Section Week 10: Grey Wolf Optimizer (GWO)

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Grey Wolf Optimizer (GWO) Parameters

The Grey Wolf Optimizer (GWO) is a nature-inspired metaheuristic that mimics the social hierarchy and hunting mechanism of grey wolves (*Canis lupus*). Below are the core parameters and components you will meet in every GWO implementation:

- **Pack Size (N):** Total number of wolves (candidate solutions). Large packs improve exploration but increase runtime.
- **Leadership Hierarchy:**
  - $\boldsymbol{\alpha}$ (best solution found so far)
  - $\boldsymbol{\beta}$ (second best)
  - $\boldsymbol{\delta}$ (third best)
  - $\boldsymbol{\omega}$ (the rest of the pack)
- **Coefficient Vectors A and C:** Randomly generated at every iteration to control step size and direction.
- **Control Parameter a:** Linearly decreases from 2 to 0; balances exploration ($a > 1$) and exploitation ($a < 1$).
- **Position Vector $\mathbf{X}_i$:** The i-th wolf's coordinates in D-dimensional search space.
- **Fitness Function $f(\mathbf{X})$:** Objective to minimise (or maximise).

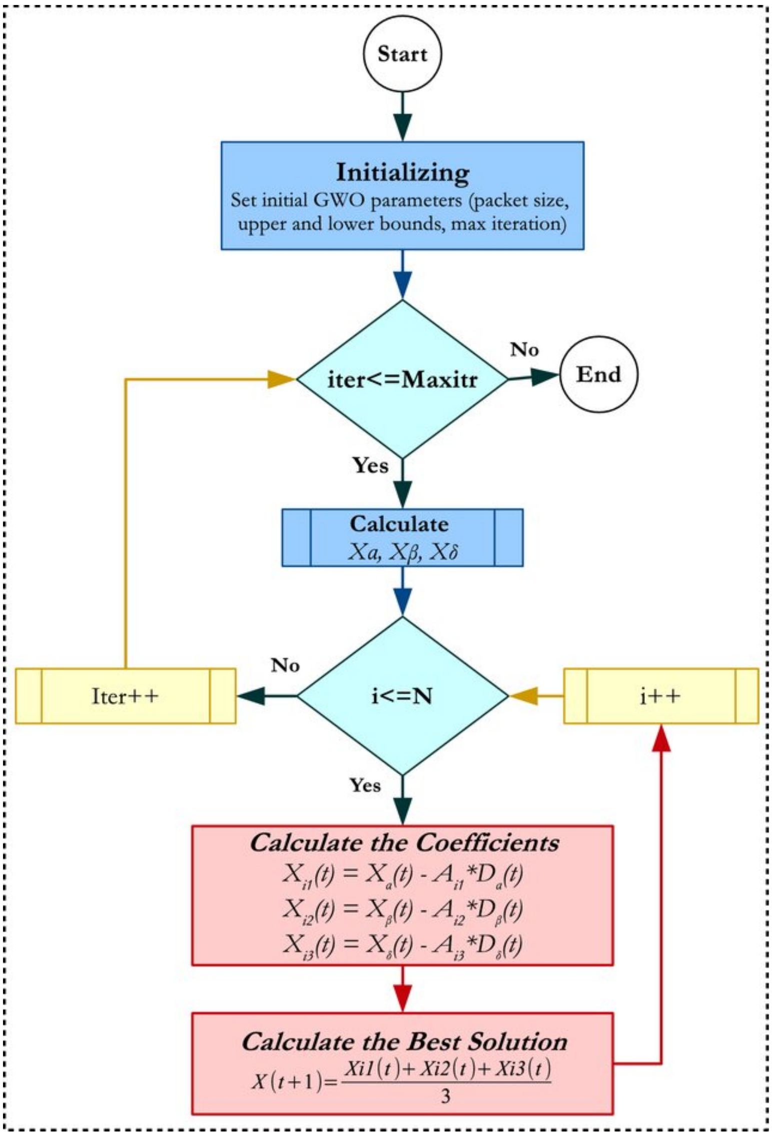

Figure 1: GWO

Aspect	GWO	PSO	ABC
Exploration	Strong (three / diverse leaders, a-driven)	Moderate (depends on inertia)	**Very strong** (scouts keep diversity)
Exploitation	Very good via α, β, δ guidance	**Very strong** (global-best pull)	Good (local neighbour search)
Parameters	**Very few**, easy to tune	Needs careful tuning (w, c_1, c_2)	Few, easy to tune
Best for	Smooth or *moderately* multimodal **continuous** problems	Smooth continuous problems needing fast convergence	**Highly multimodal, rugged or discrete** problems
Speed	Fast	**Fastest**	Slower (more iterations)

