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FINANCIAL ENGINEERING
ASSIGNMENT-2

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2K18/MC/008

1. (i) One call and one put option.

* if $S(T) > K$
Call option will be exercised
Put option will not be exercised.
Payoff for call option = $S(T) - K$
Payoff for put option = 0
Total = $S(T) - K$

* if $S(T) < K$
Call option will not be exercised.
Put option will be exercised
Payoff for call option = 0
Payoff for put option = $K - S(T)$
Total = $K - S(T)$

* if $S(T) = K$
Both options will not be exercised
payoff for both = 0
Total = 0

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(ii) Two calls and has sold short one share of stock.

* if $S(T) > K$

Call option will be exercised.
Since due to the call option being exercised, we will have two stock, out of which we will return the stock which we sold short

$$\text{Payoff} = 2(S(T) - K) - S(T)$$
$$= S(T) - 2K$$

* if $S(T) \leq K$

Call option will not be exercised

$$\text{Payoff} = -S(T)$$

(iii) One share of stock and has sold one call.

* if $S(T) \leq K$

Call option will not be exercised

$$\text{Payoff} = S(T)$$

* if $S(T) > K$

Call option is exercised

$$\text{Payoff} = (K - S(T)) + S(T) = K$$

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(2) $S(0) = Rs\ 50$

$u = 0.1$ $\quad u' = 1.1$ $\quad u' = 1+u$

$d = -0.1$ $\quad d' = 0.9$ $\quad d' = 1+d$

$r = 0.01$ $\quad R = 1+r = 1.01$

$K = Rs\ 48$

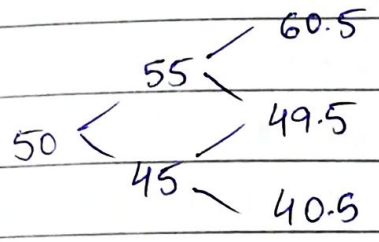
$S^u = S(0) \times u' = 50 \times 1.1 = Rs\ 55$

$S^d = S(0) \times d' = 50 \times 0.9 = Rs\ 45$

$S^{uu} = S(0) \times u' \times u' = 50 \times 1.1 \times 1.1 = Rs\ 60.5$

$S^{ud} = S(0) \times u' \times d' = 50 \times 1.1 \times 0.9 = Rs\ 49.5$

$S^{dd} = S(0) \times d' \times d' = 50 \times 0.9 \times 0.9 = Rs\ 40.5$



$$p^* = \frac{R-d'}{u'-d'} = \frac{1.01-0.9}{1.1-0.9} = \frac{0.11}{0.2} = \frac{11}{20}$$

$$1-p^* = \frac{9}{20}$$

$C^{uu} = [S^{uu} - K]^+ = Rs\ 12.5$

$C^{ud} = [S^{ud} - K]^+ = Rs\ 1.5$

$C^{dd} = [S^{dd} - K]^+ = Rs\ 0$

$$C(0) = \frac{1}{R}\left[p^{*2} C^{uu} + 2p^*(1-p^*) C^{ud} + (1-p^*)^2 C^{dd}\right]$$

$$= \frac{1}{1.01}\left[\left(\frac{11}{20}\right)^2 \times 12.5 + 2 \times \frac{11}{20} \times \frac{9}{20} \times 1.5 + \left(\frac{9}{20}\right)^2 \times 0\right]$$

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$$C(0) = \frac{1 \times 1}{1.01\,400}\left[1572.5 + 297\right] = Rs\ 4.4789$$

