CHAPTERS 5, 7: SEARCH PART IV, AND CSP, PART II

DIT411/TIN175, Artificial Intelligence

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REPETITION OF SEARCH

CLASSICAL SEARCH (R&N 3.1–3.6)

Generic search algorithm, tree search, graph search, depth-first search, breadth-first search, uniform cost search, iterative deepending, bidirectional search, greedy best-first search, A* search, heuristics, admissibility, consistency, dominating heuristics, …

NON-CLASSICAL SEARCH (R&N 4.1, 4.3–4.4)

Hill climbing, random moves, random restarts, beam search, nondeterministic actions, contingency plan, and-or search trees, partial observations, belief states, sensor-less problems, …

ADVERSARIAL SEARCH (R&N 5.1–5.4)

Cooperative, competetive, zero-sum games, game trees, minimax, α-β pruning, H-minimax, evaluation function, cutoff test, features, weighted linear function, quiescence search, horizon effect, …

MORE GAMES

STOCHASTIC GAMES (R&N 5.5)

REPETITION: MINIMAX SEARCH FOR ZERO-SUM GAMES

Given two players called MAX and MIN:

- MAX wants to maximise the utility value,
- MIN wants to minimise the same value.

$\Rightarrow$ MAX should choose the alternative that maximises, assuming MIN minimises.

function Minimax(*state*):
    if TerminalTest(*state*) **then return** Utility(*state*)
    A := Actions(*state*)
    if *state* is a MAX node **then return** $\max_{a \in A}$ Minimax(Result(*state*, *a*))
    if *state* is a MIN node **then return** $\min_{a \in A}$ Minimax(Result(*state*, *a*))

REPETITION: H-MINIMAX ALGORITHM

The *Heuristic* Minimax algorithm is similar to normal Minimax

- it replaces **TerminalTest** with **CutoffTest**, and **Utility** with **Eval**
- the cutoff test needs to know the current search depth

function H-Minimax(*state*, *depth*):
    if CutoffTest(*state*, *depth*) **then return** Eval(*state*)
    A := Actions(*state*)
    if *state* is a MAX node **then return** $\max_{a \in A}$ H-Minimax(Result(*state*, *a*), *depth*+1)
    if *state* is a MIN node **then return** $\min_{a \in A}$ H-Minimax(Result(*state*, *a*), *depth*+1)

REPETITION: THE α–β ALGORITHM

function AlphaBetaSearch(*state*):
    v := MaxValue(*state*, $-\infty$, $+\infty$))
    return the *action* in Actions(*state*) that has value v

function MaxValue(*state*, α, β):
    if TerminalTest(*state*) **then return** Utility(*state*)
    $v := -\infty$
    for each *action* in Actions(*state*):
        v := max(v, MinValue(Result(*state*, *action*), α, β))
        if $v \geq \beta$ **then return** v
        α := max(α, v)
    return v

function MinValue(*state*, α, β):
    same as MaxValue but reverse the roles of α/β and *min*/*max* and $-\infty/+\infty$

STOCHASTIC GAMES (R&N 5.5)

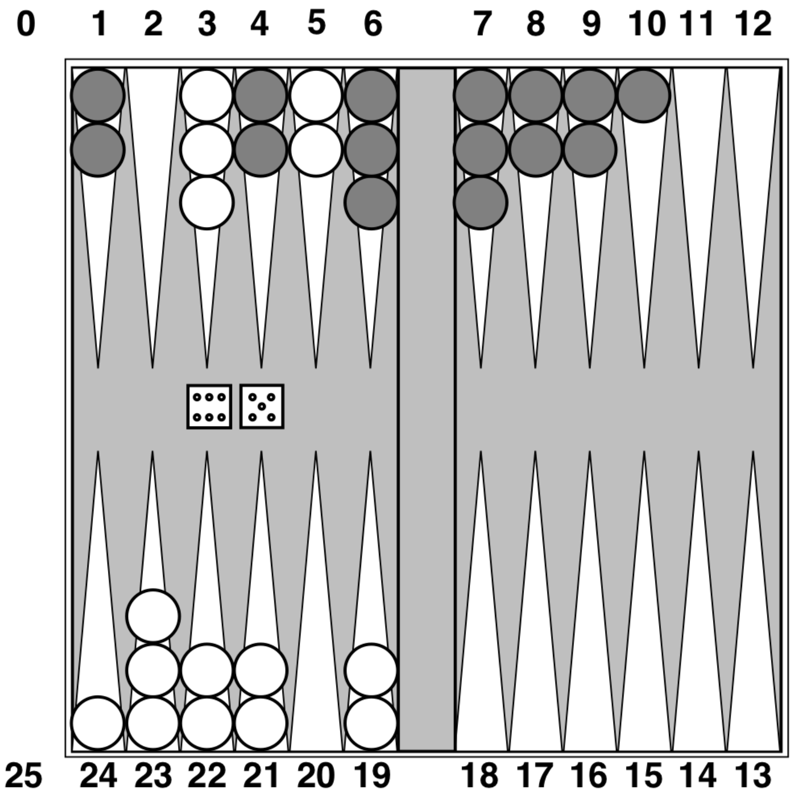
- chance nodes
- expected value
- expecti-minimax algorithm

