Assignment-4

Manikanta vallepu - AI20BTECH11014

Download all latex-tikz codes from

https://github.com/AI20BTECH11014/EE3900−Linear−Systems−and−Signal−processing/blob/main/Assignment_4/Assignment_4.tex

1 Question: Linear Forms Q.2.14

The sum of the perpendicular distances of a variable point $\mathbf{P}$ from the lines

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 0$$

$$\begin{pmatrix} 3 & -2 \end{pmatrix}\mathbf{x} = -7$$

is always 10. Show that $\mathbf{P}$ must move on a line.

2 Solution

The foot of perpendicular from point $\mathbf{P}$ to line $\mathbf{n}^\top\mathbf{x} = c$ is given as $\mathbf{P} + \alpha\mathbf{n}$,where $\alpha \in \mathbb{R}$

$$\mathbf{n}^\top(\mathbf{P} + \alpha\mathbf{n}) = c \tag{2.0.1}$$

$$\mathbf{n}^\top\mathbf{P} + \|\mathbf{n}\|^2\alpha = c \tag{2.0.2}$$

The perpendicular distance from a point $\mathbf{P}$ to line $\mathbf{n}^\top\mathbf{x} = c$ is given as,

$$= \|\mathbf{P} - (\mathbf{P} + \alpha\mathbf{n})\| \tag{2.0.3}$$

$$= |\alpha|\|\mathbf{n}\| \tag{2.0.4}$$

using (2.0.2) in (2.0.4), perpendicular distance is given as,

$$= \frac{|c - \mathbf{n}^\top\mathbf{P}|}{\|\mathbf{n}\|} \tag{2.0.5}$$

The sum of the perpendicular distances of a variable point $\mathbf{P}$ from the lines

$$\mathbf{n_1}^\top\mathbf{x} = c_1$$

$$\mathbf{n_2}^\top\mathbf{x} = c_2$$

is always d,then,

$$\frac{|c_1 - \mathbf{n_1}^\top\mathbf{P}|}{\|\mathbf{n_1}\|} + \frac{|c_2 - \mathbf{n_2}^\top\mathbf{P}|}{\|\mathbf{n_2}\|} = d \tag{2.0.6}$$

$$\pm\left(\frac{c_1 - \mathbf{n_1}^\top\mathbf{P}}{\|\mathbf{n_1}\|}\right) \pm \left(\frac{c_2 - \mathbf{n_2}^\top\mathbf{P}}{\|\mathbf{n_2}\|}\right) = d \tag{2.0.7}$$

given,

$$\pm\left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{P}\right) \pm \left(\frac{1}{\sqrt{13}}\left(\begin{pmatrix} 3 & -2 \end{pmatrix}\mathbf{P} + 7\right)\right) = 10 \tag{2.0.8}$$

$\therefore$ point $\mathbf{P}$ lies on either of the lines

$$L_1 : \begin{pmatrix} \frac{1}{\sqrt{2}} + \frac{3}{\sqrt{13}} & \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{13}} \end{pmatrix}\mathbf{P} = 10 - \frac{7}{\sqrt{13}} \tag{2.0.9}$$

$$L_2 : \begin{pmatrix} \frac{1}{\sqrt{2}} - \frac{3}{\sqrt{13}} & \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{13}} \end{pmatrix}\mathbf{P} = 10 + \frac{7}{\sqrt{13}} \tag{2.0.10}$$

$$L_3 : \begin{pmatrix} \frac{1}{\sqrt{2}} + \frac{3}{\sqrt{13}} & \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{13}} \end{pmatrix}\mathbf{P} = -10 - \frac{7}{\sqrt{13}} \tag{2.0.11}$$

$$L_4 : \begin{pmatrix} -\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{13}} & -\frac{1}{\sqrt{2}} - \frac{2}{\sqrt{13}} \end{pmatrix}\mathbf{P} = 10 - \frac{7}{\sqrt{13}} \tag{2.0.12}$$