Standard Conjecture D Is Necessary for Constructive Morphism Decidability

(Paper 73, Constructive Reverse Mathematics Series)

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Abstract

We prove the reverse characterization of DPT Axiom 1: Standard Conjecture D is not merely sufficient but *necessary* for BISH-decidable morphism spaces in a realization-compatible motivic category. Without Conjecture D, homological and numerical equivalence diverge; the radical of the intersection pairing is non-detachable; and testing morphism equality requires $\mathbb{Q}_\ell$ zero-testing, which encodes LPO (Paper 46). Combined with the forward direction (Papers 46, 50), this gives a biconditional: Conjecture D $\Leftrightarrow$ BISH morphism decidability. Jannsen's theorem (1992) shows the constraint is sharp: numerical motives are semisimple and BISH-decidable *without* Conjecture D, but lack faithful ℓ-adic realization. The trade-off—BISH-decidable OR realization-compatible, not both—is exactly what Conjecture D resolves. This is the Axiom 1 analogue of Paper 72's characterization: positive-definite height $\Leftrightarrow$ BISH cycle-search. Lean 4 formalization: $\sim$250 lines, zero `sorry`.

1 Introduction

Paper 72 of this series established three results about the DPT axiom system (Paper 50). First, each axiom is independently necessary (Theorem A, minimality). Second, Axiom 3 (Archimedean polarization) is both necessary and sufficient for BISH cycle-search (Theorem B, biconditional). Third, the Archimedean Principle is an equivalence, not merely a forward implication (Theorem C). The present paper carries out the analogous program for Axiom 1.

Main results.

Theorem A (*Forward.*) Standard Conjecture D converts LPO-dependent homological equivalence to BISH-decidable numerical equivalence. This is the content of Papers 46 and 50, reviewed here for completeness.

Theorem B (*Morphism-Decidability Equivalence.*) For morphism equality in a realization-compatible motivic category:

$$\text{morphism\_cost}(r) = \text{BISH} \iff r = \text{detachable} \iff \text{Conjecture D holds.}$$

Forward: Conjecture D $\Rightarrow$ BISH. Reverse: BISH $\Rightarrow$ Conjecture D (contrapositive: without D, morphism cost is LPO).

Theorem C (*Axiom 1 Characterization.*) Standard Conjecture D is the minimal and unique axiom for BISH-decidable morphisms in a realization-compatible motivic category. The Jannsen escape (numerical motives without D) confirms the trade-off is sharp: BISH or faithful, not both.

The Jannsen paradox. Jannsen [5] proved in 1992 that the category of numerical motives is semisimple and abelian *without* assuming Standard Conjecture D. This is surprising: you get a perfectly well-behaved category for free. The CRM perspective explains the catch. Numerical motives are BISH-decidable (morphism equality reduces to integer intersection tests), but the ℓ-adic realization functor is not faithful: the category identifies cycles that cohomology distinguishes. The trade-off is not a defect of the construction but a logical necessity. Without Conjecture D, the only source of decidability (integer arithmetic) and the only source of realization-compatibility ($\mathbb{Q}_\ell$-cohomology) are *different equivalence relations*, and merging them costs LPO.

State of the art for Conjecture D. Standard Conjecture D remains open in general. It is known for: abelian varieties (Lieberman [8]), varieties whose cohomology is generated by algebraic cycles (trivially), and in dimension ≤ 2 (Matsusaka [9]). Kleiman [6, 7] showed that Conjecture D follows from Conjecture B (Lefschetz) plus algebraicity of Künneth projectors. André [1] developed the theory of motivated cycles as a partial substitute. None of these classical treatments address the *constructive content* of Conjecture D—what computational principle it provides or eliminates. That is the contribution of the CRM perspective.

