Assignment-4

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1 QUESTION: LINEAR FORMS Q.2.14

The sum of the perpendicular distances of a variable point $\mathbf{P}$ from the lines

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 0$$

$$\begin{pmatrix} 3 & -2 \end{pmatrix} \mathbf{x} = -7$$

is always 10. Show that $\mathbf{P}$ must move on a line.

2 SOLUTION

The foot of perpendicular from point $\mathbf{P}$ to line $\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 0$ is given as $\mathbf{P} + \alpha_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, where $\alpha_1 \in \mathbb{R}$

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\left(\mathbf{P} + \alpha_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = 0 \qquad (2.0.1)$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{P} + 2\alpha_1 = 0 \qquad (2.0.2)$$

The foot of perpendicular from point $\mathbf{P}$ to line $\begin{pmatrix} 3 & -2 \end{pmatrix} \mathbf{x} = -7$ is given as $\mathbf{P} + \alpha_2 \begin{pmatrix} 3 \\ -2 \end{pmatrix}$, where $\alpha_2 \in \mathbb{R}$

$$\begin{pmatrix} 3 & -2 \end{pmatrix}\left(\mathbf{P} + \alpha_2 \begin{pmatrix} 3 \\ -2 \end{pmatrix}\right) = -7 \qquad (2.0.3)$$

$$\begin{pmatrix} 3 & -2 \end{pmatrix} \mathbf{P} + 13\alpha_2 = -7 \qquad (2.0.4)$$

Given,

$$\left\|\mathbf{P} - \left(\mathbf{P} + \alpha_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)\right\| + \left\|\mathbf{P} - \left(\mathbf{P} + \alpha_2 \begin{pmatrix} 3 \\ -2 \end{pmatrix}\right)\right\| = 10 \qquad (2.0.5)$$

$$\sqrt{2}|\alpha_1| + \sqrt{13}|\alpha_2| = 10 \qquad (2.0.6)$$

From equations (2.0.2) , (2.0.4) and (2.0.6),

$$\frac{1}{\sqrt{2}}|\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{P}| + \frac{1}{\sqrt{13}}|\begin{pmatrix} 3 & -2 \end{pmatrix} \mathbf{P} + 7| = 10 \quad (2.0.7)$$

$\therefore$ point $\mathbf{P}$ lies on either of the lines

$$L_1 : \left(\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{13}} \quad \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{13}}\right)\mathbf{P} = 10 - \frac{7}{\sqrt{13}} \qquad (2.0.8)$$

$$L_2 : \left(\frac{1}{\sqrt{2}} - \frac{3}{\sqrt{13}} \quad \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{13}}\right)\mathbf{P} = 10 + \frac{7}{\sqrt{13}} \qquad (2.0.9)$$

$$L_3 : \left(\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{13}} \quad \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{13}}\right)\mathbf{P} = -10 - \frac{7}{\sqrt{13}} \qquad (2.0.10)$$

$$L_4 : \left(-\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{13}} \quad -\frac{1}{\sqrt{2}} - \frac{2}{\sqrt{13}}\right)\mathbf{P} = 10 - \frac{7}{\sqrt{13}} \qquad (2.0.11)$$