GAMES OF IMPERFECT INFORMATION

Imperfect information games

- e.g., card games, where the opponent’s initial cards are unknown
- typically we can calculate a probability for each possible deal
- seems just like having one big dice roll at the beginning of the game
- main idea: compute the minimax value of each action in each deal, then choose the action with highest expected value over all deals

STOCHASTIC GAME EXAMPLE: BACKGAMMON

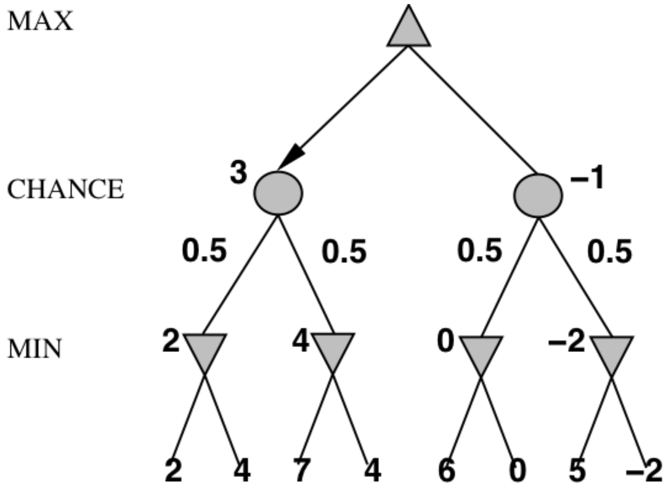


STOCHASTIC GAMES IN GENERAL

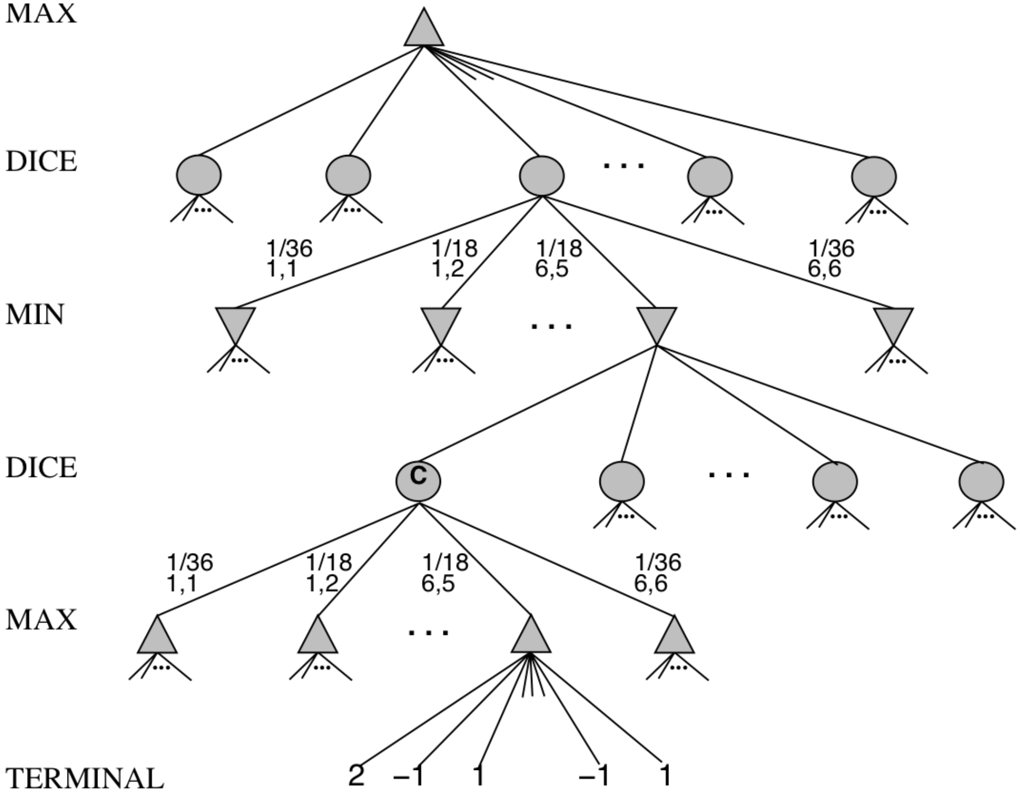
In stochastic games, chance is introduced by dice, card-shuffling, etc.

- We introduce *chance nodes* to the game tree.
- We can't calculate a definite minimax value,
  instead we calculate the *expected value* of a position.
- The expected value is the average of all possible outcomes.

