Building a High-Resolution Earthquake Catalog from Raw Waveforms: A Step-by-Step Guide
Template Matching

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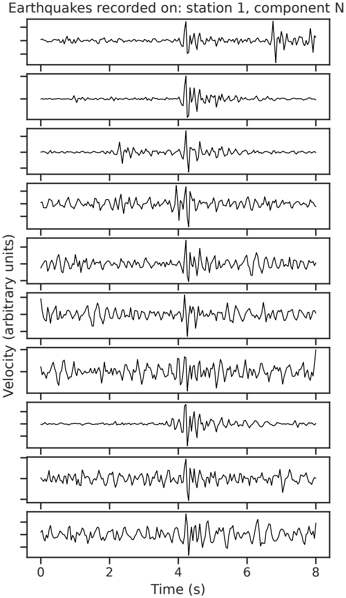
Outline for section 1

1. Introduction

2. Demonstration
   - Notebook 1
   - Notebook 2
   - Notebook 3 (bonus)

Earthquake signals often come in "families"



Family = near-repeats of the same waveform.

Can we use the similarity of these signals to our advantage to design an earthquake detection algorithm?

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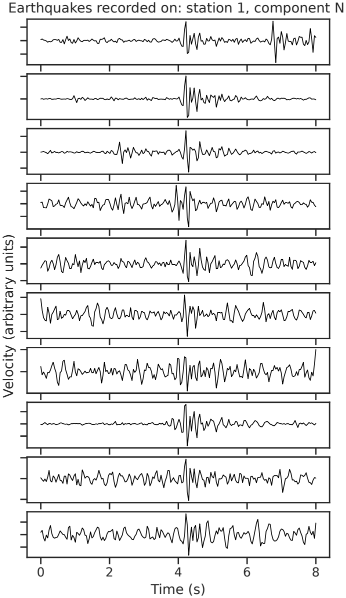

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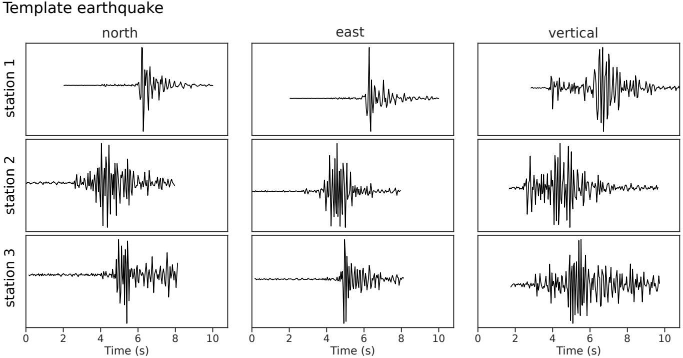
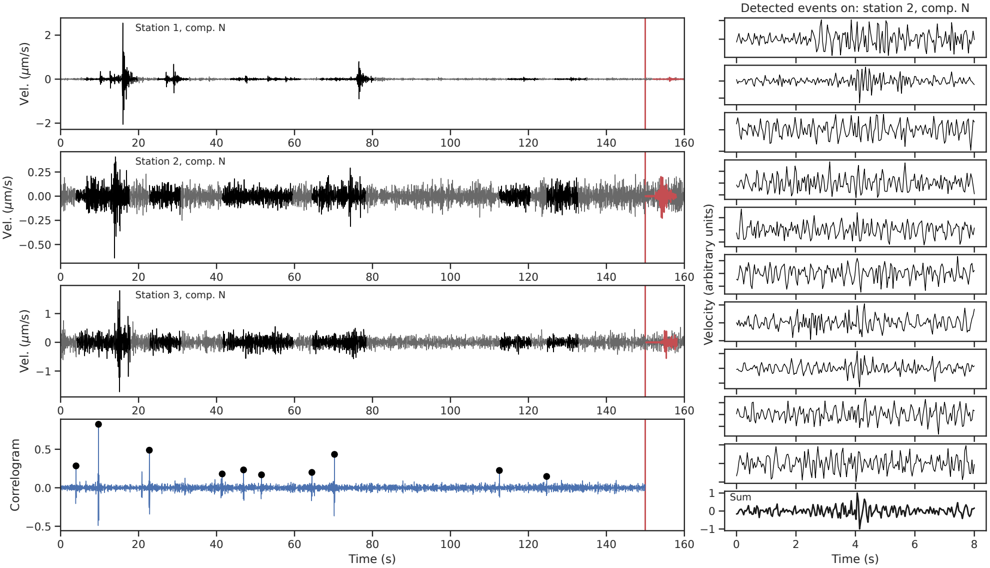


Looking for repeating patterns (template matching)

All earthquakes detected by the same template are **co-located**.



