自动控制原理II
(Principle of Automatic Control Theory)

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学时：48学时

考试：闭卷

References

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Chapter 7 Analysis and Design of Linear Discrete-Time System

7.1 Introduction

7.2 The Sampling Process and Sampling Theorem

7.3 Signal Recovery and Zero-Order Hold

7.4 Z-Transform and Inverse Z Transform

7.5 Mathematical Models of Discrete-Time Systems

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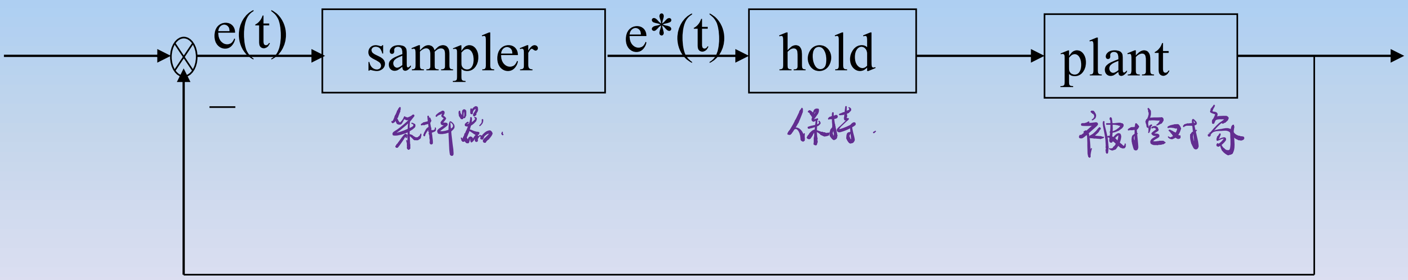
7.1 Introduction

Discrete-Time Systems:

Types:
- **Sampled data systems:** Discrete Time, Continuous Value
- **Digital systems:** Discrete Time, Quantized Value

Sampled Data System: a system that is continuous except for one or more *sampling operations*.

Digital System: There is one or more *digital signals* in the system.



Sampled-data control system

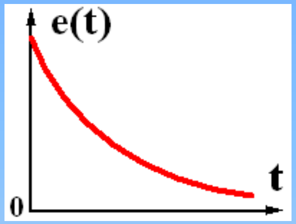
e*(t) is obtained by sampling a continuous signal e(t).

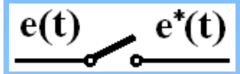
A/D : analog to digital converter

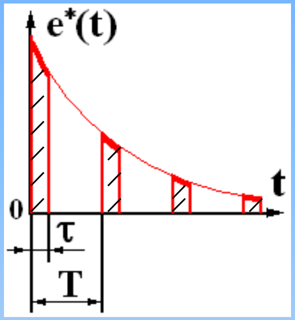
D/A : digital to analog converter

A/D process

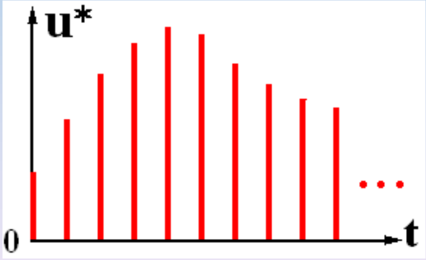
- **Sampling** — Time sampled
- **Quantization** — Value quantized

