

# CHAPTERS 4–5: NON-CLASSICAL AND ADVERSARIAL SEARCH

DIT411/TIN175, Artificial Intelligence

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# REPETITION

## UNINFORMED SEARCH (R&N 3.4)

Search problems, graphs, states, arcs, goal test, generic search algorithm, tree search, graph search, depth-first search, breadth-first search, uniform cost search, iterative deepening, bidirectional search, ...

## HEURISTIC SEARCH (R&N 3.5–3.6)

Greedy best-first search, A\* search, heuristics, admissibility, consistency, dominating heuristics, ...

## LOCAL SEARCH (R&N 4.1)

Hill climbing / gradient descent, random moves, random restarts, beam search, simulated annealing, ...

# **NON-CLASSICAL SEARCH**

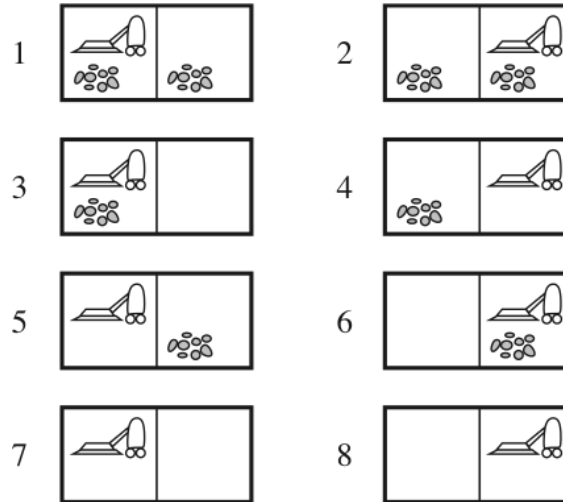
**NONDETERMINISTIC SEARCH (R&N 4.3)**

**PARTIAL OBSERVATIONS (R&N 4.4)**

## NONDETERMINISTIC SEARCH (R&N 4.3)

- Contingency plan / strategy
- *And-or* search trees (not in the written exam)

# AN ERRATIC VACUUM CLEANER



The eight possible states of the vacuum world; states 7 and 8 are goal states.

There are three actions: *Left*, *Right*, *Suck*.

Assume that the *Suck* action works as follows:

- if the square is dirty, it is cleaned but sometimes also the adjacent square is
- if the square is clean, the vacuum cleaner sometimes deposits dirt

# NONDETERMINISTIC OUTCOMES, CONTINGENCY PLANS

Assume that the *Suck* action is nondeterministic:

- if the square is dirty, it is cleaned but sometimes also the adjacent square is
- if the square is clean, the vacuum cleaner sometimes deposits dirt

Now we need a more general *result* function:

- instead of returning a single state, it returns a set of possible outcome states
- e.g.,  $\text{Results}(\text{Suck}, 1) = \{5, 7\}$  and  $\text{Results}(\text{Suck}, 5) = \{1, 5\}$

We also need to generalise the notion of a *solution*:

- instead of a single sequence (path) from the start to the goal, we need a *strategy* (or a *contingency plan*)
- i.e., we need **if-then-else** constructs
- this is a possible solution from state 1:
  - $[\text{Suck}, \text{if } \text{State}=5 \text{ then } [\text{Right}, \text{Suck}] \text{ else } []]$

# HOW TO FIND CONTINGENCY PLANS

*(will not be in the written examination)*

We need a new kind of nodes in the search tree:

- **and nodes**:  
these are used whenever an action is nondeterministic
- normal nodes are called **or nodes**:  
they are used when we have several possible actions in a state

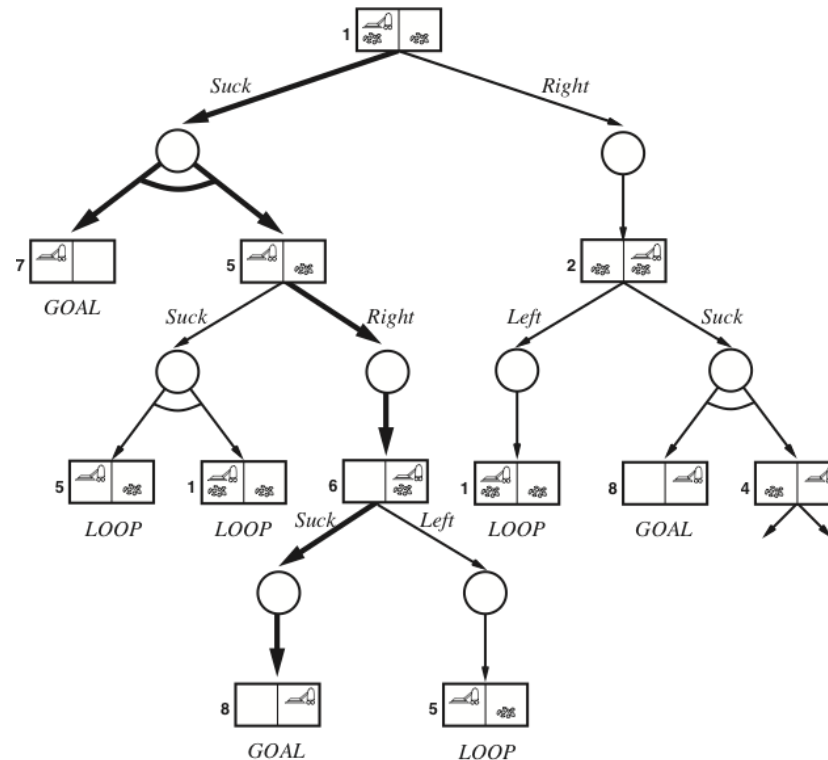
A solution for an **and-or** search problem is a subtree that:

- has a goal node at every leaf
- specifies exactly one action at each of its **or node**
- includes every branch at each of its **and node**



