Reading

Reduced-Form Models

In a reduced-form model, the framework does not consider the causes of default; that is, default is exogenous. Key drivers in a reduced-form model include time to default (or default time) and recovery in the event of default (conversely, loss given default). The difference between various reduced-form models typically involves the processes of modeling when default occurs and estimating the recovery if default happens.

Default Intensity in Reduced-Form Models

Under certain assumptions about the nature of the random process that leads to default, the probability of survival for a given number of years can be shown to have an exponential distribution. Assuming an exponential distribution and denoting $p(t)$ as the probability that a firm has survived for t years, this probability can be expressed as:

$$p(t) = \exp(-\lambda \times t)$$

where λ is called the default intensity of the model. The parameter λ determines both the expected time to default and the probability of survival. Specifically, given a default intensity of $\lambda = 0.2$, the expected time to default is $(1/\lambda)$, or 5.00. The higher the **default intensity,** the shorter the expected time to default. The probability of default at or before t is given by $1 - p(t)$. The default intensity that appears in Equation 1 may refer to the actual or physical default intensity or to the risk-neutral version of the same variable. If the actual default intensity is used, then the analyst obtains an estimate of the actual expected time of default. The risk-neutral version of the default intensity is used exclusively for valuation purposes and to estimate credit spreads on investments exposed to credit risk.

Default Intensity and the Probabilities of Default

Suppose default time is continuous, meaning that default can take place at any time, and not just at discrete points of time (e.g., at the end of each quarter). Then, given Equation 1, the probability of default in the time interval of $(t, t + \Delta t)$, assuming there has been no default up to time t, is given by $\lambda \times \Delta t$, where Δt is a relatively small length of time. This is the conditional probability of default. On the other hand, the unconditional probability of default in the time interval of $(t, t + \Delta t)$ is given by $\exp(-\lambda \times t)\lambda \times \Delta t$, which is equal to the probability of surviving up to time t multiplied by the conditional probability of defaulting between t and Δt. It is important to note that λ is not the probability of default within 1 year, as 1 year is a relatively long length of time. Finally, the probability that default could take place between s and t, assuming that no default has taken place up to time s, is given by:

$$p(s) - p(t) = \exp(-\lambda \times s) - \exp(-\lambda \times t)$$

For example, suppose a start-up company has an actual default intensity of 5%. At the time the firm is established (i.e., time 0), the probability that the firm will default within Year 3, assuming that it has already survived for 2 years, is:

$$\begin{aligned} \text{Probability of Default in Year3} &= \exp(-0.05 \times 2) - \exp(-0.05 \times 3) \\ &= 0.905 - 0.861 = 4.4\% \end{aligned}$$

That is, at time zero, there is a 4.4% chance that the firm will not survive beyond the third year, provided that it has already survived for 2 years. Also, the expected time to default is $(1/0.05) = 20$ years.

A reduced-form model can be used to relate default intensities to various financial and economic variables. Credit analysts have built models in which the default intensity is related to financial conditions of a firm as well as to macroeconomic conditions. The model is typically calibrated by selecting its parameters so that it can explain historical default patterns as well as current conditions in credit markets (e.g., credit spreads for various credit ratings). The resulting model is then used to value new issues or credit instruments, which is crucial to the development of some credit investment strategies.

Valuing Risky Debt with Default Intensity

The default intensity model can be incorporated into the valuation model for risky debt. To see the intuition of this model, consider a zero-coupon bond with a face value of K and time to maturity of T. If the bond is risk-free, then its current price is given by $K \times e^{-r \times T}$. Suppose the bond is exposed to default risk with a default intensity of λ. The probability of survival up to time T is $e^{-\lambda \times T}$, which means the probability of default by time T is $(1 - e^{-\lambda \times T})$. Assume that in the case of default the bond will have zero recovery. Then the price is given by:

$$
\begin{aligned}
D_0 &= e^{-r \times T}\left(\text{Prob}_{\text{No Default}} \times K + \text{Prob}_{\text{Default}} \times 0\right) \\
&= e^{-r \times T}\left(e^{-\lambda \times T} \times K + \left(1 - e^{-\lambda \times T}\right) \times 0\right) = Ke^{-(r+\lambda) \times T}
\end{aligned}
$$

It is important to note that Equation 3 was obtained under the assumption that investors do not care about potential systematic risk of the bond, and the only risk that enters into their calculations is the default risk. In other words, the default intensity and the resulting probability of default are assumed to be under risk neutrality (i.e., are Q-Measures).

The calculation in Application B can be reversed and, using observed market prices, one can calculate the implied risk-neutral default intensity. For example, suppose the market price of the bond in the previous example is actually \$45.24 million. The implied risk-neutral default intensity would be 8%. That is,

$$45.24 = e^{-(0.02+0.08) \times 1} \times 50$$

Relating the Credit Spread to Default Intensity and the Recovery Rate

Finally, let's consider the case where in the event of default there is some recovery of face value. This amount is represented by $K \times RR$, where RR is the recovery rate. For simplicity, assume that the recovered amount is paid at time T. In this case, the value of the bond is given by:

$$
\begin{aligned}
D_0 &= e^{-r \times T}\left(RR \times K \times \left(1 - e^{-\lambda \times T}\right) + K \times e^{-\lambda \times T}\right) \\
&\approx e^{-(r+\lambda(1-RR)) \times T} \times K
\end{aligned}
$$

When there is some recovery in case of default, the default intensity is reduced by the factor related to the recovery rate. The higher the recovery rate, the lower the impact of a default. The term $\lambda(1 - RR)$ in the second line of Equation 4 could be interpreted as an approximation to the credit spread that is added to the risk-free rate to obtain the appropriate discount rate:

$$\lambda \times (1 - RR) \approx \text{Credit Spread}$$

The credit spread is smaller for bonds with higher recovery rates or lower default intensity.

The Two Predominant Reduced-Form Credit Models

Two models are most commonly cited when referring to the reduced-form models: the Jarrow–Turnbull (1995) model and the Duffie–Singleton (2003) model. The Jarrow–Turnbull model assumes that regardless of timing of default, recovery is received at the maturity date. The Jarrow–Lando–Turnbull (1997) model extends the original model further by taking into account various credit ratings beyond just two simple states of default or survival. Here, the model takes into account that there is migration risk (i.e., that the bond will be downgraded rather than experiencing outright default). The probability of moving from one rating to the next can be obtained from rating transition tables, published by the credit rating agencies. The Duffie–Singleton model allows the recovery process to occur at any time and sets the recovery amount to be a fraction of the nondefaulting bond price at the time of default.