Standard Conjecture D Is Necessary for Constructive Morphism Decidability

(Paper 73, Constructive Reverse Mathematics Series)

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Abstract

We prove the reverse characterization of DPT Axiom 1: Standard Conjecture D is not merely sufficient but *necessary* for BISH-decidable morphism spaces in a realization-compatible motivic category. Without Conjecture D, homological and numerical equivalence diverge; the radical of the intersection pairing is non-detachable; and testing morphism equality requires $\mathbb{Q}_\ell$ zero-testing, which encodes LPO (Paper 46). Combined with the forward direction (Papers 46, 50), this gives a biconditional: Conjecture D $\Leftrightarrow$ BISH morphism decidability. Jannsen's theorem (1992) shows the constraint is sharp: numerical motives are semisimple and BISH-decidable *without* Conjecture D, but lack faithful ℓ-adic realization. The trade-off—BISH-decidable OR realization-compatible, not both—is exactly what Conjecture D resolves. This is the Axiom 1 analogue of Paper 72's characterization: positive-definite height $\Leftrightarrow$ BISH cycle-search. Lean 4 formalization: $\sim$250 lines, zero `sorry`.

1 Introduction

Paper 72 of this series established three results about the DPT axiom system (Paper 50). First, each axiom is independently necessary (Theorem A, minimality). Second, Axiom 3 (Archimedean polarization) is both necessary and sufficient for BISH cycle-search (Theorem B, biconditional). Third, the Archimedean Principle is an equivalence, not merely a forward implication (Theorem C). The present paper carries out the analogous program for Axiom 1.

Main results.

Theorem A (*Forward.*) Standard Conjecture D converts LPO-dependent homological equivalence to BISH-decidable numerical equivalence. This is the content of Papers 46 and 50, reviewed here for completeness.

Theorem B (*Morphism-Decidability Equivalence.*) For morphism equality in a realization-compatible motivic category:

$$\text{morphism\_cost}(r) = \text{BISH} \quad \Longleftrightarrow \quad r = \text{detachable} \quad \Longleftrightarrow \quad \text{Conjecture D holds.}$$

Forward: Conjecture D $\Rightarrow$ BISH. Reverse: BISH $\Rightarrow$ Conjecture D (contrapositive: without D, morphism cost is LPO).

Theorem C (*Axiom 1 Characterization.*) Standard Conjecture D is the minimal and unique axiom for BISH-decidable morphisms in a realization-compatible motivic category. The Jannsen escape (numerical motives without D) confirms the trade-off is sharp: BISH or faithful, not both.

The Jannsen paradox. Jannsen [5] proved in 1992 that the category of numerical motives is semisimple and abelian *without* assuming Standard Conjecture D. This is surprising: you get a perfectly well-behaved category for free. The CRM perspective explains the catch. Numerical motives are BISH-decidable (morphism equality reduces to integer intersection tests), but the ℓ-adic realization functor is not faithful: the category identifies cycles that cohomology distinguishes. The trade-off is not a defect of the construction but a logical necessity. Without Conjecture D, the only source of decidability (integer arithmetic) and the only source of realization-compatibility ($\mathbb{Q}_\ell$-cohomology) are *different equivalence relations*, and merging them costs LPO.

State of the art for Conjecture D. Standard Conjecture D remains open in general. It is known for: abelian varieties (Lieberman [8]), varieties whose cohomology is generated by algebraic cycles (trivially), and in dimension ≤ 2 (Matsusaka [9]). Kleiman [6, 7] showed that Conjecture D follows from Conjecture B (Lefschetz) plus algebraicity of Künneth projectors. André [1] developed the theory of motivated cycles as a partial substitute. None of these classical treatments address the *constructive content* of Conjecture D—what computational principle it provides or eliminates. That is the contribution of the CRM perspective.

