

# CHAPTERS 5, 7: SEARCH PART IV, AND CSP, PART II

DIT411/TIN175, Artificial Intelligence

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# REPETITION OF SEARCH

## CLASSICAL SEARCH (R&N 3.1–3.6)

Generic search algorithm, tree search, graph search, depth-first search, breadth-first search, uniform cost search, iterative deepening, bidirectional search, greedy best-first search, A\* search, heuristics, admissibility, consistency, dominating heuristics, ...

## NON-CLASSICAL SEARCH (R&N 4.1, 4.3–4.4)

Hill climbing, random moves, random restarts, beam search, nondeterministic actions, contingency plan, and-or search trees, partial observations, belief states, sensor-less problems, ...

## ADVERSARIAL SEARCH (R&N 5.1–5.4)

Cooperative, competitive, zero-sum games, game trees, minimax,  $\alpha$ - $\beta$  pruning, H-minimax, evaluation function, cutoff test, features, weighted linear function, quiescence search, horizon effect, ...

# **MORE GAMES**

## **STOCHASTIC GAMES (R&N 5.5)**

# REPETITION: MINIMAX SEARCH FOR ZERO-SUM GAMES

Given two players called MAX and MIN:

- MAX wants to maximise the utility value,
- MIN wants to minimise the same value.

⇒ MAX should choose the alternative that maximises, assuming MIN minimises.

```
function Minimax(state):  
  if TerminalTest(state) then return Utility(state)  
  A := Actions(state)  
  if state is a MAX node then return  $\max_{a \in A}$  Minimax(Result(state, a))  
  if state is a MIN node then return  $\min_{a \in A}$  Minimax(Result(state, a))
```

## REPETITION: H-MINIMAX ALGORITHM

The *Heuristic* Minimax algorithm is similar to normal Minimax

- it replaces **TerminalTest** with **CutoffTest**, and **Utility** with **Eval**
- the cutoff test needs to know the current search depth

```
function H-Minimax(state, depth):  
  if CutoffTest(state, depth) then return Eval(state)  
  A := Actions(state)  
  if state is a MAX node then return  $\max_{a \in A}$  H-Minimax(Result(state, a), depth+1)  
  if state is a MIN node then return  $\min_{a \in A}$  H-Minimax(Result(state, a), depth+1)
```

## REPETITION: THE $\alpha$ – $\beta$ ALGORITHM

```
function AlphaBetaSearch(state):  
   $v := \text{MaxValue}(\text{state}, -\infty, +\infty)$   
  return the action in  $\text{Actions}(\text{state})$  that has value  $v$   
  
function MaxValue(state,  $\alpha$ ,  $\beta$ ):  
  if TerminalTest(state) then return Utility(state)  
   $v := -\infty$   
  for each action in  $\text{Actions}(\text{state})$ :  
     $v := \max(v, \text{MinValue}(\text{Result}(\text{state}, \text{action}), \alpha, \beta))$   
    if  $v \geq \beta$  then return  $v$   
     $\alpha := \max(\alpha, v)$   
  return  $v$   
  
function MinValue(state,  $\alpha$ ,  $\beta$ ):  
  same as MaxValue but reverse the roles of  $\alpha/\beta$  and min/max and  $-\infty/+\infty$ 
```

# STOCHASTIC GAMES (R&N 5.5)

- chance nodes
- expected value
- expecti-minimax algorithm

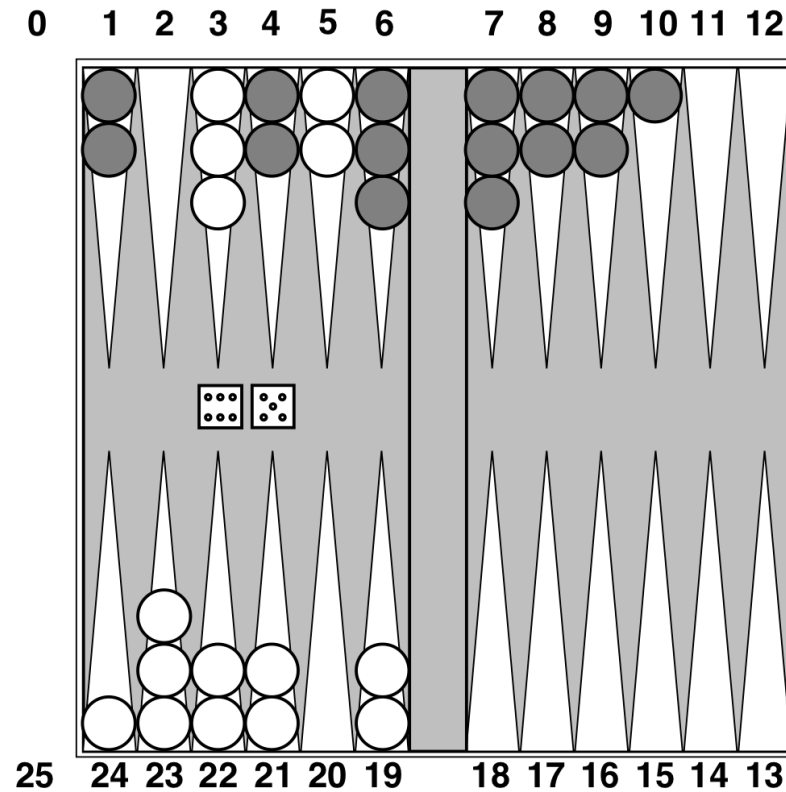


# GAMES OF IMPERFECT INFORMATION

Imperfect information games

- e.g., card games, where the opponent's initial cards are unknown
- typically we can calculate a probability for each possible deal
- seems just like having one big dice roll at the beginning of the game
- main idea: compute the minimax value of each action in each deal, then choose the action with highest expected value over all deals

