

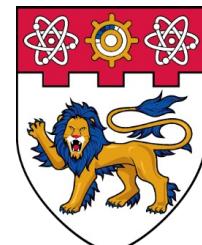
# Towards End-to-End GPS Localization with Neural Pseudorange Correction

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**NANYANG  
TECHNOLOGICAL  
UNIVERSITY**  
**SINGAPORE**



# Background (1/4)

## ■ High-performance Positioning/Localization Equipment:

NovAtel's leading SPAN® GNSS+INS Technology



PwrPak7-E1  
from HEXAGON



VEXXIS® GNSS-800 Series  
Antennas from HEXAGON

Waymo's Integrated System of Sensors



Lidar + Camera + Radar  
(From YouTube)

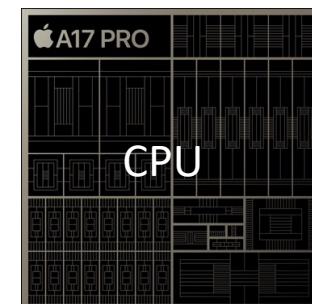
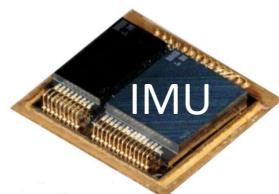
## ■ Inaccessibility in daily life

- *Heavy*
- *Large*
- *Expensive*

# Background (2/4)

## ■ By contrast, smartphones are:

- *Affordable*
- *Portable*
- *Integrated with other sensors*



Navigation



Bike Sharing



Emergency Location Service



Mobile AR

(From Google Images)

# Background (3/4)



**GNSS Satellite**

GNSS Signals



**Android Smartphone**



**GnssLogger App**  
Developed with Google



**GnssLogger**



**Analysis Software**

← Phone



Phone > gnss\_log



Sort by name ▾



gnss\_log\_2021\_06\_25\_17\_09\_43.txt

25/06/2021 - 22.91 KB

**GNSS Data File**

# Background (4/4)

- In this work, we focus on the pseudorange-based localization solutions.

```
• Clock Field  
  'TimeNanos'  
  'TimeUncertaintyNanos'  
  'LeapSecond'  
  'FullBiasNanos'  
  ....  
  
• Measurement Field  
  'Cn0DbHz'  
  'ConstellationType'  
  'MultipathIndicator'  
  'PseudorangeRateMetersPerSecond'  
  ....
```

## Android Raw GNSS Measurements

The  $n^{\text{th}}$  satellite at the  $k^{\text{th}}$  time step:



$$\text{Pseudorange } \rho_k^{(n)} = r_k^{(n)} + \delta t_{u_k} + \varepsilon_k^{(n)}$$

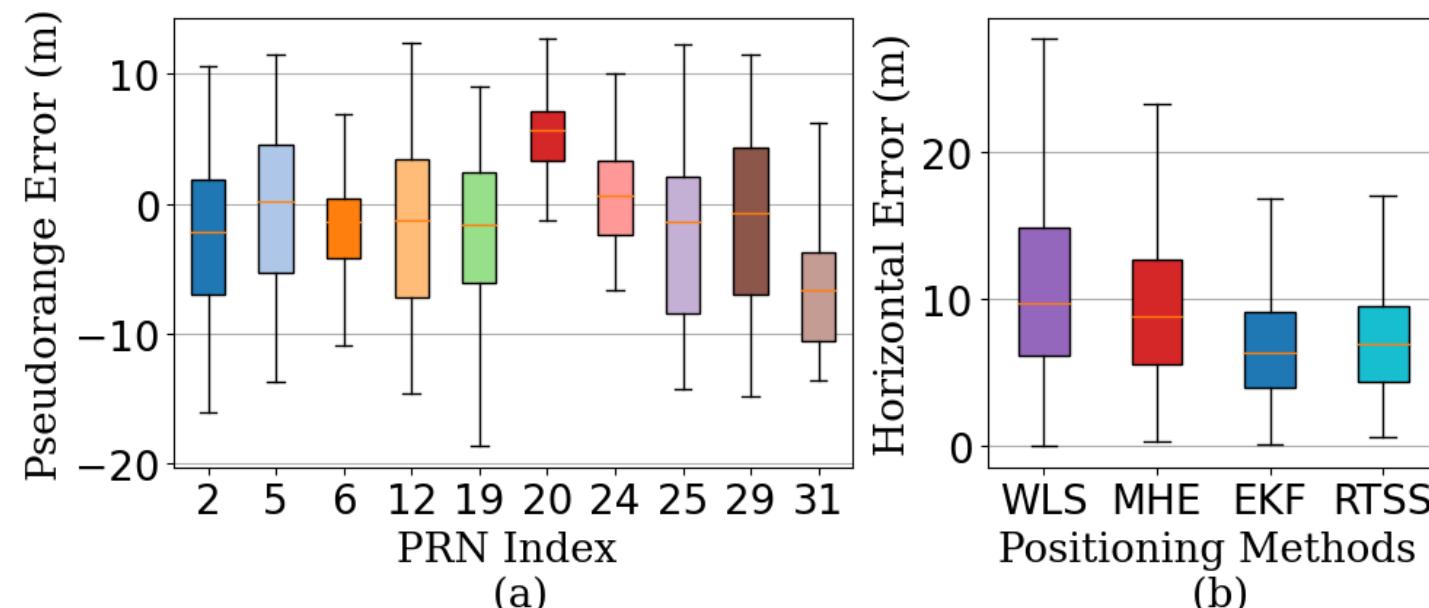
Geometry distance	User's clock offset	Pseudorange noise and errors
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We have modeled and removed:

1. Satellite clock errors
2. Relativistic effect
3. Group delays
4. Ionospheric delays
5. Tropospheric delays

# Motivation (1/3)

- GNSS measurements from smartphones have:
  - **Pseudorange errors, impacting positioning errors as a result**
  - The errors might be caused by multipath, non-line-of-sight (NLOS) propagation, modeling residuals, smartphone hardware delays, etc.

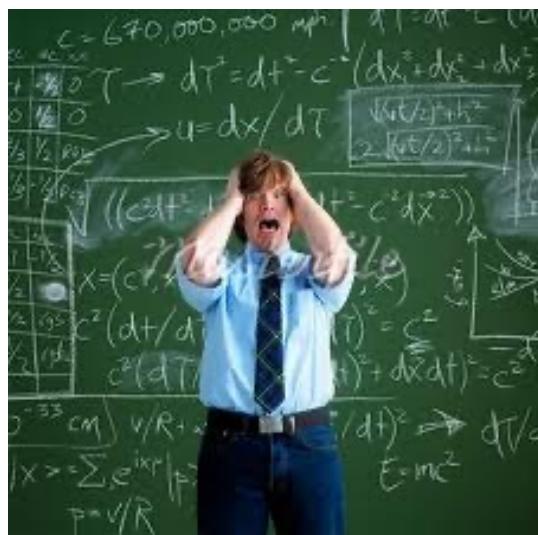


[1] X. Weng, K. Ling and H. Liu (2024),  
"PrNet: A Neural Network for  
Correcting Pseudoranges to Improve  
Positioning With Android Raw GNSS  
Measurements," in IEEE Internet of  
Things Journal, doi:  
10.1109/IJOT.2024.3392302.

