GATE Assignment 3

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Download all latex codes from

https://github.com/AI20BTECH11014/EE3900-Linear-Systems-and-Signal-processing/tree/ main/Gate Assignment 3

1 GATE EC-2005 Q.62

For a signal x(t) the Fourier transform is X(f), then the inverse Fourier transform of X(3f + 2) is given by

(A)
$$\frac{1}{2}x(\frac{t}{2})e^{j3\pi t}$$

(B)
$$\frac{1}{3}x(\frac{t}{3})e^{\frac{-j4\pi t}{3}}$$

(C) $3x(3t)e^{-j4\pi t}$

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(D)
$$x(3t + 2)$$

Solution

By the time scaling property of Fourier transform,

$$x(\alpha t) \stackrel{\mathcal{F}}{\rightleftharpoons} \frac{1}{|\alpha|} X\left(\frac{f}{|\alpha|}\right)$$
 (1.0.1)

$$x(\frac{t}{3}) \stackrel{\mathcal{F}}{\rightleftharpoons} 3X(3f)$$
 (1.0.2)

By the time shifting property of Fourier transform,

$$e^{-j2\pi f_0 t} x(t) \stackrel{\mathcal{F}}{\rightleftharpoons} X(f + f_0)$$
 (1.0.3)

$$\frac{1}{3}e^{\frac{-j4\pi t}{3}}x(\frac{t}{3}) \stackrel{\mathcal{F}}{\rightleftharpoons} X(3f+2) \tag{1.0.4}$$

Let x(t) = rect(t) and $\text{sinc}(t) = \frac{\sin(t)}{t}$

$$X(f) = \operatorname{sinc}(f) \tag{1.0.5}$$

For $X(f) = \operatorname{sinc} (3f + 2)$

$$\frac{1}{3}e^{\frac{-j4\pi t}{3}}x(\frac{t}{3}) = \frac{1}{3}e^{\frac{-j4\pi t}{3}}\operatorname{rect}\left(\frac{t}{3}\right)$$
 (1.0.6)

The correct option is (B).