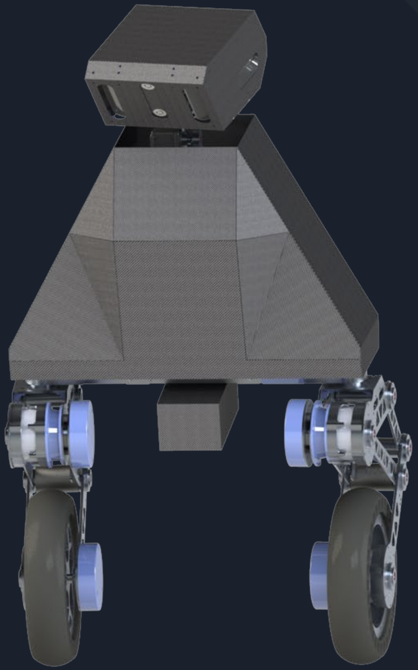
EKF+LQR+IMU =TWR

Skoltech 2022

Vladimir Guneavoy



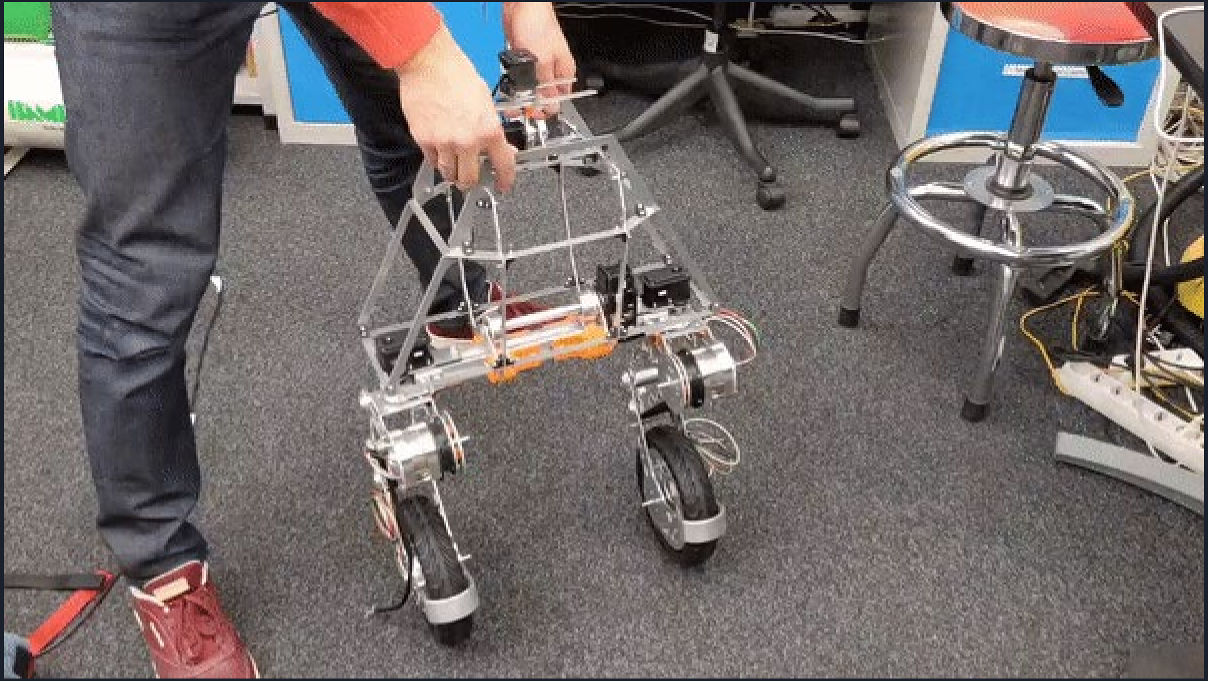
Problem statement

- Robot is an inverted pendulum
- Inverted pendulum is unstable

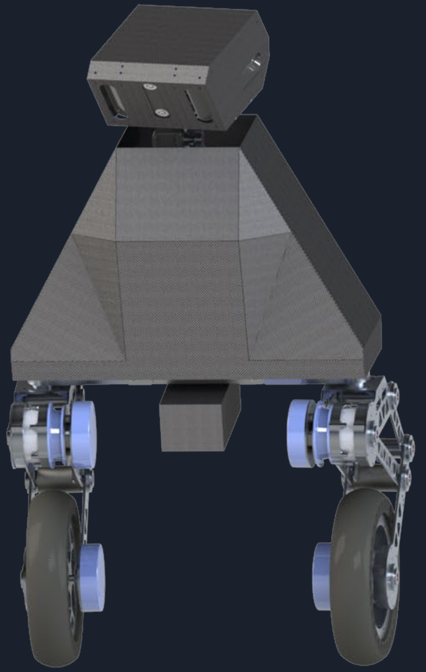
Methodology

- Develop LQR control for given system of inverted pendulum with given:
    - Geometry
    - Mass distribution
    - Kinematics

