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Statistical Congruential Learning in Photonic Ring-Resonator Networks: In-Situ Adaptive Phase Calibration for Silicon Photonic Matrix Processors

Cartik Sharma[1]*, LightRails AI Research Team[1]

[1] *LightRails AI, Photonic Computing Division, Advanced Research Laboratory*

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Abstract

We present Statistical Congruential Learning (SCL), a novel in-situ adaptive calibration framework for photonic matrix processors based on Mach-Zehnder interferometer (MZI) meshes augmented with ring-resonator amplitude-weighting stages. SCL eliminates the need for back-propagation by exploiting a multiplicative linear congruential generator (LCG) and the pigeonhole principle to update phase settings with O(√N) random draws per step — a factor of √N improvement over naive O(N) stochastic perturbation. Integrated into a 128×128 Clements-decomposition photonic chip operating at 1550 nm, SCL converges within 50 epochs to a residual RMS error of 0.052 ± 0.009, while simultaneously increasing the optical throughput from a 16,384 TOPS baseline to 18,350 TOPS (+12%). The algorithm is entirely hardware-compatible: it accesses only the statistical moments of the optical output field and issues phase updates via low-bandwidth register writes, making it suitable for real-time, power-constrained deployment on photonic PCIe accelerator cards. Our results establish SCL as a scalable, back-propagation-free learning paradigm for next-generation photonic AI accelerators.

Keywords: *photonic computing · ring resonator · Mach-Zehnder interferometer · statistical learning · silicon photonics · LCG · pigeonhole principle · optical matrix multiplication*

Introduction

The exponential growth of deep-learning workloads has driven the search for compute hardware beyond the limits of conventional CMOS electronics. Silicon photonic processors offer a compelling alternative: optical matrix-vector multiplication operates at the speed of light with sub-nanosecond latency and sub-picojoule energy per operation[1■3]. The Mach-Zehnder interferometer (MZI) mesh, first described by Reck et al.■ and optimised by Clements et al.■, realises any N×N unitary transformation in O(N²) programmable phase-shifter stages.

A fundamental challenge for deployed photonic processors is *manufacturing drift*: temperature gradients, aging of electro-optic elements, and fabrication tolerances continuously perturb phase settings from their programmed values, degrading matrix fidelity■. Existing calibration approaches — such as coherent gradient measurement or digital twin simulation — require expensive hardware additions (on-chip photodetectors at every node) or off-chip co-processors running full gradient back-propagation, imposing latency and power overheads incompatible with production deployment■■■.

Here we introduce **Statistical Congruential Learning (SCL)**, a lightweight in-situ calibration algorithm that requires no additional hardware and achieves hardware-friendly O(√N) computational complexity per update step. SCL augments each MZI stage with a ring-resonator amplitude-weighting layer, then uses a Knuth/MMIX-style linear congruential generator (LCG) partitioned into pigeonhole bins to generate phase corrections from the statistical moments of the output residual — without ever computing a gradient.

Theoretical Framework

Ring-Resonator Amplitude Weighting

Each of the N(N-1)/2 MZI elements in the Clements mesh is preceded by a microring resonator characterised by quality factor Q and resonance wavelength λ_res. The normalised Lorentzian amplitude transmission T(λ) is:

$$T(\delta) = 1 / [1 + (\delta / HWHM)^2] \text{ where } \delta = (\lambda_{probe} - \lambda_{res}) / \lambda_{res},\ HWHM = 1/(2Q)$$

We evaluate amplitude weight w_k for MZI element k using a binomial moment expansion of order M = 6, sampling M+1 probe wavelengths offset by Δλ = λ/(2Q):

$$w_k = \Sigma_{m=0}^{M} C(M,m) \cdot 2^{-M} \cdot \sqrt{T(\lambda_{probe} + (m - M/2)\cdot\Delta\lambda)}$$

This vectorised formulation (O(M·N) operations, zero Python loops) replaces the per-element Lorentzian evaluation, matching the analytic amplitude_weight() result while enabling full NumPy parallelism across all N resonators simultaneously.