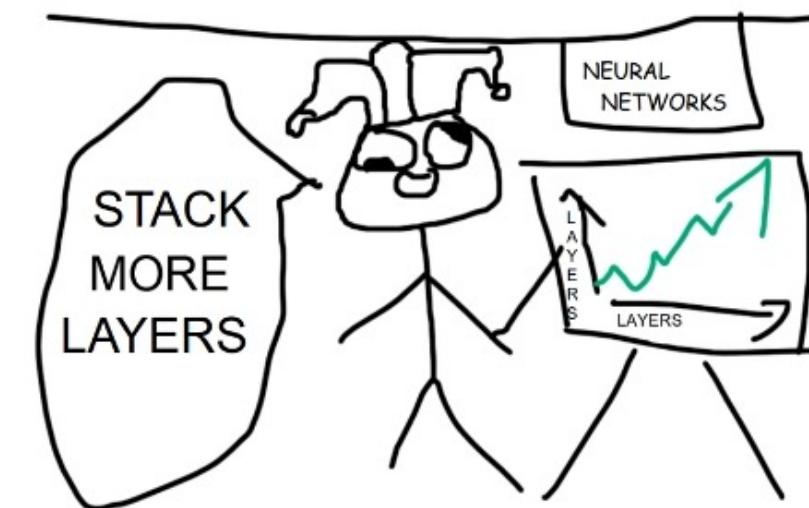
Trace "2021-04-28-US-MTV-1" collected by Pixel4 in GSDC dataset<sup>[1]</sup>

- (a) Pseudorange errors of all visible satellites throughout the trace.
- (b) Horizontal errors calculated with Vincenty's formulae using Weighted Least Squares (WLS), Moving Horizon Estimation (MHE), Extended Kalman Filter (EKF), and Rauch-Tung-Striebel Smoother (RTSS).

## Correcting Pseudoranges From Model-based to Data-driven Methods



From Google Images



From Google Images

## The High Cost of Labeling Training Data in CPS/IoT Tasks (Embedded AI)



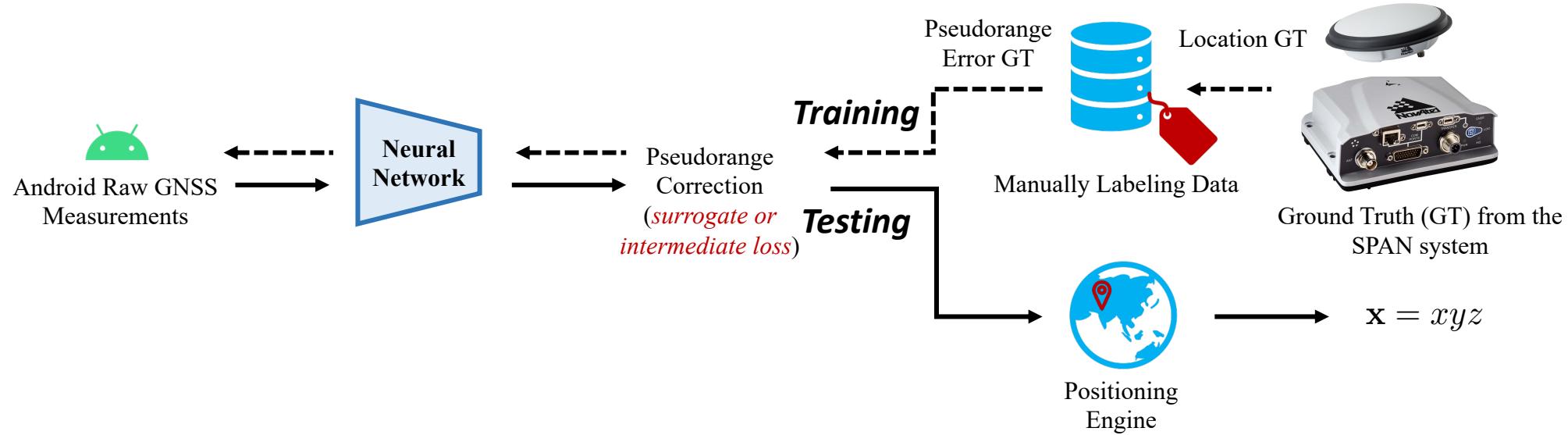
From Google Images



From Google Images

## Related Work (1/3)

- Neural pseudorange correction uses surrogate or intermediate loss.
  - FCNN-LSTM<sup>[1]</sup>: double-differentiated pseudorange errors (with noise)
  - PBC-RF<sup>[2]</sup>: pseudorange errors (with noise)
  - PrNet<sup>[3]</sup>: smoothed pseudorange errors (filtered noise)



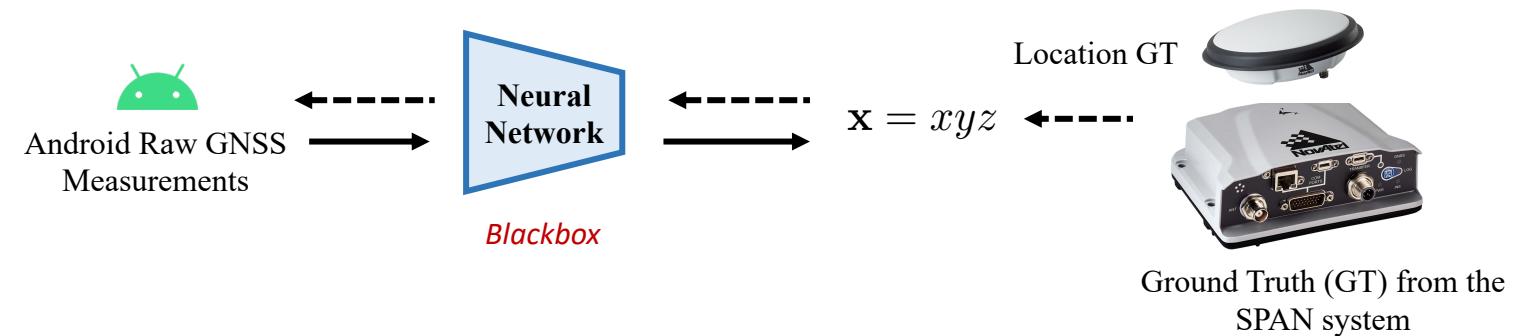
[1] Zhang, G., Xu, P., Xu, H., & Hsu, L. T. (2021). Prediction on the urban GNSS measurement uncertainty based on deep learning networks with long short-term memory. *IEEE Sensors Journal*, 21(18), 20563-20577.

[2] Rui Sun, Linxia Fu, Qi Cheng, Kai-Wei Chiang, and Wu Chen. Resilient pseudorange error prediction and correction for GNSS positioning in urban areas. *IEEE Internet of Things Journal*, 2023.

[3] X. Weng, K. Ling and H. Liu (2024), "PrNet: A Neural Network for Correcting Pseudoranges to Improve Positioning With Android Raw GNSS Measurements," in *IEEE Internet of Things Journal*, doi: 10.1109/JIOT.2024.3392302.

## Related Work (2/3)

- End-to-end learning for GNSS localization without using a model.
  - Set Transformer<sup>[1]</sup>: replaces classical positioning engines with set transformer
  - Graph Convolution Neural Network (GCNN) <sup>[2]</sup>: replaces classical positioning engines with GCNN

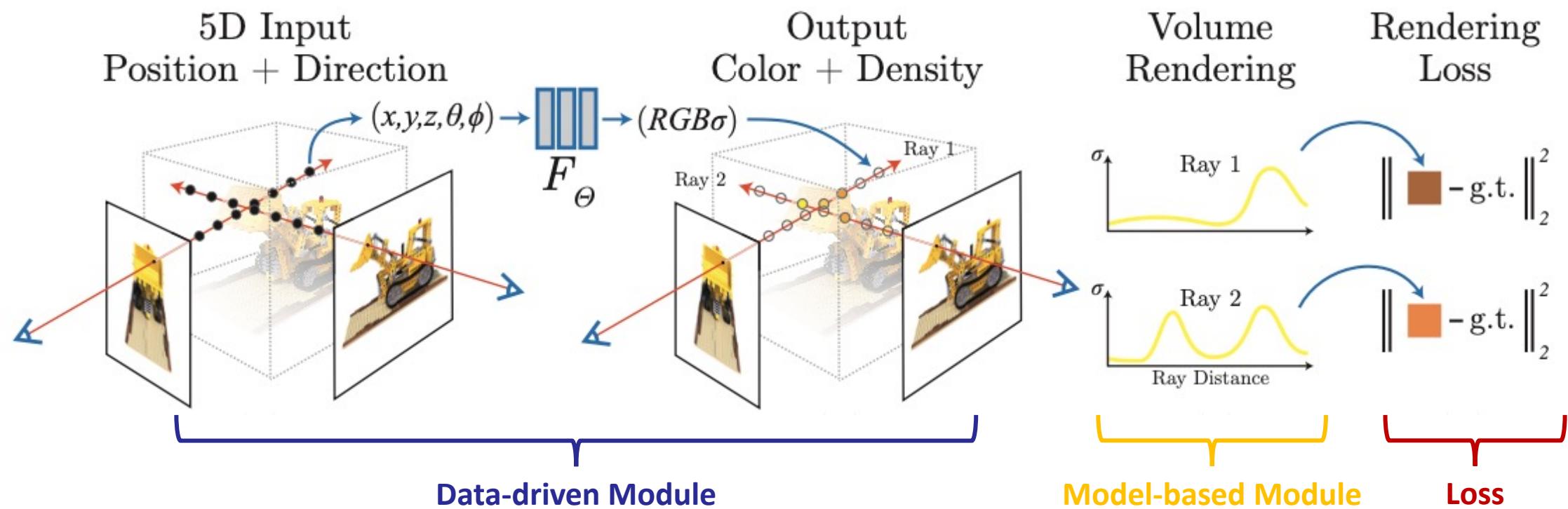


[1] Kanhere, A. V., Gupta, S., Shetty, A., & Gao, G. (2022). Improving GNSS positioning using neural-network-based corrections. NAVIGATION: Journal of the Institute of Navigation, 69(4).

