QUALITATIVE STUDY OF PROPERTIES OF NONLNEAR DIFFERENTAL EQUATIONS USING PARAMETRIC SYSTEM OF COUPLED CIRCUITS IN WOLFRAM MATHEMATICA

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5TH NOVEMBER, 2023

Abstract

This research work is concerned with *Qualitative Study Of Properties Of Nonlinear Systems Using Parametric Systems Of Coupled Circuits* where the unknown functions X and Y are matrix valued ($X, Y \in \mathbb{R}^{n\times m}$) functions. The most effective method to determine the stability and boundedness of solutions of differential equations is the Lyapunov Second or Direct Method. This involves the construction of a scalar function, V(X,Y) which is positive definite and its derivatives $\dot{V}(X, Y)$ which is negative definite along the trajectory of the system considered. When these properties of V(X,Y) and $\dot{V}(X, Y)$ are shown to be satisfied then the behaviors of the system are known.

BACKGROUND TO THE STUDY

Calculus was one of the major achievement around seventeenth century. Through its invention, major branches of mathematics came into existence; examples are Differential equations, Infinite Series, Complex Analysis, Calculus of Variation, Differential Geometry and many others. Among all these, Differential equations are the most widely used in social sciences, management, medicine and applied sciences. Differential Equations is a mathematical equations that relates some functions with its derivatives, the functions usually represent physical quantities, the derivatives represents rates of changes, the equation defines the relationship between the two. Differential equations serve as a very useful tool in engineering, medicine agricultural sciences, physical and social sciences etc

On the other hand Wolframe Mathematica is a soft ware system with in- built libraries for several areas of technical computing that allow machine learning, statistics, symbolic computation, data manipulation, network analysis, optimization,plotting functions, time series analysis, etc.

BACKGROUND TO THE STUDY

The analysis of differential equations is subdivided into quantitative and the qualitative theory. The quantitative theory of differential equations is much concerned with finding solutions of differential equations while qualitative study involves the study of the properties and the behavior of the solutions of equations without finding the solutions. The qualitative study was initiated in the 19th century by H.Poincare and A.M Lyapunov. We are to use Wolfram Mathematical to aid some of these analysis.

BACKGROUND TO THE STUDY

One of the central properties in system and control theory engineering is stability. Practically, one of the most important properties that a system must satisfy is that it has to be stable, otherwise the system is useless and potentially chaotic [19]. The theory of stability has got rich result and could be widely used in concrete problems of the real world.

Our method of approach to study the qualitative properties is the use of Lyapunov's second method which comes from the original work of Lyapunov in 1892, more than a century ago, but only in the half century has this concept been appreciated to the point where workers in the area of stability of dynamical systems and automatic control are aware of its application [19]. The application of the Lyapunov method lies in constructing a scalar function (say V and its derivatives such that they possess certain properties. When these properties of V and $\dot{V}$ are shown to be satisfied, the stability behavior of the system is known [9]. This research is concerned with the Qualitative study of parametric system of two coupled circuit by Lyapunov Methods in terms of the use of Machine learning.

Our motivation came from Biryuk et al[6] where he used Lyapunov method to obtain the Stability Of Parametric System Of two Coupled Circuits with External Conductive Connection where he considered only the homogeneous vector equations (i.e natural process) and left the heterogeneous vector equations as an open problem . With reference to our observation in the relevant literature, the result in this direction do not exist! Our aim in this project is to obtain stability property of the parametrical system of two coupled circuits using an approach different from the one used by Biryuk et al [6] and also extend his results to heterogeneous system of differential equations by obtaining new conditions for stability and boundedness of the state variables q_1, q_2, Φ_1, Φ_2. In this project we make use of Lyapunov Second or Direct Method.

The purpose of this research study include using machine learning Wolfram Mathematica

1. To review and understand Lyapunov Stability Theory in non-linear and dynamical systems .
2. To obtain sufficient conditions that guarantee the stability of state variables, q_1,q_2, Φ_1,Φ_2 describing the system of two coupled circuit .
3. To obtain conditions that guarantees boundedness of the state variables q_1,q_2, Φ_1,Φ_2 describing the system of two coupled circuits.

The significance of the study is to further improve the stability of a given system and the boundedness of the state variables describing the parametric system of two circuits and the effect on the external conductive connection.

