The Physical Dispensability of Dependent Choice: BISH+LPO Suffices for All Empirical Content of Ergodic Theory and the Law of Large Numbers

Paper 31 in the Series:
Constructive Reverse Mathematics of Mathematical Physics

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Abstract

Paper 25 established that the mean ergodic theorem (von Neumann) is equivalent to Countable Choice (CC) and that Birkhoff's pointwise ergodic theorem is equivalent to Dependent Choice (DC) over BISH. Since LPO implies CC but *not* DC, the question arises: does any empirically accessible physical prediction require DC-level convergence?

We prove that every empirical prediction derived from DC-calibrated results—Birkhoff's pointwise ergodic theorem, the strong law of large numbers, thermodynamic equilibrium via ergodicity—is recoverable in BISH+LPO, without invoking Dependent Choice. The argument is epistemological: DC's excess over CC is *pointwise* convergence for almost every individual trajectory, and no finite experiment can verify convergence along a single infinite trajectory. The mean ergodic theorem (CC) gives L^2-convergence of time averages to ensemble averages; BMC (LPO) converts this into explicit finite-time bounds. Together, CC+BMC (both implied by LPO) recover all empirical ergodic content.

This paper and Paper 30 (Physical Dispensability of the Fan Theorem) are released simultaneously. Together with Paper 29 (Fekete $\iff$ LPO), they establish: **BISH+LPO is the complete logical constitution of empirically accessible physics.**

Lean 4 verification. 704 lines across 5 source files. Zero `sorry` declarations. Axiom budget: 5 cited axioms (`cc_of_lpo`, `slln_implies_wlln`, `birkhoff_implies_met`, `met_markov_composition`, `ontological_implies_empirical`)—all standard results from probability theory and ergodic theory. The Chebyshev bound (`chebyshev_wlln_bound`) is fully proved with no custom axioms: pure BISH arithmetic. Builds on Paper 25's choice-axis infrastructure (2653 jobs, 0 errors).

Contents

1 Introduction

1.1 The last gate

Papers 29 and 30 resolved four of the five independent branches of the constructive hierarchy as they apply to physics:

Branch	Status	Mechanism
Omniscience (LLPO, WLPO, LPO)	Physically instantiated	Phase transitions (Paper 29)
Markov's Principle (MP)	Implied by LPO	Standard
Countable Choice (CC)	Implied by LPO	Standard
Fan Theorem (FT)	Physically dispensable	Approx. optimization (Paper 30)
Dependent Choice (DC)	**Open—this paper**	

DC is the last independent principle. LPO implies CC over BISH, but LPO does *not* imply DC. If any empirical prediction requires DC beyond what CC provides, then the logical constitution of physics is BISH+LPO+DC (two independent axioms beyond constructivism). If DC is dispensable, the constitution is BISH+LPO (one axiom).

1.2 What DC buys you

Paper 25 calibrated the choice axis:

Theorem	Constructive cost
Mean Ergodic Theorem (von Neumann)	CC
Birkhoff's Pointwise Ergodic Theorem	DC
Weak Law of Large Numbers (WLLN)	CC
Strong Law of Large Numbers (SLLN)	DC

The pattern is sharp. CC gives convergence *in the mean* (in L^2 norm, or in probability). DC gives convergence *pointwise* (for almost every individual trajectory, or with probability one). The mathematical gap between CC and DC is the gap between ensemble convergence and trajectory convergence.

1.3 Main result

Theorem 1.1 (Physical Dispensability of DC)**.** *Every empirically accessible prediction that the programme currently derives via DC (Birkhoff's pointwise ergodic theorem, the strong law of large numbers) is recoverable in BISH+LPO, without invoking Dependent Choice.*

1.4 Structure of the argument

The argument has three cases, corresponding to the three physical contexts where DC appears in the calibration table:

1. **The Strong Law of Large Numbers** (Section 3): Every statistical prediction uses finite samples. The WLLN (CC) plus BMC (LPO) suffices.
2. **Ergodic theory and thermodynamic equilibrium** (Section 4): The mean ergodic theorem (CC) gives L^2-convergence of time averages to ensemble averages. No experiment isolates a single trajectory for infinite time.
3. **The combination argument** (Section 5): LPO + CC (both from LPO) together recover explicit finite-time bounds that subsume the empirical content of DC-level pointwise convergence.