(3) $S(0) = 60$

$K = 62$

$u = 1.1$

$d = 0.95$

$r = 0.03 \quad R = 1 + r = 1.03$

$T = 3$

$S^{u} = S(0) \times u = 60 \times 1.1 = 66$

$S^{d} = S(0) \times d = 60 \times 0.95 = 57$

$S^{uu} = S(0) \times u^2 = 60 \times (1.1)^2 = 72.6$

$S^{ud} = S(0) \times u \times d = 60 \times 1.1 \times 0.95 = 62.7$

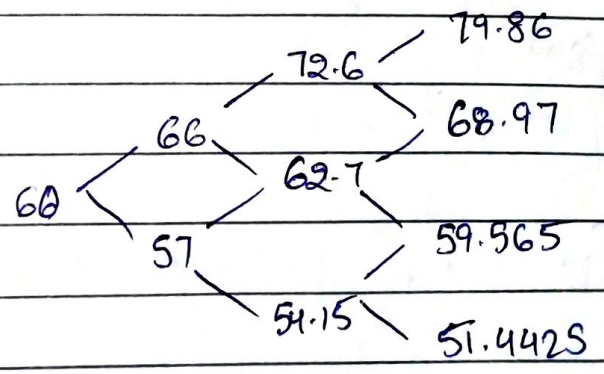
$S^{dd} = S(0) \times d^2 = 60 \times (0.95)^2 = 54.15$

$S^{uuu} = S(0) \times u^3 = 60 \times (1.1)^3 = 79.86$

$S^{uud} = S(0) \times u^2 \times d = 60 \times (1.1)^2 \times 0.95 = 68.97$

$S^{udd} = S(0) \times u \times d^2 = 60 \times (1.1) \times (0.95)^2 = 59.565$

$S^{ddd} = S(0) \times d^3 = 60 \times (0.95)^3 = 51.4425$



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$$p^* = \frac{R-d}{u-d} = \frac{1.03-0.95}{1.1-0.95} = \frac{8}{15}$$

$$1-p^* = \frac{7}{15}$$

$$C^{uuu} = [S^{uuu}-K]^+ = 17.86$$
$$C^{uud} = [S^{uud}-K]^+ = 6.97$$
$$C^{udd} = [S^{udd}-K]^+ = 0$$
$$C^{ddd} = [S^{ddd}-K]^+ = 0$$

$$C(0) = \frac{1}{R}\Big[(p^*)^3 C^{uuu} + 3p^{*2}(1-p^*)C^{uud} + 3p^*(1-p^*)^2 C^{udd} + (1-p^*)^3 C^{ddd}\Big]$$

$$= \frac{1}{1.03}\left[\left(\frac{8}{15}\right)^3 \times 17.86 + 3\times\left(\frac{8}{15}\right)^2 \times \frac{7}{15} \times 6.9 + 3\times\frac{8}{15}\times\left(\frac{7}{15}\right)^2 \times 0 + \left(\frac{7}{15}\right)^3 \times 0\right]$$

$$= \frac{1}{1.03} \times \frac{1}{3375} \times 18512$$

$$= 5.3253$$

$$P(0) = C(0) - S(0) + Ke^{-rT}$$
$$= 5.3253 - 60 + 62 \times e^{-0.03\times 3}$$
$$= 1.989$$

(4.) $S(0) = 120$

$u = 1.2 \quad d = 0.9$

$r = 0.01 \quad R = 1.01$

$K = 120 \quad T = 2$

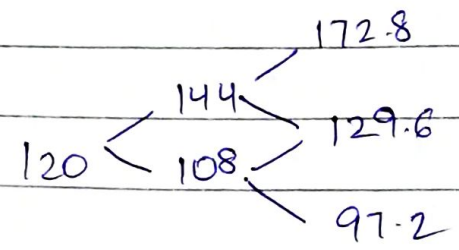
$S^u = S(0) \times u = 120 \times 1.2 = 144$