# A SOLUTION TO THE ERRATIC VACUUM CLEANER

*(will not be in the written examination)*



The solution subtree is shown in bold, and corresponds to the plan:

**[Suck, if State=5 then [Right, Suck] else []]**

# AN ALGORITHM FOR FINDING A CONTINGENCY PLAN

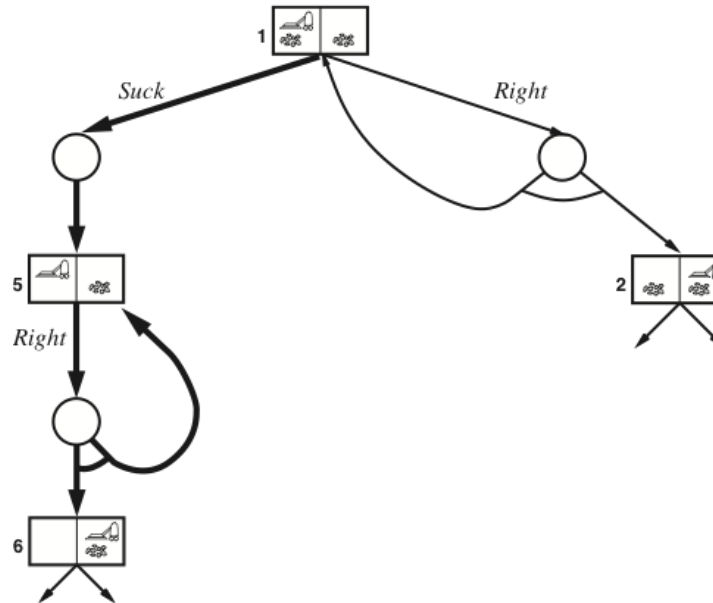
*(will not be in the written examination)*

This algorithm does a depth-first search in the *and-or* tree, so it is not guaranteed to find the best or shortest plan:

```
function AndOrGraphSearch(problem):  
    return OrSearch(problem.InitialState, problem, [])  
  
function OrSearch(state, problem, path):  
    if problem.GoalTest(state) then return []  
    if state is on path then return failure  
    for each action in problem.Actions(state):  
        plan := AndSearch(problem.Results(state, action), problem, [state] ++ path)  
        if plan ≠ failure then return [action] ++ plan  
    return failure  
  
function AndSearch(states, problem, path):  
    for each  $s_i$  in states:  
        plani := OrSearch( $s_i$ , problem, path)  
        if plani = failure then return failure  
    return [if  $s_1$  then plan1 else if  $s_2$  then plan2 else ... if  $s_n$  then plann]
```

# WHILE LOOPS IN CONTINGENCY PLANS

*(will not be in the written examination)*



If the search graph contains cycles, **if-then-else** is not enough in a contingency plan:

- we need **while** loops instead

In the slippery vacuum world above, the cleaner don't always move when told:

- the solution above translates to [*Suck*, *while* *State*=5 *do* *Right*, *Suck*]

# PARTIAL OBSERVATIONS (R&N 4.4)

- Belief states: goal test, transitions, ...
- Sensor-less (conformant) problems
- Partially observable problems

# OBSERVABILITY VS DETERMINISM

A problem is *nondeterministic* if there are several possible outcomes of an action

- deterministic — nondeterministic (chance)

It is *partially observable* if the agent cannot tell exactly which state it is in

- fully observable (perfect info.) — partially observable (imperfect info.)

A problem can be either nondeterministic, or partially observable, or both:

|                       | deterministic                   | chance                                 |
|-----------------------|---------------------------------|--|
| perfect information   | chess, checkers,<br>go, othello | backgammon<br>monopoly                 |
| imperfect information | battleships,<br>blind tictactoe | bridge, poker, scrabble<br>nuclear war |

# BELIEF STATES

Instead of searching in a graph of states, we use *belief states*

- A belief state is a *set of states*

In a sensor-less (or conformant) problem, the agent has *no information at all*

- The initial belief state is the set of all problem states
  - e.g., for the vacuum world the initial state is {1,2,3,4,5,6,7,8}

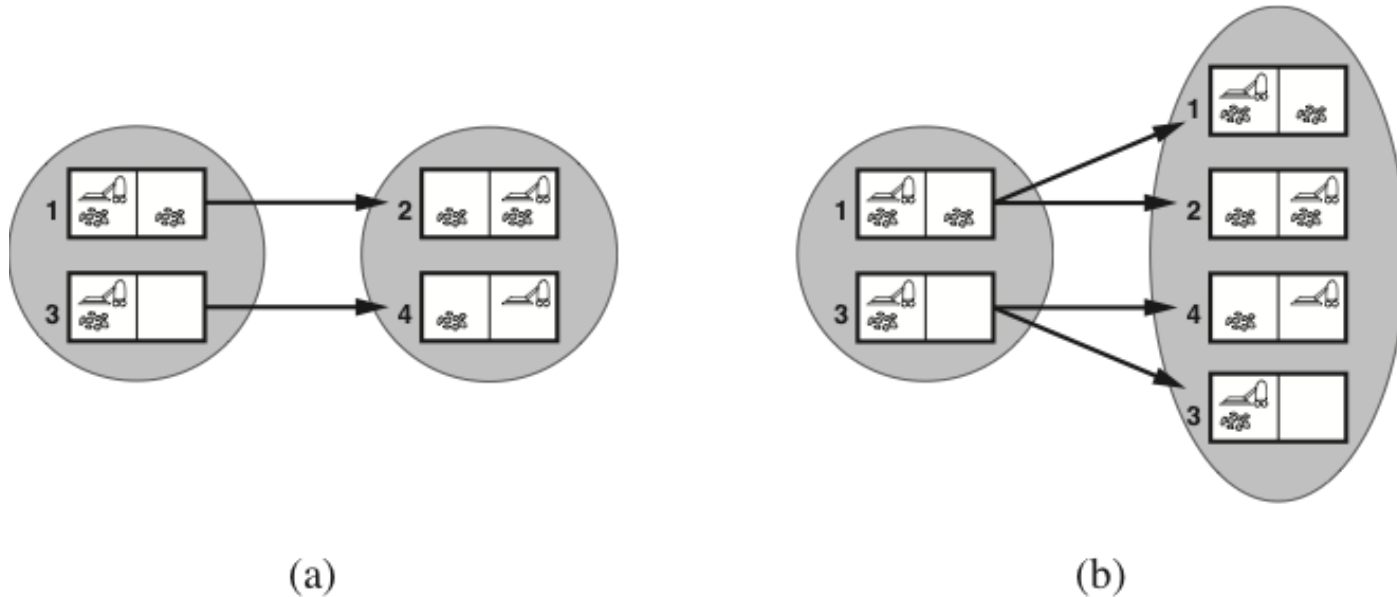
The goal test has to check that *all* members in the belief state is a goal