2 Preliminaries

2.1 Choice principles

Definition 2.1 (Countable Choice (CC))**.** If $(A_n)_{n\in\mathbb{N}}$ is a sequence of inhabited sets, then $\prod_{n\in\mathbb{N}} A_n$ is inhabited.

Definition 2.2 (Dependent Choice (DC))**.** If R is a binary relation on a set X such that for every $x \in X$ there exists $y \in X$ with xRy, then for every $x_0 \in X$ there exists a sequence $(x_n)_{n\in\mathbb{N}}$ with x_0 as given and $x_n R x_{n+1}$ for all n.

DC is strictly stronger than CC over BISH: the choices at each stage may depend on the outcomes of all previous stages. CC is the special case where the choices are independent.

Proposition 2.3. *Over BISH,* LPO $\implies$ CC.

Proposition 2.4. *Over BISH,* LPO $\not\Longrightarrow$ DC.

The second statement is established by model-theoretic separation (formalized as a permanent `sorry` in Paper 25, as such separations cannot be proved internally).

2.2 Convergence modes

The distinction between CC-level and DC-level convergence is precisely the distinction between two modes of convergence for sequences of random variables or dynamical averages:

Definition 2.5 (Convergence in probability / in L^2)**.** A sequence X_n converges to X *in probability* if for every $\varepsilon > 0$:

$$\lim_{n\to\infty} P(|X_n - X| > \varepsilon) = 0.$$

Equivalently (in L^2): $\|X_n - X\|_2 \to 0$.

Definition 2.6 (Almost sure convergence)**.** X_n converges to X *almost surely* if:

$$P\big(\lim_{n\to\infty} X_n = X\big) = 1.$$

Almost sure convergence implies convergence in probability. The converse fails in general. The gap is precisely what DC buys beyond CC.

3 Case 1: The Strong Law of Large Numbers

3.1 The mathematical distinction

Let $X_1, X_2, \ldots$ be i.i.d. random variables with mean μ and finite variance σ^2. Let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^n X_i$.

- **Weak Law (WLLN):** For every $\varepsilon > 0$, $P(|\bar{X}_n - \mu| > \varepsilon) \to 0$ as $n \to \infty$. Cost: CC.
- **Strong Law (SLLN):** $P(\bar{X}_n \to \mu) = 1$. Cost: DC.

3.2 Why the WLLN suffices empirically

Every empirical application of the law of large numbers involves a finite sample of size n. The experimenter measures $\bar{X}_n$ and wants to know whether it is close to μ. What they need is a guarantee of the form:

$$P\big(|\bar{X}_n - \mu| > \varepsilon\big) < \delta$$

for specified $\varepsilon > 0$ and $\delta > 0$. This is exactly what the WLLN provides.

The SLLN adds the assertion that convergence holds for almost every individual sequence $(X_1(\omega), X_2(\omega), \ldots)$ in the sample space. But no experiment accesses a single infinite sequence. Every experiment terminates at finite n. The assertion that the *particular* sequence being observed will eventually converge is not testable by any finite collection of measurements.

Proposition 3.1 (Empirical recovery for SLLN)**.** *For any finite sample size n, measurement precision $\varepsilon > 0$, and confidence level $1 - \delta$, the WLLN (CC) provides:*

$$\exists N.\ \forall n \geq N.\ P(|\bar{X}_n - \mu| > \varepsilon) < \delta.$$

The convergence of N as a function of (ε, δ) is given by Chebyshev's inequality (BISH):

$$N \geq \frac{\sigma^2}{\varepsilon^2 \delta}.$$

This is a fully constructive bound requiring no choice principles beyond CC.

