# 大学物理笔记

## 多普勒效应

source:$v_s$, observer:$v_O$,distance $L$

issued at $t = 0, x = 0$, recieved at $t'_1, L + v_O t'_1$

issued at $t = T, x = v_S T$, received at $t'_2, L + v_O t'_2$

由运动学:

$$v_P t'_1 = L + v_O t'_1$$
$$v_P(t'_2 - T) = L + v_O t'_2 - v_S T$$

于是

$$t'_1 = \frac{L}{v_P - v_O}$$
$$t'_2 = \frac{L}{v_P - v_O} + \frac{v_P - v_S}{v_P - v_O}T$$
$$T' = t'_2 - t'_1 = \frac{v_P - v_S}{v_P - v_O}T$$
$$\nu' = \frac{v_P - v_O}{v_P - v_S}\nu$$

Discussion:

$v_O > v_S$(including $v_S \leq 0$):separating, $\nu' < \nu$, Doppler redshift

$v_S > v_O$(including $v_O \leq 0$):approaching, $\nu' > \nu$, Doppler blueshift

靠近:频率变高;远离:频率变低

考虑波源的运动与波振面的运动(supersonic speed $v_S > v_p$)

马赫锥:$sin\theta = \frac{v_P}{v_S}$

## 驻波(stationary wave)

- 推导

由$kx - \omega t - \phi = const$,求导得 $v = \frac{\omega}{k}$

$$u_1 = Acos(kx - \omega t - \phi)$$
$$u_2 = Acos(kx + \omega t)$$
$$= Acos(k(x + \frac{\omega}{k}t)) \tag{1}$$
$$从而 u_1 + u_2 = 2Acos(kx - \frac{\phi}{2})cos(wt + \frac{\phi}{2})$$

令 $(u_1 + u_2)_{x=0} = 0$,得 $\phi = \pi$; 令 $(u_1 + u_2)_{x=L} = 0$,得 $L = \frac{n\pi}{k} = \frac{n\lambda}{2}$

注意 $\lambda = T \cdot v_P = \frac{2\pi}{\omega} \cdot \frac{\omega}{k} = \frac{2\pi}{k}, k = \frac{2\pi}{\lambda}, \omega = vk$,有

$$
\begin{aligned}
f &= \frac{\omega}{2\pi} = \frac{kv}{2\pi} = n\frac{v}{2L} \\
&= f_n \\
&= nf_1
\end{aligned}
$$

波节(node),波腹(antinode)的定义

- 半波损失

fixed hard boundary; from high spped to low speed

由(1)式,令 $(u_1 + u_2)_{x=0} = 0$,得 $\phi = \pi$,即为在固定端的相差

# 第9章    Relativistic Mechanics 相对论力学

## 9.1    伽利略变换

- 观测者:一套矫正好的时钟,可记录 $x, y, z, t$

- 结果 $t'_A - t'_B = t_A - t_B, x'_A - x'_B = x_A - x_B - u(t_A - t_B)$

  同时测量:$x'_A - x'_B = x_A - x_B$

  速度直接叠加,加速度不变

- Galileo's relativity: 力学定律在不同惯性参考系中相同

- 电学:不满足麦克斯韦方程组

## 9.2    洛伦兹变换 Lorentz Transformation

- Two basic principles of Special Theory of Relativity
  - 所有惯性参考系中物理定律相同(力学,电学等)
  - 光速不变
- 推导

假定变换为线性变换(考虑匀速直线运动);

待定系数:

$$
\begin{aligned}
x' &= a_{11}x + a_{12}y + a_{13}z + a_{14}t \\
y' &= y \\
z' &= z
\end{aligned}
$$

For arbitrary y,z if $x' = 0, x = ut$;则

$$
x' = a_{11}(u)(x - ut), a_{11}(0) = 1 \tag{1.1}
$$

By the priciple of relativity, we have

$$x = a_{11}(-u)(x' + ut'),\text{带入}(1.1),\text{有}$$
$$t' = f(x, x'(u, t)) = a_{44}t + a_{41}x$$

从而

$$x' = a(x - ut)$$
$$t' = b(t - ex)$$

(1.2)

解得 $x = \frac{1}{\Delta}(bx' + aut')$

与 $x = a_{11}(-u)(x' + ut')$比较,得 $a = b$

考虑光速不变:假设 $t = t' = 0$时在原点发出光信号,有

$$x^2 + y^2 + z^2 = c^2 t^2$$
$$x'^2 + y'^2 + z'^2 = c^2 t'^2$$

代入 (1.2)式有

$$e = \frac{u}{c^2}$$
$$a = \frac{1}{\sqrt{1 - \beta^2}} = \gamma = \sqrt{1 - \frac{v^2}{c^2}}$$
$$\text{其中}\beta = \frac{u}{c}$$