- e.g., for the vacuum world, the following are goal states: {7}, {8}, and {7,8}

The result of performing an action is the *union* of all possible results

- i.e.,  $\text{Predict}(b, a) = \{\text{Result}(s, a) \text{ for each } s \in b\}$
- if the problem is also nondeterministic:
  - $\text{Predict}(b, a) = \bigcup \{\text{Results}(s, a) \text{ for each } s \in b\}$

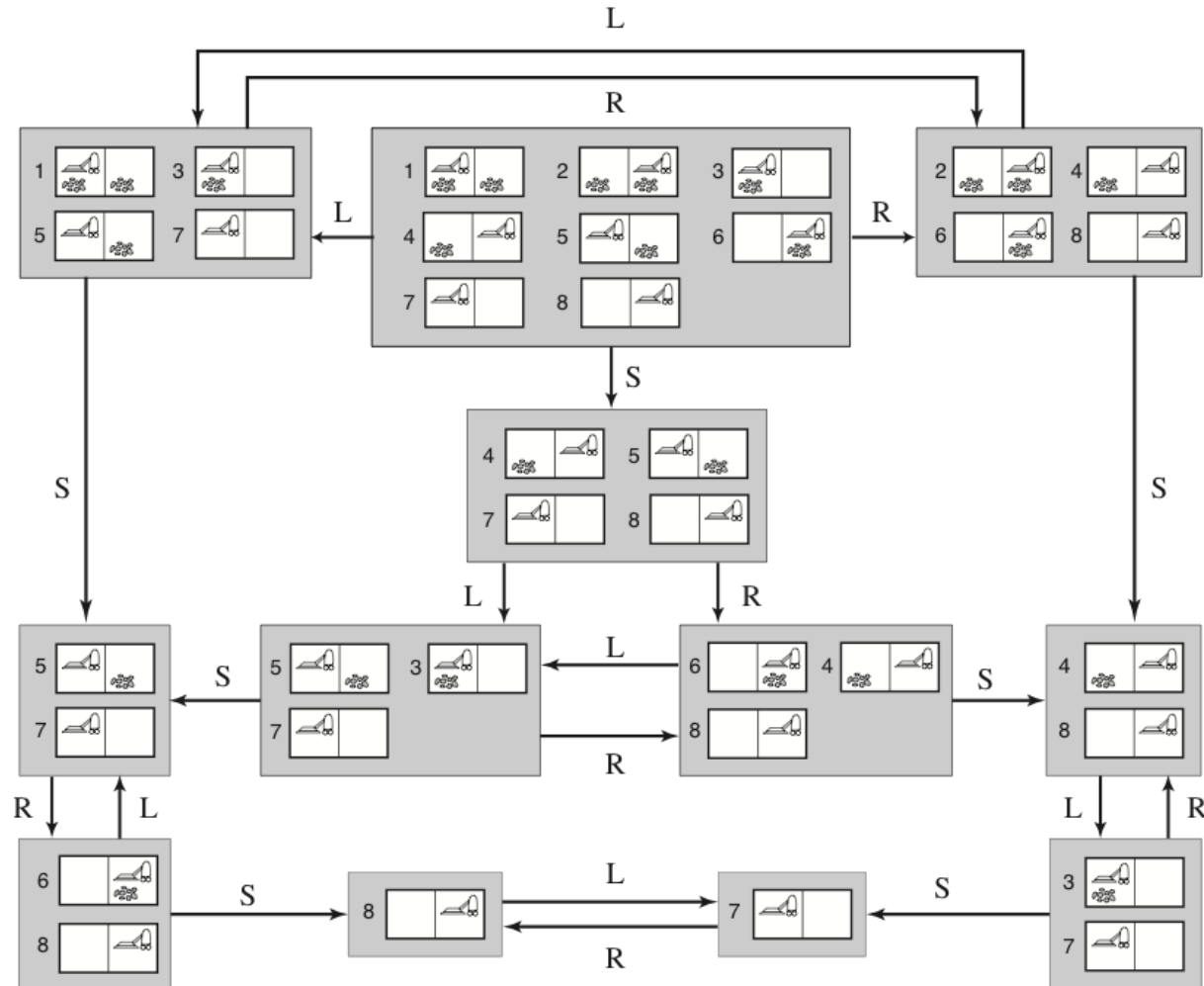
# PREDICTING BELIEF STATES IN THE VACUUM WORLD



(a) Predicting the next belief state for the sensorless vacuum world with a deterministic action, *Right*.

(b) Prediction for the same belief state and action in the nondeterministic slippery version of the sensorless vacuum world.

