DATA SCIENCE

HYPOTHESIS TESTING

Introduction to Hypothesis Testing

Basic Framework of a Hypothesis Test

Distance Measures

Central Limit Theorem

Types of Hypothesis Tests

Multiple Sample Tests

Agenda

Anova

- One Way
- Two Way
- Post Hoc Tests

Chi Square

- Association Tests
- Goodness-of-fit Tests

Chi Square Parametric

- Tests of Variance

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Anova

- One Way
- Two Way
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Anova

We have reviewed hypothesis tests of two types:

1. Single Sample: Testing a sample outcome against an expected population outcome

2. Two Sample: Testing the difference between two sample means

In situations where we want to compare means across multiple samples -

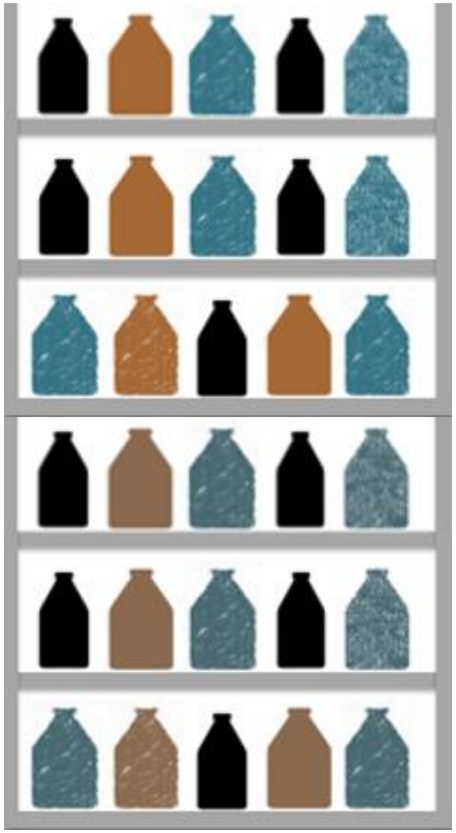
Can we use multiple sets of t-tests? For example, to test for difference between three samples:

Mean 1 = Mean 2,
Mean 2 = Mean 3,
Mean 1 = Mean 3.

Anova

Example:

- A retailer wants to understand shelving height impacts on sales. That is, do sales of a particular brand change significantly if they are placed at eye level, or at lower levels or higher levels?



Anova

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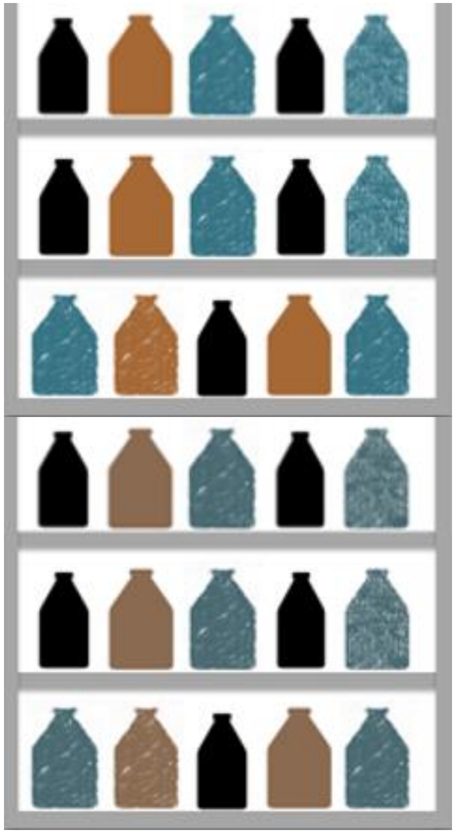
- A retailer wants to understand shelving height impacts on sales. That is, do sales of a particular brand change significantly if they are placed at eye level, or at lower levels or higher levels?

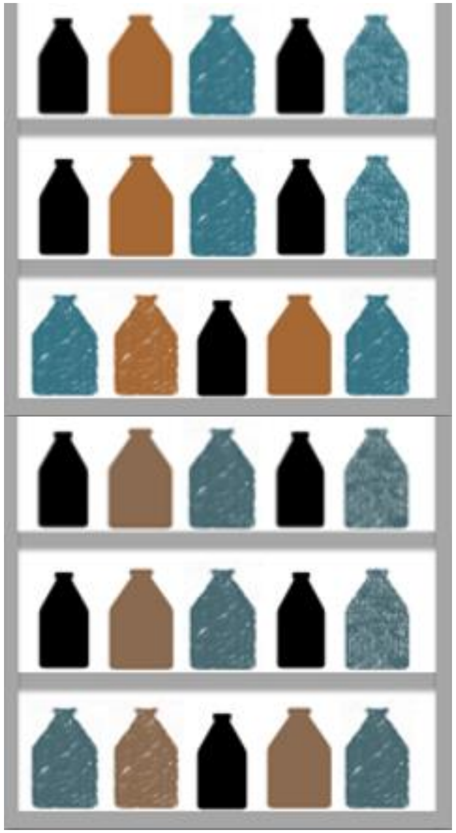
- One way to test this “hypothesis” – store the same product at different shelves and record sales for a fixed number of days at each height



Anova

Example:

- A retailer wants to understand shelving height impacts on sales. That is, do sales of a particular brand change significantly if they are placed at eye level, or at lower levels or higher levels?
- One way to test this "hypothesis" – store the same product at different shelves and record sales for a fixed number of days at each height
- Look at sales averages for each height, and then run a test to see if any observed differences are **statistically significant**



Anova

Below table lists total sales for 10 days, when the brand was stocked in shelves at different heights

We need to determine if height has an impact on total sales, i.e., are the differences observed in the sample means statistically significant?

	Shelf 1	Shelf 2	Shelf 3	Shelf 4	Shelf 5
	210.5	198.1	170.5	167.1	188.5
	198.1	189	225.5	167.9	167.9
	145.3	210.3	158	175.5	176.5
	185.5	254.4	139.4	175	152
	189.1	210.3	156.4	149.1	164.5
	135.9	160.9	217.1	189.3	171.7
	180	120.8	189.1	198.2	158.9
	149.4	167.8	158.2	205	177.9
	176.4	148.9	218.1	233.5	189.1
	229	190.4	178.9	167.9	187.1
	179.92	185.09	181.12	182.85	173.41
Avg	179.92	185.09	181.12	182.85	173.41

Anova

Analysis of Variance (ANOVA) uses variance to reach a conclusion about group means

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Overall Variance	Total sum of squared differences between observation the overall mean of all observations	Total Sum of Squares SST
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Within Group Variance	Sum of squared differences between each observation and the mean of the group it belongs to	Sum of Squares Within SSW
Between Groups Variance		

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Between Groups Variance	Sum of squared differences between each group mean and the overall mean	Sum of Squares Between SSB

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It can be established (mathematically) that there are two independent ways of establishing the standard error of the mean (essentially a measure of variance)

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ANOVA looks at a ratio of the two methods of estimating variance – if the ratio is similar, then the null hypothesis is unlikely to be rejected

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Any observation in an experiment can be broken down into

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If the independent variable has no impact, then within group variation and between group variation should be similar with any small differences attributable to random sampling error

Anova – Test Statistic

Test stat for ANOVA = MSB/MSW

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SSB:

$$SSB = \sum_{k=1}^{K} N_k (\overline{Y}_k - \overline{Y})^2$$

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DFB: k-1, - k # of groups

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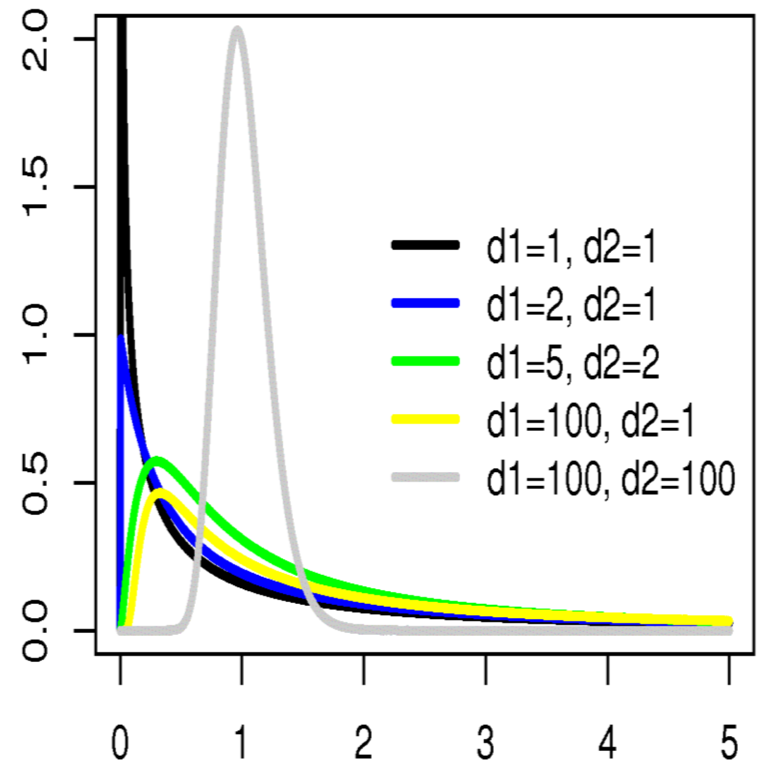
DFB: k-1, - k # of groups

