The Weight 1 Boundary:
Constructivising the GL_2 Langlands Programme

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Abstract

Papers 68–70 showed that every modularity theorem for $\mathrm{GL}_2/\mathbb{Q}$ calibrates at $\mathsf{BISH}+\mathsf{WLPO}$, with the WLPO entering through the Arthur–Selberg trace formula in three roles: Langlands–Tunnell (base change), Jacquet–Langlands (quaternionic transfer), and Jacquet–Langlands (level-lowering). We investigate whether these WLPO atoms can be eliminated.

Two of the three are eliminated. By restructuring the proof to work on compact Shimura curves (attaching Galois representations via étale cohomology rather than transferring to GL_2), the Jacquet–Langlands transfer becomes unnecessary for the modularity lifting step. For the Jacquet–Langlands comparison at weight ≥ 2, we prove a *decidability descent*: the trace formula identity reduces to an equation between algebraic numbers, verifiable in the decidable first-order theory ACF. The analytic proof is classical scaffolding; the algebraic content is BISH.

The third WLPO atom—Langlands–Tunnell at weight 1—is irreducible. We identify three independent obstructions, all specific to weight 1: (i) Archimedean orbital integrals involve transcendental regulators that do not cancel; (ii) the spectral side mixes holomorphic forms with Maass forms of eigenvalue $\lambda = 1/4$, whose Hecke eigenvalues are generically transcendental; (iii) the dimension of the space of weight 1 forms has no algebraic formula (Riemann–Roch gives only the Euler characteristic). The decidability descent fails on all three fronts.

The WLPO in Fermat's Last Theorem is localised to a single irreducible atom: extracting a weight 1 holomorphic form from the L^2 spectral decomposition of a non-compact locally symmetric space. This atom lies at the exact boundary between algebra and analysis, where the discrete and continuous spectra of the Laplacian meet at $\lambda = 1/4$.

Contents

1 Introduction

Papers 68–70 audited the $\mathrm{GL}_2/\mathbb{Q}$ Langlands programme and found a uniform classification: BISH+WLPO. The WLPO entered through three structurally distinct applications of the Arthur–Selberg trace formula. The present paper asks: are all three necessary?

The answer is *no* for two of them and *yes* for the third. The constructive profile of Wiles's proof can be improved, but not to BISH. The irreducible obstruction is the spectral theory of weight 1 automorphic forms.

1.1 Summary of results

WLPO source (Paper 70)	Weight	Status (Paper 71)	Method
Langlands–Tunnell (base change)	1	WLPO (irreducible)	—
Jacquet–Langlands (icosahedral)	≥ 2	BISH	Shimura curve, Route 3
Jacquet–Langlands (level-lowering)	≥ 2	BISH	Decidability descent

2 The Decidability Descent

The central technique is a meta-mathematical argument that converts a classical analytic proof into a constructive algebraic verification.

Definition 2.1 (Decidability descent)**.** Let φ be a sentence in the first-order theory of algebraically closed fields (ACF). If φ is proved classically (using WLPO or stronger principles), it nevertheless holds constructively, because ACF is decidable (Tarski–Seidenberg).

More concretely: if the Arthur–Selberg trace formula, applied with specific test functions, produces an identity $\sum_{i=1}^{N} a_i = \sum_{j=1}^{M} b_j$ where all a_i, b_j are algebraic numbers and the bounds N,

M are algebraically computable, then the identity is BISH-verifiable regardless of whether its proof used non-constructive analysis.

For the descent to succeed, three conditions must hold:

(i) The terms on both sides are algebraic numbers.

(ii) The index bounds N, M are computable by algebraic geometry (e.g., Riemann–Roch, Brandt matrix dimensions).

(iii) No transcendental quantities remain after cancellation.

This is a special case of the de-omniscientising descent identified in Paper 50: classical logic establishes a result whose content is constructively verifiable at a lower logical level.

3 The Algebraic Trace Formula for Quaternion Algebras

Theorem 3.1 ($D^\times$ trace formula is BISH)**.** *For a definite quaternion algebra D over a totally real field F, the Selberg trace formula with algebraic test functions is an identity between algebraic numbers.*

Proof sketch. Non-Archimedean places: The orbital integrals of characteristic functions of maximal compact subgroups are non-negative integers (counting fixed vertices in the Bruhat–Tits tree).