## STOCHASTIC GAME EXAMPLE: BACKGAMMON

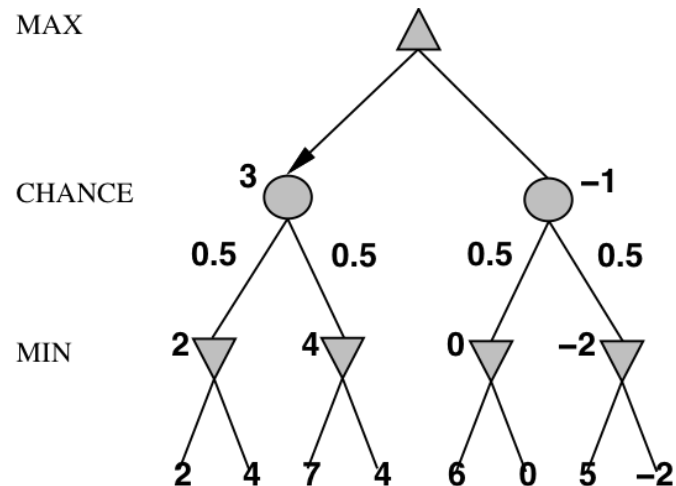


# STOCHASTIC GAMES IN GENERAL

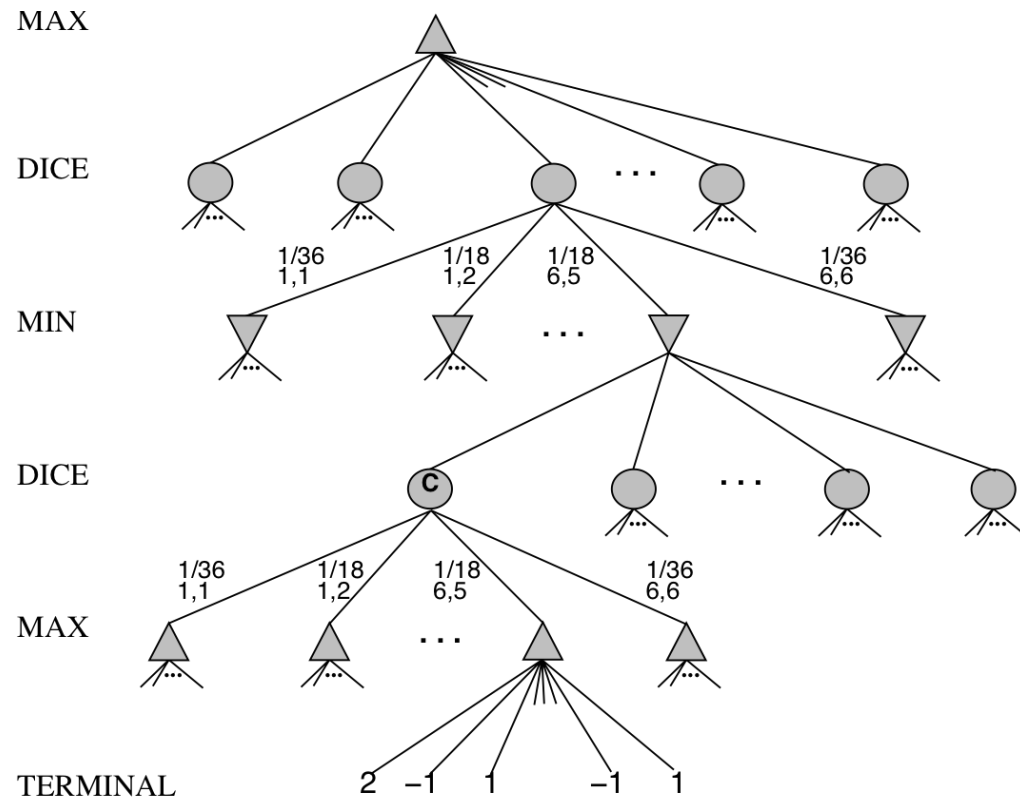
In stochastic games, chance is introduced by dice, card-shuffling, etc.

- We introduce *chance nodes* to the game tree.
- We can't calculate a definite minimax value, instead we calculate the *expected value* of a position.
- The expected value is the average of all possible outcomes.