DFW: n-k, -n # of observations

Anova – Test Statistic

The Test Stat follows an **F-Distribution**



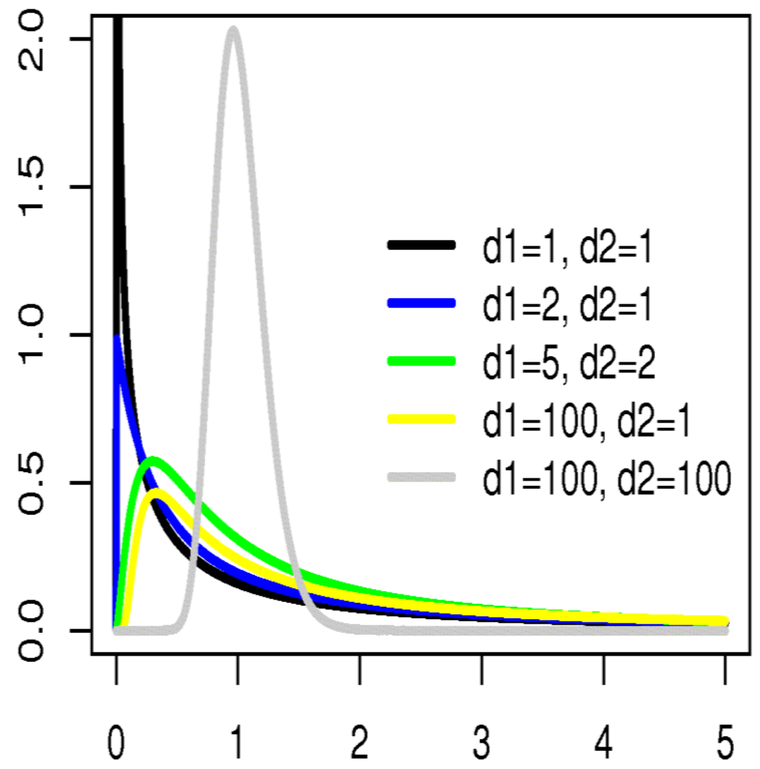
Anova – Test Statistic

The Test Stat follows an **F-Distribution**

Any random variate of F-distribution can be characterized as the ratio of two Chi Square Distributions

$$\frac{U_1/d_1}{U_2/d_2}$$

where U_1 and U_2 are Chi Square Dist with d_1 and d_2 df



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- The Alternate: At least one pair of means are unequal
- What would be constructed as a test-statistic?
- Ratio of Within Group Variation to Between Group Variation

Coming Up

Anova:

Tests Statistic Calculations

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- **One Way**
- Two Way
- Post Hoc Tests

Chi Square
- Association Tests
- Goodness-of-fit Tests

Chi Square Parametric
- Tests of Variance

Anova Calculations

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How do we calculate the within group variations?

- Calculate the variance for each group, and then calculate an average across groups

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Between group variation?

- Calculate the average of the square variations of each population mean from the mean for all the data (**Grand Mean**)

Anova Calculations

Within Group Variance

Anova Calculations

Within Group Variance

1. Calculate the Mean for each group

Anova Calculations

Within Group Variance

1. Calculate the Mean for each group
2. Subtract each sample mean from every score in that group

Anova Calculations

Within Group Variance

1. Calculate the Mean for each group
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3. Square the difference

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Within Group Variance

1. Calculate the Mean for each group
2. Subtract each sample mean from every score in that group
3. Square the difference
4. Add up all the squared Differences

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Within Group Variance

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2. Subtract each sample mean from every score in that group
3. Square the difference
4. Add up all the squared Differences

The SSW (Sum of Squares, Within) can be written as

$$SSW = \sum_{K} \sum_{I} (Y_{ik} - \bar{Y}_{K})^2$$

Anova Calculations

Between Group Variance

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Between Group Variance

1. Calculate a Grand Mean for all observations across all groups

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2. Subtract each grand mean from each sample mean

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Anova Calculations

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1. Calculate a Grand Mean for all observations across all groups
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5. Add them all up

Anova Calculations

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The SSW (Sum of Squares, Within) can be written as

$$SSB = \sum_{k=1}^{K} N_k (\overline{Y}_k - \overline{Y})^2$$

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We have:

SSW (Sum of squares, within)

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MSW = SSW/DFW, where DFW = n-k

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We need to divide each quantity by the appropriate degrees of freedom:

MSW = SSW/DFW, where DFW = n-k

MSB = SSB/DFB, where DFB = k-1

Anova Calculations

<u>Retail example</u>

Data				
Shelf 1	**Shelf 2**	**Shelf 3**	**Shelf 4**	**Shelf 5**
210.5	198.1	170.5	167.1	188.5
198.1	189	225.5	167.9	177.7
145.3	210.3	158	175.5	176.5
185.5	254.4	139.4	175	158
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180	120.8	189.1	198.2	176.2
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176.4	148.9	218.1	233.5	189.1
229	190.4	178.9	167.9	187.1
179.92	185.09	181.12	182.85	178.72
Total sum of squared differences: Within				34735.02

Anova Calculations

Retail example

SSW = 34735

Data				
Shelf 1	**Shelf 2**	**Shelf 3**	**Shelf 4**	**Shelf 5**
210.5	198.1	170.5	167.1	188.5
198.1	189	225.5	167.9	177.7
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Total sum of squared differences: Within				**34735.02**

Anova Calculations

Retail example

SSW = 34735

DFW = (50- 5) = 45

Data				
Shelf 1	**Shelf 2**	**Shelf 3**	**Shelf 4**	**Shelf 5**
210.5	198.1	170.5	167.1	188.5
198.1	189	225.5	167.9	177.7
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Total sum of squared differences: Within				**34735.02**

Anova Calculations

Retail example

SSW = 34735

DFW = (50- 5) = 45

MSW = 34735/45 = 771.88

Data				
Shelf 1	Shelf 2	Shelf 3	Shelf 4	Shelf 5
210.5	198.1	170.5	167.1	188.5
198.1	189	225.5	167.9	177.7
145.3	210.3	158	175.5	176.5
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Total sum of squared differences: Within				34735.02

Anova Calculations

SSB = 250.71	Shelf 1	Shelf 2	Shelf 3	Shelf 4	Shelf 5
	210.5	198.1	170.5	167.1	188.5
	198.1	189	225.5	167.9	177.7
	145.3	210.3	158	175.5	176.5
	185.5	254.4	139.4	175	158
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	229	190.4	178.9	167.9	187.1
	179.92	185.09	181.12	182.85	178.72
Grand Mean	181.54				
Squared Difference	2.6244	12.6025	0.1764	1.7161	7.9524
Squared Difference * Sample Size	26.244	126.025	1.764	17.161	79.524
Sum of total squared diff * Sample Size	250.718				

Anova Calculations

SSB = 250.71

DFB = (5-1) = 4

	Shelf 1	Shelf 2	Shelf 3	Shelf 4	Shelf 5
	210.5	198.1	170.5	167.1	188.5
	198.1	189	225.5	167.9	177.7
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Anova Calculations

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	145.3	210.3	158	175.5	176.5
MSB = 250.71/4 = 62.7	185.5	254.4	139.4	175	158
	189.1	210.3	156.4	149.1	174.5
	135.9	160.9	217.1	189.3	181.7
	180	120.8	189.1	198.2	176.2
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Anova Calculations