A very simple example with coin-flipping and arbitrary values:



BACKGAMMON GAME TREE



ALGORITHM FOR STOCHASTIC GAMES

The ExpectiMinimax algorithm gives perfect play;
it's just like Minimax, except we must also handle chance nodes:

function ExpectiMinimax(*state*):
    if TerminalTest(*state*) **then return** Utility(*state*)
    A := Actions(*state*)
    if *state* is a MAX node **then return** $\max_{a \in A}$ ExpectiMinimax(Result(*state*, a))
    if *state* is a MAX node **then return** $\min_{a \in A}$ ExpectiMinimax(Result(*state*, a))
    if *state* is a chance node **then return** $\sum_{a \in A} P(a) \cdot$ExpectiMinimax(Result(*state*, a))

where $P(a)$ is the probability that action a occurs.

STOCHASTIC GAMES IN PRACTICE

Dice rolls increase the branching factor:

- there are 21 possible rolls with 2 dice

Backgammon has ≈20 legal moves:

- depth $4 \Rightarrow 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$ nodes

As depth increases, the probability of reaching a given node shrinks:

- value of lookahead is diminished
- α-β pruning is much less effective

TD-Gammon (1995) used depth-2 search + very good Eval function:

- the evaluation function was learned by self-play
- world-champion level

REPETITION OF CSP

CONSTRAINT SATISFACTION PROBLEMS (R&N 7.1)

Variables, domains, constraints (unary, binary, n-ary), constraint graph

CSP AS A SEARCH PROBLEM (R&N 7.3–7.3.2)

Backtracking search, heuristics (minimum remaining values, degree, least constraining value), forward checking, maintaining arc-consistency (MAC)

CONSTRAINT PROPAGATION (R&N 7.2–7..2.2)

Consistency (node, arc, path, k, …), global constratints, the AC-3 algorithm

CSP: CONSTRAINT SATISFACTION PROBLEMS (R&N 7.1)

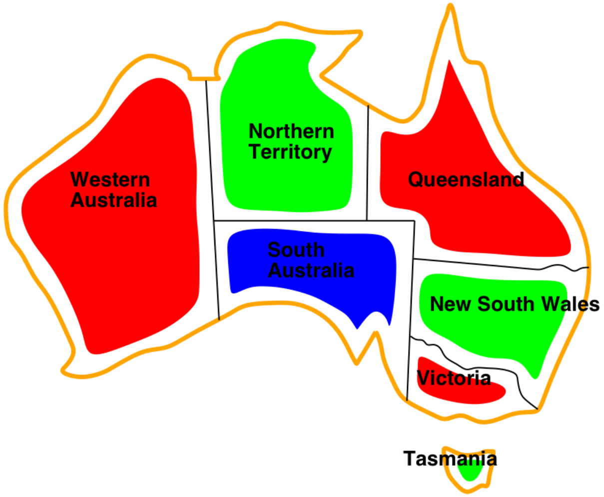
CSP is a specific kind of search problem:

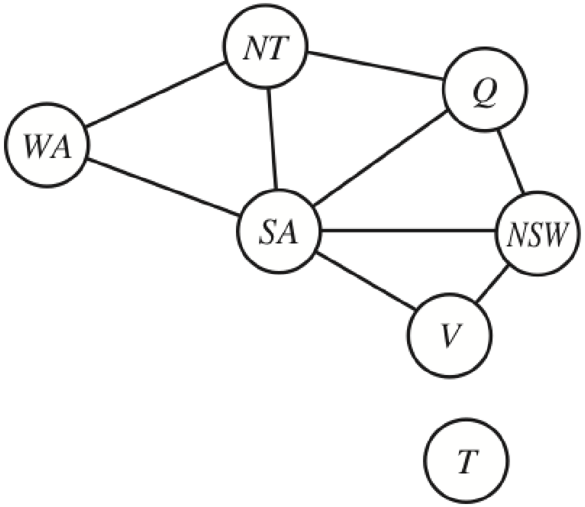
- the *state* is defined by *variables* X_i, each taking values from the domain D_i
- the *goal test* is a set of *constraints*:
  - each constraint specifies allowed values for a subset of variables
  - all constraints must be satisfied

Differences to general search problems:

- the path to a goal isn't important, only the solution is.
- there are no predefined starting state
- often these problems are huge, with thousands of variables, so systematically searching the space is infeasible

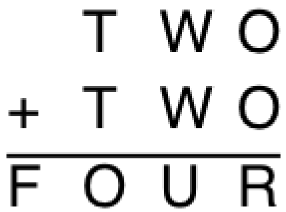
EXAMPLE: MAP COLOURING (BINARY CSP)

