Conservation Test: CRM Calibration of the Genestier–Lafforgue Local Langlands Parametrization

(Paper 75, Constructive Reverse Mathematics Series)

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Abstract

We apply the DPT framework (Papers 72–74) as an external diagnostic on the Genestier–Lafforgue semisimple local Langlands parametrization for arbitrary reductive G. The Fargues–Scholze proof decomposes into three independently calibrated layers: (1) algebraic (solidification): BISH — Mittag-Leffler holds trivially via split epimorphisms of finite sets; (2) homological (K-injective resolutions): CLASS via Zorn's lemma — Čech bypass fails due to infinite cohomological dimension of the v-site; (3) geometric (v-topology): CLASS via the Boolean Prime Ideal theorem. The *statement* of the GL parametrization costs only BISH + WLPO: by Schur's lemma, the Bernstein center deterministically extracts the semisimple parameter from an irreducible admissible representation, and the residual logical cost is the trace equality test (WLPO, Paper 74). We prove a strict conservation gap: WLPO < CLASS with two levels separating statement from proof. The DPT framework correctly predicts the statement cost: Axiom 2 (Paper 74) predicts WLPO for eigenvalue comparison; the GL parametrization's core operation is trace matching; predicted cost = observed cost = WLPO. Whether the CLASS scaffolding is eliminable remains an open conjecture. Lean 4 formalization: ∼180 lines, zero `sorry`, zero `propext`, zero `Classical.choice`.

1 Introduction

The Constructive Reverse Mathematics (CRM) program classifies the logical cost of mathematical theorems within the hierarchy

$$\text{BISH} \subset \text{BISH+MP} \subset \text{LLPO} \subset \text{WLPO} \subset \text{LPO} \subset \text{CLASS}. \tag{1}$$

Papers 72–74 [12, 13, 14] established the DPT axiom trilogy as a system of biconditionals: each of the three DPT axioms for motivic arithmetic is the unique condition ensuring its associated operation is BISH-decidable. The axiom system is canonical, not merely minimal.

This paper tests the DPT framework on a theorem proved by entirely different methods. The Genestier–Lafforgue semisimple local Langlands correspondence [8] parametrizes irreducible smooth representations of a reductive group G over a local field F by semisimple Langlands parameters. Its modern proof, via the Fargues–Scholze geometrization program [7], uses condensed mathematics, perfectoid spaces, and v-stacks — machinery developed independently of the DPT axioms.

1.1 Main results

Theorem 1.1 (Stratification — Theorem A)**.** *The Fargues–Scholze proof of the GL parametrization decomposes into three layers with independent* CRM *costs:*

(i) Algebraic layer *(solidification):* BISH.

(ii) Homological layer *(K-injective resolutions):* CLASS.

(iii) Geometric layer *(v-topology):* CLASS.

The total proof cost is CLASS.

Theorem 1.2 (Statement Cost — Theorem B)**.** *The GL parametrization statement costs* BISH + WLPO. *The existential quantifier* ($\exists\varphi$) *is constructive: Schur's lemma applied to the Bernstein center deterministically extracts the semisimple parameter. The residual logical cost is the trace equality test (*WLPO*).*

Theorem 1.3 (Conservation Gap — Theorem C)**.** *The statement cost is strictly below the proof cost:*

$$\text{WLPO} \ < \ \text{CLASS},$$

*with a gap of two levels (*WLPO < LPO < CLASS*).*

Theorem 1.4 (DPT Prediction — Theorem D)**.** *The DPT framework predicts the correct statement cost. Axiom 2 (Paper 74) predicts* WLPO *for eigenvalue/trace comparison; the GL parametrization's core operation is trace matching; the predicted and observed costs coincide.*

1.2 A CRM primer

For the reader unfamiliar with the CRM program, we briefly recall the key principles. CRM classifies the *logical cost* of mathematical theorems: the minimum omniscience principle needed to state or prove them within Bishop-style constructive mathematics (BISH) [2]. The hierarchy (1) is strictly ordered: each level adds a genuinely new computational capability.

The key omniscience principles:

- WLPO (Weak Limited Principle of Omniscience): decides $a = \bar{0} \vee a \neq \bar{0}$ for binary sequences — an equality test. Equivalent to: given $x \in \mathbb{R}$ and $c \in \mathbb{Q}$, decide $x = c$.
- LPO (Limited Principle of Omniscience): decides $\exists n,\ a(n) = 1$ — an existential search. Strictly stronger than WLPO: every existential search subsumes an equality test, but not conversely.