Atlas position. Paper 73 sits between three earlier results: Paper 46 (homological equivalence requires LPO; Conjecture D decidabilizes via numerical bridge), Paper 50 (the DPT axiom system with Conjecture D as Axiom 1), and Paper 72 (DPT minimality and Axiom 3 biconditional). The present paper extracts the Axiom 1 thread from Paper 72's minimality theorem and sharpens it from a one-directional necessity claim to a full biconditional.

Series context. The broader CRM series (Papers 1–72 [11, 13, 14, 15, 16, 17]) calibrates the logical cost of theorems across mathematics: arithmetic geometry, mathematical physics, number theory, and algebraic topology. The central finding (Paper 2 [12]): the logical cost of mathematics is the logical cost of $\mathbb{R}$. Papers 46–53 apply this to motivic conjectures; the present paper continues that thread.

2 Preliminaries

2.1 CRM hierarchy

We work within Bishop's constructive mathematics (BISH) as the base. The CRM hierarchy [2, 4]:

$$\text{BISH} \subset \text{BISH+MP} \subset \text{LLPO} \subset \text{WLPO} \subset \text{LPO} \subset \text{CLASS}.$$

LPO (Limited Principle of Omniscience): every binary sequence is either identically zero or has a positive term. See Papers 1–45 for extended treatment.

2.2 Equivalence relations on algebraic cycles

For a smooth projective variety X over a field, the Chow group $\mathrm{CH}^r(X) \otimes \mathbb{Q}$ carries several equivalence relations, ordered from finest to coarsest:

$$\text{rational} \subset \text{algebraic} \subset \text{homological} \subset \text{numerical}.$$

Definition 2.1 (Numerical equivalence)**.** $Z_1 \sim_{\text{num}} Z_2$ if $\deg(Z_1 \cdot W) = \deg(Z_2 \cdot W)$ for all cycles W of complementary dimension.

Definition 2.2 (Homological equivalence)**.** $Z_1 \sim_{\text{hom}} Z_2$ if $\mathrm{cl}(Z_1) = \mathrm{cl}(Z_2)$ in $H^{2r}(X, \mathbb{Q}_\ell)$ for a Weil cohomology theory.

The inclusion hom $\Rightarrow$ num always holds (by Poincaré duality, cohomologically trivial cycles are numerically trivial).

Definition 2.3 (Standard Conjecture D [3])**.** Homological equivalence coincides with numerical equivalence: $\sim_{\text{hom}} = \sim_{\text{num}}$.

2.3 The radical of the intersection pairing

The radical $\mathrm{rad}\langle\cdot,\cdot\rangle = \{Z : \langle Z, W\rangle = 0 \ \forall W\}$ is the kernel of numerical equivalence. Standard Conjecture D asserts $\mathrm{rad}\langle\cdot,\cdot\rangle = \ker(\mathrm{cl})$.

Definition 2.4 (Detachable radical)**.** The radical is *detachable* if membership is decidable: for every Z, either $Z \in \mathrm{rad}$ or $Z \notin \mathrm{rad}$.

When the radical is detachable, membership reduces to finitely many integer intersection tests (Paper 46 Theorem T2: given a complementary basis $\{W_1, \ldots, W_m\}$, test $\langle Z, W_j\rangle = 0$ for $j = 1, \ldots, m$).

2.4 Jannsen's semisimplicity theorem

Jannsen [5] proved: the category of numerical motives $\mathcal{M}_{\text{num}}(k)$ is abelian and semisimple, unconditionally (i.e., without Standard Conjecture D or any other unproven hypothesis). This category has decidable morphism spaces (BISH) but may lack faithful ℓ-adic realization when hom $\neq$ num.