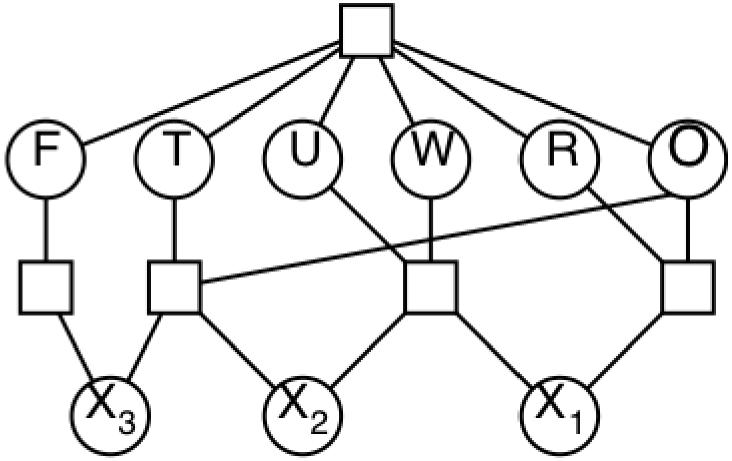




Variables:	*WA*, *NT*, *Q*, *NSW*, *V*, *SA*, *T*
Domains:	D_i = {red, green, blue}
Constraints:	*SA≠WA, SA≠NT, SA≠Q, SA≠NSW, SA≠V,* *WA≠NT, NT≠Q, Q≠NSW, NSW≠V*
Constraint graph:	Every variable is a node, every binary constraint is an arc.

EXAMPLE: CRYPTARITHMETIC PUZZLE (HIGHER-ORDER CSP)





Variables:	*F, T, U, W, R, O,* X_1, X_2, X_3
Domains:	D_i = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
Constraints:	*Alldiff*(*F,T,U,W,R,O*), *O*+*O*=*R*+10·X_1, etc.
Constraint graph:	This is not a binary CSP! The graph is a *constraint hypergraph*.

CSP AS A SEARCH PROBLEM (R&N 7.3–7.3.2)

- backtracking search
- select variable: minimum remaining values, degree heuristic
- order domain values: least constraining value
- inference: forward checking and arc consistency

ALGORITHM FOR BACKTRACKING SEARCH

At each depth level, decide on one single variable to assign:

- this gives branching factor $b = d$, so there are d^n leaves

Depth-first search with single-variable assignments is called *backtracking search*:

```
function BacktrackingSearch(csp):
    return Backtrack(csp, { })

function Backtrack(csp, assignment):
    if assignment is complete then return assignment
    var := SelectUnassignedVariable(csp, assignment)
    for each value in OrderDomainValues(csp, var, assignment):
        if value is consistent with assignment:
            inferences := Inference(csp, var, value)
            if inferences ≠ failure:
                result := Backtrack(csp, assignment ∪ {var=value} ∪ inferences)
                if result ≠ failure then return result
    return failure
```

IMPROVING BACKTRACKING EFFICIENCY

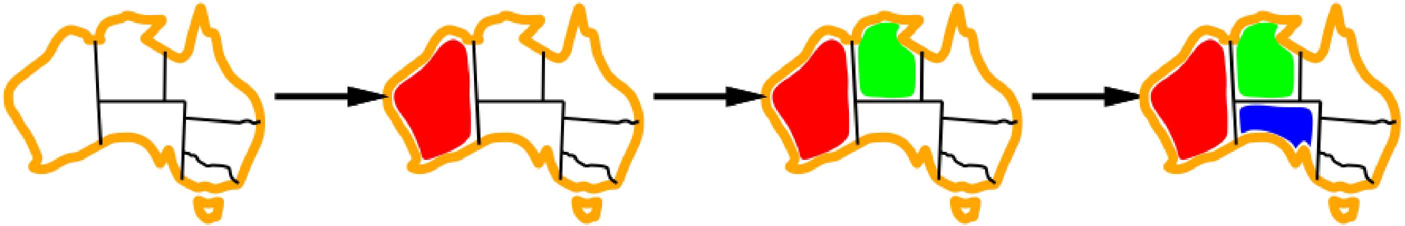
The general-purpose algorithm gives rise to several questions:

- Which variable should be assigned next?
  - $\text{SelectUnassignedVariable}(csp, assignment)$
- In what order should its values be tried?
  - $\text{OrderDomainValues}(csp, var, assignment)$
- What inferences should be performed at each step?
  - $\text{Inference}(csp, var, value)$
- Can the search avoid repeating failures?
  - Conflict-directed backjumping, constraint learning, no-good sets (R&N 7.3.3, not covered in this course)

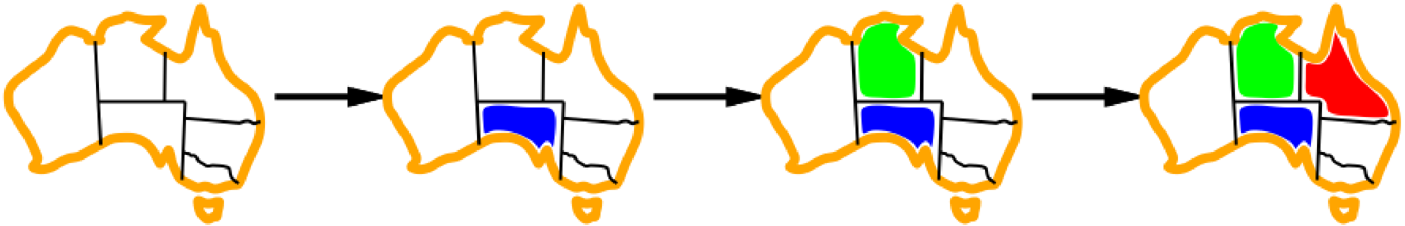
SELECTING UNASSIGNED VARIABLES

Heuristics for selecting the next unassigned variable:

- Minimum remaining values (MRV):
  $\Longrightarrow$ choose the variable with the fewest legal values



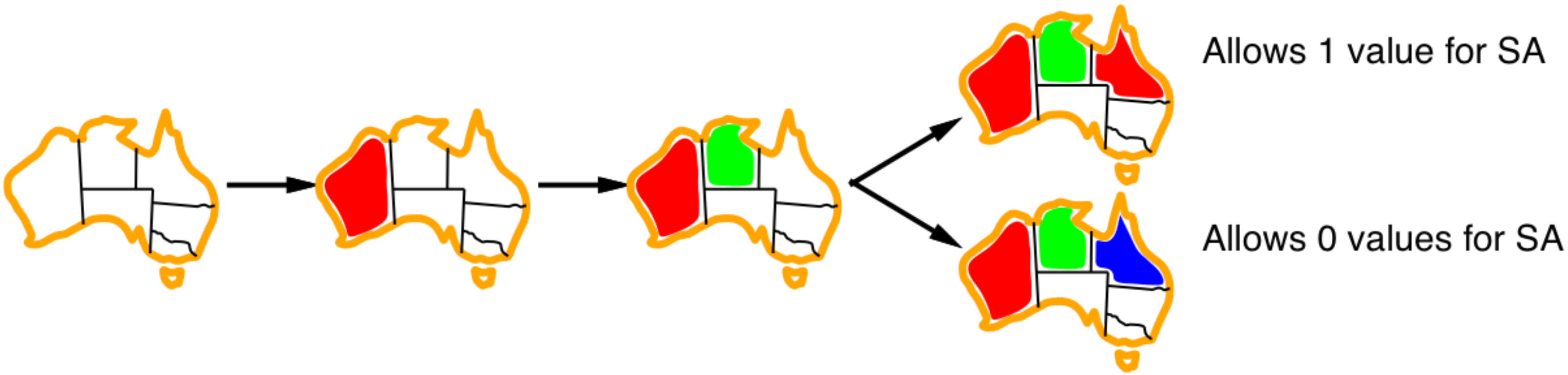
- Degree heuristic (if there are several MRV variables):
  $\Longrightarrow$ choose the variable with most constraints on remaining variables



ORDERING DOMAIN VALUES

Heuristics for ordering the values of a selected variable:

- Least constraining value:
  $\Longrightarrow$ prefer the value that rules out the fewest choices for the neighboring variables in the constraint graph



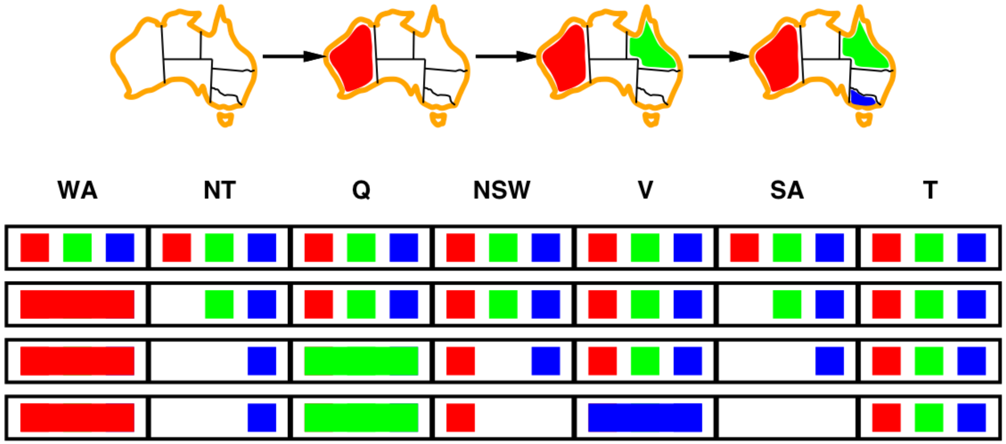
CONSTRAINT PROPAGATION (R&N 7.2–7.2.2)

- consistency (node, arc, path, k, …)
- global constratints
- the AC-3 algorithm
- maintaining arc consistency

INFERENCE: FORWARD CHECKING AND ARC CONSISTENCY

Forward checking is a simple form of inference:

- Keep track of remaining legal values for unassigned variables
- When a new variable is assigned, recalculate the legal values for its neighbors



Arc consistency: $X \rightarrow Y$ is ac iff for every x in X, there is some allowed y in Y

- since NT and SA cannot both be blue, the problem becomes arc inconsistent before forward checking notices
- arc consistency detects failure earlier than forward checking

ARC CONSISTENCY ALGORITHM, AC-3

Keep a set of arcs to be considered: pick one arc (X, Y) at the time and make it consistent (i.e., make X arc consistent to Y).

- Start with the set of all arcs $\{(X, Y), (Y, X), (X, Z), (Z, X), \ldots\}$.