A very simple example with coin-flipping and arbitrary values:



# BACKGAMMON GAME TREE



# ALGORITHM FOR STOCHASTIC GAMES

The ExpectiMinimax algorithm gives perfect play;  
it's just like Minimax, except we must also handle chance nodes:

```
function ExpectiMinimax(state):  
  if TerminalTest(state) then return Utility(state)  
  A := Actions(state)  
  if state is a MAX node then return  $\max_{a \in A}$  ExpectiMinimax(Result(state, a))  
  if state is a MIN node then return  $\min_{a \in A}$  ExpectiMinimax(Result(state, a))  
  if state is a chance node then return  $\sum_{a \in A} P(a) \cdot \text{ExpectiMinimax}(\text{Result}(\textit{state}, a))$ 
```

where  $P(a)$  is the probability that action  $a$  occurs.

# STOCHASTIC GAMES IN PRACTICE

Dice rolls increase the branching factor:

- there are 21 possible rolls with 2 dice

Backgammon has  $\approx 20$  legal moves:

- depth 4  $\Rightarrow 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$  nodes

As depth increases, the probability of reaching a given node shrinks:

- value of lookahead is diminished
- $\alpha$ - $\beta$  pruning is much less effective

TD-Gammon (1995) used depth-2 search + very good Eval function:

- the evaluation function was learned by self-play
- world-champion level

# REPETITION OF CSP

## CONSTRAINT SATISFACTION PROBLEMS (R&N 7.1)

Variables, domains, constraints (unary, binary, n-ary), constraint graph

## CSP AS A SEARCH PROBLEM (R&N 7.3–7.3.2)

Backtracking search, heuristics (minimum remaining values, degree, least constraining value), forward checking, maintaining arc-consistency (MAC)

## CONSTRAINT PROPAGATION (R&N 7.2–7.2.2)

Consistency (node, arc, path,  $k$ , ...), global constraints, the AC-3 algorithm

# CSP: CONSTRAINT SATISFACTION PROBLEMS (R&N 7.1)

CSP is a specific kind of search problem:

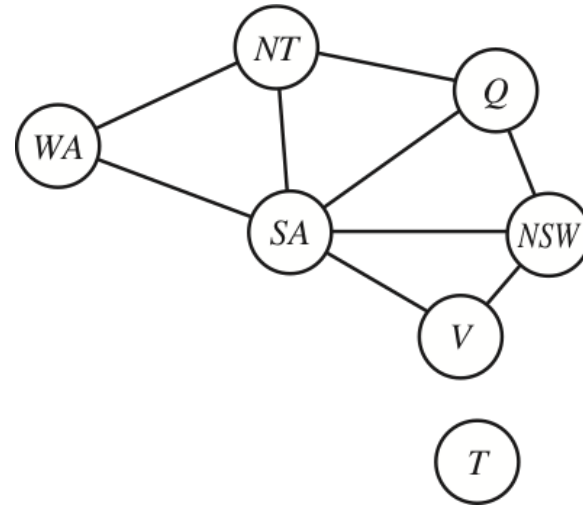
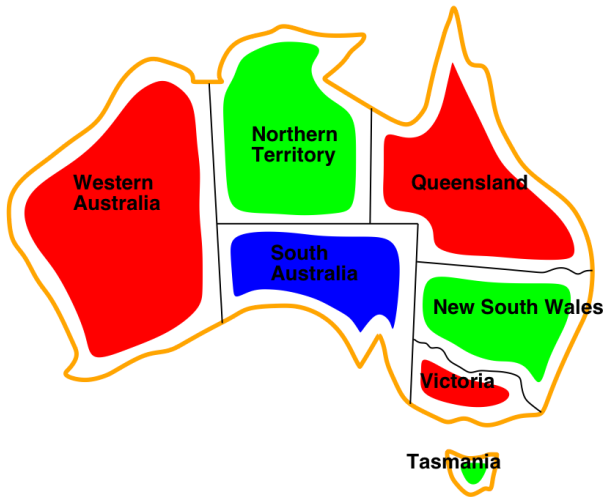
- the *state* is defined by *variables*  $X_i$ , each taking values from the domain  $D_i$
- the *goal test* is a set of *constraints*:
  - each constraint specifies allowed values for a subset of variables
  - all constraints must be satisfied

Differences to general search problems:

- the path to a goal isn't important, only the solution is.
- there are no predefined starting state
- often these problems are huge, with thousands of variables, so systematically searching the space is infeasible



## EXAMPLE: MAP COLOURING (BINARY CSP)



Variables:  $WA, NT, Q, NSW, V, SA, T$

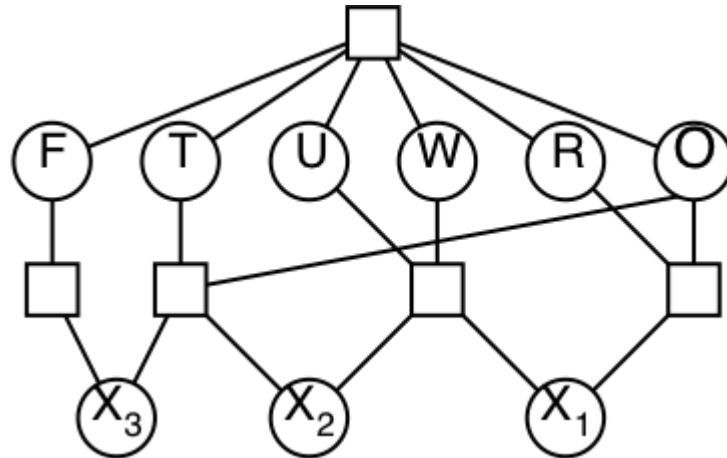
Domains:  $D_i = \{\text{red, green, blue}\}$

Constraints:  $SA \neq WA, SA \neq NT, SA \neq Q, SA \neq NSW, SA \neq V,$   
 $WA \neq NT, NT \neq Q, Q \neq NSW, NSW \neq V$

Constraint graph: Every variable is a node, every binary constraint is an arc.