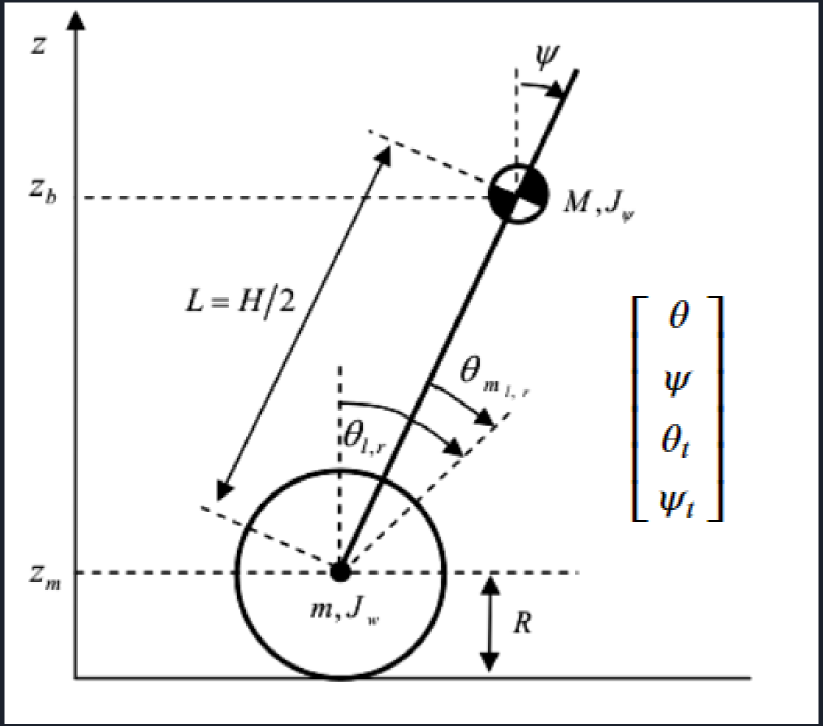
Robot’s Frame



3D Model of the Robot



Robot 3D model



Considering the robot as a inverted pendulum

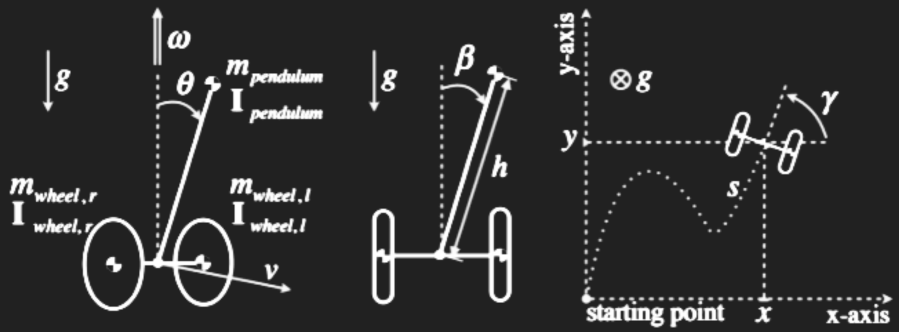
System equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i^n$$

Lagrange equation (2nd type)

$$\mathcal{L} = T - V$$

Lagrange function



Robot physical model

Equations

$$\begin{cases} F_\theta &= MR^2\ddot{\theta} + MRL\ddot{\psi}\cos\psi - MRL\dot{\psi}^2\sin\psi, \\ F_\psi &= MRL\ddot{\theta}\cos\psi + (ML^2 + J_\psi)\ddot{\psi} - MgL\sin\psi. \end{cases}$$