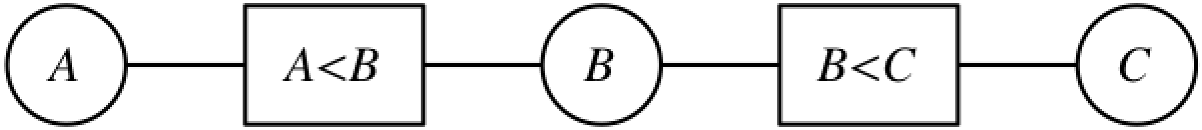
When an arc has been made arc consistent, does it ever need to be checked again?

- An arc (Z, X) needs to be revisited if the domain of X is revised.

```
function AC-3(inout csp):
    initialise queue to all arcs in csp
    while queue is not empty:
        (X, Y) := RemoveOne(queue)
        if Revise(csp, X, Y):
            if  D_X = ∅  then return failure
            for each Z in X.neighbors–{Y} do add (Z, X) to queue

function Revise(inout csp, X, Y):
    delete every x from D_X such that there is no value y in D_Y satisfying the constraint C_XY
    return true if D_X was revised
```

AC-3 EXAMPLE



remove	D_A	D_B	D_C	add	queue
	1234	1234	1234		*A<B, B<C, C>B, B>A*
A<B	***123***	1234	1234		*B<C, C>B, B>A*
B<C	123	***123***	1234	*A<B*	*C>B, B>A,* ***A<B***
C>B	123	123	***234***		*B>A, A<B*
B>A	123	***23***	234	*C>B*	*A<B,* ***C>B***
A<B	***12***	23	234		*C>B*
C>B	12	23	***34***		$\emptyset$

COMBINING BACKTRACKING WITH AC-3

What if some domains have more than one element after AC?

We can resort to backtracking search:

- Select a variable and a value using some heuristics
  (e.g., minimum-remaining-values, degree-heuristic, least-constraining-value)
- Make the graph arc-consistent again
- Backtrack and try new values/variables, if AC fails
- Select a new variable/value, perform arc-consistency, etc.

Do we need to restart AC from scratch?

- no, only some arcs risk becoming inconsistent after a new assignment
- restart AC with the queue $\{(Y_i, X) \mid X \rightarrow Y_i\}$,
  i.e., only the arcs (Y_i, X) where Y_i are the neighbors of X
- this algorithm is called *Maintaining Arc Consistency* (MAC)

CONSISTENCY PROPERTIES

There are several kinds of consistency properties and algorithms:

- *Node consistency*: single variable, unary constraints (straightforward)
- *Arc consistency*: pairs of variables, binary constraints (AC-3 algorithm)
- *Path consistency*: triples of variables, binary constraints (PC-2 algorithm)
- *k-consistency*: k variables, k-ary constraints (algorithms exponential in k)
- Consistency for global constraints:
  Special-purpose algorithms for different constraints, e.g.:
    - *Alldiff($X_1, \ldots, X_m$)* is inconsistent if $m > |D_1 \cup \cdots \cup D_m|$
    - *Atmost($n, X_1, \ldots, X_m$)* is inconsistent if $n < \sum_i \min(D_i)$

MORE ABOUT CSP

LOCAL SEARCH FOR CSP (R&N 7.4)

PROBLEM STRUCTURE (R&N 7.5)

LOCAL SEARCH FOR CSP (R&N 7.4)

Given an assignment of a value to each variable:

- A conflict is an unsatisfied constraint.
- The goal is an assignment with zero conflicts.

Local search / Greedy descent algorithm:

- Start with a complete assignment.
- Repeat until a satisfying assignment is found:
  - select a variable to change
  - select a new value for that variable

MIN CONFLICTS ALGORITHM

Heuristic function to be minimised: the number of conflicts.

- this is the *min-conflicts* heuristics

Note: this does not always work!