F – Stat = MSB/MSW

= 62.7/771.8 = 0.08

Anova Calculations

F – Stat = MSB/MSW

= 62.7/771.8 = 0.08

	Degrees of Freedom for Numerator											
	1	2	3	4	5	6	7	8	9	10	11	12
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.30	2.25	2.20	2.16	2.13
	7.68	**5.49**	**4.60**	**4.11**	**3.79**	**3.56**	**3.39**	**3.26**	**3.14**	**3.06**	**2.98**	**2.93**
28	4.20	3.34	2.95	2.71	2.56	2.44	2.36	2.29	2.24	2.19	2.15	2.12
	7.64	**5.45**	**4.57**	**4.07**	**3.76**	**3.53**	**3.36**	**3.23**	**3.11**	**3.03**	**2.95**	**2.90**
29	4.18	3.33	2.93	2.70	2.54	2.43	2.35	2.28	2.22	2.18	2.14	2.10
	7.60	**5.52**	**4.54**	**4.04**	**3.73**	**3.50**	**3.32**	**3.20**	**3.08**	**3.00**	**2.92**	**2.87**
30	4.17	3.32	2.92	2.69	2.53	2.42	2.34	2.27	2.21	2.16	2.12	2.09
	7.56	**5.39**	**4.51**	**4.02**	**3.70**	**3.47**	**3.30**	**3.17**	**3.06**	**2.98**	**2.90**	**2.84**
32	4.15	3.30	2.90	2.67	2.51	2.40	2.32	2.25	2.19	2.14	2.10	2.07
	7.50	**5.34**	**4.46**	**3.97**	**3.66**	**3.42**	**3.25**	**3.12**	**3.01**	**2.94**	**2.86**	**2.80**
34	4.13	3.28	2.88	2.65	2.49	2.38	2.30	2.23	2.17	2.12	2.08	2.05
	7.44	**5.29**	**4.42**	**3.93**	**3.61**	**3.38**	**3.21**	**3.08**	**2.97**	**2.89**	**2.82**	**2.76**
36	4.11	3.26	2.86	2.63	2.48	2.36	2.28	2.21	2.15	2.10	2.06	2.03
	7.39	**5.25**	**4.38**	**3.89**	**3.58**	**3.35**	**3.18**	**3.04**	**2.94**	**2.86**	**2.78**	**2.72**
38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09	2.05	2.02
	7.35	**5.21**	**4.34**	**3.86**	**3.54**	**3.32**	**3.15**	**3.02**	**2.91**	**2.82**	**2.75**	**2.69**
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.07	2.04	2.00
	7.31	**5.18**	**4.31**	**3.83**	**3.51**	**3.29**	**3.12**	**2.99**	**2.88**	**2.80**	**2.73**	**2.66**
42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.06	2.02	1.90
	7.27	**5.15**	**4.29**	**3.80**	**3.49**	**3.26**	**3.10**	**2.96**	**2.86**	**2.77**	**2.70**	**2.64**
44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05	2.01	1.98
	7.24	**5.12**	**4.26**	**3.78**	**3.46**	**3.24**	**3.07**	**2.94**	**2.84**	**2.75**	**2.68**	**2.62**
46	4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.14	2.09	2.04	2.00	1.97
	7.21	**5.10**	**4.24**	**3.76**	**3.44**	**3.22**	**3.05**	**2.92**	**2.82**	**2.73**	**2.66**	**2.60**
48	4.04	3.19	2.80	2.56	2.41	2.30	2.21	2.14	2.08	2.03	1.99	1.96
	7.19	**5.08**	**4.22**	**3.74**	**3.42**	**3.20**	**3.04**	**2.90**	**2.80**	**2.71**	**2.64**	**2.58**
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.02	1.98	1.95
	7.17	**5.06**	**4.20**	**3.72**	**3.41**	**3.18**	**3.02**	**2.88**	**2.78**	**2.70**	**2.62**	**2.56**
55	4.02	3.17	2.78	2.54	2.38	2.27	2.18	2.11	2.05	2.00	1.97	1.93
	7.12	**5.01**	**4.16**	**3.68**	**3.37**	**3.15**	**2.98**	**2.85**	**2.75**	**2.66**	**2.59**	**2.53**
60	4.00	3.15	2.76	2.52	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	7.08	**4.98**	**4.13**	**3.65**	**3.34**	**3.12**	**2.95**	**2.82**	**2.72**	**2.63**	**2.56**	**2.50**
65	3.99	3.14	2.75	2.51	2.36	2.24	2.15	2.08	2.02	1.98	1.94	1.90

Anova Calculations

F – Stat = MSB/MSW

= 62.7/771.8 = 0.08

F-Critical: 2.57

	Degrees of Freedom for Numerator											
	1	2	3	4	5	6	7	8	9	10	11	12
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.30	2.25	2.20	2.16	2.13
	7.68	**5.49**	**4.60**	**4.11**	**3.79**	**3.56**	**3.39**	**3.26**	**3.14**	**3.06**	**2.98**	**2.93**
28	4.20	3.34	2.95	2.71	2.56	2.44	2.36	2.29	2.24	2.19	2.15	2.12
	7.64	**5.45**	**4.57**	**4.07**	**3.76**	**3.53**	**3.36**	**3.23**	**3.11**	**3.03**	**2.95**	**2.90**
29	4.18	3.33	2.93	2.70	2.54	2.43	2.35	2.28	2.22	2.18	2.14	2.10
	7.60	**5.52**	**4.54**	**4.04**	**3.73**	**3.50**	**3.32**	**3.20**	**3.08**	**3.00**	**2.92**	**2.87**
30	4.17	3.32	2.92	2.69	2.53	2.42	2.34	2.27	2.21	2.16	2.12	2.09
	7.56	**5.39**	**4.51**	**4.02**	**3.70**	**3.47**	**3.30**	**3.17**	**3.06**	**2.98**	**2.90**	**2.84**
32	4.15	3.30	2.90	2.67	2.51	2.40	2.32	2.25	2.19	2.14	2.10	2.07
	7.50	**5.34**	**4.46**	**3.97**	**3.66**	**3.42**	**3.25**	**3.12**	**3.01**	**2.94**	**2.86**	**2.80**
34	4.13	3.28	2.88	2.65	2.49	2.38	2.30	2.23	2.17	2.12	2.08	2.05
	7.44	**5.29**	**4.42**	**3.93**	**3.61**	**3.38**	**3.21**	**3.08**	**2.97**	**2.89**	**2.82**	**2.76**
36	4.11	3.26	2.86	2.63	2.48	2.36	2.28	2.21	2.15	2.10	2.06	2.03
	7.39	**5.25**	**4.38**	**3.89**	**3.58**	**3.35**	**3.18**	**3.04**	**2.94**	**2.86**	**2.78**	**2.72**
38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09	2.05	2.02
	7.35	**5.21**	**4.34**	**3.86**	**3.54**	**3.32**	**3.15**	**3.02**	**2.91**	**2.82**	**2.75**	**2.69**
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.07	2.04	2.00
	7.31	**5.18**	**4.31**	**3.83**	**3.51**	**3.29**	**3.12**	**2.99**	**2.88**	**2.80**	**2.73**	**2.66**
42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.06	2.02	1.90
	7.27	**5.15**	**4.29**	**3.80**	**3.49**	**3.26**	**3.10**	**2.96**	**2.86**	**2.77**	**2.70**	**2.64**
44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05	2.01	1.98
	7.24	**5.12**	**4.26**	**3.78**	**3.46**	**3.24**	**3.07**	**2.94**	**2.84**	**2.75**	**2.68**	**2.62**
46	4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.14	2.09	2.04	2.00	1.97
	7.21	**5.10**	**4.24**	**3.76**	**3.44**	**3.22**	**3.05**	**2.92**	**2.82**	**2.73**	**2.66**	**2.60**
48	4.04	3.19	2.80	2.56	2.41	2.30	2.21	2.14	2.08	2.03	1.99	1.96
	7.19	**5.08**	**4.22**	**3.74**	**3.42**	**3.20**	**3.04**	**2.90**	**2.80**	**2.71**	**2.64**	**2.58**
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.02	1.98	1.95
	7.17	**5.06**	**4.20**	**3.72**	**3.41**	**3.18**	**3.02**	**2.88**	**2.78**	**2.70**	**2.62**	**2.56**
55	4.02	3.17	2.78	2.54	2.38	2.27	2.18	2.11	2.05	2.00	1.97	1.93
	7.12	**5.01**	**4.16**	**3.68**	**3.37**	**3.15**	**2.98**	**2.85**	**2.75**	**2.66**	**2.59**	**2.53**
60	4.00	3.15	2.76	2.52	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	7.08	**4.98**	**4.13**	**3.65**	**3.34**	**3.12**	**2.95**	**2.82**	**2.72**	**2.63**	**2.56**	**2.50**
65	3.99	3.14	2.75	2.51	2.36	2.24	2.15	2.08	2.02	1.98	1.94	1.90