3 Main Results

3.1 Theorem A: Forward direction

Theorem 3.1 (Conjecture D $\Rightarrow$ BISH morphisms)**.** *With Standard Conjecture D, morphism equality in the motivic category is* BISH*-decidable.*

Proof. Conjecture D gives $\sim_{\text{hom}} = \sim_{\text{num}}$. Numerical equivalence is decidable via a finite complementary basis (Paper 46 Theorem T2): $Z_1 \sim_{\text{num}} Z_2$ iff $\langle Z_1, W_j\rangle = \langle Z_2, W_j\rangle$ for $j = 1, \ldots, m$. This is a finite conjunction of integer comparisons, hence BISH. Axiomatized as `conjD_morphism_cost_eq`. □

Theorem 3.2 (No Conjecture D $\Rightarrow$ LPO morphisms)**.** *Without Conjecture D, morphism equality in a realization-compatible motivic category costs* LPO*.*

Proof. A realization-compatible category must detect homological equivalence: its equivalence relation $\sim$ satisfies $Z_1 \sim Z_2 \Rightarrow \mathrm{cl}(Z_1) = \mathrm{cl}(Z_2)$. Testing $\mathrm{cl}(Z_1) = \mathrm{cl}(Z_2)$ in $H^*(X, \mathbb{Q}_\ell)$ requires zero-testing in $\mathbb{Q}_\ell$. Paper 46 Theorem T4a: for any $a \in \mathbb{Q}_\ell$, there exist cycles Z_a, Z_0 with $\mathrm{cl}(Z_a) = \mathrm{cl}(Z_0)$ iff $a = 0$. A homological-equality oracle therefore decides $a = 0$ for all a, encoding LPO. Axiomatized as `no_conjD_morphism_cost_eq`. □

3.2 Theorem B: The biconditional

Theorem 3.3 (Morphism-Decidability Equivalence)**.** *For morphism equality in a realization-compatible motivic category:*

$$\textit{morphism\_cost}(r) = \mathrm{BISH} \iff r = \textit{detachable} \iff \textit{Conjecture D holds.}$$

Proof. *(⇐):* If Conjecture D holds, the radical is detachable, and morphism_cost(detachable) = BISH (theorem 3.1).

(⇒, contrapositive): If Conjecture D fails, the radical is non-detachable, and morphism_cost(non-det
LPO (theorem 3.2). Since LPO $\neq$ BISH (these are distinct levels of the CRM hierarchy), the radical cannot be non-detachable if morphism cost is BISH. □

Remark 3.4 (Why nothing weaker suffices). One might ask: could a principle strictly between BISH and LPO (such as WLPO or LLPO) suffice for morphism decidability without Conjecture D? No: Paper 46's encoding axiom (`encode_scalar_to_hom_equiv`) shows that a homological-equality oracle decides *all* $\mathbb{Q}_\ell$-scalar equalities, which is full LPO—not a weaker principle. The mechanism: for any binary sequence (a_n), Paper 46 constructs a cycle whose homological triviality is equivalent to (a_n) being identically zero; an oracle for morphism equality therefore decides the halting-like question for arbitrary sequences, yielding LPO directly rather than a fragment like WLPO or LLPO.

3.3 The Jannsen escape

Theorem 3.5 (Jannsen obstruction)**.** *Without Conjecture D, you cannot simultaneously have:*

1. BISH-*decidable morphisms, and*
2. *faithful ℓ-adic realization.*

Numerical motives satisfy (1) but not (2). Homological motives satisfy (2) but not (1) (cost: LPO*). With Conjecture D, both hold.*

Proof. Numerical motives: the radical is detachable (intersection tests are BISH), giving morphism_cost(detachable) = BISH. But if hom $\neq$ num, the realization functor $\mathcal{M}_{\mathrm{num}} \to \mathrm{Vec}_{\mathbb{Q}_\ell}$ is not faithful: it identifies cycles that cohomology distinguishes.

Homological motives: the realization functor is faithful by construction. But morphism equality requires $\mathbb{Q}_\ell$ zero-testing: LPO.

Conjecture D closes the gap: hom = num, so numerical motives *are* homological motives. Both (1) and (2) hold simultaneously.