Properties of Solutions of non-linear Ordinary Differential Equations

So many researchers and authors have studied properties of solutions of differential equations of various order (second, third, fourth etc) in scalar and in vector form but only few results we were obtained using coupled circuit analysis See ([1], [2], [3],[4][5]... Olutimo [16] also considered the theory of stability and boundedness analysis using a system of RLC circuit model with a time varying state space method. The Lyapunov method gives an avenue to analyse the behavior of the system of RLC circuit. He considered a system of heterogeneous differential equation and reduced it to a stability problem. He proved boundedness of the variables characterising the system of RLC circuit based on some assumptions.

Biryuk et.al [6] based on Lyapunov's stability theory of a homogeneous differential equation, he analysed a stability provision problem using a Parametric System of two coupled circuits of a communication system. He obtained a four system of differential equations of first order which he reduced to a stability problem. These equations he represented in the form

$$\frac{d}{dt}(X) = A(t) + f(t),$$

, where X is a column vector, A(t) a 4×4 matrix and f(t) a free column vector. Using a natural process he considered a homogeneous differential equations where $f(t) = 0$. He proved stability and asymptotic stability using an incomplete Lyapunov function to obtain certain conditions that gives the stability of the parametric system of the coupled circuits. Works on Lyapunov Stability theory using electrical or coupled circuits are few (see []). Hence, further study on the subject is worthwhile. In this research, we will be improving and extending the above results (Olutimo [16],[17], Biryuk et.al [6]) to a system of

Theorem 1 & 2

Theorem

Assume there exists a function V defined for $t \geq T$, δ_0 a non - negative constant, $|X| < \delta_0$ with the following properties;

1. *V(t,X)* $\equiv$ *0, if* $X = 0$
2. *V(t,X)* $\geq$ *a(*$|X|$*) where a(r) is monotonically increasing and a(0) = 0*
3. $\dot{V}(t, X) \leq 0$

then the zero solution $X \equiv 0$ *is stable. [15]*

Theorem

In addition to the assumptions of theorem 1, assumed that

$$V(\dot{t}, X) \quad \leq \quad -c(|\ X\ |),$$

then the zero solution above is asymptotically stable (AS)

Theorem

Suppose there exists a Lyapunov function V(t,X) defined on $I \times \Re_n$ *which satisfies the following conditions :*

1. $a(\mid X \mid \leq V(t,X), where a(r) is continuously monotone increasing and$ *a(0)=0,t*
   $0,$

then the solution of equations are bounded.

.

An electrical circuit is said to be coupled when there exist a mutual inductance between the coils present in that circuit. Coil is nothing but seen as combination of resistors and inductors. In the absence of resistor coil becomes an inductor. Coupling can either be be electrical coupling or magnetic coupling. An electrical coupling is a physical connection between two inductors and coils while magnetic coupling involves no physical connection. Also coupling is the transfer of electrical energy from one circuit segment to the other. In a coupled circuit current flow is considered. The flow of current in a circuit involves the following components; Resistor is an element that opposes current flow. It is a current limiting device in an electrical circuit. Capacitor stores electric charges in form of static field. Inductor is a charge storing device in form of magnetic field. A transistor is a semiconductor for amplification and sometimes for rectification.

A PARAMETRICAL SYSTEM OF TWO COUPLED CIRCUITS

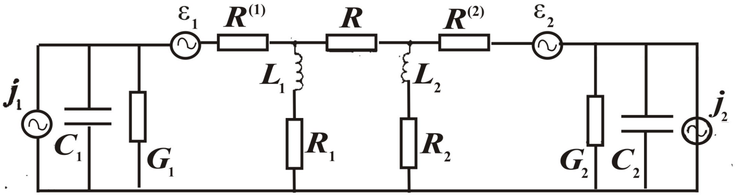

Fig. 1.

2

Figure: Fig.1

The qualitative Theory of Parametric System of Coupled Circuit is a complicated one. If we consider the input, the state variables, and their response, they are varied. The out response in the coupled circuit tends to zero as time tends to infinity. A situation where out response in the circuit tends to infinity the coupled circuit is said to be unstable. The undesirable case can be detected theoretically and eliminated at development stage using Lyapunov Stability Theory. In this chapter we consider the stability of the state variables present in the coupled circuit. The energy storage elements of a system are what make a system dynamic. The capacitor stores energy in the electric field while the inductor stores energy in the magnetic field. According to [18] state variables are minimum set of variables that fully describes the circuit system and its response to any set of inputs. The qualitative properties of the parametric system in Fig.1 is thus characterized by the stability of the state variables q_1, Φ_1, Φ_2, q_2.

