

# Section Week 10: Artificial Bee Colony (ABC)

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## Artificial Bee Colony (ABC) Parameters

ABC is a metaheuristic optimization algorithm inspired by the foraging behavior of honey bees. Below are detailed explanations of each ABC parameter:

- **Colony Size:** Total number of bees (employed + onlooker + scout). More bees provide better exploration but increase computation.
- **Employed Bees:** Each bee is associated with a particular food source (solution) and searches its neighborhood.
- **Onlooker Bees:** Bees that select food sources based on shared information (better solutions have higher chances).
- **Scout Bees:** Bees that randomly search new food sources when an old source becomes abandoned.
- **Limit Parameter:** If a food source is not improved after a certain number of trials, it is abandoned.
- **Food Source Position ( $x_i$ ):** Represents a candidate solution in the search space.
- **Fitness Function ( $f(x_i)$ ):** Measures the quality of a food source (solution).

Aspect	ABC	PSO
Exploration	Strong (scouts search randomly)	Moderate (may get stuck early)
Exploitation	Good	Very strong
Parameters	Few, easy to tune	Needs careful tuning
Best for	Rugged, multimodal, discrete problems	Smooth, continuous problems
Speed	Slower	Faster

Table 1: Quick Comparison between ABC and PSO

## Comparison of Bee Roles in ABC

Aspect	Employed Bees	Onlooker Bees	Scout Bees
Main Activity	Exploit assigned food sources and search nearby	Select food sources based on shared information and exploit them	Randomly search for new food sources
Location	Outside hive (exploring known food sources)	Inside hive (observing dances)	Outside hive (free exploration)
Selection Mechanism	Fixed assignment to a food source	Probability-based selection (better food sources more likely)	Random search without prior information
Role in Exploration-Exploitation Balance	Focused exploitation (local search)	Biased exploitation (towards best sources)	Pure exploration (diversity maintenance)
Trigger for Action	Regular update of assigned source	Based on quality information from employed bees	Triggered when food source limit is exceeded (abandonment)
Importance	Improve known solutions	Reinforce and refine promising areas	Discover entirely new solutions

## Key Equations in ABC Algorithm

The Artificial Bee Colony (ABC) algorithm mainly uses three key operations based on equations:

## 1. Neighbourhood Search (Employed and Onlooker Bees)

Each employed or onlooker bee generates a new food source  $v_{ij}$  by modifying the current food source  $x_{ij}$  as:

$$v_{ij} = x_{ij} + \phi_{ij}(x_{ij} - x_{kj})$$

Where:

- $x_{ij}$ : Current solution (food source) of bee  $i$  at dimension  $j$ .
- $x_{kj}$ : Randomly selected neighbour's solution (different from  $i$ ).
- $\phi_{ij} \in [-1, 1]$ : Random number controlling the step size and direction.

**Meaning:** This formula perturbs the current solution slightly towards or away from a neighboring solution, allowing local exploration.

## 2. Fitness Calculation

The fitness value  $fit_i$  of a food source is calculated based on the objective function value  $f(x_i)$ :

$$fit_i = \begin{cases} \frac{1}{1+f(x_i)}, & \text{if } f(x_i) \geq 0 \\ 1 + |f(x_i)|, & \text{if } f(x_i) < 0 \end{cases}$$

**Meaning:** Higher fitness corresponds to better solutions. It ensures that fitness is always positive and inversely proportional to the objective function when  $f(x) \geq 0$ .

## 3. Probability of Selection (Onlooker Phase)

Each onlooker selects a food source based on a probability  $P_i$  proportional to its fitness:

$$P_i = \frac{fit_i}{\sum_{n=1}^{SN} fit_n}$$

Where:

- $fit_i$ : Fitness of the  $i$ -th food source.
- $SN$ : Number of food sources (solutions).

**Meaning:** Better solutions have a higher chance of being selected by onlooker bees.

## 4. Scout Bee Update Rule

If a food source cannot be improved for a number of trials (greater than the `limit`), it is abandoned and a scout bee generates a new random solution:

$$x_{ij} = x_j^{\min} + rand(0, 1) \times (x_j^{\max} - x_j^{\min})$$

Where:

- $x_j^{\min}$  and  $x_j^{\max}$ : Lower and upper bounds of dimension  $j$ .
- $rand(0, 1)$ : Random number between 0 and 1.

