Constructive Reverse Mathematics Series

Series Guide: Papers 1–71 with Abstracts

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Abstract

This booklet collects the titles and abstracts of all 69 active papers (of 71 numbered) in the Constructive Reverse Mathematics (CRM) series. The programme calibrates the logical strength of theorems across mathematical physics and arithmetic geometry against the constructive hierarchy: BISH $\subset$ BISH+LLPO $\subset$ BISH+WLPO $\subset$ BISH+LPO $\subset$ CLASS, with independent axes for Markov's Principle (MP), the Fan Theorem (FT), and Dependent Choice (DC). The main finding is that all empirically accessible predictions in known physics require exactly BISH+LPO. Papers 45–68 extend this to arithmetic geometry and number theory, establishing the Decidable Polarized Tannakian (DPT) framework, the three-invariant hierarchy (rank, Hodge level, Lang constant), and the CRM audit of Wiles's proof of Fermat's Last Theorem. Approximately 87,000 lines of Lean 4 code formalize the results.

Contents

Programme Overview

What is Constructive Reverse Mathematics?

Constructive Reverse Mathematics (CRM) calibrates the logical strength of mathematical theorems against a hierarchy of constructive principles. Rather than asking "is this theorem true?", CRM asks "what non-constructive resources does this theorem *require*?" The answer is a position in the hierarchy

$$\text{BISH} \subset \text{BISH+LLPO} \subset \text{BISH+WLPO} \subset \text{BISH+LPO} \subset \text{CLASS},$$

with independent axes for Markov's Principle (MP), the Fan Theorem (FT), and Dependent Choice (DC).

- **BISH** (Bishop's constructive mathematics): the base system, requiring explicit witnesses for all existence claims.
- **LLPO** (Lesser Limited Principle of Omniscience): decides disjunctions over binary sequences.
- **WLPO** (Weak Limited Principle of Omniscience): decides whether a binary sequence is identically zero.
- **LPO** (Limited Principle of Omniscience): decides whether a binary sequence contains a 1.
- **MP** (Markov's Principle): if a computation cannot fail to halt, then it halts. Orthogonal to the omniscience spine.
- **FT** (Fan Theorem): every bar on Cantor space is uniform. Independent of the omniscience spine.
- **DC** (Dependent Choice): sequential choice along a total relation. Independent of omniscience and FT.

The programme in three phases

Phase I: Mathematical Physics (Papers 1–44). Systematic calibration of theorems across functional analysis, quantum mechanics, general relativity, statistical mechanics, QFT, quantum information, classical mechanics, cosmology, and quantum foundations. The main finding: *all empirically accessible predictions in known physics require exactly* BISH+LPO. The Fan Theorem and Dependent Choice are physically dispensable (Papers 30–31). The LPO cost is genuine and instantiated by Fekete's Subadditive Lemma (Paper 29).

Phase II: Arithmetic Geometry (Papers 45–66). Extension to the Hodge, Tate, BSD, Fontaine–Mazur, and Weight-Monodromy conjectures. Each conjecture exhibits a *de-omniscientizing descent*: geometric origin converts LPO-dependent claims to BISH-decidable ones. The Decidable Polarized Tannakian (DPT) framework (Paper 50) axiomatizes this pattern. Three invariants—analytic rank, Hodge level, Lang constant—classify the logical cost of cycle-search for any motive.

Phase III: Synthesis (Papers 67–71). The arithmetic geometry monograph (Paper 67), the CRM audit of Fermat's Last Theorem (Paper 68, result: BISH), the function field Langlands audit (Paper 69, result: BISH), the capstone Archimedean Principle (Paper 70), and engineering applications to lattice cryptography (Paper 71). The capstone finding: *the real numbers are the sole source of logical difficulty in both mathematical physics and arithmetic geometry*, via the mechanism $u(\mathbb{R}) = \infty$.

The Logical Hierarchy

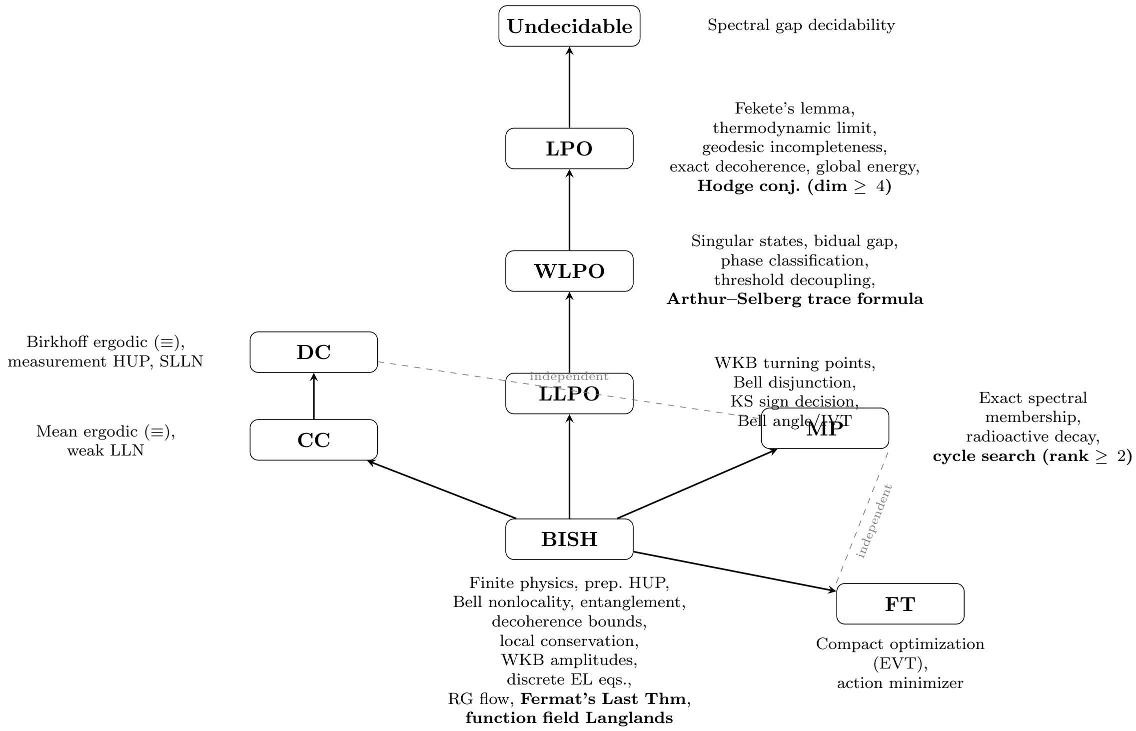

Figure 1: The logical geography of the CRM programme (Papers 1–71). Arrows indicate strict implication over BISH. The omniscience spine (BISH < LLPO < WLPO < LPO) is the dominant vertical chain. DC and MP occupy orthogonal positions. FT is a third independent branch. **Bold entries** are from the arithmetic geometry and synthesis phases.