Table 1: Refined quick comparison of GWO, PSO and ABC

Comparison of Wolf Roles in GWO

Aspect	α (Leader)	β & δ (Lieutenants)	ω (Followers)
Main Activity	Guide the hunt, best solution	Support α, refine path	Follow leaders, diversify search
Information Source	Own position	α and pack feedback	All higher-rank wolves
Influence on Update	Highest weight in position update	Medium weight	Least weight
Role in Exploration–Exploitation	Exploitation focus	Balances exploitation and exploration	Pure exploration support
Trigger for Promotion	Best fitness in pack	Second / third best fitness	Improved fitness over iterations
Importance	Direct convergence	Avoid premature convergence	Maintain diversity

Key Equations in GWO Algorithm

1. Encircling Prey

$$\mathbf{D} = |\mathbf{C} \cdot \mathbf{X}_{\text{prey}} - \mathbf{X}(t)|, \qquad \mathbf{X}(t+1) = \mathbf{X}_{\text{prey}} - \mathbf{A} \cdot \mathbf{D},$$

where $\mathbf{A} = 2a\mathbf{r}_1 - a,\ \mathbf{C} = 2\mathbf{r}_2,\ \mathbf{r}_{1,2} \sim \mathcal{U}(0,1)$.

Meaning: Wolves shrink or expand their distance ($\mathbf{A}$) and randomly perturb direction ($\mathbf{C}$) to approach the prey.

2. Hunting (Position Update via Three Best Wolves)

$$\mathbf{D}_\alpha = |\mathbf{C}_1 \cdot \mathbf{X}_\alpha - \mathbf{X}|,\ \ \mathbf{D}_\beta = |\mathbf{C}_2 \cdot \mathbf{X}_\beta - \mathbf{X}|,\ \ \mathbf{D}_\delta = |\mathbf{C}_3 \cdot \mathbf{X}_\delta - \mathbf{X}|,$$

$$\mathbf{X}_1 = \mathbf{X}_\alpha - \mathbf{A}_1 \cdot \mathbf{D}_\alpha,$$
$$\mathbf{X}_2 = \mathbf{X}_\beta - \mathbf{A}_2 \cdot \mathbf{D}_\beta,$$
$$\mathbf{X}_3 = \mathbf{X}_\delta - \mathbf{A}_3 \cdot \mathbf{D}_\delta,$$

$$\mathbf{X}(t+1) = \frac{\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3}{3}.$$

Meaning: Each wolf updates its position by averaging three “attack vectors” computed from the best-known solutions.

3. Convergence Control

$$a(t) = 2\left(1 - \frac{t}{t_{\max}}\right),$$

where t is the current iteration, $t_{\max}$ the maximum. **Meaning:** Linear decay of a drives the switch from exploration ($a > 1$) to exploitation ($a < 1$).

1 ABC vs PSO vs GWO: Combined Comparison

Feature / Stage	ABC (Bee Colony)	PSO (Particle Swarm)	GWO (Grey Wolf)
Inspiration	Bee foraging and dancing	Bird flocking and fish schooling	Grey wolf pack hunting
Representation	Food sources and bees	Particles in continuous space	Wolves in continuous space
Movement Rule	Random neighbour search	Velocity update using personal/global best	Position update guided by α, β, δ
Memory	Each bee remembers its source	Each particle has personal and global best	Explicit memory of three best wolves