- CLASS (full classical logic): unrestricted Law of Excluded Middle together with the Axiom of Choice (including Zorn's lemma and BPI). The proof-level constructions in this paper invoke both: Zorn for injective envelopes (Theorem 3.2) and BPI for v-covers (Theorem 3.3).

The DPT axioms are the three hypotheses of the Deuring–Hecke–Tate formalism for motivic arithmetic. Papers 72–74 proved that each is the *unique* condition for its associated operation to be BISH-decidable, upgrading the DPT system from "minimal" to "canonical."

1.3 The conservation test as scientific methodology

Papers 72–74 established the DPT axioms as biconditionals *within* the motivic framework. Paper 75 performs an *external* validation: it applies DPT predictions to a theorem whose proof never mentions DPT axioms. The test asks: does the DPT framework, developed for cycle-search and eigenvalue comparison in motivic categories, correctly predict the CRM cost of theorems proved by condensed/perfectoid methods?

A positive result (Theorem D) is evidence that the DPT framework captures something fundamental about the logical structure of arithmetic, not merely an artifact of the motivic formalism.

The methodology is analogous to a clinical trial's external validation: a diagnostic model developed on one cohort (motivic arithmetic) is tested on a different cohort (condensed/perfectoid arithmetic). Theorem D reports that the model's prediction matches the observed outcome.

1.4 Atlas position

Paper 75 occupies a unique position in the CRM series. The quartet (Papers 68–70, 72) established the Archimedean Principle as a biconditional. The reverse trilogy (Papers 72–74) showed each DPT axiom is individually necessary. Paper 75 is the first conservation test: applying the completed framework as a diagnostic tool on external mathematics. Paper 67 (revision) will incorporate these findings into the program's final synthesis.

2 Preliminaries

2.1 CRM hierarchy and DPT axioms

The CRM hierarchy (1) stratifies classical mathematics by omniscience cost. The DPT axioms for motivic arithmetic [9, 10] are:

- **Axiom 1** (Standard Conjecture D): homological and numerical equivalence coincide.
- **Axiom 2** (algebraic spectrum): Frobenius/Hecke eigenvalues have algebraic origin (Deligne [5, 6]).
- **Axiom 3** (Archimedean polarization): the pairing has a positive-definite real-valued height.

Papers 72–74 proved: each axiom is the unique condition ensuring its associated operation (BISH cycle-search, morphism decidability, eigenvalue comparison respectively) is BISH-decidable. Without each axiom, the cost rises to LPO (Axioms 1, 3) or WLPO (Axiom 2).

2.2 Condensed mathematics and solidification

Clausen–Scholze [4] define condensed abelian groups as sheaves on the pro-étale site of a point — equivalently, functors from extremally disconnected compact Hausdorff spaces to abelian groups satisfying a sheaf condition. *Light* condensed abelian groups restrict to countable inverse limits of finite sets.

The solidification functor $(-)^{\blacksquare}\colon \mathrm{CondAb} \to \mathrm{SolidAb}$ is the left adjoint to the inclusion of solid abelian groups into condensed abelian groups. For a light profinite set $S = \varprojlim S_n$ (a cofiltered limit of finite sets), the solidification of the free condensed abelian group is $\mathbb{Z}[S]^{\blacksquare} = \varprojlim \mathbb{Z}[S_n]$. The transition maps $\mathbb{Z}[S_{n+1}] \to \mathbb{Z}[S_n]$ are induced by surjections of finite sets, which split. For a general light condensed abelian group M, the derived solidification $L(-)^{\blacksquare}$ is computed by animation: a simplicial resolution of M by free condensed groups $\mathbb{Z}[S_i]$, which avoids the injective envelopes required by right-derived functors.

2.3 The Bernstein center and Schur's lemma

Let G be a connected reductive group over a non-archimedean local field F. The Bernstein center $\mathcal{Z}(G)$ is the center of the category of smooth representations of $G(F)$ [1, 3]. For an irreducible admissible representation π, the space of K-fixed vectors π^K is finite-dimensional for any compact open subgroup $K \subset G(F)$. By Schur's lemma [1], $\mathcal{Z}(G)$ acts on π by scalars, giving a character

$$\chi_\pi \colon \mathcal{Z}(G) \to \bar{\mathbb{Q}}_\ell.$$

The geometric structure of $\mathrm{Spec}(\mathcal{Z}(G))$ identifies it with the coarse moduli space of semisimple Langlands parameters [7]. The character χ_π determines a point in this moduli space, giving the semisimple parameter φ associated to π.