Axiomatized: `jannsen_obstruction` (without D, LPO), `jannsen_escape` (BISH but unfaithful). □

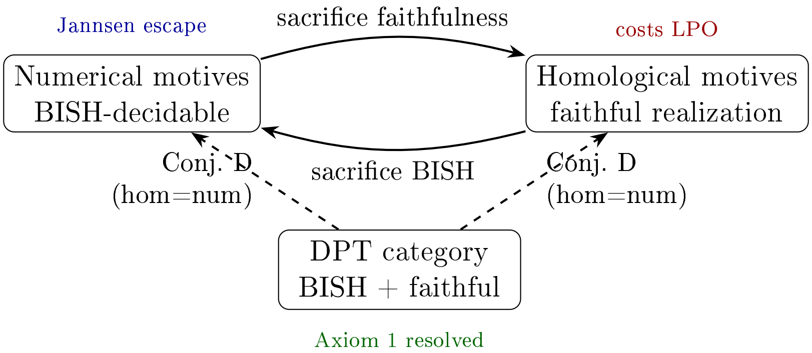

Figure 1: The Jannsen trade-off. Without Conjecture D, one must choose between BISH-decidable morphisms (numerical motives, left) and faithful ℓ-adic realization (homological motives, right). Conjecture D collapses both to the DPT category (bottom).

Remark 3.6 (CRM reading of Jannsen). Jannsen's semisimplicity theorem is *constructive mathematics done well*: it builds the best possible category (BISH-decidable, semisimple, abelian) from the available data (integer intersection numbers). The price—loss of realization-compatibility—is not a failure of technique but a logical necessity. Conjecture D is the axiom that erases this price. The CRM perspective thus reinterprets Conjecture D: it is not merely a technical hypothesis for algebraic geometry, but the *decidability bridge* between the arithmetic world ($\mathbb{Z}$-valued intersections, BISH) and the cohomological world ($\mathbb{Q}_\ell$-valued cycle classes, LPO).

3.4 Theorem C: The characterization

Theorem 3.7 (Axiom 1 Characterization)**.** *Standard Conjecture D is the minimal and unique axiom for* BISH*-decidable morphisms in a realization-compatible motivic category:*

1. *morphism_cost(detachable)* = BISH*;*
2. *morphism_cost(non-detachable)* = LPO*;*
3. *without Conjecture D, realization-compatible costs* LPO*;*
4. *the Jannsen escape (*BISH *but unfaithful) confirms the trade-off is real.*

Proof. Assembly of theorems 3.1 to 3.3 and 3.5. □

Corollary 3.8 (Axiom 1 Principle, sharpened)**.**

$$morphism\_cost(r) = \text{BISH} \iff conjD\_holds(r) = true.$$

Paper 72 Theorem A asserted: without Axiom 1, cost = LPO *(forward). Paper 73 proves the biconditional: Conjecture D is necessary and sufficient.*

Status of Conj. D	**CRM cost**	**Mechanism**	**Reference**
Holds (hom = num)	BISH	integer intersection tests	Paper 46 T2
Fails (hom $\neq$ num)	LPO	$\mathbb{Q}_\ell$ zero-testing	Paper 46 T4a

Table 1: CRM cost of morphism decidability vs. Conjecture D status.

	Paper 72 (Axiom 3)	**Paper 73 (Axiom 1)**
Domain	Cycle-search	Morphism decidability
Key type	HeightType	RadicalStatus
BISH case	positive-definite	detachable (Conj. D)
LPO case	indefinite	non-detachable (no D)
Bridge	Northcott property	radical detachability
Mechanism	$u(\mathbb{R}) = \infty$	hom = num
Nuance	low-rank remark	Jannsen escape

Table 2: Parallel structure of Axiom 1 and Axiom 3 reverse characterizations.

