INTEREST FORMULAS AND THEIR APPLICATIONS

3.1 INTRODUCTION

Interest rate is the rental value of money. It represents the growth of capital per unit period. The period may be a month, a quarter, semiannual or a year. An interest rate 15% compounded annually means that for every hundred rupees invested now, an amount of Rs. 15 will be added to the account at the end of the first year. So, the total amount at the end of the first year will be Rs. 115. At the end of the second year, again 15% of Rs. 115, i.e. Rs. 17.25 will be added to the account. Hence the total amount at the end of the second year will be Rs. 132.25. The process will continue thus till the specified number of years.

3.2 TIME VALUE OF MONEY

If an investor invests a sum of Rs. 100 in a fixed deposit for five years with an interest rate of 15% compounded annually, the accumulated amount at the end of every year will be as shown in Table 3.1.

Table 3.1 Compound Amounts

(amount of deposit = Rs. 100.00)

Year end	*Interest* (Rs.)	*Compound amount* (Rs.)
0		100.00
1	15.00	115.00
2	17.25	132.25
3	19.84	152.09
4	22.81	174.90
5	26.24	201.14

The formula to find the future worth in the third column is

$$F = P \quad (1 + i)^n$$

where

P = principal amount invested at time 0,

F = future amount,

i = interest rate compounded annually,
n = period of deposit.

The maturity value at the end of the fifth year is Rs. 201.14. This means that the amount Rs. 201.14 at the end of the fifth year is equivalent to Rs. 100.00 at time 0 (i.e. at present). This is diagrammatically shown in Fig. 3.1. This explanation assumes that the inflation is at zero percentage.

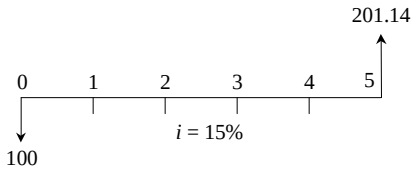


Fig. 3.1 Time value of money.

Alternatively, the above concept may be discussed as follows: If we want Rs. 100.00 at the end of the *n*th year, what is the amount that we should deposit now at a given interest rate, say 15%? A detailed working is shown in Table 3.2.

Table 3.2 Present Worth Amounts

(rate of interest = 15%)

End of year (n)	*Present worth*	*Compound amount after n year(s)*
0		100
1	86.96	100
2	75.61	100
3	65.75	100
4	57.18	100
5	49.72	100
6	43.29	100
7	37.59	100
8	32.69	100
9	28.43	100
10	24.72	100

The formula to find the present worth in the second column is

$$P = \frac{F}{(1 + i)^n}$$

From Table 3.2, it is clear that if we want Rs. 100 at the end of the fifth year, we should now deposit an amount of Rs. 49.72. Similarly, if we want Rs. 100.00 at the end of the 10th year, we should now deposit an amount of Rs. 24.72.

Also, this concept can be stated as follows:

A person has received a prize from a finance company during the recent festival contest. But the prize will be given in either of the following two modes:

1. Spot payment of Rs. 24.72 or
2. Rs. 100 after 10 years from now (this is based on 15% interest rate compounded annually).

If the prize winner has no better choice that can yield more than 15% interest rate compounded annually, and if 15% compounded annually is the common interest rate paid in all the finance companies, then it makes no difference whether he receives Rs. 24.72 now or Rs. 100 after 10 years.

On the other hand, let us assume that the prize winner has his own business wherein he can get a yield of 24% interest rate (more than 15%) compounded annually, it is better for him to receive the prize money of Rs. 24.72 at present and utilize it in his business. If this option is followed, the equivalent amount for Rs. 24.72 at the end of the 10th year is Rs. 212.45. This example clearly demonstrates the time value of money.

3.3 INTEREST FORMULAS

While making investment decisions, computations will be done in many ways. To simplify all these computations, it is extremely important to know how to use interest formulas more effectively. Before discussing the effective application of the interest formulas for investment-decision making, the various interest formulas are presented first.

Interest rate can be classified into *simple interest rate* and *compound interest rate*.

In simple interest, the interest is calculated, based on the initial deposit for every interest period. In this case, calculation of interest on interest is not applicable. In compound interest, the interest for the current period is computed based on the amount (principal plus interest up to the end of the previous period) at the beginning of the current period.

The notations which are used in various interest formulae are as follows:

P = principal amount

n = No. of interest periods

i = interest rate (It may be compounded monthly, quarterly, semiannually or annually)

F = future amount at the end of year n

A = equal amount deposited at the end of every interest period

G = uniform amount which will be added/subtracted period after period to/from the amount of deposit A1 at the end of period 1

3.3.1 Single-Payment Compound Amount

Here, the objective is to find the single future sum (F) of the initial payment (P) made at time 0 after n periods at an interest rate i compounded every period. The cash flow diagram of this situation is shown in Fig. 3.2.

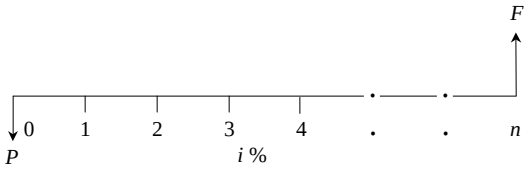


Fig. 3.2 Cash flow diagram of single-payment compound amount.

The formula to obtain the single-payment compound amount is

$$F = P(1 + i)^n = P(F/P, i, n)$$

where

$(F/P, i, n)$ is called as single-payment compound amount factor.

EXAMPLE 3.1 A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

Solution

P = Rs. 20,000
i = 18% compounded annually
n = 10 years
$F = P(1 + i)^n = P(F/P, i, n)$
= 20,000 $(F/P, 18\%, 10)$
= 20,000 5.234 = Rs. 1,04,680

The maturity value of Rs. 20,000 invested now at 18% compounded yearly is equal to Rs. 1,04,680 after 10 years.

3.3.2 Single-Payment Present Worth Amount

Here, the objective is to find the present worth amount (P) of a single future sum (F) which will be received after n periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 3.3.

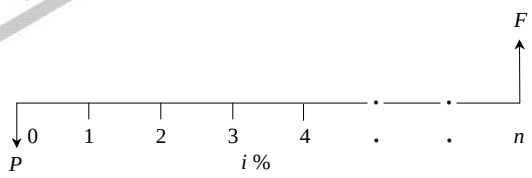


Fig. 3.3 Cash flow diagram of single-payment present worth amount.

The formula to obtain the present worth is

$$P = \frac{F}{(1 + i)^n} = F(P/F, i, n)$$

where

$(P/F, i, n)$ is termed as *single-payment present worth factor.*

EXAMPLE 3.2 A person wishes to have a future sum of Rs. 1,00,000 for his son's education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

Solution

F = Rs. 1,00,000
i = 15%, compounded annually
n = 10 years
$P = F/(1 + i)^n = F(P/F, i, n)$
= 1,00,000 $(P/F, 15\%, 10)$
= 1,00,000 0.2472
= Rs. 24,720

The person has to invest Rs. 24,720 now so that he will get a sum of Rs. 1,00,000 after 10 years at 15% interest rate compounded annually.

3.3.3 Equal-Payment Series Compound Amount

In this type of investment mode, the objective is to find the future worth of n equal payments which are made at the end of every interest period till the end of the nth interest period at an interest rate of i compounded at the end of each interest period. The corresponding cash flow diagram is shown in Fig. 3.4.

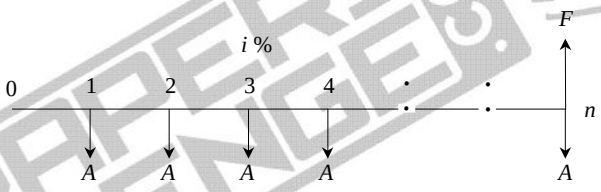


Fig. 3.4 Cash flow diagram of equal-payment series compound amount.

In Fig. 3.4,
A = equal amount deposited at the end of each interest period
n = No. of interest periods
i = rate of interest
F = single future amount

The formula to get F is

$$F = A\,\frac{(1+i)^n - 1}{i} = A(F/A, i, n)$$

where
$(F/A, i, n)$ is termed as *equal-payment series compound amount factor*.

EXAMPLE 3.3 A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for

the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

Solution

A = Rs. 10,000
n = 25 years
i = 20%
F = ?