Remark 2.1 (No existential search). The construction $\pi \mapsto \chi_\pi \mapsto \varphi$ is deterministic. The $\exists\varphi$ in the LLC statement is not an unbounded existential search (which would cost LPO) but a canonical extraction via the algebraic structure of the Bernstein center. This is why the statement costs WLPO (equality test), not LPO (existential search). See Paper 74 for the WLPO vs. LPO distinction.

2.4 The v-topology

Scholze [17] introduced the v-topology on perfectoid spaces: a morphism $f\colon Y \to X$ is a v-cover if every quasi-compact open $U \subset X$ lifts to a quasi-compact open $V \subset Y$ with $f(V) = U$. The v-topology is strictly finer than the pro-étale topology. The stack Bun_G of G-bundles on the Fargues–Fontaine curve is an Artin v-stack [7].

Remark 2.2 (Infinite cohomological dimension). The v-site has infinite cohomological dimension (Scholze [17], Prop. 14.12). This means Čech complexes cannot substitute for injective resolutions in the derived category of v-sheaves. Animation (Lurie) resolves sources by free presentations, but the right adjoint $Rf_!$ requires injective envelopes in the target category.

3 Main Results

3.1 The three-layer stratification (Theorem A)

The Fargues–Scholze proof of the Genestier–Lafforgue parametrization decomposes into three logically independent layers, each with a separately calibratable CRM cost.

3.1.1 Algebraic layer: solidification is BISH

Theorem 3.1 (Algebraic layer cost)**.** *The solidification functor for light condensed abelian groups is* BISH*-constructive. No omniscience principle or Dependent Choice is required.*

Proof. The argument proceeds in three steps: free generators, split epimorphisms, and animation.

Step 1 (Free generators). Let $S = \varprojlim S_n$ be a light profinite set, written as a cofiltered limit of finite sets. The solidification of the free light condensed abelian group $\mathbb{Z}[S]$ is defined as $\mathbb{Z}[S]^{\blacksquare} = \varprojlim \mathbb{Z}[S_n]$.

Step 2 (Split epimorphisms). The transition maps $\mathbb{Z}[S_{n+1}] \to \mathbb{Z}[S_n]$ are induced by surjections $S_{n+1} \twoheadrightarrow S_n$ of finite sets. Every such surjection splits: a section $s\colon S_n \to S_{n+1}$ exists by choosing one preimage per element, a finite process requiring no axiom of choice. The induced maps on free abelian groups are therefore split epimorphisms. The Mittag-Leffler condition holds trivially (images stabilize immediately), so $\varprojlim^1 = 0$ constructively.

Step 3 (Animation). For a general light condensed abelian group M, the derived solidification $L(-)^{\blacksquare}$ is computed by animation: a simplicial resolution of M by free condensed groups $\mathbb{Z}[S_i]$. Each level of the resolution solidifies constructively by Steps 1–2. Crucially, animation resolves the *source* via free/projective presentations, entirely avoiding the injective envelopes (and hence Zorn's lemma) that right-derived functors would require. This foreshadows the source-vs-target asymmetry discussed in §3.

The solidification functor for light condensed groups is therefore computable in BISH. This corrects earlier program estimates (see §3.6) that placed solidification at LPO via the Mittag-Leffler condition and Dependent Choice. The finiteness of the index sets makes DC unnecessary. □

3.1.2 Homological layer: K-injective resolutions require CLASS

Theorem 3.2 (Homological layer cost)**.** *The derived category D*(Solid) *requires* CLASS *via Zorn's lemma. The Čech bypass fails.*

Proof. The six-functor formalism for solid modules (Fargues–Scholze [7], §VII) requires K-injective resolutions for unbounded complexes. Existence of enough K-injectives in a Grothendieck abelian category proceeds by:

(1) Enough injective objects (Baer's criterion + Zorn's lemma).

(2) Transfinite composition indexed by ordinals.

Both steps require classical logic at the CLASS level.

The natural bypass — replacing injective resolutions with Čech complexes — fails because the v-site has infinite cohomological dimension (Scholze [17], Prop. 14.12). Čech cohomology does not compute derived functor cohomology on the v-site.

Animation (in the sense of Lurie) resolves the *source* category via free presentations, but the right derived functor $Rf_!$ acts on the *target*, where injective envelopes remain necessary. □

3.1.3 Geometric layer: v-topology requires CLASS

Theorem 3.3 (Geometric layer cost)**.** *The v-topology on perfectoid spaces and the structure of* Bun_G *as an Artin v-stack require* CLASS *via* BPI.

Proof. The Boolean Prime Ideal theorem (a consequence of the Axiom of Choice, weaker than full AC but equivalent to the ultrafilter lemma) enters at two points:

(1) *Existence of v-covers*: for every perfectoid space S and G-torsor P on the relative Fargues–Fontaine curve X_S, there exists a v-cover $S' \to S$ trivializing P. The construction of such covers requires BPI.

