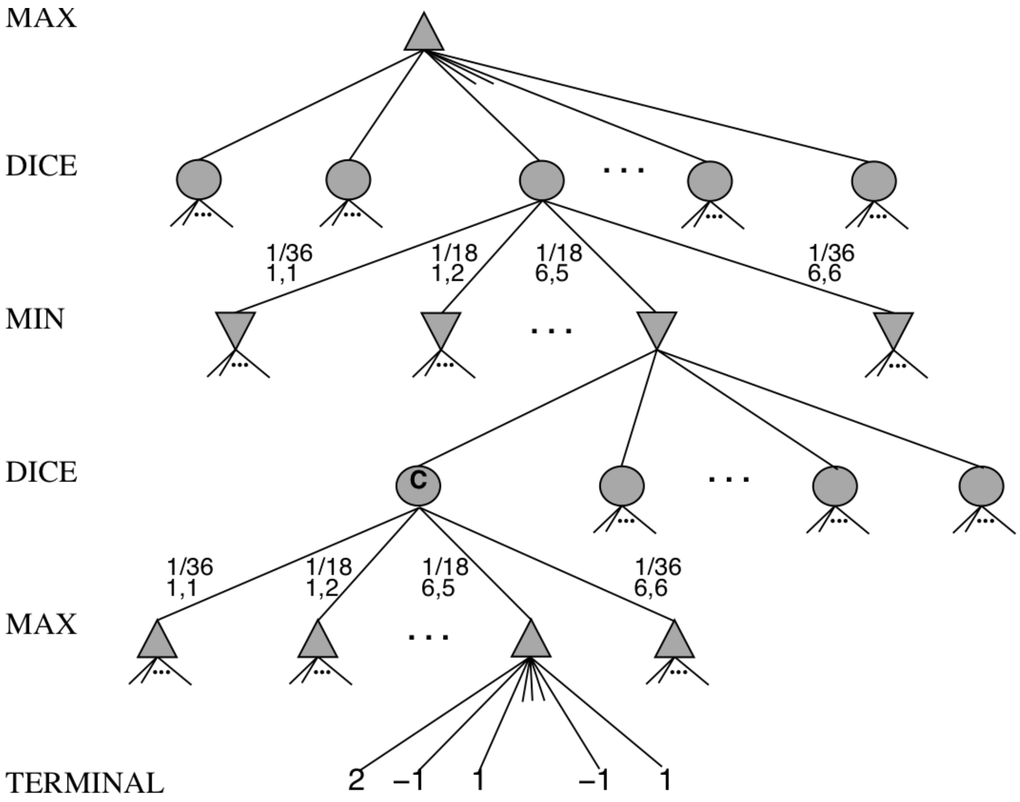
In stochastic games, chance is introduced by dice, card-shuffling, etc.

- We introduce *chance nodes* to the game tree.
- We can't calculate a definite minimax value,
  instead we calculate the *expected value* of a position.
- The expected value is the average of all possible outcomes.

A very simple example with coin-flipping and arbitrary values:



BACKGAMMON GAME TREE



ALGORITHM FOR STOCHASTIC GAMES

The ExpectiMinimax algorithm gives perfect play;
it's just like Minimax, except we must also handle chance nodes:

function ExpectiMinimax(*state*):
  if TerminalTest(*state*) **then return** Utility(*state*)
  A := Actions(*state*)
  if *state* is a MAX node **then return** $\max_{a \in A}$ Minimax(*state*, a)
  if *state* is a MAX node **then return** $\min_{a \in A}$ Minimax(*state*, a)
  if *state* is a chance node **then return** $\sum_{a \in A} P(a)$ Minimax(*state*, a)

where $P(a)$ is the probability that action a occurs.

STOCHASTIC GAMES IN PRACTICE

Dice rolls increase the branching factor *b*:

- there are 21 possible rolls with 2 dice

Backgammon has ≈20 legal moves:

- depth $4 \Rightarrow 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$ nodes

As depth increases, the probability of reaching a given node shrinks:

- value of lookahead is diminished
- α-β pruning is much less effective

TDGammon (1995) used depth-2 search + very good Eval:

- the evaluation function was learned by self-play
- world-champion level