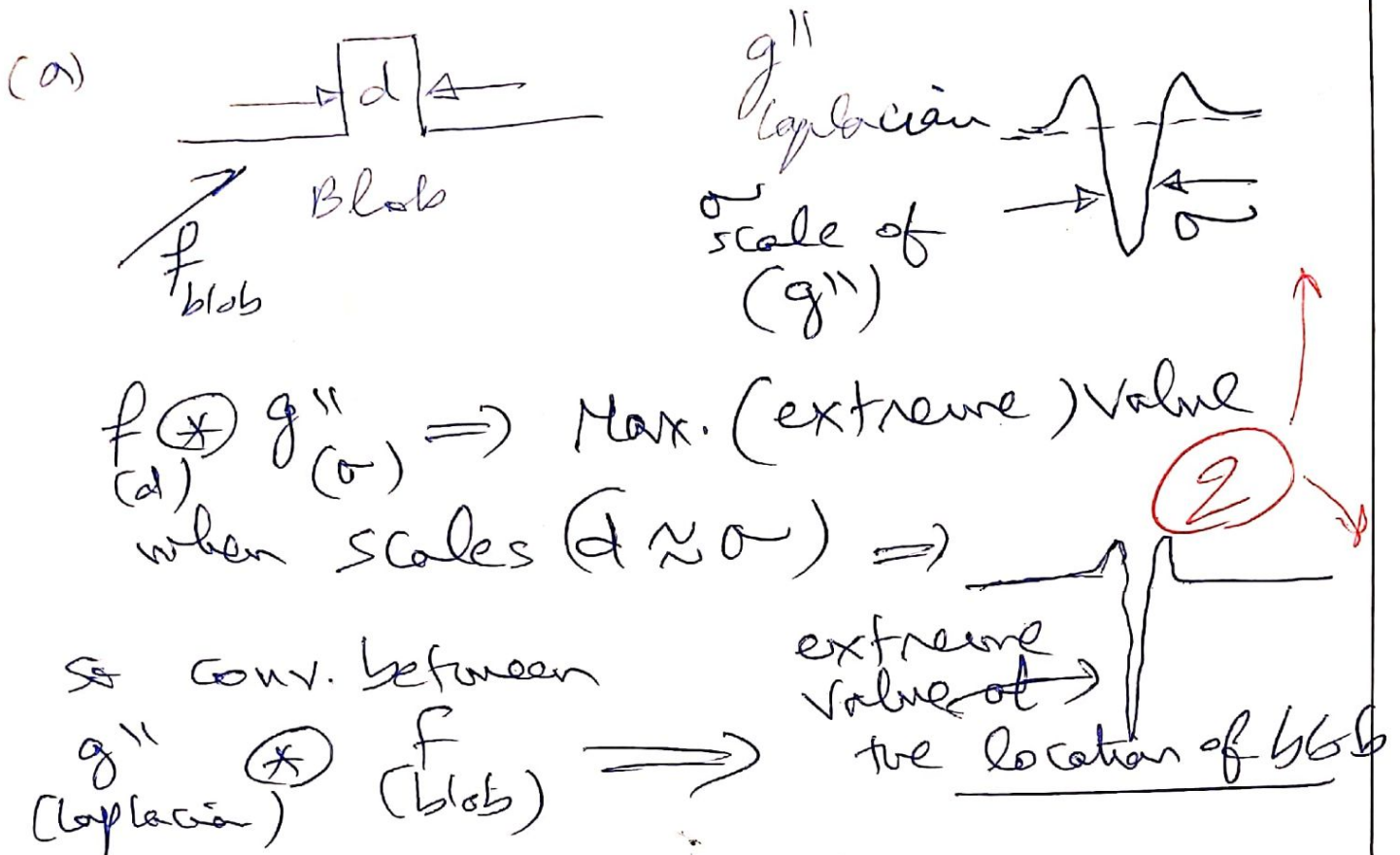


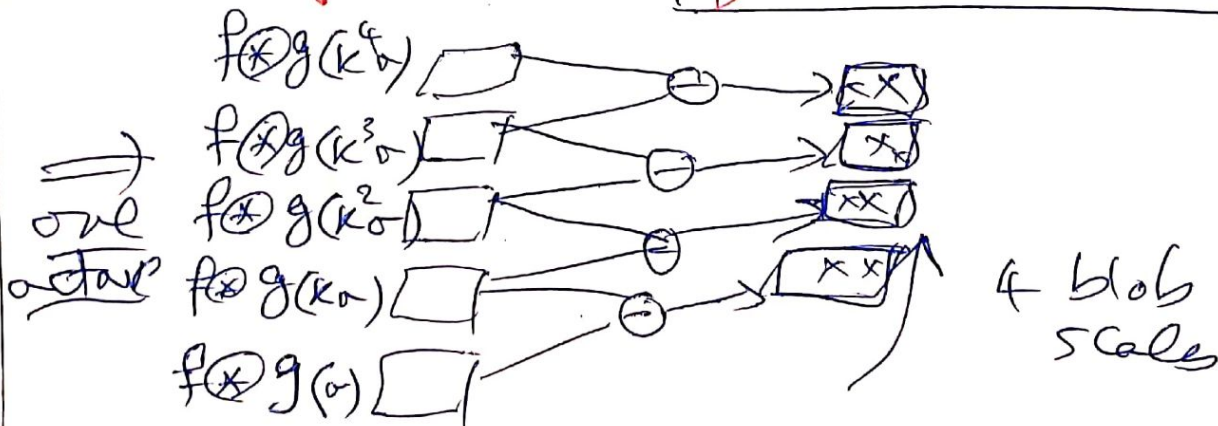
**Question 3:**

- (a) Explain using equations: Why and How the Laplacian of Gaussian is used to detect blobs in images and how the Gaussian width " $\sigma$ " determines the scale of the detected blob (2)  
 (b) Explain using a figure how many blob-scales can be detected in the standard SIFT algorithm? (1)



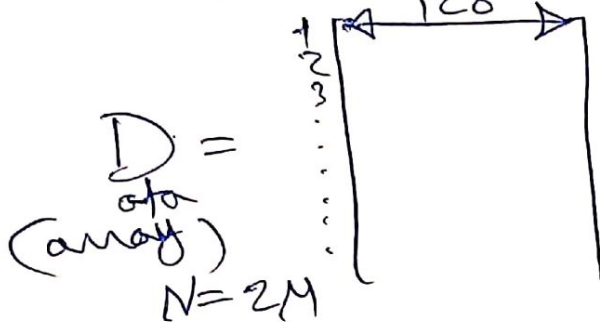
(b) 4 octaves, each octave detects 4 SIFT (blob) scales

then, resize image to half rows, columns and Repeat  $\rightarrow$  octave 2, 3, 4  
 so, we get 16 scales