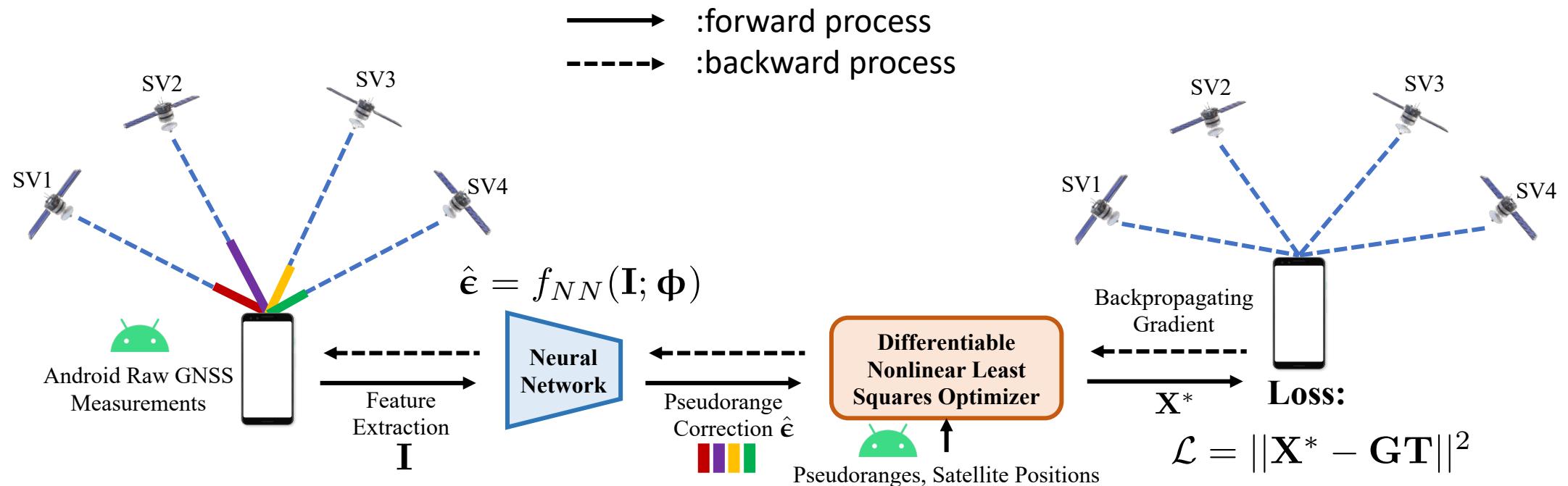
[2] Mohanty, A., & Gao, G. (2023). Learning GNSS positioning corrections for smartphones using graph convolution neural networks. NAVIGATION: Journal of the Institute of Navigation, 70(4).

- Can we combine Data-driven and Model-based approaches so that they can cooperate and be in tune with one another?
  - Successful examples in other domains: NeRF (ECCV, 2020 Best Paper), EPro-PnP (CVPR, 2022 Best Student Paper), etc.

### The Pipeline of Neural Radiance Fields



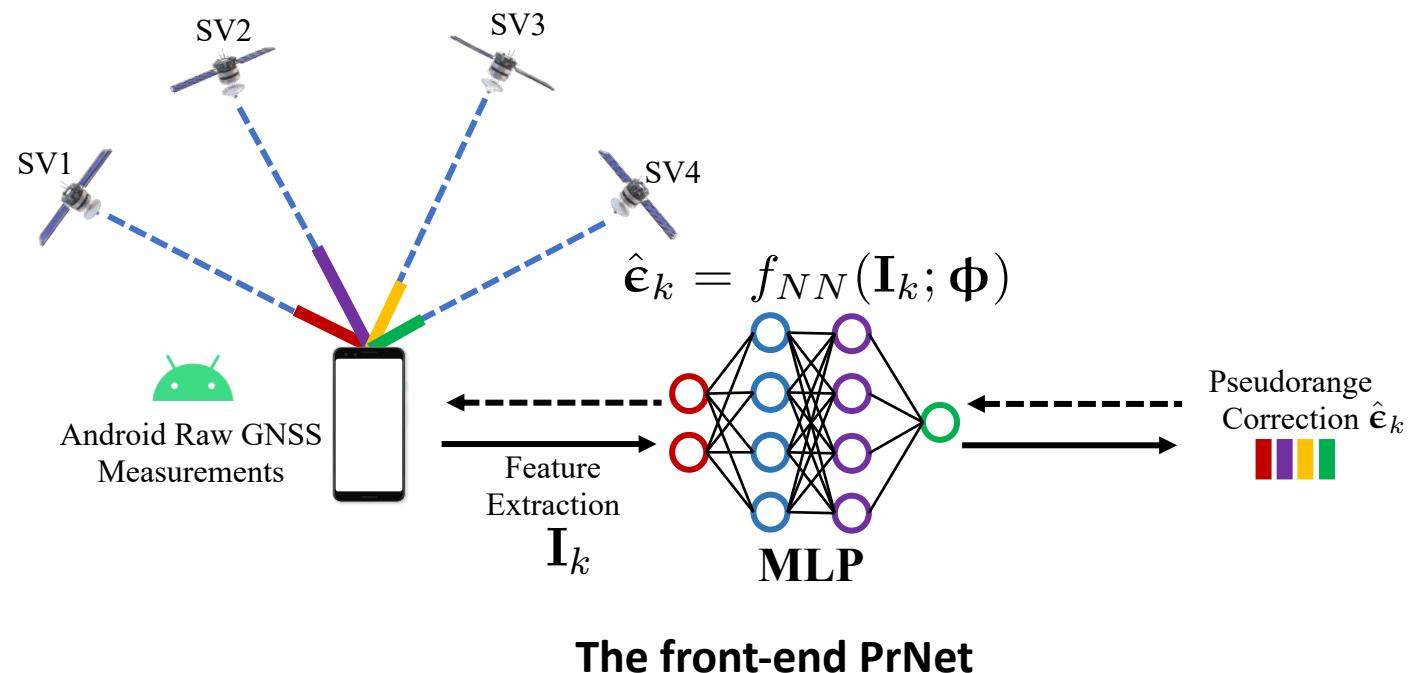
# Our Proposed Framework: E2E-PrNet



- End-to-End Neural Pseudorange Correction (E2E-PrNet) consists of
  - The front-end neural network for pseudorange correction (PrNet)
  - The model-based differentiable nonlinear least squares (DNLS) optimizer
- Pseudorange correction is learned via minimizing positioning errors.