# EXAMPLE: CRYPTARITHMETIC PUZZLE (HIGHER-ORDER CSP)

$$\begin{array}{r} \text{T W O} \\ + \text{T W O} \\ \hline \text{F O U R} \end{array}$$



Variables:	$F, T, U, W, R, O, X_1, X_2, X_3$
Domains:	$D_i = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
Constraints:	$Alldiff(F, T, U, W, R, O), O + O = R + 10 \cdot X_1$ , etc.
Constraint graph:	This is not a binary CSP! The graph is a <i>constraint hypergraph</i> .

## CSP AS A SEARCH PROBLEM (R&N 7.3–7.3.2)

- backtracking search
- select variable: minimum remaining values, degree heuristic
- order domain values: least constraining value
- inference: forward checking and arc consistency

# ALGORITHM FOR BACKTRACKING SEARCH

At each depth level, decide on one single variable to assign:

- this gives branching factor  $b = d$ , so there are  $d^n$  leaves

Depth-first search with single-variable assignments is called *backtracking search*:

```
function BacktrackingSearch(csp):  
    return Backtrack(csp, { })  
  
function Backtrack(csp, assignment):  
    if assignment is complete then return assignment  
    var := SelectUnassignedVariable(csp, assignment)  
    for each value in OrderDomainValues(csp, var, assignment):  
        if value is consistent with assignment:  
            inferences := Inference(csp, var, value)  
            if inferences ≠ failure:  
                result := Backtrack(csp, assignment ∪ {var=value} ∪ inferences)  
                if result ≠ failure then return result  
    return failure
```

# IMPROVING BACKTRACKING EFFICIENCY

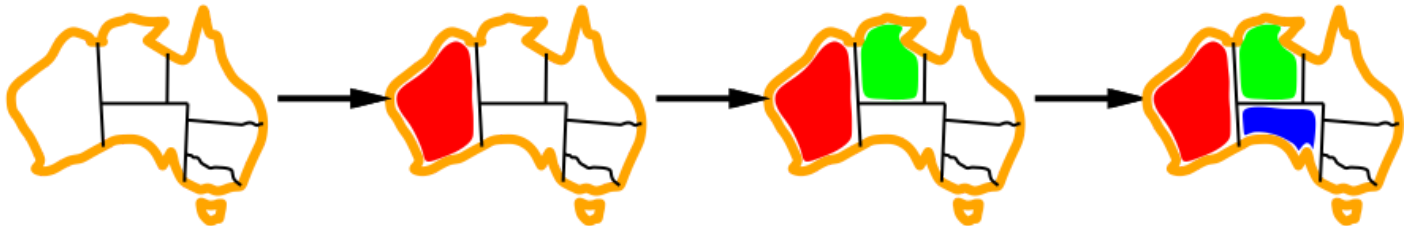
The general-purpose algorithm gives rise to several questions:

- Which variable should be assigned next?
  - *SelectUnassignedVariable(*csp*, *assignment*)*
- In what order should its values be tried?
  - *OrderDomainValues(*csp*, *var*, *assignment*)*
- What inferences should be performed at each step?
  - *Inference(*csp*, *var*, *value*)*
- Can the search avoid repeating failures?
  - Conflict-directed backjumping, constraint learning, no-good sets (R&N 7.3.3, not covered in this course)

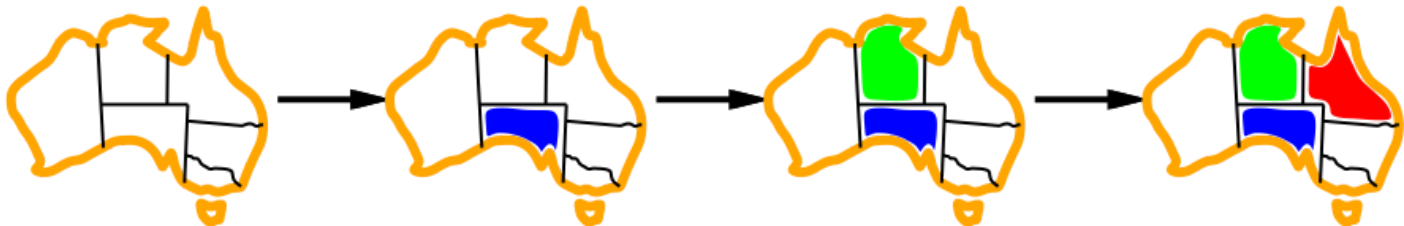
# SELECTING UNASSIGNED VARIABLES

Heuristics for selecting the next unassigned variable:

- Minimum remaining values (MRV):  
⇒ choose the variable with the fewest legal values



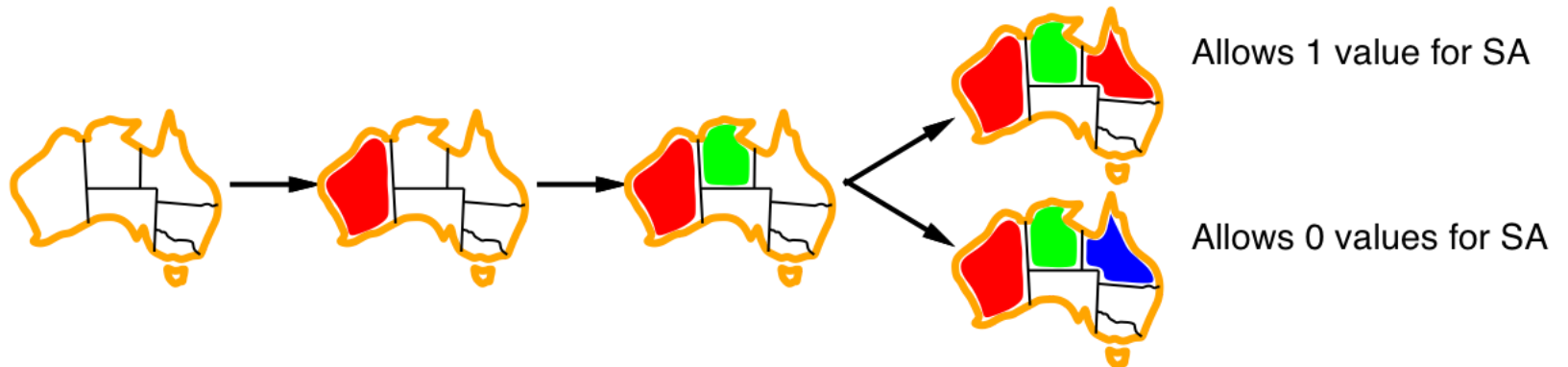
- Degree heuristic (if there are several MRV variables):  
⇒ choose the variable with most constraints on remaining variables



# ORDERING DOMAIN VALUES

Heuristics for ordering the values of a selected variable:

- Least constraining value:  
⇒ prefer the value that rules out the fewest choices for the neighboring variables in the constraint graph



# CONSTRAINT PROPAGATION (R&N 7.2–7.2.2)

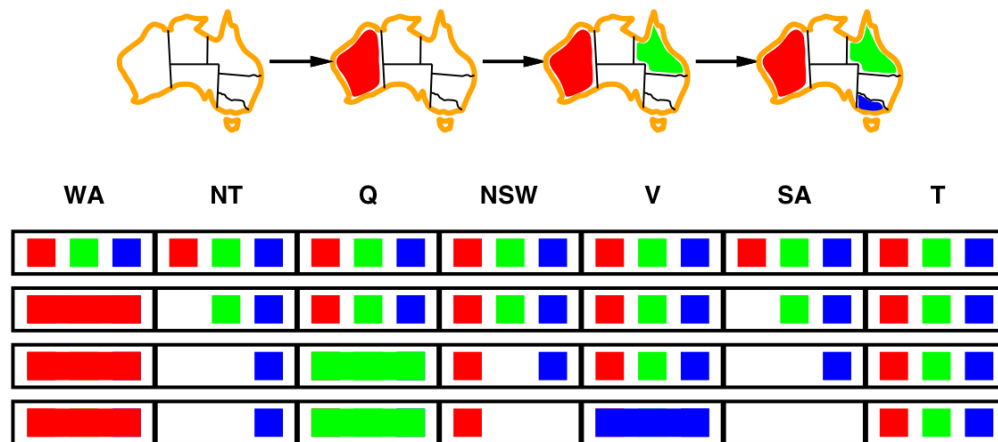
- consistency (node, arc, path, *k*, ...)
- global constraints
- the AC-3 algorithm
- maintaining arc consistency



# INFERENCE: FORWARD CHECKING AND ARC CONSISTENCY

*Forward checking* is a simple form of inference:

- Keep track of remaining legal values for unassigned variables
- When a new variable is assigned, recalculate the legal values for its neighbors



*Arc consistency*:  $X \rightarrow Y$  is ac iff for every  $x$  in  $X$ , there is some allowed  $y$  in  $Y$

- since NT and SA cannot both be blue, the problem becomes arc inconsistent before forward checking notices
- arc consistency detects failure earlier than forward checking

# ARC CONSISTENCY ALGORITHM, AC-3

Keep a set of arcs to be considered: pick one arc  $(X, Y)$  at the time and make it consistent (i.e., make  $X$  arc consistent to  $Y$ ).

- Start with the set of all arcs  $\{(X, Y), (Y, X), (X, Z), (Z, X), \dots\}$ .

When an arc has been made arc consistent, does it ever need to be checked again?