$S^d = S(0) \times d = 120 \times 0.9 = 108$

$S^{uu} = S(0) \times u^2 = 120 \times (1.2)^2 = 172.8$

$S^{ud} = S(0) \times u \times d = 129.6$

$S^{dd} = S(0) \times d^2 = 97.2$



$C^{uu} = [S^{uu} - K]^+ = 52.8$

$C^{ud} = [S^{ud} - K]^+ = 9.6$

$C^{dd} = [S^{dd} - K]^+ = 0.$

$$p^* = \frac{R-d}{u-d} = \frac{1.01-0.9}{1.2-0.9} = \frac{0.11}{0.3} = \frac{11}{30}$$

$$1-p^* = \frac{19}{30}$$

$$C(0) = \frac{1}{R}\left[p^{*2} C^{uu} + 2p^*(1-p^*) C^{ud} + (1-p^*)^2 C^{dd}\right]$$

$$= \frac{1}{1.01}\left[\left(\frac{11}{30}\right)^2 \times 52.8 + 2 \times \frac{11}{30} \times \frac{9}{30} \times 9.6 + \left(\frac{9}{30}\right)^2 \times 0\right]$$

$$= 11.4429$$

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(5) $S(0) = Rs\ 100$

$\sigma = 0.18$ (18%)

$r = 0.04$ (continuously compounded)

$K = Rs\ 80$

$T = 3$ years.

$\Delta t = 2$

$u = e^{\sigma\sqrt{\Delta t}} = e^{0.18 \times \sqrt{2}} = 1.2899$

$d = \frac{1}{u} = 0.7753$

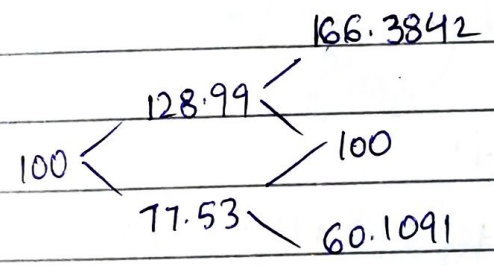
$S^u = S(0) \times u = 100 \times 1.2899 = 128.99$

$S^d = S(0) \times d = 100 \times 0.7753 = 77.53$

$S^{uu} = S(0) \times u^2 = 100 \times (1.2899)^2 = 166.3842$

$S^{ud} = S(0) \times u \times d = 100 \times 1.2899 \times 0.7753 = 100$

$S^{dd} = S(0) \times d^2 = 100 \times (0.7753)^2 = 60.1091$



$C^{uu} = [S^{uu} - K]^+ = 86.3842$

$C^{ud} = [S^{ud} - K]^+ = 20$

$C^{dd} = [S^{dd} - K]^+ = 0$

$R = e^{r\Delta t} = e^{0.04 \times 2} = 1.0833$

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$$p^* = \frac{R-d}{u-d} = \frac{1.0833 - 0.7753}{1.2899 - 0.7753} = 0.5985$$

$$C(0) = \frac{1}{R}\left[p^{*2} C_{uu} + 2p^*(1-p^*) C_{ud} + (1-p^*)^2 C_{dd}\right]$$

$$= \frac{1}{\cancel{0.5985}} \frac{1}{1.0833}\left[(0.5985)^2 \times 86.3842 + 2 \times 0.5985 \times (1-0.5985) \times 20 + 0\right]$$

$$= 37.5882$$

(6.) $S(0) = Rs\ 51$

$K = Rs\ 50$

$\sigma = 30\% = 0.3$

$r = 8\% = 0.08$

$T = 3 \text{ months} = 3/12 = 1/4$

$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{51}{50}\right) + \left(0.08 + \frac{(0.3)^2}{2}\right) \times \frac{1}{4}}{0.3 \times \frac{1}{\sqrt{4}}}$$

$$= 0.3404$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$= 0.3404 - 0.3 \times \frac{1}{\sqrt{4}}$$

$$= 0.1904$$

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$\phi(d_1) = 0.6332$
$\phi(d_2) = 0.5755$

$C(0) = S(0)\,\phi(d_1) - Ke^{-rT}\,\phi(d_2)$
$= 51 \times 0.6332 - 50 \times e^{-0.08 \times \frac{1}{4}} \times 0.5755$
$= Rs\ 4.08798$

$P(0) = Ke^{-rT}\,\phi(-d_2) - S(0)\,\phi(-d_1)$

$\phi(-d_1) = 0.3668$
$\phi(-d_2) = 0.4245$

$P(0) = 50 \times e^{-0.08 \times \frac{1}{4}} \times 0.4245 - 51 \times 0.3668$
$Rs\ 2.0979$

Using Put call Parity
$P(0) = C(0) - S(0) + Ke^{-rT}$
$= 4.08798 - 51 + 50 \times e^{-0.08 \times \frac{1}{4}}$
$= Rs\ 2.0979$

Hence both values are same.