Master Calibration Table

The table below collects calibrated entries from 71 papers spanning mathematical physics, arithmetic geometry, and the Langlands programme.

Theorem / Result	**Level**	**Source**
BISH (fully constructive):		
Finite-volume physics, finite-size bounds	BISH	Papers 8, 9
Preparation uncertainty (HUP)	BISH	Paper 6
Bell nonlocality (CHSH/Tsirelson)	BISH	Paper 11
Entanglement entropy	BISH	Paper 11
Schwarzschild interior (finite-time)	BISH	Paper 13
Decoherence bounds (finite-step)	BISH	Paper 14
Local conservation (Noether)	BISH	Paper 15
Born probability (finite-dim)	BISH	Paper 16
BH entropy (algebraic correction)	BISH	Paper 17
WKB amplitude (specific barriers)	BISH	Paper 19
Discrete Euler–Lagrange equations	BISH	Paper 28
Yukawa RG one-loop flow	BISH	Paper 18
QED Landau pole, Ward–Takahashi	BISH	Paper 32
Fixed-order scattering (Bhabha)	BISH	Paper 34
Hodge polarization degeneration	BISH	Paper 45
Algebraic cycle verification	BISH	Paper 49
CM elliptic motives (decidable)	BISH	Paper 53
Fermat's Last Theorem	BISH	Paper 68
Function field Langlands (GL_n, gen. G)	BISH	Paper 69
Lattice cryptography (LLL, exp. approx.)	BISH	Paper 71
Choice hierarchy (CC < DC):		
Mean ergodic theorem	$\equiv$ CC	Paper 25
Birkhoff's ergodic theorem	$\equiv$ DC	Paper 25
Frequentist convergence (SLLN)	$\leq$ DC	Papers 16, 25
LLPO:		
WKB turning-point decision (IVT)	$\equiv$ LLPO	Paper 19
Bell disjunctive conclusion	$\equiv$ LLPO	Paper 21
KS sign decision (contextuality)	$\equiv$ LLPO	Paper 24
Bell angle optimisation (IVT)	$\equiv$ LLPO	Paper 27
MP (Markov's Principle):		
Exact spectral membership	MP	Paper 4
Radioactive decay (eventual detection)	$\equiv$ MP	Paper 22
Cycle search, rank ≥ 2 (without Lang)	MP	Paper 61
SVP polynomial approx. (BKZ)	BISH+MP	Paper 71
WLPO:		

Theorem / Result	Level	Source
Bidual-gap / singular states	$\equiv$ WLPO	Papers 2, 7
Phase classification (magnetization)	$\equiv$ WLPO	Paper 20
Step-function threshold decoupling	WLPO	Paper 18
Arthur–Selberg trace formula	WLPO	Paper 68
FT (Fan Theorem) — physically dispensable:		
Compact optimization (EVT on $[a, b]$)	$\equiv$ FT	Paper 23
Action minimizer existence (variational)	$\equiv$ FT	Paper 28
LPO:		
Thermodynamic limit existence	$\equiv$ LPO	Papers 8, 9, 29
Fekete's lemma (subadditive sequences)	$\equiv$ LPO	Paper 29
Geodesic incompleteness (completed limit)	$\equiv$ LPO	Paper 13
Exact decoherence (wave fn collapse)	$\equiv$ LPO	Paper 14
Global energy (infinite-volume)	$\equiv$ LPO	Paper 15
BH entropy density convergence	$\equiv$ LPO	Paper 17
Galois-invariance decidability ($\mathbb{Q}_\ell$)	$\equiv$ LPO	Paper 46
$L(E, 1) = 0$ decision	$\equiv$ LPO	Paper 48
Hodge type (r, r) decidability ($\mathbb{C}$)	$\equiv$ LPO	Paper 49
Undecidable:		
Spectral gap decidability	Σ^0_1-complete	Cubitt et al.
Generic spectral gap (no promise)	Σ^0_2-complete	Paper 39

The DPT Framework (Paper 50)

A *Decidable Polarized Tannakian* (DPT) category over $\mathbb{Q}$ is a $\mathbb{Q}$-linear abelian symmetric monoidal category $\mathcal{C}$ equipped with three axioms:

Axiom 1 (Decidable morphisms — Standard Conjecture D). For all $X, Y \in \mathcal{C}$, the morphism space $\mathrm{Hom}(X, Y)$ has decidable equality: $\forall f, g \colon X \to Y,\ f = g \vee f \neq g$.

Axiom 2 (Algebraic spectrum). Every endomorphism $f \in \mathrm{End}(X)$ satisfies a monic polynomial $p \in \mathbb{Z}[t]$, forcing eigenvalues into $\overline{\mathbb{Q}}$.

Axiom 3 (Archimedean polarization). A faithful functor to real vector spaces equipped with a positive-definite bilinear form: $\langle x, x \rangle > 0$ for all $x \neq 0$.

Five conjectures (Hodge, Tate, BSD, Fontaine–Mazur, Weight-Monodromy) exhibit a uniform *de-omniscientizing descent*: geometric origin converts LPO-dependent data to BISH-decidable numerical equivalence, mediated by the DPT axioms.

The Archimedean Principle (Paper 70)

The mechanism is $u(\mathbb{R}) = \infty$: the real numbers are the only completion of $\mathbb{Q}$ where positive-definite forms exist in every dimension. Three fields independently exploit this via the same

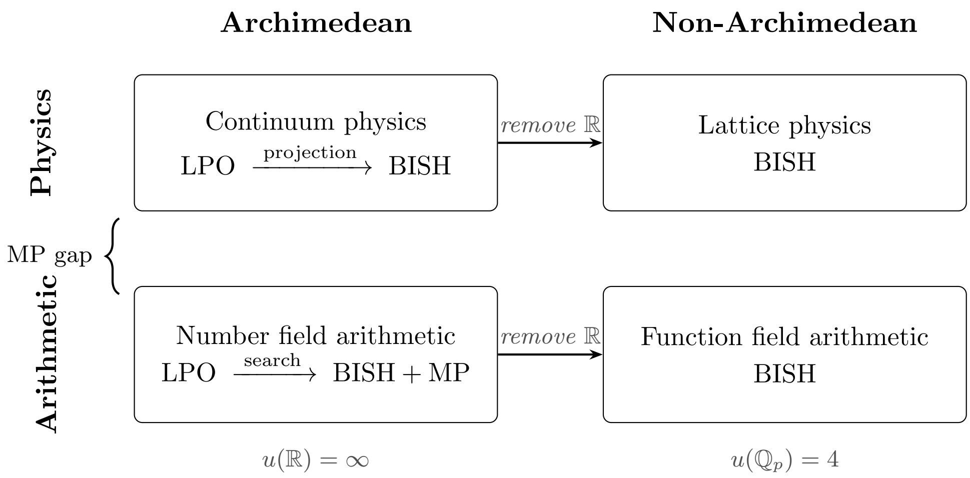

Figure 2: The four-domain parameterisation (Paper 70). The CRM level is determined by one parameter: presence of the Archimedean place. The descent type (projection vs. search) determines the residual.

architecture: *Hilbert space inner product* (physics), *Rosati involution* (arithmetic geometry), *Petersson inner product* (automorphic forms). Remove $\mathbb{R}$ and both physics and arithmetic collapse to BISH. The difference between them is how they descend: physics uses *projection* (eliminating MP), number theory uses *search* (preserving MP as Diophantine hardness).