# E2E-PrNet: the Front-end PrNet

- We use PrNet as the front-end neural network.
  - Input feature vector:  $\mathbf{I}_k$
  - Learnable weights of PrNet:  $\Phi$
  - Pseudorange correction:  $\hat{\epsilon}_k$
  - The mapping performed by PrNet:  $\hat{\epsilon}_k = f_{NN}(\mathbf{I}_k; \Phi)$



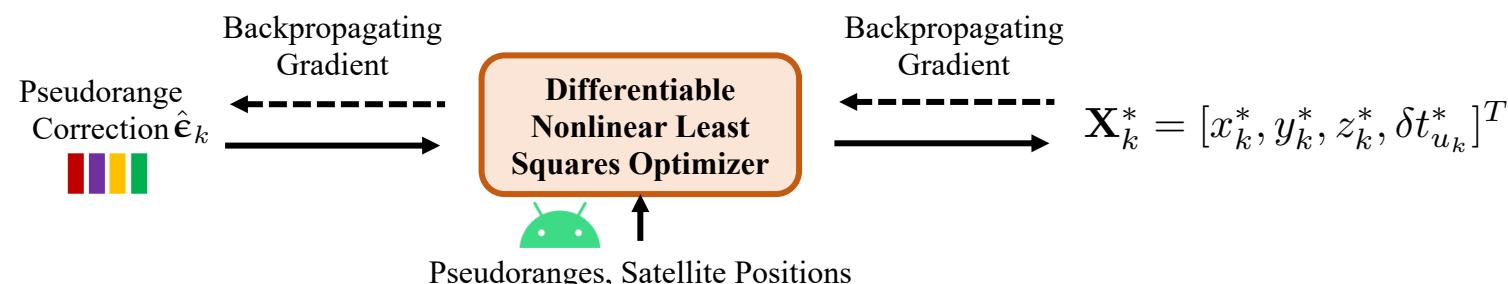
- GNSS localization is formulated as a nonlinear least squares optimization problem.

- Input of the DNLS optimizer:

- Satellite positions:  $[\mathbf{x}_k^{(1)}, \mathbf{x}_k^{(2)}, \dots, \mathbf{x}_k^{(M)}]^T$
  - Pseudorange measurements:  $[\rho_k^{(1)}, \rho_k^{(2)}, \dots, \rho_k^{(M)}]^T$
  - Pseudorange correction from the front-end PrNet:  $\hat{\epsilon}_k = [\hat{\epsilon}_k^{(1)}, \hat{\epsilon}_k^{(2)}, \dots, \hat{\epsilon}_k^{(M)}]^T$

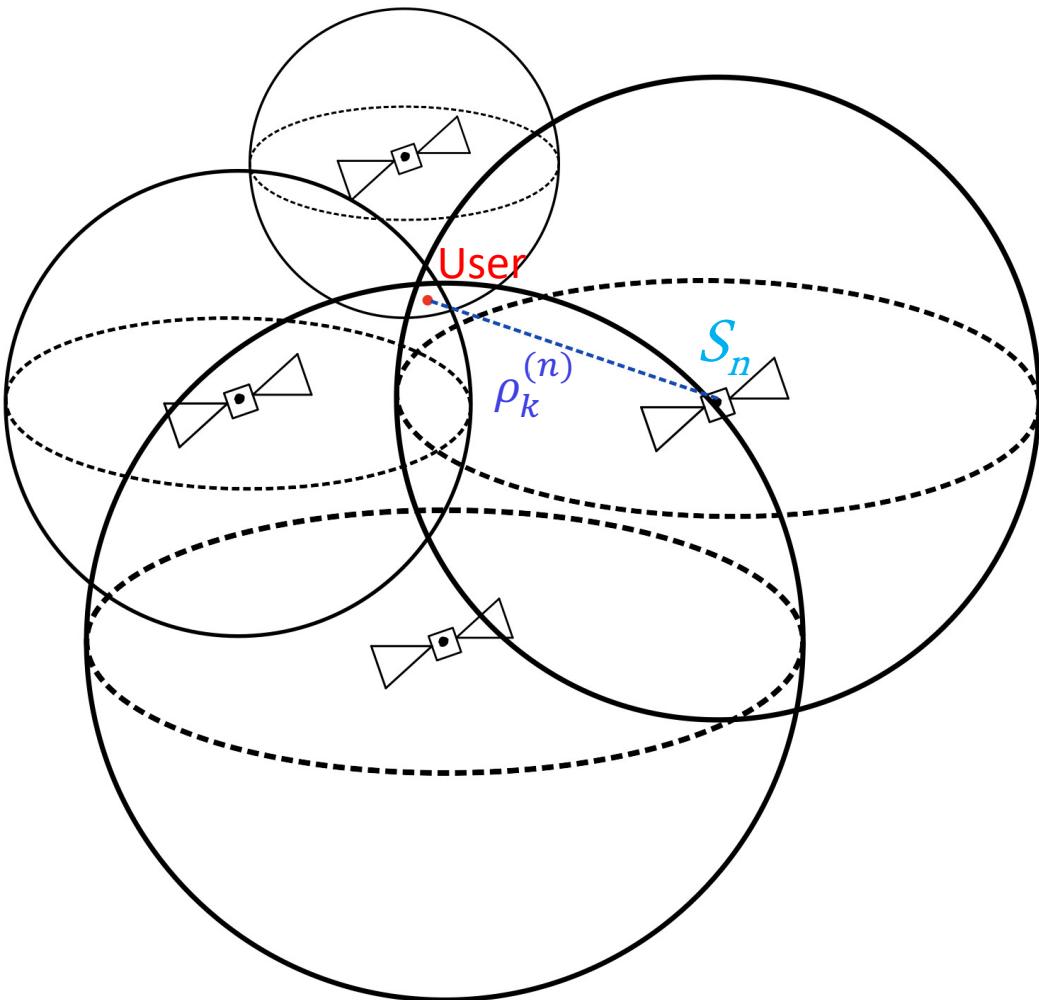
- Output of the DNLS optimizer:

- Optimal user location estimates:  $x_k^*, y_k^*, z_k^*$
  - Optimal user receiver clock offset estimate:  $\delta t_{u_k}^*$



The back-end DNLS Optimizer

## E2E-PrNet: the Back-end DNLS Optimizer (2/3)



- We can establish  $M$  nonlinear equations using pseudorange measurements to calculate positions and clock offsets:

$$\sqrt{\left(x_k - x_k^{(1)}\right)^2 + \left(y_k - y_k^{(1)}\right)^2 + \left(z_k - z_k^{(1)}\right)^2} + \delta t_{u_k} = \rho_k^{(1)}$$
$$\sqrt{\left(x_k - x_k^{(2)}\right)^2 + \left(y_k - y_k^{(2)}\right)^2 + \left(z_k - z_k^{(2)}\right)^2} + \delta t_{u_k} = \rho_k^{(2)}$$
$$\vdots$$
$$\sqrt{\left(x_k - x_k^{(M)}\right)^2 + \left(y_k - y_k^{(M)}\right)^2 + \left(z_k - z_k^{(M)}\right)^2} + \delta t_{u_k} = \rho_k^{(M)}$$

- Problem Formulation:

- Correct pseudorange measurements using the output of the front-end PrNet:

$$\rho_{c_k}^{(n)} = \rho_k^{(n)} - \hat{\varepsilon}_k^{(n)}$$

- Recall the pseudorange equation for the  $n^{\text{th}}$  satellite at time  $k$ :

$$\sqrt{(x_k - x_k^{(n)})^2 + (y_k - y_k^{(n)})^2 + (z_k - z_k^{(n)})^2 + \delta t_{u_k}} = \rho_{c_k}^{(n)}$$

- The objective is written as:

$$\mathbf{X}_k^* = \min_{\mathbf{X}_k = [x_k, y_k, z_k, \delta t_{u_k}]^T} S(\mathbf{X}_k)$$

where

$$S(\mathbf{X}_k) = \frac{1}{2} \|\mathbf{l}_k(\mathbf{X}_k)\|^2$$

$$\mathbf{l}(\mathbf{X}_k) = \left[ l_k^{(1)}, l_k^{(2)}, \dots, l_k^{(M)} \right]^T$$

$$l_k^{(n)} = (\rho_k^{(n)} - \hat{\varepsilon}_k^{(n)}) - (\sqrt{(x_k - x_k^{(n)})^2 + (y_k - y_k^{(n)})^2 + (z_k - z_k^{(n)})^2} + \delta t_{u_k}).$$