Similar earthquakes share same mechanism and location

Seismic signals result from the combination of:
source + **propagation** (including local site effects) + **instrument response**

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$$\underbrace{u(t;r)}_{\textit{seismogram}} = \underbrace{S(t;\xi,t_0)}_{\textit{source}} * \underbrace{G(t-t_0,r-\xi)}_{\textit{propagation}} * \underbrace{I(t)}_{\textit{instrument}} \tag{1}$$

- t: recording time.
- r: recording location.
- ξ: source location.
- t_0: source origin time.

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$\Rightarrow$ Similar waveforms and moveouts **=** Similar source location and mechanism

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Template matching uses the Pearson correlation coefficient, CC, to measure the similarity between the template waveforms, T, and the continuous seismograms, u:

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$$CC(t) = \sum_{s,c} w_{s,c} \sum_{i=1}^{N} \frac{T^*_{s,c}(n\Delta t) u^*_{s,c}(t + \tilde{\tau}_{s,c} + n\Delta t)}{\sqrt{\sum_{i=1}^{N} {T^*_{s,c}}^2(n\Delta t) \sum_{i=1}^{N} {u^*_{s,c}}^2(t + \tilde{\tau}_{s,c} + n\Delta t)}} \tag{2}$$

- Δt: Sampling time.
- $\tilde{\tau}_{s,c}$: Moveout on station s and channel c.
- $T^*_{s,c}$: Centered template waveform, $T^*_{s,c} = T_{s,c} - \frac{1}{N} \sum_{i=1}^{N} T_{s,c}(n\Delta t)$.
- $u^*_{s,c}(t)$: Centered continuous seismogram, $u^*_{s,c}(t) = u_{s,c}(t) - \frac{1}{N} \sum_{i=1}^{N} u_{s,c}(t + n\Delta t)$.
- $w_{s,c}$: Weight, e.g., $w_{s,c} = \frac{1}{N_s N_c}$.

$CC(t) = +1$
perfect match

$CC(t) = 0$
nothing in commmon

$CC(t) = -1$
perfect match with inverse polarity

Measuring waveform similarity

Template matching uses the Pearson correlation coefficient, CC, to measure the similarity between the template waveforms, T, and the continuous seismograms, u:

$$CC(t) = \sum_{s,c} w_{s,c} \sum_{i=1}^{N} \frac{T^*_{s,c}(n\Delta t)\, u^*_{s,c}(t + \tilde{\tau}_{s,c} + n\Delta t)}{\sqrt{\sum^{N} T^{*\,2}_{s,c}(n\Delta t) \sum^{N} u^{*\,2}_{s,c}(t + \tilde{\tau}_{s,c} + n\Delta t)}} \qquad (2)$$

Template matching is extremely computation intensive!

$\rightarrow$ requires **High Performance Computing** $\leftarrow$

- $u^*_{s,c}(t)$: Centered continuous seismogram, $u^*_{s,c}(t) = u_{s,c}(t) - \frac{1}{N} \sum_{i=1} u_{s,c}(t + n\Delta t)$.
- $w_{s,c}$: Weight, e.g., $w_{s,c} = \frac{1}{N_s N_c}$.

$CC(t) = +1$	$CC(t) = 0$	$CC(t) = -1$
perfect match	nothing in commmon	perfect match with inverse polarity

Why template matching? And when can I use it?

