Question 1:

135	135	129	133	130	134	134	137
133	133	132	132	135	127	55	119
132	127	222	200	65	55	96	110
110	104	210	65	55	103	129	160
105	112	65	45	250	201	219	231
167	65	55	223	216	231	240	238
221	55	240	223	214	216	218	219
224	217	222	214	215	217	219	220

210	65	55
65	45	250
55	223	216

0.75

$$\Delta x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \longrightarrow \Delta y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

a) For the (8-by-8) image block given, It is required to do edge detection using Prewitt operators (assume image already smoothed with Gaussian filter). Is the given above the Δ_y *or* Δ_x operator? **Deduce the other one**. Which of them estimates horizontal edges and which estimates vertical edges? **(1.5)**

b) Apply the x-gradient and y-gradient Prewitt operators to pixel $[f(5,4) = 45]$ → **Find the values for** Δ_y *and* Δ_x, Hence Calculate the **strength and orientation** of the gradient at that pixel. **Does the orientation angle make sense according to the edge direction you see in the block**? **(2.5)**

(a) $\Delta x \longrightarrow$ shows vertical edges, $\Delta y \longrightarrow$ shows horiz. edges

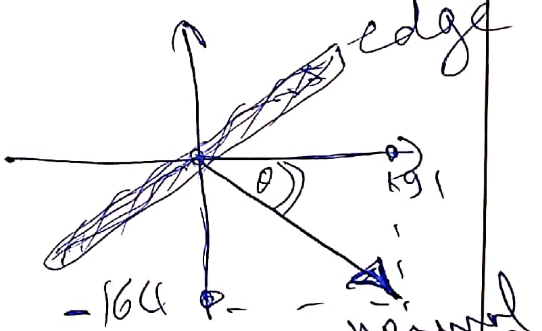
0.75

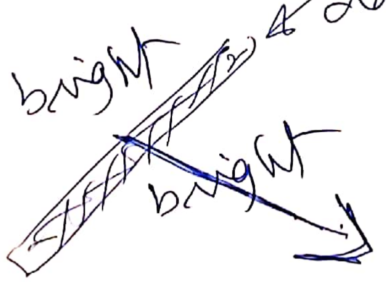
(b)
$$\Delta x = -(210+65+55) + (55+250+216) = +191$$

0.75
$$\Delta y = (210+65+55) - (55+223+216) = -164$$

0.75
$$|\Delta| = \sqrt{(191)^2 + (164)^2} = 251.7$$

1
$$\theta = \tan^{-1}\left(\frac{-164}{191}\right) = -40.65^\circ$$





makes sense as corresponds to ⊥ edge-direction

c) In the Prewitt edge detection we apply the following processes $\Delta_x = M_x^{prewitt} * \{g(x,y) * f(x,y)\}$ and $\Delta_y = M_y^{prewitt} * \{g(x,y) * f(x,y)\}$. Explain **what will be different** if you want to use the Canny edge detection (**Re-Write the equations in this case**). **(2)**

d) Explain why the Canny approach is better than the Prewitt approach in edge detection? **(1)**

(c) Canny: we take advantage of

$$\frac{\partial}{\partial x}(f \circledast g) \Rightarrow f \circledast g_x'$$

$$\frac{\partial}{\partial y}(f \circledast g) \Rightarrow f \circledast g_y'$$

2

so, instead of $M_x \circledast (g \circledast f) \Rightarrow f \circledast g_x'$

$M_y \circledast (g \circledast f) \Rightarrow f \circledast g_y'$

(b) better in (i) less convolutions instead of 4 convs → only two

0.5

0.5 (ii) No derivative operator approxim. (M_x, M_y) (Prewitt, Roberts, sobel) ..... are replaced by the real derivative of the gaussian

$\Rightarrow g'$

Question 2:

(a) For the 8-by-8 image above in (Question-1), apply the given normalized Gaussian filter on pixel $f(5,4) = 45$ and find the pixel's new value. Has it been smoothed? Does this smoothing preserves edges? And why? **(2)**

0.075	0.124	0.075
0.124	0.204	0.124
0.075	0.124	0.075

210	65	55
65	45	250
55	223	216

(b) Explain briefly the main differences between the Median Filter, Gaussian Filter, Bilateral Filter and the Non-local-means filter for **Noise Removal** and show which of them preserves the edges and how. **(2)**

(a) new pixel value $\cong$ 124 $(\sum w_{ij} * P_{ij})$

1

it has been smoothed → the dark value has been brightened & this is harmful

1

to the edge information → blur occured.

