

Question: Find: (1) the probability of default; (2) the probability of survival; and (3) the expected time to default for Company A over the next two periods and the default intensity ( $\lambda$ ) for Company B.

Company A has an actual default intensity of 0.10, and Company B has a 17% probability of defaulting in the next three periods.

Answer: The expected time to default of any company is simply  $(1/\lambda)$ , which in the case of Company A is 10 periods. Both remaining problems for Company A are solved using Equation 1. For Company A, the probability of survival is  $e^{-0.10 \times 2}$ , which equals 0.819. The probability of default is simply 1.0 minus the probability of survival:  $1.0 - 0.819$ , or 0.181. Company B has a probability of survival of 83%, or 0.83 (since its default probability is 17%). Therefore, the default intensity can be found by inserting 0.83 into the left side of Equation 1 and taking the natural logarithm of both sides. This generates the value that is equal to  $-\lambda t$ , specifically,  $\ln(0.83)$ , or  $-0.18633$ . Dividing that quantity by  $-3$  (i.e.,  $-t$ ) generates a default intensity ( $\lambda$ ) of 6.21%.

Question: What is the current price of a 1-year zero-coupon bond issued by a start-up firm with a default intensity of 5%? Assume the riskless rate is 2%.

Answer: The face value of the bond is \$50 million. The value (\$46.62 million) can be found using Equation 3:

$$D_0 = e^{-(0.02+0.05) \times 1} \times 50 = \$46.62 \text{ million}$$

Note that Equation 3 can be solved for any of its variables given the others.

Question: Find the risk-neutral default intensity.

Suppose the credit spread on a 1-year zero-coupon bond is 2%, and the recovery rate is estimated to be 80%.

Answer: Using Equation 5, the approximated implied risk-neutral default intensity is 0.10:

$$0.02 \approx \lambda \times (1 - 0.8) \quad \lambda \approx 10\%$$

Note that Equation 5 contains three variables, any one of which can be solved using the other two.