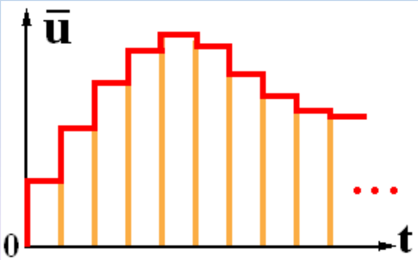


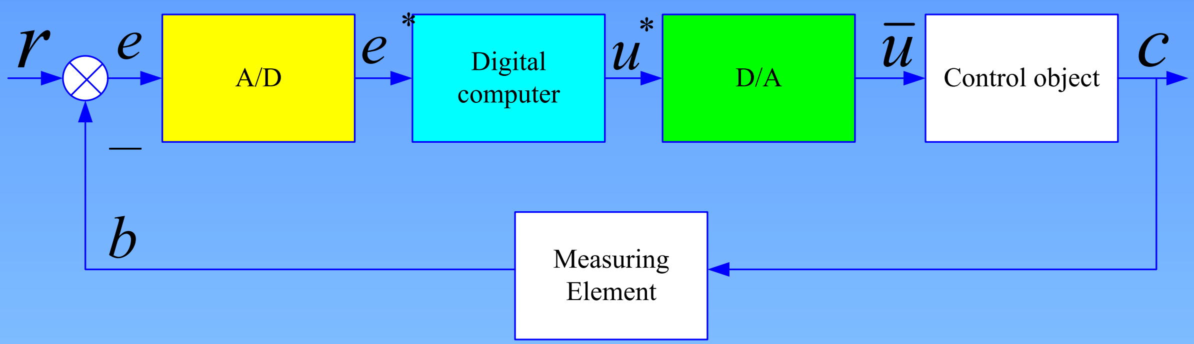




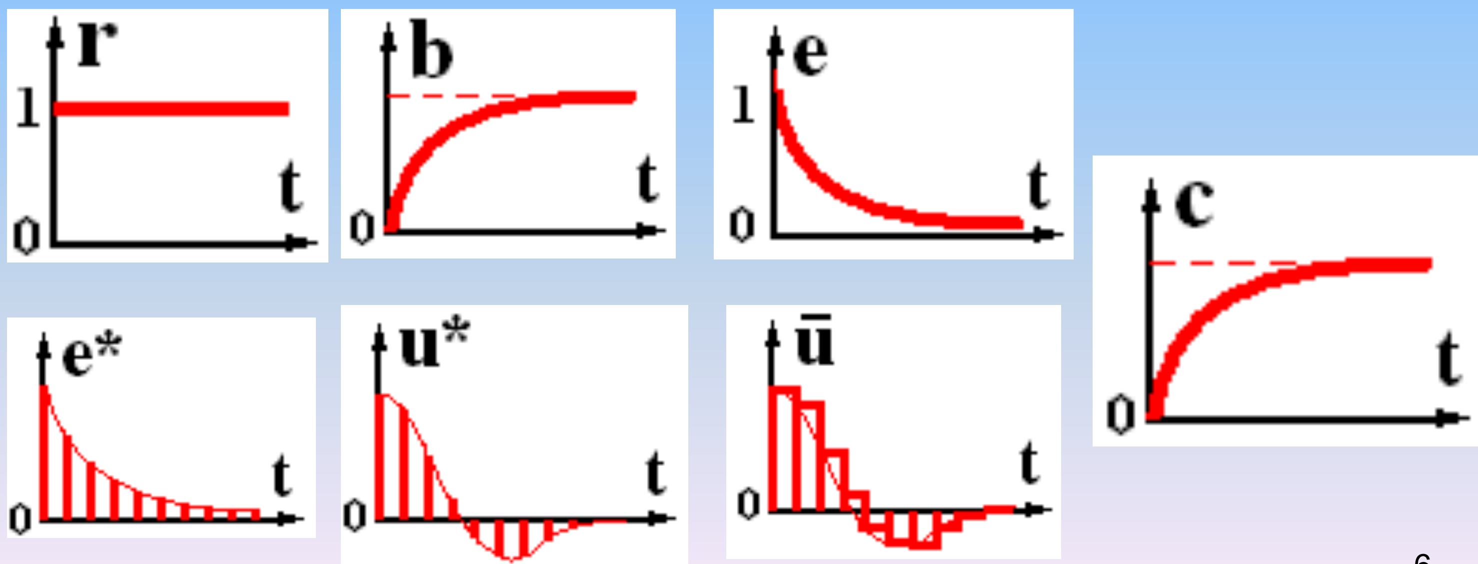
D/A process







Computer Controlled Systems



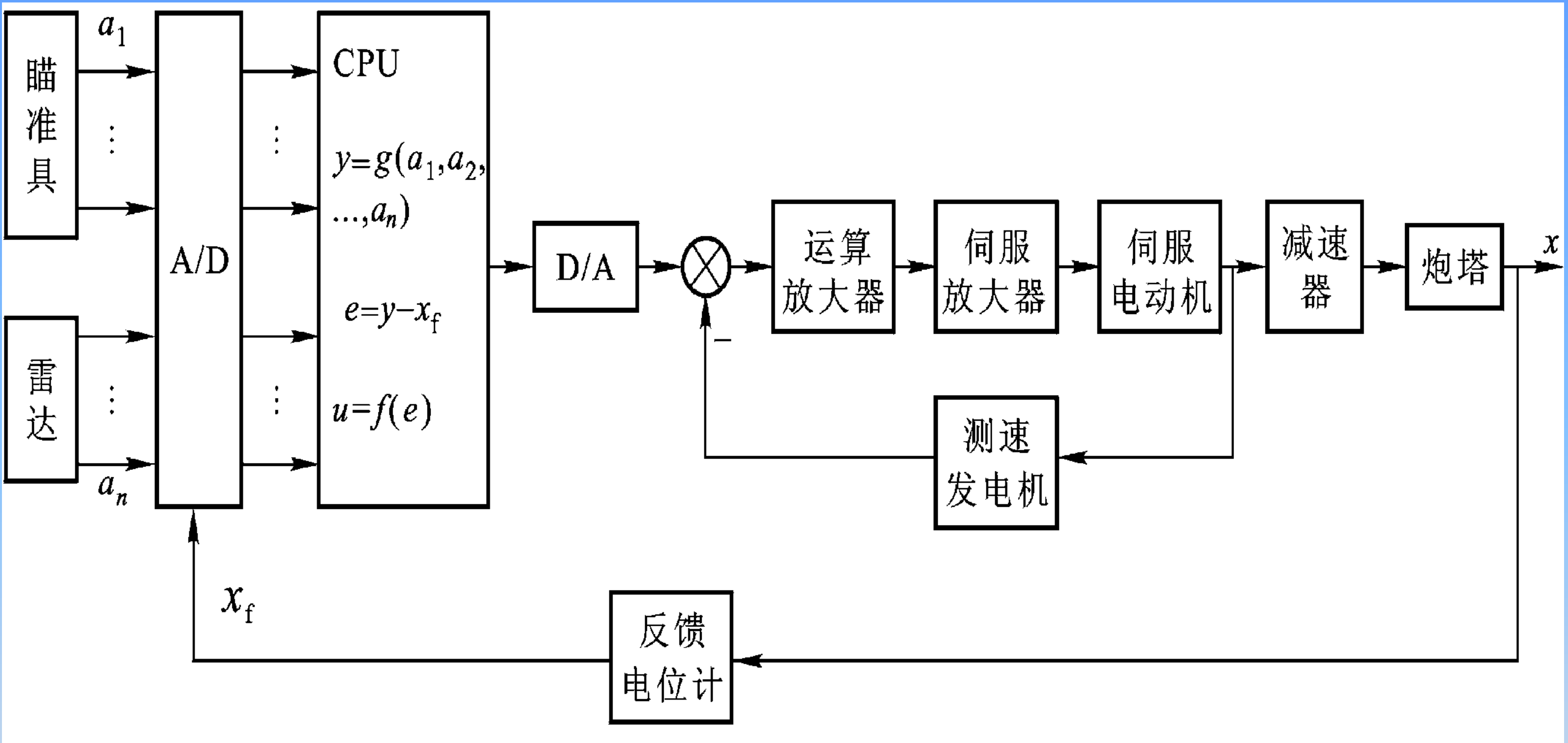


Fig 7-1 机载火力控制系统原理

History of Discrete-time system (p. 212-213)

DDC-Direct Digital Control （直接数字控制系统 ）

SCC- Surveillance Computer Control System(计算机监督控制系统)

