

GATE Assignment 3

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Download all latex codes from

https://github.com/AI20BTECH11014/EE3900-Linear-Systems-and-Signal-processing/tree/main/Gate_Assignment_3

1 GATE EC-2005 Q.62

For a signal $x(t)$ the Fourier transform is $X(f)$. then the inverse Fourier transform of $X(f+2)$ is given by

- (A) $\frac{1}{2}x\left(\frac{t}{2}\right)e^{j3\pi t}$
- (B) $\frac{1}{3}x\left(\frac{t}{3}\right)e^{-\frac{j4\pi t}{3}}$
- (C) $3x(3t)e^{-j4\pi t}$
- (D) $x(3t+2)$

SOLUTION

By the time scaling property of Fourier transform,

$$x(\alpha t) \xrightarrow{\mathcal{F}} \frac{1}{|\alpha|} X\left(\frac{f}{|\alpha|}\right) \quad (1.0.1)$$

$$x\left(\frac{t}{3}\right) \xrightarrow{\mathcal{F}} 3X(3f) \quad (1.0.2)$$

By the time shifting property of Fourier transform,

$$e^{-j2\pi f_0 t} x(t) \xrightarrow{\mathcal{F}} X(f+f_0) \quad (1.0.3)$$

$$e^{-\frac{j4\pi t}{3}} x(t) \xrightarrow{\mathcal{F}} X\left(f+\frac{2}{3}\right) \quad (1.0.4)$$

using (1.0.2),

$$e^{-\frac{j4\pi t}{3}} x\left(\frac{t}{3}\right) \xrightarrow{\mathcal{F}} 3X\left(3\left(f+\frac{2}{3}\right)\right) \quad (1.0.5)$$

$$\frac{1}{3}e^{-\frac{j4\pi t}{3}} x\left(\frac{t}{3}\right) \xrightarrow{\mathcal{F}} X(3f+2) \quad (1.0.6)$$

Let $x(t) = \text{rect}(t)$ and $\text{sinc}(t) = \frac{\sin(t)}{t}$

$$X(f) = \text{sinc}(f) \quad (1.0.7)$$

For $X(f) = \text{sinc}(3f+2)$

$$\frac{1}{3}e^{-\frac{j4\pi t}{3}} x\left(\frac{t}{3}\right) = \frac{1}{3}e^{-\frac{j4\pi t}{3}} \text{rect}\left(\frac{t}{3}\right) \quad (1.0.8)$$

The correct option is (B).