Archimedean places: At each real place, $D_v^\times/F_v^\times \cong \mathrm{SU}(2)$. The test function is a character of an irreducible representation $\mathrm{Sym}^n(\mathbb{C}^2)$, which is a class function. The orbital integral factors as $f_v(\gamma) \cdot \mathrm{vol}(T_v\backslash\mathrm{SU}(2))$. The character $\chi_n(\theta) = U_n(\cos\theta)$, a Chebyshev polynomial with integer coefficients, evaluated at $\cos\theta = \mathrm{tr}(\gamma)/(2\sqrt{N(\gamma)})$, which is algebraic since $\gamma \in D^\times(F)$. With canonical normalisations ($\mathrm{vol}(\mathrm{SU}(2)) = 1$), the integral is algebraic.

Volumes: For regular semisimple $\gamma \notin F$, the centraliser $K = F(\gamma)$ is a CM extension of F. The volume $\mathrm{vol}(D_\gamma^\times(F)\backslash D_\gamma^\times(\mathbb{A}_F))$ reduces to the relative class number h_K/h_F divided by the number of relative roots of unity. This is rational: the transcendental regulators R_K and R_F cancel by Dirichlet's unit theorem (the unit ranks of a CM field and its totally real subfield coincide).

Spectral side: The space of automorphic forms is finite-dimensional (compact quotient). Hecke operators act via Brandt matrices with integer entries. Traces are integers.

Both sides are algebraic. Transcendental factors from Tamagawa measure normalisation (powers of π, L-values) appear identically on both sides and cancel by the analytic class number formula and the Klingen–Siegel theorem. □

4 Eliminating Jacquet–Langlands: The Shimura Curve Strategy

4.1 The restructuring

Paper 70 identified the $D^\times \to \mathrm{GL}_2$ transfer (Jacquet–Langlands correspondence) as a WLPO bottleneck. We eliminate it by restructuring the proof to avoid GL_2 automorphic forms entirely during the lifting step.

Proposition 4.1 (Shimura curve restructuring)**.** *Let D/F be an indefinite quaternion algebra (split at exactly one Archimedean place). The associated Shimura curve Sh_D is a compact algebraic curve over F.*

(i) The étale cohomology $H^1_{\mathrm{et}}(\mathrm{Sh}_D \times_F \bar{F}, \mathbb{Q}_\ell)$ carries a 2-dimensional Galois representation attached to each automorphic representation of $D^\times(\mathbb{A}_F)$ via the Eichler–Shimura relation and Carayol's theorem.

(ii) *The Taylor–Wiles patching method can be executed entirely on* $H^1_{\mathrm{et}}(\mathrm{Sh}_D)$*, without transferring to* GL_2*.*

(iii) *The Galois representation is attached directly to the* $D^\times$ *form. No Jacquet–Langlands transfer is needed.*

This is precisely how Taylor's potential modularity theorem operates: modularity lifting on the cohomology of compact Shimura varieties, where the compactness eliminates boundary contributions and makes the deformation theory clean.

4.2 The Jacquet–Langlands comparison at weight ≥ 2

For the applications in the Khare–Wintenberger induction (level-lowering, where a form must be transferred between inner forms of GL_2), we need the Jacquet–Langlands comparison. At weight ≥ 2, the decidability descent applies.

Theorem 4.2 (Jacquet–Langlands at weight ≥ 2 is BISH)**.** *The Jacquet–Langlands correspondence for* GL_2*, restricted to automorphic representations of weight* $k \geq 2$*, is* BISH*-verifiable via decidability descent.*

Proof sketch. Geometric side: The Archimedean test function is a pseudo-coefficient of the discrete series of weight k. By Harish-Chandra's theory, its orbital integrals on hyperbolic (split) elements vanish. The surviving elliptic orbital integrals evaluate to algebraic character values. The non-Archimedean orbital integrals are integers (§3). Volume factors (powers of π) cancel between the GL_2 and $D^\times$ sides.

Spectral side: The traces are products of Hecke eigenvalues of holomorphic modular forms, which are algebraic integers.

Index bounds: The dimension of the space of cusp forms of weight k and level N is computable by the Riemann–Roch theorem (or the Eichler–Selberg dimension formula): $\dim S_k(\Gamma_0(N))$ is an explicit arithmetic function of k and N.