We consider the parametric system of coupled circuit with external conductive connection in Fig.1, the term $R^{(1)}$, $R^{(2)}$ are the internal resistances ; the energy sources are the current sources, j_1, j_2 and the voltage sources, ξ_1, ξ_2. C_1, C_2 and L_1, L_2 are respectively Capacitances and Inductances. G_1, G_2 are Conductances. R, R_1, R_2 are resistances and q_1, q_2, are the electric charges from the capacitors and Φ_1, Φ_2 are magnetic fluxes in the inductor coils. We suppose the circuit elements : resistances, capacitances, inductances are positive. The first time derivative of inductance and capacitance exists. j_1, j_2, and ξ_1, ξ_2 are undefined.

Mathematical equation of the ($Fig.1$) can be obtained by applying Kirchoff's law to the coupled circuit in ($Fig.1$) and by KCL and KVL in [18], we obtain a system of differential equations ;

$$
\begin{aligned}
\frac{dq_1}{dt} &= -\left(G_1 + \frac{1}{R+R^{(1)}+R^{(2)}}\right)\frac{q_1}{C_1} - \frac{R+R^{(2)}}{R+R^{(1)}+R^{(2)}}\frac{\Phi_1}{L_1} + \frac{R^2}{R+R^1+R^2}\frac{\Phi_2}{L_2} \\
&+ \frac{1}{R+R^{(1)}+R^{(2)}}\frac{q_2}{C_2} + j_1 + \frac{-\xi_1+\xi_2}{R+R^{(1)}+R^{(2)}} \\
\frac{d\Phi_1}{dt} &= \frac{R+R^{(2)}}{R+R^{(1)}+R^{(2)}}\frac{q_1}{C_1} - \left(R_1 + R^{(1)}\frac{R+R^{(2)}}{R+R(1)+R^{(2)}}\right)\frac{\Phi_1}{L_1} \\
&- \frac{R^{(1)}R^{(2)}}{R+R^{(1)}+R^{(2)}}\frac{\Phi^2}{L_2} + \frac{R^{(1)}}{R+R^{(1)}+R^{(2)}}\frac{q_2}{C_2}\frac{(R+R^1)\xi_1 + R^{(1)}\xi_2}{R+R^{(1)}+R^{(2)}} \\
\frac{d\Phi_2}{dt} &= \frac{R^{(2)}}{R+R^{(1)}+R^{(2)}}\frac{q_1}{C_1} - \frac{R^{(1)}R^{(2)}}{R+R^{(1)}+R^{(2)}}\frac{\Phi_1}{L_1} - \left(R_2 + R^{(2)}\frac{R+R^{(1)}}{R+R^{(1)}+R^{(2)}}\right)\frac{\Phi_2}{L_2} \\
&+ \frac{R+R^{(1)}}{R+R^{(1)}+R^{(2)}}\frac{q_2}{C_2} + \frac{R^{(2)}\xi_1 + (R+R^{(1)})\xi_2}{R+R^{(1)}+R^{(2)}} \\
\frac{dq_2}{dt} &= \frac{1}{R+R^{(1)}+R^{(2)}}\frac{q_1}{C_1} - \frac{R^{(1)}}{R+R^{(1)}+R^{(2)}}\frac{\Phi_1}{L_1} + \frac{R+R^{(1)}}{R+R^{(1)}+R^{(2)}}\frac{\Phi_2}{L_2} \\
&- \left(G_2 + \frac{1}{R+R^{(1)}+R^{(2)}}\right)\frac{\Phi_2}{L_2} + j_2 + \frac{\xi_1\xi_2}{R+R^{(1)}+R^{(2)}}.
\end{aligned}
$$

