EE3900 : Assignment-3

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Download all latex-tikz codes from

https://github.com/AI20BTECH11014/EE3900−Linear−Systems−and−Signal−processing/blob/main/Assignment_4/Assignment_4.tex

1 QUESTION: Linear Forms Q.2.14

The sum of the perpendicular distances of a variable point $\mathbf{P}$ from the lines

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 0$$

$$\begin{pmatrix} 3 & -2 \end{pmatrix} \mathbf{x} = -7$$

is always 10. Show that $\mathbf{P}$ must move on a line.

2 SOLUTION

The foot of perpendicular from point $\mathbf{P}$ to line $\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 0$ is given as $\mathbf{P} + \alpha_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$,where $\alpha_1 \in \mathbb{R}$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \left(\mathbf{P} + \alpha_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right) = 0 \tag{2.0.1}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{P} + 2\alpha_1 = 0 \tag{2.0.2}$$

The foot of perpendicular from point $\mathbf{P}$ to line $\begin{pmatrix} 3 & -2 \end{pmatrix} \mathbf{x} = -7$ is given as $\mathbf{P}+\alpha_2 \begin{pmatrix} 3 \\ -2 \end{pmatrix}$,where $\alpha_2 \in \mathbb{R}$

$$\begin{pmatrix} 3 & -2 \end{pmatrix} \left(\mathbf{P} + \alpha_2 \begin{pmatrix} 3 \\ -2 \end{pmatrix} \right) = -7 \tag{2.0.3}$$

$$\begin{pmatrix} 3 & -2 \end{pmatrix} \mathbf{P} + 13\alpha_2 = -7 \tag{2.0.4}$$

Given,

$$\|\mathbf{P} - \left(\mathbf{P} + \alpha_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right)\| + \|\mathbf{P} - \left(\mathbf{P} + \alpha_2 \begin{pmatrix} 3 \\ -2 \end{pmatrix} \right)\| = 10 \tag{2.0.5}$$

$$\sqrt{2}|\alpha_1| + \sqrt{13}|\alpha_2| = 10 \tag{2.0.6}$$

From equations (2.0.2) , (2.0.4) and (2.0.6),

$$\frac{1}{\sqrt{2}} |\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{P}| + \frac{1}{\sqrt{13}} |\begin{pmatrix} 3 & -2 \end{pmatrix} \mathbf{P} + 7| = 10 \tag{2.0.7}$$

$\therefore$ point $\mathbf{P}$ lies on either of the lines

$$L_1 : \begin{pmatrix} \frac{1}{\sqrt{2}} + \frac{3}{\sqrt{13}} & \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{13}} \end{pmatrix} \mathbf{P} = 10 - \frac{7}{\sqrt{13}} \tag{2.0.8}$$

$$L_2 : \begin{pmatrix} \frac{1}{\sqrt{2}} - \frac{3}{\sqrt{13}} & \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{13}} \end{pmatrix} \mathbf{P} = 10 + \frac{7}{\sqrt{13}} \tag{2.0.9}$$

$$L_3 : \begin{pmatrix} \frac{1}{\sqrt{2}} + \frac{3}{\sqrt{13}} & \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{13}} \end{pmatrix} \mathbf{P} = -10 - \frac{7}{\sqrt{13}} \tag{2.0.10}$$

$$L_4 : \begin{pmatrix} -\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{13}} & -\frac{1}{\sqrt{2}} - \frac{2}{\sqrt{13}} \end{pmatrix} \mathbf{P} = 10 - \frac{7}{\sqrt{13}} \tag{2.0.11}$$