Atlas position. Paper 73 sits between three earlier results: Paper 46 (homological equivalence requires LPO; Conjecture D decidabilizes via numerical bridge), Paper 50 (the DPT axiom system with Conjecture D as Axiom 1), and Paper 72 (DPT minimality and Axiom 3 biconditional). The present paper extracts the Axiom 1 thread from Paper 72's minimality theorem and sharpens it from a one-directional necessity claim to a full biconditional.

Series context. The broader CRM series (Papers 1–72 [11, 13, 14, 15, 16, 17]) calibrates the logical cost of theorems across mathematics: arithmetic geometry, mathematical physics, number theory, and algebraic topology. The central finding (Paper 2 [12]): the logical cost of mathematics is the logical cost of $\mathbb{R}$. Papers 46–53 apply this to motivic conjectures; the present paper continues that thread.

2 Preliminaries

2.1 CRM hierarchy

We work within Bishop's constructive mathematics (BISH) as the base. The CRM hierarchy [2, 4]:

$$\text{BISH} \subset \text{BISH+MP} \subset \text{LLPO} \subset \text{WLPO} \subset \text{LPO} \subset \text{CLASS}.$$

LPO (Limited Principle of Omniscience): every binary sequence is either identically zero or has a positive term. See Papers 1–45 for extended treatment.

2.2 Equivalence relations on algebraic cycles

For a smooth projective variety X over a field, the Chow group $\mathrm{CH}^r(X) \otimes \mathbb{Q}$ carries several equivalence relations, ordered from finest to coarsest:

$$\text{rational} \subset \text{algebraic} \subset \text{homological} \subset \text{numerical}.$$

Definition 2.1 (Numerical equivalence)**.** $Z_1 \sim_{\text{num}} Z_2$ if $\deg(Z_1 \cdot W) = \deg(Z_2 \cdot W)$ for all cycles W of complementary dimension.

Definition 2.2 (Homological equivalence)**.** $Z_1 \sim_{\text{hom}} Z_2$ if $\mathrm{cl}(Z_1) = \mathrm{cl}(Z_2)$ in $H^{2r}(X, \mathbb{Q}_\ell)$ for a Weil cohomology theory.

The inclusion hom $\Rightarrow$ num always holds (by Poincaré duality, cohomologically trivial cycles are numerically trivial).

Definition 2.3 (Standard Conjecture D [3])**.** Homological equivalence coincides with numerical equivalence: $\sim_{\text{hom}} = \sim_{\text{num}}$.

2.3 The radical of the intersection pairing

The radical $\mathrm{rad}\langle\cdot,\cdot\rangle = \{Z : \langle Z, W\rangle = 0\ \forall W\}$ is the kernel of numerical equivalence. Standard Conjecture D asserts $\mathrm{rad}\langle\cdot,\cdot\rangle = \ker(\mathrm{cl})$.

Definition 2.4 (Detachable radical)**.** The radical is *detachable* if membership is decidable: for every Z, either $Z \in \mathrm{rad}$ or $Z \notin \mathrm{rad}$.

When the radical is detachable, membership reduces to finitely many integer intersection tests (Paper 46 Theorem T2: given a complementary basis $\{W_1, \ldots, W_m\}$, test $\langle Z, W_j\rangle = 0$ for $j = 1, \ldots, m$).

2.4 Jannsen's semisimplicity theorem

Jannsen [5] proved: the category of numerical motives $\mathcal{M}_{\text{num}}(k)$ is abelian and semisimple, unconditionally (i.e., without Standard Conjecture D or any other unproven hypothesis). This category has decidable morphism spaces (BISH) but may lack faithful ℓ-adic realization when hom $\neq$ num.