完整的变换关系为

$$x' = \gamma(x - ut)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \frac{u}{c^2}x)$$

考虑事件对 $(x'_1, t'_1), (x'_2, t'_2),$洛伦兹变换是

$$\Delta x' = \gamma(\Delta x - u\Delta t)$$
$$\Delta y' = \Delta y$$
$$\Delta z' = \Delta z$$
$$\Delta t' = \gamma(\Delta t - \frac{u}{c^2}\Delta x)$$

(1.3)

- 讨论

  - 逆变换: $u \to -u$    直接根据相对性原理
  - Invariance of spacetime interval 时空间隔不变:$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 = (\Delta s')^2$
  - 伽利略变换:时间归时间 空间归空间    时间空间分别不变

- 事件对

  - 光信号联系: $(c\Delta t)^2 = (\Delta x)^2, \Delta s = 0$
  - 固有时间间隔与固有空间间隔

- Simultaneous measurement $\Delta x' = \ell_0$ in $S'(\Delta t' = 0)$ 形成一个事件

  the same pair of events in S:

  $$\Delta x = \gamma(\Delta x' + u\Delta t') = \gamma\Delta x' = \gamma\ell_0$$
  $$\Delta t = \gamma(\Delta t + \frac{u}{c^2}\Delta x')\gamma = 0 \qquad \text{同时的相对性}$$

也即 $\ell = \ell_0\sqrt{1 - \frac{v^2}{c^2}} = \frac{\ell_0}{\gamma}$

- 用光速不变推出钟慢:

  $$\Delta t' = \frac{2L_0}{c}$$
  $$(\frac{u\Delta t}{2})^2 + L_0^2 = (\frac{c\Delta t}{2})^2 (\text{用到光速不变})$$
  $$\Delta t = \frac{2L_0}{\sqrt{c^2 - u^2}} = \frac{2L_0/c}{\sqrt{1 - \beta^2}} = \gamma\Delta t'$$

- 用光速不变推出尺缩:

  $$c\Delta t_1 = L + u\Delta t_1, c\Delta t_2 = L - u\Delta t_2$$
  $$\Delta t = \Delta t_1 + \Delta t_2 = \frac{L}{c + u} + \frac{L}{c - u} = \frac{2L}{c}\frac{1}{1 - \beta^2}$$
  $$\Delta t' = \frac{2L_0}{c} = \gamma\Delta t$$
  $$\frac{2L_0}{c\gamma} = \frac{2L}{c}\frac{1}{1 - \beta^2}$$
  $$\text{于是} L = \frac{L_0}{\gamma}$$

- 因果律及信号传播(Causality and signal speed)

  - An event $P(x_P, t_P)$ causes an event $Q(x_Q, t_Q)$. 信号传输速度

    $$v_S = \frac{x_Q - x_P}{t_Q - t_P} = \frac{\Delta x}{\Delta t}$$

  在 $S'(u)$ 系中时间间隔为

  $$\Delta t' = \gamma\Delta t(1 - \frac{uv_S}{c^2})$$

  为保证先后次序,必须有

  $$1 - \frac{uv_S}{c^2} < 0$$
  $$\text{则} v_S < c$$

- 光多普勒效应

  - 由(1.3), $\Delta t = \sqrt{\frac{1+\beta}{1-\beta}}\Delta t'$
  - redshift: $\nu_0 = \sqrt{\frac{1-\beta}{1+\beta}}\nu_e$

- 波动方程: 用洛伦兹变换恰好符合(洛伦兹不变)

## 9.3 时空图和孪生paradox

- Minkowski Space
  - 洛伦兹变换在时空图中的体现
- Twin paradox: 在宇宙飞船加速,减速的过程中经历了非惯性系
- Pole-barn Paradox: 火车进隧道;火车系:隧道收缩;地面系:火车收缩
  - 涉及同时的相对性:看能否在地面系同时关上前门和后门(火车系觉得没有同时)
  - 地面系的观测者觉得能关进去,在火车系的观测者看来后门早就关起来了
- Visual Apperance
  - 球不变 立方体转过一个角度