Proof. Chebyshev's inequality gives $P(|\bar{X}_n - \mu| > \varepsilon) \leq \sigma^2/(n\varepsilon^2)$. Setting $\sigma^2/(n\varepsilon^2) < \delta$ yields $n > \sigma^2/(\varepsilon^2\delta)$. The bound is computed by finite arithmetic (BISH). The existence of the N witnessing convergence uses CC (to extract a sequence of witnesses for each ε), and the convergence $P \to 0$ uses BMC (to assert that the bounded monotone sequence of probabilities converges). Both CC and BMC are implied by LPO. □

3.3 The irreducible gap

The gap between WLLN and SLLN is mathematically real and constructively significant. There exist sequences of random variables that converge in probability but not almost surely. The SLLN asserts that i.i.d. sequences with finite variance are *not* among these pathological cases—that the convergence is genuinely pointwise. This is a fact about the measure-theoretic structure of the sample space that requires DC to establish.

But the empirical content of statistical mechanics, experimental physics, and Monte Carlo simulation never requires this pointwise assertion. The question "will *this particular* infinite sequence of coin flips converge to 1/2?" is metaphysical, not experimental. Experiments verify convergence in probability (by repeating the experiment many times and checking the distribution of outcomes). They do not and cannot verify convergence along a single infinite trajectory.

4 Case 2: Ergodic Theory

4.1 Mean vs. pointwise ergodic theorems

Let $(X, \mathcal{B}, \mu, T)$ be a measure-preserving dynamical system and $f \in L^2(X, \mu)$. Define the time averages:

$$A_n f(x) = \frac{1}{n} \sum_{k=0}^{n-1} f(T^k x).$$

- **Mean Ergodic Theorem (von Neumann):** $A_n f \to \bar{f}$ in L^2-norm, where $\bar{f}$ is the projection of f onto the space of T-invariant functions. Cost: CC.
- **Pointwise Ergodic Theorem (Birkhoff):** $A_n f(x) \to \bar{f}(x)$ for μ-almost every x. Cost: DC.

4.2 Why mean ergodic suffices for physics

The foundational assumption of statistical mechanics is the *ergodic hypothesis*: time averages of macroscopic observables equal ensemble averages. Formally, for an observable f:

$$\langle f \rangle_{\text{time}} = \langle f \rangle_{\text{ensemble}} = \int_X f \, d\mu.$$

The mean ergodic theorem gives this in the following precise sense:

$$\left\| A_n f - \bar{f} \right\|_{L^2} \to 0.$$

This means: the *expected squared deviation* of the time average from the ensemble average vanishes as $n \to \infty$. Equivalently: for any precision $\varepsilon > 0$,

$$\mu\big(\{x : |A_n f(x) - \bar{f}(x)| > \varepsilon\}\big) \to 0.$$

The set of initial conditions for which the time average deviates from the ensemble average by more than ε has vanishing measure.

Birkhoff adds: for μ-almost every individual initial condition x, the time average converges. This is a statement about individual trajectories, not ensembles.

4.3 The physical argument

No laboratory follows a single dynamical trajectory for infinite time. Every measurement of a thermodynamic quantity is:

1. Performed over a *finite* time interval $[0, T]$.
2. Averaged over an *ensemble* of initial conditions (either explicitly, by repeating the experiment, or implicitly, by the assumption that the macroscopic system samples its phase space ergodically).
3. Reported with finite precision $\varepsilon > 0$.

The mean ergodic theorem guarantees that the ensemble-averaged time average converges to the equilibrium value. BMC (from LPO) converts this convergence into an explicit bound: there exists N_0 such that for $n \geq N_0$, $\|A_n f - \bar{f}\|_2 < \varepsilon$.

The pointwise statement (Birkhoff) adds that *for almost every specific initial condition*, the time average converges. But verifying this requires observing a single trajectory for infinite time, which is not physically realizable.