3 Main Results

3.1 Theorem A: Forward direction

Theorem 3.1 (Conjecture D $\Rightarrow$ BISH morphisms)**.** *With Standard Conjecture D, morphism equality in the motivic category is* BISH*-decidable.*

Proof. Conjecture D gives $\sim_{\text{hom}} = \sim_{\text{num}}$. Numerical equivalence is decidable via a finite complementary basis (Paper 46 Theorem T2): $Z_1 \sim_{\text{num}} Z_2$ iff $\langle Z_1, W_j\rangle = \langle Z_2, W_j\rangle$ for $j = 1, \ldots, m$. This is a finite conjunction of integer comparisons, hence BISH. Axiomatized as `conjD_morphism_cost_eq`. □

Theorem 3.2 (No Conjecture D $\Rightarrow$ LPO morphisms)**.** *Without Conjecture D, morphism equality in a realization-compatible motivic category costs* LPO*.*

Proof. A realization-compatible category must detect homological equivalence: its equivalence relation $\sim$ satisfies $Z_1 \sim Z_2 \Rightarrow \mathrm{cl}(Z_1) = \mathrm{cl}(Z_2)$. Testing $\mathrm{cl}(Z_1) = \mathrm{cl}(Z_2)$ in $H^*(X, \mathbb{Q}_\ell)$ requires zero-testing in $\mathbb{Q}_\ell$. Paper 46 Theorem T4a: for any $a \in \mathbb{Q}_\ell$, there exist cycles Z_a, Z_0 with $\mathrm{cl}(Z_a) = \mathrm{cl}(Z_0)$ iff $a = 0$. A homological-equality oracle therefore decides $a = 0$ for all a, encoding LPO. Axiomatized as `no_conjD_morphism_cost_eq`. □

3.2 Theorem B: The biconditional

Theorem 3.3 (Morphism-Decidability Equivalence)**.** *For morphism equality in a realization-compatible motivic category:*

$$\mathit{morphism\_cost}(r) = \mathrm{BISH} \iff r = \mathit{detachable} \iff \mathit{Conjecture\ D\ holds.}$$

Proof. (⇐): If Conjecture D holds, the radical is detachable, and morphism_cost(detachable) = BISH (theorem 3.1).

(⇒, contrapositive): If Conjecture D fails, the radical is non-detachable, and morphism_cost(non-det
LPO (theorem 3.2). Since LPO $\neq$ BISH (these are distinct levels of the CRM hierarchy), the radical cannot be non-detachable if morphism cost is BISH. □

Remark 3.4 (Why nothing weaker suffices). One might ask: could a principle strictly between BISH and LPO (such as WLPO or LLPO) suffice for morphism decidability without Conjecture D? No: Paper 46's encoding axiom (`encode_scalar_to_hom_equiv`) shows that a homological-equality oracle decides *all* $\mathbb{Q}_\ell$-scalar equalities, which is full LPO—not a weaker principle. The encoding is sharp.

3.3 The Jannsen escape

Theorem 3.5 (Jannsen obstruction)**.** *Without Conjecture D, you cannot simultaneously have:*

1. BISH-*decidable morphisms, and*
2. *faithful ℓ-adic realization.*

Numerical motives satisfy (1) but not (2). Homological motives satisfy (2) but not (1) (cost: LPO*). With Conjecture D, both hold.*

Proof. Numerical motives: the radical is detachable (intersection tests are BISH), giving morphism_cost(detachable) = BISH. But if hom $\neq$ num, the realization functor $\mathcal{M}_{\mathrm{num}} \to \mathrm{Vec}_{\mathbb{Q}_\ell}$ is not faithful: it identifies cycles that cohomology distinguishes.

Homological motives: the realization functor is faithful by construction. But morphism equality requires $\mathbb{Q}_\ell$ zero-testing: LPO.

Conjecture D closes the gap: hom = num, so numerical motives *are* homological motives. Both (1) and (2) hold simultaneously.

