Section 2: Ant Colony Optimization (ACO) :: Equations and Rules

Eng. Ahmed Métwalli

Ant Colony Optimization (ACO) is a metaheuristic inspired by real ants' foraging behavior. Ants deposit pheromones on paths they take to food, reinforcing shorter, more efficient routes. In ACO, simulated ants construct solutions (e.g., TSP tours) by probabilistically selecting paths based on pheromone levels and heuristic factors (like inverse distances). Over successive iterations, pheromones are updated strengthening good solutions and evaporating to avoid premature convergence leading the algorithm toward an optimal or near optimal solution.

1 Transition (Choice) Rule

When an ant is at node i and must choose the next node j (from the set of allowed nodes), the probability P_{ij} is computed as:

$$P_{ij} = \frac{[\tau_{ij}]^{\alpha}\,[\eta_{ij}]^{\beta}}{\sum_{k\in \text{Allowed}}[\tau_{ik}]^{\alpha}\,[\eta_{ik}]^{\beta}}$$

where:

- τ_{ij}: Current pheromone level on the edge from i to j.
- η_{ij}: Heuristic information (commonly $1/\text{distance}_{ij}$ for TSP).
- α: Parameter controlling the influence of the pheromone (intensity).
- β: Parameter controlling the influence of the heuristic (inverse of distance).
- *Allowed*: The set of cities not yet visited by the ant.

This equation is used during the tour construction phase in both the *Evaluate Fitness* and *Update Swarm* steps.

2 Pheromone Evaporation

After each iteration, the pheromone levels on all edges are reduced to simulate evaporation:

$$\tau_{ij} \leftarrow (1-\rho)\,\tau_{ij}$$

where:

- ρ: The evaporation rate (e.g., 0.1 for 10% evaporation).

This mechanism prevents the unlimited accumulation of pheromone and ensures that older information gradually fades.

3 Pheromone Deposit

After evaporation, ants deposit additional pheromone along the paths they have traversed. The amount of pheromone deposited by ant k on edge (i, j) is given by:

$$\Delta\tau_{ij}^{(k)} = \begin{cases} \frac{Q}{L_k} & \text{if ant } k \text{ used edge } (i,j) \text{ in its tour,} \\ 0 & \text{otherwise,} \end{cases}$$

where:

- Q: A constant representing the total pheromone quantity deposited.
- L_k: The length (or cost) of the tour constructed by ant k.

Then, the pheromone level is updated by adding contributions from all ants:

$$\tau_{ij} \leftarrow \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{(k)}$$

where m is the total number of ants. This update is performed in the *Communication* step, reinforcing the edges that form part of successful tours.

4 Summary of the ACO Cycle

1. **Initialize Swarm:** Set initial pheromone levels (e.g., $\tau_{ij} = 1.0$) and randomly assign starting positions to the ants.
2. **Evaluate Fitness (Construct Tours):** Each ant constructs a complete tour using the transition rule.
3. **Communication (Pheromone Update):**
   - **Evaporation:** Update pheromones as
   $$\tau_{ij} \leftarrow (1 - \rho)\,\tau_{ij}.$$
   - **Deposit:** Update pheromones by adding the deposits:
   $$\tau_{ij} \leftarrow \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{(k)}.$$

4. **Update Swarm:** Use the updated pheromone values to construct new tours.

5. **Termination:** Check stopping criteria (e.g., maximum iterations, convergence threshold) to decide whether to continue or end the process.

These equations and rules work together to balance exploration (via probabilistic decisions based on pheromone and heuristic information) and exploitation (reinforcing good solutions through pheromone deposits), guiding the swarm toward an optimal or near-optimal solution.