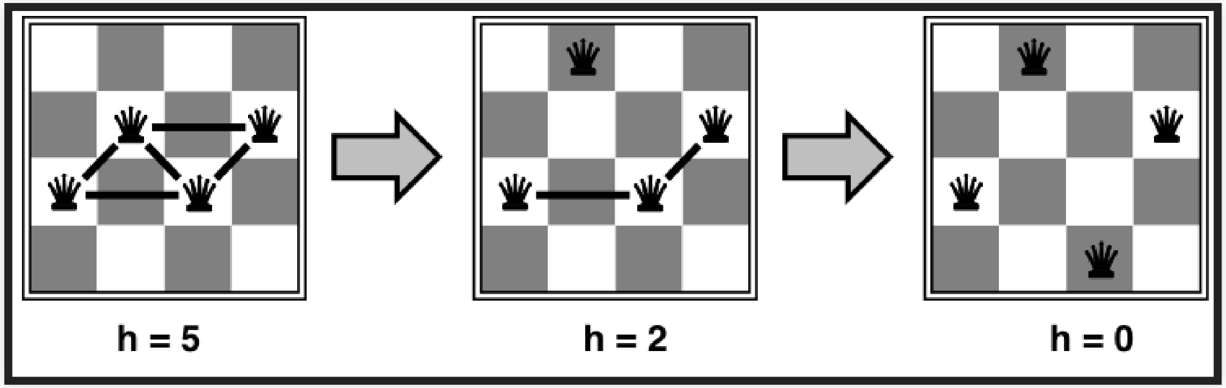
- it can get stuck in a *local minimum*

function MinConflicts(*csp*, *max_steps*)
    current := an initial complete assignment for *csp*
    repeat *max_steps* times:
        if *current* is a solution for csp **then return** *current*
        var := a randomly chosen conflicted variable from *csp*
        value := the value *v* for *var* that minimises Conflicts(*var*, *v*, *current*, *csp*)
        current[*var*] = *value*
    return failure

EXAMPLE: n-QUEENS (REVISITED)

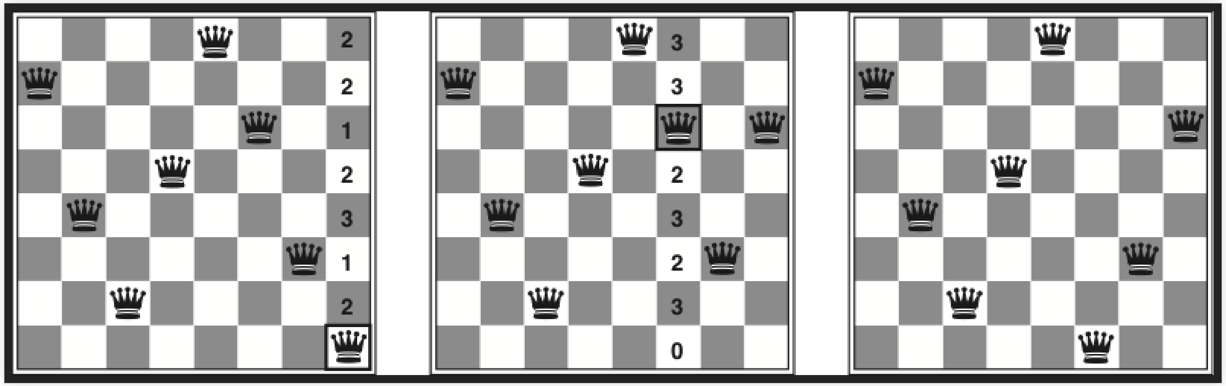
Do you remember this example?

- Put n queens on an $n \times n$ board, in separate columns
- Conflicts = unsatisfied constraints = n:o of threatened queens
- Move a queen to reduce the number of conflicts
  - repeat until we cannot move any queen anymore
  - then we are at a local maximum — hopefully it is global too



EASY AND HARD PROBLEMS

Two-step solution using min-conflicts for an 8-queens problem:



The runtime of min-conflicts on *n-queens* is *independent of problem size*!

- it solves even the *million*-queens problem ≈50 steps

Why is *n*-queens easy for local search?

- because solutions are *densely distributed* throughout the state space!

VARIANTS OF GREEDY DESCENT

To choose a variable to change and a new value for it:

- Find a variable-value pair that minimises the number of conflicts.
- Select a variable that participates in the most conflicts.
  Select a value that minimises the number of conflicts.
- Select a variable that appears in any conflict.
  Select a value that minimises the number of conflicts.
- Select a variable at random.
  Select a value that minimises the number of conflicts.
- Select a variable and value at random;
  accept this change if it doesn't increase the number of conflicts.

All local search techniques from section 4.1 can be applied to CSPs, e.g.:

- random walk, random restarts, simulated annealing, beam search, …

PROBLEM STRUCTURE (R&N 7.5)

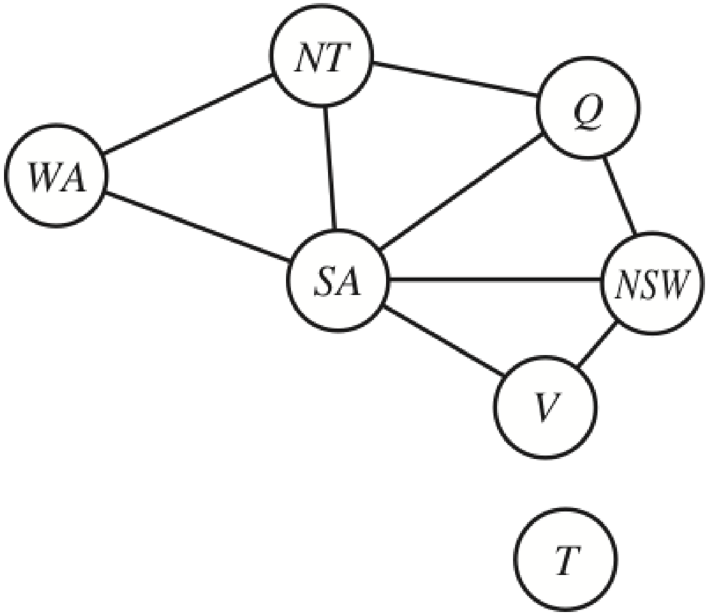
(will not be in the written examination)

- independent subproblems, connected components
- tree-structured CSP, topological sort
- converting to tree-structured CSP, cycle cutset, tree decomposition

INDEPENDENT SUBPROBLEMS

Tasmania is an *independent subproblem*:

- there are efficient algorithms for finding *connected components* in a graph



Suppose that each subproblem has c variables out of n total. The cost of the worst-case solution is $n/c \cdot d^c$, which is linear in n.