TDC- Total and Distributed Control(集散控制系统)：muti-agent robots

Advantages and Disadvantages

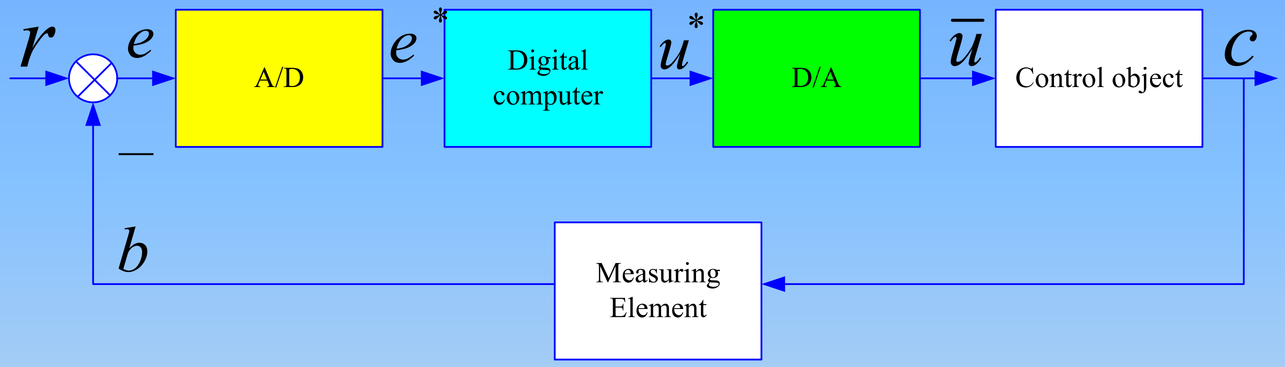
Computer Control System

- (1) Calculations are performed in the software. Easy for modification.
  (2) Complex control laws easily realized;
  (3) Reduced sensitivity to noise;
  (4) One computer for multi-tasks, high utilization ;
  (5) Network for process automation, macro-management and remote control.

- (1) Information between samples is lost. Compared with continuous system in the similar condition, the performance is reduced
  (2) Needs A/D and D/A conversion devices

7.2 The Sampling Process and Sampling Theorem

7.2.1 The Sampling Process

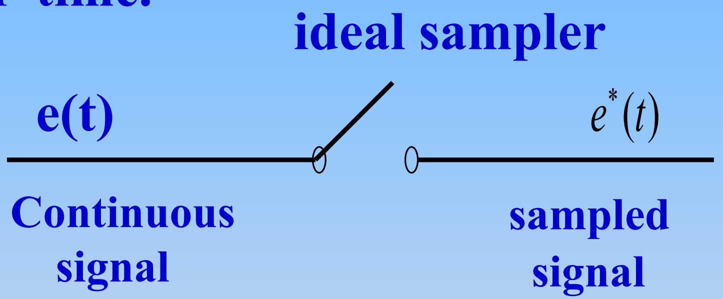


Computer Control System

Question: In the above computer control system, which signals are discrete, which signals are continuous?

- **Sampling Process: Continuous signal → Discrete Signal**
- **Holding Process: Discrete Signal → Continuous Signal.**
- **The two are inverse process to each other.**

Sampler: A switch that closes every T seconds for one instant of time.



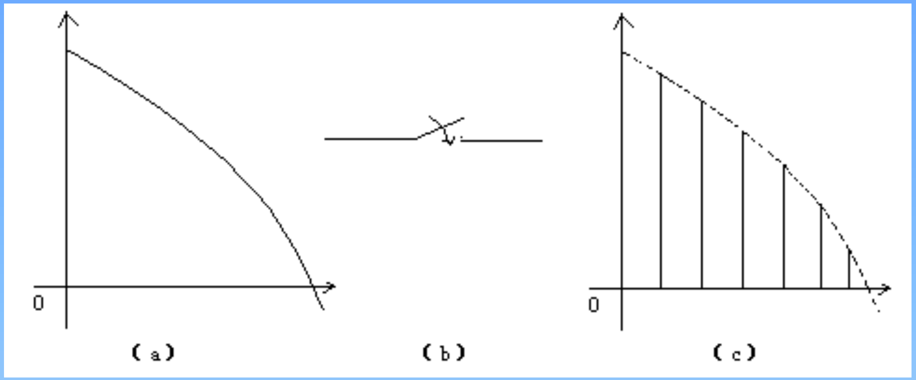
Where T is called the sampling period.

Ideal sampling process:

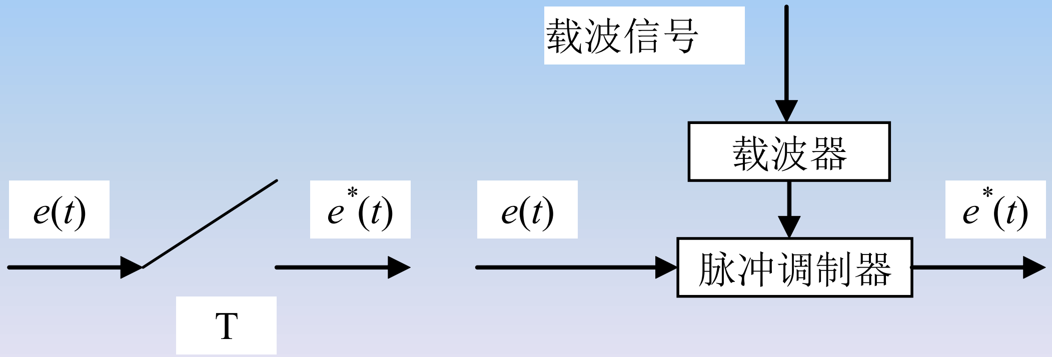
(1) t <<T. The sampling process is completed instantaneously

(2) Word Length is enough, thus e*(Kt)=e(Kt)

Types of Samplers: ideal, periodical, random,…



Sampling Process



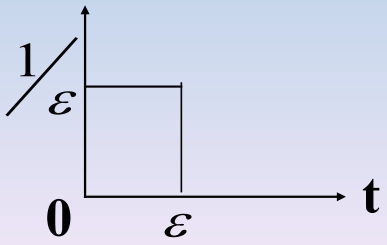
7.2.2 Mathematical Model for Sampling Signals

1、 Some ideal assumptions

- The sampler can be connected and cut down immediately；
- The signals in and out the sampler have no error/noise；
- $\tau \ll T$, that is $\tau \to 0$；
- The output is constant when sampler shuts down；
- Sample Period T is a constant。