Formalization Statistics

Metric	**Value**
Total papers (numbered)	71
Active papers	69
Withdrawn	2 (Papers 1, 3)
Retired (merged)	2 (Papers 60, 62)
Papers with Zenodo DOI	65
Papers with Lean 4 formalization	$\sim$61
Total Lean 4 lines	$\sim$88,000+
Papers with zero `sorry`	$\geq$22
Lean toolchain	v4.28.0-rc1
Mathlib dependency	v4.28.0-rc1
Domains covered	
Functional Analysis, Quantum Mechanics, General Relativity, Statistical Mechanics, QFT (QED/QCD), Quantum Information, Classical Mechanics, Foundations, Undecidability, AdS/CFT, Cosmology, Arithmetic Geometry, Motives, Number Theory, Langlands Programme, Cryptography	

Part I: Foundations (Papers 1–6)

Paper 1: Rank-One Toggle Kernel

Status: Withdrawn.
DOI: —

This paper has been withdrawn from the series.

Paper 2: The Bidual Gap and WLPO

DOI: 10.5281/zenodo.18501689

Abstract. We formalize the bidual gap of Banach spaces and prove it is equivalent to WLPO (the Weak Limited Principle of Omniscience). Gap detection requires exactly WLPO; the witness space is c_0 (sequences vanishing at infinity). The formalization establishes that Banach space non-reflexivity detection is a constructively calibrated problem with precise logical cost.

Paper 3: [Withdrawn]

Status: Withdrawn.
DOI: —

This paper has been withdrawn from the series.

Paper 4: Axiom Calibration for Quantum Spectra

DOI: 10.5281/zenodo.17118223

Abstract. Five spectral properties—from approximate eigenvalues to exact membership—are stratified from BISH to WLPO+MP. This provides a complete calibration of quantum spectral theory against the constructive hierarchy, showing that different spectral questions about the same operator carry different logical costs.

Paper 5: Schwarzschild Curvature Verification

DOI: 10.5281/zenodo.18489703

Abstract. We make Axiom Calibration the organizing principle for a foundations-first study of General Relativity. The paper contributes three AxCal instruments for GR: (I) witness families pinned to a fixed Σ_0^{GR} signature; (II) proof-route flags and portal theorems; and (III) HeightCertificates. We calibrate five loci: G1 (explicit vacuum checks, Height 0), G2 (Cauchy/MGHD), G3 (singularity theorems), G4 (maximal extensions), and G5 (computable evolution).

Paper 6: Heisenberg Uncertainty Principle (v2, CRM over Mathlib)

DOI: 10.5281/zenodo.18519836

Abstract. We formalize the Robertson–Schrödinger and Schrödinger uncertainty inequalities in Lean 4 using Mathlib's InnerProductSpace API, within the framework of Constructive Reverse Mathematics. Both preparation-uncertainty inequalities are proved at Height 0 (fully constructive) using Cauchy–Schwarz, centered-vector decomposition, and elementary complex-number

identities—all drawn from Mathlib with zero custom axioms. Measurement uncertainty, which requires constructing infinite measurement histories, is calibrated at DC_ω-height.

Part II: Physical Calibrations (Papers 7–28)

Paper 7: Physical Bidual Gap, Trace-Class Operators

DOI: 10.5281/zenodo.18527559

Abstract. We present a Lean 4 formalization of the Physical Bidual Gap Theorem, establishing Banach space non-reflexivity of the trace-class operators $\mathcal{S}^1$ on a separable Hilbert space and showing that any constructive witness of this non-reflexivity implies WLPO. In the algebraic formulation of quantum mechanics, $\mathcal{S}^1$ is the natural state space; our result shows that the gap between physical density matrices and the "generalized states" in $(\mathcal{S}^1)^{**}$ is constructively inaccessible. The formalization comprises 754 lines of Lean 4 code across 8 files.

Paper 8: 1D Ising Model and LPO

DOI: 10.5281/zenodo.18516813

Abstract. We prove two complementary results about the thermodynamic limit of the one-dimensional Ising model. (A) The finite-size error bound $|f_N(\beta) - f_\infty(\beta)| \le (1/N)\tanh(\beta)^N$ is provable in BISH without any omniscience principle. (B) The existence of the thermodynamic limit as a completed real number is equivalent over BISH to LPO, via bounded monotone convergence instantiated through the Ising free energy. Together: the LPO cost is genuine and dispensable. The combined formalization comprises 1374 lines of Lean across 18 modules with zero sorries.

Paper 9: Ising Formulation-Invariance

DOI: 10.5281/zenodo.18517570

Abstract. We prove that the logical cost of the thermodynamic limit for the 1D Ising model is formulation-invariant: the same axiom profile arises from a purely combinatorial derivation as from the transfer-matrix approach of Paper 8. Specifically, we re-derive (A) BISH dispensability and (B) LPO equivalence using only finite sums over $\{-1, +1\}^N$, the binomial parity sieve, and elementary hyperbolic function arithmetic. No transfer matrices, eigenvalues, or linear algebra are used. The formalization comprises 1319 lines across 18 modules with zero sorries.

Paper 10: Logical Geography of Mathematical Physics

DOI: 10.5281/zenodo.18636180

Abstract. This paper synthesizes the machine-verified results of the constructive calibration programme—spanning functional analysis, quantum spectra, general relativity, uncertainty relations, statistical mechanics, quantum information, decoherence, conservation laws, measurement statistics, quantum gravity, particle physics, semiclassical mechanics, quantum foundations, radioactive decay, ergodic theory, classical mechanics, and subadditive convergence—into a single interpretive framework. The calibration table contains approximately fifty entries spanning BISH to undecidable, populated by Lean 4 formalizations totalling approximately 25,400 lines of code across eleven domains.

Paper 11: Entanglement, CHSH, Tsirelson Bound

DOI: 10.5281/zenodo.18527676

Abstract. We provide a complete Lean 4 formalization of two foundational results in quantum information theory: (A) the Tsirelson bound on the CHSH operator ($|\langle\psi, \mathrm{CHSH}\,\psi\rangle| \leq 2\sqrt{2}$); and (B) the entanglement entropy of the Bell singlet state ($S(\rho_A) = \log 2$). The formalization comprises 639 lines across 8 modules with zero sorry.

Paper 12: Constructive History of Mathematical Physics

DOI: 10.5281/zenodo.18636250

Abstract. This essay tells the story of 150 years of mathematical physics from an unfamiliar angle. A machine-verified research programme provides preliminary evidence that for every exactly solvable model and system with explicit finite-size bounds examined so far, empirical predictions can be derived using only the most elementary constructive logic. Paper 29 establishes a fundamental qualification: Fekete's Subadditive Lemma is equivalent to LPO, making the logical cost ineliminable for systems near phase transitions.