Anova Calculations

F – Stat = MSB/MSW

= 62.7/771.8 = 0.08

F-Critical: 2.57

We fail to reject the null hypothesis -

	Degrees of Freedom for Numerator											
	1	2	3	4	5	6	7	8	9	10	11	12
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.30	2.25	2.20	2.16	2.13
	7.68	**5.49**	**4.60**	**4.11**	**3.79**	**3.56**	**3.39**	**3.26**	**3.14**	**3.06**	**2.98**	**2.93**
28	4.20	3.34	2.95	2.71	2.56	2.44	2.36	2.29	2.24	2.19	2.15	2.12
	7.64	**5.45**	**4.57**	**4.07**	**3.76**	**3.53**	**3.36**	**3.23**	**3.11**	**3.03**	**2.95**	**2.90**
29	4.18	3.33	2.93	2.70	2.54	2.43	2.35	2.28	2.22	2.18	2.14	2.10
	7.60	**5.52**	**4.54**	**4.04**	**3.73**	**3.50**	**3.32**	**3.20**	**3.08**	**3.00**	**2.92**	**2.87**
30	4.17	3.32	2.92	2.69	2.53	2.42	2.34	2.27	2.21	2.16	2.12	2.09
	7.56	**5.39**	**4.51**	**4.02**	**3.70**	**3.47**	**3.30**	**3.17**	**3.06**	**2.98**	**2.90**	**2.84**
32	4.15	3.30	2.90	2.67	2.51	2.40	2.32	2.25	2.19	2.14	2.10	2.07
	7.50	**5.34**	**4.46**	**3.97**	**3.66**	**3.42**	**3.25**	**3.12**	**3.01**	**2.94**	**2.86**	**2.80**
34	4.13	3.28	2.88	2.65	2.49	2.38	2.30	2.23	2.17	2.12	2.08	2.05
	7.44	**5.29**	**4.42**	**3.93**	**3.61**	**3.38**	**3.21**	**3.08**	**2.97**	**2.89**	**2.82**	**2.76**
36	4.11	3.26	2.86	2.63	2.48	2.36	2.28	2.21	2.15	2.10	2.06	2.03
	7.39	**5.25**	**4.38**	**3.89**	**3.58**	**3.35**	**3.18**	**3.04**	**2.94**	**2.86**	**2.78**	**2.72**
38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09	2.05	2.02
	7.35	**5.21**	**4.34**	**3.86**	**3.54**	**3.32**	**3.15**	**3.02**	**2.91**	**2.82**	**2.75**	**2.69**
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.07	2.04	2.00
	7.31	**5.18**	**4.31**	**3.83**	**3.51**	**3.29**	**3.12**	**2.99**	**2.88**	**2.80**	**2.73**	**2.66**
42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.06	2.02	1.90
	7.27	**5.15**	**4.29**	**3.80**	**3.49**	**3.26**	**3.10**	**2.96**	**2.86**	**2.77**	**2.70**	**2.64**
44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05	2.01	1.98
	7.24	**5.12**	**4.26**	[illegible]	**3.46**	**3.24**	**3.07**	**2.94**	**2.84**	**2.75**	**2.68**	**2.62**
46	4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.14	2.09	2.04	2.00	1.97
	7.21	**5.10**	**4.24**	**3.76**	**3.44**	**3.22**	**3.05**	**2.92**	**2.82**	**2.73**	**2.66**	**2.60**
48	4.04	3.19	2.80	2.56	2.41	2.30	2.21	2.14	2.08	2.03	1.99	1.96
	7.19	**5.08**	**4.22**	**3.74**	**3.42**	**3.20**	**3.04**	**2.90**	**2.80**	**2.71**	**2.64**	**2.58**
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.02	1.98	1.95
	7.17	**5.06**	**4.20**	**3.72**	**3.41**	**3.18**	**3.02**	**2.88**	**2.78**	**2.70**	**2.62**	**2.56**
55	4.02	3.17	2.78	2.54	2.38	2.27	2.18	2.11	2.05	2.00	1.97	1.93
	7.12	**5.01**	**4.16**	**3.68**	**3.37**	**3.15**	**2.98**	**2.85**	**2.75**	**2.66**	**2.59**	**2.53**
60	4.00	3.15	2.76	2.52	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	7.08	**4.98**	**4.13**	**3.65**	**3.34**	**3.12**	**2.95**	**2.82**	**2.72**	**2.63**	**2.56**	**2.50**
65	3.99	3.14	2.75	2.51	2.36	2.24	2.15	2.08	2.02	1.98	1.94	1.90

Anova Calculations

F – Stat = MSB/MSW

= 62.7/771.8 = 0.08

F-Critical: 2.57

We fail to reject the null hypothesis -

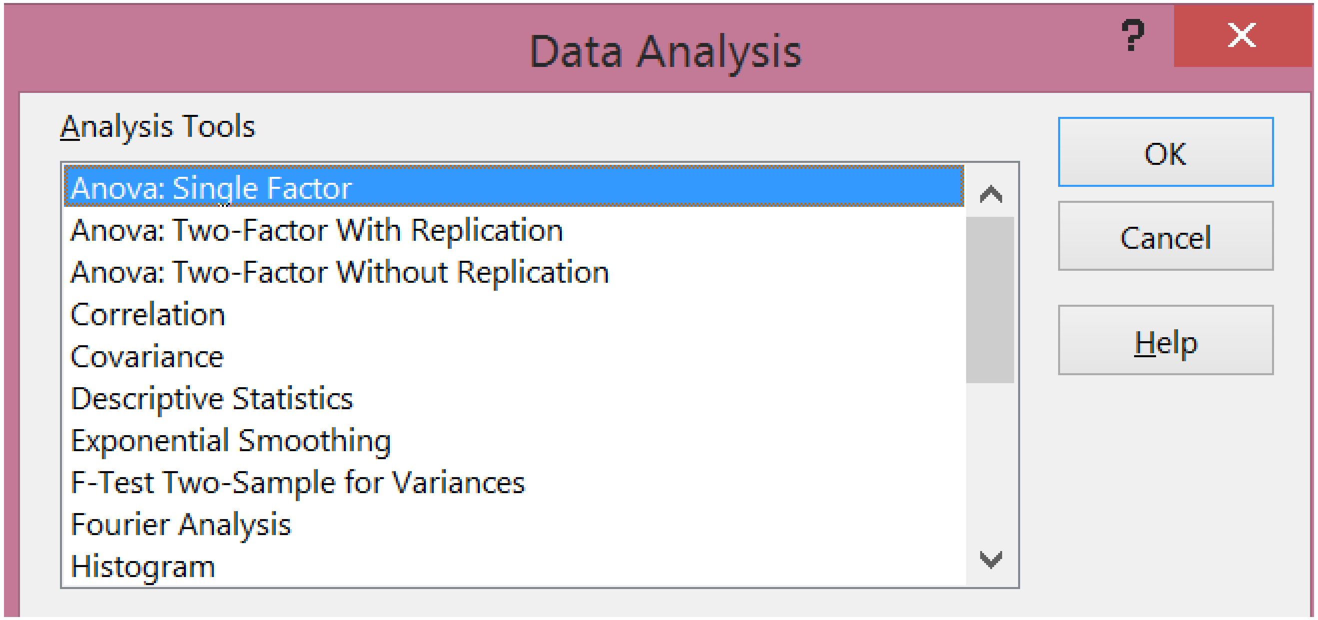
the variation we see is simply due to random chance, and therefore we cannot conclude that shelf height has any impact on sales

