1

(1)$\forall x\exists y\forall z\exists u.P(x,y,z,u)$

(2)$\forall x\exists z\exists u.P(x,f(x),z,u)$,这里$f$为一元新函数.

(3)$\forall x\forall zP(x,f(x),z,g(x,z))$,这里$g$为二元新函数。

这是原式子的Skolem范式.

2

(1)$(\forall xP(x)\wedge\forall yQ(y))\rightarrow\exists zP(z)$

(2)$\neg(\forall xP(x)\wedge\forall yQ(y)\vee\exists zP(z)$

(3)$\neg\forall xP(x)\wedge\neg\forall yQ(y)\vee\exists zP(z)$

(4)$\exists x\neg P(x)\wedge\exists y\neg Q(y)\vee\exists zP(z)$

(5)$\exists x\exists y\exists z(\neg P(x)\vee\neg Q(y))\wedge P(z))$

(6)$\exists x\exists y\exists z(\neg P(x)\wedge\neg Q(y))\rightarrow P(z))$

(6)中的原式的前束型范式.

3

由定义即得.

4

证：设m为任何model

设$m\vDash\exists x\forall yP(x,y)$,以下证明

$m\vDash\forall y\exists xP(x,y)$

$m\vDash\exists x\forall yP(x,y)\Rightarrow$

存在$a\in M$ 对任何$b\in M$, 有$<a,b>\in P_M\Rightarrow$

对任何$b\in M$, 有$a\in M<a,b>\in P_M\Rightarrow m\vDash\forall y\exists xP(x,y)$

5

证：欲证,$\nvDash\forall x\exists xP(x,y)\rightarrow\forall y\exists xP(x,y)$

只需构作m使$m\vDash\forall x\exists yP(x,y)$ ...(*)

但$m\nvDash\exists y\forall xP(x,y)$ ...(**)

令$m=(N,<)$,这里$P_M=\{(a,b)|a<b\}$

易见(*),(**)成立。

6

证：设m为任何model

因为$m\vDash\forall xP(x,f(f))\Rightarrow$

对任何$a\in M.<a,f_M(a)>\in P\Rightarrow$

对任何$a \in M$有$b \in M$使$< a, b > \in P$
$\Rightarrow m \vDash \forall x \exists y P(x, y)$
所以$m \vDash \forall x P(x, f(x)) \to \forall x \exists y P(x, y)$

7

证：$\forall x \exists y P(x, y)$可满足$< a, b > \in P_M \Rightarrow$
$\{$ 由AC知,存在$\tau : \mathcal{P}(M) - \{\phi\} \to M$使对$Z \subseteq M$,且$Z \neq \phi, \tau(Z) \in Z.\}$
对任何$a \in M$, $M_a = \{b| < a, b > \in P_M\} \neq \phi \Rightarrow$
对任何$a \in M$, $\tau(M_a) \in M_a \Rightarrow$
对任何$a \in M$, $< a, \tau(M_a) > \in P_M \Rightarrow$
$\{$令$f_M = \{< a, \tau(M_a) > |a \in M\}\}$
$m^{'} \vDash \forall x P(x, f(x))$,这里$m^{'} = m + f_M$

8

解：8.1 A 为$P(f(c))$
$H_0 = \{c\}$
$H_1 = H_0 \cup \{f(t)|t \in H_0\} = \{c\} \cup \{f(c)\} = \{c, f(c)\}$
$H_2 = \{c, f(c), f^2(c)\}$
...
$H_n = \{c, f(c), ..., f^n(c)\}$
$H = \{f^n(c)|n \in N\}$

9

证：对n归纳证明H_n有穷
Basis.$n = 0, H_0 = \{c_0\}$或$\{c|c$为α中常元$\}$
易见H_n有穷
I.H. H_n有穷
I. step $H_{n+1} = H_n \cup \{f(t_1, ..., t_k)|t_i \in H_n$
且f 为α中k元素$\}$
$|H_{n+1}| \leq |H_n| + |\{f|f$为$\alpha$中函数$\} \times H_n$
$< \aleph_0 + \aleph_0 \cdot \aleph_0 = \aleph_0$
又因为$\aleph_0 \leq |H| \leq \aleph_0 \cdot \aleph_0 = \aleph_0$
所以$\aleph_0 = |H|$