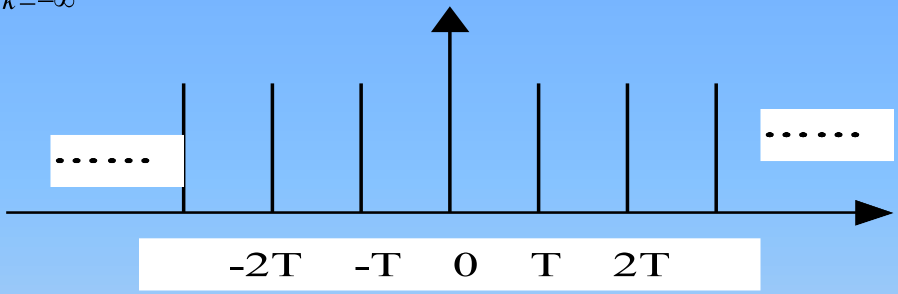
2、 Unit Impulsive Signal $\delta(t)$

$$\delta(t) = \begin{cases} 1/\varepsilon & 0 \le t \le \varepsilon \\ 0 & t<0 \text{ or } t>\varepsilon \end{cases}$$



3、Unit Impulsive Sequence Signal

$$\delta_T(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT) = \delta(t) + \delta(t-T) + \cdots + \delta(t-kT) + \cdots$$



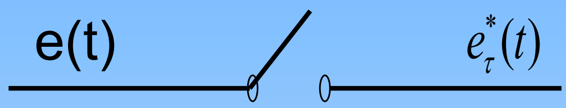
Unit Impulsive sequential signal

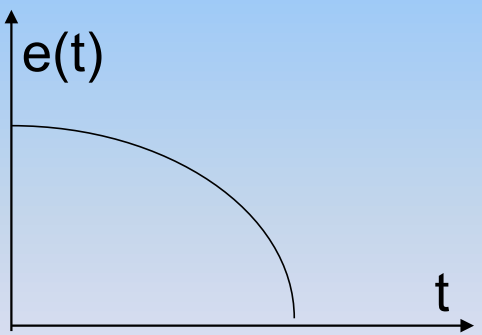
4、Sampling Signal

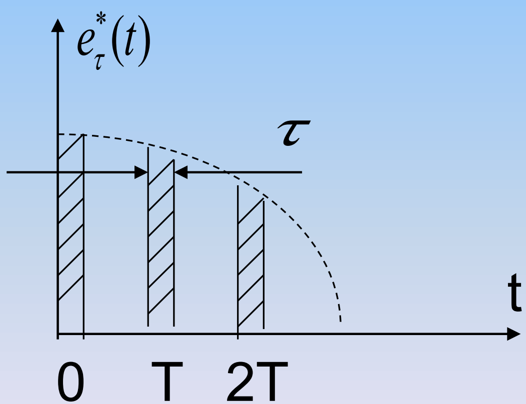
$$e^*(t) = \sum_{k=-\infty}^{\infty} e(t)\delta(t-kT) = e(t)\,\delta_T(t)$$

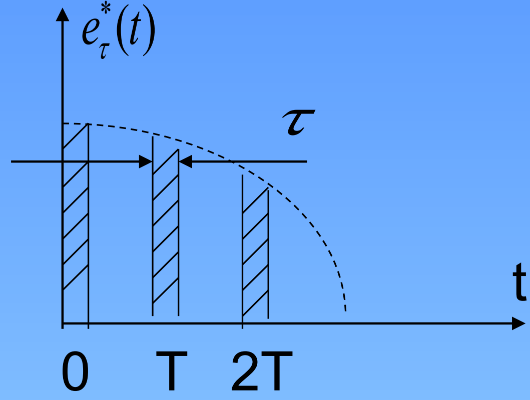
$$e^*(t) = \sum_{k=0}^{\infty} e(kT)\delta(t - kT)$$