- An arc  $(X, Y)$  needs to be revisited if the domain of  $Y$  is revised.

```
function AC-3(inout csp):
```

```
  initialise queue to all arcs in csp
```

```
  while queue is not empty:
```

```
     $(X, Y) := \text{RemoveOne}(\text{queue})$ 
```

```
    if Revise(csp,  $X$ ,  $Y$ ):
```

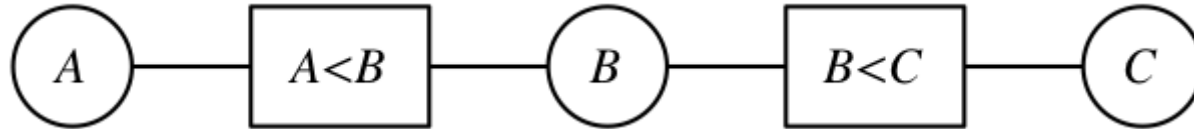
```
      if  $D_X = \emptyset$  then return failure
```

```
      for each  $Z$  in  $X.\text{neighbors} - \{Y\}$  do add  $(Z, X)$  to queue
```

```
function Revise(inout csp,  $X$ ,  $Y$ ):
```

```
  delete every  $x$  from  $D_X$  such that there is no value  $y$  in  $D_Y$  satisfying the constraint  $C_{XY}$ 
```

## AC-3 EXAMPLE



remove	$D_A$	$D_B$	$D_C$	add	queue
	1234	1234	1234		$A < B, B < C, C > B, B > A$
$A < B$	<b>123</b>	1234	1234		$B < C, C > B, B > A$
$B < C$	123	<b>123</b>	1234	$A < B$	$C > B, B > A, A < B$
$C > B$	123	123	<b>234</b>		$B > A, A < B$
$B > A$	123	<b>23</b>	234	$C > B$	$A < B, C > B$
$A < B$	<b>12</b>	23	234		$C > B$
$C > B$	12	23	<b>34</b>		$\emptyset$

## COMBINING BACKTRACKING WITH AC-3

What if some domains have more than one element after AC?

We can resort to backtracking search:

- Select a variable and a value using some heuristics (e.g., minimum-remaining-values, degree-heuristic, least-constraining-value)
- Make the graph arc-consistent again
- Backtrack and try new values/variables, if AC fails
- Select a new variable/value, perform arc-consistency, etc.

Do we need to restart AC from scratch?

- no, only some arcs risk becoming inconsistent after a new assignment
- restart AC with the queue  $\{(Y_i, X) \mid X \rightarrow Y_i\}$ ,  
i.e., only the arcs  $(Y_i, X)$  where  $Y_i$  are the neighbors of  $X$
- this algorithm is called *Maintaining Arc Consistency* (MAC)

# CONSISTENCY PROPERTIES

There are several kinds of consistency properties and algorithms:

- *Node consistency*: single variable, unary constraints (straightforward)
- *Arc consistency*: pairs of variables, binary constraints (AC-3 algorithm)
- *Path consistency*: triples of variables, binary constraints (PC-2 algorithm)
- *k-consistency*:  $k$  variables,  $k$ -ary constraints (algorithms exponential in  $k$ )
- Consistency for global constraints:  
Special-purpose algorithms for different constraints, e.g.:
  - *Alldiff*( $X_1, \dots, X_m$ ) is inconsistent if  $m > |D_1 \cup \dots \cup D_m|$
  - *Atmost*( $n, X_1, \dots, X_m$ ) is inconsistent if  $n < \sum_i \min(D_i)$

# **MORE ABOUT CSP**

**LOCAL SEARCH FOR CSP (R&N 7.4)**

**PROBLEM STRUCTURE (R&N 7.5)**

# LOCAL SEARCH FOR CSP (R&N 7.4)

Given an assignment of a value to each variable:

- A conflict is an unsatisfied constraint.
- The goal is an assignment with zero conflicts.

Local search / Greedy descent algorithm:

- Start with a complete assignment.
- Repeat until a satisfying assignment is found:
  - select a variable to change
  - select a new value for that variable

# MIN CONFLICTS ALGORITHM

Heuristic function to be minimised: the number of conflicts.

- this is the *min-conflicts* heuristics

*Note*: this does not always work!