The corresponding cash flow diagram is shown in Fig. 3.5.

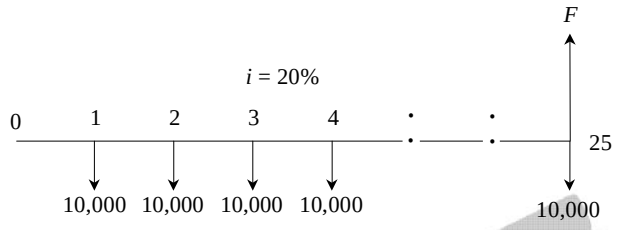


Fig. 3.5 Cash flow diagram of equal-payment series compound amount.

$$F = A\,\frac{(1+i)^n - 1}{i}$$
$$= A(F/A,\ i,\ n)$$
$$= 10{,}000(F/A,\ 20\%,\ 25)$$
$$= 10{,}000 \quad 471.981$$
$$= \text{Rs. } 47{,}19{,}810$$

The future sum of the annual equal payments after 25 years is equal to Rs. 47,19,810.

3.3.4 Equal-Payment Series Sinking Fund

In this type of investment mode, the objective is to find the equivalent amount (A) that should be deposited at the end of every interest period for n interest periods to realize a future sum (F) at the end of the nth interest period at an interest rate of i.

The corresponding cash flow diagram is shown in Fig. 3.6.

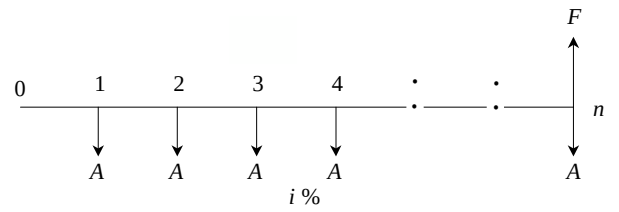


Fig. 3.6 Cash flow diagram of equal-payment series sinking fund.

In Fig. 3.6,

A = equal amount to be deposited at the end of each interest period
n = No. of interest periods
i = rate of interest
F = single future amount at the end of the nth period

The formula to get F is

$$A = F \frac{i}{(1+i)^n - 1} = F(A/F, i, n)$$

where

$(A/F, i, n)$ is called as *equal-payment series sinking fund factor.*

EXAMPLE 3.4 A company has to replace a present facility after 15 years at an outlay of Rs. 5,00,000. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.

Solution

F = Rs. 5,00,000
n = 15 years
i = 18%
A = ?

The corresponding cash flow diagram is shown in Fig. 3.7.

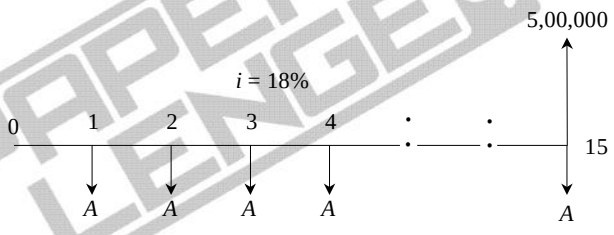


Fig. 3.7 Cash flow diagram of equal-payment series sinking fund.

$$A = F \frac{i}{(1+i)^n - 1} = F(A/F, i, n)$$
$$= 5,00,000(A/F, 18\%, 15)$$
$$= 5,00,000 \quad 0.0164$$
$$= \text{Rs. } 8,200$$

The annual equal amount which must be deposited for 15 years is Rs. 8,200.

3.3.5 Equal-Payment Series Present Worth Amount

The objective of this mode of investment is to find the present worth of an equal

payment made at the end of every interest period for n interest periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 3.8. Here,

P = present worth
A = annual equivalent payment
i = interest rate
n = No. of interest periods

The formula to compute P is

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i, n)$$

where

$(P/A, i, n)$ is called *equal-payment series present worth factor.*

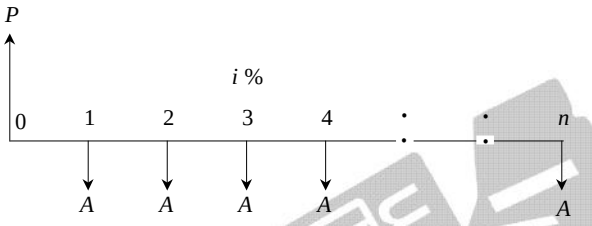


Fig. 3.8 Cash flow diagram of equal-payment series present worth amount.

EXAMPLE 3.5 A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

Solution

A = Rs. 10,00,000
i = 15%
n = 20 years
P = ?

The corresponding cash flow diagram is illustrated in Fig. 3.9.

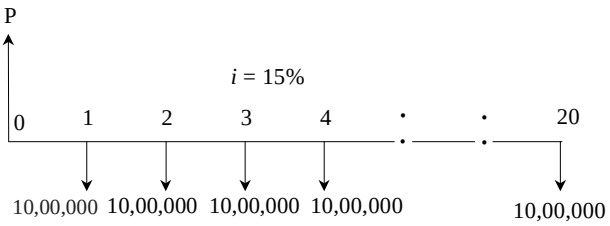


Fig. 3.9 Cash flow diagram of equal-payment series present worth amount.

$$P = A\frac{(1 + i)^n - 1}{i(1 + i)^n} = A(P/A, i, n)$$

$$= 10,00,000 \quad (P/A, 15\%, 20)$$

$$= 10,00,000 \quad 6.2593$$

$$= \text{Rs. } 62,59,300$$

The amount of reserve which must be set-up now is equal to Rs. 62,59,300.

3.3.6 Equal-Payment Series Capital Recovery Amount

The objective of this mode of investment is to find the annual equivalent amount (A) which is to be recovered at the end of every interest period for n interest periods for a loan (P) which is sanctioned now at an interest rate of i compounded at the end of every interest period (see Fig. 3.10).

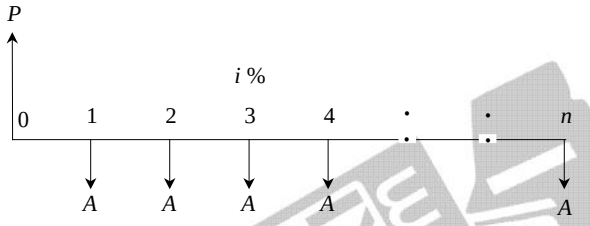


Fig. 3.10 Cash flow diagram of equal-payment series capital recovery amount.

In Fig. 3.10,

P = present worth (loan amount)
A = annual equivalent payment (recovery amount)
i = interest rate
n = No. of interest periods

The formula to compute P is as follows:

$$A = P\frac{i(1 + i)^n}{(1 + i)^n - 1} = P(A/P, i, n)$$

where,

$(A/P, i, n)$ is called *equal-payment series capital recovery factor*.

EXAMPLE 3.6 A bank gives a loan to a company to purchase an equipment worth Rs. 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

Solution

P = Rs. 10,00,000
i = 18%
n = 15 years
A = ?

The corresponding cash flow diagram is shown in Fig. 3.11.

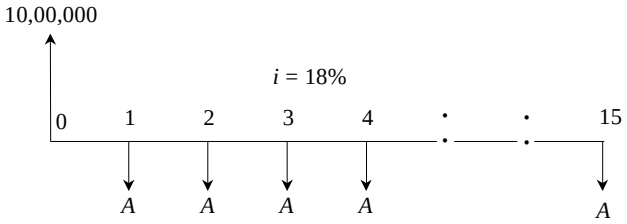


Fig. 3.11 Cash flow diagram of equal-payment series capital recovery amount.

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P, i, n)$$

$$= 10,00,000 \quad (A/P, 18\%, 15)$$

$$= 10,00,000 \quad (0.1964)$$

$$= \text{Rs. } 1,96,400$$

The annual equivalent installment to be paid by the company to the bank is Rs. 1,96,400.

3.3.7 Uniform Gradient Series Annual Equivalent Amount

The objective of this mode of investment is to find the annual equivalent amount of a series with an amount $A1$ at the end of the first year and with an equal increment (G) at the end of each of the following $n - 1$ years with an interest rate i compounded annually.

The corresponding cash flow diagram is shown in Fig. 3.12.