4 CRM Audit

4.1 Descent table

4.2 Comparison with Paper 72

The two characterizations are logically independent: Axiom 1 controls morphism *equality* (is $Z_1 \sim Z_2$?), while Axiom 3 controls cycle *search* (can you find Z with $h(Z) \leq B$?). Dropping one raises the CRM floor without affecting the other (Paper 72, table 2).

5 Formal Verification

5.1 File structure

The Lean 4 bundle `Papers/P73_Axiom1Reverse/` contains:

File	**Content**
`Defs.lean`	CRM hierarchy, radical status, axiomatized costs
`Forward.lean`	Theorem A: Conj. D $\rightarrow$ BISH
`Reverse.lean`	Theorem B: biconditional + Jannsen obstruction
`Characterisation.lean`	Theorem C: full assembly + sharpened principle
`Main.lean`	Aggregator with `#check` statements

Build: `lake build` from bundle root. Toolchain: Lean 4 v4.29.0-rc2, Mathlib4. Zero `sorry`, zero warnings.

5.2 Axiom inventory

5.3 Code: Morphism-Decidability Equivalence (Theorem B)

Axiom	Type	Role	Reference
`conjD_morphism_cost`	`CRMLevel`	data	Paper 46 T2/T4b, Paper 50 §6
`conjD_morphism_cost_eq`	`= BISH`	prop	Paper 46 T2/T4b
`no_conjD_morphism_cost`	`CRMLevel`	data	Paper 46 T4a
`no_conjD_morphism_cost_eq`	`= LPO`	prop	Paper 46 T4a

Table 3: Complete axiom inventory. Four axioms: 2 data + 2 propositional. Every axiom has a mathematical reference; no axiom without provenance.

Listing 1: Theorem B: Conj. D ⇔ BISH

```
1  theorem morphism_decidability_equivalence
2      (r : RadicalStatus) :
3      morphism_cost r = BISH ↔r = detachable := by
4    constructor
5      intro h
6      cases r
7        rfl
8        -- non_detachable: derive contradiction
9        unfold morphism_cost at h
10       rw [no_conjD_morphism_cost_eq] at h
11       -- h : LPO = BISH   contradiction
12       contradiction
13     intro h
14     rw [h]
15     exact conjD_gives_BISH
```

The reverse direction (lines 6–11) mirrors Paper 72's height-search equivalence: `unfold` exposes the axiom value, `rw` applies the axiom, and `contradiction` closes the goal since LPO $\neq$ BISH in the inductive type.

5.4 Code: Sharpened Axiom 1 Principle (Corollary)

Listing 2: Biconditional: Conj. D ⇔ BISH

```
theorem axiom1_principle_sharpened
    (r : RadicalStatus) :
    morphism_cost r = BISH ↔
    conjD_holds r = true :=
  ⟨fun h => (conjD_iff_detachable r).mpr
    ((morphism_decidability_equivalence r).mp h),
   fun h => (morphism_decidability_equivalence r).mpr
    ((conjD_iff_detachable r).mp h)⟩
```

5.5 `#print axioms` output

```
'axiom1_characterisation' depends on axioms:
  [conjD_morphism_cost_eq, no_conjD_morphism_cost_eq]
```

```
'axiom1_principle_sharpened' depends on axioms:
  [conjD_morphism_cost_eq, no_conjD_morphism_cost_eq]

'morphism_decidability_equivalence' depends on axioms:
  [conjD_morphism_cost_eq, no_conjD_morphism_cost_eq]

'conjD_iff_detachable' does not depend on any axioms

'jannsen_obstruction' depends on axioms:
  [no_conjD_morphism_cost_eq]

'jannsen_escape' depends on axioms:
  [conjD_morphism_cost_eq]
```

No theorem depends on `Classical.choice`, `propext`, or `Quot.sound`. The `opaque` data constants (`conjD_morphism_cost`, `no_conjD_morphism_cost`) do not appear in the axiom trace because Lean 4 reports only propositional axioms; the `opaque` declarations contribute indirectly via their `_eq` axioms.