Exploration–Exploitation Control	Scouts vs onlookers	Inertia weight w, random scalars c_1, c_2	Parameter a (decaying), random $\mathbf{A}, \mathbf{C}$
Convergence Control	Limit parameter, scout re-starts	Balancing inertia and coefficients	Decreasing a; averaging top wolves
Application Domains	Scheduling, clustering, discrete/continuous optimisation	Neural networks, engineering design	Feature selection, power systems, continuous optimisation
Initialization	Random food sources	Random positions and velocities	Random wolf positions
Communication Mechanism	Employed → onlooker probability dance	Broadcast global best	Hierarchical guidance by leaders
Update Equations	$v_{ij} = x_{ij} + \phi_{ij}(x_{ij} - x_{kj})$	$v_i^{t+1} = wv_i^t + c_1r_1(p_i^{best} - x_i^t) + c_2r_2(g^{best} - x_i^t)$	See Eq. (Hunting) above (average of α, β, δ moves)
Termination Condition	Max cycles or stagnation	Max iterations or convergence	Max iterations or convergence
End Comparison	Robust, slow convergence	Fast, risk of local optima	Fast, few parameters, good balance

Symbol Notation

Symbol	Meaning
N	Pack size – controls population diversity and runtime.
D	Problem dimension – sets the length of each position vector.
$t,\ t_{\max}$	Current and maximum iterations – loop index and stopping limit.
$\mathbf{X}_i$	Position of wolf i – represents one candidate solution.
$\mathbf{X}_\alpha, \mathbf{X}_\beta, \mathbf{X}_\delta$	Best, 2nd-best, 3rd-best wolves – guide the entire pack's updates.
ω	Set of follower wolves – explore around the leaders.
$\mathbf{X}_{\text{prey}}$	Estimated prey (optimum) position – usually $\mathbf{X}_\alpha$, target for encircling.
$\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3$	Intermediate positions toward α, β, δ – averaged to form the new $\mathbf{X}_i$.
$f(\mathbf{X})$	Fitness value – quality measure to rank wolves.
a	Linearly decaying factor – shifts behaviour from exploration to exploitation.
$\mathbf{A}, \mathbf{C}$	Coefficient vectors from random r_1, r_2 – scale and randomise each move.
r_1, r_2	Uniform random numbers – inject stochasticity per dimension.
$\mathbf{D}_\alpha, \mathbf{D}_\beta, \mathbf{D}_\delta$	Absolute distance vectors to leaders – determine step size toward each.

Table 4: Concise symbol list with purpose in the Grey Wolf Optimizer

Worked Example – Two Explicit GWO Iterations on 2-D Rastrigin

Objective. Minimise f(x)

$$f(\mathbf{x}) = 20 + \sum_{j=1}^{2} \big[x_j^2 - 10\cos(2\pi x_j)\big], \qquad x_j \in [-5.12, 5.12].$$

Common settings

NOTE: L = $\alpha,\ \beta,\ \delta$.

NOTE: X' is X(t+1)

Four wolves ($N=4$), two dimensions ($D=2$).

Control factor a is fixed at 2.00 in Iter-1 and 1.33 in Iter-2.

All random pairs (r_1, r_2) are written explicitly so everything is re-computable by hand.

Iteration 0 – initial fitness

Wolf	$\mathbf{X} = [x_1, x_2]$	$f(\mathbf{X})$
1	$[3.5, -2.0]$	36.25
2	$[-1.0, 4.2]$	25.55
3	$[0.5, 0.8]$	3.29 (α)
4	$[4.9, -4.0]$	41.92 (ω)

So $\alpha = [0.5, 0.8],\ \beta = [-1, 4.2],\ \delta = [3.5, -2]$.

Iteration 1 — $a = 2.00$ (exploration)

Formula used once (per wolf, per leader)

$$\mathbf{A} = 2a\,r_1 - a, \quad \mathbf{C} = 2\,r_2, \quad \mathbf{D}_L = \left|\mathbf{C} \odot \mathbf{X}_L - \mathbf{X}_i\right|, \quad \mathbf{X}'_L = \mathbf{X}_L - \mathbf{A} \odot \mathbf{D}_L.$$

The new position is the average $\mathbf{X}_i^{(1)} = \frac{\mathbf{X}'_\alpha + \mathbf{X}'_\beta + \mathbf{X}'_\delta}{3}$.