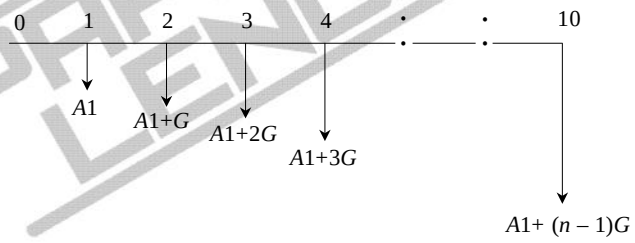


Fig. 3.12 Cash flow diagram of uniform gradient series annual equivalent amount.

The formula to compute A under this situation is

$$A = A1 + G\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$

$$= A1 + G\ (A/G, i, n)$$

where

(A/G, i, n) is called *uniform gradient series factor.*

EXAMPLE 3.7 A person is planning for his retired life. He has 10 more years

of service. He would like to deposit 20% of his salary, which is Rs. 4,000, at the end of the first year, and thereafter he wishes to deposit the amount with an annual increase of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

Solution Here,

$A1$ = Rs. 4,000
G = Rs. 500
i = 15%
n = 10 years
A = ? & F = ?

The cash flow diagram is shown in Fig. 3.13.

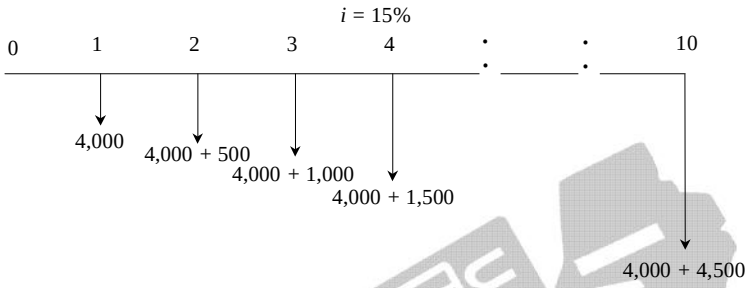


Fig. 3.13 Cash flow diagram of uniform gradient series annual equivalent amount.

$$A = A1 + G\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$
$$= A1 + G(A/G, i, n)$$
$$= 4{,}000 + 500(A/G, 15\%, 10)$$
$$= 4{,}000 + 500 \quad 3.3832$$
$$= \text{Rs. } 5{,}691.60$$

This is equivalent to paying an equivalent amount of Rs. 5,691.60 at the end of every year for the next 10 years. The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$$F = A(F/A, i, n)$$
$$= A(F/A, 15\%, 10)$$
$$= 5{,}691.60(20.304)$$
$$= \text{Rs. } 1{,}15{,}562.25$$

At the end of the 10th year, the compound amount of all his payments will be Rs. 1,15,562.25.

EXAMPLE 3.8 A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs. 8,500 at the end of the first year and

thereafter he wishes to deposit the amount with an annual decrease of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

Solution Here,

$A1$ = Rs. 8,500
G = –Rs. 500
i = 15%
n = 10 years
A = ? & F = ?

The cash flow diagram is shown in Fig. 3.14.

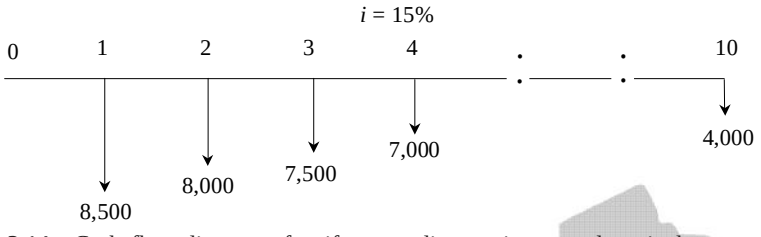


Fig. 3.14 Cash flow diagram of uniform gradient series annual equivalent amount.

$$A = A1 - G\,\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$
$$= A1 - G\,(A/G, i, n)$$
$$= 8{,}500 - 500(A/G, 15\%, 10)$$
$$= 8{,}500 - 500 \quad 3.3832$$
$$= \text{Rs. } 6{,}808.40$$

This is equivalent to paying an equivalent amount of Rs. 6,808.40 at the end of every year for the next 10 years.

The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$$F = A(F/A, i, n)$$
$$= A(F/A, 15\%, 10)$$
$$= 6{,}808.40(20.304)$$
$$= \text{Rs. } 1{,}38{,}237.75$$

At the end of the 10th year, the compound amount of all his payments is Rs. 1,38,237.75.

3.3.8 Effective Interest Rate

Let i be the nominal interest rate compounded annually. But, in practice, the compounding may occur less than a year. For example, compounding may be monthly, quarterly, or semi-annually. Compounding monthly means that the interest is computed at the end of every month. There are 12 interest periods in

a year if the interest is compounded monthly. Under such situations, the formula to compute the effective interest rate, which is compounded annually, is

$$\text{Effective interest rate, } R = 1 + i/C^{C} - 1$$

where,

i = the nominal interest rate

C = the number of interest periods in a year.

EXAMPLE 3.9 A person invests a sum of Rs. 5,000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find the maturity amount of the deposit after 10 years.

Solution

P = Rs. 5,000

n = 10 years

i = 12% (Nominal interest rate)

F = ?

METHOD 1

No. of interest periods per year = 4

No. of interest periods in 10 years = 10 4 = 40

Revised No. of periods (No. of quarters), N = 40

Interest rate per quarter, r = 12%/4

= 3%, compounded quarterly.

$$F = P(1 + r)^{N} = 5{,}000(1 + 0.03)^{40}$$

= Rs. 16,310.19

METHOD 2

No. of interest periods per year, C = 4

Effective interest rate, $R = (1 + i/C)^{C} - 1$

$= (1 + 12\%/4)^{4} - 1$

= 12.55%, compounded annually.

$$F = P(1 + R)^{n} = 5{,}000(1 + 0.1255)^{10}$$

= Rs. 16,308.91

3.4 BASES FOR COMPARISON OF ALTERNATIVES

In most of the practical decision environments, executives will be forced to select the best alternative from a set of competing alternatives. Let us assume that an organization has a huge sum of money for potential investment and there are three different projects whose initial outlay and annual revenues during their lives are known. The executive has to select the best alternative among these three competing projects.

There are several bases for comparing the worthiness of the projects. These bases are:

1. Present worth method
2. Future worth method
3. Annual equivalent method
4. Rate of return method

These methods are discussed in detail in Chapters 4–7.

QUESTIONS

1. Explain the time value of money.

2. Give practical applications of various interest formulas.

3. A person deposits a sum of Rs. 1,00,000 in a bank for his son's education who will be admitted to a professional course after 6 years. The bank pays 15% interest rate, compounded annually. Find the future amount of the deposited money at the time of admitting his son in the professional course.

4. A person needs a sum of Rs. 2,00,000 for his daughter's marriage which will take place 15 years from now. Find the amount of money that he should deposit now in a bank if the bank gives 18% interest, compounded annually.

5. A person who is just 30 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 30 years starting from the end of next year. The bank gives 15% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

6. A company is planning to expand its business after 5 years from now. The expected money required for the expansion programme is Rs. 5,00,00,000. The company can invest Rs. 50,00,000 at the end of every year for the next five years. If the assured rate of return of investment is 18% for the company, check whether the accumulated sum in the account would be sufficient to meet the fund for the expansion programme. If not, find the difference in amounts for which the company should make some other arrangement after 5 years.

7. A financial institution introduces a plan to pay a sum of Rs. 15,00,000 after 10 years at the rate of 18%, compounded annually. Find the annual equivalent amount that a person should invest at the end of every year for the next 10 years to receive Rs. 15,00,000 after 10 years from the institution.

8. A company is planning to expand its business after 5 years from now. The money required for the expansion programme is Rs. 4,00,00,000. What annual equivalent amount should the company deposit at the end of every year at an interest rate of 15% compounded annually to get Rs. 4,00,00,000 after 5 years from now?

9. A company wants to set-up a reserve which will help it to have an annual equivalent amount of Rs. 15,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made as the reserve amount now.