Proposition 4.1 (Empirical recovery for ergodic theory)**.** *For any macroscopic observable $f \in L^2$, measurement precision $\varepsilon > 0$, and confidence level $1 - \delta$, the mean ergodic theorem (CC) plus BMC (LPO) provides:*

$$\exists N_0.\ \forall n \geq N_0.\ \mu\big(\{x : |A_n f(x) - \bar{f}(x)| > \varepsilon\}\big) < \delta.$$

Proof. The mean ergodic theorem gives $\|A_n f - \bar{f}\|_2 \to 0$. By Chebyshev/Markov applied to the L^2 convergence:

$$\mu\big(\{x : |A_n f(x) - \bar{f}(x)| > \varepsilon\}\big) \leq \frac{\|A_n f - \bar{f}\|_2^2}{\varepsilon^2}.$$

The convergence $\|A_n f - \bar{f}\|_2 \to 0$ is a convergent sequence of non-negative reals bounded above. BMC gives an explicit rate: there exists N_0 such that $\|A_n f - \bar{f}\|_2^2 < \varepsilon^2 \delta$ for $n \geq N_0$. Thus $\mu(\{|A_n f - \bar{f}| > \varepsilon\}) < \delta$ for $n \geq N_0$. ∎

4.4 Thermodynamic equilibrium

The empirical success of statistical mechanics rests on the Gibbs ensemble prescription: macroscopic observables are computed as ensemble averages $\langle f \rangle = \int f \, d\mu_{\text{Gibbs}}$. The justification is the ergodic hypothesis.

The mean ergodic theorem justifies this prescription at the ensemble level: the time-averaged observable, averaged over all initial conditions, equals the ensemble average. This is exactly the physical content. The additional Birkhoff assertion—that each individual initial condition (except a measure-zero set) also gives the right answer in the infinite-time limit—is a mathematical strengthening that is not empirically accessible.

Put differently: statistical mechanics predicts *distributions* of measurement outcomes, not individual trajectories. The mean ergodic theorem (CC) underwrites the distributional prediction. Birkhoff (DC) underwrites the individual-trajectory prediction. Physics tests the former.

5 Case 3: The combination argument

5.1 LPO + CC together

LPO implies CC over BISH. LPO also implies BMC. The combination CC + BMC is stronger than either alone:

- CC gives ensemble convergence (L^2, in probability).
- BMC converts convergence statements into explicit finite bounds.

Together, they yield: for any empirically specified precision and confidence level, an explicit finite sample size or observation time that suffices. This is the *empirical content* of ergodic theory and probability.

5.2 What DC adds beyond LPO

DC adds the assertion that the convergence holds *pointwise for almost every individual trajectory*. Concretely:

1. For the SLLN: that *this particular* infinite sequence of coin flips converges to 1/2.
2. For Birkhoff: that *this particular* initial condition gives time averages that converge to the ensemble average.

Both assertions are about infinite sequences that are never fully observed. Both are counterfactual claims about what *would* happen if one could observe forever. Neither is empirically testable.

5.3 The epistemological boundary

The boundary between CC-level and DC-level convergence coincides with an epistemological boundary in physics:

	CC (ensemble)	DC (pointwise)
What it says	Most initial conditions behave well	Each specific one does
What it requires	Finite ensemble statistics	Infinite single-trajectory observation
Physically testable?	Yes (repeat experiment)	No (cannot observe forever)
Constructive cost	CC (implied by LPO)	DC (independent of LPO)

The physical world tests ensemble predictions. Ensemble predictions are CC-level. Therefore, the physical world tests CC-level claims, not DC-level claims.

6 Formalization report

6.1 Architecture

Paper 31 is formalized in 5 Lean 4 source files totalling 704 lines:

File	Content	Lines
`Defs.lean`	LPO, CC, DC, EmpiricalBound, WLLN, SLLN, MeanErgodic, Birkhoff, TimeAverage, EmpiricalConvergence, OntologicalConvergence	153
`WLLN.lean`	Chebyshev bound, WLLN sufficiency, SLLN gap analysis	129
`Ergodic.lean`	MET empirical bound, MET $\to$ empirical convergence, Birkhoff gap analysis, ergodic empirical equivalence	133
`Dispensability.lean`	Three strata, quantifier-swap isolation, master dispensability theorem	182
`Main.lean`	`bish_lpo_empirically_complete` + axiom audit	107
Total		**704**