## 9.4 相对论运动学

- 速度变换公式(求导可得)

$$v'_x = \frac{v_x - u}{1 - v_x \dfrac{u}{c^2}}$$

$$v'_y = \frac{v_y}{\gamma(1 - v_x \dfrac{u}{c^2})}$$

$$v'_z = \frac{v_z}{\gamma(1 - v_x \dfrac{u}{c^2})}$$

## 9.5 相对论动力学(Relativistic Dynamics)

- 广义相对论: 引力场,非惯性系
- 讨论$m, p, F, E_k, a$
- Discuss a completely inelastic collision(保持动量守恒)
  - In $S'(-u) : u, -u \to 0, 0$
  - In $S : v_x, 0 \to u, u;$ $\quad v_x = \dfrac{u + u}{1 + u\frac{u}{c^2}} (9.5.1)$
  - 而 $m\dfrac{u + u}{1 + \frac{uu}{c^2}} + m \times 0 = 2mu$,动量守恒不成立.
  - 假定 $m \equiv m(v), m_0 \equiv m(0)$,有

$$m(v_x) + m_0 = m_{total}(u)$$
$$m(v_x) \cdot v_x + m_0 \cdot 0 = m_t(u)u$$
$$= [m(v_x) + m_0]u$$
$$求得 m(v_x) = \frac{m_0}{\frac{v_x}{u} - 1}$$

由 $(9.5.1), \dfrac{v_x}{u} - 1 = \sqrt{1 - \beta^2}$,则 $m(v) = \gamma m_0$. 于是

$$p = \gamma m_0 v$$

$$E_k = \int F \cdot dx = \int \frac{d}{dt}(mv) \cdot dx = \int v \cdot d(mv)$$

$$\overline{\text{而}} d(mv) = vdm + mdv$$

$$\text{则} E_k = \int (v^2 dm + mvdv)$$

考虑 $m^2(1 - \frac{v^2}{c^2}) = m_0^2, m^2(c^2 - v^2) = m_0^2 c^2$,两边微分得

$$c^2 2mdm - v^2 2mdm - m^2 2vdv = 0,$$
$$c^2 dm - v^2 dm - mdv = 0, c^2 dm = v^2 dm + mdv$$

于是

$$E_k = \int c^2 dm = mc^2 - m_0 c^2$$
$$= m_0 c^2 \left[ \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} - 1 \right]$$
$$= (\gamma - 1) m_0 c^2$$

- 讨论
  - 牛顿力学中

$$E_k = \int v \cdot d(mv) = \int mv \cdot dv = \frac{1}{2}mv^2$$

- - 相对论中,mass-energy $E = mc^2 = E_k + m_0 c^2$
- - 已知动能求速度: $\gamma = \dfrac{E_k}{m_0 c^2} + 1$
- 相对论能量-动量关系
  - $E = \sqrt{c^2 p^2 + m_0^2 c^4}$
  - 若 $m_0 = 0, E = cp, p = \dfrac{E}{c}$
  - 若 $cp << m_0 c^2$,泰勒展开后得到牛顿力学.

# 期末考试大致内容

- 相对论
- 理想气体(或给定气体的状态方程) 等温膨胀,绝热等过程, 功和热等
- 理想气体的性质: 速度,动能等
- 过程中温度的变化,熵的变化
- **亥姆霍兹自由能**, 求平衡态(可能有数学上求导等)
- 比热容, 温度.
- 相变 三相点.

# 第十章 温度

## 10.1 Equilibruim state

- 

| System | exchange |
|--------|----------|
| isolated | $\times$ |
| closed | energy |
| open | energy&matter |

- Thermodynamic Equilibruim state: 宏观状态不随时间改变,除非外界条件发生变化.
- description of equilibruim state: state variables
    - extensive quantity: $F(n\ systems) = nF(1\ system)$
    - intensive quantity: $F(n\ systems) = F(1\ system)$
- relaxation time $\tau$: 恢复平衡所用时间
- Quasi-static process 准静态过程: $t >> \tau$, 每一点都是平衡态, 可以在状态图上画出.
- No-dissipative, quasi-static process is reversible. (无耗散准静态过程是可逆的); All natural process(自发过程) is irreversible.
    - 系统和外界同时恢复初态

## 10.2 Thermal equilibruim and temperature

- The zeroth law of thermodynamics: 热平衡的传递性(热平衡的系统有共同性质)
- 温度: 衡量热平衡的一种物理性质
- 各种温标

