

WLPO Constructive Completion Status (v3)

P2_BidualGap Audit

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Summary

- **Lean proof status:** `HB/WLPO_to_Gap_HB.lean` remains complete (no `sorry`s). The direct dual witness `G` now uses the constructive σ_ε coefficients, avoiding sign case splits.
- **WLPO usage:** norm attainment for ℓ^1 now calls `exists_coeff_near_sup_WLPO`; it uses WLPO to decide the “all false vs not all false” case, but still uses classical extraction of a witness index.
- **Build status:** full `lake` build succeeded. There are still `sorry` declarations in `Papers/P1_GBC/RankOneTogg`. P2 HB/Slim files are `sorry`-free (script pass).
- **Small folder:** added `LeanProof/` as a lightweight wrapper that re-exports `HB/WLPO_to_Gap_HB.lean` and provides `LeanProof/All.lean`.

Lean Proof: What Changed

- `HB/DirectDual.lean`: replaced sign-vector coefficients with σ_ε ; finite-sum bounds now use $2\|f\|$. This keeps the direct witness constructive in spirit.
- `HB/WLPO_DualBanach.lean`: replaced the classical `exists_coeff_near_sup` with `exists_coeff_near_sup_WLPO`. WLPO decides the bound; a classical step still extracts a witness.

CRM Constructivity Status

The HB path is *improved but not fully CRM-constructive*. Remaining classical dependencies include:

- witness extraction from `~forall` in `exists_coeff_near_sup_WLPO`;
- summability via `summable_of_sum_le` (tsum existence in Lean’s classical reals);
- Hahn–Banach style dual isometries in `HB/DualIsometriesComplete.lean`.

Latest LaTeX Draft: Incomplete Places

Latest draft: `documentation/paper-v5.tex` (newest mtime).

- Line 757: `sorry` in the WLPO-to-gap sketch.

- Line 802: `sorry` in the algebra structure on ℓ^∞/c_0 .
- Lines 817–820: `sorry` placeholders in the Stone window equivalence construction.

Other drafts: `paper-final.tex` still contains explicit `sorry` blocks (e.g., lines 899, 905).

Next Actions for Full CRM Compliance

- Replace classical witness extraction in `exists_coeff_near_sup_WLPO` with a constructive witness principle (or explicitly assume LPO/Markov).
- Replace classical summability lemmas with constructive Cauchy bounds where possible.
- Isolate Hahn–Banach dependencies in a classical-only namespace and keep the WLPO path free of them.