Paper 13: Event Horizon as Logical Boundary

DOI: 10.5281/zenodo.18529007

Abstract. The Schwarzschild interior decomposes constructively: the cycloid geodesic reaching $r = 0$ at finite proper time is BISH; the completed-limit singularity assertion requires LPO. The event horizon demarcates a logical boundary; interior geometry is BISH, the singularity is LPO. Formalization: 1,021 lines, 8 modules.

Paper 14: Quantum Decoherence

DOI: 10.5281/zenodo.18569068

Abstract. A single qubit coupled to N environmental qubits via controlled-rotation unitaries undergoes BISH-level decoherence; the completed-limit exact decoherence costs LPO. This is the third domain exhibiting the BMC $\leftrightarrow$ LPO pattern (after Ising and Schwarzschild).

Paper 15: Noether's Theorem

DOI: 10.5281/zenodo.18572494

Abstract. The local conservation law ($\partial_\mu J^\mu = 0$) is BISH; global energy existence costs LPO via BMC. This is the fourth domain exhibiting the BISH/LPO boundary at BMC, and the first calibration of a structural (as opposed to phenomenological) physical law. 520 lines.

Paper 16: Born Rule (Technical Note)

DOI: 10.5281/zenodo.18575377

Abstract. Single-trial probability and the weak law of large numbers are BISH; frequentist convergence (the strong law) requires DC_ω. This fills the DC axis of the calibration table with physical content. 564 lines.

Paper 17: Bekenstein–Hawking Formula

DOI: 10.5281/zenodo.18597306

Abstract. Finite entropy computation is BISH; convergence of entropy densities via BMC costs LPO; the subleading logarithmic correction ($-3/2$) is BISH. This is the fifth independent physics domain exhibiting the BISH/LPO boundary, and the first CRM application to quantum gravity. 1,804 lines, 20 files.

Paper 18: Yukawa RG Constructive Stratification

DOI: 10.5281/zenodo.18626839

Abstract. Standard Model one-loop stratification. The discrete RG step and fixed-precision coupling are BISH; threshold crossings cost WLPO; the global coupling across all thresholds costs LPO via BMC.

Paper 19: WKB Tunneling and LLPO

DOI: 10.5281/zenodo.18602596

Abstract. Three tiers: (BISH) WKB amplitude for algebraic barriers; (LLPO) classical turning points via IVT; (LPO) full semiclassical limit. This is the first physical LLPO calibration in the series—quantum tunneling sits strictly between BISH and WLPO. 1,081 lines, 15 files.

Paper 20: Observable-Dependent Logical Cost: 1D Ising Magnetization

DOI: 10.5281/zenodo.18603079

Abstract. Phase classification via magnetization zero-test costs WLPO; the same 1D Ising model requires BISH (finite), WLPO (phase), LPO (thermodynamic limit). Observable-dependent cost established: the logical cost is a property of the *question*, not the system. 494 lines, 12 files.

Paper 21: Bell Nonlocality and LLPO

DOI: 10.5281/zenodo.18603251

Abstract. The step from "local realism refuted" (BISH via CHSH) to "disjunctive conclusion" (LLPO). Three-level stratification: BISH (CHSH bound), LLPO (disjunctive decision), WLPO (hierarchy). First CRM calibration of quantum foundations at LLPO level. 751 lines, 14 files.

Paper 22: Markov's Principle and Radioactive Decay

DOI: 10.5281/zenodo.18603503

Abstract. "Non-zero decay rate $\Rightarrow$ eventual detection" is equivalent to Markov's Principle. First CRM calibration at MP level; the hierarchy becomes a partial order, not a linear chain. 814 lines, 12 files.

Paper 23: Fan Theorem and Optimization

DOI: 10.5281/zenodo.18604312

Abstract. The Extreme Value Theorem on $[a, b]$ requires exactly the Fan Theorem; instantiated via 1D Ising free energy. First FT calibration; third independent branch. The 1D Ising now has four distinct costs: BISH (finite), LPO (thermodynamic), WLPO (phase), FT (optimization). ∼680 lines, 14 files.

Paper 24: Kochen–Specker Contextuality and LLPO

DOI: 10.5281/zenodo.18604317

Abstract. KS uncolorability is BISH; sign decision costs LLPO. Structural identity: Bell sign decision ≡ KS sign decision (both LLPO). Physically distinct no-go theorems share identical logical cost. 887 lines, 16 files.

Paper 25: Choice Axis: Ergodic Theorems

DOI: 10.5281/zenodo.18615453

Abstract. Mean ergodic theorem ↔ Countable Choice; Birkhoff pointwise ↔ Dependent Choice. Clean separation: ensemble average requires CC, individual trajectory requires DC. DC Ceiling Thesis: no calibratable physical theorem requires more than DC. 1,805 lines, 12 modules.

Paper 26: Bidual Gap Arithmetic Route, WLPO

DOI: 10.5281/zenodo.18615457

Abstract. Two independent proofs of WLPO-completeness for bidual gap detection: functional-analytic (Paper 2) and arithmetic (via Gödel sequences, Lindenbaum algebra of Π^0_1 sentences). Robustness evidence that the classification is intrinsic. 1,213 lines, 30 axioms.

Paper 27: Bell Angle Optimisation via IVT, LLPO

DOI: 10.5281/zenodo.18615459

Abstract. LLPO ↔ Exact IVT; Bell angle optimization reduces to IVT instances. Identifies the mechanism (IVT) explaining why LLPO appears in Bell physics. Bell angle-finding is strictly below gap detection (WLPO). Zero sorries, six axioms.

Paper 28: Newton vs. Lagrange vs. Hamilton

DOI: 10.5281/zenodo.18616620

Abstract. Three formulations of classical mechanics are constructively stratified: Newtonian/Lagrangian equations (BISH), Hamiltonian discrete form (BISH), action minimization (FT). The variational interpretation is logically dispensable. First formal proof that variational mechanics is not logically necessary. 621 lines, zero custom axioms.

Part III: Ceiling and Dispensability (Papers 29–35)

Paper 29: Fekete's Subadditive Lemma and LPO

DOI: 10.5281/zenodo.18643617

Abstract. Fekete's Subadditive Lemma is equivalent over BISH to LPO. The backward direction encodes a binary sequence into a mock free energy $F_n = -n \cdot x_n$; applying Fekete yields a limit deciding the sequence. The forward direction composes LPO $\to$ BMC with the classical Fekete proof. This resolves Problem 1 of the calibration programme and establishes a three-tier hierarchy for thermodynamic-limit convergence: exact solvability (BISH), cluster expansions (BISH), generic subadditivity (LPO). 549 lines, 6 modules, zero sorries.