# THE DETERMINISTIC SENSORLESS VACUUM WORLD





## PARTIAL OBSERVATIONS: STATE TRANSITIONS

With partial observations, we can think of belief state transitions in three stages:

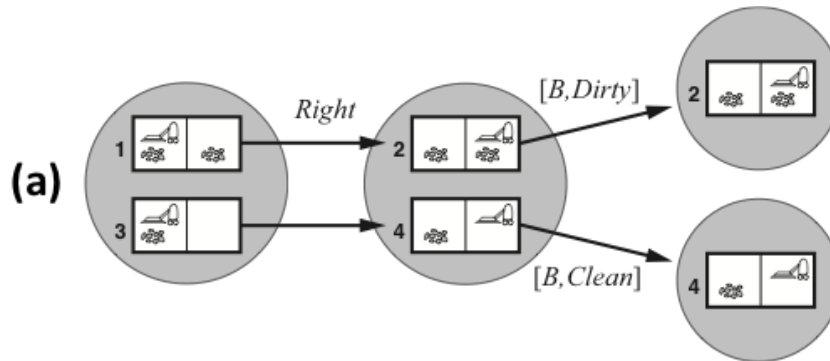
- **Prediction**, the same as for sensorless problems:
  - $b' = \text{Predict}(b, a) = \{\text{Result}(s, a) \text{ for each } s \in b\}$
- **Observation prediction**, determines the percepts that can be observed:
  - $\text{PossiblePercepts}(b') = \{\text{Percept}(s) \text{ for each } s \in b'\}$
- **Update**, filters the predicted states according to the percepts:
  - $\text{Update}(b', o) = \{s \text{ for each } s \in b' \text{ such that } o = \text{Percept}(s)\}$

Belief state transitions:

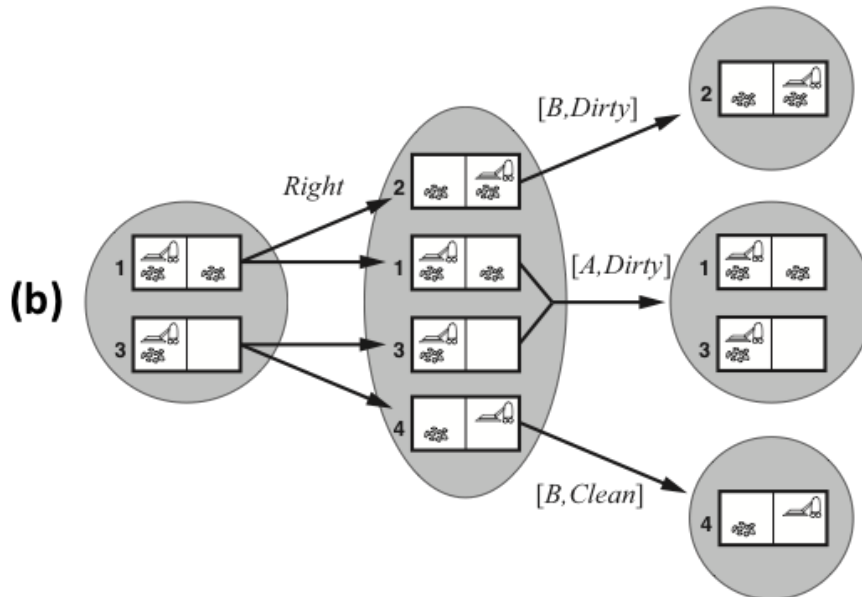
- $\text{Results}(b, a) = \{\text{Update}(b', o) \text{ for each } o \in \text{PossiblePercepts}(b')\}$   
where  $b' = \text{Predict}(b, a)$

# TRANSITIONS IN PARTIALLY OBSERVABLE VACUUM WORLDS

The percepts return the current position and the dirtiness of that square.

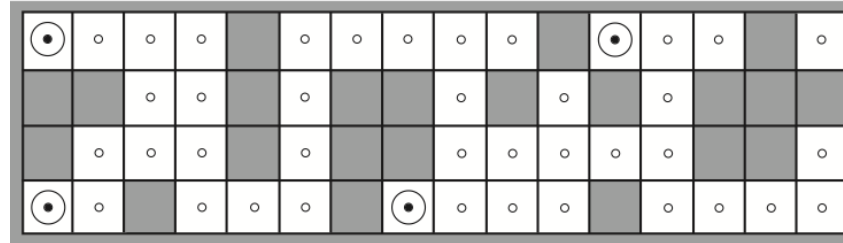


The deterministic world:  
*Right* always succeeds.

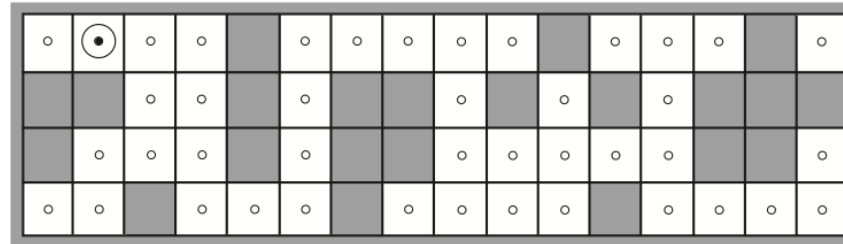


The slippery world:  
*Right* sometimes fails.

# EXAMPLE: ROBOT LOCALISATION



(a) Possible locations of robot after  $E_1 = \text{NSW}$



(b) Possible locations of robot After  $E_1 = \text{NSW}, E_2 = \text{NS}$

The percepts return whether there is a wall in each of the directions.