All three conditions of the decidability descent (Definition 2.1) are satisfied. The trace formula identity is a sentence in ACF, verified classically, hence constructively valid. □

Remark 4.3 (Weight ≥ 2 versus weight 1)*.* At weight ≥ 2, the pseudo-coefficient kills the hyperbolic orbital integrals (eliminating transcendental regulators), the spectral side contains only holomorphic forms (no Maass form contamination), and Riemann–Roch computes the exact dimension (no H^1 obstruction). All three properties fail at weight 1. The weight 1 / weight ≥ 2 boundary is the exact logical fault line.

5 The Octahedral Reduction

Langlands–Tunnell proves modularity of 2-dimensional Artin representations with solvable projective image. The three solvable cases are dihedral (D_n, handled by Hecke theta series, BISH), tetrahedral (A_4, Langlands), and octahedral (S_4, Tunnell).

Tunnell's original proof uses the Gelbart–Jacquet symmetric square lifting to GL_3, which invokes the converse theorem for GL_3. This would introduce an additional WLPO source.

Proposition 5.1 (Octahedral modularity without GL_3)**.** *Over* $\mathbb{Q}$*, octahedral modularity reduces to iterated cyclic base change for* GL_2*. The* GL_3 *converse theorem is not needed.*

Proof sketch. The key obstruction is the *quadratic twist ambiguity*: descending from A_4 to S_4 via the quadratic quotient $S_4/A_4 \cong \mathbb{Z}/2$ produces two candidates (π and $\pi \otimes \eta$) that are indistinguishable by central character (since $\eta^2 = 1$). Over arbitrary number fields, Tunnell resolves this via the symmetric square (which annihilates the ambiguity since $\mathrm{Sym}^2(\pi \otimes \eta) = \mathrm{Sym}^2(\pi)$).

Over $\mathbb{Q}$, the Deligne–Serre theorem provides an algebraic resolution: since $\bar{\rho}$ is odd, the descended form is classical holomorphic of weight 1, and Deligne–Serre attaches a Galois representation. Both twist candidates π and $\pi \otimes \eta$ are modular (twisting a modular form by a Dirichlet character preserves modularity). The ambiguity is harmless.

The $V_4 \cong \mathbb{Z}/2\times\mathbb{Z}/2$ subgroup of A_4 and S_4 is not an obstruction: representations with projective image V_4 are dihedral, modular by Hecke theta series (BISH).

Consequently, octahedral modularity over $\mathbb{Q}$ uses: dihedral modularity (BISH), cubic base change (WLPO), quadratic base change (WLPO), and Deligne–Serre (BISH). No GL_3. ☐

The WLPO in Langlands–Tunnell is therefore concentrated in the cyclic base change steps, which use the GL_2 trace formula at weight 1.

6 The Weight 1 Obstruction

The Langlands–Tunnell theorem requires cyclic base change for GL_2 at weight 1. We show that the decidability descent fails for weight 1 on three independent fronts.

6.1 Failure 1: Transcendental Archimedean integrals

To isolate weight 1 forms, the Archimedean test function is the character of a limit of discrete series. Unlike the discrete series for weight $k \geq 2$, the limit of discrete series does *not* vanish on hyperbolic (split) elements. The orbital integral over the hyperbolic torus introduces the volume of the split torus modulo the centraliser, which evaluates to $\log \varepsilon_K$, the logarithm of the fundamental unit of a real quadratic field—a transcendental number.

These transcendental terms do not cancel between the two sides of the base change comparison (they depend on the specific splitting behaviour at each place).

6.2 Failure 2: Maass form contamination

The limit of discrete series at weight 1 shares its infinitesimal character with the principal series at Laplacian eigenvalue $\lambda = 1/4$. The spectral side of the trace formula at weight 1 therefore receives contributions from both holomorphic weight 1 forms *and* odd Maass forms of eigenvalue 1/4:

$$\text{Spectral side} = \sum_{\pi \text{ hol, wt } 1} \mathrm{tr}\pi(f) \; - \sum_{\pi' \text{ Maass, } \lambda=1/4} \mathrm{tr}\pi'(f).$$

The Hecke eigenvalues of Maass forms are generically transcendental. The identity equates sums of real numbers, not algebraic numbers. The Tarski decidability argument does not apply.

6.3 Failure 3: Unknown index bounds

Even if the terms were algebraic, the decidability descent requires computable bounds on the number of terms. For weight 1, the dimension of $H^0(X, \omega)$ has no purely algebraic formula. The Riemann–Roch theorem gives $\dim H^0 - \dim H^1$, but H^1 is non-zero and jumps erratically (it depends on the existence of specific automorphic representations, not on arithmetic invariants of the curve). Without knowing the index set, the identity $\sum_i a_i = \sum_j b_j$ cannot be formulated as a sentence in ACF.

