Quiz 1

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Download latex-tikz codes from

https://github.com/AI20BTECH11014/EE3900-Linear-Systems-and-Signal-processing/blob/main/QUIZ_1/QUIZ_1.tex

1 Question

For the following signal determine whether the system is (1) stable, (2) casual, (3) linear and (4) time invariant.

$$T(x[n]) = x[n]\sum_{k=0}^{\infty}\delta[n-k] \tag{1.0.1}$$

2 Solution

$$\sum_{k=0}^{\infty}\delta[n-k] = u[n]$$

For $n > 0$

$$T(x[n]) = x[n]$$

For $n < 0$

$$T(x[n]) = 0$$

Definition 2.1. ***Stable*** *A system is said to be BIBO stable if the response to a bounded input is always bounded.*

As the given signal input $x[n]$ is bounded,

$$|x[n]| < M \text{ for some real } M \tag{2.0.1}$$

$$\text{Hence } |x[n]| < M \tag{2.0.2}$$

So $x[n]$ is also bounded. Hence, the system is stable i.e, bounded input bounded output stable.

Definition 2.2. ***Casual*** *The output at any instant does not depend on the future inputs i.e, for at n_0 $y[n_0]$ does not depend on $x[n]$ for $n > n_0$.*

Here, for this signal the output depends on $x[n_0]$.As the output always depends on the present time the system is casual.

Definition 2.3. ***Linear*** *The response to an arbitary linear combination of input signals is always the same linear combinations of the individual responses to these signals*

$$x_1[n] \implies x_1[n] \tag{2.0.3}$$

$$x_2[n] \implies x_2[n] \tag{2.0.4}$$

$$ax_1[n] + bx_2[n] \implies ax_1[n] + bx_2[n] \tag{2.0.5}$$

Hence the given system is linear

Definition 2.4. ***Time Invariant*** *The response to an arbitrary translated set of inputs is always the response to the original set, but translated by the same amount.*
If

$$x[n] \implies y[n] \tag{2.0.6}$$

then

$$x[n-n_0] \implies y[n-n_0] \tag{2.0.7}$$

for all x and n_0.

here

$$x[n] \implies x[n] \tag{2.0.8}$$

adding time delay(n_0) to the output signal

$$x[n] \implies x[(n-n_0)] \tag{2.0.9}$$

adding time delay(n_0) to the input signal

$$x[n] \implies x[n-n_0] \tag{2.0.10}$$

Now the ouput signal

$$x[n-n_0] \implies x[n-n_0] \tag{2.0.11}$$

As 2.0.9 and 2.0.11 are same, the given signal is time invariant.