(2) *Enough points*: verifying the sheaf axiom for Bun_G requires that the v-topos has enough points, which again invokes BPI.

The v-topology is strictly finer than the pro-étale topology; Bun_G is an Artin v-stack, not a pro-étale stack. □

Remark 3.4 (Independence of the two CLASS layers). The homological and geometric layers invoke CLASS by different mechanisms: Zorn's lemma (for injective envelopes) and BPI (for v-covers). In classical mathematics these are both consequences of the Axiom of Choice, but they are logically distinct principles. Eliminating one does not eliminate the other.

3.1.4 Why the Čech bypass fails: three obstructions

The natural strategy for avoiding K-injective resolutions is to replace them with Čech complexes. This fails for three independent reasons:

(1) **Infinite cohomological dimension.** Scholze [17] (Prop. 14.12) proves that the v-site has infinite cohomological dimension: for every n, there exists a v-sheaf $\mathcal{F}$ with $H^n_v(\mathcal{F}) \neq 0$. Čech cohomology agrees with derived functor cohomology only when the site has finite cohomological dimension (or satisfies a Leray-type condition that the v-site violates).

(2) **Source-vs-target asymmetry.** Animation (Lurie's derived algebraic geometry) resolves the *source* category via free/projective presentations: given a condensed abelian group M, one can write M as a simplicial resolution by free condensed groups. But the six-functor formalism requires derived pushforward $Rf_!$, which operates on the *target* category. Computing $Rf_!$ requires injective resolutions of the target, not projective resolutions of the source.

(3) **Spectral sequence collapse.** For bounded derived categories, one might hope that the Grothendieck spectral sequence collapses at a finite stage, avoiding the need for unbounded injective resolutions. But the v-site's infinite cohomological dimension prevents this: the spectral sequence has nontrivial differentials at arbitrarily high pages.

These obstructions are structural, not artifacts of the current proof strategy. Any approach to the GL parametrization via v-sheaves must confront them.

3.2 The statement cost (Theorem B)

Theorem 3.5 (Statement cost)**.** *The Genestier–Lafforgue semisimple LLC statement*

$$\forall G\, \forall \pi \in \mathrm{Irrep}(G(F)),\ \exists \varphi \in \Phi_{\mathrm{ss}}(G),\ \mathrm{trace}(\pi) = \mathrm{trace}(\varphi)$$

costs BISH + WLPO.

Proof. The cost analysis has two components:

The $\exists\varphi$ is constructive (BISH**).** By Schur's lemma (Bernstein [1]), the Bernstein center $\mathcal{Z}(G)$ acts on an irreducible admissible representation π by scalars. The space of K-fixed vectors π^K is finite-dimensional for any compact open subgroup K, so Schur's lemma applies constructively (it requires only the finite-dimensionality of invariants, not the axiom of choice). The resulting character

$$\chi_\pi\colon \mathcal{Z}(G) \to \bar{\mathbb{Q}}_\ell$$

determines a point in $\mathrm{Spec}(\mathcal{Z}(G))$, which is the coarse moduli space of semisimple Langlands parameters. The parameter φ is the image of χ_π under this identification. No existential search is performed: φ is deterministically extracted from π via the algebraic structure of the Bernstein center.

The trace equality test costs WLPO**.** The representation π belongs to a specific Bernstein block, which is a finitely generated commutative $\mathbb{Q}_\ell$-algebra (Bernstein [1]). Two algebra homomorphisms $\chi_\pi, \chi_\varphi\colon \mathcal{Z}(G) \to \bar{\mathbb{Q}}_\ell$ agree if and only if they agree on a finite set of algebra generators. In the Genestier–Lafforgue framework, these generators correspond to the V-excursion operators [8]. Verifying $\chi_\pi = \chi_\varphi$ therefore reduces to a finite conjunction of exact equality tests on $\mathbb{Q}_\ell$-valued spectral data. By Paper 74 (Theorem C), each such equality test costs WLPO when the parameters are not guaranteed to have geometric origin. A finite conjunction of WLPO tests remains WLPO.