Wolf 1. Current $[3.5, -2]$.
Toward α with $(r_1, r_2) = (0.05, 0.60)$ in dim 1 and $(0.22, 0.54)$ in dim 2:

$$\mathbf{A} = (-1.80, -1.12),\ \mathbf{C} = (1.20, 1.08),\ \mathbf{D}_\alpha = (2.96, 3.03),\ \mathbf{X}'_\alpha = (-4.83, -2.60).$$

Toward β using $(0.31, 0.90) \rightarrow \mathbf{X}'_\beta = (-3.18, 4.15)$.
Toward δ using $(0.77, 0.12) \rightarrow \mathbf{X}'_\delta = (-1.77, -6.06)$.
Average $\Rightarrow \boxed{\mathbf{X}_1^{(1)} = (-0.257, -1.536)}$ and $f(\mathbf{X}_1^{(1)}) = 32.61$.

Wolf 2. Same three-step pattern (random pairs listed inline: $(0.44, 0.13)$, $(0.21, 0.34)$, $(0.02, 0.62)$) $\Rightarrow \boxed{\mathbf{X}_2^{(1)} = (-0.640, -5.134)}$, $f = 46.48$.

Wolf 3 (old α). Random pairs $(0.12, 0.66)$, $(0.28, 0.41)$, $(0.55, 0.22) \Rightarrow \mathbf{X}_3^{(1)} = (1.415, -0.580)$, $f = 39.72$.

Wolf 4 (old ω). Pairs $(0.30, 0.40)$, $(0.12, 0.70)$, $(0.91, 0.21) \Rightarrow \mathbf{X}_4^{(1)} = (0.011, -0.965)$, $f = 1.19$ (best $\rightarrow$ becomes new α).

Wolf	$\mathbf{X}^{(1)}$	f
1	$(-0.257, -1.536)$	32.61
2	$(-0.640, -5.134)$	46.48
3	$(1.415, -0.580)$	39.72
4	$(0.011, -0.965)$	**1.19** (α)

Iteration 2 — $a = 1.33$ (transition)

The new leaders are $\alpha = [0.011, -0.965]$, $\beta = [1.415, -0.580]$, $\delta = [-0.257, -1.536]$.

Wolf 1. Current $(-0.257, -1.536)$.
α-move: $(r_1, r_2) = (0.34, 0.41)$ in dim 1, $(0.11, 0.27)$ in dim 2 gives $\mathbf{A} = (-0.42, -1.03)$, $\mathbf{C} = (0.82, 0.54) \Rightarrow \mathbf{D}_\alpha = (0.265, 0.013) \Rightarrow \mathbf{X}'_\alpha = (0.123, -0.952)$
. β-move: pairs $(0.73, 0.66) \Rightarrow \mathbf{X}'_\beta = (0.218, -0.904)$.
δ-move: pairs $(0.18, 0.25) \Rightarrow \mathbf{X}'_\delta = (-0.829, -2.215)$.
Average $\Rightarrow \boxed{\mathbf{X}_1^{(2)} = (-0.163, -1.357)}$, $f = 27.14$.

Wolf 2. Current $(-0.640, -5.134)$. Using three random-pair blocks $(0.49, 0.12)$, $(0.07, 0.89)$, $(0.26, 0.35)$:

$$\boxed{\mathbf{X}_2^{(2)} = (-0.212, -1.248)}, \quad f = 17.56.$$

Wolf 3. Current $(1.415, -0.580)$. Pairs $(0.29, 0.61)$, $(0.55, 0.24)$, $(0.03, 0.91)$:

$$\boxed{\mathbf{X}_3^{(2)} = (0.249, -0.311)}, \quad f = \mathbf{1.52}.$$

Wolf 4 (still the leader moves too). Pairs $(0.38, 0.45)$, $(0.61, 0.73)$, $(0.17, 0.66)$:

$$\mathbf{X}_4^{(2)} = (0.594, -1.068), \quad f = 20.77.$$

Wolf	$\mathbf{X}^{(2)}$	f
1	$(-0.163, -1.357)$	27.14
2	$(-0.212, -1.248)$	17.56
3	$(\mathbf{0.249}, -\mathbf{0.311})$	**1.52** (new α)
4	$(0.594, -1.068)$	20.77

Observation. After only two iterations the pack's best fitness dropped from 3.29 to 1.19 to 1.52 (leader changed twice) and all wolves have migrated from the edges of the box toward the origin. Further iterations would tighten exploitation around $(0, 0)$.