real sampler

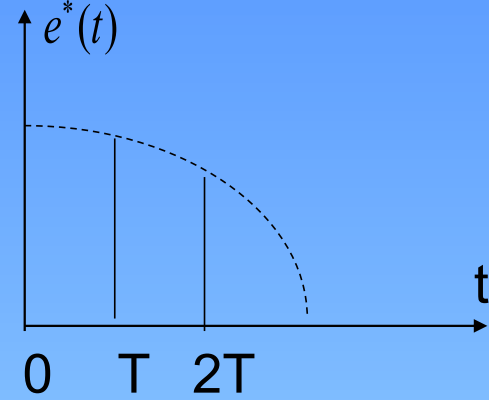








$\tau \to 0$



So the sampling operation can be expressed as

$$e^*(t) = \sum_{k=0}^{+\infty} e(kT) \cdot \delta(t - kT)$$

or

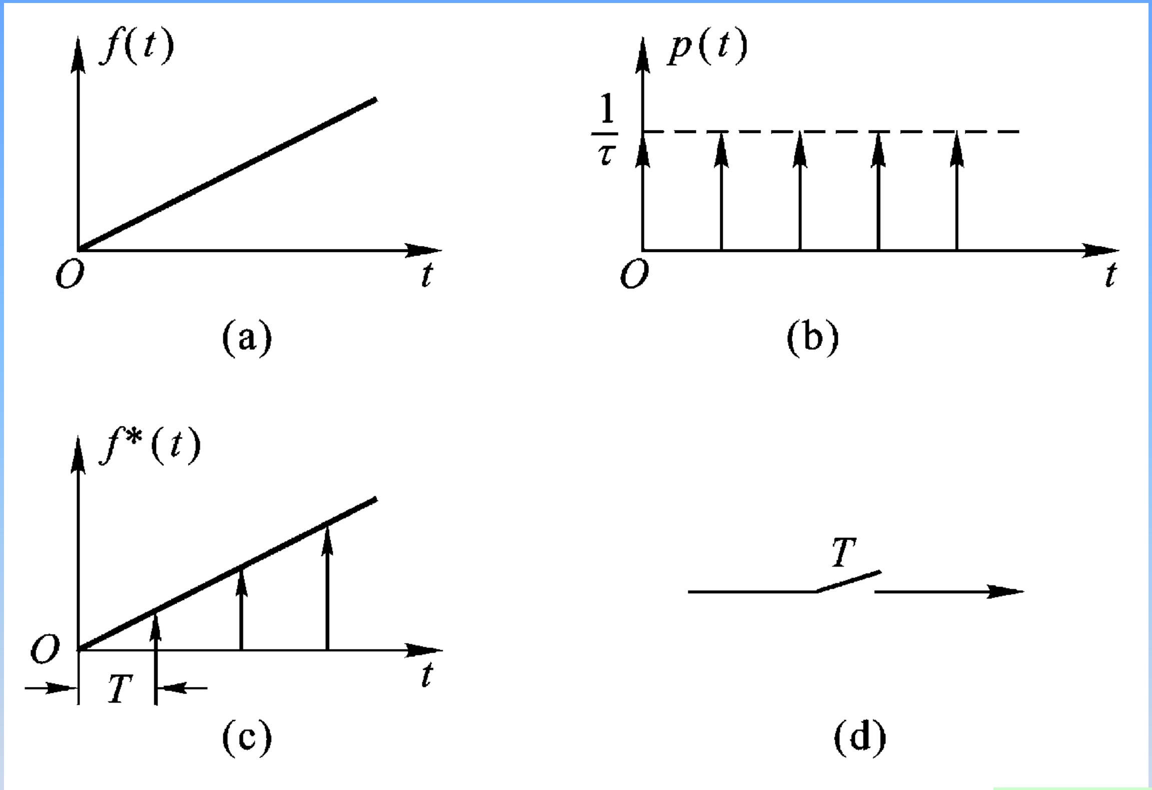
$$e^*(t) = e(t) \cdot \sum_{k=0}^{\infty} \delta(t - kT)$$

or

$$e^*(t) = e(t) \cdot \delta_T(t)$$

where

$$\delta_T(t) = \sum_{k=0}^{\infty} \delta(t - kT)$$



$$f_{\tau}^{*}(t) = p(t) \cdot f(t)$$

Fig 7 - 3 Sampling Process

Note:

$$e^{*}(t) = e(0)\delta(t) + e(T)\delta(t-T) + \cdots$$

$$\neq e(0) + e(T) + \cdots$$

Laplace Transformation

位移定理：

a.实域中的位移定理，若原函数在时间上延迟τ，则其象函数应乘以 $e^{-\tau\cdot s}$

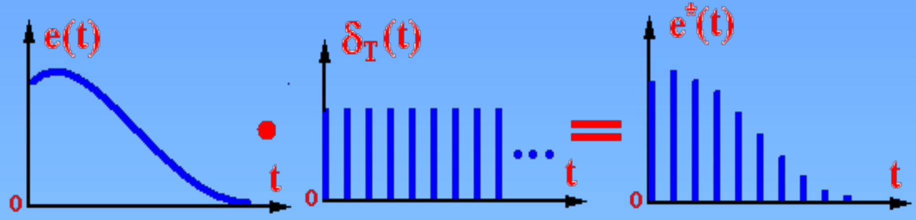
$$L[f(t-\tau)] = e^{-\tau\cdot s}F(s)$$

b.复域中的位移定理，象函数的自变量延迟a，原函数应乘以 e^{at} 即

$$L[e^{at}f(t)] = F(s-a)$$

Ideal sampling sequence

$$\delta_T(t) = \sum_{n=0}^{\infty} \delta(t - nT)$$



$$e^*(t) = e(t) \cdot \delta_T(t)$$

$$= e(t) \cdot \sum_{n=0}^{\infty} \delta(t - nT) \quad = \sum_{n=0}^{\infty} e(nT) \cdot \delta(t - nT)$$

(2) **L** :

$$E^*(s) = L\left[e^*(t)\right]$$

$$= L\left[\sum_{n=0}^{\infty} e(nT) \cdot \delta(t - nT)\right] \quad = \sum_{n=0}^{\infty} e(nT) \cdot e^{-nTs}$$

Example 7-1 $e(t) = 1(t)$ Obtain $E^*(s)$

Solution $$E^*(s) = \sum_{n=0}^{\infty} 1 \cdot e^{-nTs}$$

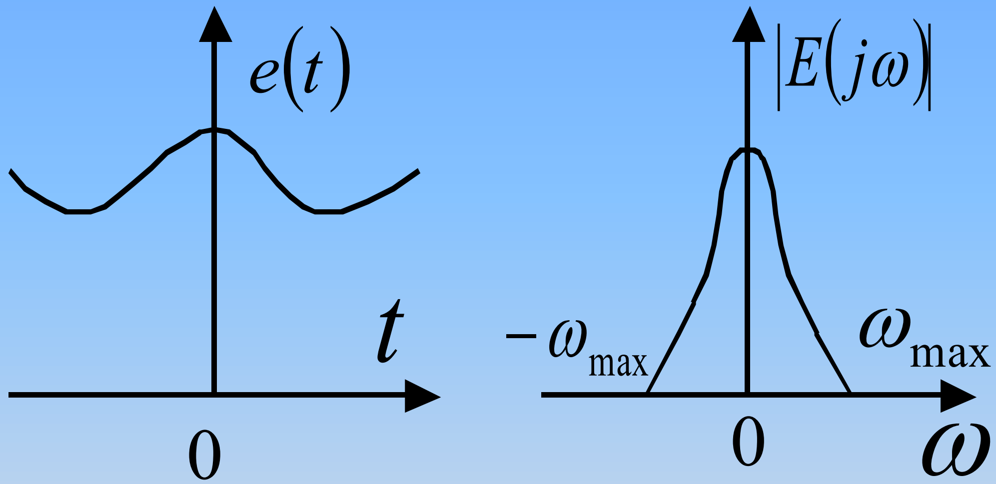
$$= 1 + e^{-Ts} + e^{-2Ts} + \cdots = \frac{1}{1-e^{-Ts}} = \frac{e^{Ts}}{e^{Ts}-1}$$

Example 7-2 $e(t) = e^{-at}$ Obtain $E^*(s)$

Solution $$E^*(s) = \sum_{n=0}^{\infty} e^{-anT} \cdot e^{-nTs} = \sum_{n=0}^{\infty} e^{-(s+a)nT}$$

$$= \frac{1}{1-e^{-(s+a)T}} = \frac{e^{Ts}}{e^{Ts}-e^{-aT}}$$

The continuous signal and its amplitude spectrum are



The Fourier-series expansion of $\delta_T(t)$:

$$\delta_T(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_s t}$$

$\omega_s = \frac{2\pi}{T}$ is the sampling freq.