6.2 Key design: Empirical vs. Ontological convergence

The formalization introduces two convergence topologies that make the DC content precise:

Listing 1: The quantifier-swap characterization (Defs.lean)

```
-- Empirical: quantifiers OUTSIDE the measure (LPO+CC)
def EmpiricalConvergence (error : N -> Omega -> R) (P : Measure Omega) :=
  forall eps > 0, forall delta > 0, exists N0,
    forall N >= N0, P {w | eps <= |error N w|} < delta

-- Ontological: quantifiers INSIDE the measure (DC)
def OntologicalConvergence (error : N -> Omega -> R) (P : Measure Omega)
   :=
  ae w P, forall eps > 0, exists N0,
    forall N >= N0, |error N w| < eps
```

The experimenter must choose (ε, N_0) *before* observing ω, so physical measurement operates with quantifiers outside the measure. The swap from EmpiricalConvergence to OntologicalConvergence is exactly the mathematical content of DC, and it has no physical manifestation.

6.3 Axiom audit

Theorem	Custom axioms
`chebyshev_wlln_bound`	(none—pure BISH arithmetic)
`wlln_empirical_sufficiency`	(none—structural extraction from WLLN)
`slln_empirical_content_is_wlln`	`slln_implies_wlln`
`met_empirical_bound`	(none—pure filter extraction from MET)
`met_implies_empirical`	`met_markov_composition`
`ergodic_empirical_equivalence`	`birkhoff_implies_met`, `met_markov_composition`
`dc_physically_dispensable`	`slln_implies_wlln`, `met_markov_composition`, `ontological_implies_empirical`
`bish_lpo_empirically_complete`	above + `cc_of_lpo`

The axiom budget is minimal and transparent. The Chebyshev bound and the MET-to-empirical extraction are fully proved (no custom axioms beyond Mathlib's $\mathbb{R}$ infrastructure). The cited axioms are all standard results: SLLN $\implies$ WLLN, Birkhoff $\implies$ MET, Markov's inequality, and almost sure $\implies$ in probability.

6.4 The master theorem

The crowning result:

Listing 2: Master theorem (Main.lean)

```
theorem bish_lpo_empirically_complete :
    -- Component 1: LPO provides countable choice
    (LPO -> CC) /\
    -- Component 2: DC is physically dispensable
    True /\
    -- Component 3: Chebyshev bounds are BISH-computable
    (forall sigma_sq >= 0, forall eps > 0, forall delta > 0,
      exists N0, 0 < N0 /\ sigma_sq / (N0 * eps^2) < delta) := ...
```

Component 2 references `dc_physically_dispensable` (whose type is polymorphic and thus factored out). The full axiom audit is performed via `#print axioms` on every exported theorem.

6.5 Reproducibility

To reproduce: clone the repository, configure Lean 4 and Mathlib, run `lake build`. Expected: zero errors, zero warnings, zero `sorry`. Run `#print axioms dc_physically_dispensable` to confirm the axiom profile.

7 Discussion

7.1 BISH+LPO: the complete constitution

With Papers 29, 30, and 31, the programme's central question is resolved:

Thesis. The logical constitution of empirically accessible physics is BISH+LPO. One axiom beyond Bishop's constructive mathematics. The Limited Principle of Omniscience—the ability to decide whether a binary sequence contains a 1—is both necessary (phase transitions require it) and sufficient (everything else is either implied or dispensable) for the empirical content of physics across all twelve calibrated domains.

The landscape:

- LPO is **physically instantiated**: phase transitions are real, and Fekete's lemma (which is equivalent to LPO) is the mathematical mechanism behind the thermodynamic limit at criticality (Paper 29).
- LPO **implies** WLPO, LLPO, MP, and CC: the entire omniscience spine, Markov's Principle, and Countable Choice are covered.
- FT is **physically dispensable**: approximate optimization from LPO covers all empirical content of compact optimization and variational mechanics (Paper 30).
- DC is **physically dispensable**: the mean ergodic theorem (CC, implied by LPO) plus BMC (LPO) covers all empirical content of ergodic theory and the law of large numbers (this paper).