E.g., $n = 80, d = 2, c = 20$:

- 2^{80} = 4 billion years at 10 million nodes/sec

If we divide it into 4 equal-size subproblems:

- $4 \cdot 2^{20}$ =0.4 seconds at 10 million nodes/sec

Note: this only has a real effect if the subproblems are (roughly) equal size!

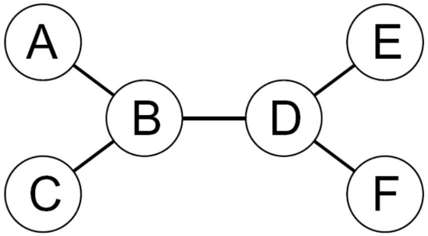
TREE-STRUCTURED CSP

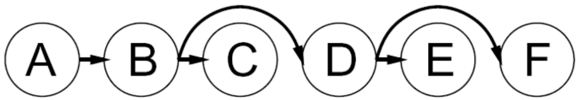
A constraint graph is a tree when any two variables are connected by only one path.

- then any variable can act as root in the tree
- tree-structured CSP can be solved in *linear time*, in the number of variables!

To solve a tree-structured CSP:

- first pick a variable to be the root of the tree
- then find a *topological sort* of the variables (with the root first)
- finally, make each arc consistent, in reverse topological order





SOLVING TREE-STRUCTURED CSP

function TreeCSPSolver(*csp*)
  n := number of variables in *csp*
  root := any variable in *csp*
  $X_1 \ldots X_n$:= TopologicalSort(*csp*, *root*)
  for $j := n, n–1, \ldots, 2$:
    MakeArcConsistent(Parent(X_j), X_j)
    if it could not be made consistent **then return** failure
  assignment := an empty assignment
  for $i := 1, 2, \ldots, n$:
    assignment[X_i] := any consistent value from D_i
  return *assignment*

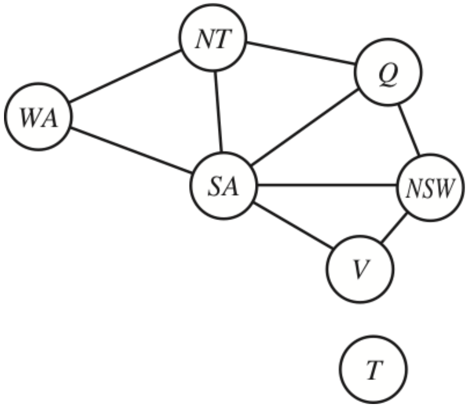
What is the runtime?

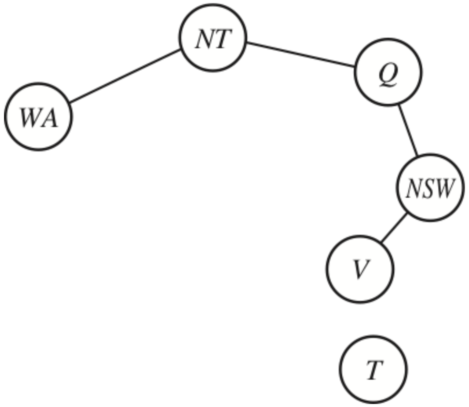
- to make an arc consistent, we must compare up to d^2 domain value pairs
- there are $n-1$ arcs, so the total runtime is $O(nd^2)$

CONVERTING TO TREE-STRUCTURED CSP

Most CSPs are *not* tree-structured, but sometimes we can reduce them to a tree

- one approach is to assign values to some variables,
  so that the remaining variables form a tree





If we assign a colour to South Australia, then the remaining variables form a tree

A (worse) alternative is to assign values to {*NT,Q,V*}

- Why is {*NT,Q,V*} a worse alternative?
- Because then we have to try 3×3×3 different assignments,
  and for each of them solve the remaining tree-CSP

SOLVING ALMOST-TREE-STRUCTURED CSP

function SolveByReducingToTreeCSP(*csp*):
  S := a cycle cutset of variables, such that *csp*–*S* becomes a tree
  for each assignment for *S* that satisfies all constraints on *S*:
    remove any inconsistent values from neighboring variables of *S*
    solve the remaining tree-CSP (i.e., *csp*–*S*)
    if there is a solution **then return** it together with the assignment for *S*
  return failure

The set of variables that we have to assign is called a *cycle cutset*

- for Australia, {*SA*} is a cycle cutset and {*NT,Q,V*} is also a cycle cutset
- finding the smallest cycle cutset is NP-hard,
  but there are efficient approximation algorithms