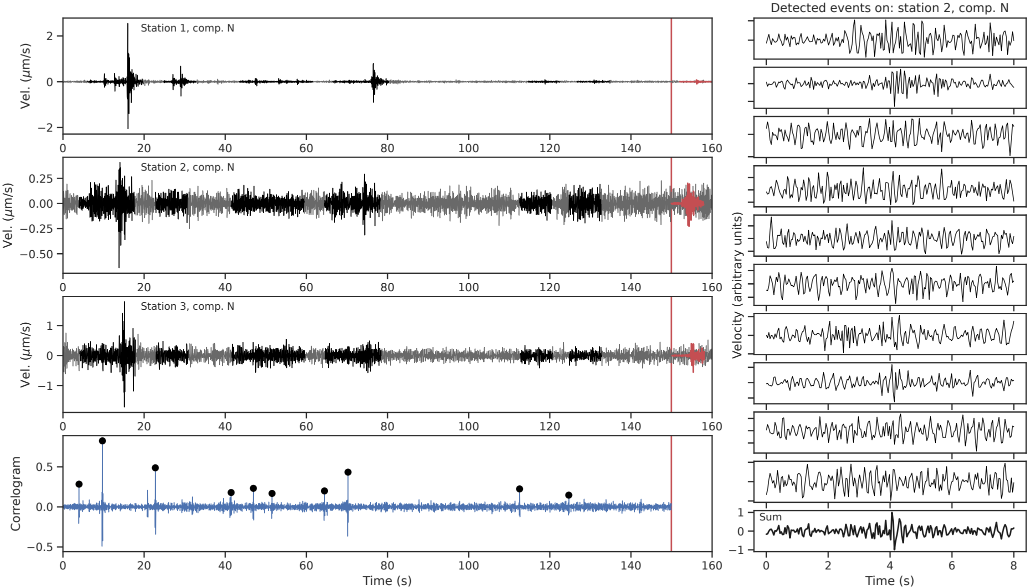
WHY?

- Searching for a *specific* seismic waveform allows to find *very small* events.
- Event location is known approximately, even for very small events.

WHEN?

- Requires a starting catalog!
- For example: densify a regional catalog or a deep learning catalog.

Next: Implementing template matching with `Fast Matched Filter`



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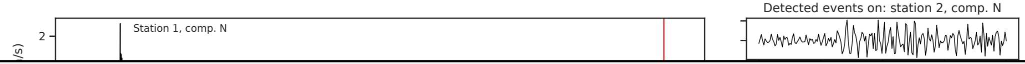


`Fast Matched Filter`

RESEARCH ARTICLE | DECEMBER 20, 2017

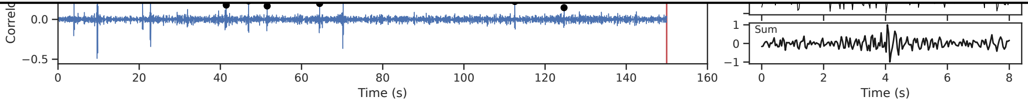
Fast Matched Filter (FMF): An Efficient Seismic Matched-Filter Search for Both CPU and GPU Architectures

Eric Beaucé; William B. Frank; Alexey Romanenko

+ Author and Article Information

Seismological Research Letters (2018) 89 (1): 165–172.
https://doi.org/10.1785/0220170181 | Article history

Other software: `EQCorrScan`, `Obspy`, `etc.`



Outline for section 2

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Hands-on demonstration with Jupyter notebooks

- Notebook 1: Introduction to template matching with a single template.
- Notebook 2: Application with multiple templates over a single day.
- Notebook 3 (bonus): Common issue with detection threshold.

Github repository:
`https://github.com/AI4EPS/Earthquake_Catalog_Workshop`

Example of template matching with a single template

`https://ai4eps.github.io/Earthquake_Catalog_Workshop/notebooks/tm_one_template/`

Github repository:

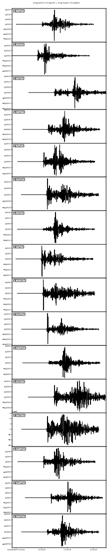
`https://github.com/AI4EPS/Earthquake_Catalog_Workshop/blob/main/notebooks/tm_one_template.ipynb`

Outline of `tm_one_template.ipynb`:

- Step 1: Build the template event from catalog data.
- Step 2: Compute the time series of correlation coefficients with `Fast Matched Filter`.
- Step 3: Choose and apply a detection threshold.
- Step 4: Extract detected events, build catalog and plot waveforms.
- Step 5: Compute relative magnitudes.