**Top:** Possible initial positions of the robot, after the first observation.

- $E_1 = \text{North, South, West}$

**Bottom:** After moving right and observing, there's only one possible position left.

- $E_1 = \text{North, South, West}$ ; **Right**;  $E_2 = \text{North, South}$

# **ADVERSARIAL SEARCH**

**TYPES OF GAMES (R&N 5.1)**

**MINIMAX SEARCH (R&N 5.2–5.3)**

**IMPERFECT DECISIONS (R&N 5.4–5.4.2)**

**STOCHASTIC GAMES (R&N 5.5)**

## TYPES OF GAMES (R&N 5.1)

- cooperative, competitive, zero-sum games
- game trees, ply/plies, utility functions

# MULTIPLE AGENTS

Let's consider problems with multiple agents, where:

- the agents select actions autonomously
- each agent has its own information state
  - they can have different information (even conflicting)
- the outcome depends on the actions of all agents
- each agent has its own utility function (that depends on the total outcome)

# TYPES OF AGENTS

There are two extremes of multiagent systems:

- **Cooperative:** The agents share the same utility function
  - *Example:* Automatic trucks in a warehouse
- **Competitive:** When one agent wins all other agents lose
  - A common special case is when  $\sum_a u_a(o) = 0$  for any outcome  $o$ . This is called a zero-sum game.
  - *Example:* Most board games

Many multiagent systems are between these two extremes.

- *Example:* Long-distance bike races are usually both cooperative (bikers form clusters where they take turns in leading a group), and competitive (only one of them can win in the end).

# GAMES AS SEARCH PROBLEMS

The main difference to chapters 3–4:  
now we have more than one agent that have different goals.

- All possible game sequences are represented in a game tree.
- The nodes are states of the game, e.g. board positions in chess.
- Initial state (root) and terminal nodes (leaves).
- States are connected if there is a legal move/ply.  
(a ply is a move by one player, i.e., one layer in the game tree)
- Utility function (payoff function). Terminal nodes have utility values  $+x$  (player 1 wins),  $-x$  (player 2 wins) and  $0$  (draw).



## TYPES OF GAMES (AGAIN)

|                       | deterministic                   | chance                                 |
|-----------------------|---------------------------------|--|
| perfect information   | chess, checkers,<br>go, othello | backgammon<br>monopoly                 |
| imperfect information | battleships,<br>blind tictactoe | bridge, poker, scrabble<br>nuclear war |

# PERFECT INFORMATION GAMES: ZERO-SUM GAMES

Perfect information games are solvable in a manner similar to fully observable single-agent systems, e.g., using forward search.

If two agents compete, so that a positive reward for one is a negative reward for the other agent, we have a two-agent *zero-sum game*.

The value of a game zero-sum game can be characterized by a single number that one agent is trying to maximize and the other agent is trying to minimize.

This leads to a *minimax strategy*:

- A node is either a MAX node (if it is controlled by the maximising agent),
- or is a MIN node (if it is controlled by the minimising agent).

# MINIMAX SEARCH (R&N 5.2–5.3)

- Minimax algorithm
- $\alpha$ - $\beta$  pruning

# MINIMAX SEARCH FOR ZERO-SUM GAMES

Given two players called MAX and MIN:

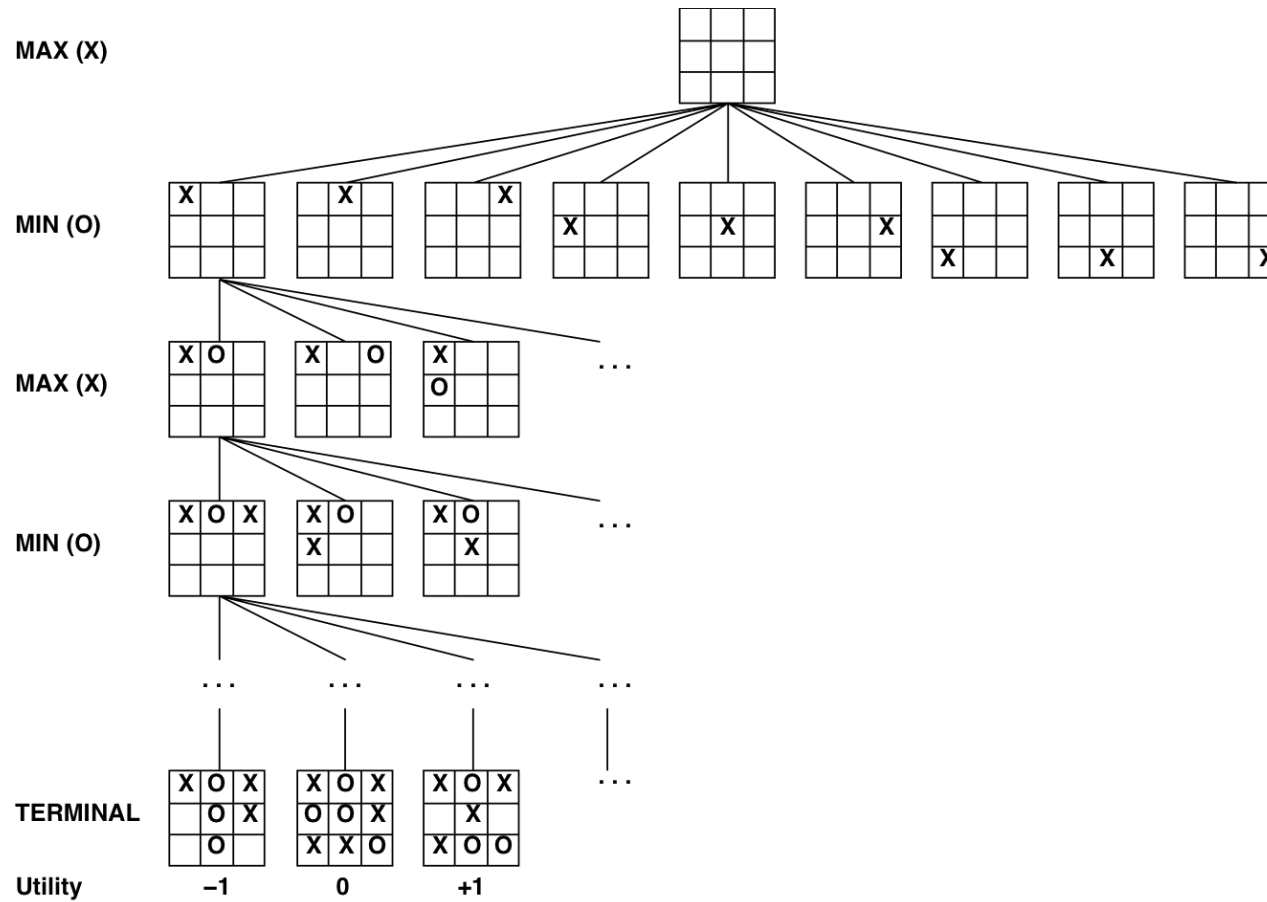
- MAX wants to maximise the utility value,
- MIN wants to minimise the same value.