Nonlinear Systems of coupled circuits

$$
\begin{aligned}
\Psi_{11} &= \frac{-G_1 - \frac{1}{(R+R^{(1)}+R^{(2)})}}{C_1}, \Psi_{12} = \frac{1}{C_2(R+R^{(1)}+R^{(2)})} \Psi_{13} = \frac{-R+R^{(2)}}{L_1(R+R^{(1)}+R^{(2)})} \\
\Psi_{14} &= \frac{R^{(2)}}{L_2(R+R^{(1)}+R^{(2)})}, \Psi_{21} = \frac{R+R^{(2)}}{C_1(R+R^{(1)}+R^{(2)})}, \Psi_{22} = \frac{R^{(1)}}{C_2(R+R^{(1)}+R^{(2)})} \\
\Psi_{23} &= \frac{-R^{(1)}(R+R^{(2)})}{L_1(R+R^{(1)}+R^{(2)})}, \Psi_{24} = \frac{R^{(1)}R^{(2)}}{L_2(R+R^{(1)}+R^{(2)})}, \Psi_{31} = \frac{R^{(2)}}{C_1(R+R^{(1)}+R^{(2)})} \\
\Psi_{32} &= \frac{R+R^{(1)}}{C_2(R+R^{(1)}+R^{(2)})}, \Psi_{33} = \frac{R^{(1)}R^{(1)}}{L_1(R+R^{(1)}+R^{(2)})}, \Psi_{34} = \frac{R_2 + \frac{(R+R^{(1)})R^{(2)}}{(R+R^{(1)}+R^{(2)})}}{L_2} \\
\Psi_{41} &= \frac{1}{C_1(R+R^{(1)}+R^{(2)})}, \Psi_{42} = \frac{G_2 + \frac{1}{(R+R^{(1)}+R^{(2)})}}{C_2}, \Psi_{43} = \frac{R^{(1)}}{C_2(R+R^{(1)}+R^{(2)})} \\
\Psi_{44} &= \frac{R+R^{(1)}}{L_2(R+R^{(1)}+R^{(2)})}.
\end{aligned}
$$

Nonlinear variables of Parametric system of the coupled circuits

We can use the mathematica code of the form $D[\{-aw(t) - 0.093x(t) - 4.333z(t) + 0.791y, \{-bx(t) + 0.907w(t) - 0.0837y(t) + 0.093z(t)\}, -cy(t) + 0.209w(t) - 0.0837x(t) + 0.791z(t), -dz(t) + 0.233w(t) - 0.093x(t) + 0.791y(t)\}, \{w, x, y, z\}, 1]$

Next we find the Jacobian matrix of (3.4), represented by

$$J\left(A\right) = \begin{pmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \frac{\partial f_1}{\Phi_1} & \frac{\partial f_1}{\partial \Phi_2} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \frac{\partial f_2}{\Phi_1} & \frac{\partial f_2}{\partial \Phi_2} \\ \frac{\partial f_3}{\partial q_1} & \frac{\partial f_3}{\partial q_2} & \frac{\partial f_3}{\Phi_1} & \frac{\partial f_3}{\partial \Phi_2} \\ \frac{\partial f_4}{\partial q_1} & \frac{\partial f_4}{\partial q_2} & \frac{\partial f_2}{\Phi_1} & \frac{\partial f_4}{\partial \Phi_2} \end{pmatrix}$$

where

$$J\left(A\right) = J(f_1, f_2, f_3, f_4).$$

Thus, we have

$$J\left(A\right) = \begin{pmatrix} \frac{-G_1 - \frac{1}{(R+R^{(1)}+R^{(2)})}}{C_1} & \frac{1}{C_2(R+R^{(1)}+R^{(2)})} & \frac{-R+R^{(2)}}{L_1(R+R^{(1)}+R^{(2)})} & \frac{R^{(2)}}{L_2(R+R^{(1)}+R^{(2)})} \\ \frac{R+R^{(2)}}{C_1(R+R^{(1)}+R^{(2)})} & \frac{R^{(1)}}{C_2(R+R^{(1)}+R^{(2)})} & \frac{-R^{(1)}(R+R^{(2)})}{L_1(R+R^{(1)}+R^{(2)})} & \frac{R^{(1)}R^{(2)}}{L_2(R+R^{(1)}+R^{(2)})} \\ \frac{R^{(2)}}{C_1(R+R^{(1)}+R^{(2)})} & \frac{R+R^{(1)}}{C_2(R+R^{(1)}+R^{(2)})} & \frac{R^{(1)}R^{(1)}}{L_1(R+R^{(1)}+R^{(2)})} & \frac{R_2 + \frac{(R+R^{(1)})R^{(2)}}{(R+R^{(1)}+R^{(2)})}}{L_2} \\ \frac{1}{C_1(R+R^{(1)}+R^{(2)})} & \frac{G_2 + \frac{1}{(R+R^{(1)}+R^{(2)})}}{C_2} & \frac{R^{(1)}}{C_2(R+R^{(1)}+R^{(2)})} & \frac{R+R^{(1)}}{L_2(R+R^{(1)}+R^{(2)})}. \end{pmatrix}$$