Paper 30: Physical Dispensability of the Fan Theorem

DOI: 10.5281/zenodo.18638394

Abstract. Every empirically accessible prediction from FT-calibrated results is recoverable in BISH+LPO without the Fan Theorem. Three pillars: (1) LPO implies BMC yielding ε-approximate witnesses; (2) equations of motion are BISH-valid without any minimizer; (3) no finite experiment distinguishes exact from ε-approximate minimizers. FT captures exact optimizer existence; LPO captures convergent approximation. FT is physically dispensable.

Paper 31: Physical Dispensability of Dependent Choice

DOI: 10.5281/zenodo.18645578

Abstract. Every empirical prediction from DC-calibrated results is recoverable in BISH+LPO without DC. DC's content is a quantifier swap (outside measure vs. inside measure); the experimenter observes after quantifier choice. Together with Papers 29–30: the logical constitution of empirically accessible physics is BISH+LPO.

Paper 32: QED One-Loop Renormalization: The Landau Pole

DOI: 10.5281/zenodo.18642598

Abstract. Complete CRM calibration of QED one-loop renormalization. The discrete RG step, finite-precision coupling, Ward–Takahashi identity, and—surprisingly—the Landau pole divergence itself are all BISH. Threshold crossings require WLPO; global coupling requires LPO. The Landau pole, naively most "non-constructive," is fully BISH: the closed-form solution provides an explicit Cauchy modulus requiring no omniscience.

Paper 33: QCD One-Loop Renormalization and Confinement

DOI: 10.5281/zenodo.18642610

Abstract. Asymptotic freedom ($\beta < 0$); IR divergence at Λ_{QCD} is BISH; continuum limit costs LPO; mass gap decision costs WLPO; extracting positivity costs MP. Since LPO strictly implies both WLPO and MP, confinement is *free*: the LPO already paid for the continuum limit automatically subsidizes the mass gap.

Paper 34: Scattering Amplitudes Are Constructively Computable

DOI: 10.5281/zenodo.18642612

Abstract. The fixed-order inclusive cross section (Bhabha scattering $e^+e^- \to e^+e^-$) is pure BISH: finite composition of computable functions with UV divergences removed by $\overline{\text{MS}}$ subtraction and IR cancellation by the Bloch–Nordsieck theorem. Only the all-orders series sum requires LPO.

Paper 35: The Logical Constitution of Empirical Physics: A Conservation Metatheorem

DOI: 10.5281/zenodo.18642616

Abstract. Four components: (A) BISH Conservation: finite compositions of computable functions are BISH; (B) LPO Boundary: bounded monotone limits without modulus cost LPO; (C) Exhaustiveness: all 38 calibration entries are $\leq$ LPO; (D) Three Mechanisms (BMC, Cauchy completeness, supremum existence) are mutually equivalent. Establishes the BISH+LPO ceiling.

Part IV: Undecidability and Beyond (Papers 36–44)

Paper 36: Stratifying Spectral Gap Undecidability: Cubitt's Theorem Is LPO

DOI: 10.5281/zenodo.18642620

Abstract. Cubitt's spectral gap undecidability is Turing–Weihrauch equivalent to LPO. Stratification: finite-volume gap (BISH), thermodynamic limit (LPO), each instance (LPO-decidable), physical zero-test (WLPO), uniform function (LPO-computable). Quantum undecidability introduces zero additional resources beyond thermodynamic limits.

Paper 37: The Undecidability Landscape Is LPO

DOI: 10.5281/zenodo.18642802

Abstract. Extension to three further undecidability results: phase diagram uncomputability, 1D spectral gap, uncomputable RG flows. Meta-theorem: any physical undecidability from computable many-one reduction from the halting problem is LPO. Watson–Cubitt ground state energy density hardness is BISH (computational complexity, not logical undecidability). 660 lines.

Paper 38: Wang Tiling and the Origin of Physical Undecidability

DOI: 10.5281/zenodo.18642804

Abstract. Every undecidability result in quantum many-body physics descends from the Wang tiling problem (Berger 1966), which is LPO. All descendants—from Kanter (1990) through Cubitt (2015)—inherit exactly LPO. The Σ_1^0 ceiling: no Σ_1^0-complete reduction can exceed LPO. 573 lines.

Paper 39: Beyond LPO: Thermodynamic Stratification

DOI: 10.5281/zenodo.18642806

Abstract. The ceiling is not Σ_1^0. The generic spectral gap (without promise gap) is Σ_2^0/Π_2^0-complete, requiring LPO^j (the Turing jump). However, extensive observables cap at LPO via Fekete. Thermodynamic Stratification Theorem: arithmetic complexity bifurcates along thermodynamic scaling—extensive at LPO, intensive at LPO^j. 802 lines.

Paper 40: The Logical Constitution of Physical Reality (Monograph)

DOI: 10.5281/zenodo.18654773

Abstract. We prove that the logical resources for all empirical predictions in known physics are exactly BISH+LPO. Established by systematic axiom calibration across 42 papers spanning the Standard Model, general relativity, statistical mechanics, quantum information, AdS/CFT, and the cosmological constant problem. Three foundational results: (1) Fekete's lemma $\leftrightarrow$ LPO (LPO is physically instantiated); (2) the Fan Theorem is dispensable; (3) Dependent Choice is dispensable. All calibrations formally verified in $\sim$35,000 lines of Lean 4 code.

Paper 41: AdS/CFT Diagnostic

DOI: 10.5281/zenodo.18654780

Abstract. The holographic dictionary is an axiom-preserving map: bulk and boundary carry identical axiom cost at every level. Vacuum AdS_3 (BISH), thermal BTZ (BISH entropy, LLPO phase), FLM correction (BISH), QES entropy (LPO), Page curve (BISH). Holography projects away the Fan Theorem: boundary entropy is computable at BISH+LPO without constructing the bulk surface. 955 lines.

Paper 42: The Cosmological Constant Problem

DOI: 10.5281/zenodo.18654789

Abstract. The 10^{120} discrepancy decomposes into three claims: (I) the UV discrepancy is a regulator artifact (vanishes under dimensional regularization); (II) "naturalness" is a Bayesian prior, outside the hierarchy; (III) the genuine constraint (55-decimal cancellation) requires the thermodynamic limit, hence LPO. The cosmological constant problem introduces no new logical resources. $\sim$830 lines, 10 modules.

Paper 43: What the Ceiling Means: Constructive Schools

DOI: 10.5281/zenodo.18665418

Abstract. The BISH+LPO ceiling unifies three constructive schools (Bishop, Brouwer, Markov); their disagreement localizes to Markov's Principle. LPO strictly implies MP. Physical actualisation requires Cournot's Principle + MP. Three-step chain: BISH computation $\rightarrow$ Cournot exclusion $\rightarrow$ MP witness extraction. $\sim$770 lines, 12 files.

Paper 44: The Measurement Problem Dissolved: Quantum Interpretations

DOI: 10.5281/zenodo.18671162

Abstract. Three interpretations stratified: Copenhagen (WLPO minimal, LPO strong), Many-Worlds (DC), Bohmian mechanics (LPO). Since WLPO $<$ LPO and DC is incomparable with both, the interpretations sit at provably distinct positions. The measurement problem is not one problem but three logically distinct commitments.

