	Anonymity, neutrality Monotonicity	Majority, Condorcet	Consistency	Complexity	Min. Bayesian risk
Kemeny	Y	Y	N	NP-hard, $\mathsf{P}_{\|\|}^{\mathsf{NP}}$-hard	N
Bayesian est. of $\mathcal{M}_\varphi^1$ (uni. prior)	Y	N	N	NP-hard, $\mathsf{P}_{\|\|}^{\mathsf{NP}}$-hard (Theorem 3)	Y
Bayesian est. of $\mathcal{M}_\varphi^2$ (uni. prior)	Y	N	N	P (Theorem 4)	Y

Table 1: Kemeny for winners vs. Bayesian estimators of $\mathcal{M}_\varphi^1$ and $\mathcal{M}_\varphi^2$ to choose *winners*.

Given these, we can adopt *Bayesian estimators* as social choice mechanisms, which make decisions to minimize the expected loss w.r.t. the posterior distribution on the parameters (called the *Bayesian risk*). This provides a principled methodology for the design and analysis of new voting rules.

To show the viability of the framework, we focus on selecting multiple alternatives (the alternatives that can be thought of as being "tied" for the first place) under a natural extension of the 0-1 loss function for two models: let $\mathcal{M}_\varphi^1$ denote the *Mallows model* with fixed dispersion [22], and let $\mathcal{M}_\varphi^2$ denote the *Condorcet model* proposed by Condorcet in the 18th century [9, 34]. In both models the dispersion parameter, denoted φ, is taken as a fixed parameter. The difference is that in the Mallows model the parameter space is composed of all linear orders over alternatives, while in the Condorcet model the parameter space is composed of all possibly cyclic rankings over alternatives (irreflexive, antisymmetric, and total binary relations). $\mathcal{M}_\varphi^2$ is a natural model that captures real-world scenarios where the ground truth may contain cycles, or agents' preferences are cyclic, but they have to report a linear order due to the protocol. More importantly, as we will show later, a Bayesian estimator on $\mathcal{M}_\varphi^2$ is superior from a computational viewpoint.

Through this approach, we obtain two voting rules as Bayesian estimators and then evaluate them with respect to various normative properties, including *anonymity, neutrality, monotonicity, the majority criterion, the Condorcet criterion* and *consistency*. Both rules satisfy anonymity, neutrality, and monotonicity, but fail the majority criterion, Condorcet criterion,[1] and consistency. Admittedly, the two rules do not enjoy outstanding normative properties, but they are not bad either. We also investigate the computational complexity of the two rules. Strikingly, despite the similarity of the two models, the Bayesian estimator for $\mathcal{M}_\varphi^2$ can be computed in polynomial time, while computing the Bayesian estimator for $\mathcal{M}_\varphi^1$ is $\mathsf{P}_{||}^{\mathsf{NP}}$-hard, which means that it is at least NP-hard. Our results are summarized in Table 1.

We also compare the asymptotic outcomes of the two rules with the Kemeny rule for winners, which is a natural extension of the maximum likelihood estimator of $\mathcal{M}_\varphi^1$ proposed by Fishburn [14]. It turns out that when n votes are generated under $\mathcal{M}_\varphi^1$, all three rules select the same winner asymptotically almost surely (a.a.s.) as $n \to \infty$. When the votes are generated according to $\mathcal{M}_\varphi^2$, the rule for $\mathcal{M}_\varphi^1$ still selects the same winner as Kemeny a.a.s.; however, for some parameters, the winner selected by the rule for $\mathcal{M}_\varphi^2$ is different with non-negligible probability. These are confirmed by experiments on synthetic datasets.

Related work. Along the second perspective in social choice (to make an objectively correct decision), in addition to Condorcet's statistical approach to social choice [9, 34], most previous work in economics, political science, and statistics focused on extending the theorem to heterogeneous, correlated, or strategic agents for two alternatives, see [25, 1] among many others. Recent work in computer science views agents' votes as i.i.d. samples from a statistical model, and computes the MLE to estimate the parameters that maximize the likelihood [10, 11, 33, 32, 2, 29, 7]. A limitation of these approaches is that they estimate the parameters of the model, but may not directly inform the right *decision* to make in the multi-agent context. The main approach has been to return the modal rank order implied by the estimated parameters, or the alternative with the highest, predicted marginal probability of being ranked in the top position.

There have also been some proposals to go beyond MLE in social choice. In fact, Young [34] proposed to select a winning alternative that is *"most likely to be the best (i.e., top-ranked in the true ranking)"* and provided formulas to compute it for three alternatives. This idea has been formalized

[1] The new voting rule for $\mathcal{M}_\varphi^1$ fails them for all $\varphi < 1/\sqrt{2}$.