Fig12 *Jacobian Matrix*

We now find the determinant of the Jacobian matrix represented by $\text{Det}\Big(J(A)\Big)$.

$$\begin{aligned}
DetJ\Big((A)\Big) &= \frac{1}{C_1C_2L_1L_2(R+R^{(1)}+R^{(2)})^2}((1+G_1R^{(1)})(-R(R+R^1)\\
&+ R(1+G_2(R+R^{(1)}))R^{(2)}+RG_2(R^{(2)})^2+R_2(R+R(1)+R^{(2)})(1\\
&+ G_2(R+R^{(2)})))+R_1((-1+G_2R_2)(R+R_{(1)})\\
&+ (-1+G_2(R+R(1)+R^{(2)}))R^{(2)}-G_2(R^{(2)})^2+G_1(R_2(R+R(1)\\
&+ R^{(2)})(1+G_2(R+R(1)+R^{(2)}))+(R+R^{(1)})(-R+R^{(1)}+G_2R^{(2)}\\
&+ R^{(2)}(1+G_2(R+R^2)))))).
\end{aligned}$$

$$Det\Big(J(A)\Big) > 0.$$

Fig12 *Determinant of the Nonlinear Systems of coupled circuits*

$$Det\left(J(A)\right) > 0.$$

$$Trace\left(J(A)\right) = -\frac{G_1 + \frac{1}{R+R(1)+R^{(2)}}}{C_1} + \frac{L_1L_2 + C_2(L_1(R+R^{(1)})) - L_2R^{(1)}R^{(2)}}{C_2L_1L_2(R+R(1)+R^{(2)})} \quad (11.1)$$

If the equilibrium points of the differential equations is such that the Determinant, Det$\left(\text{J(A)}\right) > 0$ and the Trace of the Jacobian matrix, Trace$\left(\text{J(A)}\right) < 0$ the system of the equation above is asymptotically stable based on the condition,

$$R^{(1)} + G_2R^{(2)} + R^{(2)} > R$$

$$G_2R_2 > 1$$

```
s = NDSolve[{w'[t] == -0.6174*w[t] - 0.4565*x[t] + 0.3261*y[t] + 0.2174*z[t], x'[t] == 0.4565*w[t] - 5.1413*x[t] - 0.8152*y[t] + 0.5335*z[t],
   y'[t] == 0.3261*w[t] - 0.8152*x[t] - 4.011*y[t] + 0.6739*z[t], z'[t] == 0.217*w[t] - 0.5435*x[t] + 0.6739*y[t] - 0.4174*z[t], w[0] == -37.24,
   x[0] == -13.08, y[0] == -19.64, z[0] == -99.87}, {w, x, y, z}, {t, 0, 100}]
a = Plot[Evaluate[w[t] /. s], {t, 0, 100}, PlotStyle → {Thick, Red}, PlotLabels → Placed[{"w(t)"}, Above], PlotLegends → {"w(t)"}]
b = Plot[Evaluate[x[t] /. s], {t, 0, 100}, PlotStyle → {Thick, Blue}, PlotLabels → Placed[{"x(t)"}, Below], PlotLegends → {"x(t)"}]
c = Plot[Evaluate[y[t] /. s], {t, 0, 100}, PlotStyle → {Thick, Green}, PlotLabels → Placed[{"y(t)"}, Above], PlotLegends → {"y(t)"}]
d = Plot[Evaluate[z[t] /. s], {t, 0, 100}, PlotStyle → {Thick, Black}, PlotLabels → Placed[{"z(t)"}, Above], PlotLegends → {"z(t)"}]
Show[{a, b, c, d}]
```