⇒ MAX should choose the alternative that maximises, assuming MIN minimises.

Minimax gives perfect play for deterministic, perfect-information games:

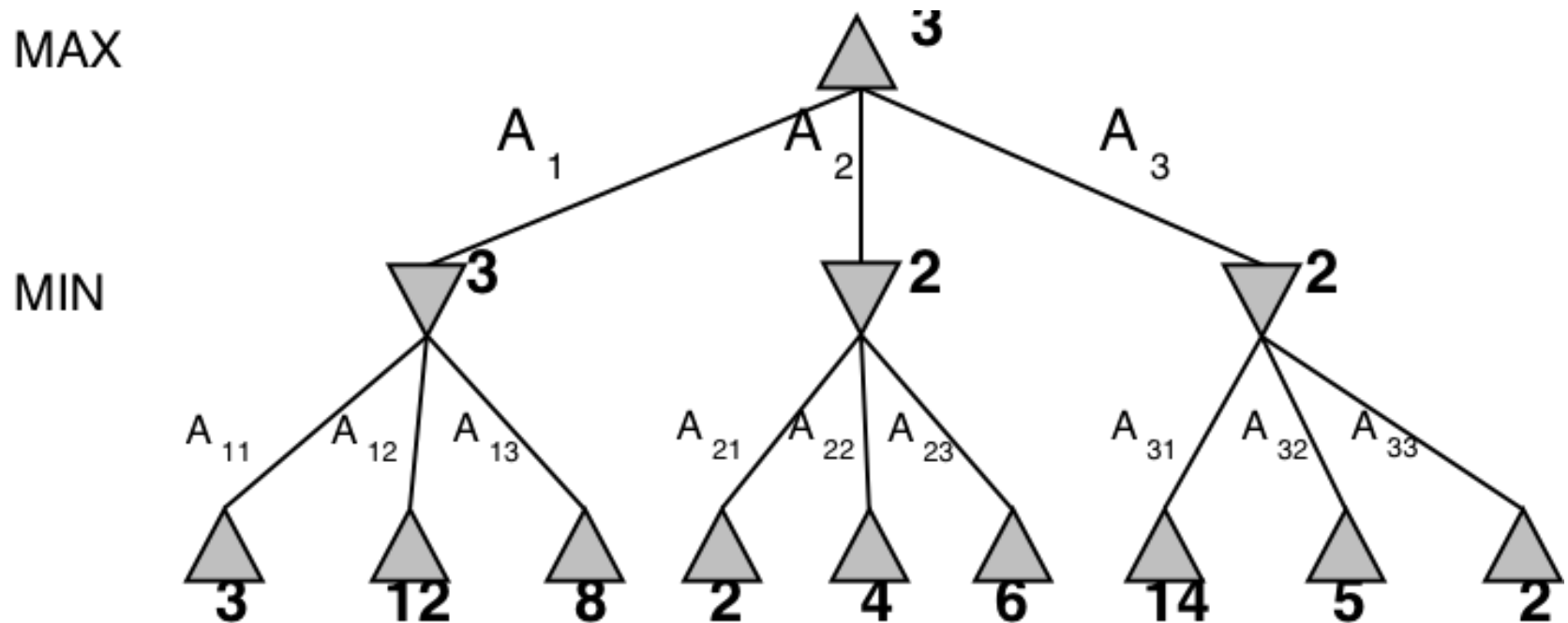
```
function Minimax(state):  
  if TerminalTest(state) then return Utility(state)  
  A := Actions(state)  
  if state is a MAX node then return  $\max_{a \in A}$  Minimax(Result(state, a))  
  if state is a MIN node then return  $\min_{a \in A}$  Minimax(Result(state, a))
```