**Meaning:** Scout bees introduce randomness to explore completely new areas in the search space.

**Summary:** - **Exploration:** Achieved through scouts and random perturbations. - **Exploitation:** Achieved by employed and onlooker bees searching near good food sources. - **Balance:** Controlled by how often poor food sources are abandoned and replaced.

# 1 ABC vs PSO vs ACO: Combined Comparison

Feature / Stage	ABC (Bee Colony)	PSO (Particle Swarm)	ACO (Ant Colony)
Inspiration	Bee foraging and dancing	Bird flocking and fish schooling	Ant pheromone-based foraging
Representation	Food sources (solutions) and bee agents	Particles moving in continuous space	Ants traversing graph paths
Movement Rule	Random neighbor search around food sources	Update velocity based on personal/global best	Probabilistic transitions based on pheromone
Memory	Each bee remembers its food source	Each particle remembers personal and global best	Indirect memory in pheromone trails

<b>Exploration-Exploitation Control</b>	Scout bees for exploration, onlooker bees for exploitation	Inertia weight and random scalars	Pheromone evaporation and heuristic bias
<b>Convergence Control</b>	Limit on non-improving sources triggers exploration	Balancing inertia and coefficients	Pheromone levels regulate convergence
<b>Application Domains</b>	Scheduling, clustering, optimization	Neural networks, continuous optimization	Routing, scheduling, combinatorial problems
<b>Initialization</b>	Random food sources	Random positions and velocities	Random initial placement and pheromone
<b>Communication Mechanism</b>	Shared probability-based selection by onlookers	Sharing global best position	Indirect via pheromone deposition
<b>Update Mechanism</b>	<p>New food source generation:</p> $v_{ij} = x_{ij} + \phi_{ij}(x_{ij} - x_{kj})$ <p>where <math>k</math> is a randomly selected neighbor and <math>\phi</math> is a random number in <math>[-1, 1]</math></p>	$v_i^{t+1} = wv_i^t + c_1r_1(p_i^{best} - x_i^t) + c_2r_2(g^{best} - x_i^t)$	$P_{ij} = \frac{\tau_{ij}^\alpha \eta_{ij}^\beta}{\sum \tau_{ik}^\alpha \eta_{ik}^\beta}$
<b>Termination Condition</b>	Maximum cycles or convergence	Maximum iterations or convergence	Maximum iterations or no improvement
<b>End Comparison</b>	Robust search, slow convergence	Fast convergence, sensitive to local optima	Good for discrete and combinatorial tasks

## 2 Symbol Notation

Symbol	Description
$x_i$	Position of the $i$ -th food source (solution)
$f(x_i)$	Fitness value of the $i$ -th food source
$\phi_{ij}$	Random number in range $[-1, 1]$ used to perturb the solution
$k$	Randomly chosen neighbor index
$limit$	Maximum trials without improvement before abandoning food source