	Degrees of Freedom for Numerator											
	1	2	3	4	5	6	7	8	9	10	11	12
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.30	2.25	2.20	2.16	2.13
	7.68	**5.49**	**4.60**	**4.11**	**3.79**	**3.56**	**3.39**	**3.26**	**3.14**	**3.06**	**2.98**	**2.93**
28	4.20	3.34	2.95	2.71	2.56	2.44	2.36	2.29	2.24	2.19	2.15	2.12
	7.64	**5.45**	**4.57**	**4.07**	**3.76**	**3.53**	**3.36**	**3.23**	**3.11**	**3.03**	**2.95**	**2.90**
29	4.18	3.33	2.93	2.70	2.54	2.43	2.35	2.28	2.22	2.18	2.14	2.10
	7.60	**5.52**	**4.54**	**4.04**	**3.73**	**3.50**	**3.32**	**3.20**	**3.08**	**3.00**	**2.92**	**2.87**
30	4.17	3.32	2.92	2.69	2.53	2.42	2.34	2.27	2.21	2.16	2.12	2.09
	7.56	**5.39**	**4.51**	**4.02**	**3.70**	**3.47**	**3.30**	**3.17**	**3.06**	**2.98**	**2.90**	**2.84**
32	4.15	3.30	2.90	2.67	2.51	2.40	2.32	2.25	2.19	2.14	2.10	2.07
	7.50	**5.34**	**4.46**	**3.97**	**3.66**	**3.42**	**3.25**	**3.12**	**3.01**	**2.94**	**2.86**	**2.80**
34	4.13	3.28	2.88	2.65	2.49	2.38	2.30	2.23	2.17	2.12	2.08	2.05
	7.44	**5.29**	**4.42**	**3.93**	**3.61**	**3.38**	**3.21**	**3.08**	**2.97**	**2.89**	**2.82**	**2.76**
36	4.11	3.26	2.86	2.63	2.48	2.36	2.28	2.21	2.15	2.10	2.06	2.03
	7.39	**5.25**	**4.38**	**3.89**	**3.58**	**3.35**	**3.18**	**3.04**	**2.94**	**2.86**	**2.78**	**2.72**
38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09	2.05	2.02
	7.35	**5.21**	**4.34**	**3.86**	**3.54**	**3.32**	**3.15**	**3.02**	**2.91**	**2.82**	**2.75**	**2.69**
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.07	2.04	2.00
	7.31	**5.18**	**4.31**	**3.83**	**3.51**	**3.29**	**3.12**	**2.99**	**2.88**	**2.80**	**2.73**	**2.66**
42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.06	2.02	1.90
	7.27	**5.15**	**4.29**	**3.80**	**3.49**	**3.26**	**3.10**	**2.96**	**2.86**	**2.77**	**2.70**	**2.64**
44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05	2.01	1.98
	7.24	**5.12**	**4.26**	[illegible]	**3.46**	**3.24**	**3.07**	**2.94**	**2.84**	**2.75**	**2.68**	**2.62**
46	4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.14	2.09	2.04	2.00	1.97
	7.21	**5.10**	**4.24**	**3.76**	**3.44**	**3.22**	**3.05**	**2.92**	**2.82**	**2.73**	**2.66**	**2.60**
48	4.04	3.19	2.80	2.56	2.41	2.30	2.21	2.14	2.08	2.03	1.99	1.96
	7.19	**5.08**	**4.22**	**3.74**	**3.42**	**3.20**	**3.04**	**2.90**	**2.80**	**2.71**	**2.64**	**2.58**
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.02	1.98	1.95
	7.17	**5.06**	**4.20**	**3.72**	**3.41**	**3.18**	**3.02**	**2.88**	**2.78**	**2.70**	**2.62**	**2.56**
55	4.02	3.17	2.78	2.54	2.38	2.27	2.18	2.11	2.05	2.00	1.97	1.93
	7.12	**5.01**	**4.16**	**3.68**	**3.37**	**3.15**	**2.98**	**2.85**	**2.75**	**2.66**	**2.59**	**2.53**
60	4.00	3.15	2.76	2.52	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	7.08	**4.98**	**4.13**	**3.65**	**3.34**	**3.12**	**2.95**	**2.82**	**2.72**	**2.63**	**2.56**	**2.50**
65	3.99	3.14	2.75	2.51	2.36	2.24	2.15	2.08	2.02	1.98	1.94	1.90

Anova in Excel

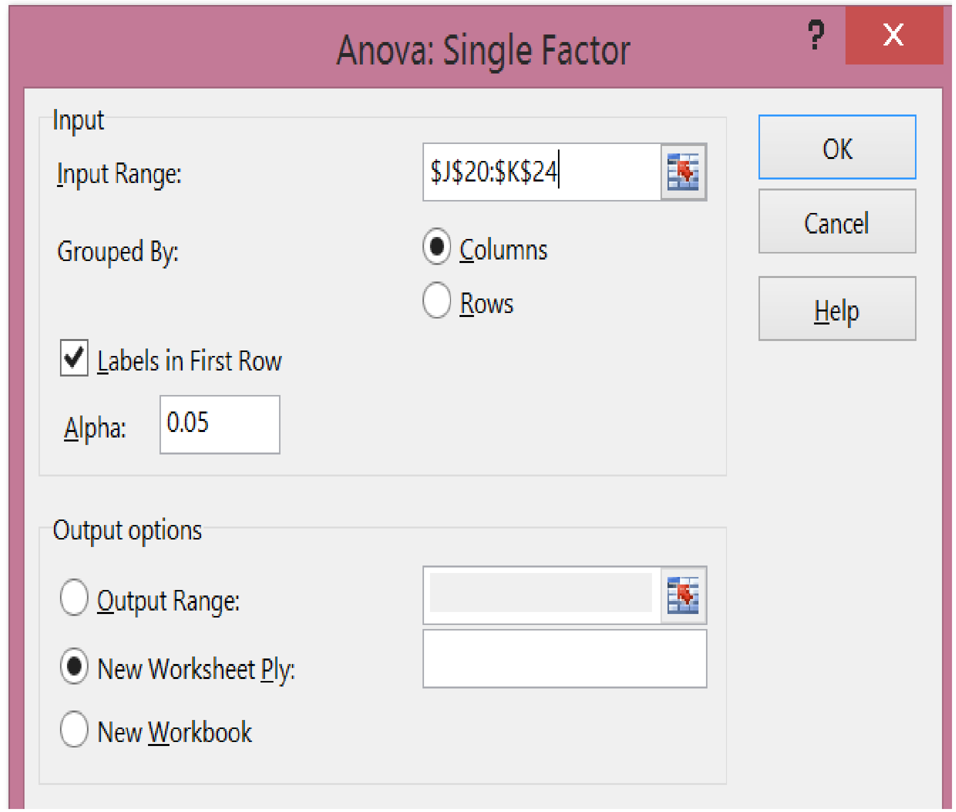
Of course, we can also use tools for ANOVA.

In Excel: Data\Data Analysis\ANOVA Single Factor



Anova in Excel

Shelf 1	Shelf 2	Shelf 3	Shelf 4	Shelf 5
210.5	198.1	170.5	167.1	188.5
198.1	189	225.5	167.9	177.7
145.3	210.3	158	175.5	176.5
185.5	254.4	139.4	175	158
189.1	210.3	156.4	149.1	174.5
135.9	160.9	217.1	189.3	181.7
180	120.8	189.1	198.2	176.2
149.4	167.8	158.2	205	177.9
176.4	148.9	218.1	233.5	189.1
229	190.4	178.9	167.9	187.1



Anova in Excel

Anova: Single Factor

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Shelf 1	10	1799.2	179.92	874.4529
Shelf 2	10	1850.9	185.09	1401.637
Shelf 3	10	1811.2	181.12	913.8396
Shelf 4	10	1828.5	182.85	587.005
Shelf 5	10	1787.2	178.72	82.51289

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	250.718	4	62.6795	0.081203	0.987743	2.578739
Within Groups	34735.02	45	771.8894			
Total	34985.74	49				

Anova

Conclusion:

- Fail to reject the Null Hypothesis
    - Shelf height has no impact on sales

Coming Up

Anova:

Two Way Tests

Post Hoc Tests

Anova in Excel

Anova: Single Factor

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Shelf 1	10	1799.2	179.92	874.4529
Shelf 2	10	1850.9	185.09	1401.637
Shelf 3	10	1811.2	181.12	913.8396
Shelf 4	10	1828.5	182.85	587.005
Shelf 5	10	1787.2	178.72	82.51289

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	250.718	4	62.6795	0.081203	0.987743	2.578739
Within Groups	34735.02	45	771.8894			
Total	34985.74	49				

Anova

Total Variation = Between Variance + Within Variance

Anova

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Another way of looking at total variation is:

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2. Calculate Difference between each observation and the Grand Mean
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Should we check if total variation calculated this way will add up to the sum of between variance and within variance?

SST (Total Sum of Squares) = 459.775

SSB (Sum of Squares Between) = 13.875

SSW (Sum of Squares Within) = 445.9

Anova Assumptions

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1. The populations from which the samples were obtained must be normally or approximately normally distributed

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2. The samples must be independent

3. The variances of the populations must be equal

An ANOVA is used when the DV(outcome) is continuous, and the IVs (factors) are discrete

Agenda

Anova
- One Way
- **Two Way**
- Post Hoc Tests

Chi Square
- Association Tests
- Goodness-of-fit Tests

Chi Square Parametric
- Tests of Variance

Two Way Anova

Example – 2 Factors Influencing Outcome

Two Way Anova

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- Let's say we are interested in understanding the impact of both shelf level as well as aisle placement on sales for Brand A

- That is, not only the height of the product placed, but also other brands / categories that the product is placed in are hypothesized to have an impact on Brand A sales

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- If there are three different aisles, we have 3*5 different placements for Brand A

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- That is, not only the height of the product placed, but also other brands / categories that the product is placed in are hypothesized to have an impact on Brand A sales
- If there are three different aisles, we have 3*5 different placements for Brand A
- How do we determine if mean sales rates are different between the groups?

Two Way Anova

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- The population means of the second factor are equal. This is like the one-way ANOVA for the column factor.

Two Way Anova

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<u>There are three null hypothesis that can be tested in a two-way ANOVA</u>

- The population means of the first factor are equal. This is like the one-way ANOVA for the row factor.
- The population means of the second factor are equal. This is like the one-way ANOVA for the column factor.
- There is no interaction between the two factors. This is similar to performing a test for independence with contingency tables.

Output Interpretation

Look at interaction p-value first:

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Output Interpretation

Look at interaction p-value first:

- If significant, implies impact of factor 1 depends on levels of factor 2
- May not then matter if individual impacts are significant

If interaction p value is NS:

- Re run ANOVA dropping the interaction term

Two Way Anova

Example:

Is there a difference in energy expended (calories burned) based on stretching before exercise and weights during exercise?