- We use the squared loss function:

$$\mathcal{L} = \|\mathbf{X}_k^* - \mathbf{X}_{GT_k}\|^2$$

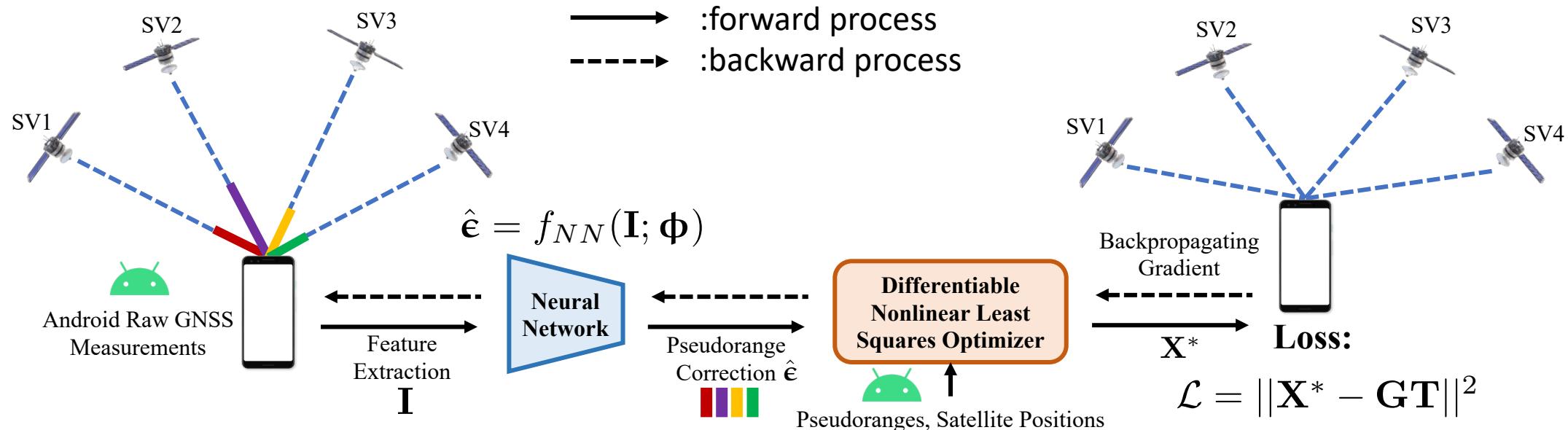
where  $\mathbf{X}_{GT_k} = [x_{GT_k}, y_{GT_k}, z_{GT_k}, \delta t_{u_{GT_k}}]^T$  is the ground truth of user location and clock offset.

- The ground truth location can be obtained using high-performance geodetic GNSS receivers.
- However, the user clock offset is hardly labeled.
  - ✓ We use its Weighted Least Squares (WLS)-based estimate to label it, i.e.,

$$\delta t_{u_{GT_k}} \leftarrow \hat{\delta t}_{u_k}^{(WLS)}.$$

# Training E2E-PrNet (1/2)

- When we train E2E-PrNet, the gradient backpropagation is performed as



the first derivative is easy to compute considering its explicit form.

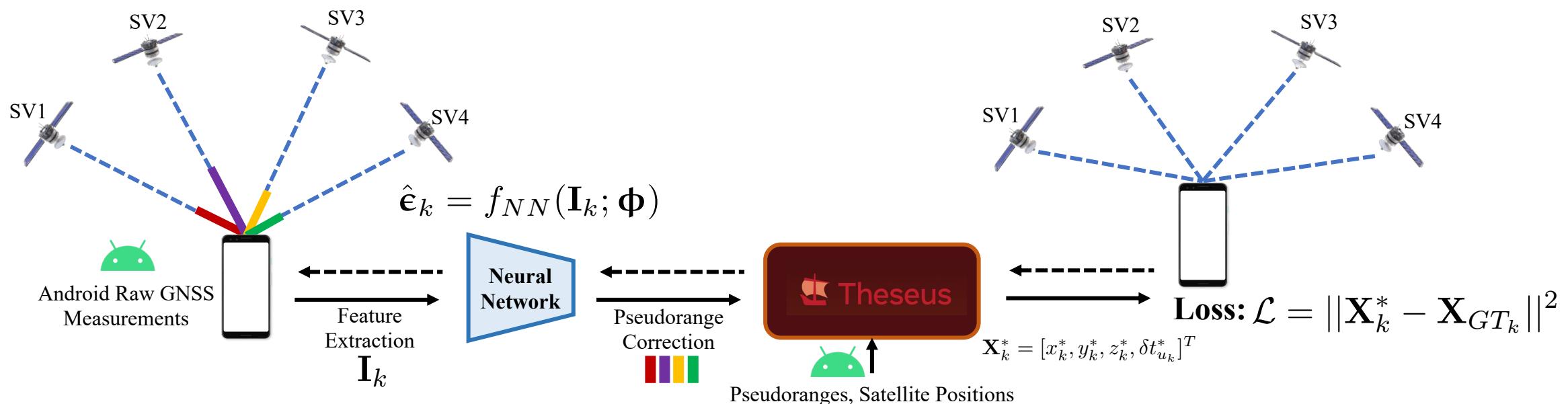
$$\frac{\partial \mathcal{L}}{\partial \Phi} = \boxed{\frac{\partial \mathcal{L}}{\partial \mathbf{X}_k}} \cdot \boxed{\frac{\partial \mathbf{X}_k^*}{\partial \hat{\epsilon}_k}} \cdot \boxed{\frac{\partial \hat{\epsilon}_k}{\partial \Phi}}$$

the last one can be solved in a standard way by PyTorch or TensorFlow.

The differentiation through the nonlinear least squares optimization is difficult!!!

## Training E2E-PrNet (2/2)

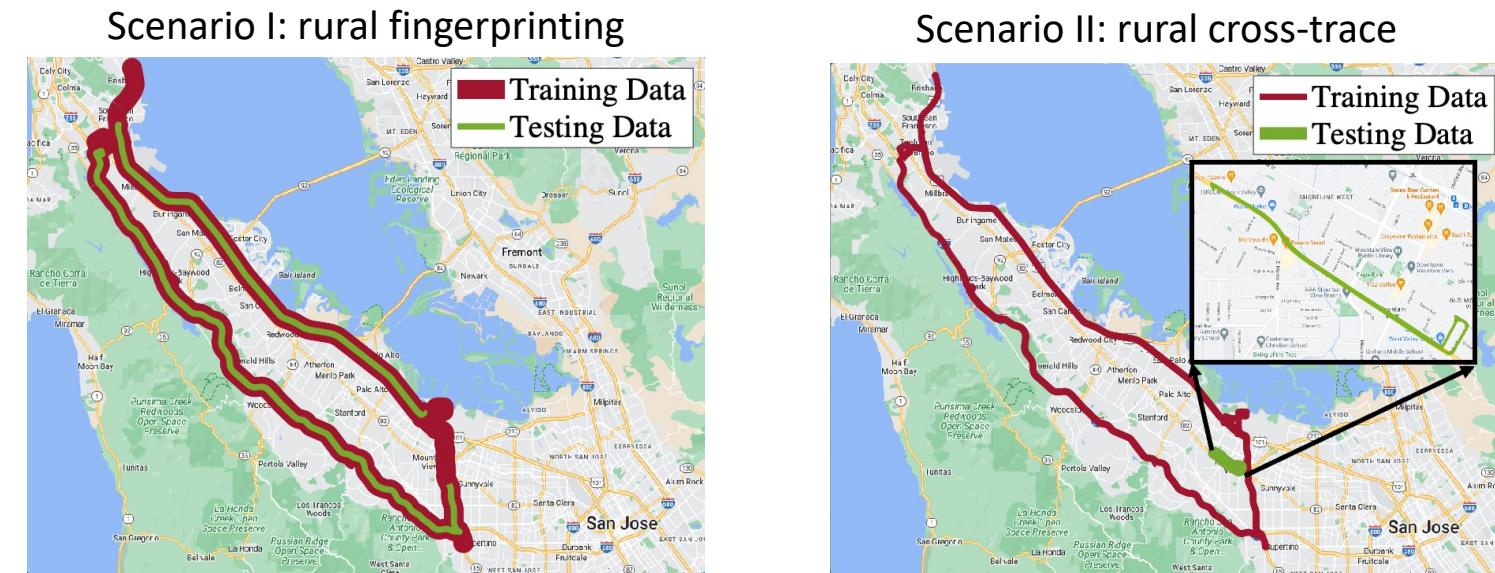
- We use Theseus, a general *differentiable nonlinear least squares solver from Meta AI*, to solve the nonlinear least squares optimization problem and backpropagate gradients.