Linear Quadratic Regulator

$$\dot{x} = A(t)x + B(t)u$$

State equation

$$J = \int_0^\infty \left(x^T Q(t) x + u^T R(t) u\right) dt$$

Optimization criteria

$$u = -R^{-1} B^T P x$$

Optimal solution

$$A^T P + PA - PBR^{-1} B^T P + Q = -\dot{P}$$

Riccati equation

Matrices

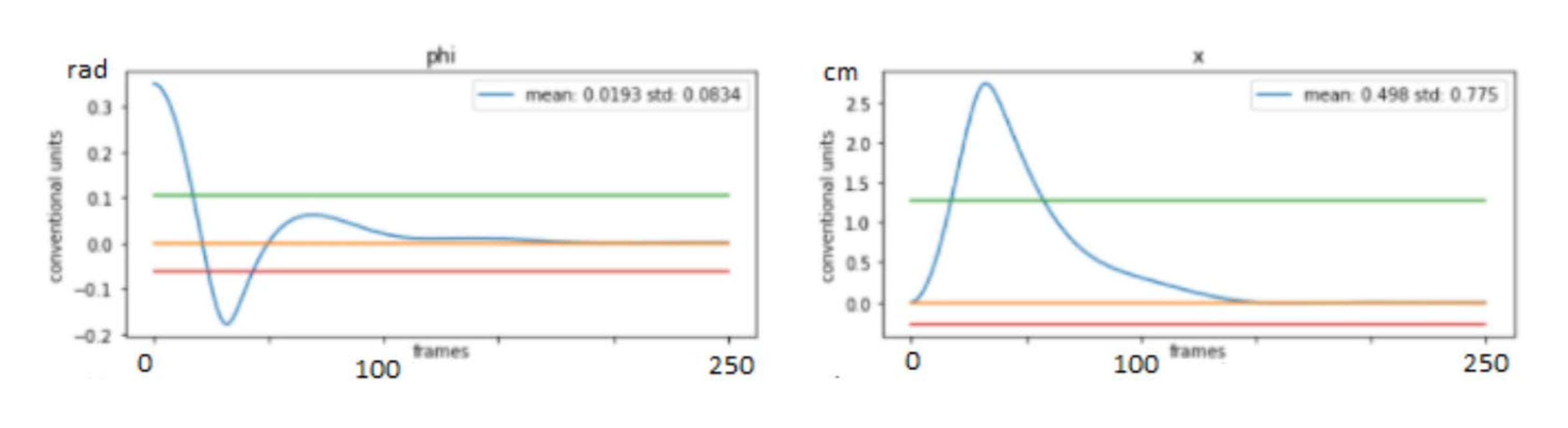
```
A
[[ 0.      0.      0.      1.      0.      0.    ]
 [ 0.      0.      0.      0.      1.      0.    ]
 [ 0.      0.      0.      0.      0.      1.    ]
 [ 0.      0.     -0.001   0.      0.      0.    ]
 [ 0.      0.     -0.      0.      0.      0.    ]
 [ 0.      0.      0.004   0.      0.      0.    ]]

B
[[ 0.          0.        ]
 [ 0.          0.        ]
 [ 0.          0.        ]
 [ 0.05760363  0.05760363]
 [ 0.00928153 -0.00928153]
 [-0.00234554 -0.00234554]]
```

```
Q
[[1. 0. 0. 0. 0. 0.]
 [0. 1. 0. 0. 0. 0.]
 [0. 0. 1. 0. 0. 0.]
 [0. 0. 0. 1. 0. 0.]
 [0. 0. 0. 0. 1. 0.]
 [0. 0. 0. 0. 0. 1.]]

R
[[1. 0.]
 [0. 1.]]
```

Theoretical result



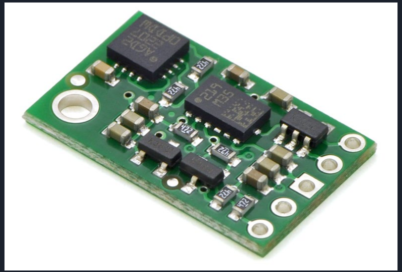
First try

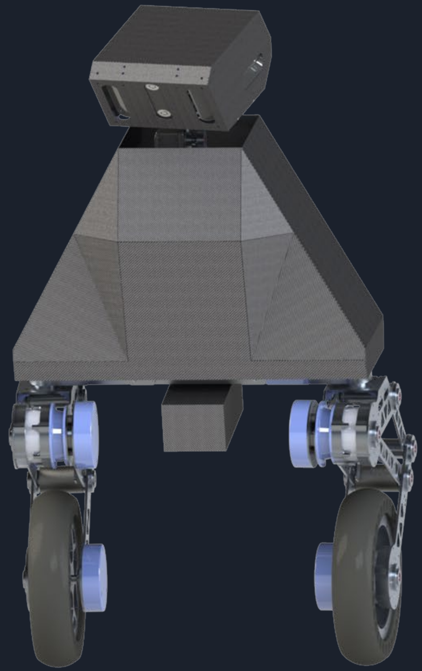
Demonstration

Problem

- IMU
  - angular velocity
  - acceleration

- Encoder
  - ground position





Kalman Filter

$$\hat{\boldsymbol{x}}_{k|k-1} = f(\hat{\boldsymbol{x}}_{k-1|k-1}, \boldsymbol{u}_k)$$

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_k \boldsymbol{P}_{k-1|k-1} \boldsymbol{F}_k^\top + \boldsymbol{Q}_k$$

$$\tilde{\boldsymbol{y}}_k = \boldsymbol{z}_k - h(\hat{\boldsymbol{x}}_{k|k-1})$$

$$\boldsymbol{S}_k = \boldsymbol{H}_k \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^\top + \boldsymbol{R}_k$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^\top \boldsymbol{S}_k^{-1}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k \tilde{\boldsymbol{y}}_k$$

$$\boldsymbol{P}_{k|k} = (\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k) \boldsymbol{P}_{k|k-1}$$

$$\boldsymbol{F}_k = \left.\frac{\partial f}{\partial \boldsymbol{x}}\right|_{\hat{\boldsymbol{x}}_{k-1|k-1}, \boldsymbol{u}_k}$$

$$\boldsymbol{H}_k = \left.\frac{\partial h}{\partial \boldsymbol{x}}\right|_{\hat{\boldsymbol{x}}_{k|k-1}}$$

Observation description

Accelerations are **not** the part of state vector!

$$\begin{bmatrix} L\psi_{tt} + R\theta_{tt}\cos(\psi) + g\sin(\psi) \\ R\theta_{tt}\sin(\psi) - g\cos(\psi) \\ \theta \\ \psi_t \end{bmatrix}$$

BUT!