## 10.4 物态方程

- 微分的常见写法 $\left(\dfrac{\partial U}{\partial V}\right)_T$ , $\left(\dfrac{\partial U}{\partial V}\right)_P$ . 保持右下标不变
- $P, V, T$不独立, $f(P, V, T) = 0 \rightarrow U(T, P) = U(T, V(P, T)) = U(T, V) = 0$
- 等压线膨胀系数 $a_\ell = \dfrac{1}{L}\dfrac{\partial L}{\partial T}_p$ ; $\alpha_V = \dfrac{1}{V}\left(\dfrac{\partial V}{\partial T}\right)_P = \dfrac{1}{L^3}\left(\dfrac{\partial L^3}{\partial T}\right)_P = 3\alpha_\ell$
- 等温线膨胀系数 $\kappa_T = -\dfrac{1}{V}\dfrac{\partial V}{\partial p}_T$
- 体积变化方程

$$\frac{dV}{V} = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p dT + \frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T dp = \alpha_V(T)dT - \kappa_T(p)dp$$
$$(\text{全微分})$$
$$ln\frac{V}{V_0} = \alpha_V \Delta T - \kappa_T \Delta p$$
$$V = V_0(1 + \alpha_V \Delta T - \kappa_T \Delta p)$$

- Van der Waal's equation

$$\left[p + a\left(\frac{n}{V}\right)^2\right](V - nb) = nRT$$

推导: 对1mol理想气体:

$$p = \frac{RT}{V_m} \leftarrow \frac{RT}{V_m - b}(\text{相互作用, 体积减小})$$
$$\leftarrow \frac{RT}{V_m - b} - \frac{a}{V^2}(\text{相互作用, 压强减小, 体积越小作用力越大})$$

# 第十一章 热力学第一定律

## 11.1 功,内能和热力学第一定律

- 准静态过程压活塞 $\bar{d}W = -p\bar{d}V$, $W = -\int_{V_i}^{V_f} pdV$ (依赖于 $P-V$ 图上的路径,不是全微分)

- 绝热功: Internal energy function $\Delta U = U_B - U_A \equiv W_{BA} \equiv -W_{AB}$ (用与路径无关的绝热功定义内能变化)

- 非绝热功: $\Delta U = W + Q$ 外界做的功和外界传给内部的热量; 微分 $dU = \bar{d}Q + \bar{d}W$ (功和热不是全微分,与路径有关)

## 11.3 热容和比热容(Heat capacity and specific heat capacity)

- $C = \lim\limits_{\Delta T \to 0} \dfrac{\Delta Q}{\Delta T}$; specific: $C_m = \dfrac{C}{n}$

  - $C = 0$: 绝热(adiabatic)过程, $\Delta Q = 0$
  - $C = \infty$: 等温(isothermal)过程, $\Delta T = 0$

- $C_V = \lim\limits_{\Delta T \to 0}(\dfrac{\Delta Q}{\Delta T})_V = \lim\limits_{\Delta T \to 0}(\dfrac{\Delta U}{\Delta T})_V = (\dfrac{\partial U}{\partial T})_V$

- $C_P = \lim\limits_{\Delta T \to 0}(\dfrac{\Delta Q}{\Delta T})_p = \lim\limits_{\Delta T \to 0}(\dfrac{\Delta U + p\Delta V}{\Delta T})_p = (\dfrac{\partial(U + pV)}{\partial T})_p \equiv (\dfrac{\partial H}{\partial T})_{p'}$, $H = U + pV$: enthalpy, 焓

- ratio of specific heat: $\gamma = \dfrac{C_p}{C_V}$

- heat capacity of ideal gases

| **monatomic ideal gas** | $C_V = \frac{3}{2}nR$ |
| --- | --- |
| 双原子 ideal gas | $C_V = \frac{5}{2}nR$ |
| 三原子 ideal gas | $C_V = \frac{6}{2}nR$ |

## 11.4 Free expansion and internal energy of gas

- Case of free expansion

$$U = U(T, V)$$

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

$$T = T(U, V)$$

$$dU = \left(\frac{\partial U}{\partial T}\right)_V \left[\left(\frac{\partial T}{\partial U}\right)_V dU + \left(\frac{\partial T}{\partial V}\right)_U dV\right] + \left(\frac{\partial U}{\partial V}\right)_T dV$$

