

Assignment-4

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Download all latex-tikz codes from

https://github.com/AI20BTECH11014/EE3900-Linear-Systems-and-Signal-processing/blob/main/Assignment_4/Assignment_4.tex

Download all python codes from

https://github.com/AI20BTECH11014/EE3900-Linear-Systems-and-Signal-processing/blob/main/Assignment_4/Assignment_4.py

1 QUESTION: LINEAR FORMS Q.2.14

The sum of the perpendicular distances of a variable point \mathbf{P} from the lines

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 0$$

$$\begin{pmatrix} 3 & -2 \end{pmatrix} \mathbf{x} = -7$$

is always 10. Show that \mathbf{P} must move on a line.

2 SOLUTION

The foot of perpendicular from point \mathbf{P} to line $\mathbf{n}^\top \mathbf{x} = c$ is given as $\mathbf{P} + \alpha \mathbf{n}$, where $\alpha \in \mathbb{R}$

$$\mathbf{n}^\top (\mathbf{P} + \alpha \mathbf{n}) = c \quad (2.0.1)$$

$$\mathbf{n}^\top \mathbf{P} + \|\mathbf{n}\|^2 \alpha = c \quad (2.0.2)$$

The perpendicular distance from a point \mathbf{P} to line $\mathbf{n}^\top \mathbf{x} = c$ is given as,

$$= \|\mathbf{P} - (\mathbf{P} + \alpha \mathbf{n})\| \quad (2.0.3)$$

$$= |\alpha| \|\mathbf{n}\| \quad (2.0.4)$$

using (2.0.2) in (2.0.4), perpendicular distance is given as,

$$= \frac{|c - \mathbf{n}^\top \mathbf{P}|}{\|\mathbf{n}\|} \quad (2.0.5)$$

The sum of the perpendicular distances of a variable point \mathbf{P} from the lines

$$\mathbf{n}_1^\top \mathbf{x} = c_1$$

$$\mathbf{n}_2^\top \mathbf{x} = c_2$$

is always d, then,

$$\frac{|c_1 - \mathbf{n}_1^\top \mathbf{P}|}{\|\mathbf{n}_1\|} + \frac{|c_2 - \mathbf{n}_2^\top \mathbf{P}|}{\|\mathbf{n}_2\|} = d \quad (2.0.6)$$

$$\pm \left(\frac{c_1 - \mathbf{n}_1^\top \mathbf{P}}{\|\mathbf{n}_1\|} \right) \pm \left(\frac{c_2 - \mathbf{n}_2^\top \mathbf{P}}{\|\mathbf{n}_2\|} \right) = d \quad (2.0.7)$$

$$- \left(\frac{\mathbf{n}_1^\top}{\|\mathbf{n}_1\|} \pm \frac{\mathbf{n}_2^\top}{\|\mathbf{n}_2\|} \right) \mathbf{P} = \pm d - \left(\frac{c_1}{\|\mathbf{n}_1\|} \pm \frac{c_2}{\|\mathbf{n}_2\|} \right) \quad (2.0.8)$$

\therefore point \mathbf{P} lies on either of the lines

$$L_1 : \left(\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{13}} \quad \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{13}} \right) \mathbf{P} = 10 - \frac{7}{\sqrt{13}} \quad (2.0.9)$$

$$L_2 : \left(\frac{1}{\sqrt{2}} - \frac{3}{\sqrt{13}} \quad \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{13}} \right) \mathbf{P} = 10 + \frac{7}{\sqrt{13}} \quad (2.0.10)$$

$$L_3 : \left(\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{13}} \quad \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{13}} \right) \mathbf{P} = -10 - \frac{7}{\sqrt{13}} \quad (2.0.11)$$

$$L_4 : \left(-\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{13}} \quad -\frac{1}{\sqrt{2}} - \frac{2}{\sqrt{13}} \right) \mathbf{P} = 10 - \frac{7}{\sqrt{13}} \quad (2.0.12)$$