They are functions of state vector!

Observation description

$$\begin{bmatrix} L\psi_{tt} + R\theta_{tt}\cos(\psi) + g\sin(\psi) \\ R\theta_{tt}\sin(\psi) - g\cos(\psi) \\ \theta \\ \psi_t \end{bmatrix}$$

$$\begin{bmatrix} \theta_{tt} \\ \psi_{tt} \end{bmatrix}$$

$$\begin{bmatrix} -\dfrac{LMR(LMg\sin(\psi)-T)\cos(\psi)}{2I_\psi I_w+I_\psi MR^2+2I_\psi R^2m+2I_wL^2M-L^2M^2R^2\cos^2(\psi)+L^2M^2R^2+2L^2MR^2m}+\dfrac{(I_\psi+L^2M)(LMR\psi_t^2\sin(\psi)+T)}{2I_\psi I_w+I_\psi MR^2+2I_\psi R^2m+2I_wL^2M-L^2M^2R^2\cos^2(\psi)+L^2M^2R^2+2L^2MR^2m} \\ -\dfrac{LMR(LMR\psi_t^2\sin(\psi)+T)\cos(\psi)}{2I_\psi I_w+I_\psi MR^2+2I_\psi R^2m+2I_wL^2M-L^2M^2R^2\cos^2(\psi)+L^2M^2R^2+2L^2MR^2m}+\dfrac{(LMg\sin(\psi)-T)(2I_w+MR^2+2R^2m)}{2I_\psi I_w+I_\psi MR^2+2I_\psi R^2m+2I_wL^2M-L^2M^2R^2\cos^2(\psi)+L^2M^2R^2+2L^2MR^2m} \end{bmatrix}$$

Observation description

$$\begin{bmatrix} L\psi_{tt} + R\theta_{tt}\cos(\psi) + g\sin(\psi) \\ R\theta_{tt}\sin(\psi) - g\cos(\psi) \\ \theta \\ \psi_t \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{\begin{array}{l} 2I_\psi I_w g\sin(\psi) + I_\psi L M R^2 \psi_t^2 \sin(\psi)\cos(\psi) + I_\psi M R^2 g\sin(\psi) + 2I_\psi R^2 g m\sin(\psi) + I_\psi R T\cos(\psi) + 4I_w L^2 M g\sin(\psi) - 2I_w L T - 2L^2 M^2 R^2 g\sin(\psi)\cos^2(\psi) + 2L^2 M^2 R^2 g\sin(\psi) \\ +4L^2 M R^2 g m\sin(\psi) + L M R^2 T\cos^2(\psi) - L M R^2 T - 2 L R^2 T m \end{array}}{2I_\psi I_w + I_\psi M R^2 + 2I_\psi R^2 m + 2I_w L^2 M - L^2 M^2 R^2\cos^2(\psi) + L^2 M^2 R^2 + 2L^2 M R^2 m} \\ \dfrac{\begin{array}{l} -2I_\psi I_w g\cos(\psi) + I_\psi L M R^2 \psi_t^2 \sin^2(\psi) - I_\psi M R^2 g\cos(\psi) - 2I_\psi R^2 g m\cos(\psi) + I_\psi R T\sin(\psi) - 2I_w L^2 M g\cos(\psi) + L^3 M^2 R^2 \psi_t^2 \sin^2(\psi) - L^2 M^2 R^2 g\sin^2(\psi)\cos(\psi) + L^2 M^2 R^2 g\cos^3(\psi) \\ -L^2 M^2 R^2 g\cos(\psi) - 2L^2 M R^2 g m\cos(\psi) + L^2 M R T\sin(\psi) + L M R^2 T\sin(\psi)\cos(\psi) \end{array}}{2I_\psi I_w + I_\psi M R^2 + 2I_\psi R^2 m + 2I_w L^2 M - L^2 M^2 R^2\cos^2(\psi) + L^2 M^2 R^2 + 2L^2 M R^2 m} \\ \theta \\ \psi_t \end{bmatrix}$$