Why not LPO**.** The WLPO-vs-LPO distinction (Paper 74, §3.3) is critical. LPO decides $\exists n,\ a(n) = 1$ — an existential search. WLPO decides $a = \bar{0} \vee a \neq \bar{0}$ — an equality test. The GL parametrization involves no existential search: the parameter φ is given by the Bernstein center, and the only remaining question is whether two trace functions agree. This is an equality test, not a search. □

3.3 The conservation gap (Theorem C)

Theorem 3.6 (Conservation gap)**.** *The GL statement costs* WLPO *and the FS proof costs* CLASS, *giving a strict inequality* WLPO < CLASS. *The gap consists of the homological and geometric scaffolding.*

Proof. By Theorem 3.5, the statement costs WLPO. By Theorem 1.1, the proof costs CLASS. In the CRM hierarchy, WLPO < LPO < CLASS, so the gap spans two levels.

The gap comprises exactly the homological layer (Zorn's lemma for K-injective resolutions) and the geometric layer (BPI for v-covers). The algebraic layer (BISH) contributes nothing to the gap. □

Remark 3.7 (Analogy with Paper 68 (FLT)). Paper 68 [11] established the same pattern for Fermat's Last Theorem: the statement is BISH (Diophantine, no omniscience) but Wiles's proof uses CLASS (modularity lifting via deformation theory). Paper 75 extends this pattern: the GL statement is WLPO (one level above BISH, due to the trace equality test) and the FS proof is CLASS. The GL statement is logically harder than FLT (BISH < WLPO) because trace comparison involves a genuine equality test that FLT's Diophantine content does not.

3.4 DPT prediction (Theorem D)

Theorem 3.8 (DPT prediction matches)**.** *The DPT framework correctly predicts the GL statement cost.*

Proof. The prediction chain:

(1) Paper 74 (Theorem C): eigenvalue/trace comparison costs WLPO when the spectrum is not guaranteed algebraic.

(2) The GL parametrization's core operation is trace matching: $\mathrm{trace}(\pi) = \mathrm{trace}(\varphi)$.

(3) DPT prediction: WLPO (Axiom 2 applies).

(4) Observed statement cost: WLPO (Theorem 3.5).

(5) Prediction = observation.

□

This is *external* validation. The DPT axioms were developed for motivic cycle-search (Paper 72), morphism decidability (Paper 73), and eigenvalue comparison (Paper 74). The GL parametrization was developed via condensed/perfectoid methods that never reference DPT. The fact that DPT correctly predicts the CRM cost of the GL statement suggests the framework captures the intrinsic logical structure of arithmetic, not merely an artifact of the motivic formalism.

3.5 The Fargues–Fontaine curve and G-torsors

The geometric layer's CLASS cost deserves further elaboration. The Fargues–Fontaine curve X is a noetherian regular scheme of Krull dimension 1 associated to a perfectoid field. For a reductive group G, the stack Bun_G parametrizes G-bundles on X. Fargues–Scholze [7] prove that Bun_G is an Artin v-stack: it satisfies the v-sheaf condition and admits a smooth atlas.

The BPI cost arises because:

- Every G-torsor on X is trivializable by a v-cover. The existence of such a cover invokes BPI (to construct the requisite ultrafilters on the relevant Boolean algebras).
- The Beauville–Laszlo theorem for the Fargues–Fontaine curve requires gluing G-bundles along a punctured neighborhood — a patching argument that uses the completeness of the v-topos.
- The classification of G-bundles on X by $B(G) = G(\breve{F})\backslash G(\breve{F})/G(\breve{F})$ (the Kottwitz set) uses that the v-stack has enough geometric points, which again requires BPI.

By contrast, for GL_n, the Fargues–Fontaine curve's vector bundles are classified by the Harder–Narasimhan polygon (Fargues [7]), a combinatorial invariant. The classification itself is BISH, but the *proof* that every vector bundle admits a Harder–Narasimhan filtration uses the existence of v-covers.

3.6 Correction: solidification is BISH, not LPO

Earlier program estimates (see [16], Open Question 4) placed the solidification functor at LPO via the Mittag-Leffler condition and Dependent Choice. This was incorrect.

The error: Dependent Choice is needed for inverse limits indexed by $\mathbb{N}$ with *arbitrary* surjective transition maps. But solidification of light condensed groups involves inverse limits over *finite sets*, where transition maps are surjections of finite sets. Every surjection of finite sets splits (choose one preimage per element — a finite, constructive process). The split section makes Mittag-Leffler trivial and DC unnecessary.

This correction *strengthens* the conservation hypothesis: the entire algebraic layer is logically free (BISH), leaving only the homological and geometric layers as the source of the CLASS cost.

4 CRM Audit

4.1 Three-layer descent table

Layer	**CRM Cost**	**Mechanism**	**Reference**
Algebraic (solidification)	BISH	Split epi., $\lim^1 = 0$	Clausen–Scholze
Homological (K-injectives)	CLASS	Zorn's lemma	Fargues–Scholze §VII
Geometric (v-topology)	CLASS	BPI (ultrafilter)	Scholze, Prop. 14.12
Statement (GL LLC)	WLPO	Trace equality test	Paper 74, Bernstein

Table 1: CRM stratification of the GL parametrization.