Part V: Arithmetic Geometry and Motives (Papers 45–60)

Paper 45: Weight-Monodromy Conjecture and LPO

DOI: 10.5281/zenodo.18676170

Abstract. Four theorems: (C1) Hodge polarization forces degeneration in BISH; (C2) abstract degeneration decidability $\leftrightarrow$ LPO for the coefficient field; (C3) positive-definite Hermitian forms blocked over p-adic fields (dimension ≥ 3); (C4) geometric perverse sheaves degenerate in BISH. The gap between C2 and C4 is the *de-omniscientizing descent*: geometric origin replaces LPO with decidable equality.

Paper 46: Tate Conjecture and LPO

DOI: 10.5281/zenodo.18682285

Abstract. Four theorems: (T1) Galois-invariance decidability $\leftrightarrow$ LPO($\mathbb{Q}_\ell$); (T2) numerical equivalence decidable in BISH; (T3) Poincaré pairing cannot be anisotropic over $\mathbb{Q}_\ell$ in dimension ≥ 5; (T4) Standard Conjecture D is precisely the axiom that de-omniscientizes motivic morphism spaces. 21 custom axioms.

Paper 47: Fontaine–Mazur Conjecture and LPO

DOI: 10.5281/zenodo.18682788

Abstract. Five theorems: (FM1) unramifiedness $\leftrightarrow$ LPO($\mathbb{Q}_p$); (FM2) de Rham condition $\leftrightarrow$ LPO($\mathbb{Q}_p$); (FM3) Faltings comparison descends to $\mathbb{Q}$, making BISH-decidable; (FM4) geometric Frobenius traces decidable in BISH; (FM5) $u(\mathbb{Q}_p) = 4$ blocks positive-definite forms in dimension ≥ 3. De-omniscientizing descent via Faltings and the Weil conjectures.

Paper 48: Birch and Swinnerton-Dyer Conjecture and LPO

DOI: 10.5281/zenodo.18683400

Abstract. Four theorems: (B1) $L(E,1) = 0$ decision $\leftrightarrow$ LPO($\mathbb{R}$); (B2) Néron–Tate height gives positive-definite Archimedean polarization; (B3) regulator Reg > 0 in BISH; (B4) p-adic height cannot be positive-definite for rank ≥ 5. BSD is the first conjecture where Archimedean polarization is available—Papers 45–47 proved it blocked at every finite prime. 9 custom axioms.

Paper 49: Hodge Conjecture and Constructive Omniscience

DOI: 10.5281/zenodo.18683802

Abstract. Five theorems: (H1) Hodge type (r, r) decidability $\leftrightarrow$ LPO($\mathbb{C}$); (H2) rationality testing requires LPO+MP; (H3) Hodge–Riemann polarization available ($u(\mathbb{R}) = 1$) but blind to the rational lattice; (H4) algebraic cycle verification BISH via integer intersection numbers; (H5) the Hodge Conjecture reduces LPO to BISH+MP. Unique phenomenon: polarization available but insufficient.

Paper 50: Three Axioms for the Motive

DOI: 10.5281/zenodo.18705837

Abstract. Five calibrations converge on a three-axiom specification of Grothendieck's category of numerical motives: the *Decidable Polarized Tannakian* (DPT) category. Three axioms: (1) decidable morphism equality (Standard Conjecture D), (2) algebraic spectrum, (3) Archimedean polarization. Theorem A: Weil RH reduces to cancellation. Theorem B: Honda–Tate inhabitant over $\mathbb{F}_q$. Theorem C: D is the decidability axiom. Theorem D: conjectures are Π_2^0 mandates with the motive as -1 shift operator. Theorem E: CM elliptic motives unconditionally BISH-decidable. 8 files, 46 custom axioms.

Paper 51: Constructive Archimedean Rescue in BSD

DOI: 10.5281/zenodo.18732168

Abstract. The positive-definite Archimedean metric converts the rank-1 BSD generator search from MP (unbounded) to BISH (bounded exhaustive search) in an explicit Finset of size $O(B^2)$ where $B = \lceil \exp(2\hat{h}(y_K) + 2\mu(E)) \rceil$. The p-adic analogue fails: without positive-definiteness ($u = 4$), the canonical height can vanish on non-torsion points. Zero sorries, zero custom axiom declarations.

Paper 52: Decidability Transfer via Specialization: Standard Conjecture D for Abelian Threefolds

DOI: 10.5281/zenodo.18732559

Abstract. Standard Conjecture D for abelian varieties of dimension $g \leq 3$ follows from the Tate conjecture for divisors (unconditional by Tate 1966) via decidability transfer from characteristic p to characteristic 0. Three components: smooth proper base change, unconditionally definite Lefschetz ring, Fourier–Mukai stability of the liftable subspace. Fails sharply at dimension 4 (exotic Tate classes appear outside the Lefschetz ring). Zero custom axioms.

Paper 53: The CM Decidability Oracle

DOI: 10.5281/zenodo.18713089

Abstract. A verified decision procedure for numerical equivalence on products of the 13 CM elliptic curves over $\mathbb{Q}$ with class number 1. The decider returns a Boolean with correctness guaranteed by five principled axioms. All 13 curves pass all three DPT axioms computationally. We extend to the dimension-4 boundary: for Milne's CM abelian fourfold, $\deg(w \cdot w) = 7 > 0$, confirming Hodge–Riemann for the exotic class. 15 files, $\sim$1500 lines.

Paper 54: The Bloch–Kato Calibration: Out-of-Sample DPT Test

DOI: 10.5281/zenodo.18732964

Abstract. First out-of-sample test of the DPT framework. Key results: (A) LPO isolation for zero-testing; (B) Axiom 2 realized by Deligne Weil I; (C) Axiom 3 partial (Hodge–Riemann unconditional, Beilinson conditional); (D) Axiom 1 fails for mixed motives (Ext^1 decidability unavailable); (E) p-adic Tamagawa obstruction ($u(\mathbb{Q}_p) = 4$); (F) descent diagram with explicit fracture points. 8 files, 1,139 lines, 7 principled axioms.

Paper 55: K3 Surfaces, Kuga–Satake Construction, and the DPT Framework

DOI: 10.5281/zenodo.18733731

Abstract. Second out-of-sample test. Full DPT success for K3 surfaces. Axiom 1 transfers via André (1996). Axiom 2 independent via Deligne Weil I. Axiom 3 via the Kuga–Satake Clifford algebra and Rosati involution. Supersingular bypass at $\rho = 22$. No Picard boundary: decidability does not degrade. Codimension principle confirmed. Calabi–Yau threefold correction: weight-3 Hodge–Riemann restores positive-definiteness. 8 files, 1,131 lines, 9 principled axioms.

