EE3900 : Assignment-3

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Download all python codes from

```
https://github.com/AI20BTECH11014/EE3900−Linear−Systems−and−Signal−processing/blob/main/Assignment_3/Assignment_3.py
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and latex-tikz codes from

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https://github.com/AI20BTECH11014/EE3900−Linear−Systems−and−Signal−processing/blob/main/Assignment_3/Assignment_3.tex
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1 Question: Ramsey 4.2 Tangent and Normal Q.14

Find the points of contact of the tangents to the circle

$$\|\mathbf{x}\| = 5 \tag{1.0.1}$$

that pass through the point $\begin{pmatrix}7\\1\end{pmatrix}$ and write down the equations of the tangents.

2 Solution

The general equation of a circle can be expressed as:

$$\mathbf{x}^{\mathbf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathbf{T}}\mathbf{x} + f = 0 \tag{2.0.1}$$

Let the equation of the tangent be

$$\begin{pmatrix}-m & 1\end{pmatrix}\mathbf{x} = c \tag{2.0.2}$$

We know that, for a circle,

$$\mathbf{V} = \mathbf{I} \tag{2.0.3}$$

$$\mathbf{c} = -\mathbf{u} \tag{2.0.4}$$

Comparing the equation (1.0.1) and (2.0.1) we get

$$\mathbf{u} = \begin{pmatrix}0 & 0\end{pmatrix}, f = -25 \tag{2.0.5}$$

$$\mathbf{c} = \begin{pmatrix}0 & 0\end{pmatrix} \tag{2.0.6}$$

The normal vector to the line is obtained as

$$\lambda\mathbf{n} = \mathbf{q} + \mathbf{u} \tag{2.0.7}$$

$$\mathbf{q} = \lambda\mathbf{n} - \mathbf{u} \tag{2.0.8}$$

from the equation (2.0.2)

$$\mathbf{n} = \begin{pmatrix}-m & 1\end{pmatrix}^{\top} \tag{2.0.9}$$

from (2.0.5) and (2.0.9)

$$\mathbf{q} = \lambda\mathbf{n} \tag{2.0.10}$$

$$\mathbf{q} = \begin{pmatrix}-\lambda m\\ \lambda\end{pmatrix} \tag{2.0.11}$$

The point $\mathbf{q}$ satisfies the equation of the circle

$$\mathbf{q}^{\top}\mathbf{q} = 25 \tag{2.0.12}$$

$$\|\mathbf{q}\|^2 = 25 \tag{2.0.13}$$

$$\lambda^2(m^2+1) = 25 \tag{2.0.14}$$

$$\lambda^2 = \frac{25}{m^2+1} \tag{2.0.15}$$

If $\mathbf{P} = \begin{pmatrix}7\\1\end{pmatrix}$ be a point on the line and $\mathbf{n}$ is the normal vector, the equation of the line can be expressed as

$$\mathbf{n}^{\top}(\mathbf{x} - \mathbf{P}) = 0 \tag{2.0.16}$$

$$\tag{2.0.17}$$

The point $\mathbf{q}$ satisfies the equation of the tangent

$$\mathbf{n}^{\top}(\mathbf{q} - \mathbf{P}) = 0 \tag{2.0.18}$$

$$\mathbf{n}^{\top}\mathbf{q} - \mathbf{n}^{\top}\mathbf{P} = 0 \tag{2.0.19}$$

$$\|\mathbf{n}\|^2\lambda - \mathbf{n}^{\top}\mathbf{P} = 0 \tag{2.0.20}$$

$$\lambda = \frac{1-7m}{1+m^2} \tag{2.0.21}$$

from (2.0.15) and (2.0.21)

$$\left(\frac{1-7m}{1+m^2}\right)^2 = \frac{25}{m^2+1} \tag{2.0.22}$$

$$1+49m^2-14m = 25 \tag{2.0.23}$$

$$12m^2-7m-12 = 0 \tag{2.0.24}$$

$$m_1 = \frac{-3}{4}, m_2 = \frac{4}{3} \tag{2.0.25}$$

from (2.0.21) and (2.0.25)

$$\lambda_1 = 4, \lambda_2 = -3 \tag{2.0.26}$$

$$\tag{2.0.27}$$

the point of contact of the tangents are given as,

$$\mathbf{q}_1 = \begin{pmatrix} -\lambda_1 m_1 \\ \lambda_1 \end{pmatrix}, \mathbf{q}_2 = \begin{pmatrix} -\lambda_2 m_2 \\ \lambda_2 \end{pmatrix} \tag{2.0.28}$$

$$\mathbf{q}_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \mathbf{q}_2 = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \tag{2.0.29}$$

The required equations of tangents are

$$L_1 : \begin{pmatrix} \frac{3}{4} & 1 \end{pmatrix} \mathbf{x} = \frac{25}{4} \tag{2.0.30}$$

$$L_2 : \begin{pmatrix} \frac{-4}{3} & 1 \end{pmatrix} \mathbf{x} = \frac{-25}{3} \tag{2.0.31}$$

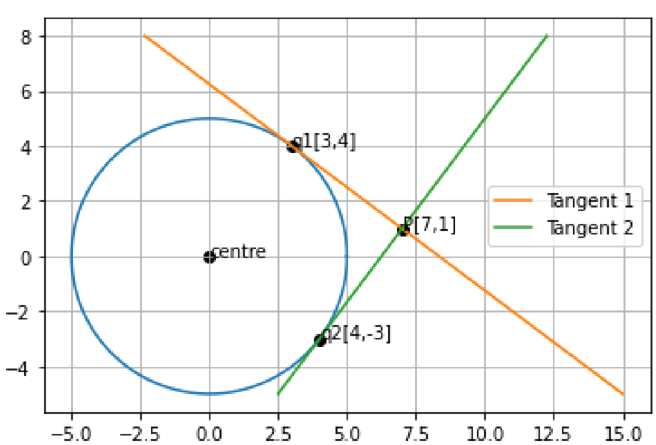

Fig. 0: The plot of tangents to the circle