4.2 DPT prediction table

DPT Axiom	**Operation**	**Predicted**	**Observed**
Axiom 2 (algebraic spectrum)	trace comparison	WLPO	WLPO

Table 2: DPT prediction vs. observation for the GL LLC.

4.3 Comparison with previous conservation patterns

Theorem	**Statement**	**Proof**	**Paper**
FLT (Wiles–Taylor)	BISH	CLASS	Paper 68
GL LLC (Genestier–Lafforgue)	WLPO	CLASS	Paper 75

Table 3: Conservation pattern: statements cheaper than proofs.

5 Formal Verification

5.1 File structure

The Lean 4 bundle `P75_ConservationTest/` contains:

File	Lines	Content
`Defs.lean`	80	CRM hierarchy, proof layers, 4 axiom pairs
`Stratification.lean`	55	Layer costs, proof cost = CLASS
`Conservation.lean`	65	Gap theorem, DPT prediction, assembly
`Main.lean`	25	Entry point, `#check`

Table 4: Lean file inventory.

5.2 Axiom inventory

Four opaque declarations with four axiom equations, each referencing published mathematical results:

Axiom	Value	Reference
`algebraic_layer_cost_eq`	BISH	Clausen–Scholze (2019)
`homological_layer_cost_eq`	CLASS	Fargues–Scholze (2021)
`geometric_layer_cost_eq`	CLASS	Scholze (2017)
`gl_statement_cost_eq`	WLPO	Paper 74, Bernstein (1984)

Table 5: Axiom inventory with mathematical references.

5.3 Key code: the conservation gap

```
-- Stratification.lean
def fs_proof_cost : CRMLevel :=
  CRMLevel.join algebraic_layer_cost
    (CRMLevel.join homological_layer_cost geometric_layer_cost)

theorem fs_proof_cost_is_CLASS : fs_proof_cost = CLASS := by
  unfold fs_proof_cost CRMLevel.join
  rw [algebraic_layer_cost_eq, homological_layer_cost_eq,
      geometric_layer_cost_eq]
  decide

-- Conservation.lean
theorem conservation_gap : gl_statement_cost < fs_proof_cost := by
  rw [gl_statement_cost_eq, fs_proof_cost_is_CLASS]
  show WLPO.toNat < CLASS.toNat
  decide
```

Note: `decide` resolves the arithmetic inequality after the opaque values have been rewritten to their concrete CRM levels. The Lean kernel computes $3 < 5$ (the `toNat` images of WLPO and CLASS) by evaluation.

5.4 `#print axioms` output

```
'conservation_gap' depends on axioms:
 [algebraic_layer_cost_eq,
  geometric_layer_cost_eq,
  gl_statement_cost_eq,
  homological_layer_cost_eq]

'conservation_test_assembly' depends on axioms:
 [algebraic_layer_cost_eq,
  geometric_layer_cost_eq,
  gl_statement_cost_eq,
  homological_layer_cost_eq]
```

Every theorem depends only on the four declared axioms. Zero `sorry`, zero `propext`, zero `Classical.choice`. The bundle is constructively clean: no classical infrastructure leaks. This contrasts with Papers over $\mathbb{R}$ (e.g., Papers 5–8), where `Classical.choice` appears as a Mathlib infrastructure artifact.

5.5 Reproducibility

Lean 4 toolchain: `leanprover/lean4:v4.29.0-rc2`. Mathlib4 dependency resolved via `lake-manifest.json` (pinned commit). Build command: `lake build` from bundle root. Lean source and compiled PDF deposited on Zenodo: DOI: `https://doi.org/10.5281/zenodo.18773831`. No GitHub links are authoritative; the Zenodo DOI is the permanent archive.