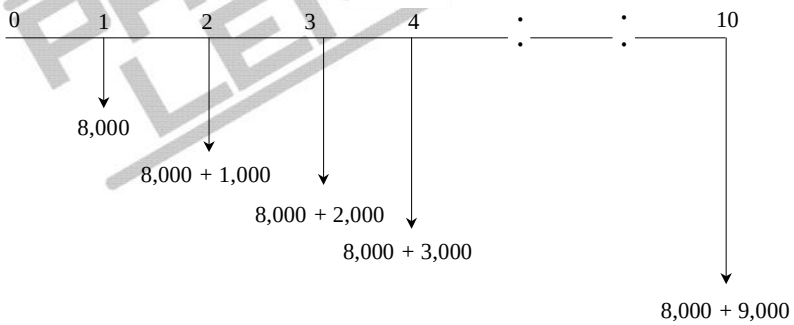
10. An automobile company recently advertised its car for a down payment of Rs. 1,50,000. Alternatively, the car can be taken home by customers without making any payment, but they have to pay an equal yearly amount of Rs. 25,000 for 15 years at an interest rate of 18%, compounded annually.
Suggest the best alternative to the customers.

11. A company takes a loan of Rs. 20,00,000 to modernize its boiler section. The loan is to be repaid in 20 equal installments at 12% interest rate, compounded annually. Find the equal installment amount that should be paid for the next 20 years.

12. A bank gives loan to a company to purchase an equipment which is worth of Rs. 5,00,000, at an interest rate of 18% compounded annually. This amount should be repaid in 25 yearly equal installments. Find the installment amount that the company has to pay to the bank.

13. A working woman is planning for her retired life. She has 20 more years of service. She would like to deposit 10% of her salary which is Rs. 5,000 at the end of the first year and thereafter she wishes to deposit the same amount (Rs. 5,000) with an annual increase of Rs. 1,000 for the next 14 years with an interest rate of 18%. Find the total amount at the end of the 15th year of the above series.

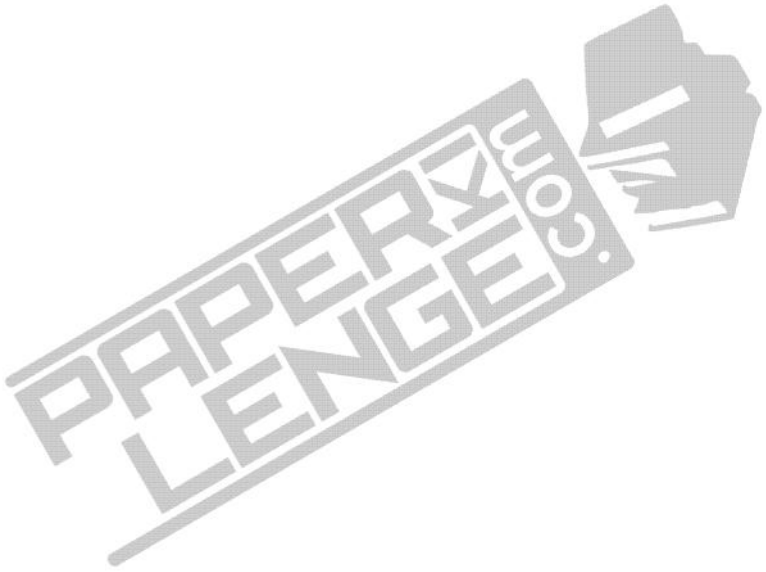
14. Consider the following cash flow diagram. Find the total amount at the end of the 10th year at an interest rate of 12%, compounded annually.



15. A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs. 10,000, at the end of the first year and thereafter he wishes to deposit the same amount (Rs. 10,000) with an annual increase of Rs. 2,000 for the next 9 years with an interest rate of 20%. Find the total amount at the end of the 10th year of the above series.

16. A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs. 30,000 at the end of the first year and thereafter he wishes to deposit the same amount (Rs. 30,000) with an annual decrease of Rs. 2,000 for the next 9 years with an interest rate of 18%. Find the total amount at the end of the 10th year of the above series.

17. A person invests a sum of Rs. 50,000 in a bank at a nominal interest rate of 18% for 15 years. The compounding is monthly. Find the maturity amount of the deposit after 15 years.



PRESENT WORTH METHOD OF COMPARISON

4.1 INTRODUCTION

In this method of comparison, the cash flows of each alternative will be reduced to time zero by assuming an interest rate *i*. Then, depending on the type of decision, the best alternative will be selected by comparing the present worth amounts of the alternatives.

The sign of various amounts at different points in time in a cash flow diagram is to be decided based on the type of the decision problem.

In a cost dominated cash flow diagram, the costs (outflows) will be assigned with positive sign and the profit, revenue, salvage value (all inflows), etc. will be assigned with negative sign.

In a revenue/profit-dominated cash flow diagram, the profit, revenue, salvage value (all inflows to an organization) will be assigned with positive sign. The costs (outflows) will be assigned with negative sign.

In case the decision is to select the alternative with the minimum cost, then the alternative with the least present worth amount will be selected. On the other hand, if the decision is to select the alternative with the maximum profit, then the alternative with the maximum present worth will be selected.

4.2 REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in Fig. 4.1.

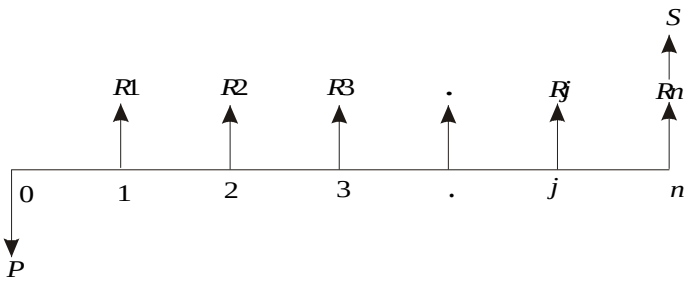


Fig. 4.1 Revenue-dominated cash flow diagram.

In Fig. 4.1, P represents an initial investment and Rj the net revenue at the end of the jth year. The interest rate is i, compounded annually. S is the salvage value at the end of the nth year.

To find the present worth of the above cash flow diagram for a given interest rate, the formula is

$$PW(i) = -P + R1[1/(1+i)^1] + R2[1/(1+i)^2] + \ldots + Rj[1/(1+i)^j] + Rn[1/(1+i)^n] + S[1/(1+i)^n]$$

In this formula, expenditure is assigned a negative sign and revenues are assigned a positive sign.

If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared. Finally, the alternative with the maximum present worth amount should be selected as the best alternative.

4.3 COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in Fig. 4.2.

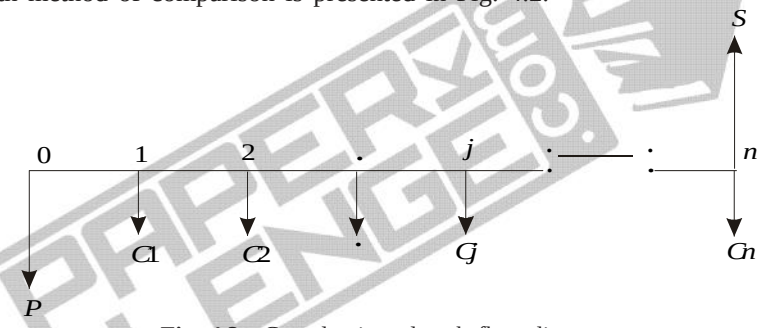


Fig. 4.2 Cost-dominated cash flow diagram.

In Fig. 4.2, P represents an initial investment, Cj the net cost of operation and maintenance at the end of the jth year, and S the salvage value at the end of the nth year.

To compute the present worth amount of the above cash flow diagram for a given interest rate i, we have the formula

$$PW(i) = P + C1[1/(1+i)^1] + C2[1/(1+i)^2] + \ldots + Cj[1/(1+i)^j] + Cn[1/(1+i)^n] - S[1/(1+i)^n]$$

In the above formula, the expenditure is assigned a positive sign and the revenue a negative sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared. Finally, the alternative with the minimum present worth amount should be selected as the best alternative.

4.4 EXAMPLES

In this section, the concept of present worth method of comparison applied to the selection of the best alternative is demonstrated with several illustrations.

EXAMPLE 4.1 Alpha Industry is planning to expand its production operation. It has identified three different technologies for meeting the goal. The initial outlay and annual revenues with respect to each of the technologies are summarized in Table 4.1. Suggest the best technology which is to be implemented based on the present worth method of comparison assuming 20% interest rate, compounded annually.