PreStretch	AnkleWeights	Energy
No stretch	No weights	106.9
No stretch	No weights	84
No stretch	No weights	97.5
No stretch	No weights	97.1
No stretch	No weights	99.5
No stretch	Weights	100.2
No stretch	Weights	101
No stretch	Weights	118.5
No stretch	Weights	104.5
No stretch	Weights	111.2
Stretch	No weights	82.8
Stretch	No weights	80.4
Stretch	No weights	95.6
Stretch	No weights	82
Stretch	No weights	83.2
Stretch	Weights	89.1
Stretch	Weights	106.4
Stretch	Weights	98.3
Stretch	Weights	89.2
Stretch	Weights	104.6

Two Way Anova

Example:

Is there a difference in energy expended (calories burned) based on stretching before exercise and weights during exercise?

Factors (IVs) – 2: Pre Stretch and Ankle Weights

PreStretch	AnkleWeights	Energy
No stretch	No weights	106.9
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2 Way ANOVA: With Replication
Multiple observations for same combination of factors

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Two Way Anova - Excel

We do not need to get into all the calculation details:

– Run a two-way analysis in Excel:

Two Way Anova - Excel

We do not need to get into all the calculation details:

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Two Way Anova - Excel

We do not need to get into all the calculation details:

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– Data has to be arranged in a specific manner

	No weights	Weights
No stretch	106.9	100.2
	84	101
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	95.6	98.3
	82	89.2
	83.2	104.6

Two Way Anova - Excel

Anova: Two-Factor With Replication			
SUMMARY	No weights	Weights	Total
No stretch			
Count	5	5	10
Sum	485	535.4	1020.4
Average	97	107.08	102.04
Variance	68.38	59.587	85.09822
Stretch			
Count	5	5	10
Sum	424	487.6	911.6
Average	84.8	97.52	91.16
Variance	37.6	67.427	91.62267
Total			
Count	10	10	
Sum	909	1023	
Average	90.9	102.3	
Variance	88.44667	81.83778	

Two Way Anova - Excel

ANOVA						
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Sample	591.872	1	591.872	10.16115	0.005724	4.493998
Columns	649.8	1	649.8	11.15565	0.004154	4.493998
Interaction	8.712	1	8.712	0.149566	0.704045	4.493998
Within	931.976	16	58.2485			
Total	2182.36	19				

Agenda

Anova

- One Way
- Two Way
- **Post Hoc Tests**

Chi Square

- Association Tests
- Goodness-of-fit Tests

Chi Square Parametric

- Tests of Variance

Post Hoc Tests

Note that the ANOVA only tells us if at least one group mean is unequal, but not which one

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http://pages.uoregon.edu/stevensj/posthoc.pdf

Recap

Anova

- One Way
- Two Way
- Post Hoc Tests

Coming Up

Chi Square Tests

HYPOTHESIS TESTING

Introduction to Hypothesis Testing

Basic Framework of a Hypothesis Test

Distance Measures

Central Limit Theorem

Types of Hypothesis Tests

Agenda

Anova
- One Way
- Two Way
- Post Hoc Tests

Chi Square
- Association Tests
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Chi Square Parametric
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Chi-Square Tests

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  - Tests of association

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  - Goodness-of-Fit

Chi-Square Tests

Example using categorical or tabular Data

As a retailer you look at brand ROI to assess shelf space effectiveness. Looking at a particular category, carbonated beverages, you know across all your stores the share of wallet for top Brands A, B and all other (C) is as listed in the first table.

Chi-Square Tests

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Brand	Transaction Share
A	52%
B	35%
C (All Other)	13%

Chi-Square Tests

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Brand	Transaction Share
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You take a random sample of data from a particular store – 300 purchases of carbonated beverages.

Before you can start on any analysis, you first need to check if this difference implies this store is not like the population

Brand	# of Transactions	%
A	177	59%
B	78	26%
C (All Other)	45	15%

Random sample of 300 transactions from Store XXX

Chi-Square Tests

- The idea is to check the difference between what you see in your sample v/s what you expected in your sample, and then assess the chances of seeing that difference purely by chance

Column 1	Brand A	Brand B	Brand C
Observed	177	78	45
Expected	156	105	39

Chi-Square Tests

- The idea is to check the difference between what you see in your sample v/s what you expected in your sample, and then assess the chances of seeing that difference purely by chance

- If there was no difference between this store and all the other stores, what would be expect to see as the # of transactions for Brands A, B and all other (C)?

Column 1	Brand A	Brand B	Brand C
Observed	177	78	45
Expected	156	105	39

Chi-Square Tests

- A chi square test uses these “observed” and “expected” frequencies, to generate a conclusion about the statistical significance of the observed differences

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Mathematically, the quantity

$$\Sigma \frac{(f_o - f_e)^2}{f_e}$$

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Mathematically, the quantity

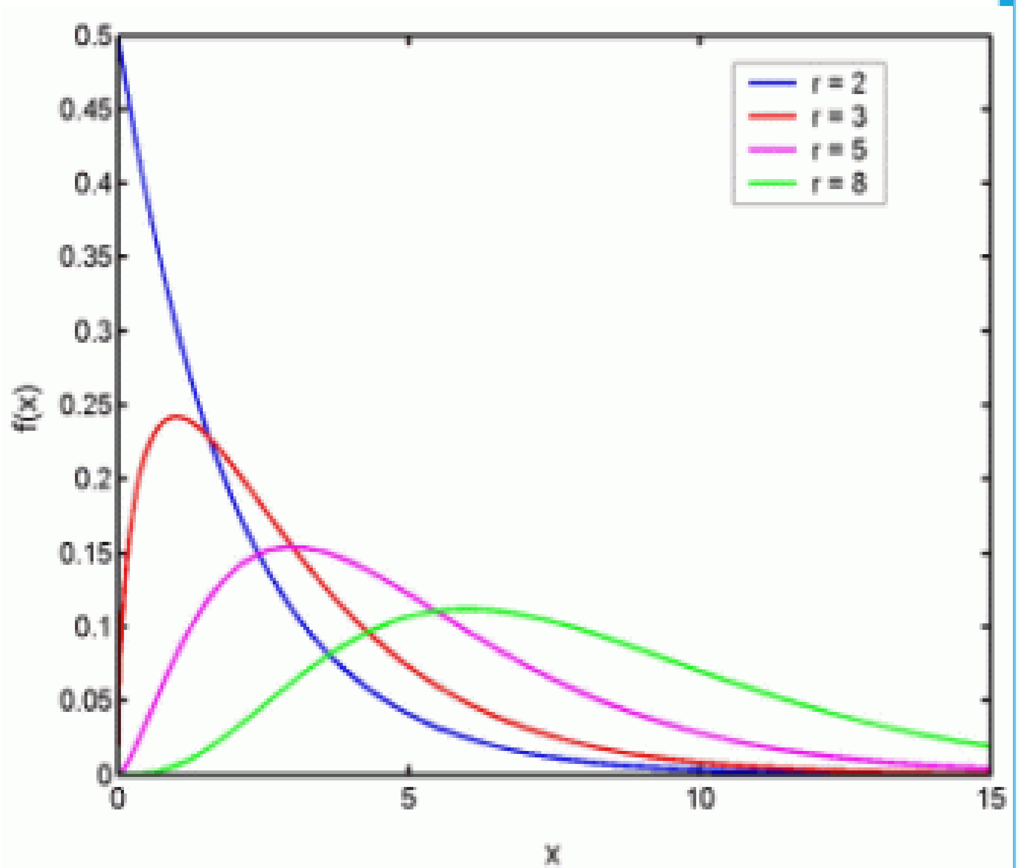
$$\sum \frac{(f_o - f_e)^2}{f_e}$$

follows a Chi Square Distribution, with k -1 degrees of freedom

Column 1	Brand A	Brand B	Brand C
Observed	177	78	45
Expected	156	105	39

Chi-Square Tests

- A Chi Square distribution is an asymmetric distribution that depends only on sample size
- It is generated as the square of std scores (Z) from a normal distribution
- As sample size increases, Chi Square tends to normal