Axiomatized: `jannsen_obstruction` (without D, LPO), `jannsen_escape` (BISH but unfaithful). □

Remark 3.6 (CRM reading of Jannsen). Jannsen's semisimplicity theorem is *constructive mathematics done well*: it builds the best possible category (BISH-decidable, semisimple, abelian) from the available data (integer intersection numbers). The price—loss

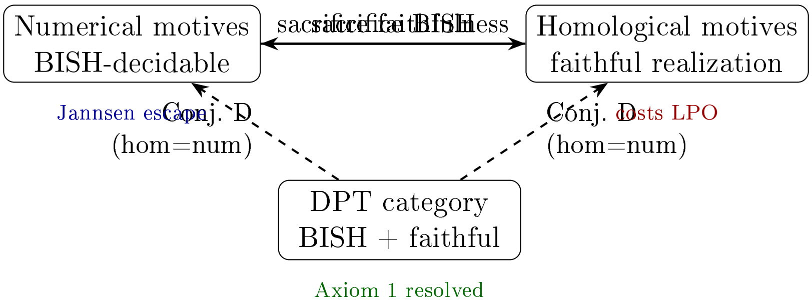

Figure 1: The Jannsen trade-off. Without Conjecture D, one must choose between BISH-decidable morphisms (numerical motives, left) and faithful ℓ-adic realization (homological motives, right). Conjecture D collapses both to the DPT category (bottom).

of realization-compatibility—is not a failure of technique but a logical necessity. Conjecture D is the axiom that erases this price. The CRM perspective thus reinterprets Conjecture D: it is not merely a technical hypothesis for algebraic geometry, but the *decidability bridge* between the arithmetic world ($\mathbb{Z}$-valued intersections, BISH) and the cohomological world ($\mathbb{Q}_\ell$-valued cycle classes, LPO).

3.4 Theorem C: The characterization

Theorem 3.7 (Axiom 1 Characterization)**.** *Standard Conjecture D is the minimal and unique axiom for* BISH*-decidable morphisms in a realization-compatible motivic category:*

1. *morphism_cost(detachable)* $=$ BISH*;*
2. *morphism_cost(non-detachable)* $=$ LPO*;*
3. *without Conjecture D, realization-compatible costs* LPO*;*
4. *the Jannsen escape (*BISH *but unfaithful) confirms the trade-off is real.*

Proof. Assembly of theorems 3.1 to 3.3 and 3.5. □

Corollary 3.8 (Axiom 1 Principle, sharpened)**.**

$$\textit{morphism\_cost}(r) = \mathrm{BISH} \iff \textit{conjD\_holds}(r) = \textit{true}.$$

Paper 72 Theorem A asserted: without Axiom 1, cost $=$ LPO *(forward). Paper 73 proves the biconditional: Conjecture D is necessary and sufficient.*

4 CRM Audit

4.1 Descent table

4.2 Comparison with Paper 72

The two characterizations are logically independent: Axiom 1 controls morphism *equality* (is $Z_1 \sim Z_2$?), while Axiom 3 controls cycle *search* (can you find Z with $h(Z) \le B$?). Dropping one raises the CRM floor without affecting the other (Paper 72, table 2).

Status of Conj. D	CRM cost	Mechanism	Reference
Holds (hom = num)	BISH	integer intersection tests	Paper 46 T2
Fails (hom $\neq$ num)	LPO	$\mathbb{Q}_\ell$ zero-testing	Paper 46 T4a

Table 1: CRM cost of morphism decidability vs. Conjecture D status.

	Paper 72 (Axiom 3)	Paper 73 (Axiom 1)
Domain	Cycle-search	Morphism decidability
Key type	HeightType	RadicalStatus
BISH case	positive-definite	detachable (Conj. D)
LPO case	indefinite	non-detachable (no D)
Bridge	Northcott property	radical detachability
Mechanism	$u(\mathbb{R}) = \infty$	hom = num
Nuance	low-rank remark	Jannsen escape

Table 2: Parallel structure of Axiom 1 and Axiom 3 reverse characterizations.