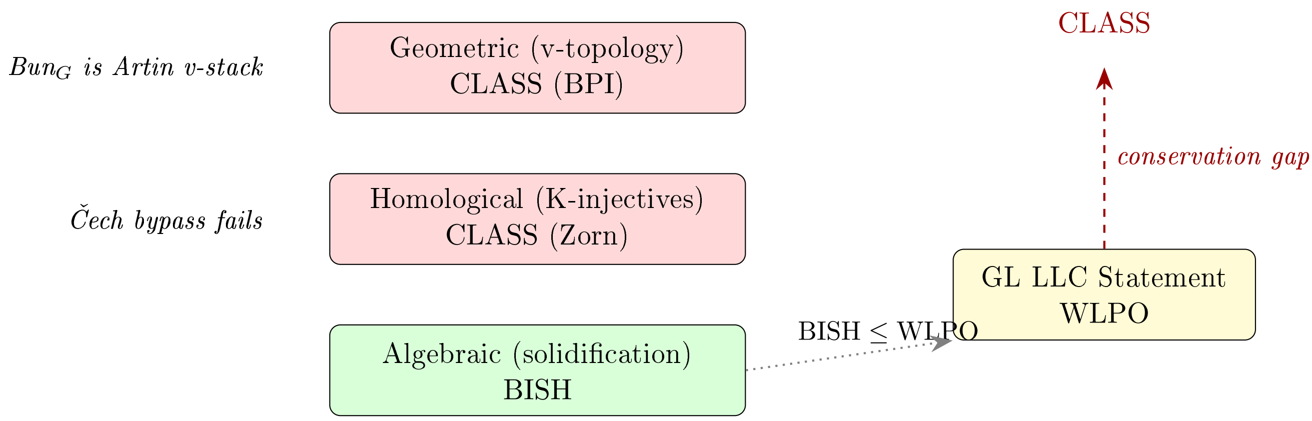

Figure 1: Three-layer stratification and conservation gap. The algebraic layer (green) is BISH; the homological and geometric layers (red) are CLASS. The GL statement (yellow) costs only WLPO, two levels below the proof.

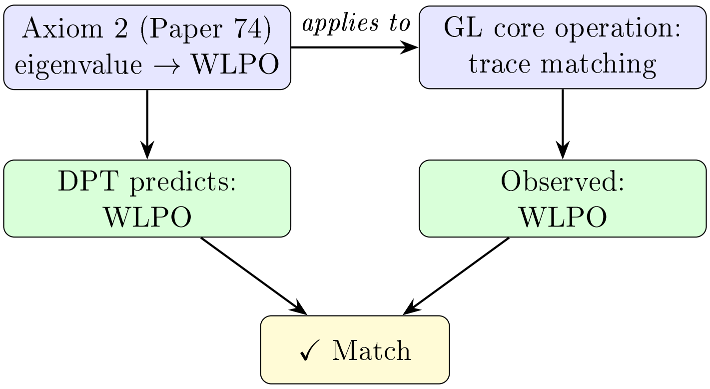

Figure 2: DPT prediction chain for the GL LLC. Axiom 2 (Paper 74) predicts WLPO for trace comparison; the GL parametrization's core operation is trace matching; the predicted and observed costs coincide.

6 Discussion

6.1 Conservation as open conjecture

The conservation gap (Theorem C) identifies a strict logical discrepancy between statement (WLPO) and proof (CLASS). Whether this gap is eliminable — whether the GL parametrization admits a BISH + WLPO proof — remains an open conjecture.

Finding such a proof would require bypassing both:

(i) Zorn's lemma in K-injective resolutions (the homological layer), and

(ii) BPI in v-covers (the geometric layer).

The contribution of Paper 75 is not proving eliminability but providing a precise map: the conservation test identifies *exactly where* CLASS enters the proof, enabling future logicians to target these specific obstructions.

6.2 The Čech obstruction

The v-site's infinite cohomological dimension (Remark 2.2) is the deeper obstruction. Even if Zorn's lemma could be avoided (e.g., by restricting to bounded derived categories), the geometric layer's use of BPI for v-covers remains. The v-topology is genuinely harder than the pro-étale topology, and this hardness is logical, not merely technical.

6.3 Connection to Paper 67 (final synthesis)

Paper 67 (the program's synthesis monograph) will incorporate three lines of evidence:

(1) The DPT biconditionals (Papers 72–74): each axiom is uniquely necessary.

(2) The conservation test (Paper 75): DPT predictions match external observations.

(3) The FLT pattern (Paper 68): classical proofs of arithmetically cheap theorems are a general phenomenon.

Together, these support the program's central thesis: the logical cost of mathematics is the logical cost of $\mathbb{R}$.

6.4 De-omniscientizing descent

The CRM program's broader project is *de-omniscientizing descent*: systematically identifying where omniscience principles enter mathematical proofs and testing whether they can be eliminated without losing the arithmetic content.

Theorem	**Classical tool**	**Statement**	**Proof**	**Gap**
FLT	Modularity lifting	BISH	CLASS	5 levels
Fun. field LLC	Trace formula	BISH	BISH*	0*
GL LLC (semisimple)	FS geometrization	WLPO	CLASS	2 levels

Table 6: De-omniscientizing descent across the program. *Paper 69 classifies the function field LLC proof as BISH. This relies on the étale site's finite cohomological dimension (enabling Čech bypass of K-injective resolutions) and Godement resolutions (avoiding Zorn for bounded derived categories). A full constructive audit of the homological layer for perverse sheaves on stacks of shtukas remains open.

