Quiz 2

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Download all python codes from

https://github.com/AI20BTECH11014/EE3900−Linear−Systems−and−Signal−processing/blob/main/QUIZ_2/QUIZ_2.py

and latex-tikz codes from

https://github.com/AI20BTECH11014/EE3900−Linear−Systems−and−Signal−processing/blob/main/QUIZ_2/QUIZ_2.tex

1 Question

For each of the following pairs of input and output Z-transforms $X(z)$ and $Y(z)$,determine the region of convergence for the system function $H(z)$:

$$X(z) = \frac{1}{1+\frac{1}{3}z^{-1}}, \qquad |z| < \frac{1}{3} \quad (1.0.1)$$

$$Y(z) = \frac{1}{\left(1-\frac{1}{6}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)}, \qquad \frac{1}{6} < |z| < \frac{1}{3} \quad (1.0.2)$$

2 Solution

Lemma 2.1. ***Properties of ROC:*** *The ROC doesnot contain any poles.*
For

$$X(s) = \frac{N(s)}{D(s)} \quad (2.0.1)$$

The poles of X(s) $\implies D(s) = 0$

Lemma 2.2. *The poles of X(s) consists of a strip parallel to jω axis in the s-plane.*

We know that the z-transform of output signal ($Y(z)$)

$$Y(z) = H(z)X(z) \quad (2.0.2)$$

we know that the z- transform of the system function $H(z)$ is given by

$$H(z) = \frac{Y(z)}{X(z)} \quad (2.0.3)$$

from (1.0.1) and (1.0.2),

$$H(z) = \frac{\frac{1}{\left(1-\frac{1}{6}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)}}{\frac{1}{1+\frac{1}{3}z^{-1}}} \quad (2.0.4)$$

$$= \frac{1}{1-\frac{1}{6}z^{-1}} \quad (2.0.5)$$

The pole of this expression is $\frac{1}{6}$ (2.0.6)

from (1.0.1),

$$X(z) = \frac{1}{1+\frac{1}{3}z^{-1}} \quad (2.0.7)$$

with ROC $|z| < \frac{1}{3}$. We shall call this ROC_1. from (1.0.2),

$$Y(z) = \frac{1}{\left(1-\frac{1}{6}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)} \quad (2.0.8)$$

with ROC $\frac{1}{6} < |z| < \frac{1}{3}$. We shall call this ROC_2.The ROC of the given expression will be the intersection of ROC_1 and ROC_2. Therefore, the ROC of the given sequence is $\frac{1}{6} < |z|$. ∴The Z-transform of the system function $H(z)$ is given by

$$H(z) = \frac{1}{1-\frac{1}{6}z^{-1}} \quad (2.0.9)$$

with ROC $|z| > \frac{1}{6}$.

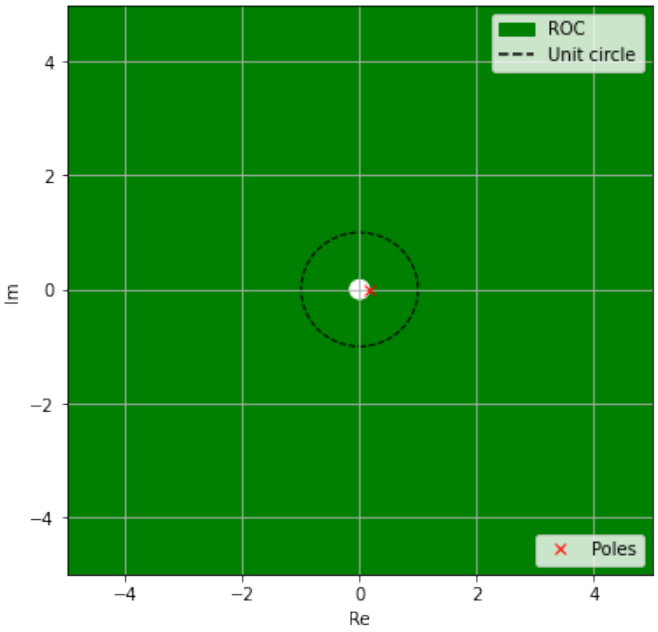

Fig. 0: Line between two points