Step 1: Build template from catalog

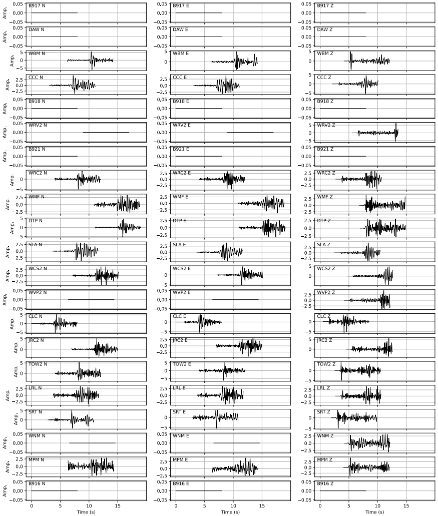
https://ai4eps.github.io/Earthquake_Catalog_Workshop/notebooks/tm_one_template/#build-template



Slice out most relevant part

⟶

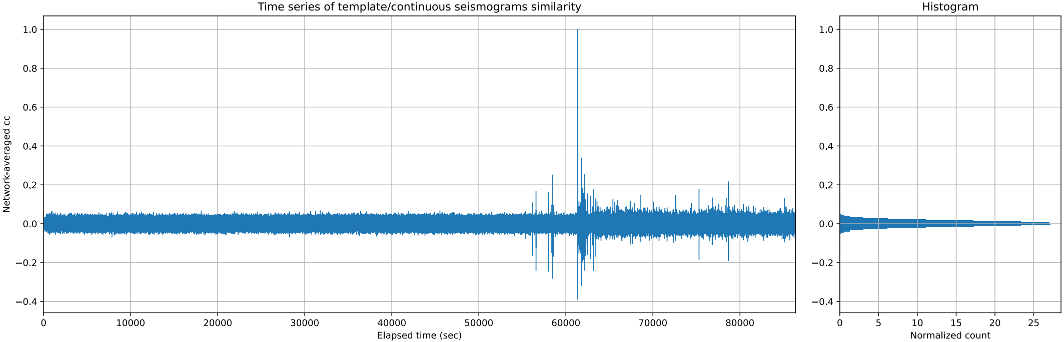
Template = Collection of waveforms and moveouts



Step 2: Compute time series of correlation coefficients

`https://ai4eps.github.io/Earthquake_Catalog_Workshop/notebooks/tm_one_template/#run-fmf`

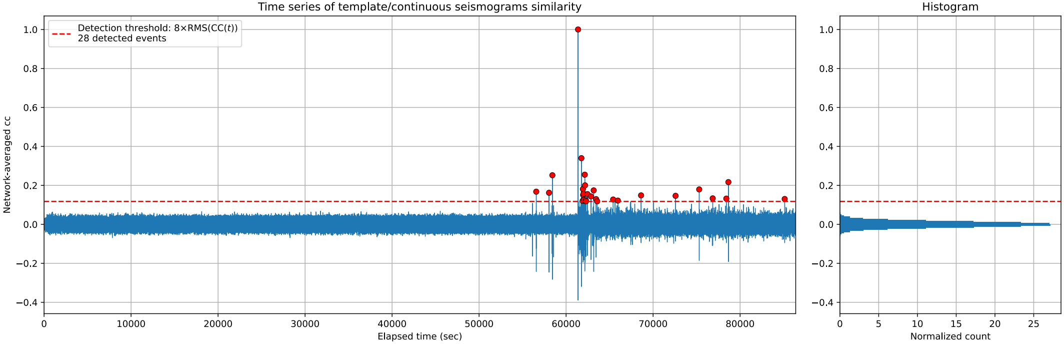
Compute $CC(t)$



Step 3: Design and apply detection threshold

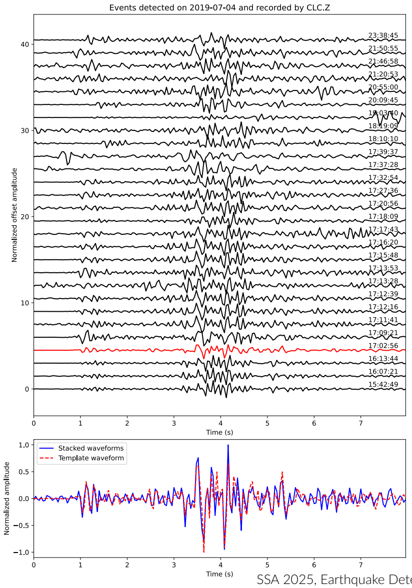
https://ai4eps.github.io/Earthquake_Catalog_Workshop/notebooks/tm_one_template/#set-detection-threshold-and-find-events