Observation jacobian

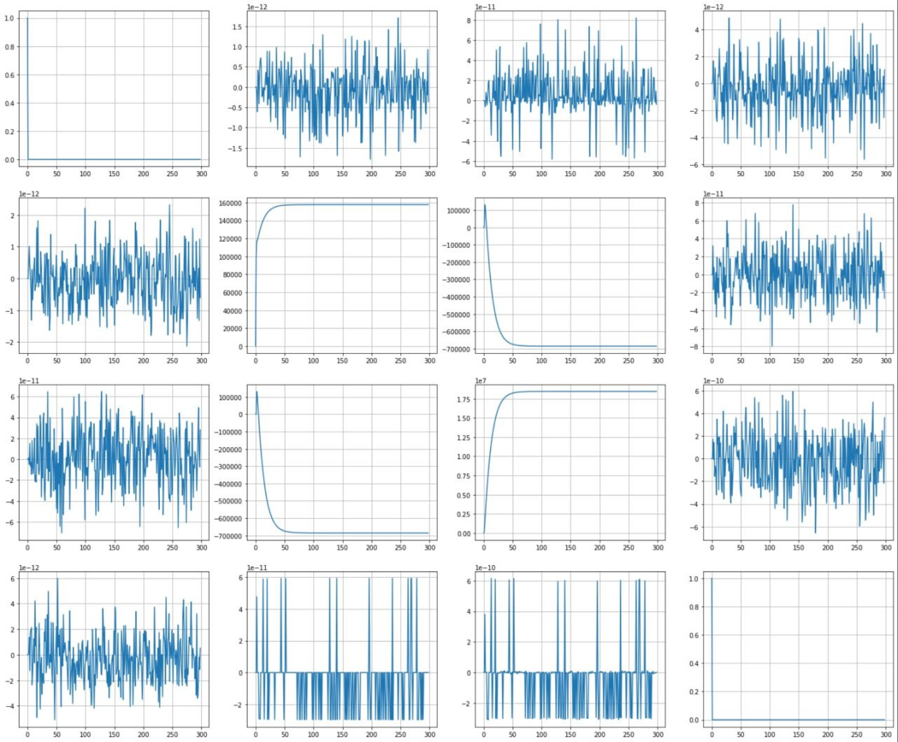
$$
\begin{bmatrix}
0 & -\dfrac{2L^2M^2R^2\cdot\left(2I_\psi I_\omega g\sin(\psi)+I_\psi LMR^2\psi_t^2\sin(\psi)\cos(\psi)+I_\psi MR^2g\sin(\psi)+2I_\psi R^2gm\sin(\psi)+I_\psi RT\cos(\psi)+4I_\omega L^2Mg\sin(\psi)-2I_\omega LT-2L^2M^2R^2g\sin(\psi)\cos^2(\psi)+2L^2M^2R^2g\sin(\psi)+4L^2MR^2gm\sin(\psi)+LMR^2T\cos^2(\psi)-LMR^2T-2LR^2Tm\right)\sin(\psi)\cos(\psi)}{\left(2I_\psi I_\omega+I_\psi MR^2+2I_\psi R^2m+2I_\omega L^2M-L^2M^2R^2\cos^2(\psi)+L^2M^2R^2+2L^2MR^2m\right)^2} \\
 & +\dfrac{2I_\psi I_\omega g\cos(\psi)-I_\psi LMR^2\psi_t^2\sin^2(\psi)+I_\psi LMR^2\psi_t^2\cos^2(\psi)+I_\psi MR^2g\cos(\psi)+2I_\psi R^2gm\cos(\psi)-I_\psi RT\sin(\psi)+4I_\omega L^2Mg\cos(\psi)+4L^2M^2R^2g\sin^2(\psi)\cos(\psi)-2L^2M^2R^2g\cos^3(\psi)+2L^2M^2R^2g\cos(\psi)+4L^2MR^2gm\cos(\psi)-2LMR^2T\sin(\psi)\cos(\psi)}{2I_\psi I_\omega+I_\psi MR^2+2I_\psi R^2m+2I_\omega L^2M-L^2M^2R^2\cos^2(\psi)+L^2M^2R^2+2L^2MR^2m} \\
0 & -\dfrac{2L^2M^2R^2\left(-2I_\psi I_\omega g\cos(\psi)+I_\psi LMR^2\psi_t^2\sin^2(\psi)-I_\psi MR^2g\cos(\psi)-2I_\psi R^2gm\cos(\psi)+I_\psi RT\sin(\psi)-2I_\omega L^2Mg\cos(\psi)+L^3M^2R^2\psi_t^2\sin^2(\psi)-L^2M^2R^2g\sin^2(\psi)\cos(\psi)+L^2M^2R^2g\cos^3(\psi)-L^2M^2R^2g\cos(\psi)-2L^2MR^2gm\cos(\psi)+L^2MRT\sin(\psi)+LMR^2T\sin(\psi)\cos(\psi)\right)\sin(\psi)\cos(\psi)}{\left(2I_\psi I_\omega+I_\psi MR^2+2I_\psi R^2m+2I_\omega L^2M-L^2M^2R^2\cos^2(\psi)+L^2M^2R^2+2L^2MR^2m\right)^2} \\
 & +\dfrac{2I_\psi I_\omega g\sin(\psi)+2I_\psi LMR^2\psi_t^2\sin(\psi)\cos(\psi)+I_\psi MR^2g\sin(\psi)+2I_\psi R^2gm\sin(\psi)+I_\psi RT\cos(\psi)+2I_\omega L^2Mg\sin(\psi)+2L^3M^2R^2\psi_t^2\sin(\psi)\cos(\psi)+L^2M^2R^2g\sin^3(\psi)-5L^2M^2R^2g\sin(\psi)\cos^2(\psi)+L^2M^2R^2g\sin(\psi)+2L^2MR^2gm\sin(\psi)+L^2MRT\cos(\psi)-LMR^2T\sin^2(\psi)+LMR^2T\cos^2(\psi)}{2I_\psi I_\omega+I_\psi MR^2+2I_\psi R^2m+2I_\omega L^2M-L^2M^2R^2\cos^2(\psi)+L^2M^2R^2+2L^2MR^2m} \\
1 & 0 \\
0 & 0
\end{bmatrix}
$$