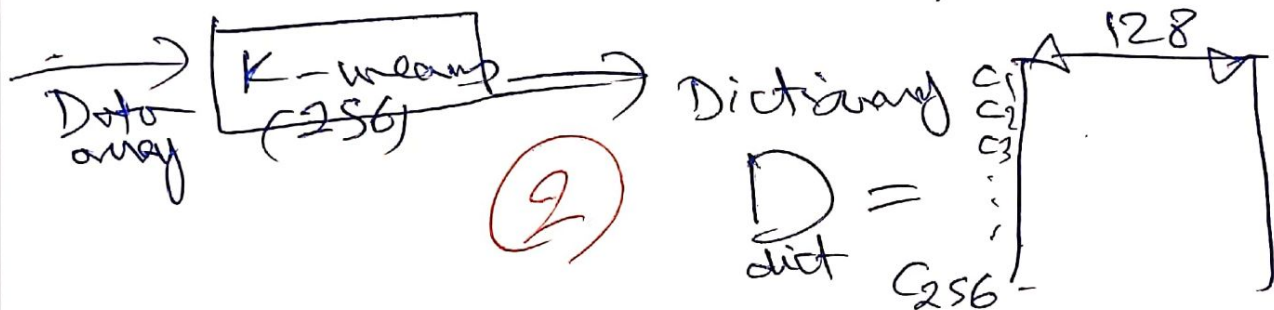


- (c) Using 10,000 images, explain the steps required to generate a SIFT codebook (dictionary) with 256 centers. Assume each image will produce 200 SIFT descriptors (Blobs), show the size of the array used before the K-means and the size of the dictionary (number of rows and number of columns) (2)
- (d) Explain how to extract a 256-dimension (bag-of-words) feature vector for a new image using the dictionary in (c). Then show the extracted vector assuming the new image has 200 SIFT-interest points, where 100 of them match dictionary center #5, 50 match center #11 and 50 match center #255. (2)