Table 4.1

	Initial outlay (Rs.)	*Annual revenue* (Rs.)	*Life* (years)
Technology 1	12,00,000	4,00,000	10
Technology 2	20,00,000	6,00,000	10
Technology 3	18,00,000	5,00,000	10

Solution In all the technologies, the initial outlay is assigned a negative sign and the annual revenues are assigned a positive sign.

TECHNOLOGY 1

Initial outlay, P = Rs. 12,00,000
Annual revenue, A = Rs. 4,00,000
Interest rate, i = 20%, compounded annually
Life of this technology, n = 10 years

The cash flow diagram of this technology is as shown in Fig. 4.3.

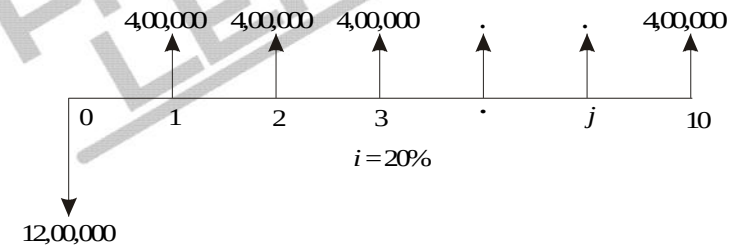


Fig. 4.3 Cash flow diagram for technology 1.

The present worth expression for this technology is

$$\begin{aligned} PW(20\%)_1 &= -12,00,000 + 4,00,000 \quad (P/A, 20\%, 10) \\ &= -12,00,000 + 4,00,000 \quad (4.1925) \\ &= -12,00,000 + 16,77,000 \\ &= \text{Rs. } 4,77,000 \end{aligned}$$

TECHNOLOGY 2

Initial outlay, P = Rs. 20,00,000
Annual revenue, A = Rs. 6,00,000
Interest rate, i = 20%, compounded annually
Life of this technology, n = 10 years

The cash flow diagram of this technology is shown in Fig. 4.4.

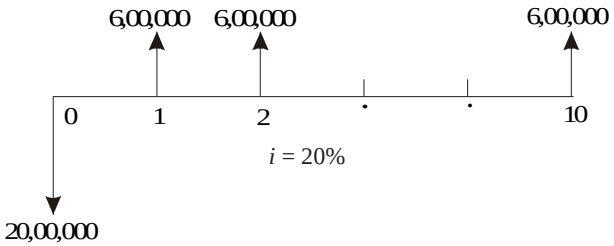


Fig. 4.4 Cash flow diagram for technology 2.

The present worth expression for this technology is

$$\begin{aligned} PW(20\%)_2 &= -20,00,000 + 6,00,000 \quad (P/A, 20\%, 10) \\ &= -20,00,000 + 6,00,000 \quad (4.1925) \\ &= -20,00,000 + 25,15,500 \\ &= \text{Rs. } 5,15,500 \end{aligned}$$

TECHNOLOGY 3

Initial outlay, P = Rs. 18,00,000
Annual revenue, A = Rs. 5,00,000
Interest rate, i = 20%, compounded annually
Life of this technology, n = 10 years

The cash flow diagram of this technology is shown in Fig. 4.5.

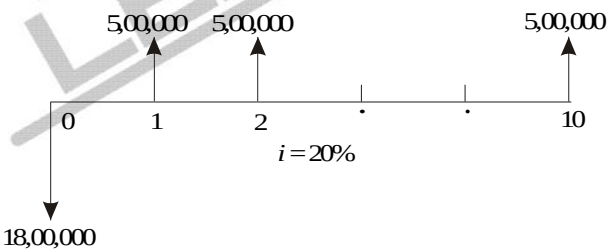


Fig. 4.5 Cash flow diagram for technology 3.

The present worth expression for this technology is

$$\begin{aligned} PW(20\%)_3 &= -18,00,000 + 5,00,000 \quad (P/A, 20\%, 10) \\ &= -18,00,000 + 5,00,000 \quad (4.1925) \\ &= -18,00,000 + 20,96,250 \\ &= \text{Rs. } 2,96,250 \end{aligned}$$

From the above calculations, it is clear that the present worth of technology 2 is the highest among all the technologies. Therefore, technology 2 is suggested for implementation to expand the production.

EXAMPLE 4.2 An engineer has two bids for an elevator to be installed in a new building. The details of the bids for the elevators are as follows:

Bid	*Engineer's estimates*		
	Initial cost (Rs.)	*Service life* (years)	*Annual operations & maintenance cost* (Rs.)
Alpha Elevator Inc.	4,50,000	15	27,000
Beta Elevator Inc.	5,40,000	15	28,500

Determine which bid should be accepted, based on the present worth method of comparison assuming 15% interest rate, compounded annually.

Solution

Bid 1: Alpha Elevator Inc.

Initial cost, P = Rs. 4,50,000
Annual operation and maintenance cost, A = Rs. 27,000
Life = 15 years
Interest rate, i = 15%, compounded annually.

The cash flow diagram of bid 1 is shown in Fig. 4.6.

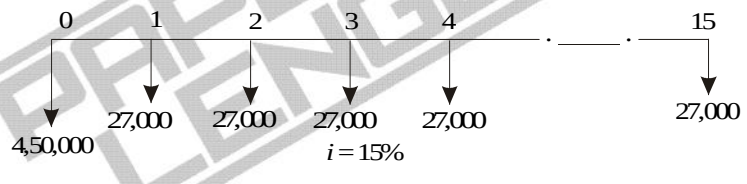


Fig. 4.6 Cash flow diagram for bid 1.

The present worth of the above cash flow diagram is computed as follows:

$$PW(15\%) = 4,50,000 + 27,000(P/A, 15\%, 15)$$
$$= 4,50,000 + 27,000 \quad 5.8474$$
$$= 4,50,000 + 1,57,879.80$$
$$= \text{Rs. } 6,07,879.80$$

Bid 2: Beta Elevator Inc.

Initial cost, P = Rs. 5,40,000
Annual operation and maintenance cost, A = Rs. 28,500
Life = 15 years
Interest rate, i = 15%, compounded annually.

The cash flow diagram of bid 2 is shown in Fig. 4.7.

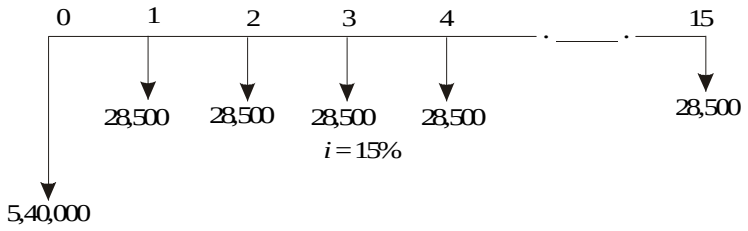


Fig. 4.7 Cash flow diagram for bid 2.

The present worth of the above cash flow diagram is computed as follows:

$$\begin{aligned} PW(15\%) &= 5{,}40{,}000 + 28{,}500(P/A,\ 15\%,\ 15) \\ &= 5{,}40{,}000 + 28{,}500 \quad 5.8474 \\ &= 5{,}40{,}000 + 1{,}66{,}650.90 \\ &= \text{Rs. } 7{,}06{,}650.90 \end{aligned}$$

The total present worth cost of bid 1 is less than that of bid 2. Hence, bid 1 is to be selected for implementation. That is, the elevator from Alpha Elevator Inc. is to be purchased and installed in the new building.

EXAMPLE 4.3 Investment proposals A and B have the net cash flows as follows:

Proposal	*End of years*				
	0	1	2	3	4
A (Rs.)	–10,000	3,000	3,000	7,000	6,000
B (Rs.)	–10,000	6,000	6,000	3,000	3,000

Compare the present worth of A with that of B at i = 18%. Which proposal should be selected?

Solution

Present worth of A at i = 18%. The cash flow diagram of proposal A is shown in Fig. 4.8.

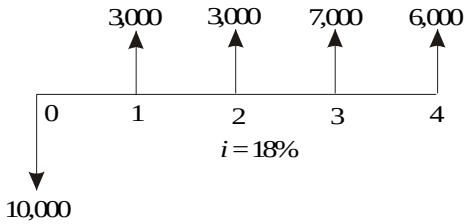


Fig. 4.8 Cash flow diagram for proposal A.

