PAKISTAN INSTITUTE OF ENGINEERING AND APPLIED SCIENCES

DEPARTMENT OF PHYSICS AND APPLIED MATHEMATICS

TERMINAL EXAM, PAM–533 COMPUTATIONAL PHYSICS, DATE: 06/09/2021

Every question carries 10 marks. Please avoid cacography.

Problem 1: Use scientific calculator to generate 10 random float points between 0.0 and 5.0 inclusive.

Use variance reduction method to estimate $\int_0^5 x^{\sin x} dx$ using the 10 generated random numbers.

You may choose $\int_0^5 x dx = 12.5$ as a known integral.

Problem 2: An ice cream vendor write downs the number of ice creams sold vs the number of hours of sunshine received per day from Monday to Friday. The data looks like this:

Sunshine Hours "x"	Ice Cream Sold "y"
2	4
3	5
5	7
7	10
9	15

Table 1: Ice cream sale vs hours of sunshine in a week

The ice cream vendor believes that there is a linear relationship between the number of ice creams sold and the number of hours of sunshine received on a given day.

What is the expected sale of the ice creams on a clear day with 10 hours of sun shine?

What is the uncertainty in the sale on this day?

Problem 3: The Fermi Dirac distribution describes the probability of finding a quantum particle of half-integer spin in energy state E:

$$f_{FD} = \int_{E_{min}}^{E_{max}} \frac{1}{e^{(E-\mu)/kT} + 1} \tag{1}$$

where μ is the Fermi energy.

Identify this problem in the context of this course and find μ such that $\int_0^2 f_{FD} dE = 1$ with $kT = \frac{1}{40}$ eV.

Generous Help: $\int \frac{1}{e^{(E-\mu)/kT}+1} dE = -kT \ln\left(e^{\frac{\mu}{kT} - \frac{E}{kT}} + 1\right) + C$

Problem 4: Write down the dynamic form of the given ODE:

$$m\ddot{\vartheta} = -\sqrt{\frac{g}{L}} \sin\vartheta - \beta\dot{\vartheta} + \gamma \sin\omega t \tag{2}$$

Problem 5: Solve $y' = (x + y)\sin(xy)$ using RK4 method for $0 \leq x \leq 2$ with step-size of $h = 0.2$ and IVP: $y(0) = 5$.

Present your solution in tabulated form showing numeric values of x, y, f_1, f_2, f_3 and f_4 at each step clearly.

THINK SMART ☺