Gate-Assignment

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Download all python codes from

```
https://github.com/AI20BTECH11014/EE3900−
    Linear−Systems−and−Signal−processing/blob/
    main/Gate_assignment/Gate_assignment.py
```

and latex-tikz codes from

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https://github.com/AI20BTECH11014/EE3900−
    Linear−Systems−and−Signal−processing/blob/
    main/Gate_assignment/Gate_assignment.tex
\vspace{0.5cm}
\section{QUESTION: Q.55 EC−GATE−2018}
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Let $X[k] = k+1, 0 \le k \le 7$ be 8-point DFT of a sequence $x[n]$,where

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{\dfrac{-j2\pi nk}{N}}$$

The value (correct to two decimal places) of $\sum_{n=0}^{3} x[2n]$

1 Solution

Given,

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{\dfrac{-j2\pi nk}{N}}$$

Considering 8-point DFT, we have

$$\begin{pmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{pmatrix} = \begin{pmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{pmatrix} \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix} \tag{1.0.1}$$

$$\text{where twiddler factor, } W_8 = \exp\left(-\frac{j2\pi}{8}\right) \tag{1.0.2}$$

Obtaining X[0] and X[4] by using (1.0.2), we get,

$$X[0] = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix} \tag{1.0.3}$$

$$X[4] = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix} \tag{1.0.4}$$

from (1.0.3) and (1.0.4),

$$X[0] + X[4] = \begin{pmatrix} 2 & 0 & 2 & 0 & 2 & 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{pmatrix} \tag{1.0.5}$$

$$X[0] + X[4] = 2\left(x[0] + x[2] + x[4] + x[6]\right) \tag{1.0.6}$$

$$X[0] + X[4] = 2\sum_{n=0}^{3} x[2n] \tag{1.0.7}$$

given,

$$X[k] = k + 1, 0 \leq k \leq 7 \tag{1.0.8}$$

from (1.0.7) and (1.0.8),

$$\sum_{n=0}^{3} x[2n] = 3 \tag{1.0.9}$$

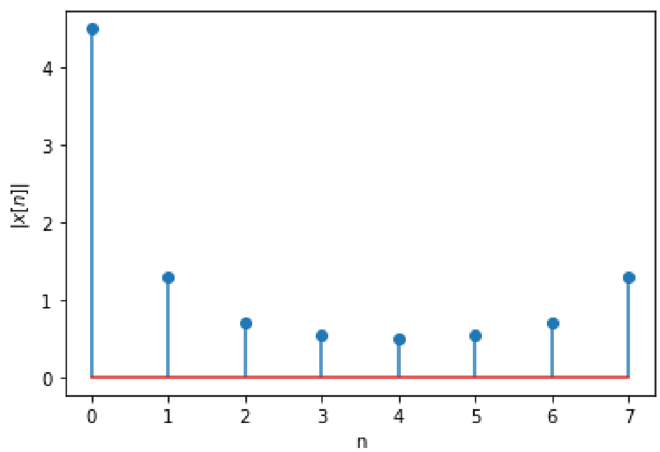

Fig. 0: Magnitude of $x[n]$ vs n