The present worth of the above cash flow diagram is computed as

$$\begin{aligned} PW_A(18\%) &= -10{,}000 + 3{,}000(P/F, 18\%, 1) + 3{,}000(P/F, 18\%, 2) \\ &\quad + 7{,}000(P/F, 18\%, 3) + 6{,}000(P/F, 18\%, 4) \\ &= -10{,}000 + 3{,}000\ (0.8475) + 3{,}000(0.7182) \\ &\quad + 7{,}000(0.6086) + 6{,}000(0.5158) \\ &= \text{Rs. } 2{,}052.10 \end{aligned}$$

Present worth of B at i = 18%. The cash flow diagram of the proposal B is shown in Fig. 4.9.

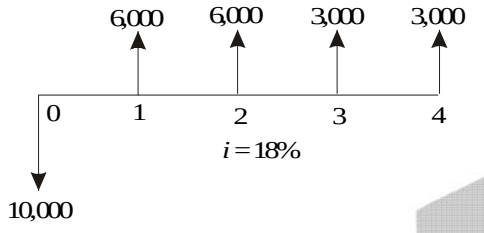


Fig. 4.9 Cash flow diagram for proposal B.

The present worth of the above cash flow diagram is calculated as

$$\begin{aligned} PW_B(18\%) &= -10{,}000 + 6{,}000(P/F, 18\%, 1) + 6{,}000(P/F, 18\%, 2) \\ &\quad + 3{,}000(P/F, 18\%, 3) + 3{,}000(P/F, 18\%, 4) \\ &= -10{,}000 + 6{,}000(0.8475) + 6{,}000(0.7182) \\ &\quad + 3{,}000(0.6086) + 3{,}000(0.5158) \\ &= \text{Rs. } 2{,}767.40 \end{aligned}$$

At i = 18%, the present worth of proposal B is higher than that of proposal A. Therefore, select proposal B.

EXAMPLE 4.4 A granite company is planning to buy a fully automated granite cutting machine. If it is purchased under down payment, the cost of the machine is Rs. 16,00,000. If it is purchased under installment basis, the company has to pay 25% of the cost at the time of purchase and the remaining amount in 10 annual equal installments of Rs. 2,00,000 each. Suggest the best alternative for the company using the present worth basis at i = 18%, compounded annually.

Solution There are two alternatives available for the company:

1. Down payment of Rs. 16,00,000

2. Down payment of Rs. 4,00,000 and 10 annual equal installments of Rs. 2,00,000 each

Present worth calculation of the second alternative. The cash flow diagram of the second alternative is shown in Fig. 4.10.

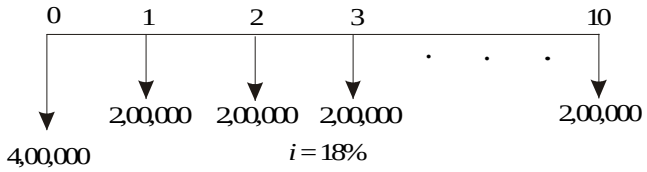


Fig. 4.10 Cash flow diagram for the second alternative.

The present worth of the above cash flow diagram is computed as

$$\begin{aligned} PW(18\%) &= 4{,}00{,}000 + 2{,}00{,}000(P/A,\ 18\%,\ 10) \\ &= 4{,}00{,}000 + 2{,}00{,}000 \quad 4.4941 \\ &= \text{Rs. } 12{,}98{,}820 \end{aligned}$$

The present worth of this option is Rs. 12,98,820, which is less than the first option of complete down payment of Rs. 16,00,000. Hence, the company should select the second alternative to buy the fully automated granite cutting machine.

EXAMPLE 4.5 A finance company advertises two investment plans. In plan 1, the company pays Rs. 12,000 after 15 years for every Rs. 1,000 invested now. In plan 2, for every Rs. 1,000 invested, the company pays Rs. 4,000 at the end of the 10th year and Rs. 4,000 at the end of 15th year. Select the best investment plan from the investor's point of view at i = 12%, compounded annually.

Solution *Plan 1.* The cash flow diagram for plan 1 is illustrated in Fig. 4.11.

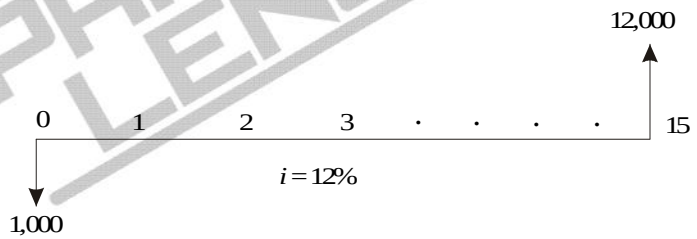


Fig. 4.11 Cash flow diagram for plan 1.

The present worth of the above cash flow diagram is calculated as

$$\begin{aligned} PW(12\%) &= -1{,}000 + 12{,}000(P/F,\ 12\%,\ 15) \\ &= -1{,}000 + 12{,}000(0.1827) \\ &= \text{Rs. } 1{,}192.40 \end{aligned}$$

Plan 2. The cash flow diagram for plan 2 is shown in Fig. 4.12.

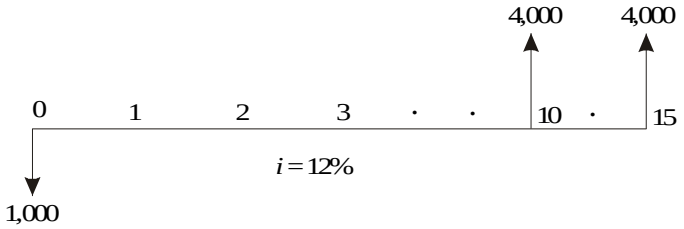


Fig. 4.12 Cash flow diagram for plan 2.

The present worth of the above cash flow diagram is computed as

$$PW(12\%) = -1,000 + 4,000(P/F, 12\%, 10) + 4,000(P/F, 12\%, 15)$$
$$= -1,000 + 4,000(0.3220) + 4,000(0.1827)$$
$$= \text{Rs. } 1,018.80$$

The present worth of plan 1 is more than that of plan 2. Therefore, plan 1 is the best plan from the investor's point of view.

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EXAMPLE 4.6 Novel Investment Ltd. accepts Rs. 10,000 at the end of every year for 20 years and pays the investor Rs. 8,00,000 at the end of the 20th year. Innovative Investment Ltd. accepts Rs. 10,000 at the end of every year for 20 years and pays the investor Rs. 15,00,000 at the end of the 25th year. Which is the best investment alternative? Use present worth base with i = 12%.

Solution *Novel Investment Ltd's plan.* The cash flow diagram of Novel Investment Ltd's plan is shown in Fig. 4.13.

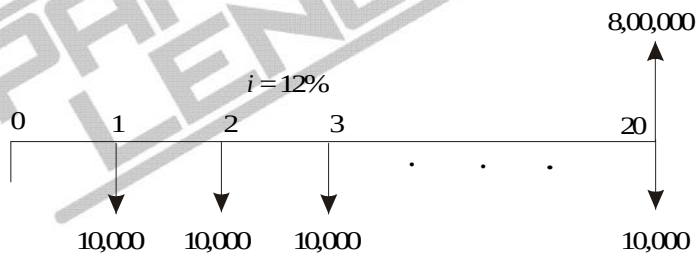


Fig. 4.13 Cash flow diagram for Novel Investment Ltd.

The present worth of the above cash flow diagram is computed as

$$PW(12\%) = -10,000(P/A, 12\%, 20) + 8,00,000(P/F, 12\%, 20)$$
$$= -10,000(7.4694) + 8,00,000(0.1037)$$
$$= \text{Rs. } 8,266$$

Innovative Investment Ltd's plan. The cash flow diagram of the Innovative Investment Ltd's plan is illustrated in Fig. 4.14.