(c) (1) apply(SIFT) on each image (10,000 \* 200)  $\approx$  2M(SIFT) descriptors each <128

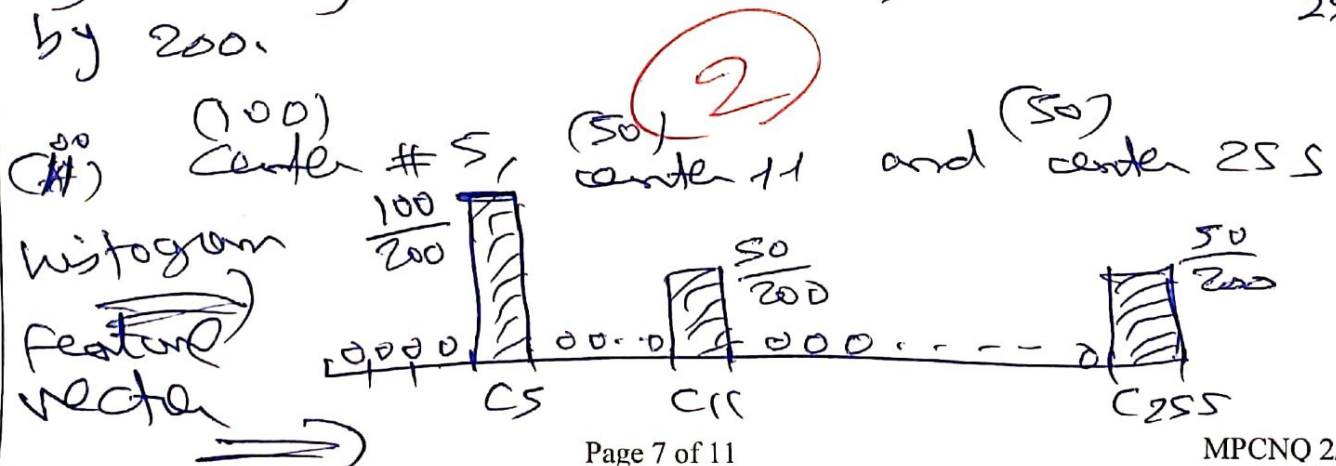
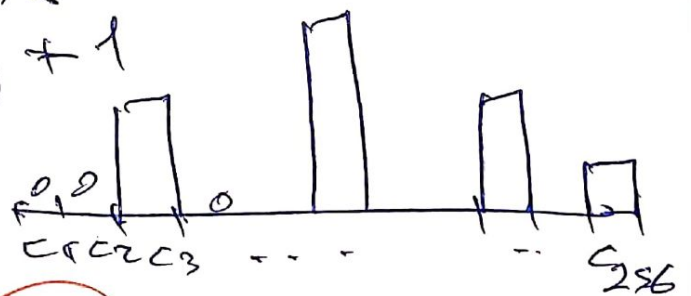


(2) Apply K-means with  $K = 256$  clusters



(d) (1) New image  $F \rightarrow$  has  $\approx 200$  interest points (blobs) for each one, search in dict for nearest center  $\leftarrow$  increment its histogram count by +1

then, Normalize by dividing all counts by 200.





