GATE ASSIGNMENT 2

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Download all python codes from

```
https://github.com/AI20BTECH11014/EE3900−
    Linear−Systems−and−Signal−processing/blob/
    main/Gate_Assignment_2/
    GATE_Assignment_2.py
```

and latex-tikz codes from

```
https://github.com/AI20BTECH11014/EE3900−
    Linear−Systems−and−Signal−processing/blob/
    main/Gate_Assignment_2/
    Gate_Assignment_2.tex
```

1 EC 2007 Q.16

If the Laplace transform of a signal $y(t)$ is

$$y(s) = \frac{1}{s(s-1)}$$

then its final value is

1) -1
2) 0
3) 1
4) unbounded

2 Solution

Theorem 2.1. *FINAL VALUE THEOREM The final value theorem states*

$$\lim_{t\to+\infty} f(t) = \lim_{s\to 0} sF(s) \tag{2.0.1}$$

It determines the steady-state value of the system response without finding the inverse transform.It is applicable only for stable systems.

Given,

$$y(s) = \frac{1}{s(s-1)}$$

from Final value theorem,

$$\lim_{t\to+\infty} y(t) = \lim_{s\to 0} sy(s) \tag{2.0.2}$$

$$= \lim_{s\to 0} s\frac{1}{s(s-1)} \tag{2.0.3}$$

$$= \lim_{s\to 0} \frac{1}{(s-1)} \tag{2.0.4}$$

s=1 is right s-plane pole of $y(s)$ with ROC $|s| > 1$ .As the system is not lining left half of s-plane,the system is unstable.Hence, it is unbounded. $\therefore$ option 4 is correct.

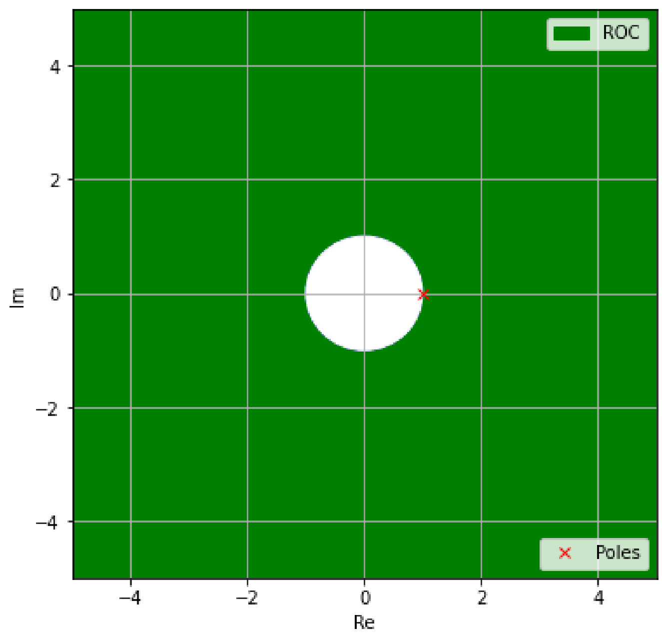

Fig. 4: ROC for $y(s)$.

Lemma 2.1 (Table of Inverse Laplace Transforms)**.**

Laplace transform of f(t) $F(s) = \mathcal{L}\{f(t)\}$	***Time Function*** $f(t) = \mathcal{L}^{-1}\{F(s)\}$
$\frac{1}{s},\ s > 0$	*1*
$\frac{1}{s-a},\ s-a > 0$	e^{at}

Lemma 2.2. *Linearity of Inverse Laplace Transform*

$$\mathcal{L}^{-1}\left\{af(t)+bg(t)\right\} = a\mathcal{L}^{-1}\left\{f(t)\right\} + b\mathcal{L}^{-1}\left\{g(t)\right\} \tag{2.0.5}$$

$$y(t) = \mathcal{L}^{-1}\left\{y(s)\right\} \tag{2.0.6}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s(s-1)}\right\} \tag{2.0.7}$$

$$= \mathcal{L}^{-1}\left\{\frac{-1}{s} + \frac{1}{s-1}\right\} \tag{2.0.8}$$

From Lemma-2.2,

$$= \mathcal{L}^{-1}\left\{\frac{-1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} \tag{2.0.9}$$

From Lemma-2.1,

$$y(t) = -1 + e^{t} \tag{2.0.10}$$

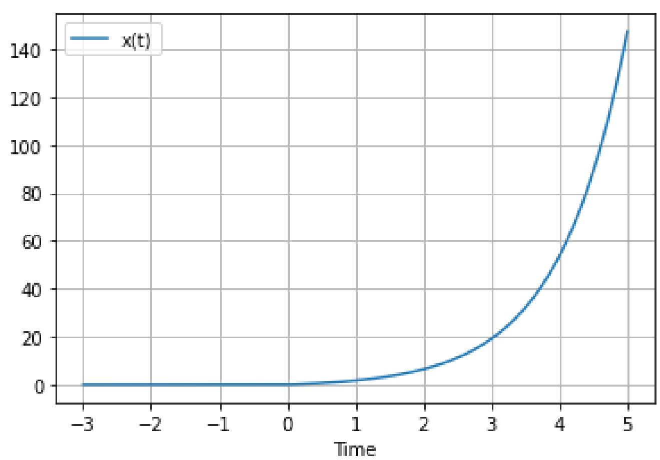

Fig. 4: plot of $y(t)$ in input domain.