7. $S(0) = Rs\ 260$
$T = 6$ months $= 1/2$
$K = Rs\ 256$
$r = 0.04$ (continuously compounded)
$\sigma = 0.25$

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$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{260}{256}\right) + \left(0.04 + \frac{(0.25)^2}{2}\right)\frac{1}{2}}{0.25 \times \frac{1}{\sqrt{2}}}$$

$$= 0.2892$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$= 0.2892 - 0.25 \times \frac{1}{\sqrt{2}} = 0.1124$$

$\phi(d_1) = 0.6138$

$\phi(d_2) = 0.5447$

$$C(0) = S(0)\phi(d_1) - Ke^{-rT}\phi(d_2)$$

$$= 260 \times 0.6138 - 256 \times e^{-0.04 \times \frac{1}{2}} \times 0.5447$$

$\Rightarrow$ Rs 22.9059

(8) $S(0) =$ Rs 42

$K =$ Rs 40

$T =$ 6 months $= 1/2$

$r = 0.05$

$\sigma = 0.22$

$x = 100$

$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{42}{40}\right) + \left(0.05 + \frac{(0.22)^2}{2}\right)\frac{1}{2}}{0.22 \times \frac{1}{\sqrt{2}}}$$

$$= 0.5521$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$= 0.5521 - 0.22 \times \frac{1}{\sqrt{2}}$$

$$= 0.3965$$

$$\phi(-d_1) = 0.2904$$

$$\phi(-d_2) = 0.3459$$

$$P(0) = Ke^{-rT}\phi(-d_2) - S(0)\phi(-d_1)$$

$$= 40 \times e^{-0.05 \times \frac{1}{2}} \times 0.3459 - 42 \times 0.2904$$

$= \text{Rs } 1.2976$ for one stock

for 100 stocks : Rs 129.76

(9.) $S(0) = \text{Rs } 20$

$K = \text{Rs } 25$

$T = 3$ months $= 1/4$

$r = 0.05$ $\sigma = 0.24$

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$r_{div} = 3\% = 0.03$ (continuously)

$S_d = S(0) * \cancel{(div)}\, e^{-rt}$

$= 20 \times e^{-0.03 \times 1/4}$

$= 19.8505$

$$d_1 = \frac{\ln\left(\frac{S_d}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{19.8505}{25}\right) + \left(0.05 + \frac{(0.24)^2}{2}\right)\frac{1}{4}}{0.24 \times \frac{1}{\sqrt{4}}}$$

$= -1.7579$

$d_2 = d_1 - \sigma\sqrt{T}$

$= -1.7579 - 0.24 \times \frac{1}{\sqrt{4}}$

$= -1.8779$

$\phi(d_1) = 0.0394$

$\phi(d_2) = 0.0302$

$C(0) = S_d \times \phi(d_1) - Ke^{-rT}\phi(d_2)$

$= 19.8505 \times 0.0394 - 25 \times e^{-0.05 \times \frac{1}{4}} \times 0.0302$

$=$ Rs 0.0365 (for one share)

for 100 shares: Rs 3.65

(10.) $S(0) = Rs\ 100$

$\sigma = 25\% = 0.25$

$r = 5\% = 0.05$

$T = 6$ months $= 1/2$

$K = Rs\ 80$

D shares of stock shorted

By assumption that the portfolio value is non-random, we have $C - DS = 0$

$$\therefore \frac{\partial C}{\partial S} = D = \phi(d_1)$$

$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) \cdot T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{100}{80}\right) + \left(0.05 + \frac{(0.25)^2}{2}\right) \times 0.5}{0.25 \times \sqrt{0.5}}$$

$$= 1.4921$$

$$D = \phi(d_1) = 0.9322$$