$$
\begin{bmatrix}
0 & \dfrac{2I_\psi LMR^2\psi_t\sin(\psi)\cos(\psi)}{2I_\psi I_\omega+I_\psi MR^2+2I_\psi R^2m+2I_\omega L^2M-L^2M^2R^2\cos^2(\psi)+L^2M^2R^2+2L^2MR^2m} \\
0 & \dfrac{2I_\psi LMR^2\psi_t\sin^2(\psi)+2L^3M^2R^2\psi_t\sin^2(\psi)}{2I_\psi I_\omega+I_\psi MR^2+2I_\psi R^2m+2I_\omega L^2M-L^2M^2R^2\cos^2(\psi)+L^2M^2R^2+2L^2MR^2m} \\
0 & 0 \\
0 & 1
\end{bmatrix}
$$

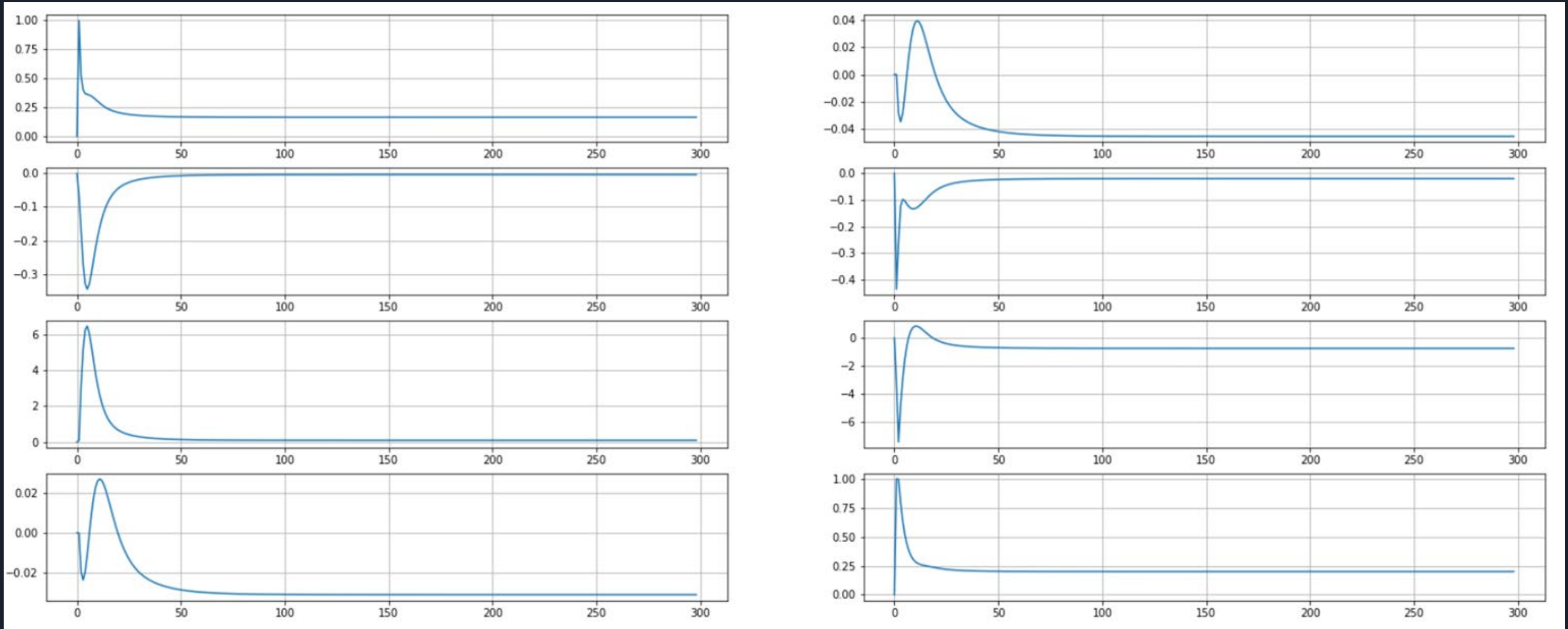
Simplification

$$\begin{bmatrix} 0 & -\dfrac{4.03448677087711\cdot 10^{-8}\psi^{2}\cdot\left(32625.0T\psi^{2}+47620T\psi-46835.0T-20883344062.5\psi^{3}+37523708617.5\psi\right)}{\left(1-0.463371875258313\psi^{2}\right)^{2}}+\dfrac{-65250.0T\psi^{2}-47620T\psi+20883344062.5\psi^{3}+37523708617.5\psi}{22970548.75-10643906.25\psi^{2}} & 0 & 0 \\ 0 & -\dfrac{4.03448677087711\cdot 10^{-8}\psi^{2}\cdot\left(32625.0T\psi^{2}+142232.5T\psi-22534108323.75\psi\right)}{\left(1-0.463371875258313\psi^{2}\right)^{2}}+\dfrac{142232.5T\psi-41766688125.0\psi^{3}+22534108323.75\psi}{22970548.75-10643906.25\psi^{2}} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