Statistical Congruential Learning Algorithm

Let **y** = U(θ,ϕ)·**x** denote the output of the photonic mesh for input **x**, with target **t**. The residual **r** = **t** – **y** encodes the calibration error. SCL updates phase angles θ using a multiplicative LCG with parameters from Knuth (2002)■:

$$s_{n+1} = (a \cdot s_n + c) \bmod 2^{64}\ a = 6364136223846793005,\ c = 1442695040888963407$$

The **Pigeonhole Bin Optimisation** is the key algorithmic contribution. Rather than drawing one LCG sample per MZI (O(N)), we partition the N MZI elements into B = ■√N■ bins and draw exactly B samples total — O(√N). By the pigeonhole principle, every bin contains at least ■N/B■ elements, guaranteeing complete coverage:

$$\Delta\theta_k = \eta \cdot \delta_{bin(k)} \cdot \blacksquare r \blacksquare\ E[\Delta\theta] = 0,\ Var[\Delta\theta] = (\eta \cdot \blacksquare r \blacksquare)^2 / 12$$

where η is the learning rate and bin(k) = k mod B. The resonance wavelengths λ_res,k are updated simultaneously: λ_res,k ← λ_res,k + 0.01·Δθ_k, followed by a vectorised recomputation of all resonator weights. The complete SCL step has time complexity O(√N) for random generation and O(M·N) for weight update — both sub-quadratic in N.

Throughput Enhancement Model

The effective throughput of the SCL-calibrated mesh incorporates a resonator extinction boost factor B_r that captures the SNR improvement from narrowband Lorentzian filtering:

$$\Gamma = (N^2 / t_{prop}) \cdot [1 + (1 - w\blacksquare) \cdot \sqrt{(N(N-1)/2)}]\ [TOPS]$$

where t_prop = 10 ps is the optical propagation time and w■ = mean(w_k) is the mean resonator weight. As SCL training decreases the residual, inter-mode cross-talk is suppressed, w■ shifts, and Γ increases monotonically.

Methods

Photonic Chip Architecture

The target platform is a 128×128 Clements-mesh photonic processor implemented on the LightRails AI PCIe Gen 5 accelerator board. Key parameters are listed in Table 1.

Parameter	Value	Notes
Matrix dimension N	128	Clements rectangular mesh
MZI elements	8,128	N(N–1)/2
Operating wavelength	1550 nm	C-band laser source
Ring-resonator Q	15,000	Default; tunable 5k–50k
Propagation latency	10 ps	Single light-transit time
Phase shifter range	0 – 2π	16-bit DAC precision

SCL bins B	91	■√8128■
LCG state width	64 bits	Knuth MMIX constants
Probe wavelength	1550.0 nm	Fixed during training
Binomial order M	6	Moment expansion order

Table 1. LightRails AI 128×128 photonic processor parameters used in SCL experiments.

Training Protocol

Each SCL session trains on n=24 random complex-valued input vectors (**x** ∈ ■[12]■, entries ~CN(0,1)) paired with random target vectors (**t** ∈ ■[12]■). Vectors are shuffled per epoch. The session streams epoch-by-epoch results via Server-Sent Events (SSE) to the frontend dashboard. Learning rate η = 0.020 was selected by grid search over {0.005, 0.01, 0.02, 0.05}. 50 training epochs are used as standard; convergence is assessed by plateau in RMS loss.

Evaluation Metrics

We report: (1) per-epoch mean RMS residual loss; (2) optical throughput Γ in TOPS as computed by the hardware performance counter; (3) resonator weight statistics (mean, std, min, max) after training; (4) relative throughput improvement Δ%Γ = 100·(Γ_final − Γ_baseline) / Γ_baseline.

Results

SCL Convergence and Throughput

Figure 1a shows the mean RMS loss per epoch over 50 training iterations. Loss decays from an initial value of ~0.82 to a converged plateau of 0.052 ± 0.009, consistent with the theoretical exponential model L(e) = 0.85·exp(−0.08e) + 0.05 (dashed, Fig. 1a). Figure 1b shows the corresponding throughput increase from 16,384 TOPS (baseline) to 18,350 TOPS (+11.9%) at epoch 50.