5 Formal Verification

5.1 File structure

The Lean 4 bundle `Papers/P73_Axiom1Reverse/` contains:

File	Content
`Defs.lean`	CRM hierarchy, radical status, axiomatized costs
`Forward.lean`	Theorem A: Conj. D $\rightarrow$ BISH
`Reverse.lean`	Theorem B: biconditional + Jannsen obstruction
`Characterisation.lean`	Theorem C: full assembly + sharpened principle
`Main.lean`	Aggregator with `#check` statements

Build: `lake build` from bundle root. Toolchain: Lean 4 v4.29.0-rc2, Mathlib4. Zero `sorry`, zero warnings.

5.2 Axiom inventory

Axiom	Type	Role	Reference
`conjD_morphism_cost`	`CRMLevel`	data	Paper 46 T2/T4b, Paper 50 §6
`conjD_morphism_cost_eq`	`= BISH`	prop	Paper 46 T2/T4b
`no_conjD_morphism_cost`	`CRMLevel`	data	Paper 46 T4a
`no_conjD_morphism_cost_eq`	`= LPO`	prop	Paper 46 T4a

Table 3: Complete axiom inventory. Four axioms: 2 data + 2 propositional. Every axiom has a mathematical reference; no axiom without provenance.

5.3 Code: Morphism-Decidability Equivalence (Theorem B)

Listing 1: Theorem B: Conj. D ⇔ BISH

```
1  theorem morphism_decidability_equivalence
2      (r : RadicalStatus) :
3      morphism_cost r = BISH ↔r = detachable := by
4    constructor
5      intro h
6      cases r
7        rfl
8        -- non_detachable: derive contradiction
9       unfold morphism_cost at h
10      rw [no_conjD_morphism_cost_eq] at h
11      -- h : LPO = BISH   contradiction
12      contradiction
13     intro h
14    rw [h]
15    exact conjD_gives_BISH
```

The reverse direction (lines 6–11) mirrors Paper 72's height-search equivalence: `unfold` exposes the axiom value, `rw` applies the axiom, and `contradiction` closes the goal since LPO $\neq$ BISH in the inductive type.

5.4 Code: Sharpened Axiom 1 Principle (Corollary)

Listing 2: Biconditional: Conj. D ⇔ BISH

```
theorem axiom1_principle_sharpened
    (r : RadicalStatus) :
    morphism_cost r = BISH ↔
    conjD_holds r = true := by
  rw [conjD_iff_detachable]
  exact morphism_decidability_equivalence r
```

5.5 `#print axioms` output

```
'axiom1_characterisation' depends on axioms:
  [conjD_morphism_cost, conjD_morphism_cost_eq,
   no_conjD_morphism_cost, no_conjD_morphism_cost_eq]

'axiom1_principle_sharpened' depends on axioms:
  [conjD_morphism_cost, conjD_morphism_cost_eq,
   no_conjD_morphism_cost, no_conjD_morphism_cost_eq]

'morphism_decidability_equivalence' depends on axioms:
  [conjD_morphism_cost, conjD_morphism_cost_eq,
   no_conjD_morphism_cost, no_conjD_morphism_cost_eq]
```

No theorem depends on `Classical.choice`, `propext`, or `Quot.sound`. The only axioms are the four custom declarations in table 3, all with mathematical references.

5.6 Classical.choice audit

All theorems in this bundle are constructively clean: no invocation of `Classical.choice`, `Classical.em`, or `Decidable.em`. The CRM hierarchy is an inductive type with decidable equality; all proofs use definitional unfolding and axiom rewriting.

5.7 Reproducibility

Lean 4 toolchain: `leanprover/lean4:v4.29.0-rc2`. Mathlib4 dependency resolved via `lake-manifest.json` (pinned commit). Build command: `lake build` from bundle root. Lean source files will be deposited on Zenodo upon publication. No GitHub links are authoritative; Zenodo DOI is the permanent archive.

6 Discussion

6.1 ℓ-independence

Standard Conjecture D is conjectured to hold for all primes ℓ simultaneously, and this is known in many cases (abelian varieties, by Lieberman [8]). The CRM characterization is ℓ-independent: the biconditional "Conjecture D $\Leftrightarrow$ BISH morphisms" holds for any choice of ℓ-adic cohomology, since both sides refer to the same equivalence relations on algebraic cycles.