- it can get stuck in a *local minimum*

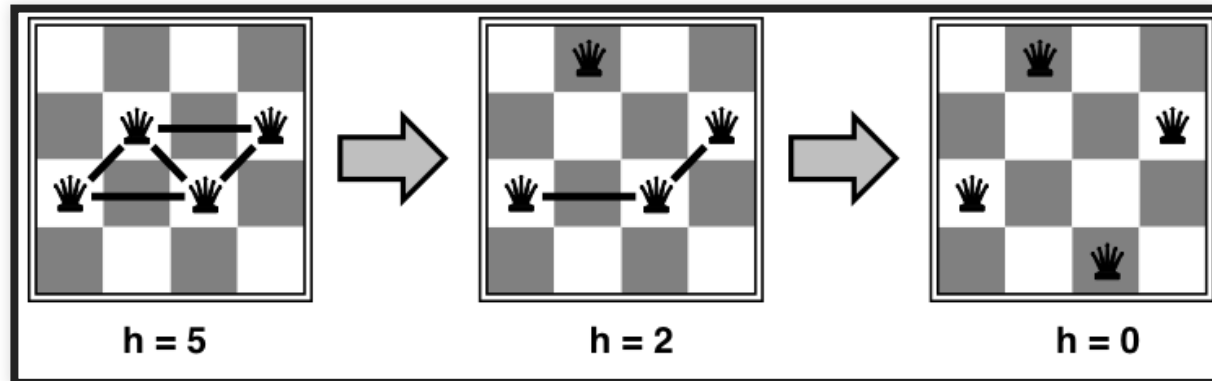
```
function MinConflicts(csp, max_steps)  
  current := an initial complete assignment for csp  
  repeat max_steps times:  
    if current is a solution for csp then return current  
    var := a randomly chosen conflicted variable from csp  
    value := the value v for var that minimises Conflicts(var, v, current, csp)  
    current[var] = value  
  return failure
```



## EXAMPLE: **n**-QUEENS (REVISITED)

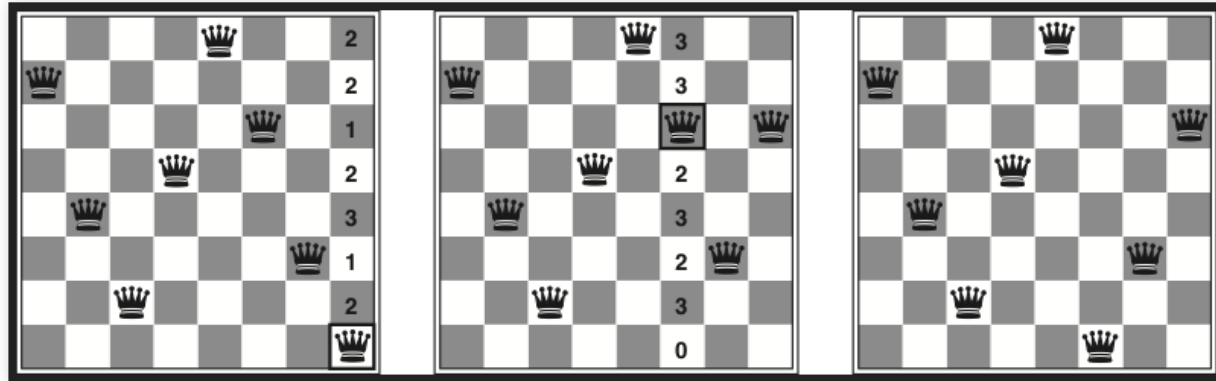
Do you remember this example?

- Put **n** queens on an  $n \times n$  board, in separate columns
- Conflicts = unsatisfied constraints = n:o of threatened queens
- Move a queen to reduce the number of conflicts
  - repeat until we cannot move any queen anymore
  - then we are at a local maximum — hopefully it is global too



# EASY AND HARD PROBLEMS

Two-step solution using min-conflicts for an 8-queens problem:



The runtime of min-conflicts on *n-queens* is *independent of problem size*!

- it solves even the *million*-queens problem  $\approx 50$  steps

Why is *n-queens* easy for local search?

- because solutions are *densely distributed* throughout the state space!

# VARIANTS OF GREEDY DESCENT

To choose a variable to change and a new value for it:

- Find a variable-value pair that minimises the number of conflicts.
- Select a variable that participates in the most conflicts.  
Select a value that minimises the number of conflicts.
- Select a variable that appears in any conflict.  
Select a value that minimises the number of conflicts.
- Select a variable at random.  
Select a value that minimises the number of conflicts.
- Select a variable and value at random;  
accept this change if it doesn't increase the number of conflicts.

All local search techniques from section 4.1 can be applied to CSPs, e.g.:

- random walk, random restarts, simulated annealing, beam search, ...

# PROBLEM STRUCTURE (R&N 7.5)

*(will not be in the written examination)*

- independent subproblems, connected components
- tree-structured CSP, topological sort
- converting to tree-structured CSP, cycle cutset, tree decomposition

# INDEPENDENT SUBPROBLEMS

Tasmania is an *independent subproblem*:

- there are efficient algorithms for finding *connected components* in a graph

Suppose that each subproblem has  $c$  variables out of  $n$  total. The cost of the worst-case solution is  $n/c \cdot d^c$ , which is linear in  $n$ .