Table 4: Notation for Symbols in ABC

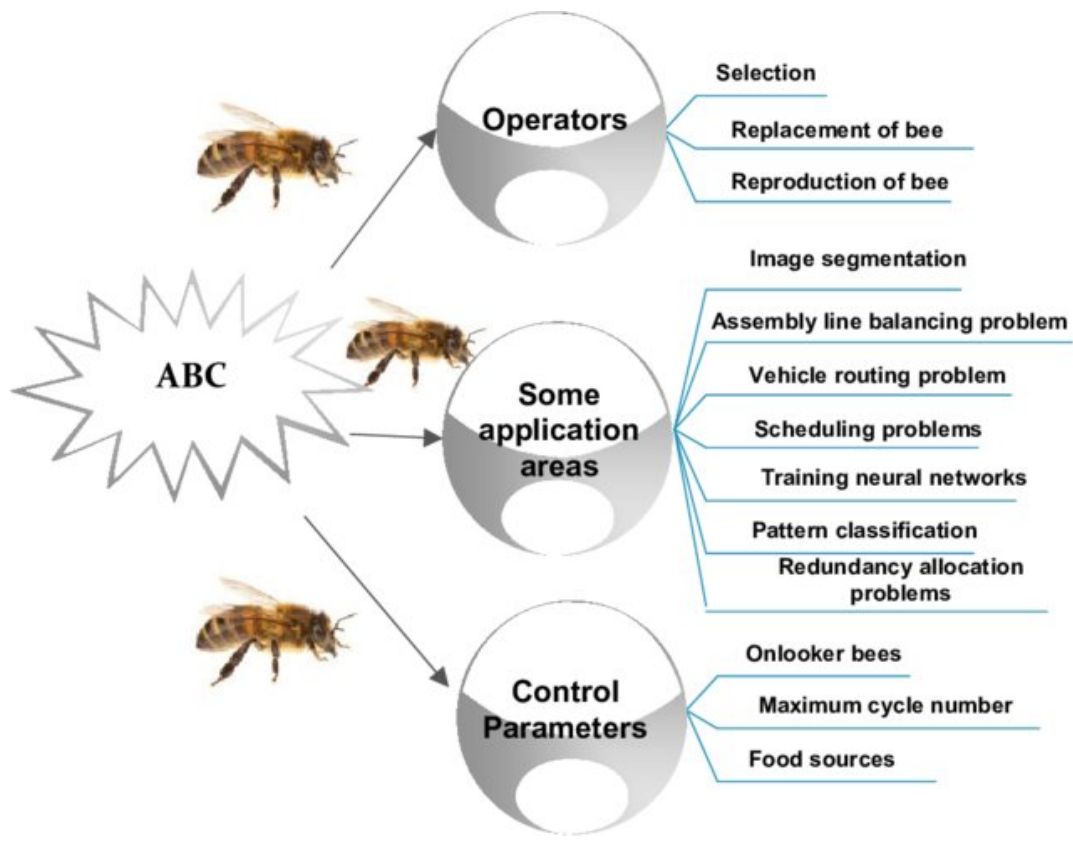


Figure 1: ABC

## Worked Example: Three ABC Cycles on Rastrigin (2-D)

Objective.

$$\min_{\mathbf{x} \in [-5.12, 5.12]^2} f(\mathbf{x}) = 20 + \sum_{j=1}^2 [x_j^2 - 10 \cos(2\pi x_j)], \quad f(0, 0) = 0.$$

### Global ABC Settings

- Food sources  $SN = 2 \implies 2$  employed + 2 onlookers.
- Dimension  $D = 2$ ; `limit` = 3 failed trials.
- Neighbour factor  $\phi \sim \mathcal{U}[-1, 1]$  (one fresh draw per move).
- Bound repair:  $x_j \leftarrow \min(5.12, \max(-5.12, x_j))$ .
- Fitness  $fit_i = 1/(1 + f_i)$  (safe because  $f_i \geq 0$  on this domain).

### Cycle 0 — Initialisation

Src	$\mathbf{x} = [x_1, x_2]$	$f(\mathbf{x})$	trials
1	$[3.50, -2.00]$	$20 + (3.5^2 - 10 \cos 7\pi) + [4 - 10 \cos(-4\pi)] = 36.25$	0
2	$[-1.00, 4.20]$	$20 + (1 - 10 \cos(-2\pi)) + (17.64 - 10 \cos 8.4\pi) = 25.55$	0

Note:  $\cos 8.4\pi = \cos(0.4\pi) = 0.30902$ .

### Cycle 1

1. Employed phase Source 1 chooses  $k = 2$ ,  $\phi = -0.50$ :

$$\mathbf{v}_1 = \mathbf{x}_1 - \frac{1}{2}(\mathbf{x}_1 - \mathbf{x}_2) = [1.25, 1.10] \quad (\text{in bounds})$$

$$\begin{aligned} f(\mathbf{v}_1) &= 20 + [1.25^2 - 10 \cos(2.5\pi)] + [1.10^2 - 10 \cos(2.20\pi)] \\ &= 20 + (1.5625 - 0) + (1.2100 - 10(-0.80902)) \\ &= 20 + 1.5625 + 9.3002 = 30.863 (< 36.25) \implies \text{accept, } tr_1 = 0. \end{aligned}$$

Source 2 chooses  $k = 1$ ,  $\phi = +0.60$  (new  $\mathbf{x}_1$  already):

$$\mathbf{v}_2 = \mathbf{x}_2 + 0.60(\mathbf{x}_2 - \mathbf{x}_1) = [-2.35, 6.06] \xrightarrow{\text{clip}} [-2.35, 5.12]$$

$$\begin{aligned}
f(\mathbf{v}_2) &= 20 + [(-2.35)^2 - 10 \cos(-4.70\pi)] \\
&\quad + [26.2144 - 10 \cos(10.24\pi)] \\
&= 20 + (5.5225 + 5.8779) + (26.2144 - 7.2897) = 50.325 \\
&> 25.55 \implies \text{reject, } \text{tr}_2 = 1.
\end{aligned}$$