Gaussian distribution: 68.2% in $\pm 1\sigma$, 95.4% in $\pm 2\sigma$, ..., $(100 - 1.22 \times 10^{-13})$ in $\pm 8\sigma$



Step 4: Extract information on detected events

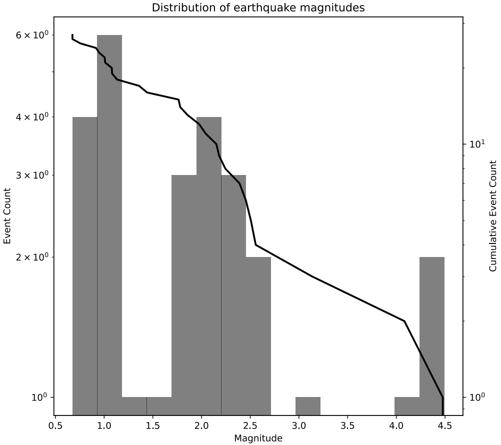
https://ai4eps.github.io/Earthquake_Catalog_Workshop/notebooks/tm_one_template/#set-detection-threshold-and-find-events



- Event timing.
- Peak waveform amplitude.
- Approximately share template location.

Step 5: Compute relative magnitudes

`https://ai4eps.github.io/Earthquake_Catalog_Workshop/notebooks/tm_one_template/#relative-magnitude`



Relative magnitudes between events sharing similar location and focal mechanism:

$$M_r = M_{ref} + \underbrace{\sum_i \log_{10} \frac{A_i}{A_{ref}}}_{\text{multiple seismometers/channels}} \tag{3}$$

Example of template matching with multiple templates

`https://ai4eps.github.io/Earthquake_Catalog_Workshop/notebooks/tm_multiple_templates/`

Github repository:
`https://github.com/AI4EPS/Earthquake_Catalog_Workshop/blob/main/notebooks/tm_multiple_templates.ipynb`

Outline of `tm_multiple_templates.ipynb`:

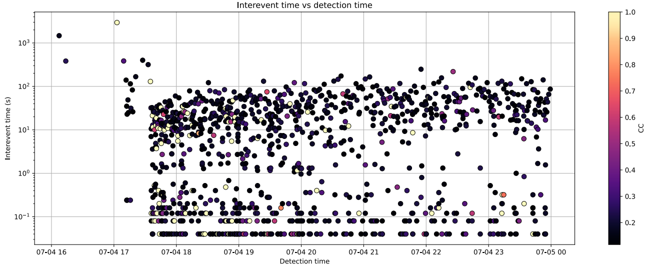
- Build the template events from catalog data.
- Compute the time series of correlation coefficients with `Fast Matched Filter`.
- Choose and apply a detection threshold.
- Extract detected events, build catalog and plot waveforms.
- **Find and remove events detected by multiple templates.**
- Compute relative magnitudes.

Template matching with multiple templates

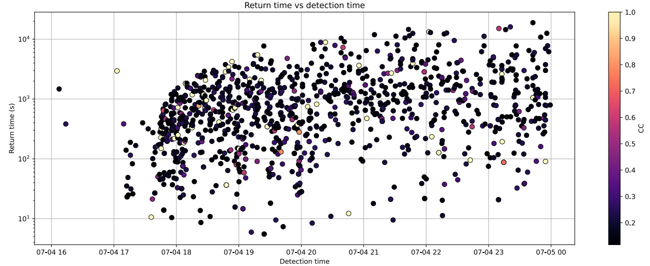
All the steps are similar to the first notebook, except at the end...

Flag and remove redundant detections I (from example with 91 templates)

`https://ai4eps.github.io/Earthquake_Catalog_Workshop/notebooks/tm_multiple_templates/#de-lumping-the-catalog`



Inter-event time:
Time between *any* two consecutive events.



Return time:
Time between two consecutive events detected by same template.