$$c_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta_T(t) e^{-jkw_s t} dt \overset{令 t=0}{=} \frac{1}{T}\int_{0^-}^{0^+} \delta(t) dt = \frac{1}{T}$$

$$\Downarrow$$

$$\delta_T(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{jk\omega_s t}$$

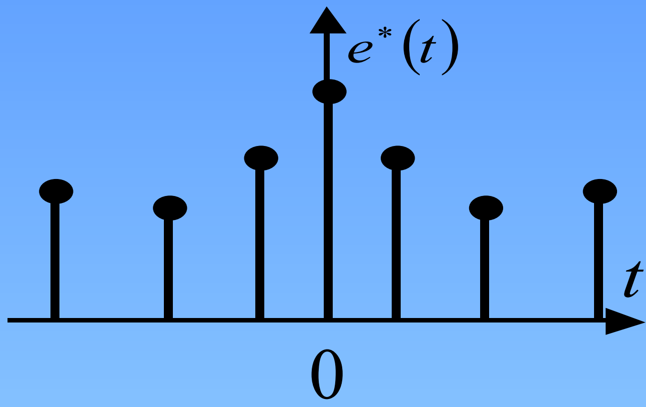
So the sampled signal is

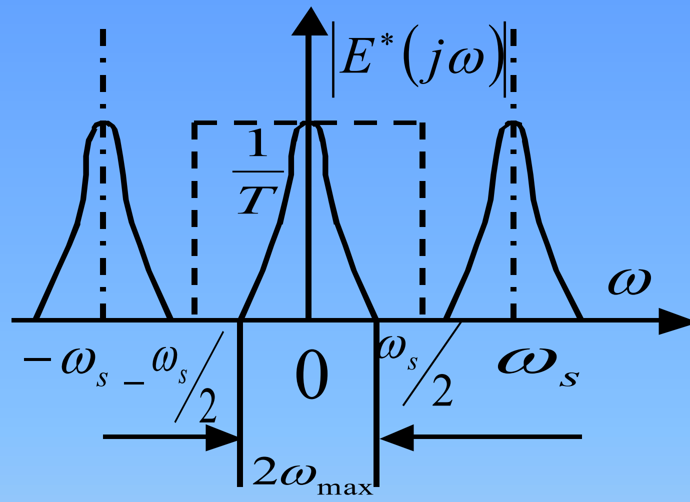
$$e^{*}(t) = e(t)\cdot\delta_{T}(t) = \frac{1}{T}\cdot\sum_{k=-\infty}^{\infty} e(t)\cdot e^{jk\omega_{s}t}$$

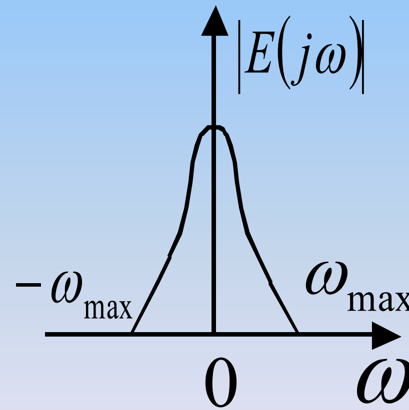
which Laplace transform is

$$E^{*}(\mathrm{j}\omega) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} E\left[j\left(\omega + k\omega_{s}\right)\right]$$

where the operator s is replaced by $j\omega$







From the figure above, we can conclude that if $\omega_s > 2\omega_{\max}$

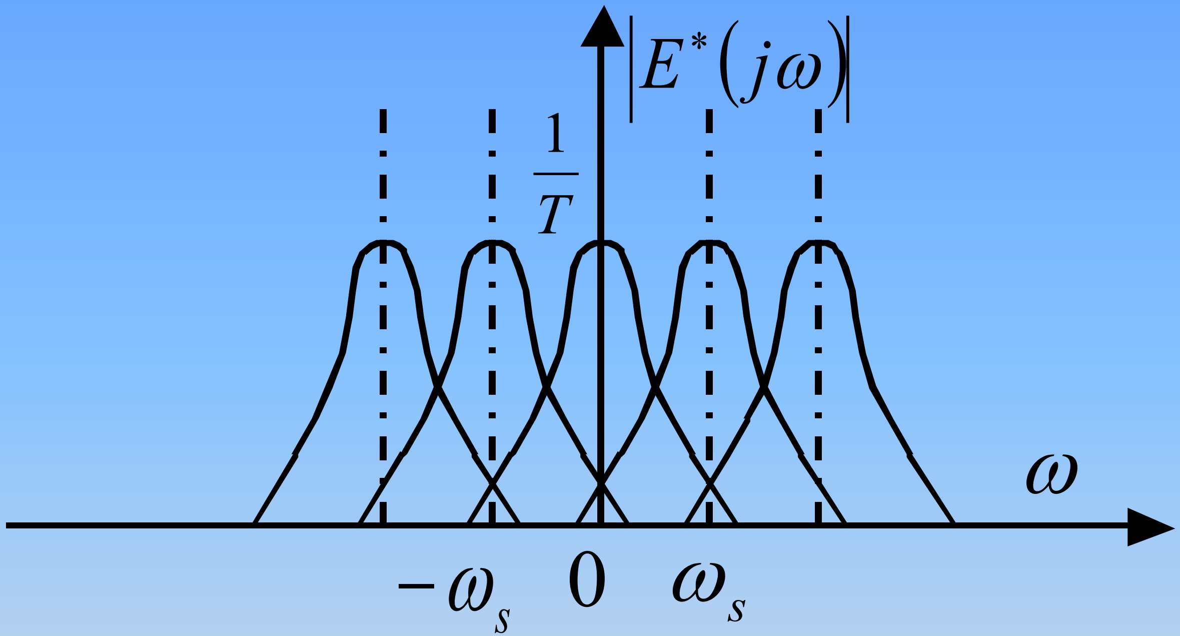
there are no overlap of each component, so the input signal can be recovered approximately. This is called sampling theorem or Shannon's Theorem

