order	true count	pure CSP count	pure CSP time	binary XORs (individual) count	binary XORs (individual) time	binary XORs (global) count	binary XORs (global) time	generalized XORs (individual) count	generalized XORs (individual) time
15	2.3×10^6	2.3×10^6	10 min	$\geq 6.6\times10^4$	41 s	$\geq 2.6\times10^5$	66 s	$\geq 3.9\times10^5$	3 s
20	3.9×10^{10}	$\geq 3.5\times10^6$	1 hr	$\geq 1.1\times10^6$	24 s	$\geq 2.1\times10^6$	17 s	$\geq 6.6\times10^8$	13 s
25	2.2×10^{15}	$\geq 2.2\times10^6$	1 hr	$\geq 3.4\times10^7$	125 s	$\geq 3.4\times10^7$	101 s	$\geq 2.0\times10^{12}$	60 s
30		$\geq 4.1\times10^6$	1 hr	$\geq 5.4\times10^8$	155 s	$\geq 5.4\times10^8$	155 s	$\geq 9.2\times10^{15}$	196 s

Table 1: *Computational results on n-Queens problems, comparing our different approaches (99% correctness confidence).*

	pb-counter		Relsat+BBR		pure CSP		CSP+XORs	
games120	$\geq 1.1\times10^6$	30 min	$\geq 1.4\times10^6$	30 min	$\geq 4.3\times10^8$	1 hr	$\geq \mathbf{4.5\times10^{42}}$	**1 min**
myciel5	$\geq 1.1\times10^7$	30 min	$\geq 3.6\times10^{11}$	30 min	$\geq 9.5\times10^8$	1 hr	$\geq \mathbf{4.1\times10^{17}}$	**12 min**
mug100_1	$\geq 2.4\times10^7$	30 min	$\geq 2.7\times10^{23}$	30 min	$\geq 7.2\times10^8$	1 hr	$\geq \mathbf{1.0\times10^{28}}$	**1 min**
2_Insertions_3	$\geq 9.0\times10^6$	30 min	$\geq 4.6\times10^8$	30 min	$\geq 1.2\times10^9$	1 hr	$\geq \mathbf{2.3\times10^{12}}$	**1 min**
					pure CSP		**CSP+XORs**	
sbls14					≥ 273	1 hr	$\geq$ **591**	**5 min**
sbls15					≥ 112	1 hr	$\geq$ **1,748**	**8 min**
sbls17					—	1 hr	$\geq$ **1,058**	**14 min**

Table 2: *Computational results on graph coloring problems and spatially balanced Latin square problems. The results for the* XOR *approach are with 99% correctness confidence.*

spatially balanced Latin squares, by using the streamlined model of Smith, Gomes, & Fernández (2005). The results are reported in Table 2 (sbls 14–17), in which we also report the solution count of pure CSP. We see, for example, that the pure CSP solver counts 112 solutions in one hour for sbls15, while our generalized XOR approach counts 1748 solutions in 8 minutes (again, with 99% correctness confidence). Moreover, sbls17 cannot be solved at all by the pure CSP solver in one hour, while we count 1058 solutions in 14 minutes. This reconfirms an interesting phenomenon observed earlier by Gomes, Sabharwal, & Selman (2006): for computationally challenging problems, randomly generated XOR constraints can sometimes prove to be effective domain-independent streamliners.

Conclusion

We introduced a new generic solution counting technique for constraint satisfaction problems. This approach builds upon a method recently proposed for Boolean satisfiability problems, and combines it with the structured representation of CSPs to quickly provide lower bounds on solution counts with strong correctness guarantees. We considered both "regular" XOR constraints on an equivalent binary representation of CSPs as well as generalized XOR constraints directly on the CSP variables. For both cases, we developed efficient complete domain filtering algorithms. Our experimental evaluation on a set of challenging combinatorial problems demonstrates the effectiveness of this approach.

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