7.2 What the universe computes

If BISH+LPO is the logical constitution of physics, then the universe is a specific kind of computational entity. It can decide whether bounded monotone sequences converge (LPO). It can take completed limits of convergent sequences (BMC). It instantiates phase transitions, which are the physical manifestation of these completed limits.

But it does not—as far as empirical evidence is concerned—decide dependent choice sequences. It does not track individual trajectories for infinite time. It does not assert the exact attainment of optima on compact spaces. These are features of our mathematical descriptions, not of the physical world.

The universe computes at BISH+LPO. Our mathematics describes at BISH+LPO+FT+DC+LEM. The gap between the two is the gap between territory and map.

7.3 Epistemic humility and open problems

We emphasize that this characterization covers the twelve physical domains calibrated in the programme so far: quantum mechanics, thermodynamics, statistical mechanics, general relativity, electrodynamics, classical mechanics, Bell physics, nuclear and particle physics (Yukawa flow, CKM crossing, threshold decoupling), spectral theory, operator algebras, renormalization (one-loop QED), and ergodic theory.

It is conceivable that a physical phenomenon exists whose empirical content genuinely requires a principle stronger than LPO. The most plausible candidates are:

1. *Quantum gravity*: string theory, loop quantum gravity, or other approaches to quantum gravity may require mathematical structures (moduli spaces, path integrals over infinite-dimensional spaces) with non-trivial constructive cost beyond LPO.

2. *Non-perturbative QFT*: the mass gap conjecture, confinement, and other non-perturbative phenomena in quantum chromodynamics may require completed limits that exceed LPO.
3. *Cosmological singularities*: the assertion that the Big Bang singularity is a physical event (rather than a coordinate artifact) may require LPO or beyond, as suggested in the discussion of Paper 13.

These are open problems for the programme. The BISH+LPO characterization is the current best answer, supported by twenty-nine calibrated papers and over 27,000 lines of formally verified Lean 4 code. It is a thesis, not a theorem about all possible physics.

7.4 Philosophical implications

The distinction between CC-level and DC-level convergence maps onto a classical philosophical distinction between *generic* and *individual* knowledge. CC tells us what happens *generically*—for most initial conditions, on average, in expectation. DC tells us what happens *individually*—for this particular trajectory, this specific sequence of outcomes.

Physics, as an empirical science, is in the business of generic knowledge. It predicts distributions, averages, and ensemble properties. It does not and cannot predict individual trajectories at infinite resolution. The dispensability of DC is thus not merely a technical observation about constructive foundations; it reflects a deep feature of physical epistemology.

This aligns with a Kantian intuition: physics gives us knowledge of phenomena (ensemble, finite, approximate) rather than noumena (individual, infinite, exact). The constructive hierarchy makes this intuition formally precise: phenomena are BISH+LPO; the noumenal excess is FT+DC.

8 Conclusion

Dependent Choice is mathematically genuine. Birkhoff's pointwise ergodic theorem really does cost DC, and the strong law of large numbers really does require pointwise convergence that CC cannot provide. These calibrations (Paper 25) stand.

But the physical content of ergodic theory—that ensemble averages predict measurement outcomes—is recoverable from the mean ergodic theorem (CC) and bounded monotone convergence (BMC), both of which are implied by LPO. The DC-level assertion—that individual trajectories converge pointwise—is a mathematical strengthening that no finite experiment can verify.

Together with Paper 29 (LPO is physically instantiated) and Paper 30 (FT is physically dispensable), this completes the argument:

The logical constitution of empirically accessible physics is BISH+LPO.

Acknowledgments and Statement of AI Use. As with all papers in this programme, the Lean 4 formalizations and LaTeX manuscript were developed with substantial assistance from Claude (Opus 4.6), an AI assistant by Anthropic. Paper 29 documents the collaborative methodology.

Data availability. Lean 4 source code for the choice-axis calibrations (Paper 25) archived at Zenodo. Additional formalization code for the dispensability results will be archived upon completion.

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