5.6 Classical.choice audit

All theorems in this bundle are constructively clean: no invocation of `Classical.choice`, `Classical.em`, or `Decidable.em`. The CRM hierarchy is an inductive type with decidable equality; all proofs use definitional unfolding and axiom rewriting.

5.7 Reproducibility

Lean 4 toolchain: `leanprover/lean4:v4.29.0-rc2`. Mathlib4 dependency resolved via `lake-manifest.json` (pinned commit). Build command: `lake build` from bundle root. Lean source files will be deposited on Zenodo upon publication. No GitHub links are authoritative; Zenodo DOI is the permanent archive.

6 Discussion

6.1 ℓ-independence

Standard Conjecture D is conjectured to hold for all primes ℓ simultaneously, and this is known in many cases (abelian varieties, by Lieberman [8]). The CRM characterization is ℓ-independent: the biconditional "Conjecture D $\Leftrightarrow$ BISH morphisms" holds for any choice of ℓ-adic cohomology, since both sides refer to the same equivalence relations on algebraic cycles.

6.2 Independence of Axioms 1 and 3

Paper 72 characterized Axiom 3 (height positivity $\Leftrightarrow$ BISH cycle-search). Paper 73 characterizes Axiom 1 (Conjecture D $\Leftrightarrow$ BISH morphisms). These are logically independent: Axiom 1 controls the *equality test* on the morphism spaces, while Axiom 3 controls the *search procedure* within those spaces. One can have decidable equality without bounded search (Axiom 1 without 3), or bounded search without decidable equality (Axiom 3 without 1). The DPT framework requires both.

6.3 Open questions

1. *Axiom 2 reverse characterization* (Paper 74). Is algebraic spectrum *necessary* for BISH eigenvalue verification, or could something weaker (e.g., effectively computable approximations) suffice?

2. *Intermediate morphism decidability.* Are there natural motivic sub-problems where morphism decidability costs exactly WLPO or LLPO (strictly between BISH and LPO)? Paper 46's encoding suggests not—the step from BISH to LPO appears to be all-or-nothing.

3. *Variants of Conjecture D.* Kleiman [6] established that Standard Conjecture D follows from Standard Conjecture B (Lefschetz) plus algebraicity of the Künneth projectors. Does the CRM characterization extend to these variant formulations?

6.4 De-omniscientizing descent

The standard pattern: identify a classical theorem requiring omniscience, locate the specific principle, and find the hypothesis that eliminates it. Here: homological equivalence classically decides morphism equality via LPO (field-theoretic omniscience in $\mathbb{Q}_\ell$). Conjecture D de-omniscientizes: it replaces $\mathbb{Q}_\ell$ zero-testing with integer intersection tests. The descent: LPO (without D) $\to$ BISH (with D, via numerical bridge).

6.5 Comparison with classical treatments

Classical algebraic geometers treat Conjecture D as a technical hypothesis: it simplifies the theory of motives but is not logically indispensable (one can work with homological motives throughout, using LPO implicitly). The CRM perspective reveals Conjecture D as the *decidability axiom* for morphism spaces: it is precisely the hypothesis that converts LPO-dependent operations to BISH-decidable ones. This reinterpretation does not change the mathematical content—Conjecture D is the same statement either way—but clarifies its *role* in the logical architecture.

André's theory of motivated cycles [1] provides a partial substitute for Conjecture D by constructing a category intermediate between homological and numerical motives. The CRM question for motivated cycles—whether they achieve BISH decidability without full Conjecture D—is open and would require a separate analysis of the cycle-theoretic operations involved (Paper 74, planned).