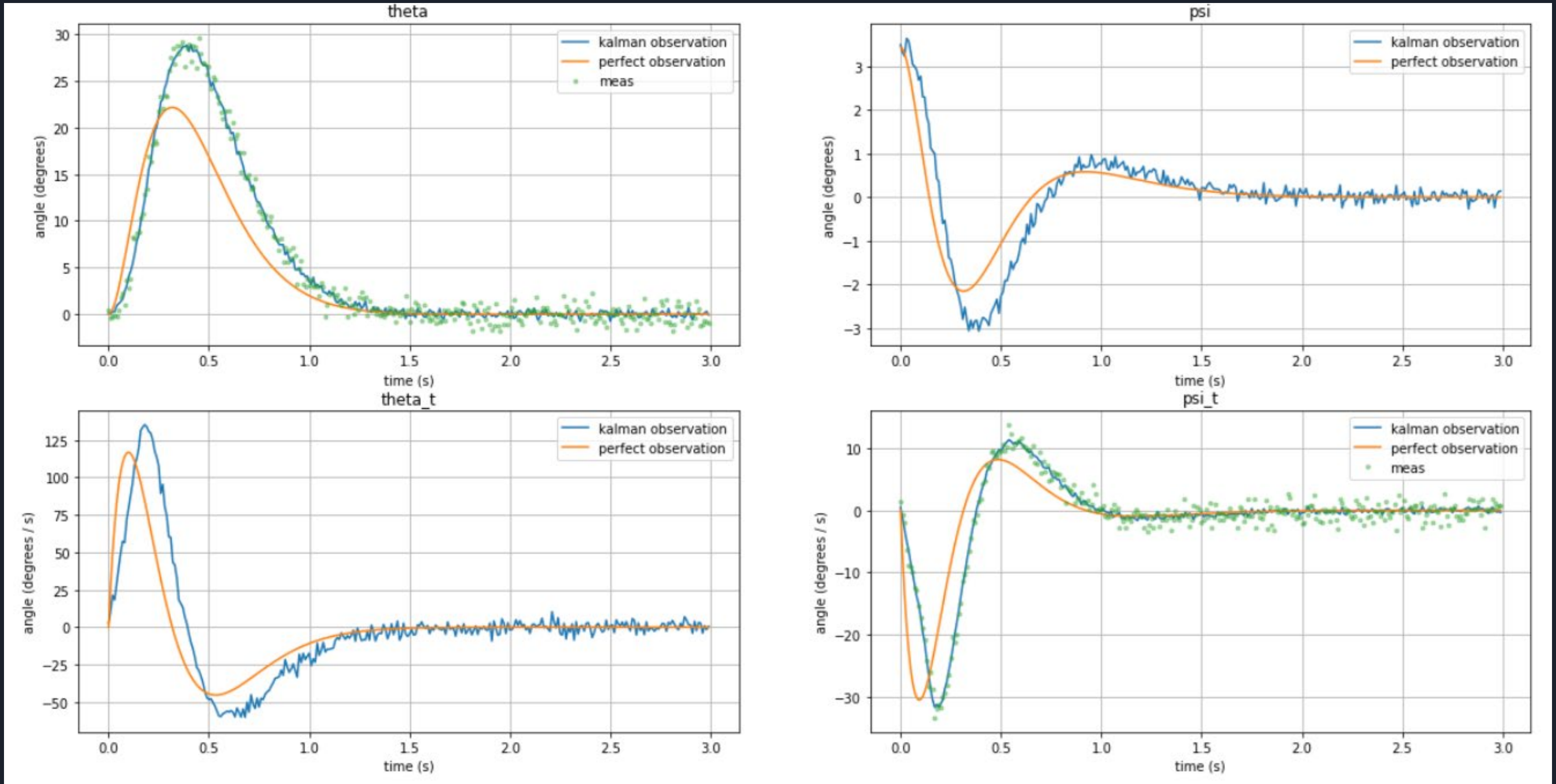
Results. Covariance Matrix



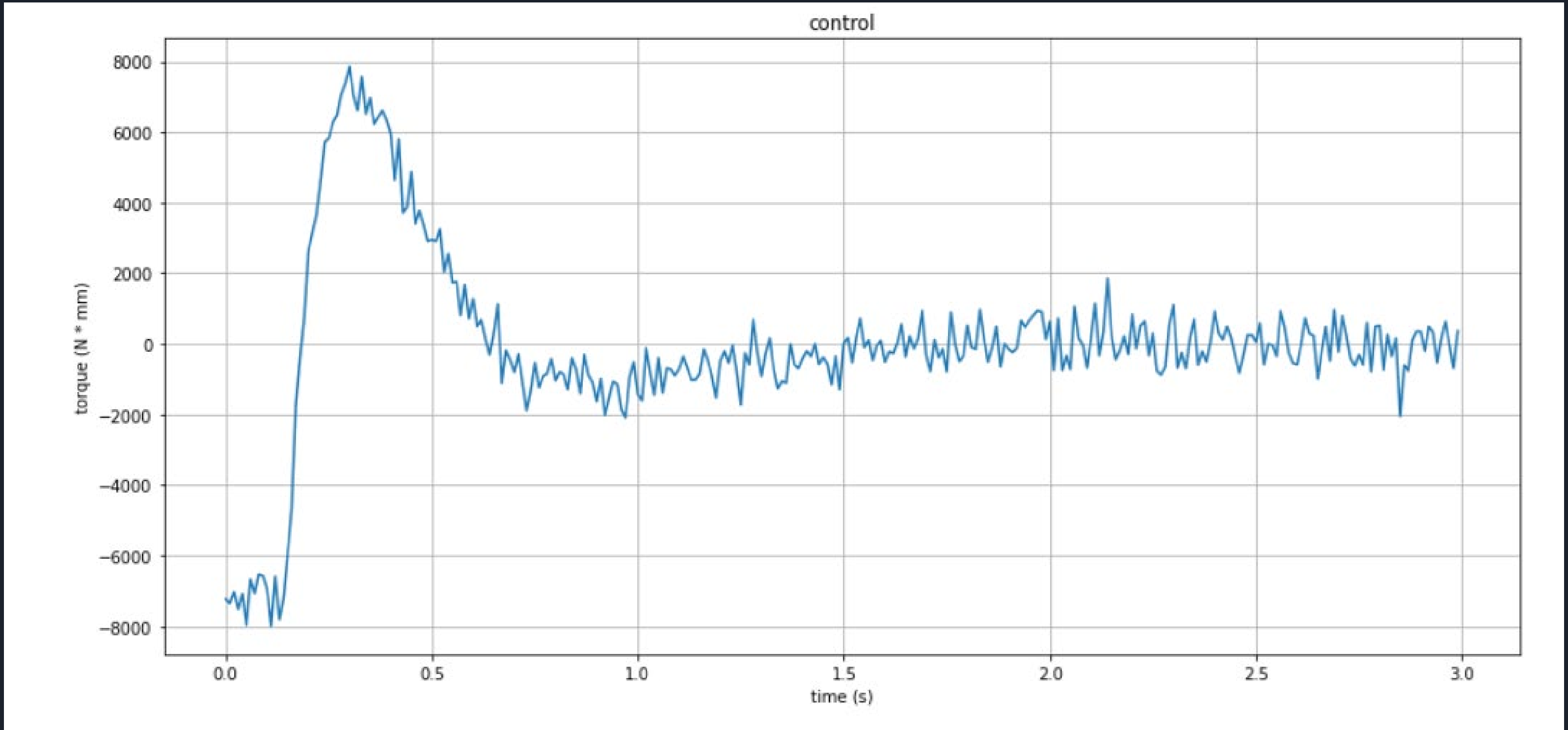
Results. Kalman Gain