Theorem 6.1 (The weight 1 obstruction is irreducible)**.** *The Langlands–Tunnell base change at weight* 1 *cannot be constructivised by decidability descent. The* WLPO *is irreducible: extracting a weight* 1 *holomorphic form from the* L^2 *spectral decomposition of* $\mathrm{GL}_2(F)\backslash\mathrm{GL}_2(\mathbb{A}_F)$ *requires separating the discrete spectrum from the continuous spectrum at the point* $\lambda = 1/4$ *where they meet.*

6.4 Failure 4: The p-adic initialisation trap

The p-adic theory of overconvergent modular forms (Buzzard–Taylor [3], Kassaei [8], Pilloni [13]) provides a purely p-adic construction of weight 1 forms that avoids all three Archimedean obstructions: Hida families are BISH, specialisation is BISH, and Kassaei–Pilloni classicality is BISH (using U_p as a contraction mapping on rigid analytic spaces, with no Archimedean input).

However, the p-adic machinery cannot *originate* modularity. To place $\bar{\rho}_3$ on the eigencurve, one must find a classical modular form of weight $k \geq 2$ whose residual Galois representation matches $\bar{\rho}_3$. This requires lifting $\bar{\rho}_3$ to characteristic zero and invoking Langlands–Tunnell to produce the initial modular form. Modularity lifting theorems (Taylor–Wiles, Kisin, Calegari–Geraghty) are similarly relative: they transform modularity from one form to another but cannot create it.

The trace formula is the unique *absolute* bridge from Galois representations to automorphic forms. All other methods are relative—they propagate modularity but cannot initialise it. The WLPO at weight 1 is not an artefact of choosing the wrong construction; it is the logical cost of *creating* a modular form from a Galois representation *ex nihilo*.

6.5 Failure 5: The universal quantifier

The preceding four failures concern the trace formula approach and the p-adic approach. A third approach—direct algebraic computation—reveals the deepest structure of the obstruction.

For any *specific* conductor N, the space $S_1(N, \chi)$ is explicitly computable by purely algebraic methods (BISH). The algorithm uses the *Eisenstein trick*: choose two Eisenstein series E_a, E_b of weight 1 with no common zeros on $X_1(N)$; then $S_1(N, \chi)$ is the intersection of the images of the two injections $f \mapsto f \cdot E_a$ and $f \mapsto f \cdot E_b$ into $S_2(N, \chi\psi)$, which is computable via modular symbols. The Sturm bound provides a finite verification criterion. For any given N, one can construct a candidate q-expansion from the Galois representation and verify its modularity by finite linear algebra.

However, Fermat's Last Theorem is a proof by contradiction about a *hypothetical* Frey curve $E_{a,b,c}$ whose conductor $N = \mathrm{rad}(abc)$ is an unbounded variable. One cannot execute a linear algebra algorithm on an unspecified input. To complete the proof, one needs a *universal* existence theorem:

> For *every* conductor N and every valid S_4 representation of conductor N, the space $S_1(N, \chi)$ contains the corresponding eigenform.

This requires proving $\dim S_1(N, \chi) \geq 1$ in the relevant eigenspace for all N. Because $H^1(X_1(N), \mathcal{O}) \neq 0$ at weight 1, the Riemann–Roch theorem gives only the Euler characteristic $h^0 - h^1$, not a lower bound on h^0. The only known method for forcing $h^0 \geq 1$ universally is the Arthur–Selberg trace formula.

Remark 6.2 (The ontology of weight 1 forms)*.* The WLPO is not the cost of constructing weight 1 forms (which are algebraic, computable objects). It is the cost of the *universal quantifier*: proving that the algebraic algorithm will succeed for every possible input. The trace formula is needed not because weight 1 forms are analytic, but because proving they *always exist* requires analytic methods. The distinction is between *verification* (checking a specific instance—BISH) and *certification* (proving the algorithm never fails—WLPO).

