Gate-Assignment

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Download all python codes from

```
https://github.com/AI20BTECH11014/EE3900−
    Linear−Systems−and−Signal−processing/blob/
    main/Gate_assignment/Gate_assignment.py
```

and latex-tikz codes from

```
https://github.com/AI20BTECH11014/EE3900−
    Linear−Systems−and−Signal−processing/blob/
    main/Gate_assignment/Gate_assignment.tex
\vspace{0.5cm}
\section{QUESTION: Q.55 EC−GATE−2018}
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Let $X[k] = k + 1, 0 \le k \le 7$ be 8-point DFT of a sequence $x[n]$,where

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi nk}{N}}$$

The value (correct to two decimal places) of $\sum_{n=0}^{3} x[2n]$

1 SOLUTION

Given,

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi nk}{N}}$$

Let $\mathbf{F}_N$ be the N-point DFT matrix.
Using the property of complex exponentials, we can express $\mathbf{F}_N$ in terms of $\mathbf{F}_{N/2}$

$$\mathbf{F}_N = \begin{pmatrix} \mathbf{I}_{N/2} & \mathbf{D}_{N/2} \\ \mathbf{I}_{N/2} & -\mathbf{D}_{N/2} \end{pmatrix} \begin{pmatrix} \mathbf{F}_{N/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{N/2} \end{pmatrix} \mathbf{P}_N \tag{1.0.1}$$

Where

$$\mathbf{F}_N = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & w & \cdots & w^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & w^{N-1} & \cdots & w^{(N-1)(N-1)} \end{pmatrix} \tag{1.0.2}$$

where $w = e^{\frac{-2\pi j}{N}}$

$$\mathbf{D}_N = \begin{pmatrix} w_{2N}^0 & 0 & \cdots \\ 0 & w_{2N}^1 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \tag{1.0.3}$$

$$\mathbf{D}_N = \text{diag}(w_{2N}^0, w_{2N}^1, \ldots, w_{2N}^{N-1}) \tag{1.0.4}$$

P_N is the permutation matrix defined as

$$\mathbf{P}_N = \left[a_{ij}\right]_{N\times N}, \; i, j \in \{0, 1, \ldots, N-1\} \tag{1.0.5}$$

$$a_{ij} = \begin{cases} 1 & j = 2i, \; i < \frac{N}{2} \\ 1 & j = 2\left(i - \frac{N}{2}\right) + 1, \; i \ge \frac{N}{2} \\ 0 & otherwise \end{cases} \tag{1.0.6}$$

For $N = 8$

$$\mathbf{F}_8 = \begin{pmatrix} \mathbf{I}_4 & \mathbf{D}_4 \\ \mathbf{I}_4 & -\mathbf{D}_4 \end{pmatrix} \begin{pmatrix} \mathbf{F}_4 & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_4 \end{pmatrix} \mathbf{P}_8 \tag{1.0.7}$$

as we know,

$$\mathbf{X}\mathbf{F}_8 = \mathbf{x} \tag{1.0.8}$$

$$\tag{1.0.9}$$

now,

$$\begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} = \mathbf{F}_4 \mathbf{x}_e + \mathbf{D}_2 \mathbf{F}_4 \mathbf{x}_o \tag{1.0.10}$$

$$\begin{pmatrix} X(4) \\ X(5) \\ X(6) \\ X(7) \end{pmatrix} = \mathbf{F}_4 \mathbf{x}_e - \mathbf{D}_2 \mathbf{F}_4 \mathbf{x}_o \tag{1.0.11}$$

from (1.0.10) and (1.0.11),

$$2\mathbf{F}_4 \mathbf{x}_e = \begin{pmatrix} X(0) + X(4) \\ X(1) + X(5) \\ X(2) + X(6) \\ X(3) + X(7) \end{pmatrix} \tag{1.0.12}$$

$$2\mathbf{F}_4^T \mathbf{F}_4 \mathbf{x}_e = \mathbf{F}_4^T \begin{pmatrix} X(0) + X(4) \\ X(1) + X(5) \\ X(2) + X(6) \\ X(3) + X(7) \end{pmatrix} \tag{1.0.13}$$

as we know ,

$$\mathbf{F}_N^T\mathbf{F}_N = N\mathbf{I} \tag{1.0.14}$$

$$8\mathbf{x}_e = \mathbf{F}_4^T\begin{pmatrix} X(0)+X(4) \\ X(1)+X(5) \\ X(2)+X(6) \\ X(3)+X(7) \end{pmatrix} \tag{1.0.15}$$

from (1.0.2),

$$\mathbf{F}_N^T = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{pmatrix} \tag{1.0.16}$$

$$8\mathbf{x}_e = \begin{pmatrix} 36 \\ -44j \\ -4 \\ -4+4j \end{pmatrix} \tag{1.0.17}$$

$$8\mathbf{I}^T\mathbf{x}_e = 24 \tag{1.0.18}$$

$$\mathbf{I}^T\mathbf{x}_e = 3 \tag{1.0.19}$$

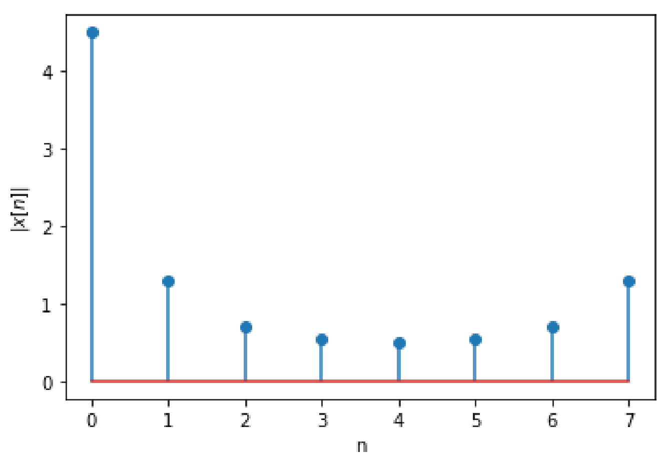

Fig. 0: Magnitude of $x[n]$ vs n