Shannon's Sampling Theorem:

Let $x(t)$ denote any continuous-time signal having a continuous Fourier transform

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Let $x^*(t)$ denote the samples of $x(t)$ at uniform intervals of T seconds. Then $x(t)$ can be exactly reconstructed from its samples $x^*(t)$ if and only if $X(j\omega) = 0$ for all $|\omega| \geq \pi/T$

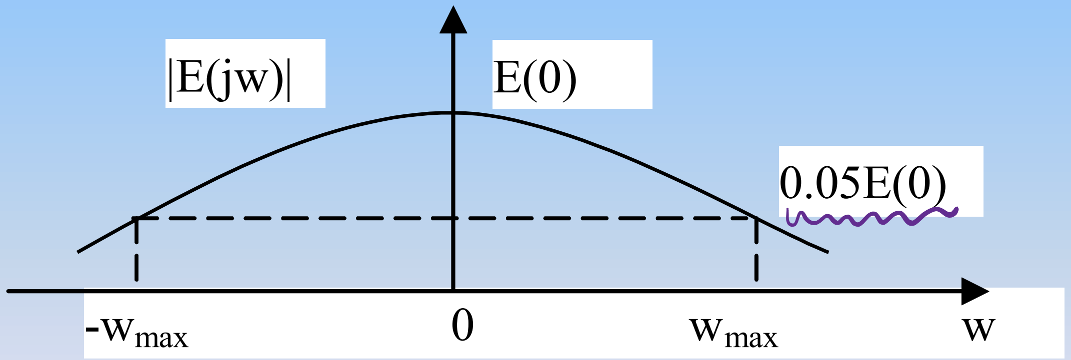


In the figure the input signal can't be recovered.

Problem: the maximum frequency ω_{max} is ***infinite*** for a non-periodic signal!

Then, how could we select the sampling frequency ω_s for it?

Solution:



Example 7-3 **Let** ***e(t)=e^-t^*****，select the sampling frequency according to Shannon's sampling therem.**

Solution.

The Laplace transform of *e*(*t*): $E(s)=\dfrac{1}{s+1}$

The Fourier transform of *e*(*t*) is: $E(j\omega)=\dfrac{1}{j\omega+1}$

The amplitude frequency characteristics:

$$\left|E(j\omega)\right|=\frac{1}{\sqrt{\omega^2+1}}$$

Assume the maximum frequency of *e*(*t*) satisfies: $\left|E(j\omega_{\max})\right|=0.05\left|E(0)\right|$

$$\frac{1}{\sqrt{\omega_{\max}^2+1}}=0.05,\quad \omega_{\max}=20rad/s$$

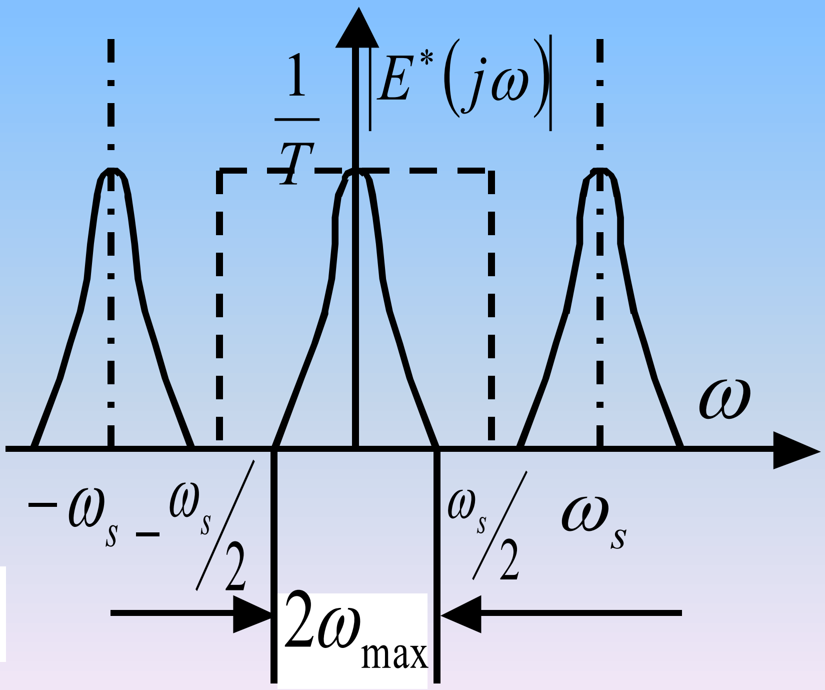
Then according to Shannon's theorem we have

$$\omega_s\geq 2\omega_{\max}=40rad/s$$

7.3 Signal Recovery and Zero-Order Hold

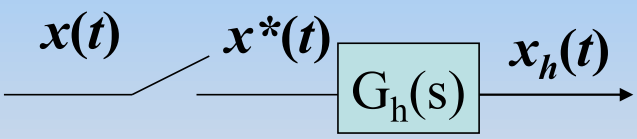
I.Signal recovery

The ideal filter is illustrated as the dotted line in the figure.

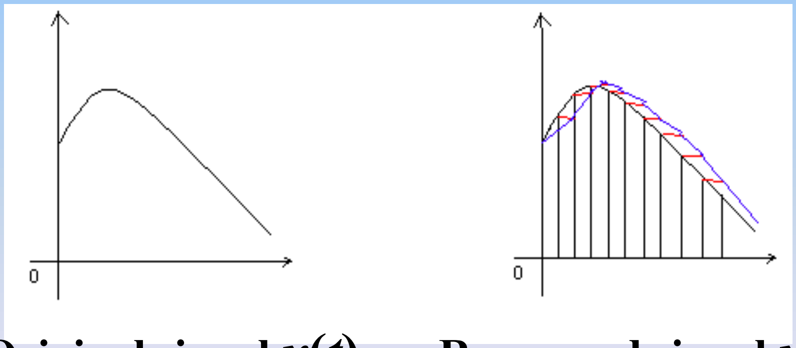


- The most common-used and simplest filter is zero-order hold filter.