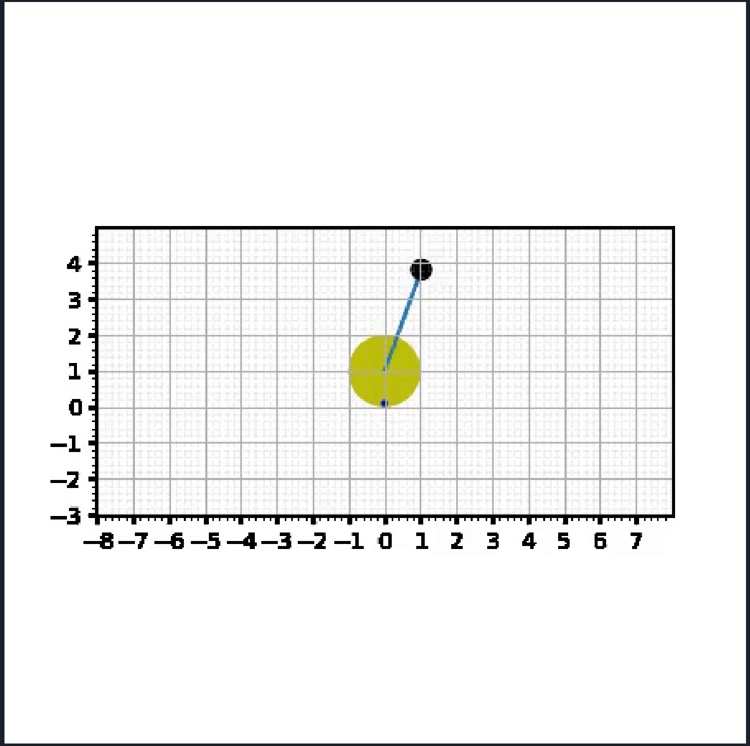
Results. Result



Results. Result



Result. Simulation



Application

Thx for attention