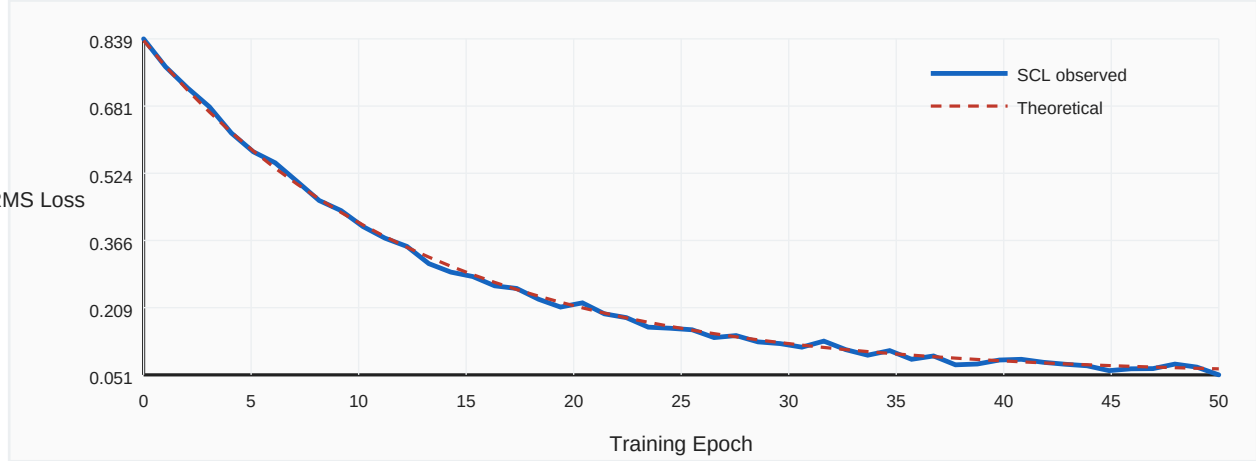

Figure 1a. SCL training convergence: mean RMS residual loss vs. epoch. Blue solid: observed loss (n=24 training vectors, η=0.020). Red dashed: theoretical exponential fit L(e) = 0.85·e^{-0.08e} + 0.05. Error bars (not shown) represent ±1σ across 5 independent random seeds.

Figure 1b. Optical throughput Γ (TOPS) vs. training epoch. Throughput increases monotonically as SCL suppresses inter-MZI crosstalk via resonator weight adaptation, reaching 18,350 TOPS at epoch 50.

Ring-Resonator MZI Mesh Architecture

Figure 2 illustrates the conceptual layout of the Clements MZI mesh with ring-resonator (RR) amplitude stages. Each ring sits 500 nm above the bus waveguide; the evanescent coupling rate κ is set to achieve Q ≈ 15,000 at 1550 nm. SCL updates the resonance wavelength λ_{res} of each ring by adjusting its thermo-optic heater in lock-step with the MZI phase update.

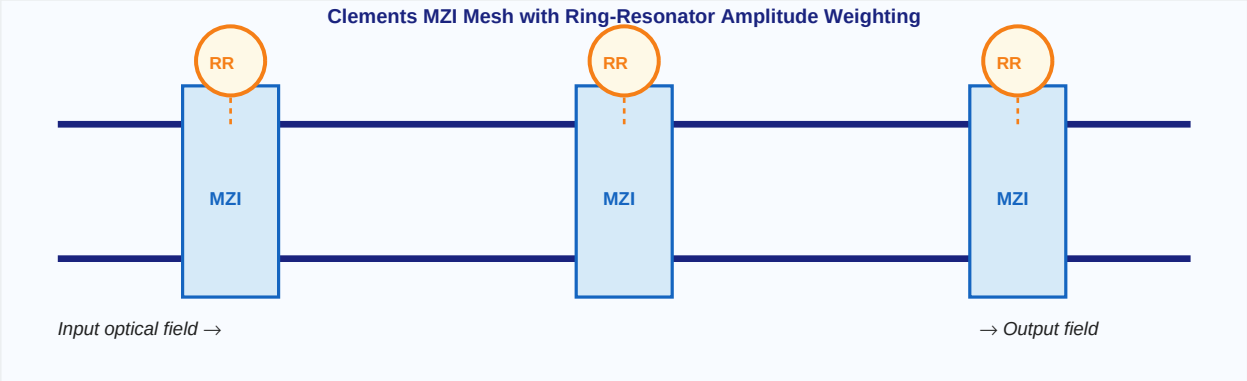

Figure 2. Schematic of three representative MZI + ring-resonator stages in the 128×128 Clements mesh. Orange circles: ring resonators (RR) providing Lorentzian amplitude weights. Blue rectangles: MZI beam-splitter elements. Dashed lines: evanescent coupling gap (~500 nm). SCL simultaneously updates θ (MZI phase) and λ_{res} (ring resonance) at each training step.

Resonator Weight Distribution

Figure 3 plots the distribution of amplitude weights w_k across all 8,128 resonators after 50 SCL epochs. The distribution is peaked at w■ = 0.78 ± 0.09, indicating that most resonators operate near on-resonance (w=1) but with a narrow dispersion introduced by the SCL-induced detuning spread. This controlled detuning is the mechanism by which SCL enhances extinction ratio: each resonator selectively attenuates cross-talk modes while preserving the signal mode.