# MINIMAX SEARCH: TIC-TAC-TOE



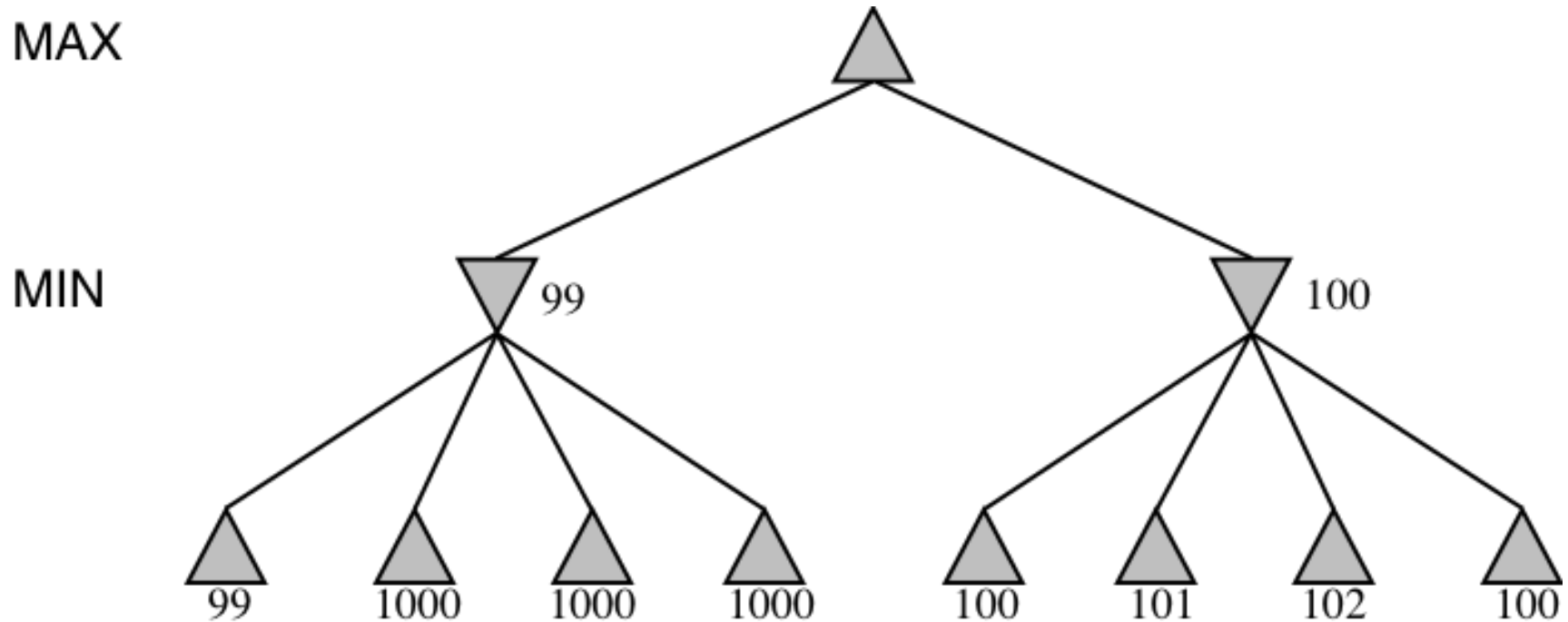
# MINIMAX EXAMPLE

The Minimax algorithm gives perfect play for deterministic, perfect-information games.



# CAN MINIMAX BE WRONG?

Minimax gives perfect play, but is that always the best strategy?



Perfect play assumes that the opponent is also a perfect player!

# 3-PLAYER MINIMAX

*(will not be in the written examination)*

Minimax can also be used on multiplayer games

to move

**A**

( 1, 2, 6) □

**B**

( 1, 2, 6) □

(-1, 5, 2) □

**C**

( 1, 2, 6) □

( 6, 1, 2) □

(-1, 5, 2) □

( 5, 4, 5) □

**A**

□  
( 1, 2, 6)

□  
( 4, 2, 3)

□  
( 6, 1, 2)

□  
( 7, 4, -1)

□  
( 5, -1, -1)

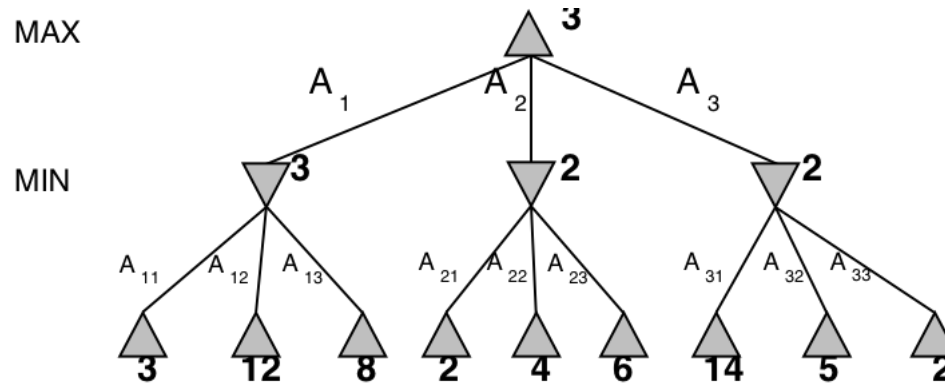
□  
(-1, 5, 2)

□  
( 7, 7, -1)

□  
( 5, 4, 5)



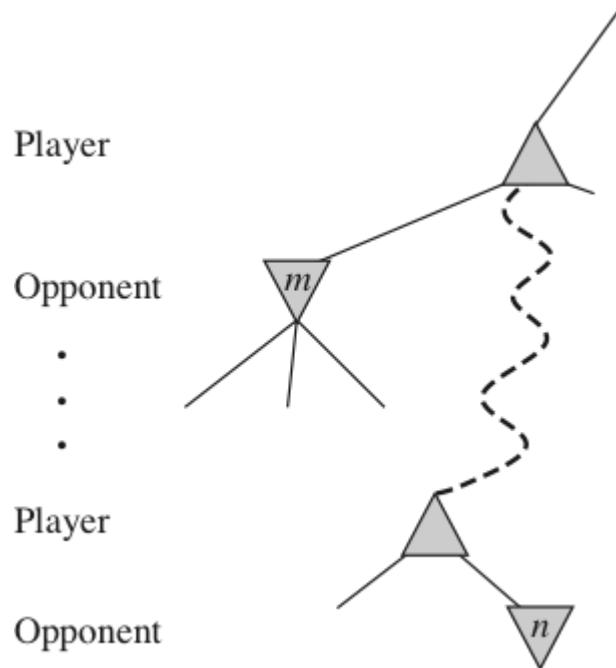
# $\alpha$ - $\beta$ PRUNING



$$\begin{aligned}
 \text{Minimax}(\text{root}) &= \max(\min(3, 12, 8), \min(2, x, y), \min(14, 5, 2)) \\
 &= \max(3, \min(2, x, y), 2) \\
 &= \max(3, z, 2) \text{ where } z = \min(2, x, y) \leq 2 \\
 &= 3
 \end{aligned}$$

I.e., we don't need to know the values of  $x$  and  $y$ !

