Balance and Filtering in Structured Satisfiable Problems

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Abstract

New methods to generate hard random problem instances have driven progress on algorithms for deduction and constraint satisfaction. Recently Achlioptas *et al.* (AAAI 2000) introduced a new generator based on Latin squares that creates only *satisfiable* problems, and so can be used to accurately test incomplete (one sided) solvers. We investigate how this and other generators are biased away from the uniform distribution of satisfiable problems and show how they can be improved by imposing a *balance* condition. More generally, we show that the generator is one member of a family of related models that generate distributions ranging from ones that are everywhere tractable to ones that exhibit a sharp hardness threshold. We also discuss the critical role of the problem encoding in the performance of both systematic and local search solvers.

1 Introduction

The discovery of methods to generate hard random problem instances has driven progress on algorithms for propositional deduction and satisfiability testing. Gomes & Selman (1997) introduced a generation model based on the quasigroup (or Latin square) completion problem (QCP). The task is to determine if a partially colored square can be completed so that no color is repeated in any row or any column. QCP is an NP-complete problem, and random instances exhibit a peak in problem hardness in the area of the phase transition in the percentage of satisfiable instances generated as the ratio of the number of uncolored cells to the total number of cells is varied. The structure implicit in a QCP problem is similar to that found in real-world domains: indeed, many problems in scheduling and experimental design take the form of a QCP. Thus, QCP complements earlier simpler generation models, such as random k-cnf (Mitchell *et al.* 1992). Like them QCP generates a mix of satisfiable and unsatisfiable instances.

In order to measure the performance of incomplete solvers, it is necessary to have benchmark instances that are known to be satisfiable. This requirement is problematic in domains where incomplete methods can solve larger instances than complete methods: it is not possible to use a complete method to filter out the unsatisfiable instances. It has proven difficult to create generators for satisfiable k-sat. Achlioptas *et al.* (2000) described a generation model for satisfiable quasigroup completion problems called "quasigroups with holes" (QWH). The QWH generation procedure basically inverts the completion task: it *begins* with a randomly-generated completed Latin square, and then erases colors or "pokes holes". The *backbone* of a satisfiable problem is the set of variables that receive the same value in all solutions to that problem. Achlioptas *et al.* (2000) showed that the hardest QWH problems arise in the vicinity of a threshold in the average size of the backbone.

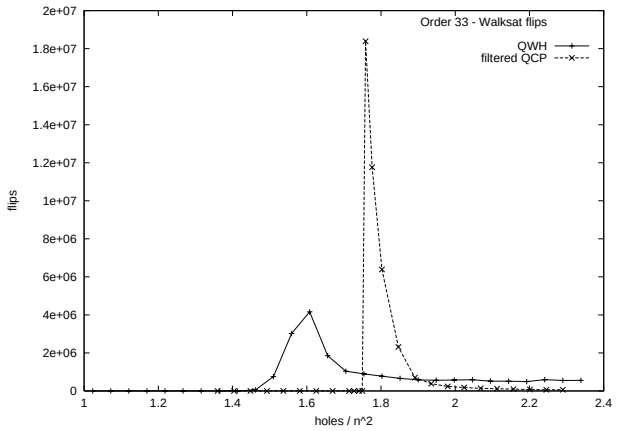

Figure 1: Comparison of the QWH and filtered QCP models for order 33 using Walksat. The x-axis is the percentage of holes (re-parameterized) and the y-axis is the number of flips.

model	Walksat flips	Satz backtracks	Sato branches
QWH	$4e+6$	41	$3e+5$
filtered QCP	$1e+8$	394	$1.5e+6$
balanced QWH	$6e+8$	4,984	$8e+6$

Table 1: Peak median hardness for different generation models for order 33 problems.

Despite the similarities between the filtered QCP and QWH models the problem distributions they generate are quite different. As noted by Achlioptas *et al.* (AAAI 2000), the threshold for QCP is at a higher ratio of holes than is the threshold for QWH. But even more significantly, we discovered that the hardest problems obtained using the filtered QCP