

Процедури от по-висок ред

Сума

$$\sum_{x=a}^b x$$

Сума... на целите числа

$$\sum_{x=a}^b x$$

```
(define (sum-integers a b)
  (if (> a b)
      0
      (+ a
         (sum-integers (+ a 1) b)))))
```

Сума... на квадратите

$$\sum_{x=a}^b x^2$$

```
(define (sum-squares a b)
  (if (> a b)
      0
      (+ ...
         (sum-squares (+ a 1) b)))))
```

Сума... на квадратите

$$\sum_{x=a}^b x^2$$

```
(define (sum-squares a b)
  (if (> a b)
      0
      (+ (square a)
          (sum-squares (+ a 1) b))))
```

Сума... на дроби

$$\sum_{x=a}^b \frac{1}{x^4 + 1}$$

```
(define (sum-fractions a b)
  (if (> a b)
      0
      (+ ...
         (sum-fractions (+ a 1) b))))
```

Сума... на дроби

$$\sum_{x=a}^b \frac{1}{x^4 + 1}$$

```
(define (sum-fractions a b)
  (if (> a b)
      0
      (+ (/ 1
            (+ (square (square a)) 1))
         (sum-fractions (+ a 1) b))))
```

Обобщение

$$\sum_{x=a}^b x$$

$$\sum_{x=a}^b x^2$$

$$\sum_{x=a}^b \frac{1}{x^4 + 1}$$

...

Обобщение

$$\sum_{x=a}^b x$$

$$\sum_{x=a}^b x^2$$

$$\sum_{x=a}^b \frac{1}{x^4 + 1}$$

...

$$\sum_{x=a}^b f(x)$$

По-абстрактна сума

$$\sum_{x=a}^b f(x)$$

```
(define (sum f a b)
  (if (> a b)
      0
      (+ (f a)
          (sum f (+ a 1) b)))))
```

По-абстрактна сума

$$\sum_{x=a}^b term(x)$$

```
(define (sum term a b)
  (if (> a b)
      0
      (+ (term a)
          (sum term (+ a 1) b))))
```

По-абстрактна сума

```
(define (identity x) x)
```

```
(define (sum-integers a b)  
  (sum identity a b))
```

```
(define (square x) (* x x))
```

```
(define (sum-squares a b)  
  (sum square a b))
```

```
(define (cube x) (* x x x))
```

```
(define (sum-cubes a b)  
  (sum cube a b))
```

```
(define (sum-fractions a b)  
  (define (term x)  
    (/ 1  
      (+ (square (square x)) 1)))  
  
  (sum term a b))
```

Произведение

$$\prod_{x=a}^b f(x)$$

Произведение

$$\prod_{x=a}^b term(x)$$

```
(define (product term a b)
  (if (> a b)
      1
      (* (term a)
         (product term (+ a 1) b))))
```

(* Произведение 0) ?

```
(define (identity x) x)  
(product identity 1 5) ; 0
```

```
(define (square x) (* x x))  
(product square 2 3) ; 0
```

```
(define (identity x) x)  
(define (factorial n)  
  (product identity 1 n))
```

(* Произведение 1)

$$\prod_{x=a}^b term(x)$$

```
(define (product term a b)
  (if (> a b)
      1
      (* (f a)
         (product term (+ a 1) b))))
```

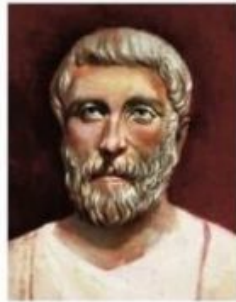



Още по-абстрактно

$$\sum_{x=a}^b f(x) \quad \prod_{x=a}^b f(x) \quad \bigwedge_{x=a}^b f(x) \quad \dots$$

```
(define (accumulate combiner null-value term a b)
  (if (> a b)
      null-value
      (combiner (term a)
                 (accumulate combiner
                             null-value
                             term
                             (+ a 1)
                             b)))))
```

От частното към общото



$$a^2 + b^2 = c^2$$



$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$



Let H be a (real) Hilbert space,
 $\| \cdot \|$ be the by the inner product induced norm.
Then we have $\forall x, y \in H$:

$$\|x + y\|^2 = \|x\|^2 + \|y\|^2 + 2\langle x, y \rangle$$



if A is an $n \times k$ matrix, then

$$\det(A^t A) = \sum_I \det(A_I)^2$$

where the sum is over all
 $k \times k$ minors of A .

Да дефинираме `sum` и `product` чрез `accumulate`

```
(define (sum term a b)
  (accumulate + 0 term a b))

(define (product term a b)
  (accumulate * 1 term a b))
```

Или нещо по-конкретно

```
(define (square x) (* x x))  
  
(define (sum-squares a b)  
  (accumulate + 0 square a b))
```

```
(define (cube x) (* x x x))  
  
(define (product-cubes a b)  
  (accumulate * 1 cube a b))
```

```
(define (average x y) ; бинарна процедура  
  (/ (+ x y) 2))  
  
(define (identity x) x)  
  
(define (fractions a b)  
  (accumulate average 0 identity a b))
```

λ

Създаване на процедури чрез `lambda`

```
(define (square x) (* x x))
```

```
(define square (lambda (x) (* x x)))
```

```
(sum 1 5 square) ; 55
```

```
(sum 1 5 (lambda (x) x)) ; 15
```

λ

Как да четем lambda

(lambda		(x)		(+ x 4))
the procedure	of an argument	x	that adds	x and 4

(define	(square	x)		(*	x	x))
To	square	something,	multiply	it	by	itself.