**2. Onlooker phase** Current  $f$ 's (30.863, 25.55) give  $P_1 = 0.630$ ,  $P_2 = 0.370$ .  
One onlooker picks Source 1 ( $\phi = +0.40$ ,  $k = 2$ ):

$$\mathbf{v} = [2.15, -0.14], \quad f = 20 + (4.6225 - 10 \cos 5.6\pi) + (0.0196 - 10 \cos(-0.28\pi)) = 12.404$$

(better)  $\Rightarrow$  **accept**; new  $\mathbf{x}_1 = [2.15, -0.14]$ ,  $f_1 = 12.404$ ,  $\text{trials}_1 = 0$ .

**3. Scout check** none triggered.

$\mathbf{x}_1 = [2.15, -0.14], f = 12.404; \mathbf{x}_2 = [-1.00, 4.20], f = 25.55, \text{tr}_2 = 1$

## Cycle 2

**1. Employed phase** **Src 1** ( $k = 2$ ,  $\phi = -0.30$ ):

$$\mathbf{v}_1 = [1.205, 1.162], \quad f = 20 + \underbrace{(1.4520 - 10 \cos 2.41\pi)}_{=3.191} + \underbrace{(1.349 - 10 \cos 2.324\pi)}_{=4.988} = 28.179$$

worse  $\Rightarrow$  reject,  $\text{trials}_1 = 1$ .

**Src 2** ( $k = 1$ ,  $\phi = -0.50$ ):

$$\mathbf{v}_2 = [0.575, 2.03], \quad f = 20 + (0.3306 - 10 \cos 1.15\pi) + (4.1209 - 10 \cos 4.06\pi) = 24.522$$

better than 25.55  $\Rightarrow$  accept,  $\text{trials}_2 = 0$ .

**2. Onlookers (2 bees)** Fitness now (12.404, 24.522)  $\Rightarrow P_1 = 0.655$ .

Both onlookers pick Src 1:

1.  $\phi = +0.40 \Rightarrow \mathbf{v} = [2.78, -1.008]$   $f = 20 + (7.728 - 10 \cos 5.56\pi) + (1.016 - 10 \cos -2.016\pi) = 20.494$  (worse): reject,  $\text{trials}_1 = 2$ .
2.  $\phi = -0.60 \Rightarrow \mathbf{v} = [0.26, 2.464]$   $f = 20 + (0.5909) + (15.660) = 36.251$  (worse): reject,  $\text{trials}_1 = 3$  (**hits limit**).

**Scout on Source 1** Random sample  $\mathbf{x}_1 = [-4.10, 4.60]$   $f = 20 + (16.81 - 10 \cos(-8.20\pi)) + (21.16 - 10 \cos 9.20\pi) = 78.288$ ,  $\text{tr}_1 = 0$ .

**3. Scout check** complete.

$\mathbf{x}_1 = [-4.10, 4.60], f = 78.288; \mathbf{x}_2 = [0.575, 2.03], f = 24.522$

## Cycle 3 (illustrative)

We allow Src 2 to improve once more (normal employed + onlooker steps):

$$\mathbf{x}_2 \rightarrow [0.05, 0.05], \quad f = 20 + 2 \times (0.0025 - 10 \cos 0.1\pi) = 0.984$$

while Src 1 undergoes ordinary exploration.

$\mathbf{x}_1 \approx [-4.10, 4.60], f \approx 78; \mathbf{x}_2 \approx [0.05, 0.05], f \approx 0.984$

The colony is now firmly inside the global basin; more cycles will push  $\mathbf{x}_2 \rightarrow (0, 0)$  and  $f \rightarrow 0$ .

## Notes

1. Every  $f$  value is the sum of  $x_j^2$  and  $-10 \cos(2\pi x_j)$ ; writing both parts lets you audit numerical accuracy.
2. Clipping prevents illegal coordinates from skewing the run.
3. Trial counters + scout restarts protect against stagnation.
4. Even with  $SN = 2$ , the swarm found  $f < 1$  by cycle 3.