7 The Optimised Classification

Theorem 7.1 (Optimised classification of FLT)**.** *Wiles's proof of Fermat's Last Theorem, restructured via the Shimura curve strategy (§4) and the decidability descent (§2), calibrates at* BISH + WLPO*. The* WLPO *consists of a single irreducible atom: the Langlands–Tunnell theorem at weight* 1*.*

Proof. The restructured proof has the following components:

Component	**Weight**	**Classification**	**Method**
Base case			
Dihedral modularity	—	BISH	Hecke theta series
Tetrahedral base change	1	WLPO	GL_2 trace formula
Octahedral descent	1	WLPO	GL_2 trace formula
$\mathrm{GL}_2 \to D^\times$ transfer	1	WLPO	JL trace formula
Lifting			
TW patching on Shimura curve	≥ 2	BISH	Brochard, eff. Chebotarev
Galois rep via étale cohom.	≥ 2	BISH	Eichler–Shimura–Carayol
Induction (Khare–Wintenberger)			
Level-lowering (JL comparison)	≥ 2	BISH	Decidability descent
Level-raising	≥ 2	BISH	Ihara, supersingular locus
Weight reduction	≥ 2	BISH	Hasse invariant, θ
Serre's recipe	—	BISH	Finite group theory
Potential modularity	≥ 2	BISH	Moret-Bailly
Overall		BISH + WLPO	

The WLPO components (tetrahedral/octahedral base change, $\mathrm{GL}_2 \to D^\times$ transfer) all occur at weight 1 and all use the GL_2 trace formula. They constitute a single irreducible atom: the Langlands–Tunnell theorem. ☐

Corollary 7.2 (Improvement over Paper 70)**.** *Paper 70 identified three* WLPO *sources. Paper 71 reduces this to one:*

WLPO ***source***	***Paper 70***	***Paper 71***
Langlands–Tunnell	WLPO	WLPO *(irreducible)*
JL: icosahedral transfer	WLPO	BISH *(Route 3)*
JL: level-lowering over F	WLPO	BISH *(descent)*

Corollary 7.3 (Algebraic weight 1 modularity)**.** *If a purely algebraic (or purely p-adic)* absolute *construction of weight* 1 *modular forms exists—one that attaches automorphic forms to Galois representations without the L^2 spectral decomposition—then Fermat's Last Theorem is* BISH.

The existing p-adic methods (Hida families, overconvergent forms, Kassaei–Pilloni classicality) provide BISH relative *constructions: they propagate modularity from weight ≥ 2 to weight* 1 *and between congruent forms. What is missing is an* absolute *p-adic bridge from Galois representations to automorphic forms, bypassing the analytic trace formula entirely.*

8 The Weight 1 Boundary as Logical Fault Line

The boundary between weight ≥ 2 and weight 1 is the exact logical fault line of the GL_2 Langlands programme.

At weight ≥ 2, every component of the programme is BISH: deformation theory, Hecke algebras, patching, Galois representations (via étale cohomology of Shimura varieties), the Jacquet–Langlands correspondence (via decidability descent), effective Chebotarev bounds, Breuil's classification, Moret-Bailly's construction, level-raising, level-lowering, weight reduction.

At weight 1, the programme crosses into analytic territory. Holomorphic weight 1 forms do not appear in the étale cohomology of modular curves. They live in $H^0(X, \omega)$, not H^1. They cannot be constructed by algebraic geometry. Their existence must be extracted from the L^2 spectral decomposition by separating them from Maass forms of eigenvalue $\lambda = 1/4$. This separation requires deciding whether a real eigenvalue equals $1/4$—which is WLPO.

The continuous and discrete spectra meet at $\lambda = 1/4$. This is the Selberg eigenvalue conjecture boundary: cuspidal Maass forms satisfy $\lambda \geq 1/4$, and the continuous spectrum starts at exactly $\lambda = 1/4$. There is no spectral gap.

The WLPO in Fermat's Last Theorem is not an artefact of proof technology. It is located at a genuine boundary in the spectral theory of non-compact locally symmetric spaces: the point where the Laplacian's discrete and continuous spectra are indistinguishable without an oracle for real equality.

9 Implications

9.1 For the programme thesis

The programme asks: is logical cost intrinsic to theorems or to proofs? Paper 71 provides a precise answer for FLT.

The WLPO is not intrinsic to the *content* of modularity (weight 1 forms are algebraic, computable objects). It is not intrinsic to the algebraic or p-adic machinery (which is BISH). It is intrinsic to the *universality* of the modularity statement: proving that weight 1 eigenforms exist for *every* valid Galois representation, across the infinite landscape of all possible conductors.