Flag and remove redundant detections I (from example with 91 templates)

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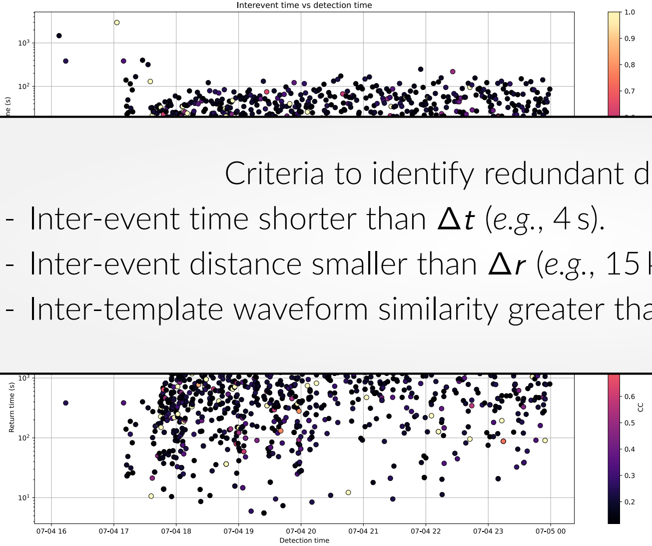


Inter-event time:
Time between *any* two consecutive events

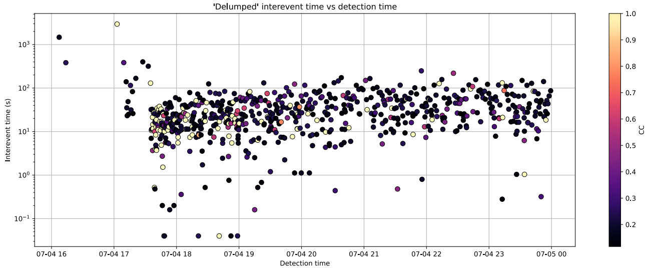
Criteria to identify redundant detections:

- Inter-event time shorter than Δt (*e.g.*, 4 s).
- Inter-event distance smaller than Δr (*e.g.*, 15 km).
- Inter-template waveform similarity greater than S (*e.g.*, 0.1).

Return time:
Time between two consecutive events detected by same template.

Flag and remove redundant detections II (from example with 91 templates)

`https://ai4eps.github.io/Earthquake_Catalog_Workshop/notebooks/tm_multiple_templates/#de-lumping-the-catalog`



Inter-event time:
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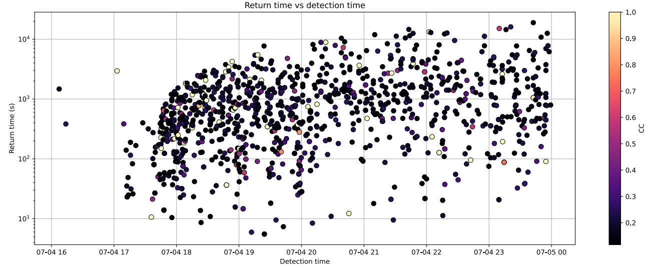

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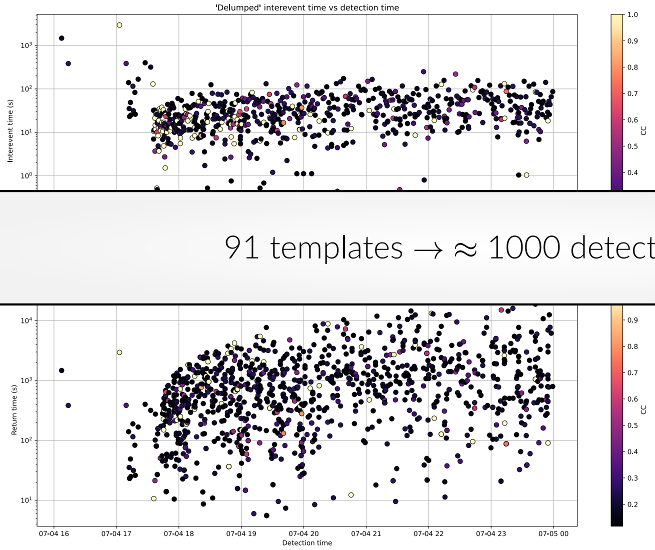


Inter-event time:
Time between *any* two consecutive events.