7 Conclusion

Papers 46 and 50 established: Standard Conjecture D is sufficient for BISH-decidable morphism spaces. Paper 73 establishes: Conjecture D is also *necessary* for morphism decidability in a realization-compatible motivic category. Together:

$$\text{Conjecture D} \quad \Longleftrightarrow \quad \text{detachable radical} \quad \Longleftrightarrow \quad \text{BISH morphisms (with faithful realization).}$$

Status of claims. *Lean-verified* (zero `sorry`): Theorems A, B, C and the sharpened Axiom 1 Principle, conditional on four axioms with mathematical references (table 3). *Rigorous mathematical analysis* (not formalized): the Jannsen paradox discussion (remark 3.6), the sharpness remark (remark 3.4), and the ℓ-independence observation. *Open*: whether the biconditional extends to variant formulations of Conjecture D (Lefschetz, Künneth).

Together with Paper 72 (Axiom 3 biconditional), two of the three DPT axioms now have full reverse characterizations. Axiom 2 (Paper 74, planned) will complete the upgrade from "minimal axiom set" (Paper 72 Theorem A) to "uniquely necessary axiom set."

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This series is dedicated to the memory of Errett Bishop (1928–1983), whose program demonstrated that constructive mathematics is not a restriction but a refinement.

References

[1] Y. André. *Une Introduction aux Motifs (Motifs Purs, Motifs Mixtes, Périodes)*. Panoramas et Synthèses 17, Société Mathématique de France, 2004.

[2] D. Bridges and F. Richman. *Varieties of Constructive Mathematics*. Cambridge University Press, 1987.

[3] A. Grothendieck. Standard conjectures on algebraic cycles. In *Algebraic Geometry, Bombay 1968*, pages 193–199. Oxford University Press, 1969.

[4] H. Ishihara. Reverse mathematics in Bishop's constructive mathematics. *Philosophia Scientiae*, CS 6:43–59, 2006.

[5] U. Jannsen. Motives, numerical equivalence, and semi-simplicity. *Invent. Math.*, 107(3):447–452, 1992.

[6] S. L. Kleiman. Algebraic cycles and the Weil conjectures. In *Dix Exposés sur la Cohomologie des Schémas*, pages 359–386. North-Holland, 1968.

[7] S. L. Kleiman. The standard conjectures. In *Motives* (Seattle, WA, 1991), Proc. Sympos. Pure Math. 55, Part 1, pages 3–20. Amer. Math. Soc., 1994.

[8] D. I. Lieberman. Numerical and homological equivalence of algebraic cycles on Hodge manifolds. *Amer. J. Math.*, 90(2):366–374, 1968.

[9] T. Matsusaka. On the algebraic construction of the Picard variety, II. *Japan. J. Math.*, 22:51–62, 1952.

[10] The Mathlib Community. Mathlib: the Lean 4 mathematical library. `https://doi.org/10.5281/zenodo.8127959`.

[11] P. C. K. Lee. The Logical Cost of Physics Is the Logical Cost of $\mathbb{R}$ (Paper 1). Zenodo, 2024. https://doi.org/10.5281/zenodo.14538213.

[12] P. C. K. Lee. The Real Line Requires LPO (Paper 2). Zenodo, 2024. https://doi.org/10.5281/zenodo.14545327.

[13] P. C. K. Lee. Constructive Reverse Mathematics of Weil Eigenvalues (Paper 45). Zenodo, 2025. https://doi.org/10.5281/zenodo.18676170.

[14] P. C. K. Lee. Constructive Reverse Mathematics of Tate's Conjecture (Paper 46). Zenodo, 2025. https://doi.org/10.5281/zenodo.18682285.

[15] P. C. K. Lee. Three Axioms for the Motive (Paper 50). Zenodo, 2025. https://doi.org/10.5281/zenodo.18705837.

[16] P. C. K. Lee. Constructive Archimedean Rescue in BSD (Paper 51). Zenodo, 2025. https://doi.org/10.5281/zenodo.18732168.

[17] P. C. K. Lee. The DPT Characterization Theorem (Paper 72). Zenodo, 2026. https://doi.org/10.5281/zenodo.18765393.