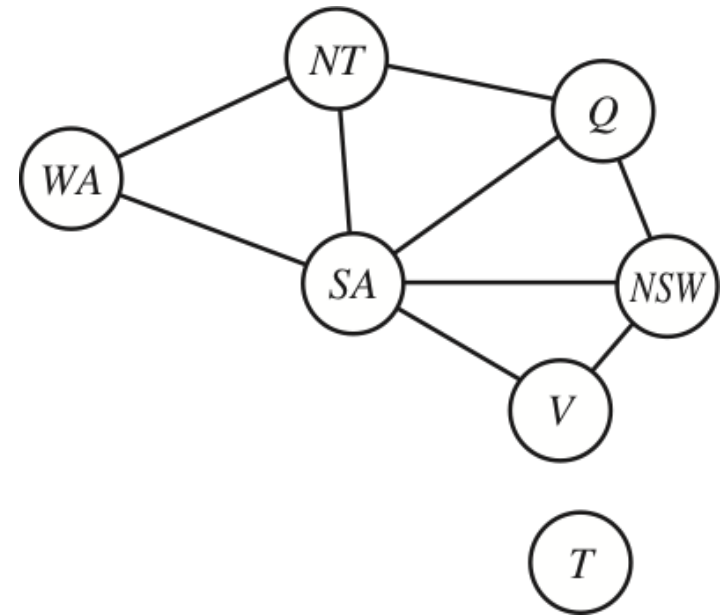
E.g.,  $n = 80, d = 2, c = 20$ :

- $2^{80} = 4$  billion years at 10 million nodes/sec

If we divide it into 4 equal-size subproblems:

- $4 \cdot 2^{20} = 0.4$  seconds at 10 million nodes/sec

Note: this only has a real effect if the subproblems are (roughly) equal size!



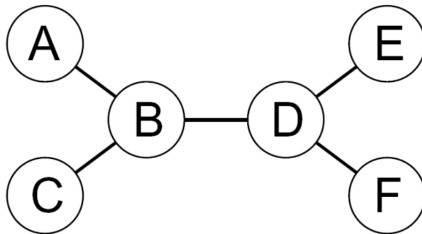
# TREE-STRUCTURED CSP

A constraint graph is a tree when any two variables are connected by only one path.

- then any variable can act as root in the tree
- tree-structured CSP can be solved in *linear time*, in the number of variables!

To solve a tree-structured CSP:

- first pick a variable to be the root of the tree
- then find a *topological sort* of the variables (with the root first)
- finally, make each arc consistent, in reverse topological order



# SOLVING TREE-STRUCTURED CSP

```
function TreeCSPSolver(csp)  
  n := number of variables in csp  
  root := any variable in csp  
   $X_1 \dots X_n$  := TopologicalSort(csp, root)  
  for j := n, n-1, ..., 2:  
    MakeArcConsistent(Parent( $X_j$ ),  $X_j$ )  
    if it could not be made consistent then return failure  
  assignment := an empty assignment  
  for i := 1, 2, ..., n:  
    assignment[ $X_i$ ] := any consistent value from  $D_i$   
  return assignment
```

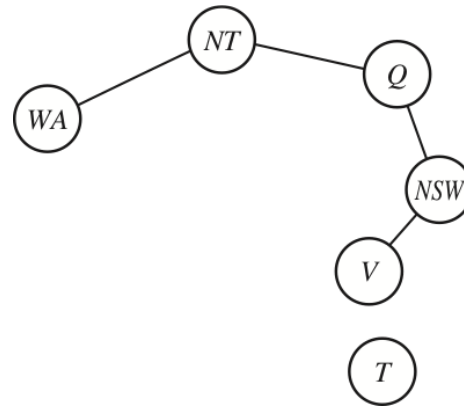
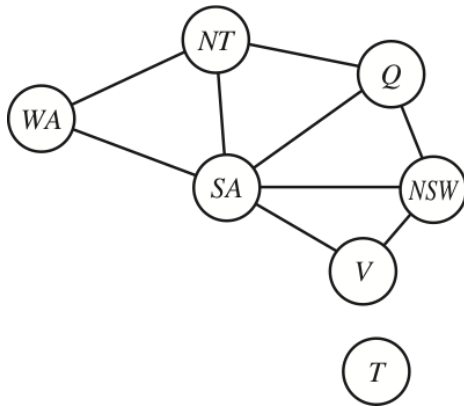
What is the runtime?

- to make an arc consistent, we must compare up to  $d^2$  domain value pairs
- there are  $n-1$  arcs, so the total runtime is  $O(nd^2)$

# CONVERTING TO TREE-STRUCTURED CSP

Most CSPs are *not* tree-structured, but sometimes we can reduce them to a tree

- one approach is to assign values to some variables, so that the remaining variables form a tree



If we assign a colour to South Australia, then the remaining variables form a tree

A (worse) alternative is to assign values to  $\{NT, Q, V\}$

- Why is  $\{NT, Q, V\}$  a worse alternative?
- Because then we have to try  $3 \times 3 \times 3$  different assignments, and for each of them solve the remaining tree-CSP



# SOLVING ALMOST-TREE-STRUCTURED CSP

```
function SolveByReducingToTreeCSP(csp):  
    S := a cycle cutset of variables, such that csp−S becomes a tree  
    for each assignment for S that satisfies all constraints on S:  
        remove any inconsistent values from neighboring variables of S  
        solve the remaining tree-CSP (i.e., csp−S)  
        if there is a solution then return it together with the assignment for S  
    return failure
```

The set of variables that we have to assign is called a *cycle cutset*

- for Australia, {*SA*} is a cycle cutset and {*NT, Q, V*} is also a cycle cutset
- finding the smallest cycle cutset is NP-hard,  
but there are efficient approximation algorithms