Fig12 *Wolfram code*

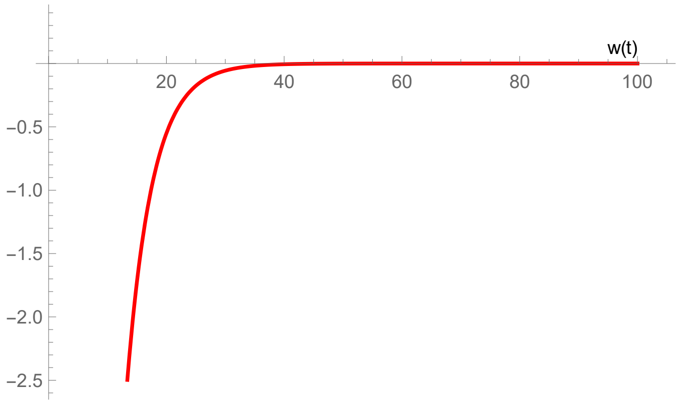

Fig1 *Plot showing the state variable q_1 of the coupled circuit is stable*

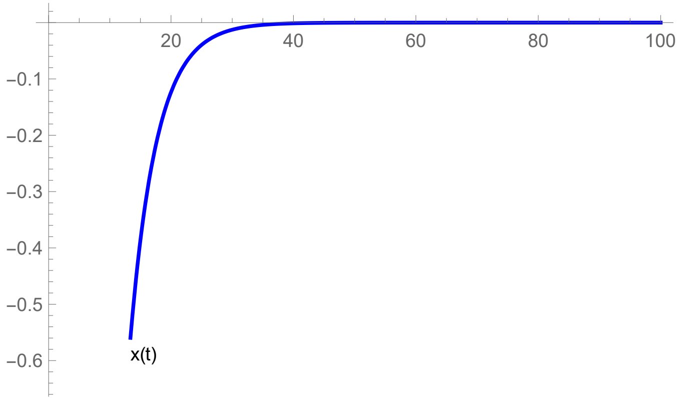

Fig2 *Plot showing the state variable q_2 of the coupled circuit is asymptotically stable*

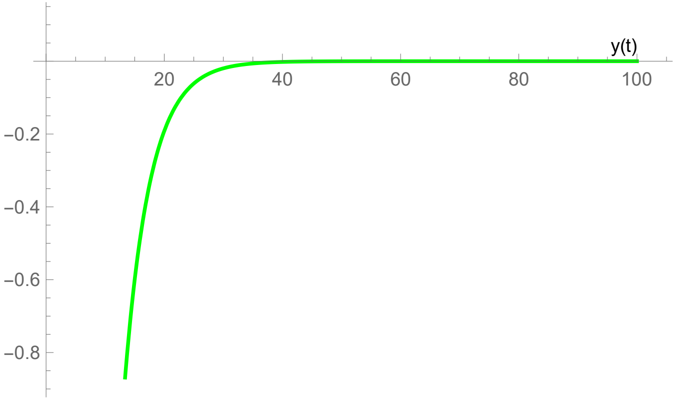

Fig3 *Plot showing the state variable Φ_1 of the coupled circuit is asymptotically stable*

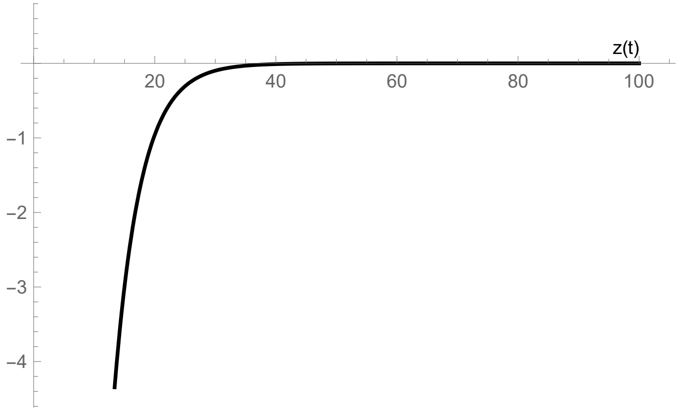

Fig4 *Plot showing the state variable Φ_2 of the coupled circuit is asymptotically stable*

Conclusion

THe state variables controlling the parametric system of coupled circuits are asymptotically stable based on the given conditions.

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Special Thanks

THANK YOU !