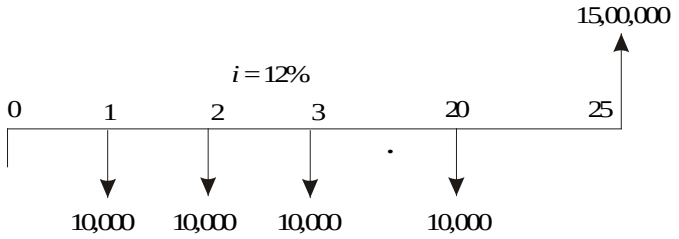


Fig. 4.14 Cash flow diagram for Innovative Investment Ltd.

The present worth of the above cash flow diagram is calculated as

$$PW(12\%) = -10{,}000(P/A, 12\%, 20) + 15{,}00{,}000(P/F, 12\%, 25)$$
$$= -10{,}000(7.4694) + 15{,}00{,}000(0.0588)$$
$$= \text{Rs. } 13{,}506$$

The present worth of Innovative Investment Ltd's plan is more than that of Novel Investment Ltd's plan. Therefore, Innovative Investment Ltd's plan is the best from investor's point of view.

EXAMPLE 4.7 A small business with an initial outlay of Rs. 12,000 yields Rs. 10,000 during the first year of its operation and the yield increases by Rs. 1,000 from its second year of operation up to its 10th year of operation. At the end of the life of the business, the salvage value is zero. Find the present worth of the business by assuming an interest rate of 18%, compounded annually.

Solution

Initial investment, P = Rs. 12,000
Income during the first year, A = Rs. 10,000
Annual increase in income, G = Rs. 1,000
n = 10 years
i = 18%, compounded annually

The cash flow diagram for the small business is depicted in Fig. 4.15.

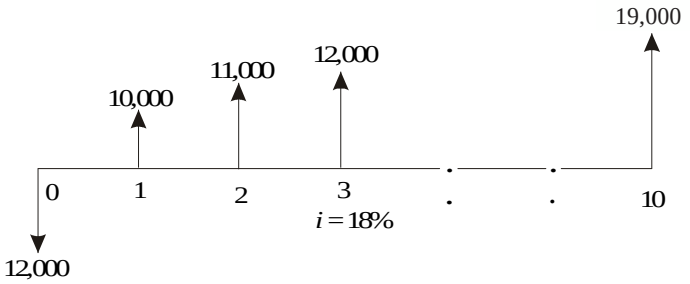


Fig. 4.15 Cash flow diagram for the small business.

The equation for the present worth is

$$PW(18\%) = -12{,}000 + (10{,}000 + 1{,}000 \quad (A/G, 18\%, 10)) \quad (P/A, 18\%, 10)$$
$$= -12{,}000 + (10{,}000 + 1{,}000 \quad 3.1936) \quad 4.4941$$
$$= -12{,}000 + 59{,}293.36$$
$$= \text{Rs. } 47{,}293.36$$

The present worth of the small business is Rs. 47,293.36.

QUESTIONS

1. A project involves an initial outlay of Rs. 30,00,000 and with the following transactions for the next five years. The salvage value at the end of the life of the project after five years is Rs. 2,00,000. Draw a cash flow diagram of the project and find its present worth by assuming i = 15%, compounded annually.

End of year	*Maintenance and operating expense* (Rs.)	*Revenue* (Rs.)
1	2,00,000	9,00,000
2	2,50,000	10,00,000
3	3,00,000	12,00,000
4	3,00,000	13,00,000
5	4,00,000	12,00,000

2. Find the present worth of the following cash flow series. Assume i = 15%, compounded annually.

End of year	0	1	2	3	4	5
Cash flow (Rs.)	–10,000	30,000	30,000	30,000	30,000	30,000

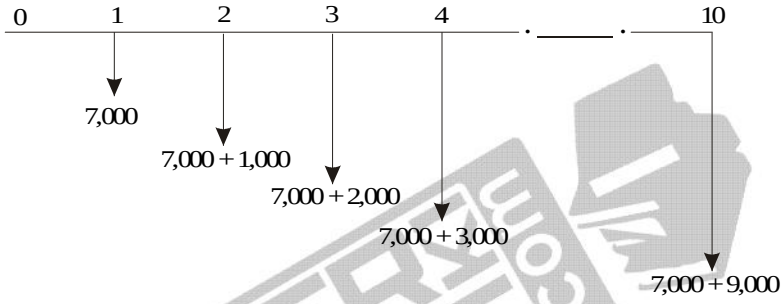
3. Consider the following cash flow series over a 20-year period. Assuming the interest rate as 18% compounded annually, compute the present worth of the series; give your comments.

End of year	*Cash flow* (Rs.)
0	–50,00,000
1	6,00,000
2	6,00,000
.	.
.	.
.	.
20	6,00,000

4. The cost of erecting an oil well is Rs. 1,50,00,000. The annual equivalent yield from the oil well is Rs. 30,00,000. The salvage value after its useful life of 10 years is Rs. 2,00,000. Assuming an interest rate of 18%, compounded annually, find out whether the erection of the oil well is financially feasible, based on the present worth method.

5. The details of the feasibility report of a project are as shown below. Check the feasibility of the project based on present worth method, using i = 20%.
Initial outlay = Rs. 50,00,000
Life of the project = 20 years.
Annual equivalent revenue = Rs. 15,00,000
Modernizing cost at the end of the 10th year = Rs. 20,00,000
Salvage value at the end of project life = Rs. 5,00,000.

6. Consider the following cash flow diagram. Find the present worth using an interest rate of 15%, compounded annually.



7. An automobile company recently advertised its car for a down payment of Rs. 1,50,000. Alternatively, the car can be taken home by customers without making any payment, but they have to pay an equal yearly amount of Rs. 25,000 for 15 years at an interest rate of 18%, compounded annually.
You are asked to advise the best alternative for the customers based on the present worth method of comparison.

8. The cash flows of two project proposals are as given below. Each of the project has an expected life of 10 years. Select the best project based on present worth method of comparison using an interest rate of 18%, compounded annually.

	Initial outlay (Rs.)	*Annual equivalent revenue* (Rs.)	*Salvage value after 10 years* (Rs.)
Project 1	–7,50,000	2,00,000	50,000
Project 2	–9,50,000	2,25,000	1,00,000

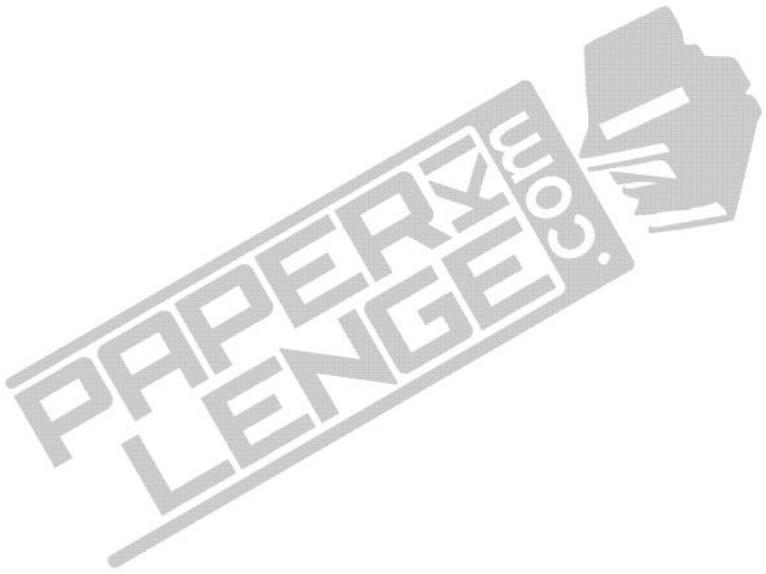
9. A company has two alternatives for satisfying its daily travel requirements of its employees for the next five years:

Alternative 1: Renting a vehicle at a cost of Rs. 10,00,000 per year.

Alternative 2: Buying a vehicle for Rs. 5,00,000 with an operating and maintenance cost of Rs. 3,50,000 per year. The salvage value of the vehicle after five years is Rs. 1,00,000.

Select the best alternative based on the present worth method of comparison using the interest rate of 20%, compounded annually.

10. A working woman is planning for her retired life. She has 20 more years of service. She would like to have an annual equivalent amount of Rs. 3,00,000, starting from the end of the first year of her retirement. Find the single amount that should be deposited now so that she receives the above mentioned annual equivalent amount at the end of every year for 20 years after her retirement. Assume i = 15%, compounded annually.



