



Figure 1. Experiment 1. The upper row shows results for  $\beta_c = 1$  and the lower row for  $\beta_c = 2$ . The three graphs in each row show  $\Delta_{KL}^s(f, \tilde{f})$ ,  $CR^s(\tilde{\beta}; \beta)$  and the model dimension respectively.

| Procedure | BreastCancer (683/9) |            |      | ClevelandHeart (294/13) |            |      | Crab (173/4)        |            |      | German (1000/24)    |            |      |
|-----------|----------------------|------------|------|-------------------------|------------|------|---------------------|------------|------|---------------------|------------|------|
|           | $-l(\tilde{\beta})$  | $CR^s(\%)$ | Dim  | $-l(\tilde{\beta})$     | $CR^s(\%)$ | Dim  | $-l(\tilde{\beta})$ | $CR^s(\%)$ | Dim  | $-l(\tilde{\beta})$ | $CR^s(\%)$ | Dim  |
| ML        | 12.5                 | 96.7       | 10.0 | 39.5                    | 83.3       | 14.0 | 56.9                | 70.4       | 5.0  | 49.4                | 76.9       | 25.0 |
| AIC       | 13.1                 | 96.4       | 6.8  | 42.4                    | 82.1       | 9.4  | 58.4                | 69.0       | 3.0  | 49.8                | 76.5       | 14.2 |
| BIC       | 14.1                 | 96.1       | 4.9  | 45.1                    | 81.4       | 5.3  | 59.5                | 67.7       | 2.1  | 51.2                | 74.5       | 6.1  |
| CIC       | 12.6                 | 96.7       | 9.7  | 39.9                    | 83.1       | 13.6 | 57.4                | 67.0       | 4.7  | 50.1                | 76.2       | 13.9 |
| RIC       | 13.9                 | 96.0       | 5.4  | 44.8                    | 81.3       | 5.4  | 59.4                | 68.1       | 2.7  | 51.3                | 74.5       | 6.3  |
| New       | 12.7                 | 96.6       | 9.5  | 39.7                    | 83.2       | 13.8 | 58.0                | 69.2       | 4.1  | 49.5                | 76.8       | 23.3 |
|           | Ionosphere (351/33)  |            |      | Pima (768/8)            |            |      | Spambase (500/57)   |            |      | WDBC (569/30)       |            |      |
|           | $-l(\tilde{\beta})$  | $CR^s(\%)$ | Dim  | $-l(\tilde{\beta})$     | $CR^s(\%)$ | Dim  | $-l(\tilde{\beta})$ | $CR^s(\%)$ | Dim  | $-l(\tilde{\beta})$ | $CR^s(\%)$ | Dim  |
| ML        | 39.8                 | 88.2       | 34.0 | 48.7                    | 77.4       | 9.0  | 32.6                | 88.7       | 58.0 | 15.2                | 95.7       | 31.0 |
| AIC       | 36.9                 | 87.6       | 15.6 | 49.0                    | 76.8       | 6.7  | 33.6                | 86.9       | 19.4 | 15.5                | 96.0       | 10.7 |
| BIC       | 40.1                 | 85.9       | 7.3  | 49.6                    | 76.4       | 4.5  | 37.2                | 86.6       | 8.6  | 16.6                | 95.6       | 4.6  |
| CIC       | 39.3                 | 86.1       | 8.0  | 48.7                    | 77.4       | 8.8  | 37.9                | 86.6       | 7.7  | 16.8                | 95.5       | 4.4  |
| RIC       | 41.5                 | 85.6       | 6.0  | 49.2                    | 76.5       | 5.3  | 38.2                | 86.7       | 7.4  | 16.7                | 95.5       | 4.4  |
| New       | 38.8                 | 87.8       | 24.7 | 48.7                    | 77.3       | 8.5  | 34.5                | 86.9       | 19.3 | 16.7                | 95.6       | 4.6  |

Table 1. Results for practical datasets in Experiment 2.

high-dimensional parameter spaces. The ideas resemble the empirical Bayes methodology, although we do not adopt a Bayesian perspective nor even assume the existence of a *random* prior distribution.

We conducted a case study on the use of this general approach to fit logistic regression models. Although all our theoretical conclusions are asymptotic, simulation results on finite datasets (both artificial and practical) are promising: the new method is nearly always