- The zero-order hold (ZOH) is a mathematical model of the practical signal reconstruction done by a conventional digital-to-analog converter (DAC). That is, it describes the effect of converting a discrete-time signal to a continuous-time signal by ***holding each sample value for one sample interval***.



ZOH



Original signal $x(t)$　　**Recovered signal $x_h(t)$**

The original signal $x(t)$ and recovered signal $x_h(t)$ satisfy :

$$x_h(t) = \sum_{k=0}^{\infty} x(kT)(1(t-kT) - 1(t-kT-T))$$

Apply Laplace transform on both sides of the equation, we have

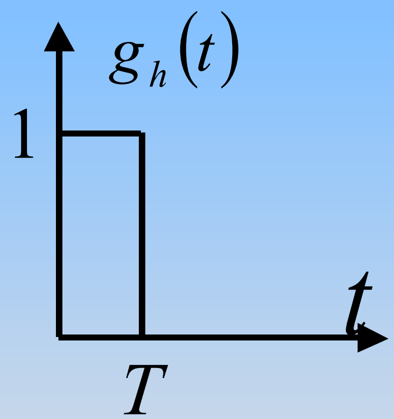
$$x_h(s) = \underbrace{\sum_{k=0}^{\infty} x(kT)e^{-kTs}}_{x^*(s)} \left[\frac{1}{s} - \frac{1}{s} e^{-Ts} \right]$$

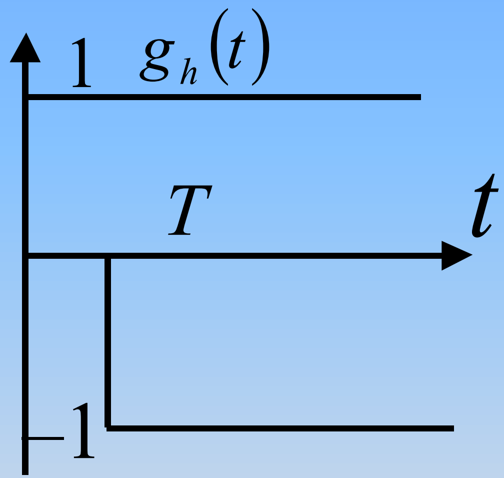
Then the ZOH equivalent transition function is

$$\frac{x_h(s)}{x^*(s)} = \frac{1-e^{-Ts}}{s} = G_h(s)$$

Corresponding time-domain function:

$$g_h(t) = 1(t) - 1(t - T)$$





Analysis of the frequency characteristics of ZOH filter :

$$G_h(j\omega)=\frac{1-e^{-j\omega T}}{j\omega}=\frac{e^{-\frac{1}{2}j\omega T}(e^{j\frac{1}{2}\omega T}-e^{-j\frac{1}{2}\omega T})}{j\omega}=\frac{2e^{-\frac{1}{2}j\omega T}\left(\frac{e^{j\frac{1}{2}\omega T}-e^{-j\frac{1}{2}\omega T}}{2j}\right)}{\omega}$$

Considering we have, $\boxed{\sin x=\frac{e^{jx}-e^{-jx}}{2j}}$ **therefore**

$$G_h(j\omega)=\frac{2e^{-\frac{1}{2}j\omega T}\sin(\frac{1}{2}\omega T)}{\omega} \Rightarrow G_h(j\omega)=T\frac{\sin(\frac{\omega T}{2})}{\frac{\omega T}{2}}e^{-\frac{1}{2}j\omega T}$$

$$G_h(j\omega)=\frac{1-e^{-j\omega T}}{j\omega}=T\cdot\frac{\sin\left(\omega T/2\right)}{\omega T/2}\cdot e^{-\frac{j\omega T}{2}}$$

$$\because T=\frac{2\pi}{\omega_s}$$

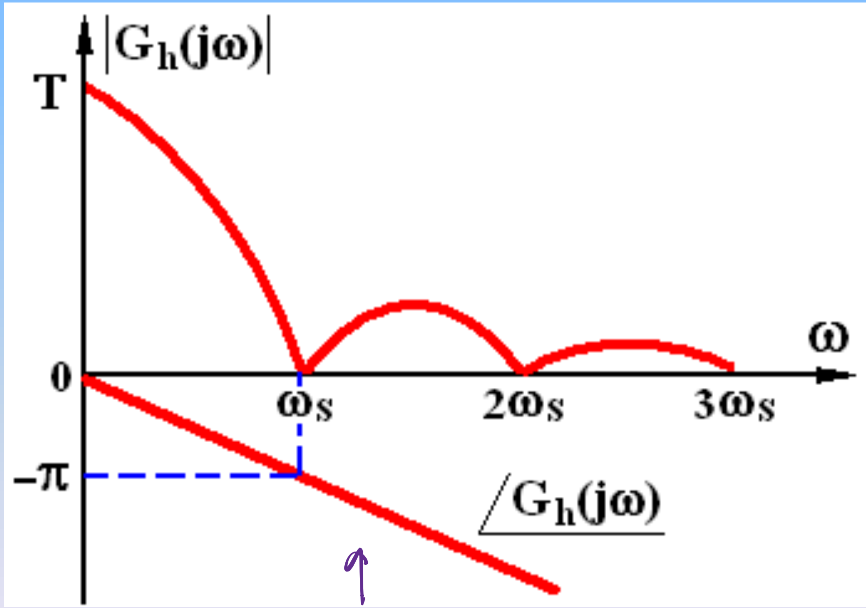
let $$S_a(x)=\frac{\sin x}{x}$$

then we have

$$G_h(j\omega)=\frac{2\pi}{\omega_s}\cdot S_a(\pi\omega/\omega_s)\cdot e^{-j\frac{\pi\omega}{\omega_s}}$$

Amplitude: $$|G_h(j\omega)| = \frac{2\pi}{\omega_s} \cdot |S_a(\pi\omega/\omega_s)|$$

Phase angle: $$\angle G_h(j\omega) = -\frac{\pi\omega}{\omega_s}$$



滞后效果

Note:

• The ZOH filter is not an ideal low-pass filter. ***Ripple error*** may occur after the filtering.

纹波

• There is a ***delayed phase angle*** when a signal is filtered by a ZOH.

相位滞后