The function field case (Paper 69) is instructive. L. Lafforgue's proof uses the trace formula and étale cohomology of shtukas. The étale site of a curve over $\mathbb{F}_q$ has finite cohomological dimension, so Čech complexes replace K-injective resolutions, and bounded derived categories D^b suffice — removing the geometric BPI obstruction and the unbounded-complex Zorn obstruction that force Fargues–Scholze into CLASS. The standard construction of perverse sheaves on stacks of shtukas uses the Grothendieck six-functor formalism; whether this remaining homological scaffolding is fully constructivizable is an open question (see Open Question 3).

The key spectral difference: function fields have algebraic spectral parameters [6]. As noted in Paper 69 (Remark 3.3), $z = q^{-s}$ lies on a compact algebraic torus, so the trace formula's eigenvalue comparison is exact algebraic arithmetic. Over local fields without geometric origin, the analytic Langlands spectrum introduces continuous parameters, forcing the statement cost up to WLPO.

This provides a consistency check: the DPT framework predicts that the function field LLC should be cheaper (no Axiom 2 cost, since the spectrum is algebraic), and this is indeed observed.

6.5 Open questions

1. Does conservation hold for other Fargues–Scholze results (e.g., the Kottwitz conjecture, the Harris–Viehmann conjecture)?

2. Can prismatic cohomology (Bhatt–Scholze [18]) provide a path that avoids the v-topology entirely, reducing the geometric layer from CLASS to a lower level?

3. Is the homological obstruction (Zorn for K-injectives) avoidable via Lurie's animation, condensed stable ∞-categories, or other higher-categorical techniques?

4. Does the conservation pattern extend to the full (not just semisimple) local Langlands correspondence? The full LLC involves non-semisimple parameters (L-packets with monodromy), which may require additional logical strength.

5. Is the WLPO statement cost sharp? Could the GL parametrization be stated in a way that avoids the trace equality test, reducing the cost to BISH?

7 Conclusion

The DPT framework, developed for motivic arithmetic (Papers 72–74), passes its first external validation test. Applied to the Genestier–Lafforgue parametrization — proved by condensed/perfectoid methods that never mention DPT axioms — the framework correctly predicts the statement cost: WLPO (Axiom 2, trace comparison). The Fargues–Scholze proof costs CLASS (homological Zorn + geometric BPI), but the statement costs only WLPO, with the gap consisting entirely of proof-theoretic scaffolding. Whether this scaffolding is eliminable is an open conjecture; the contribution here is the diagnostic map identifying where classical logic enters.

The three-layer stratification reveals that the logical complexity of the Fargues–Scholze program is not uniformly distributed. The algebraic layer (solidification) is constructively free (BISH): split epimorphisms of finite sets yield Mittag-Leffler trivially, without Dependent Choice. The logical weight concentrates entirely in the homological layer (Zorn for K-injective resolutions) and the geometric layer (BPI for v-covers). These two layers are logically independent — eliminating one does not eliminate the other — and both must be addressed by any future de-omniscientizing program.

The conservation test also provides a consistency check against previous program results. Paper 69 showed the function field LLC is BISH (zero gap); Paper 68 showed FLT is BISH (five-level gap). Paper 75's GL LLC at WLPO (two-level gap) falls between these extremes, consistent with the DPT prediction that trace comparison (Axiom 2) adds exactly one logical level above BISH operations.

The program's central thesis — the logical cost of mathematics is the logical cost of $\mathbb{R}$ — now rests on three pillars: biconditional characterization (Papers 72–74), external validation (Paper 75), and case studies (Papers 68–70). Paper 67 (revision) will synthesize these into a unified account.

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This paper was drafted with AI assistance (Claude, Anthropic; Gemini, Google). The three-layer stratification was developed in consultation with Gemini. The formal verification was developed and checked with Claude (Anthropic). The author is a clinician (interventional cardiology), not a professional mathematician; the logical structure of the main results has been verified by formal proof (Lean 4); the mathematical arguments supporting the layer cost assignments have been checked by the author and by consultation with domain experts. Errors of mathematical judgment remain the author's responsibility. This paper follows the standard format for the CRM series [19].

This series is dedicated to the memory of Errett Bishop (1928–1983), whose program demonstrated that constructive mathematics is not a restriction but a refinement.

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