[1] Pineda, L., Fan, T., Monge, M., Venkataraman, S., Sodhi, P., Chen, R. T., ... & Mukadam, M. (2022). Theseus: A library for differentiable nonlinear optimization. *Advances in Neural Information Processing Systems*, 35, 3801-3818.

# Experiments: Google Smartphone Decimeter Challenge (GSDC) Datasets

Training and testing data were collected ***along the same routes on different dates.***



Training and testing data were collected ***along the different routes on different dates.***

## DETAILS OF DATA SETS

Scenarios	Time Length	Trace Distance	Smartphones	Urban Canyon
Training Data in Scenario I & II	5.5 h	650 km	Pixel 4	Light
Testing Data in Scenario I	1 h	120 km	Pixel 4	Light
Testing Data in Scenario II	0.5 h	11 km	Pixel 4	Medium

## Experiments: Evaluation Metrics

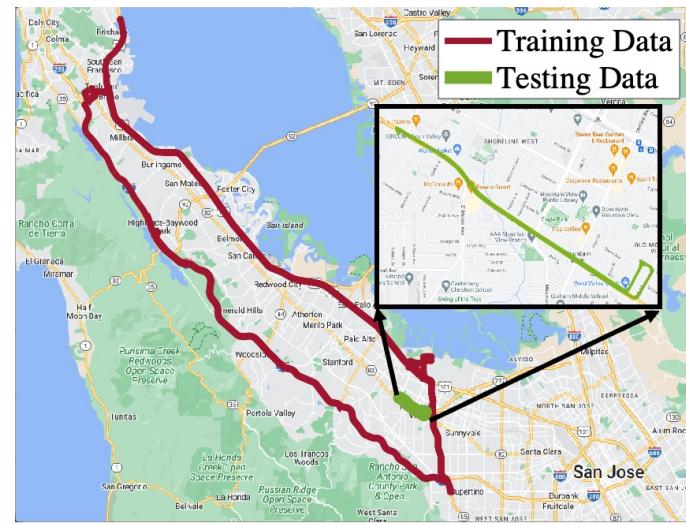
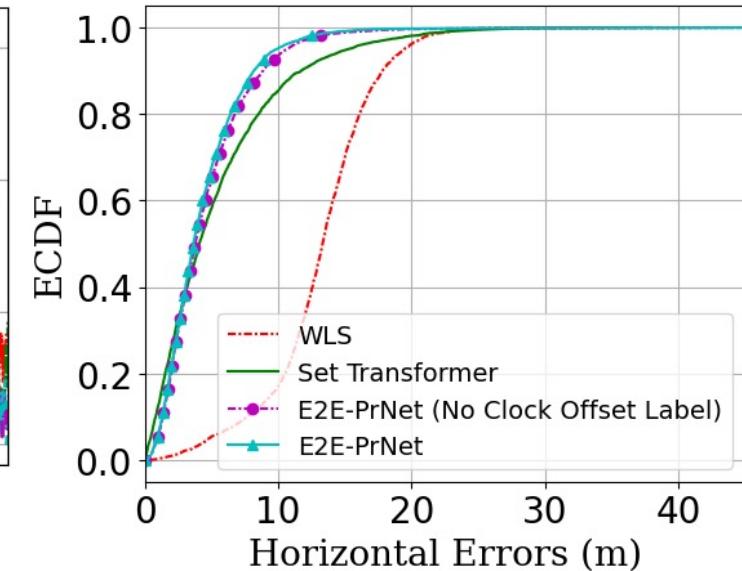
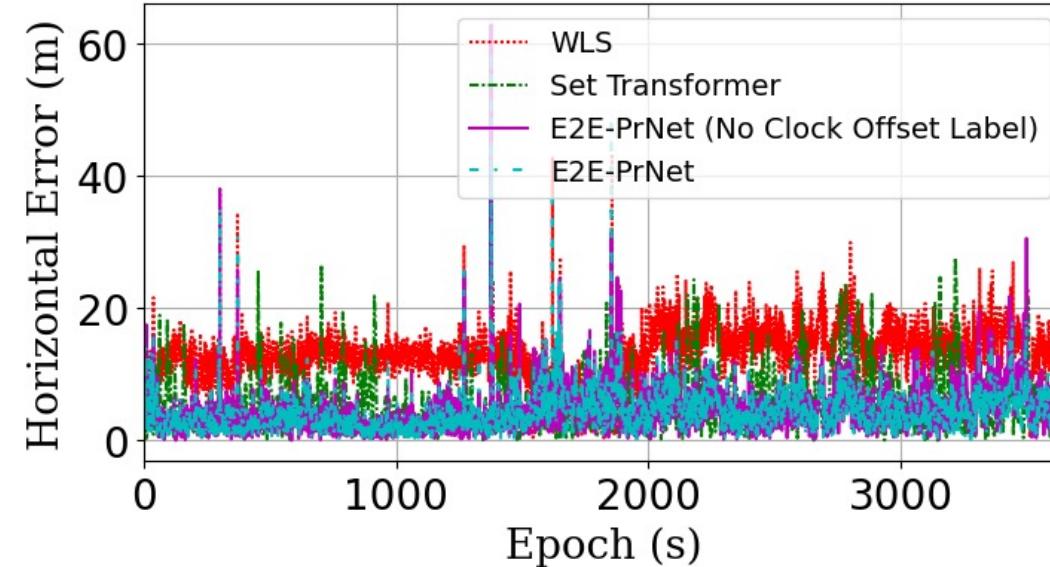
- The evaluation metrics adopted by Google:
  - Calculate the horizontal distance errors between the predicted latitude/longitude and the ground truth latitude/longitude.
  - Compute the mean of the 50<sup>th</sup> and 95<sup>th</sup> percentile horizontal distance errors.



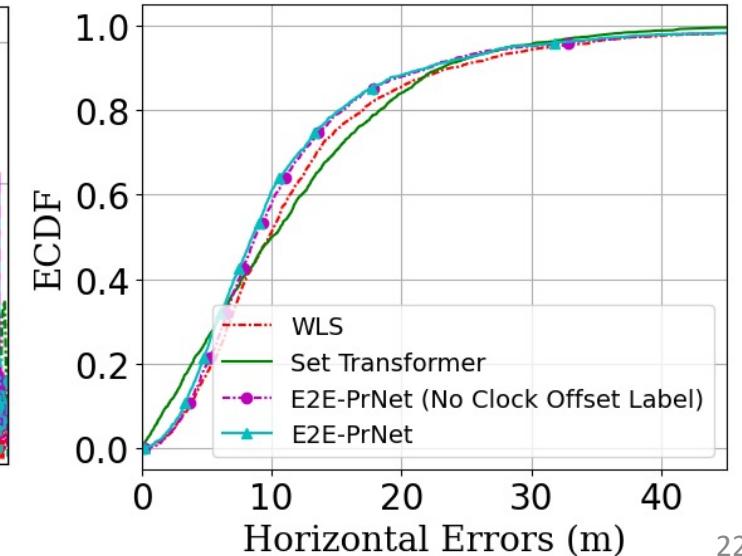
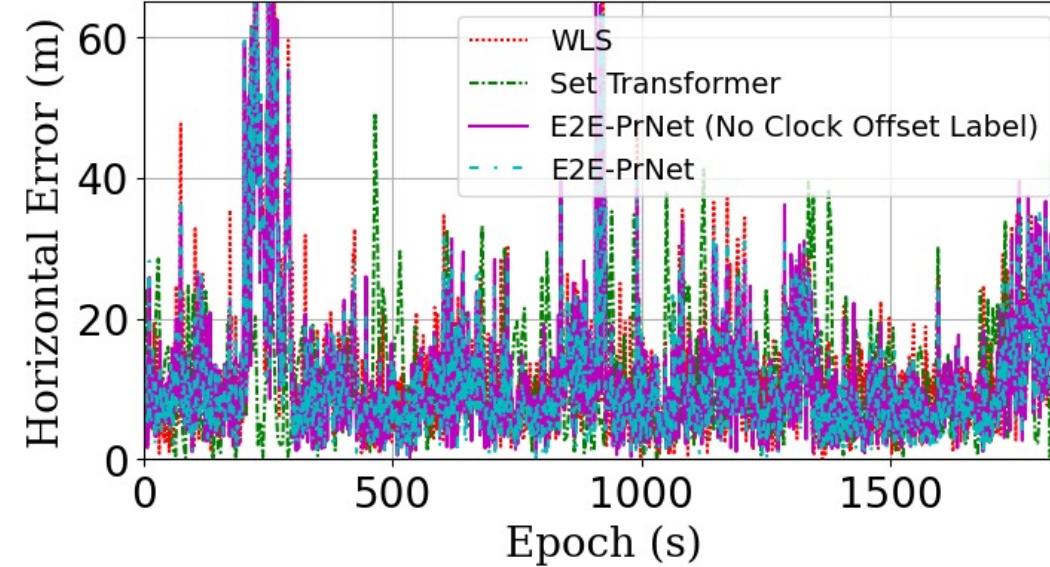
# Experiments: Results (1/2)