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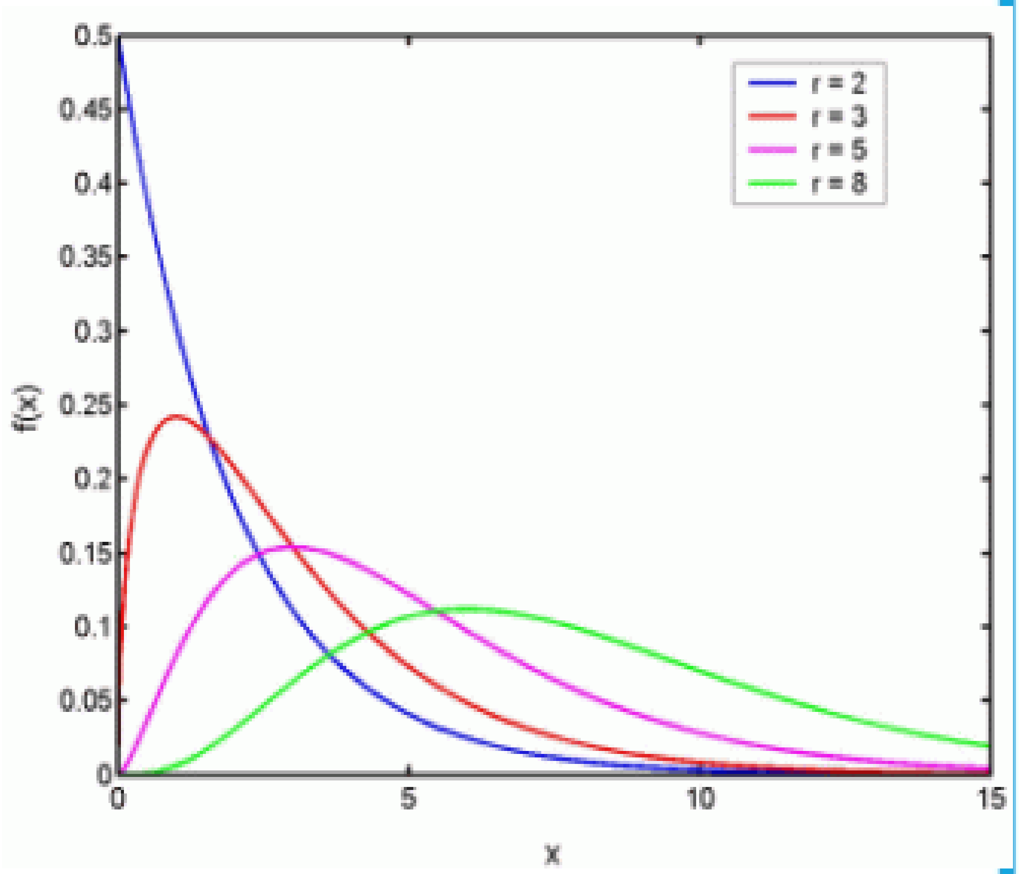

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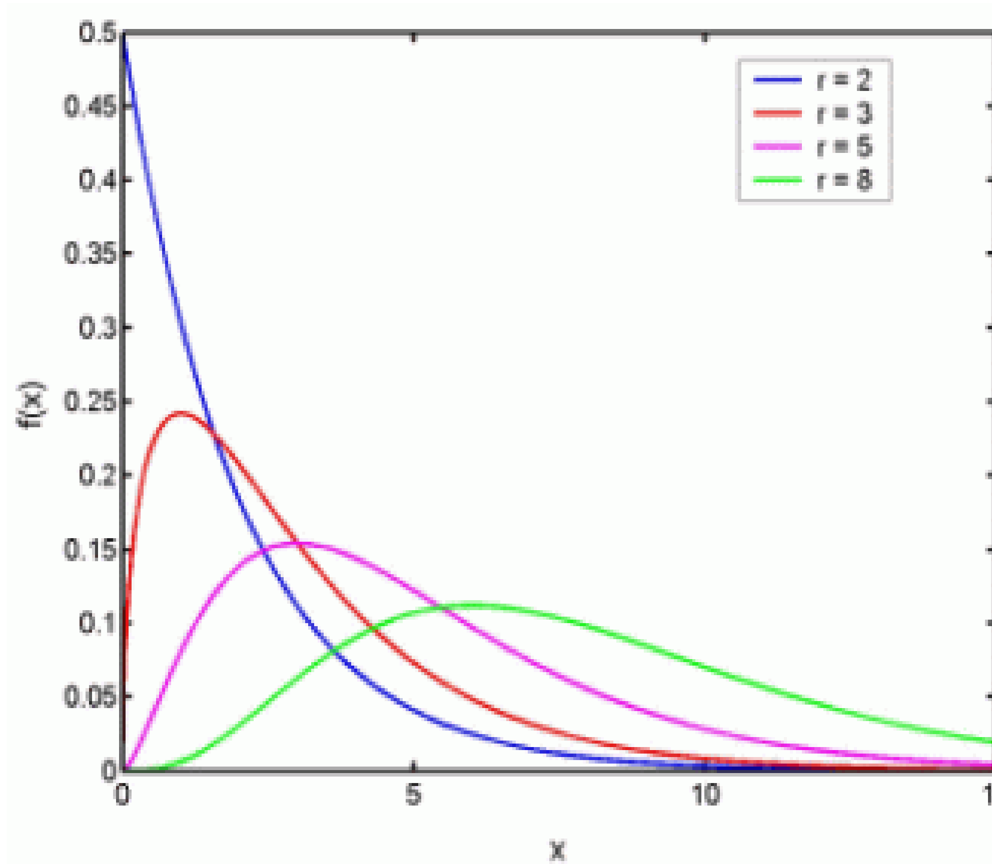

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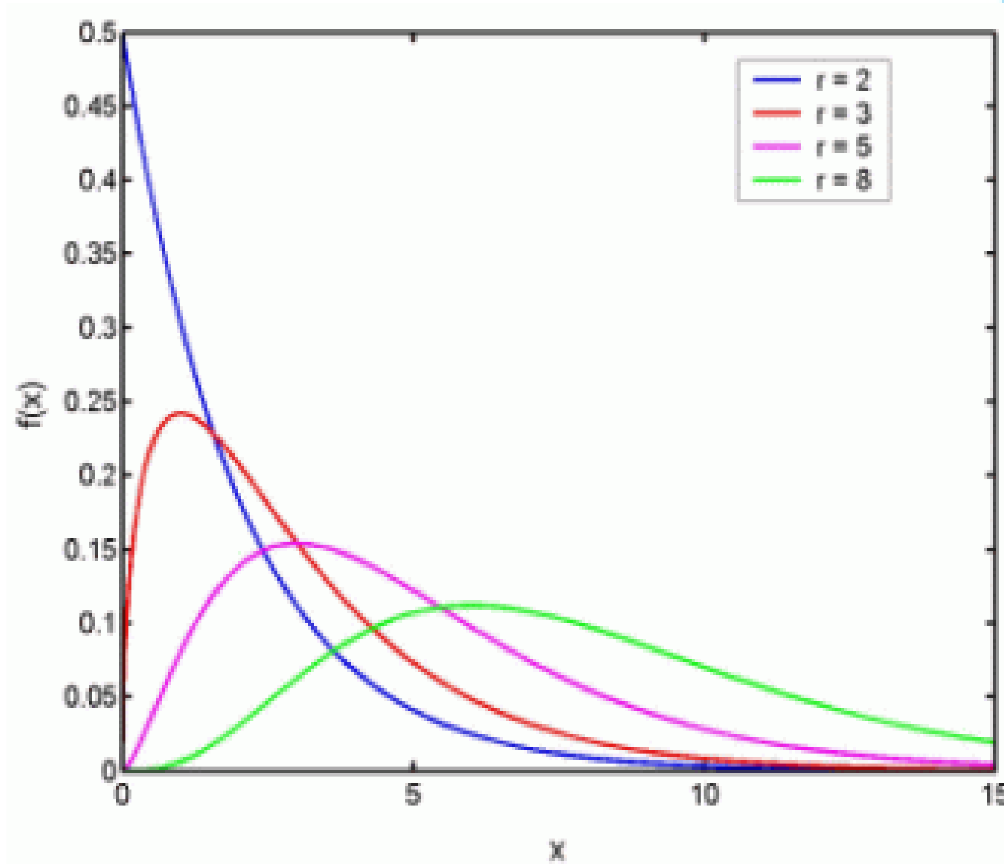

Chi-Square Tests

The Chi-Square statistic is built as:

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

For our example, Chi-Square Test Statistic:

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$$\chi^2 = \Sigma \frac{(f_o - f_e)^2}{f_e}$$

For our example, Chi-Square Test Statistic:

(177-156)^2/156 + (78-105)^2/105 + (45-39)^2/39 = 10.69

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For our example, Chi-Square Test Statistic:

(177-156)^2/156 + (78-105)^2/105 + (45-39)^2/39 = 10.69

To use a table: also need df

Degrees of Freedom = Number of cells - 1 = 3 - 1 = 2

Chi-Square Tests

Chi Square Distribution Table

d.f.	$\chi^2_{.25}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$	$\chi^2_{.001}$
1	1.32	2.71	3.84	5.02	6.63	7.88	10.8
2	2.77	4.61	5.99	7.38	9.21	10.6	13.8
3	4.11	6.25	7.81	9.35	11.3	12.8	16.3
4	5.39	7.78	9.49	11.1	13.3	14.9	18.5
5	6.63	9.24	11.1	12.8	15.1	16.7	20.5
6	7.84	10.6	12.6	14.4	16.8	18.5	22.5
7	9.04	12.0	14.1	16.0	18.5	20.3	24.3
8	10.2	13.4	15.5	17.5	20.1	22.0	26.1
9	11.4	14.7	16.9	19.0	21.7	23.6	27.9
10	12.5	16.0	18.3	20.5	23.2	25.2	29.6
11	13.7	17.3	19.7	21.9	24.7	26.8	31.3
12	14.8	18.5	21.0	23.3	26.2	28.3	32.9
13	16.0	19.8	22.4	24.7	27.7	29.8	34.5
14	17.1	21.1	23.7	26.1	29.1	31.3	36.1
15	18.2	22.3	25.0	27.5	30.6	32.8	37.7
16	19.4	23.5	26.3	28.8	32.0	34.3	39.3

1. What was our null hypothesis?

2. What is the critical value here at the 5% significance level?

3. What is the conclusion based on your test statistic?

Chi-Square Tests

Using Excel:

=CHITEST(AC3:AE3,AC4:AE4)

Column1	Brand A	Brand B	Brand C
Observed	177	78	45
Expected	156	105	39

=CHITEST(AC3:AE3,AC4:AE4)

Chi-Square Tests

Using Excel:

=CHITEST(AC3:AE3,AC4:AE4)

Column1	Brand A	Brand B	Brand C
Observed	177	78	45
Expected	156	105	39

=CHITEST(AC3:AE3,AC4:AE4)

This will generate a p-value directly. In this example: 0.004765139

Coming Up

Chi Square:

- Association Tests
- Goodness-of-Fit Tests

Agenda

Anova

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Chi Square

- **Association Tests**
- Goodness-of-Fit Tests

Chi Square Parametric

- Tests of Variance

Chi-Square Tests

A more complex example:

You look at preferences for beverages by age to understand if there is an association between age and brand preference, in order to decide if you need differentiated marketing strategies by age

Chi-Square Tests

<u>A more complex example:</u>

You look at preferences for beverages by age to understand if there is an association between age and brand preference, in order to decide if you need differentiated marketing strategies by age

You do a survey on a random sample, and get the following results:

Observed	Preference			
Brand	M 15-25	M 26-40	M 41-55	Total
Coke	49	50	69	168
Pepsi	24	36	38	98
Sprite	19	22	28	69
Total	92	108	135	335

Chi-Square Tests

<u>A more complex example:</u>

We want to check if the preference for a brand changes as age changes i.e., is there an association between brand preference and age, or are they independent?

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Brand	M 15-25	M 26-40	M 41-55	Total
Coke	49	50	69	168
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A more complex example:

We want to check if the preference for a brand changes as age changes i.e., is there an association between brand preference and age, or are they independent?

Expected value – calculate the expected values under the assumption that the null hypothesis is true

Observed	Preference			
Brand	M 15-25	M 26-40	M 41-55	Total
Coke	49	50	69	168
Pepsi	24	36	38	98
Sprite	19	22	28	69
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A more complex example:

Mathematical calculation :

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Brand	M 15-25	M 26-40	M 41-55	Total
Coke	49	50	69	168
Pepsi	24	36	38	98
Sprite	19	22	28	69
Total	92	108	135	335

Expected	Preference			
Brand	M 15-25	M 26-40	M 41-55	Total
Coke	46.14	54.16	67.70	168
Pepsi	26.91	31.59	39.49	98
Sprite	18.95	22.24	27.81	69
Total	92	108	135	335

Chi-Square Tests

A more complex example:

Mathematical calculation :

Expected Values = (Row Total * Column Total) / n,

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A more complex example:

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Expected Values = (Row Total * Column Total) / n,

where n is total number of observations in sample

Observed	Preference			
Brand	M 15-25	M 26-40	M 41-55	Total
Coke	49	50	69	168
Pepsi	24	36	38	98
Sprite	19	22	28	69
Total	92	108	135	335

Expected	Preference			
Brand	M 15-25	M 26-40	M 41-55	Total
Coke	46.14	54.16	67.70	168
Pepsi	26.91	31.59	39.49	98
Sprite	18.95	22.24	27.81	69
Total	92	108	135	335

Chi-Square Tests

A more complex example:

Now that we have observed and expected values, we can either manually calculate a Chi Square test statistic,

Or use a tool – like Excel

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Expected	Preference			
Brand	M 15-25	M 26-40	M 41-55	Total
Coke	46.13731	54.16119	67.70149	168
Pepsi	26.91343	31.59403	39.49254	98
Sprite	18.94925	22.24478	27.80597	69
Total	92	108	135	335

=chitest(B3:D5,H3:J5

CHITEST(actual_range, **expected_range**)

Agenda

Anova

- One Way
- Two Way
- Post Hoc Tests

Chi Square

- Association Tests
- **Goodness-of-Fit Tests**

Chi Square Parametric

- Tests of Variance

Chi-Square Tests

Very popular use of Chi Square, Goodness-of-fit tests if the data follows a particular distribution or not

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<u>Example:</u>

A gambler is playing a new game in a casino, which involves rolling three dice at a time. Winnings are directly proportional to the number of 6’s rolled.

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This is what is observed in 100 rolls of the dice

Number of 6's	Rolls
0	48
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Would you have cause to believe that the gambler is maybe "too" lucky, and is playing with loaded dice?

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What distribution would you expect the outcome of seeing a 6 on rolled dice to follow?

- **Binomial**

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=BINOM.DIST(O26,3,1/6,FALSE)

Expected	Number of 6's	Expected Prob	Expected 6's in 100 throws
	0	0.5787	57.8704
	1	0.34722	34.7222
	2	0.06944	6.94444
	3	0.00463	0.46296

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<u>Testing for a normal distribution or any type of distribution</u>

- Calculate expected distribution using the probability distribution formula

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  2. Bin the data into sub-intervals

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  5. Construct Chi Square and test

Coming Up

Chi Square Parametric:

- Tests of Variance

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Chi Square Parametric

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Chi-Square Test of Variance

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- The Chi Square tests reviewed so far have been **non-parametric tests**
- To apply these tests, we do not need the underlying population to follow any specific distribution
- There are many kinds of non-parametric tests, an equivalent one for every parametric test
- Which types of test are preferable? Non-parametric, or parametric?

Non-Parametric Tests

- There are tests in statistics that do not require a specific distribution for your data – **non-parametric tests**

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  - Non-parametric tests are less powerful than parametric tests in the sense that they use more information and are sometimes less flexible in terms of testing different kinds of hypothesis
  - Also, as sample size increases, it turns out that non-parametric test distributions approximate normal distributions

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- What about the variance of the samples?

  – The variance of samples will follow a Chi Square distribution

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  - Is the new approach sufficiently different from the standard to justify investment in it?

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Example:

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- If our aim is consistency, we check if there is significant reduction in variance of resolution time:

H0: Variance = 4.5 minutes

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DF = 29

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We could use a table to compare calculated Test Stat against a critical Value

OR

Directly calculate p-values in Excel

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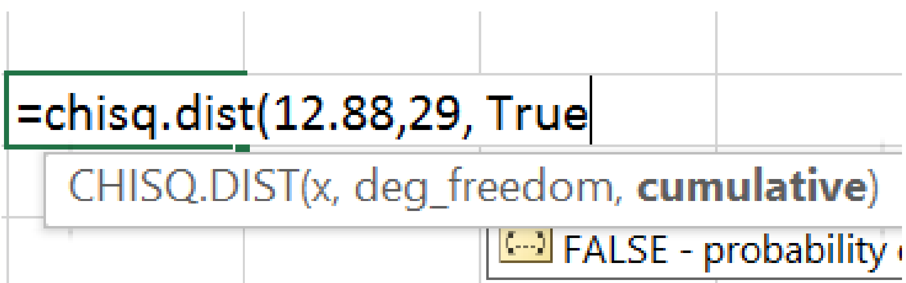

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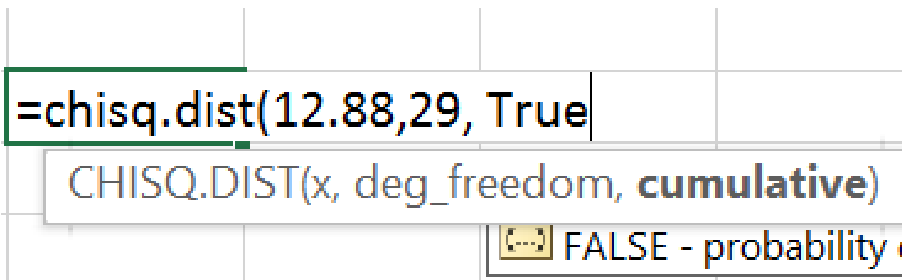

p-value = 0.002, therefore reject the null and conclude variance of calls has reduced