#### Question 4:

- (a) A quantized Image has the following 5 pixel values shown in the table below with their number of occurrences in the image:

Pixel Value	Number of occurrences
0	5
1	10
2	170
3	10
4	5

- (i) Design a Huffman code using the given image data showing the binary codes for all the 5 possible pixel values (2)  
(ii) Find the total number of bits required to store this image using the designed Huffman code and the total number of bits required if we are using a fixed-length encoder. (1)

(b) For the image given by the table above:

- (i) Plot the Normalized Histogram as well as the cumulative distribution of the image (1.5)  
(ii) Apply the Histogram Equalization Algorithm on this image and find the new pixel values and Plot the histogram for the equalized image (2.5)

(a) Huffman (1)

(1)  $S_2: 0.85$   
 $S_1: 0.05$   
 $S_3: 0.05$   
 $S_0: 0.025$   
 $S_4: 0.025$

(2)  $S_2: 0.85$   
 $S_3: 0.05$   
 $S_0: 0.025$   
 $S_4: 0.025$

(3)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(4)  $S_1(S_3(S_0, S_4)): 0.15$   
 $S_2: 0.85$

Code assign:

(5)  $\Sigma$  bits with Huffman =  
 $(170 \times 1) + (5 \times 4) + (5 \times 4) + (10 \times 3) + (10 \times 2) = 260$  bits  
VS Fixed length coding  
 $\rightarrow 5$  symbols  $\rightarrow 3$  bits  
 $3 \times 200 = 600$  bits

(6)  $S_1: 10$  2 bits  
 $S_2: 110$  3 bits  
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(7)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(8)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(9)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(10)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(11)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(12)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(13)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(14)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(15)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(16)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(17)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(18)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(19)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(20)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(21)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(22)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(23)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(24)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(25)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(26)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(27)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(28)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(29)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(30)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(31)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(32)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(33)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(34)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(35)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(36)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(37)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(38)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(39)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(40)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(41)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(42)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(43)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(44)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(45)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(46)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(47)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(48)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(49)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(50)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(51)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(52)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(53)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(54)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(55)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(56)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(57)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(58)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(59)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(60)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(61)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(62)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(63)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(64)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(65)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(66)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(67)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(68)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(69)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(70)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(71)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(72)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(73)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(74)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(75)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(76)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(77)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(78)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(79)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(80)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(81)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(82)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(83)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(84)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(85)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(86)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(87)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(88)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(89)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(90)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(91)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(92)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(93)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(94)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(95)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(96)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(97)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

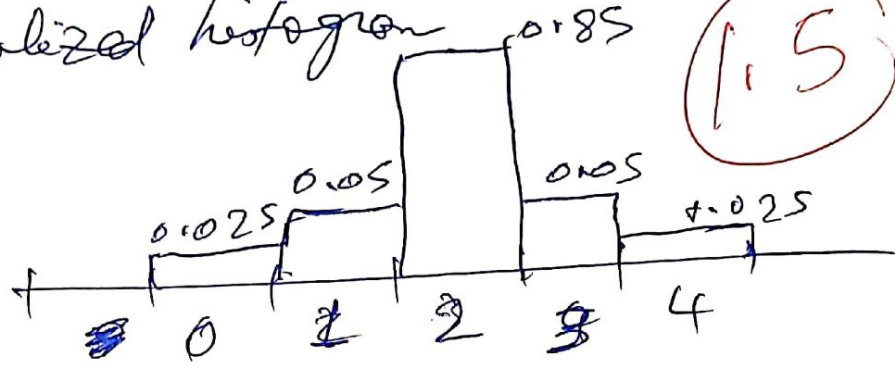
(98)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

(99)  $S_1(S_3(S_0, S_4)) \Rightarrow 1$   
 $S_3(S_0, S_4) \Rightarrow 11$   
 $S_0: 1110$  4 bits  
 $S_4: 1111$  4 bits