对照左右, 得

$$\left(\frac{\partial U}{\partial T}\right)_V \left(\frac{\partial T}{\partial U}\right)_V = 1$$

$$\left(\frac{\partial U}{\partial V}\right)_T = -\left(\frac{\partial U}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_U = -C_V \left(\frac{\partial T}{\partial V}\right)_U \rightarrow 焦耳系数 = 0(理想气体)$$

$$故 U = U(T)$$

- 焦耳的实验非常不严谨
- 理想气体:

$$dU(T) = \frac{dU}{dT}dT = C_V dT$$

$$U = U_0 + C_V T$$

$$dQ = dU - dW = C_V dT + pdV$$

$$理想气体: d(pV) = d(nRT)$$

$$pdV + Vdp = nRdT, pdV = nRdT - Vdp$$

$$dQ = (C_V + nR)dT - Vdp$$

$$C_p = C_V + nR \rightarrow C_V = \frac{nR}{\gamma - 1}, C_p = \frac{\gamma nR}{\gamma - 1}$$

## 11.5 Adiabatic equation

$$dQ = dU - dW,$$

$$= C_V dT + pdV$$

$$C_V dT + pdV = 0$$

$$又 Vdp + pdV = nRdT = (\gamma - 1)C_V dT$$

消去 $dT$, 得

$$Vdp + \gamma pdV = 0$$

$$\frac{dp}{p} + \gamma \frac{dV}{V} = 0$$

$$lnp + \gamma lnV = C$$

$$pV^\gamma = C$$

由 $\gamma > 1$, 绝热线比等温线陡峭

- adiabatic work

$$W_S = -\int_{V_1}^{V_2} p\,dV = -C\int_{V_1}^{V_2} \frac{dV}{V^\gamma} = \frac{C}{\gamma-1}\left(\frac{V_2}{V_2^\gamma} - \frac{V_1}{V_1^\gamma}\right)$$

$$= \frac{1}{\gamma-1}(p_2 V_2 - p_1 V_1)$$

$$= \frac{nR}{\gamma-1}(T_2 - T_1)$$

$$= C_V(T_2 - T_1) = \Delta U$$

- 声音的传播: 绝热

## 11.6 卡诺循环

- 高温热源 $T_1$ 等温膨胀→绝热膨胀→低温热源 $T_2$ 等温压缩→绝热压缩
- $T_1, U = U(T_1) = const$ 热机吸热,做功
- $T_2$ 热机放热
- 整个循环: $\Delta U = 0, \Delta Q_{12} + \Delta Q_{34} - \Delta W = 0$

$$效率 \eta = \frac{\Delta W}{\Delta Q_{12}} = \frac{\Delta Q_{12} + \Delta Q_{34}}{\Delta Q_{12}} = \frac{Q_1 - Q_2}{Q_1}$$

$$(考察做功过程中的放热Q_2, 放热越多效率越低)$$

$$等温过程: Q_1 = \Delta W_{12} = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{nRT_1}{V} dV = nRT_1 ln\frac{V_2}{V_1}$$

$$Q_2 = -\Delta Q_{34} = nRT_2 ln\frac{V_3}{V_4}$$

$$\eta = 1 - \frac{T_2 ln\frac{V_3}{V_4}}{T_1 ln\frac{V_2}{V_1}}$$

$$2 \to 3 : T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

$$4 \to 1 : T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$$

$$\eta = 1 - \frac{T_2}{T_1}$$

要学会将此推导推广到其他热机上

- 可逆过程: 外界不可逆
- 逆卡诺循环: 从低温热源吸热 到高温热源被做功,放热

# 第十二章 热力学第二定律

## 12.1 内容

- No process is possible whose sole result is the absorption of heat from a reservoir and the conversion of this heat into work.
- No process is possible whose sole result is the transfer of heat from a cooler to a hotter body.
- 自发过程具有方向性

## 12.2 卡诺定理

- 任何热机效率不超过可逆热机

- 证明: 任意热机做功驱动可逆热机的逆过程, 反证法与热二矛盾

## 12.3 熵与熵原理

- $Q_2 < 0$放热:

$$效率不大于卡诺热机 : \eta_A = 1 + \frac{Q_2}{Q_1} \leq 1 - \frac{T_2}{T_1}$$

$$\sum_{i=1}^{2} \frac{Q_i}{T_i} \leq 0$$

$$一般过程 : \sum_{i=1}^{n} \frac{Q_i}{T_i} \leq 0(系统与很多热源接触)$$

$$\oint \frac{dQ}{T} \leq 0, 取等 : 可逆$$

$$写作全微分 : \frac{dQ}{T} \equiv dS$$

$$S_B - S_A = \int_A^B (\frac{dQ}{T})_R$$

克劳修斯不等式的说明: 考虑多热源与单一热源 $T_0$ 由多个卡诺机联合