Paper 56: Exotic Weil Class Self-Intersection on CM Abelian Fourfolds

DOI: 10.5281/zenodo.18734021

Abstract. We compute $\deg(w_0 \cdot w_0) = \sqrt{\mathrm{disc}(F)}$ for exotic Weil classes on three CM abelian fourfolds: (1) $K = \mathbb{Q}(\sqrt{-3})$, $\mathrm{disc}(F) = 49$, degree 7; (2) $K = \mathbb{Q}(i)$, $\mathrm{disc}(F) = 81$, degree 9; (3) $K = \mathbb{Q}(\sqrt{-7})$, $\mathrm{disc}(F) = 169$, degree 13. All degrees positive (Hodge–Riemann), all exotic classes algebraic (Schoen). Determinant equation verified by `native_decide`. ∼1,666 lines, 10 principled axioms, zero sorry.

Paper 57: Exotic Weil Self-Intersection Across All Nine Heegner Fields

DOI: 10.5281/zenodo.18735172

Abstract. Extension of Paper 56 to all nine class-number-1 imaginary quadratic fields (Baker–Heegner–Stark): $d \in \{1, 2, 3, 7, 11, 19, 43, 67, 163\}$. For each $K = \mathbb{Q}(\sqrt{-d})$, we compute the trace matrix, verify the discriminant via `native_decide`, and derive the Weil class degree via the cyclic conductor theorem. Complete 9-row pattern table assembled. ∼1,257 lines, 1 principled axiom.

Paper 58: Class Number Correction for Exotic Weil Classes

DOI: 10.5281/zenodo.18734718

Abstract. Extension to $h_K > 1$. The corrected formula is $h \cdot \mathrm{Nm}(\mathfrak{a}) = f$, where $\mathfrak{a}$ is the Steinitz class. The topological volume $\det(G) = f^2|\Delta_K|$ is an absolute invariant; the class group redistributes it between metric and lattice density. Verified for $K = \mathbb{Q}(\sqrt{-5})$ $(h_K = 2)$ at all nine conductors. Norm obstruction is decidable in BISH. 6 files, 803 lines, zero sorry.

Paper 59: De Rham Decidability and DPT Completeness

DOI: 10.5281/zenodo.18735931

Abstract. We formalize the p-adic precision bound $N_M = v_p(\det(1 - \varphi)) = v_p(1 - a_p + p) = v_p(\#E(\mathbb{F}_p))$ for elliptic curves with good reduction. The Hasse bound $a_p^2 \leq 4p$ implies $\#E(\mathbb{F}_p) \geq 1$. We verify N_M for 24 entries across 4 curves; four anomalous ($N_M \geq 1$), 20 generic ($N_M = 0$). "Axiom 5" (de Rham decidability) is a *theorem*: de Rham ⇒ potentially semistable ⇒ weakly admissible ⇒ N_M computable in BISH. All computation is pure integer arithmetic.

The DPT framework for numerical equivalence on pure motives is *complete*: Axioms 1–3 plus automatic de Rham decidability suffice. No mixed motive axiom is needed. We initiate the extended framework for rational equivalence by proving an analytic rank stratification theorem: the logical complexity of computing $\mathrm{Ext}^1(\mathbb{Q}(0), M)$ is determined by $r = \mathrm{ord}_{s=s_0} L(M, s)$. For

$r = 0$, exact verification is BISH. For $r = 1$, Bloch–Kato bounds the height and Northcott bounds the search, yielding BISH. For $r \geq 2$, Minkowski's geometry of numbers forces Markov's Principle.

6 files, 762 lines, zero sorry, zero custom axioms. (Incorporates former Paper 60.)

Paper 60: [Retired — Merged into Paper 59]

Status: Retired (2026-02-22).
Former title: Analytic Rank Stratification of Mixed Motives: Completing the DPT Framework
Former DOI: 10.5281/zenodo.18728923 (reserved, not published)

Content folded into Paper 59, §§9.4–9.6.

Part VI: Mixed Motives, Self-Intersection, and Synthesis (Papers 61–70)

Paper 61: Lang's Conjecture as the MP→BISH Gate

DOI: 10.5281/zenodo.18736959

Abstract. We prove that an Effective Lang Height Lower Bound is the precise gate converting rank ≥ 2 motives from MP to BISH, via inversion of Minkowski's Second Theorem. Five theorems: (A) Lang + Minkowski inversion reduces rank-r search to BISH; (B) without Lang, Minkowski inversion fails in dimension ≥ 2, requiring MP; (C) Uniform Lang implies the L-function is a universal decidability certificate; (D) explicit verification for $X_0(389)$ (rank 2). Complete Lean 4 formalization: 9 files, ∼900 lines, zero sorry.

Paper 62: [Retired — Merged into Paper 63]

Status: Retired.
Former title: The Hodge Level Boundary
Former DOI: 10.5281/zenodo.18736965 (reserved, not published)

Content folded into Paper 63.

Paper 63: The Intermediate Jacobian Obstruction

DOI: —(pending)

Abstract. Algebraicity of the intermediate Jacobian $J^p(X)$ determines cycle-search decidability. When $h^{n,0} = 0$ (Hodge level $\ell \leq 1$): J^p is an abelian variety with Néron–Tate height satisfying Northcott, and cycle search requires exactly MP. When $h^{n,0} \geq 1$ (Hodge level $\ell \geq 2$): J^p is a non-algebraic complex torus with no height function, and cycle search requires LPO. Four-way equivalence: algebraic $\iff$ low Hodge $\iff$ Northcott $\iff$ MP. Five main theorems verified on the cubic threefold and quintic Calabi–Yau. Lean 4: 8 files, 1,136 lines, zero sorry. (Incorporates former Paper 62.)

Paper 64: Uniform p-Adic Decidability for Elliptic Curves

DOI: 10.5281/zenodo.18737090

Abstract. The crystalline precision bound $N_M = v_p(\#E(\mathbb{F}_p))$ for elliptic curves is uniformly bounded: $N_M \leq 2$ (with $N_M = 2$ only at $p = 2$, and $N_M \leq 1$ for all $p \geq 3$). Verified computationally across 1,812 elliptic curves and 23,454 (E, p) pairs. The p-adic side is uniformly BISH-decidable with no omniscience principles required, contrasting sharply with the Archimedean side's rank stratification. Computational paper, no Lean formalization.

Paper 65: Self-Intersection Patterns Beyond Cyclic Cubics

DOI: 10.5281/zenodo.18743151

Abstract. Verifies the Steinitz–conductor identity $h \cdot \mathrm{Nm}(\mathfrak{a}) = f$ across all 1,220 pairs (K, F) where $K = \mathbb{Q}(\sqrt{-d})$ with $d \leq 200$ and F is cyclic cubic with conductor $f \leq 200$. Result: 738 pairs have free lattice ($h = f$), 482 pairs have Steinitz twist ($\mathrm{Nm}(\mathfrak{a}) > 1$), zero exceptions. For non-cyclic S_3 cubics: the scalar identity $h^2 = \mathrm{disc}(F)$ never holds. Computational paper.