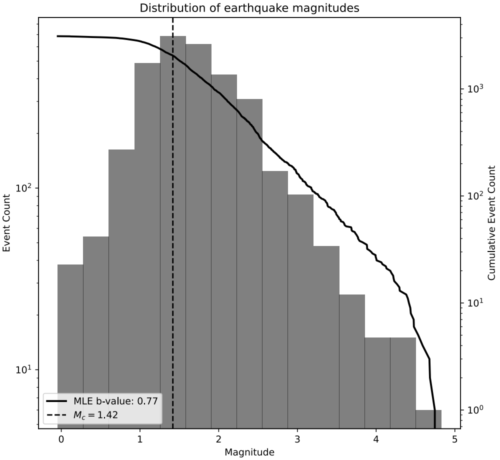
91 templates → ≈ 1000 detected events

Return time:
Time between two consecutive events detected by same template.

Run with 937 templates

https://ai4eps.github.io/Earthquake_Catalog_Workshop/notebooks/tm_multiple_templates/#relative-magnitudes

937 templates $\rightarrow \approx 3200$ detected events



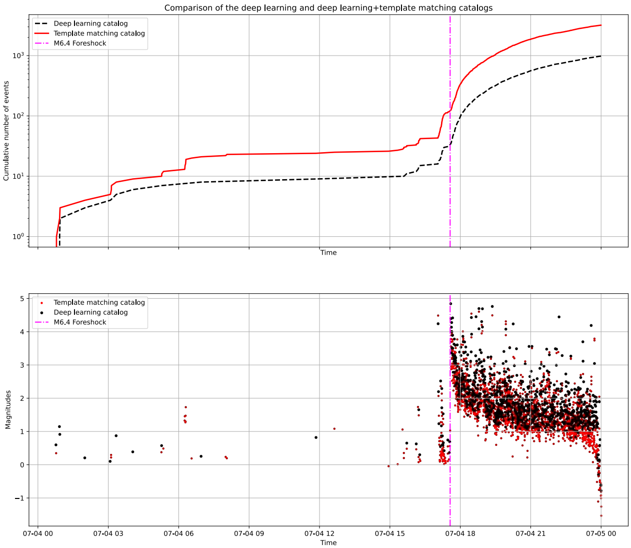
Gutenberg-Richter law:

$$\log_{10} N(m \geq M) = a - bM, \quad (4)$$

for magnitudes above M_c, the magnitude of completeness.

Run with 937 templates: Comparison against deep learning catalog

https://ai4eps.github.io/Earthquake_Catalog_Workshop/notebooks/tm_multiple_templates/#compare-with-deep-learning-catalog



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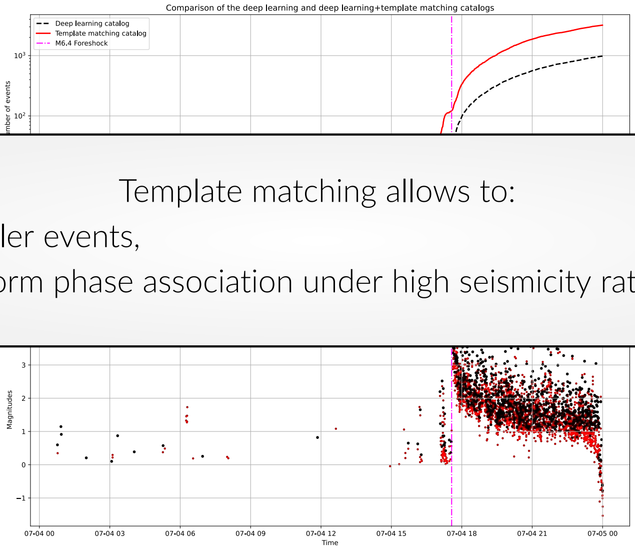

Template matching allows to:

- detect smaller events,
- better perform phase association under high seismicity rates.

Example of deffective detection threshold in template matching (BONUS)

`https://ai4eps.github.io/Earthquake_Catalog_Workshop/notebooks/tm_issues_w_detection_threshold/`

Github repository:
`https://github.com/AI4EPS/Earthquake_Catalog_Workshop/blob/main/notebooks/tm_issues_w_detection_threshold.ipynb`

Outline of `tm_issues_w_detection_threshold.ipynb`:

- Build the template events from catalog data.
- Introduce gaps in the data.
- Compute the time series of correlation coefficients with `Fast Matched Filter`.
- Choose and apply a detection threshold, illustrate gap-induced threshold bias.