# $\alpha$ - $\beta$ PRUNING, GENERAL IDEA



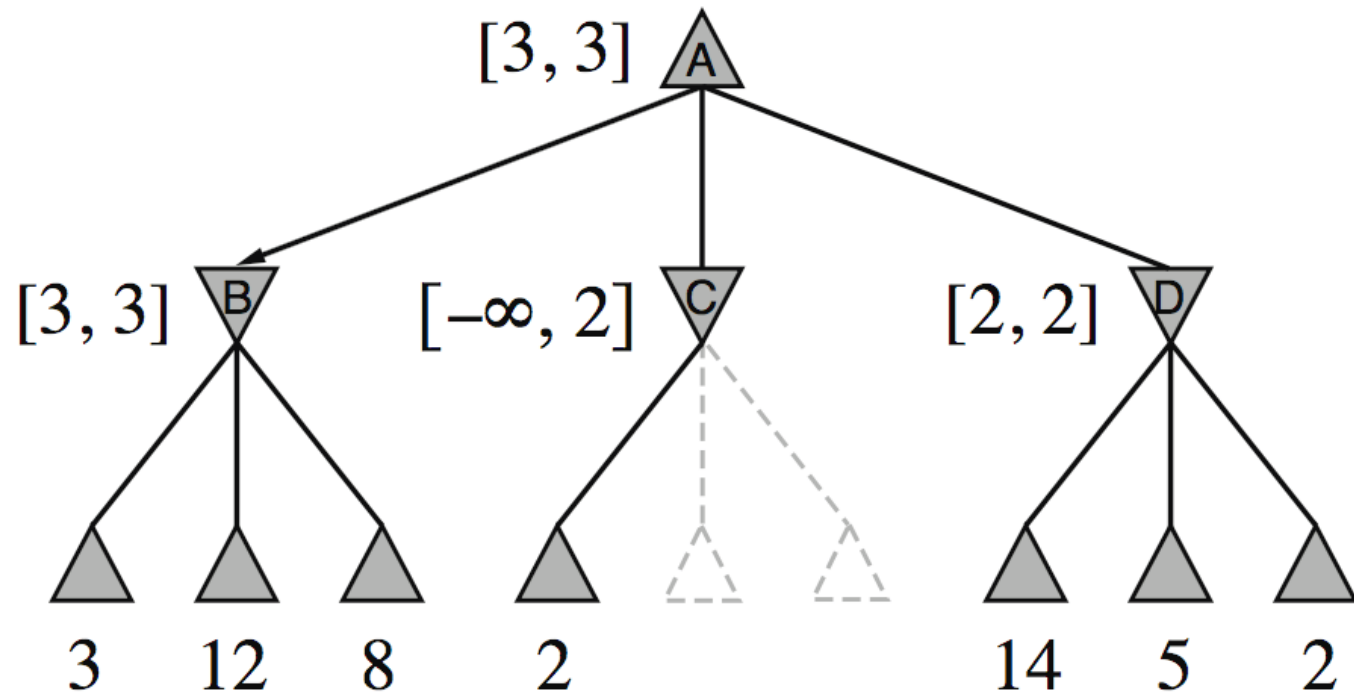
The general idea of  $\alpha$ - $\beta$  pruning is this:

- if  $m$  is better than  $n$  for Player, we don't want to pursue  $n$
- so, once we know enough about  $n$  we can prune it
- sometimes it's enough to examine just one of  $n$ 's descendants

$\alpha$ - $\beta$  pruning keeps track of the possible range of values for every node it visits; the parent range is updated when the child has been visited.

## MINIMAX EXAMPLE, WITH $\alpha$ - $\beta$ PRUNING

(f)



# THE $\alpha$ – $\beta$ ALGORITHM

```
function AlphaBetaSearch(state):  
     $v := \text{MaxValue}(\text{state}, -\infty, +\infty)$   
    return the action in  $\text{Actions}(\text{state})$  that has value  $v$   
  
function MaxValue(state,  $\alpha$ ,  $\beta$ ):  
    if TerminalTest(state) then return Utility(state)  
     $v := -\infty$   
    for each action in  $\text{Actions}(\text{state})$ :  
         $v := \max(v, \text{MinValue}(\text{Result}(\text{state}, \text{action}), \alpha, \beta))$   
        if  $v \geq \beta$  then return  $v$   
         $\alpha := \max(\alpha, v)$   
    return  $v$   
  
function MinValue(state,  $\alpha$ ,  $\beta$ ):  
    same as MaxValue but reverse the roles of  $\alpha/\beta$  and min/max and  $-\infty/+\infty$ 
```

## HOW EFFICIENT IS $\alpha$ — $\beta$ PRUNING?

The amount of pruning provided by the  $\alpha$ - $\beta$  algorithm depends on the ordering of the children of each node.

- It works best if a highest-valued child of a MAX node is selected first and if a lowest-valued child of a MIN node is selected first.
- In real games, much of the effort is made to optimise the search order.
- With a “perfect ordering”, the time complexity becomes  $O(b^{m/2})$ 
  - this doubles the solvable search depth
  - however,  $35^{80/2}$  (for chess) or  $250^{160/2}$  (for go) is still quite large...

# MINIMAX AND REAL GAMES

Most real games are too big to carry out minimax search, even with  $\alpha$ - $\beta$  pruning.

- For these games, instead of stopping at leaf nodes, we have to use a cutoff test to decide when to stop.
- The value returned at the node where the algorithm stops is an estimate of the value for this node.
- The function used to estimate the value is an evaluation function.
- Much work goes into finding good evaluation functions.
- There is a trade-off between the amount of computation required to compute the evaluation function and the size of the search space that can be explored in any given time.

# IMPERFECT DECISIONS (R&N 5.4–5.4.2)

## STOCHASTIC GAMES (R&N 5.5)

*Note: these two sections will be presented Tuesday 6th February!*