GATE Assignment 3

MANIKANTA VALLEPU AI20BTECH11014

Download all latex codes from

https://github.com/AI20BTECH11014/EE3900-

Linear-Systems-and-Signal-processing/blob/main/Gate_Assignment_3/

Gate_Assignment_3.tex

Download all python codes from

https://github.com/AI20BTECH11014/EE3900-

Linear-Systems-and-Signal-processing/blob/main/Gate Assignment 3/

Gate_Assignment_3.py

1 GATE EC-2005 Q.62

For a signal x(t) the Fourier transform is X(f), then the inverse Fourier transform of X(f+2) is given by

(A)
$$\frac{1}{2}x\left(\frac{t}{2}\right)e^{j3\pi t}$$

(B)
$$\frac{1}{3}x(\frac{t}{3})e^{\frac{-j4\pi t}{3}}$$

(C)
$$3x(3t)e^{-j4\pi t}$$

(D) x(3t+2)

SOLUTION

By the time scaling property of Fourier transform,

$$x(\alpha t) \stackrel{\mathcal{F}}{\rightleftharpoons} \frac{1}{|\alpha|} X \left(\frac{f}{|\alpha|} \right)$$
 (1.0.1)

$$x\left(\frac{t}{3}\right) \stackrel{\mathcal{F}}{\rightleftharpoons} 3X(3f) \tag{1.0.2}$$

By the time shifting property of Fourier transform,

$$e^{-j2\pi f_0 t} x(t) \stackrel{\mathcal{F}}{\rightleftharpoons} X(f + f_0)$$
 (1.0.3)

$$e^{\frac{-j4\pi t}{3}}x(t) \stackrel{\mathcal{F}}{\rightleftharpoons} X\left(f + \frac{2}{3}\right)$$
 (1.0.4)

using (1.0.2),

$$e^{\frac{-j4\pi t}{3}}x\left(\frac{t}{3}\right) \stackrel{\mathcal{F}}{\rightleftharpoons} 3X\left(3\left(f + \frac{2}{3}\right)\right)$$
 (1.0.5)

$$\frac{1}{3}e^{\frac{-j4\pi t}{3}}x\left(\frac{t}{3}\right) \stackrel{\mathcal{F}}{\rightleftharpoons} X(3f+2) \tag{1.0.6}$$

Let x(t) = rect(t) and $\text{sinc}(t) = \frac{\sin(t)}{t}$

$$X(f) = \operatorname{sinc}(f) \tag{1.0.7}$$

For $X(f) = \operatorname{sinc}(3f + 2)$

$$\frac{1}{3}e^{\frac{-j4\pi t}{3}}x\left(\frac{t}{3}\right) = \frac{1}{3}e^{\frac{-j4\pi t}{3}}\operatorname{rect}\left(\frac{t}{3}\right)$$
 (1.0.8)

The correct option is (B).