(100)  $S_2: 0.85$   
 $S_3(S_0, S_4): 0.1$   
 $S_1: 0.05$

## (b) Histogram equalization

(i) Normalized histogram



(ii) new pixel values

$$(L * \sum p) \rightarrow 4$$

$$S_0 = 4 * 0.025 = 0.1 \rightarrow 0$$

$$S_1 = 4 * (0.025 + 0.05) = 0.3 \rightarrow 0$$

$$S_2 = 4 * (0.025 + 0.05 + 0.85) = 3.7 \approx 4$$

$$S_3 \rightarrow 3.9 \approx 4$$

$$S_4 \rightarrow 4 \approx 4$$

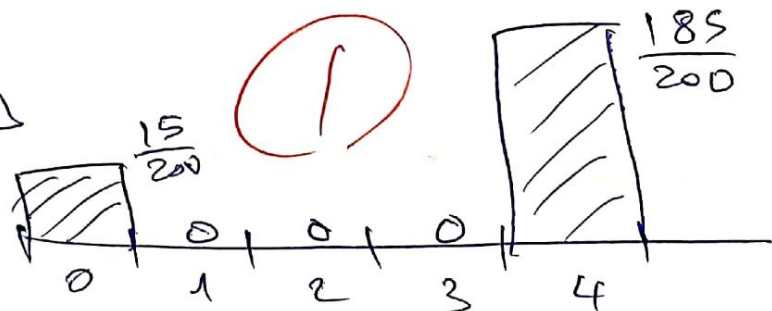
So, new count of pixels:

$$r_0 \Rightarrow 5 + 10 \Rightarrow 15 \text{ pixels}$$

$$r_1, r_2, r_3 \Rightarrow \text{Zero pixel}$$

$$r_4 \Rightarrow 5 + 10 + 170 = 185 \text{ pixel}$$

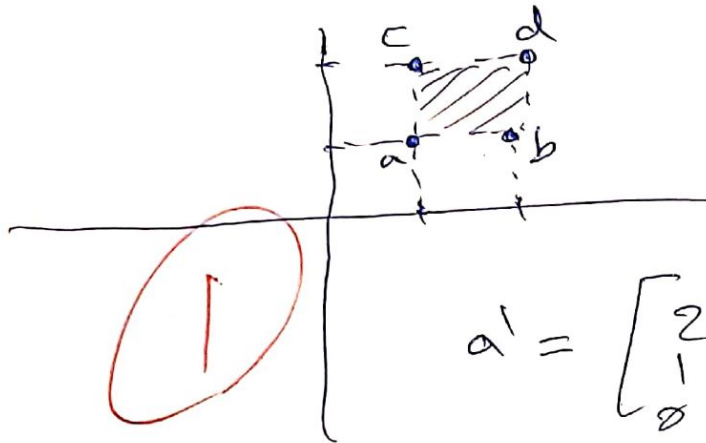
New histogram  
(Normalized)



(c) An image has a Box with pixel Coordinates:  $\{x_i, y_i\} = \{(10,10), (20,10), (10,20), (20,20)\}$ . The Homography transform below is applied to the image:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

- Draw the Box before and after the transform. Explain what happened to the shape of the Box? (2)
- Is this a translation, Euclidean, Affine or projective transform? (1)



$$\begin{aligned} a &= (10, 10, 1) \\ b &= (20, 10, 1) \\ c &= (10, 20, 1) \\ d &= (20, 20, 1) \end{aligned}$$

$$a' = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 31 \\ 22 \\ 1 \end{bmatrix}$$

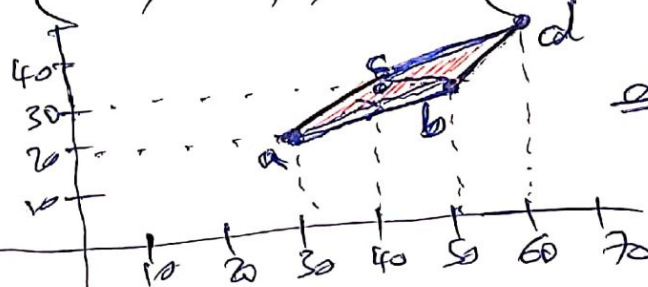
$$b' = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 51 \\ 32 \\ 1 \end{bmatrix}$$

$$c' = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 1 \end{bmatrix} = \begin{bmatrix} 41 \\ 32 \\ 1 \end{bmatrix}, \quad d' = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 20 \\ 1 \end{bmatrix} = \begin{bmatrix} 61 \\ 42 \\ 1 \end{bmatrix}$$

new box coordinate

$$a' = (31, 22), \quad b' = (51, 32), \quad c' = (41, 32)$$

$$d' = (61, 42)$$



distorted

This is an Affine transform  $\Rightarrow$  6 coeffs are allowed