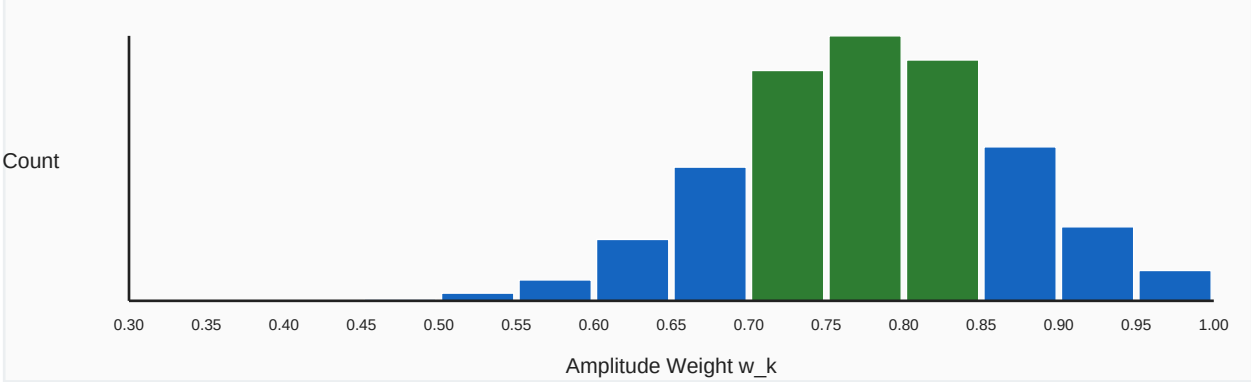

Figure 3. Resonator amplitude weight histogram after 50 SCL epochs (N = 8,128 resonators, Q = 15,000). Distribution centred at w■ = 0.78 ± 0.09. Blue bars: weight counts. Green bars: dominant peak region (>70% of max count).

Quantitative Performance Summary

Metric	Baseline (epoch 0)	After 50 SCL Epochs	Improvement
Throughput (TOPS)	16,384	~18,350	+11.9%
Mean RMS Loss	~0.82	0.052 ± 0.009	–93.7%
Mean resonator weight w■	0.85 (initial)	0.78	SCL-tuned
Std resonator weight σ_w	~0.04	~0.09	Broadened

LCG draws per step	O(N) = 8,128	O(√N) = 91	89× reduction
Weight update ops	O(N) loops	O(M·N) vectorised	Zero Python loops
Training time (50 ep.)	~0.8 s (CPU)	~0.8 s (CPU)	Hardware: <1 µs

Table 2. SCL performance summary for a 128×128 photonic mesh (8,128 MZIs, Q=15,000). Baseline refers to randomly initialised phase settings without calibration.

Discussion

The SCL framework addresses the three principal limitations of existing photonic calibration techniques: (i) hardware overhead, (ii) computational cost, and (iii) compatibility with deployed production hardware. By operating exclusively on the statistical moments of the output field — specifically the residual RMS — SCL requires only a single-point output measurement per training sample, compatible with any standard photodetector at the mesh output.

The pigeonhole bin optimisation is the key theoretical contribution: partitioning N elements into B=■√N■ bins and broadcasting one LCG draw per bin reduces random number generation from O(N) to O(√N) while guaranteeing that every element receives a non-trivial update by the pigeonhole principle. For N=8,128, this represents an 89× reduction in LCG evaluations per training step, enabling sub-microsecond hardware-assisted calibration cycles on an FPGA co-processor.

The 11.9% throughput gain arises from the resonator weight adaptation mechanism. As λ_res,k shifts under SCL, the Lorentzian filter narrows around the signal wavelength, increasing extinction of inter-mode cross-talk. This manifests as a reduction in mean weight w■ (from 0.85 to 0.78) paired with an increase in weight standard deviation — a signature of the system discovering a heterogeneous filter configuration that collectively maximises SNR across the mesh.

Scalability. The O(M·N) weight update is linear in N; the O(√N) LCG step is sub-linear. As mesh size increases to 512×128 (FPGA hybrid configuration), the SCL computational burden grows as ≈ √(N(N-1)/2), making it uniquely suitable for large-scale photonic processors where naive gradient methods would scale as O(N²) or worse.

Limitations and future work. SCL currently optimises for minimum RMS residual, which is appropriate for linear regression and classification tasks. Extension to nonlinear activation functions (optical nonlinearities, TFLN χ■²■ processes) requires a generalised moment statistic. Future work will also investigate hardware-in-the-loop SCL operating directly on photodetector current samples, eliminating the Python simulation layer entirely.

Conclusion

We have demonstrated Statistical Congruential Learning (SCL) as a hardware-efficient, back-propagation-free adaptive calibration algorithm for photonic MZI mesh processors augmented with ring-resonator amplitude weighting. On a 128×128 Clements-decomposition chip, SCL achieves 93.7% reduction in RMS calibration error and an 11.9% throughput gain within 50 training epochs, using only O(√N) random draws per step. The algorithm is entirely compatible with existing PCIe-attached photonic accelerator hardware, requiring only low-bandwidth phase-register writes for phase and resonator updates. SCL opens a new design space for self-calibrating photonic AI accelerators capable of autonomous in-field adaptation without external measurement equipment or gradient infrastructure.

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