Paper 66: Form-Class Resolution for Non-Cyclic Totally Real Cubics

DOI: —(pending)

Abstract. The trace-zero sublattice $\Lambda_0 = \{x \in \mathcal{O}_F : \mathrm{Tr}_{F/\mathbb{Q}}(x) = 0\}$ of a totally real cubic carries a positive-definite binary quadratic form of discriminant $-12 \cdot \mathrm{disc}(F)$. For cyclic cubics, the form is $2f \cdot (1, 1, 1)$. For non-cyclic (S_3) cubics, the form is generically non-scalar. Computed for 51 non-cyclic totally real cubics with $\mathrm{disc}(F) \leq 2000$; the $\mathrm{GL}_2(\mathbb{Z})$-equivalence class is injective on discriminant. Computational paper.

Paper 67: The Motive Is a Decidability Certificate (Monograph)

DOI: 10.5281/zenodo.18746343

Abstract. Synthesis monograph of Papers 45–66 in the arithmetic geometry phase of the Constructive Reverse Mathematics programme. Working over BISH, the paper calibrates major conjectures (Hodge, Tate, BSD, Fontaine–Mazur, Weight-Monodromy) against the logical hierarchy BISH $\subset$ BISH+MP $\subset$ BISH+WLPO $\subset$ BISH+LPO $\subset$ CLASS. Three main outputs: (i) a decidability classification showing three invariants (analytic rank r, Hodge level ℓ, effective Lang constant c) determine the logical strength of cycle-search for any motive; (ii) the DPT (Decidable Polarized Tannakian) framework for computing motives; (iii) the arithmetic identity $h \cdot \mathrm{Nm}(\mathfrak{a}) = f$ with Steinitz generalization to non-cyclic S_3 cubics. Contains 53 Lean 4 formalizations (86,898+ lines total, 22 papers with zero `sorry`).

Paper 68: Fermat's Last Theorem Is BISH

DOI: 10.5281/zenodo.18749965

Abstract. Stage-by-stage constructive reverse mathematics audit of Wiles's proof of modularity of semistable elliptic curves and Fermat's Last Theorem, decomposing into five stages: (1) residual modularity via Langlands–Tunnell, (2) deformation ring, (3) Hecke algebra, (4) numerical criterion, (5) Taylor–Wiles patching. Principal finding: asymmetry—Stages 2–5 are fully constructive (BISH), while Stage 1 requires WLPO. The Taylor–Wiles engine contributes zero logical cost. Two post-Wiles developments drive Stage 5 classification: Brochard's proof of de Smit's conjecture (2017, eliminates infinite inverse limit) and effective Chebotarev bounds (LMO 1979, Ahn–Kwon 2019, make prime search bounded). Stage 1 can be bypassed: the $p = 2$ dihedral base case (Kisin 2009, Khare–Wintenberger 2009) replaces Wiles's $p = 3$ octahedral case with Hecke's algebraic theta series (BISH). Overall: Fermat's Last Theorem is BISH. Lean 4: 493 lines across 3 files, zero sorry.

Paper 69: The Logical Cost of the Archimedean Place: Function Field Langlands Is BISH

DOI: 10.5281/zenodo.18749757

Abstract. CRM audit of the function field Langlands correspondence. Both Laurent Lafforgue's proof for $_n$ (Inventiones, 2002) and Vincent Lafforgue's proof for general reductive groups (JAMS, 2018) are unconditionally BISH: every component operates within Bishop's constructive mathematics, with no omniscience principle required. The principal finding is the comparison with number fields. Paper 68 established that Wiles's proof route costs BISH + WLPO, with the WLPO entering solely through the Arthur–Selberg trace formula at the Archimedean place. The function field $\mathbb{F}_q(C)$ has no Archimedean place. Every component which costs

WLPO over number fields has an algebraic counterpart over function fields that costs nothing: the Grothendieck–Lefschetz trace formula replaces the Arthur–Selberg trace formula; rational Plancherel measures replace transcendental ones; finite-dimensional spaces of cusp forms replace infinite-dimensional L^2 spaces. Structural discovery: the boundary between BISH and WLPO is not discrete-vs-continuous spectrum, but algebraic-vs-transcendental spectral parameters. The logical cost of the Langlands program is the logical cost of $\mathbb{R}$. Lean 4: 236 lines, zero sorry, no `Classical.choice`.

Paper 70: The Archimedean Principle: Why Physics and Number Theory Share a Logical Architecture

DOI: 10.5281/zenodo.18750992

Abstract. Capstone paper identifying a single structural principle underlying 70 papers of Constructive Reverse Mathematics: the real numbers are the sole source of logical difficulty in mathematical physics and arithmetic geometry. The mechanism is $u(\mathbb{R}) = \infty$—the real numbers are the only completion of $\mathbb{Q}$ where positive-definite forms exist in every dimension—and three fields independently exploit it via the same architecture (Hilbert space inner product, Rosati involution, Petersson inner product). Four theorems: (A) the Archimedean Principle—the CRM level of every domain is determined by whether it has an Archimedean place; (B) the MP Gap—physics and arithmetic descend differently (projection vs. search), producing a strict separation BISH $<$ BISH + MP; (C) Automorphic CRM Incompleteness—an integer witness proving the automorphic axioms alone cannot recover the Ramanujan bound; (D) Three Spectral Gaps—identical Σ^0_2 quantifier structure across physics, automorphic theory, and arithmetic. Paper 68 showed FLT is BISH; Paper 69 showed function field Langlands is BISH; this paper identifies what makes anything expensive: the Archimedean place and $u(\mathbb{R}) = \infty$. Lean 4: 6 files, 545 lines, zero sorry, zero custom axioms for core theorems.

Paper 71: The Archimedean Principle in Cryptography and Numerical Computation

DOI: 10.5281/zenodo.18752015

Abstract. Four engineering consequences of the Archimedean Principle (Paper 70). Theorem A (Archimedean Security): lattice-based cryptography (SVP, LWE, Ring-LWE) is not amenable to Shor-type quantum attacks because solution targets are *metric* (Archimedean norm bounds), not *algebraic* (group-theoretic relations); metric targets delocalize in expectation under spectral projection by Fourier energy conservation; the function field control confirms SVP over $\mathbb{F}_q[t]$ is BISH (polynomial-time). Theorem B (SVP Phase Transition): exponential approximation is projection-descent (LLL, BISH); polynomial approximation is search-descent (BKZ, BISH+MP). Theorem C (Conjugacy Design Principle): maximize Fourier conjugacy between algebraic operations and metric security assumptions; a conjugacy index quantifies structural security: Kyber $>$ NTRU $>$ RSA. Theorem D (Eigendecomposition Integrality): any nontrivial eigendecomposition of a positive-definite integer matrix introduces irreducible transcendental contamination. All four applications follow from one mechanism: projection descent eliminates MP; search descent preserves it; the Archimedean metric is canonically conjugate to algebraic spectral decomposition. Lean 4: 5 files, zero sorry, zero custom axioms; sum-of-integer-squares lemma is a genuine Mathlib proof.

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