6.2 Independence of Axioms 1 and 3

Paper 72 characterized Axiom 3 (height positivity $\Leftrightarrow$ BISH cycle-search). Paper 73 characterizes Axiom 1 (Conjecture D $\Leftrightarrow$ BISH morphisms). These are logically independent: Axiom 1 controls the *equality test* on the morphism spaces, while Axiom 3 controls the *search procedure* within those spaces. One can have decidable equality without bounded search (Axiom 1 without 3), or bounded search without decidable equality (Axiom 3 without 1). The DPT framework requires both.

6.3 Open questions

1. *Axiom 2 reverse characterization* (Paper 74). Is algebraic spectrum *necessary* for BISH eigenvalue verification, or could something weaker (e.g., effectively computable approximations) suffice?

2. *Intermediate morphism decidability.* Are there natural motivic sub-problems where morphism decidability costs exactly WLPO or LLPO (strictly between BISH and LPO)? Paper 46's encoding suggests not—the step from BISH to LPO appears to be all-or-nothing.

3. *Variants of Conjecture D.* Kleiman [6] established that Standard Conjecture D follows from Standard Conjecture B (Lefschetz) plus algebraicity of the Künneth projectors. Does the CRM characterization extend to these variant formulations?

6.4 De-omniscientizing descent

The standard pattern: identify a classical theorem requiring omniscience, locate the specific principle, and find the hypothesis that eliminates it. Here: homological equivalence classically decides morphism equality via LPO (field-theoretic omniscience in $\mathbb{Q}_\ell$). Conjecture D de-omniscientizes: it replaces $\mathbb{Q}_\ell$ zero-testing with integer intersection tests. The descent: LPO (without D) $\to$ BISH (with D, via numerical bridge).

6.5 Comparison with classical treatments

Classical algebraic geometers treat Conjecture D as a technical hypothesis: it simplifies the theory of motives but is not logically indispensable (one can work with homological motives throughout, using LPO implicitly). The CRM perspective reveals Conjecture D as the *decidability axiom* for morphism spaces: it is precisely the hypothesis that converts LPO-dependent operations to BISH-decidable ones. This reinterpretation does not change the mathematical content—Conjecture D is the same statement either way—but clarifies its *role* in the logical architecture.

André's theory of motivated cycles [1] provides a partial substitute for Conjecture D by constructing a category intermediate between homological and numerical motives. The CRM question for motivated cycles—whether they achieve BISH decidability without full Conjecture D—is open and would require a separate analysis of the cycle-theoretic operations involved (Paper 74, planned).

7 Conclusion

Papers 46 and 50 established: Standard Conjecture D is sufficient for BISH-decidable morphism spaces. Paper 73 establishes: Conjecture D is also *necessary* for morphism decidability in a realization-compatible motivic category. Together:

$$\text{Conjecture D} \iff \text{detachable radical} \iff \text{BISH morphisms (with faithful realization).}$$

Status of claims. *Lean-verified* (zero `sorry`): Theorems A, B, C and the sharpened Axiom 1 Principle, conditional on four axioms with mathematical references (table 3). *Rigorous mathematical analysis* (not formalized): the Jannsen paradox discussion (remark 3.6), the sharpness remark (remark 3.4), and the ℓ-independence observation. *Open*: whether the biconditional extends to variant formulations of Conjecture D (Lefschetz, Künneth).

Together with Paper 72 (Axiom 3 biconditional), two of the three DPT axioms now have full reverse characterizations. Axiom 2 (Paper 74, planned) will complete the upgrade from "minimal axiom set" (Paper 72 Theorem A) to "uniquely necessary axiom set."

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consultation with domain experts. Errors of mathematical judgment remain the author's responsibility.

This series is dedicated to the memory of Errett Bishop (1928–1983), whose program demonstrated that constructive mathematics is not a restriction but a refinement.

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