Scenario I: rural fingerprinting

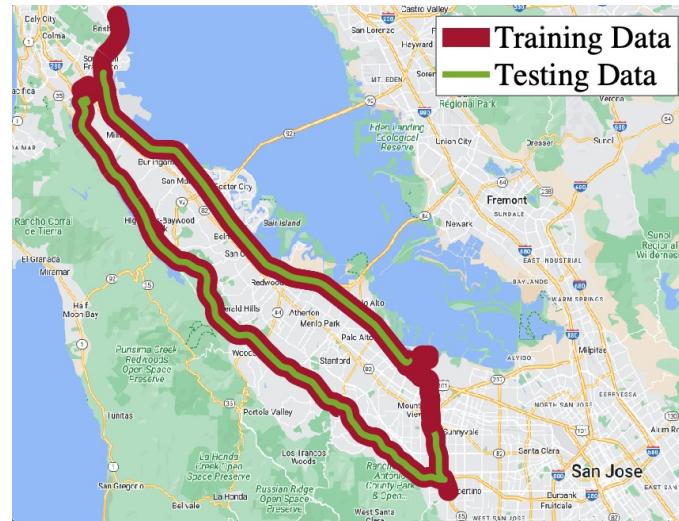


Scenario II: rural cross-trace

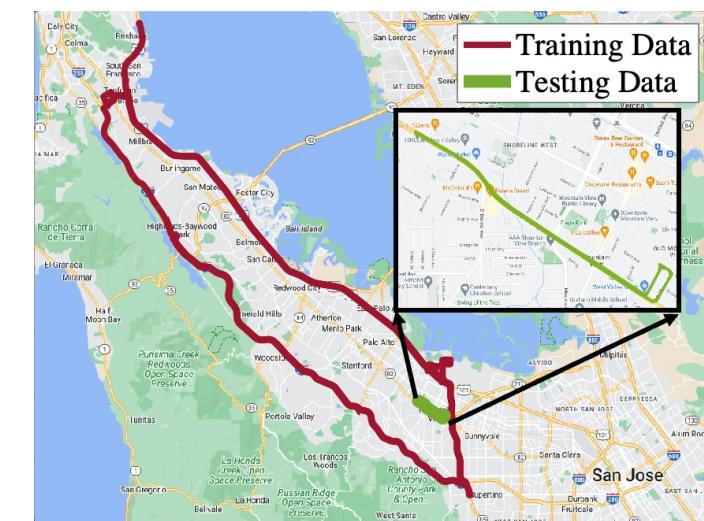


# Experiments: Results (2/2)

Scenario I: rural fingerprinting



Scenario II: rural cross-trace



## HORIZONTAL POSITIONING SCORES OF END-TO-END SOLUTIONS

Methods	Horizontal Score (meter)↓	
	Scenario I	Scenario II
<b>WLS</b>	16.390	20.666
<b>Set Transformer</b>	9.699 ( $\mu = 15m$ )	19.247 ( $\mu = 22m$ )
<b>E2E-PrNet (No RCOL)</b>	7.239	19.158
<b>E2E-PrNet</b>	<b>6.777</b>	<b>18.520</b>

RCOL: Receiver Clock Offset Label

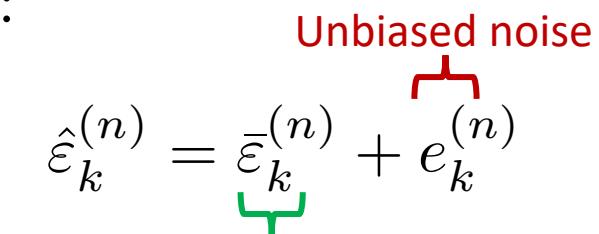
## Discussion: Does the Front-end PrNet Really Learn Pseudorange Errors? (1/2)

- We use the ground truth location and the WLS-based estimation of the user clock offset to train E2E-PrNet.

Substituting  $x_k^* = x_{GT_k}$ ,  $y_k^* = y_{GT_k}$ ,  $z_k^* = z_{GT_k}$ , and  $\delta t_{u_k}^* = \hat{\delta t}_k^{(WLS)}$  into the pseudorange equation yields\*:

$$\hat{\varepsilon}_k^{(n)} = \bar{\varepsilon}_k^{(n)} + e_k^{(n)}$$

Unbiased noise  
Denoised error

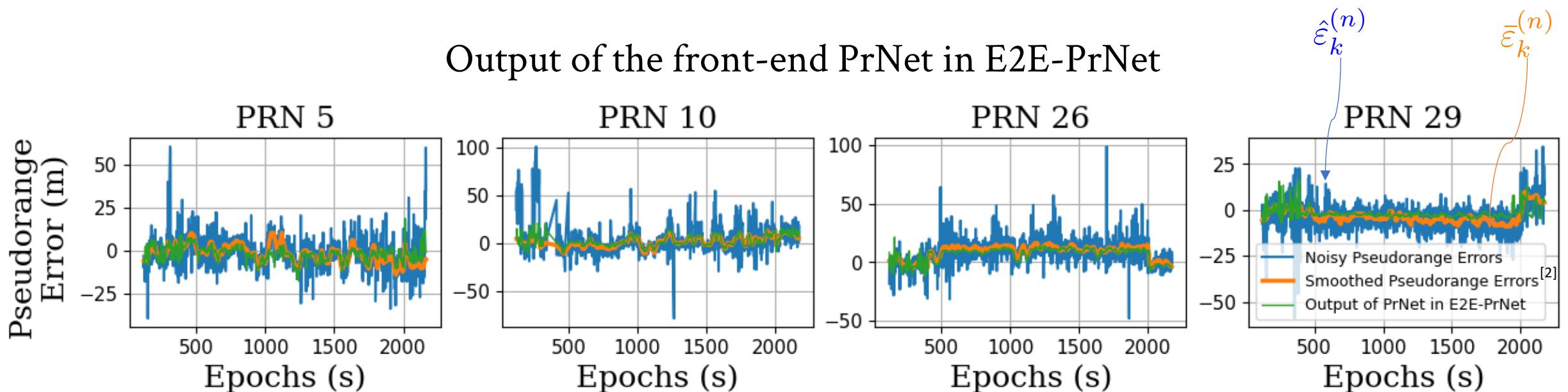


- The front-end PrNet is equivalently trained with *noisy pseudorange errors*.

\*Please refer to our paper for details

## Discussion: Does the Front-end PrNet Really Learn Pseudorange Errors? (2/2)

- Neural network can learn information from noisy labels with regularization tricks, such as dropout and early stopping, when the **signal-to-noise ratio is not too low**<sup>[1]</sup>. *Neural network can filter noisy signals!*
- Thus, the output of the front-end PrNet should be  $\hat{\varepsilon}_k^{(n)} \approx \bar{\varepsilon}_k^{(n)}$ .