(b) (i) Median → non-linear filter & order-statistics filter we take the median value → it is very good for salt + pepper Noise when it is moderate

0.5

(ii) Gaussian → smoothes according to size and σ but causes blur since it combines (averages) pixels that may be at different sides of edges

0.5

0.5

(iii) Bilateral will give a weight (Range) weight that will

200	205	200	200	200
200	200	200	200	200
200	200	10	10	10
10	10	10	10	10
	10	10	10	10

give ≃ 1 weight to pixels similar in value while ≃ 0 to pixels with different range of values

(iv) (NLM) 0.5 will average pixel with other Non_Neighbors but with similar characteristics (similar value + Neighbors)

(c) The following equation represents the **Bilateral Filter**. Use the 8-by-8 image (Question-1) and a (3-by-3) normalized Gaussian filter (above).

$$BF[I]_{\mathbf{p}} = \underbrace{\overset{\text{new}}{\frac{1}{W_{\mathbf{p}}}}}_{\text{normalization}} \sum_{\mathbf{q} \in S} \underbrace{\overset{\text{not new}}{G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)}}_{\text{space weight}} \underbrace{\overset{\text{new}}{G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|)}}_{\text{range weight}} I_{\mathbf{q}}$$

$\longrightarrow$ Call it α_{ij}

Consider that the Bilateral filter is centered at pixel $f(5,4) = 45$, Take $\sigma_r = 300$ and $W_p = 1.0$. Calculate the new value for pixel $f(5,4) = 45$ (use the pixel values from the image, and the 3-by-3 Gaussian filter given, as well as the range weights that you should calculate). **(3)**

(d) Explain the difference between **Low-pass, High-pass, Band-pass and Notch filters** in their frequency domain characteristics. Which of them would you use to get rid of a periodic 50Hz supply noise distorting an image? Justify your answer with a figure **(2)**

(c) P_{new} = new pixel value = $(210 * 0.075 * \alpha_{11} + 65 * 0.124 * \alpha_{12} + 55 * 0.075 * \alpha_{13}) + (65 * 0.124 * \alpha_{21} + 45 * 0.204 * \alpha_{22} + 250 * 0.124 * \alpha_{23}) + (55 * 0.075 * \alpha_{31} + 223 * 0.124 * \alpha_{32} + 216 * 0.075 * \alpha_{33})$

1.5

$\alpha_{11} = e^{-\frac{|210-45|}{300}}$, $\alpha_{12} = e^{-\frac{|65-45|}{300}}$, $\alpha_{13} = e^{-\frac{|55-45|}{300}}$

1

$\alpha_{21} = e^{-\frac{|65-45|}{300}}$, $\alpha_{22} = 1$, $\alpha_{23} = e^{-\frac{|250-45|}{300}}$

$\alpha_{31} = e^{-\frac{|55-45|}{300}}$, $\alpha_{32} = e^{-\frac{|223-45|}{300}}$, $\alpha_{33} = e^{-\frac{|216-45|}{300}}$

$\therefore P_{new} = \checkmark\ 81.41 \simeq \boxed{81}$ 0.5

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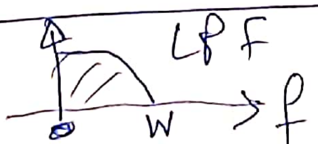
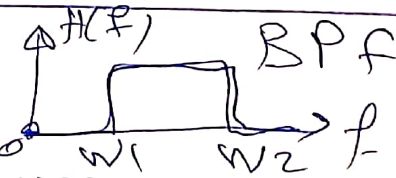
d

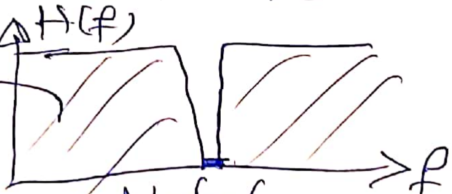


(LPF)

(HPF)

1.5





used to remove specific freq e.g. 50 Hz

Notch filter

0.5

[illegible] (2D)

→ LPF