	General	Normal	Disjunctive
General	Θ_2^p-complete	Θ_2^p-complete	Θ_2^p-complete
Prerequisite-free	Θ_2^p-complete	coNP-complete	Θ_2^p-complete

Table 1: Complexity of model checking for default, logic

	AEL	$S4F_{MDD}$	$SW5_{MDD}$	$S5_G$	MKNF
General	Θ_2^p-complete	Θ_2^p-complete	Θ_2^p-complete	Σ_2^p-complete	Σ_2^p-complete
Flat	Θ_2^p-complete	Θ_2^p-complete	Θ_2^p-complete	Θ_2^p-complete	Θ_2^p-complete

Table 2: Complexity of model checking for nonmonotonic modal logics

We start by examining the case of autoepistemic logic. In Figure 2 we report, the algorithm AEL-Check for checking whether a propositional formula represents an autoepistemic model of a modal formula. In the algorithm, $\Sigma(K\varphi \rightarrow \text{true})$ represents the formula obtained from Σ by replacing each occurrence of the subformula $K\varphi$ with true, while $\Sigma(K\varphi \rightarrow \text{false})$ represents the formula obtained from Σ by replacing each occurrence of the subformula $K\varphi$ with false.

Informally, the algorithm iteratively computes the value of all modal subformulas (without nested occurrences of the modality) in Σ according to /, until all modal subformulas have been replaced by a truth value. The resulting propositional formula is compared with /, and the algorithm returns true if and only if the two formulas are equivalent.

Correctness of the algorithm can be established by means of previous results on reasoning in autoepistemic logic [Marek and Truszczyriski, 1993].

Lemma 9 *Let $\Sigma \in \mathcal{L}_K$, $f \in \mathcal{L}$. Then, $M = \{I : I \models f\}$ is an* AEL *model of Σ iff AEL-Check(Σ, f) returns* true.

The above property allows us to prove Θ_2^p-completeness of model checking in AEL.

Theorem 10 *Let $\Sigma \in \mathcal{L}_K$, $f \in \mathcal{L}$. Then, the problem of establishing whether $M = \{I : I \models f\}$ is an* AEL *model of Σ is Θ_2^p- complete.*

Proof sketch. Membership in Θ_2^p follows from Lemma 9 and from the fact that the algorithm AEL-Check can be polynomially reduced to an NP-tree. Hardness follows from the fact that it is possible to reduce an instance of the problem of model checking for prerequisite-free default theories to model checking in AEL: the reduction is based on the correspondence* between the prerequisite-free default $\frac{:\beta}{\gamma}$ and the modal f o r $\neg K\neg\beta \supset \gamma$ n autoepistemie logic. □

It can actually be shown that model checking for AEL is Θ_2^p-hard (and thus, from the above theorem, Θ_2^p-complete) even under the restriction that the formula Σ is *flat,* i.e. each propositional symbol in Σ lies within the scope of exactly one modality. The proof of this property can be obtained through a reduction from PARITY(SAT).

A similar analysis allows for establishing the same complexity characterization for the problem of model checking in two well-known nonmonotonic modal formalisms of the McDermott and Doyle's (MDD) family, i.e. the nonmonotonic logics based on the modal systems SW5 and S4F [Marek and Truszczynski, 1993].

Theorem 11 *Let $\Sigma \in \mathcal{L}_K$, $f \in \mathcal{L}$. Then, the problem of establishing whether $M = \{I : I \models f\}$ is an* S4FMDD *model (or an $SW5_{MDD}$ m o d of Σ is Θ_2^p-p l e t e .*

As in the case of autoepistemic logic, the above property also holds if we restrict to flat formulas.

For modal logics based on the *minimal kmowledge* paradigm, we prove that model checking is harder than for the above presented nonmonotonic logics. In particular, it is a Σ_2^p-complete problem. However, logical inference in such logics of minimal knowledge is harder than in default logic and autoepistemic logic, since it is a II3-complete problem both in MKNF and in S5G [Donini *et al*., 1997; Rosati, 1997). Hence, also in such formalisms model checking is easier than logical inference. We first, analyze modal logic S5G, I.e. the logic of minimal knowledge introduced in [Halpern and Moses, 1985].

Theorem 12 *Let $\Sigma \in \mathcal{L}_K$, $f \in \mathcal{L}$. Then, the problem of establishing whether $M = \{I : I \models f\}$ is an* $S5_G$ *model of Σ is Σ_2^p- complete.*

Interestingly, if we impose that the modal formula Σ is flat, then model checking in $S5_G$ becomes easier.

Theorem 13 *Let $\Sigma \in \mathcal{L}_K^F$, $f \in \mathcal{L}$. Then, the problem of establishing whether $M = \{I : I \models f\}$ is an $S5_G$ model of Σ is Θ_2^p- -complete.*

The same computational characterization of model checking can be shown for the logic MKNF, i.e. the logic of minimal knowledge and negation as failure introduced in [Lifschitz, 1991], which extends $S5_G$ with a second modal operator interpreted in terms of negation as failure.

Comparing the above results with known computational characterizations of the inference problem in nonmonotonic modal logics, it turns out that model checking is easier than logical inference in all the cases considered. Moreover, we remark that logical inference in the flat fragment of $S5_G$; and MKNF is Π_2^p-complete. This