WLPO Constructive Completion Status (v2)

P2_BidualGap Audit

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Summary

- Lean proof status: `HB/WLPO_to_Gap_HB.lean` is *fully complete* (no `sorry`s). The current discharge of `DualIsBanach` for `c_0` and `(c_0)*` is classical (no axioms), i.e. it does not yet isolate WLPO as the sole logical input.
- A slim sharing folder now exists at `Slim/` with `Slim/WLPO_to_Gap_HB.lean` (local copy) and `Slim/All.lean` (aggregator).
- Latest LaTeX draft appears to be `documentation/paper-v5.tex` (newest mtime). It still contains explicit proof placeholders (`sorry`).

Lean Proof: What Is Complete

- **Main equivalence**: `Papers.P2.HB.gap_equiv_wlpo` proves `BidualGapStrong{0}` $\leftrightarrow$ `WLPO` in `HB/WLPO_to_Gap_HB.lean`.
- **Direct gap witness**: the functional `G` and the non-reflexivity argument for `c_0` are complete (no sorries).

WLPO-Constructive Gaps (Still Open)

The present Lean proof uses classical norm-attainment and operator-norm arguments to discharge `DualIsBanach`. To make the proof WLPO-constructive, the following pieces still need to be formalized:

- **Dual isometry** $(\ell^1)^* \cong \ell^\infty$: The skeleton exists in `WIP/DualIsometries.lean`, but the isometry and norm equality are not filled.
- **WLPO $\Rightarrow$ locatedness of sup in** ℓ^∞: implement a WLPO-based bisection for the supremum of bounded sequences (see `HB/IMPLEMENTATION_PLAN.md`).
- **DualIsBanach for** $(c_0)^*$: after the two items above, transport the operator norm/attainment to finish the WLPO-only route.

Latest LaTeX Draft: Incomplete Places

In `documentation/paper-v5.tex`, the following explicit placeholders remain:

- Line 757: `sorry` in the WLPO-to-gap sketch.
- Line 802: `sorry` in the algebra structure on ℓ^∞/c_0.
- Lines 817–820: `sorry` placeholders in the Stone window equivalence construction.

Next Actions If You Want a Fully WLPO-Constructive Lean Proof

- Complete `WIP/DualIsometries.lean` Part B ($(\ell^1)^* \cong \ell^\infty$) or add a new finished file and wire it into `HB/WLPO_to_Gap_HB.lean`.
- Implement `located_sup_norm_of_WLPO` and then replace the current classical `DualIsBanach` proofs with WLPO-based ones.
- Update `paper-v5.tex` to remove the explicit `sorry` blocks.