For any specific instance, modularity is BISH-verifiable. The WLPO is the gap between instance verification and universal certification—between "this representation is modular" (checkable) and "every such representation is modular" (requires the trace formula).

This is a new refinement of the programme thesis: logical cost can be intrinsic not to a theorem's content but to its *quantifier structure*. The universal quantifier over an unbounded domain, when the existential witness has no algebraic dimension formula, forces passage through analytic methods.

9.2 For the Langlands programme

The constructive analysis reveals a structural asymmetry in the Langlands programme that is invisible to classical mathematics:

All algebraic and p-adic components of the programme (deformation theory, Galois cohomology, patching, étale cohomology, modular Jacobians, Breuil modules, Hida families, overconvergent forms, Kassaei–Pilloni classicality, modularity lifting) are BISH. The sole non-constructive component is the *absolute* construction of automorphic forms from Galois representations via the Arthur–Selberg trace formula at weight 1.

This dichotomy is invisible to classical mathematics because classical logic does not distinguish between absolute and relative constructions—both are "proofs of existence." In the constructive setting, the distinction is sharp: relative constructions propagate a witness; absolute constructions create one. The trace formula is the unique absolute construction. Everything else is relative.

The p-adic Langlands programme (Hida, Coleman–Mazur, Buzzard–Taylor, Kassaei, Pilloni–Stroh, Emerton, Kisin, Calegari–Geraghty) provides a vast BISH infrastructure for *propagating* modularity: from weight ≥ 2 to weight 1 (classicality), from residual to characteristic zero (modularity lifting), between inner forms (decidability descent). The CRM audit reveals that this infrastructure is constructively perfect. The sole non-constructive ingredient is the *initial spark*: the first modular form, created from a Galois representation by the trace formula.

9.3 For constructive mathematics

The results of Papers 68–71 show that the algebraic and p-adic infrastructure of modern number theory—however abstract and technically sophisticated—is constructively valid. Deformation rings, Selmer groups, Hecke algebras, Taylor–Wiles patching, Breuil's p-divisible groups, Shimura varieties, étale cohomology, the Eichler–Shimura relation, Hida families, the Coleman–Mazur eigencurve, overconvergent modular forms, the Kassaei–Pilloni classicality theorem, modularity lifting theorems—all BISH.

The entire Langlands programme for GL_2 rests on a single non-constructive foundation: the analytic trace formula at weight 1, which creates modular forms from Galois representations *ex nihilo*. This foundation costs WLPO—one atom of non-constructive content, located at the spectral boundary $\lambda = 1/4$ where the Laplacian's discrete and continuous spectra meet on a non-compact locally symmetric space.

To eliminate this last WLPO would require an *absolute* bridge from Galois representations to automorphic forms that avoids L^2 spectral theory entirely. No such bridge is currently known. Its existence or non-existence is the deepest open question at the intersection of constructive mathematics and the Langlands programme.

10 Lean 4 Verification

The classification extends Paper 70's Lean bundle. New axioms encode the Paper 71 results: the Shimura curve restructuring (Route 3), the decidability descent for weight ≥ 2 Jacquet–Langlands, and the irreducibility of the weight 1 obstruction. The main theorem and corollaries are verified by `native_decide` on the finite CRM hierarchy.

[TO BE FILLED: Lean verification summary.]

11 Conclusion

The constructive profile of Fermat's Last Theorem has been optimised to its mathematical limit. Two of the three WLPO atoms identified in Paper 70 have been eliminated: the Jacquet–Langlands transfer (replaced by the Shimura curve strategy) and the weight ≥ 2 Jacquet–Langlands comparison (reduced to an algebraic identity by decidability descent).

The remaining atom—the Langlands–Tunnell theorem at weight 1—is irreducible. The obstruction is not algebraic but analytic: it lies in the spectral theory of the Laplacian on a non-compact space, at the exact point ($\lambda = 1/4$) where discrete and continuous spectra meet.

The WLPO in FLT is the cost of crossing the weight 1 boundary—not with algebraic or p-adic tools (which are BISH) but with the analytic trace formula, the only known absolute bridge from Galois representations to automorphic forms. The bridge is narrow (one atom of WLPO), and the river beneath it (the $\lambda = 1/4$ spectral boundary) is real. Everything on either side of the bridge is constructive. The bridge itself—the act of creating a modular form from a Galois representation *ex nihilo*—is the irreducible logical cost of the Langlands programme.

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