[1] M. Li, M. Soltanolkotabi, and S. Oymak, "Gradient descent with early stopping is provably robust to label noise for overparameterized neural networks," in International conference on artificial intelligence and statistics. PMLR, 2020, pp. 4313–4324.

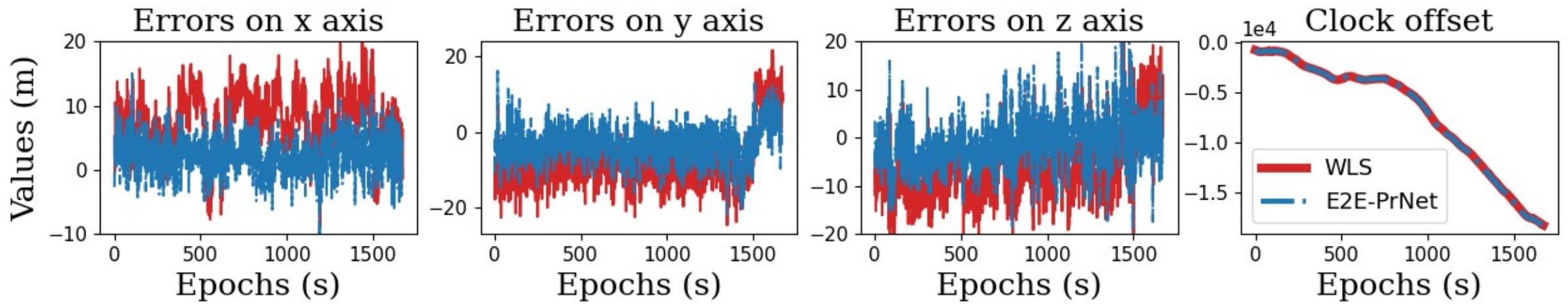
[2] X. Weng, K. Ling and H. Liu (2024), "PrNet: A Neural Network for Correcting Pseudoranges to Improve Positioning With Android Raw GNSS Measurements," in IEEE Internet of Things Journal, doi: 10.1109/JIOT.2024.3392302.

## Discussion: How good results can E2E-PrNet deliver? (1/2)

- With the pseudorange error correction from the front-end PrNet, we get the state estimate of E2E-PrNet\*:

$$\hat{x}_k = x_{GT_k} + \mathbf{h}_{x_k}^T \mathbf{U}_k \quad \hat{y}_k = y_{GT_k} + \mathbf{h}_{y_k}^T \mathbf{U}_k \quad \hat{z}_k = z_{GT_k} + \mathbf{h}_{z_k}^T \mathbf{U}_k \quad \hat{\delta t}_{u_k} = \hat{\delta t}_{u_k}^{(WLS)}$$

Errors caused by unbiased noise

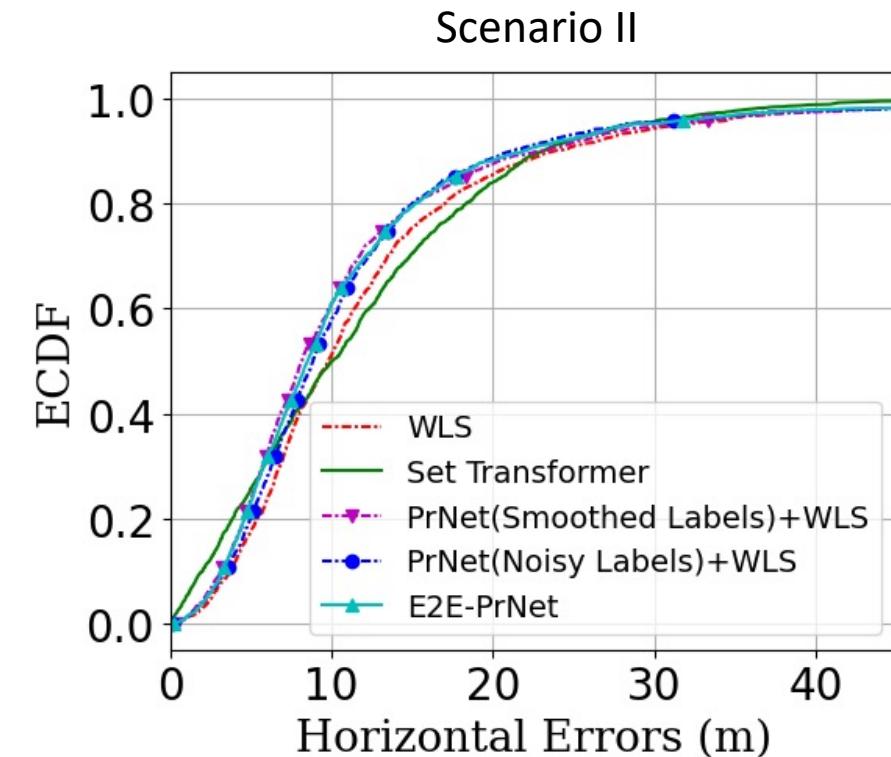
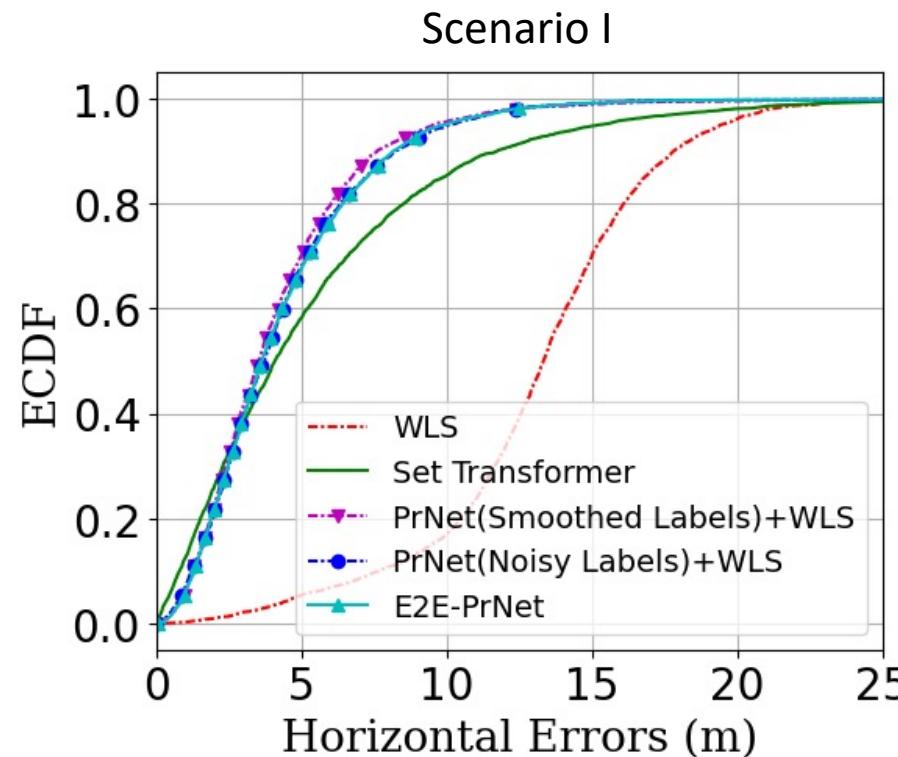


\*Please refer to our paper for details

E2E-PrNet removes the biased positioning errors! 😊

## Discussion: How good results can E2E-PrNet deliver? (2/2)

- Compare E2E-PrNet and PrNet:



Methods	Horizontal Score (meter)↓	
	Scenario I	Scenario II
<b>E2E-PrNet</b>	6.777	18.520
<b>PrNet+Noisy Labels</b>	6.922	18.434
<b>PrNet+Smoothed Labels</b>	6.537	19.524

- Contribution:
  - We proposed an end-to-end learning framework, E2E-PrNet, to regress pseudorange errors by minimizing the final localization loss.
  - The key is ***the fusion of data-driven and model-based modules*** via a differentiable nonlinear least squares optimizer.
- Results:
  - E2E-PrNet has ***better*** performance than the classical WLS and the SOTA end-to-end data-driven method on the GSDC datasets.
  - E2E-PrNet is ***highly automatic*** and has an equivalent performance to PrNet trained with hand-crafted labels.

*Thank you !*  
*and*  
*Q&A!*