FUTURE WORTH METHOD

5.1 INTRODUCTION

In the future worth method of comparison of alternatives, the future worth of various alternatives will be computed. Then, the alternative with the maximum future worth of net revenue or with the minimum future worth of net cost will be selected as the best alternative for implementation.

5.2 REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the future worth method of comparison is presented in Fig. 5.1.

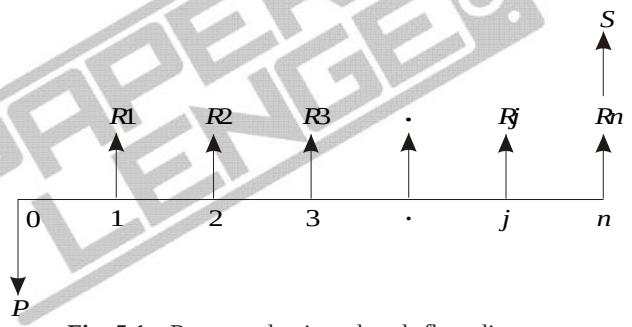


Fig. 5.1 Revenue-dominated cash flow diagram.

In Fig. 5.1, P represents an initial investment, Rj the net-revenue at the end of the jth year, and S the salvage value at the end of the nth year.

The formula for the future worth of the above cash flow diagram for a given interest rate, i is

$$FW(i) = -P(1 + i)^n + R1(1 + i)^{n-1} + R2(1 + i)^{n-2} + \ldots + Rj(1 + i)^{n-j} + \ldots + Rn + S$$

In the above formula, the expenditure is assigned with negative sign and the revenues are assigned with positive sign.

If we have some more alternatives which are to be compared with this

alternative, then the corresponding future worth amounts are to be computed and compared. Finally, the alternative with the maximum future worth amount should be selected as the best alternative.

5.3 COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the future worth method of comparison is given in Fig. 5.2.

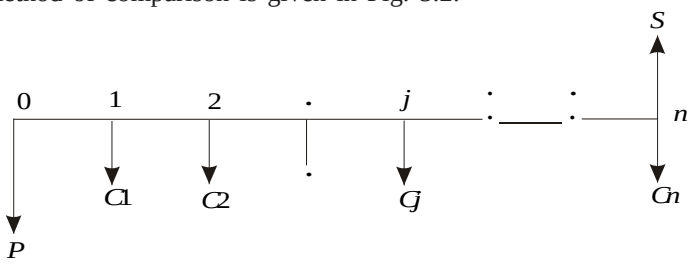


Fig. 5.2 Cost-dominated cash flow diagram.

In Fig. 5.2, P represents an initial investment, Cj the net cost of operation and maintenance at the end of the jth year, and S the salvage value at the end of the nth year.

The formula for the future worth of the above cash flow diagram for a given interest rate, i is

$$FW(i) = P(1 + i)^n + C1(1 + i)^{n-1} + C2(1 + i)^{n-2} + \ldots + Cj(1 + i)^{n-j} + \ldots + Cn - S$$

In this formula, the expenditures are assigned with positive sign and revenues with negative sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding future worth amounts are to be computed and compared. Finally, the alternative with the minimum future worth amount should be selected as the best alternative.

5.4 EXAMPLES

In this section, several examples highlighting the applications of the future worth method of comparison are presented.

EXAMPLE 5.1 Consider the following two mutually exclusive alternatives:

		End of year			
Alternative	0	1	2	3	4
A (Rs.)	–50,00,000	20,00,000	20,00,000	20,00,000	20,00,000
B (Rs.)	–45,00,000	18,00,000	18,00,000	18,00,000	18,00,000

At i = 18%, select the best alternative based on future worth method of comparison.

Solution Alternative A

Initial investment, P = Rs. 50,00,000
Annual equivalent revenue, A = Rs. 20,00,000
Interest rate, i = 18%, compounded annually
Life of alternative A = 4 years

The cash flow diagram of alternative A is shown in Fig. 5.3.

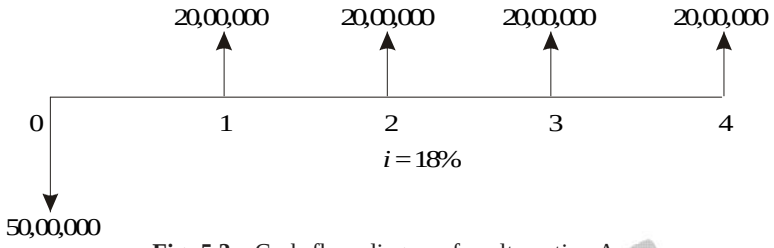


Fig. 5.3 Cash flow diagram for alternative A.

The future worth amount of alternative B is computed as

$$FW_A(18\%) = -50,00,000(F/P, 18\%, 4) + 20,00,000(F/A, 18\%, 4)$$
$$= -50,00,000(1.939) + 20,00,000(5.215)$$
$$= \text{Rs. } 7,35,000$$

Alternative B

Initial investment, P = Rs. 45,00,000
Annual equivalent revenue, A = Rs. 18,00,000
Interest rate, i = 18%, compounded annually
Life of alternative B = 4 years

The cash flow diagram of alternative B is illustrated in Fig. 5.4.

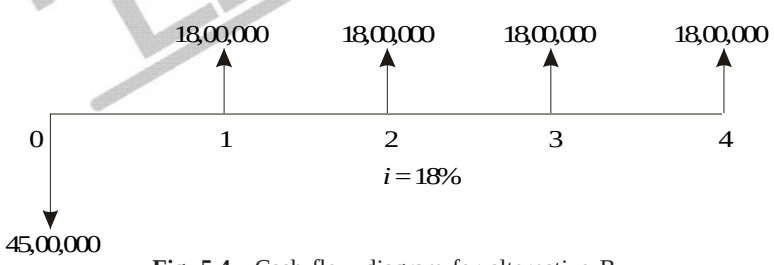


Fig. 5.4 Cash flow diagram for alternative B.

The future worth amount of alternative B is computed as

$$FW_B(18\%) = -45,00,000(F/P, 18\%, 4) + 18,00,000\ (F/A, 18\%, 4)$$
$$= -45,00,000(1.939) + 18,00,000(5.215)$$
$$= \text{Rs. } 6,61,500$$

The future worth of alternative A is greater than that of alternative B. Thus, alternative A should be selected.

EXAMPLE 5.2 A man owns a corner plot. He must decide which of the several alternatives to select in trying to obtain a desirable return on his investment. After much study and calculation, he decides that the two best alternatives are as given in the following table:

	Build gas station	*Build soft ice-cream stand*
First cost (Rs.)	20,00,000	36,00,000
Annual property taxes (Rs.)	80,000	1,50,000
Annual income (Rs.)	8,00,000	9,80,000
Life of building (years)	20	20
Salvage value (Rs.)	0	0

Evaluate the alternatives based on the future worth method at i = 12%.

***Alternative 1*—Build gas station**

First cost = Rs. 20,00,000

Net annual income = Annual income – Annual property tax

= Rs. 8,00,000 – Rs. 80,000

= Rs. 7,20,000

Life = 20 years

Interest rate = 12%, compounded annually

The cash flow diagram for this alternative is depicted in Fig. 5.5.

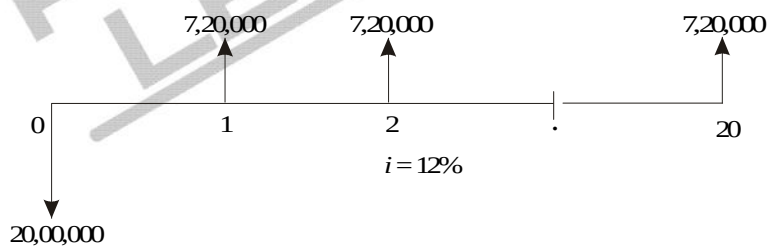


Fig. 5.5 Cash flow diagram for alternative 1.

The future worth of alternative 1 is computed as

$$FW_1(12\%) = -20,00,000\,(F/P,\ 12\%,\ 20) + 7,20,000\,(F/A,\ 12\%,\ 20)$$

$$= -20,00,000\,(9.646) + 7,20,000\,(72.052)$$

$$= \text{Rs. } 3,25,85,440$$

***Alternative 2*—Build soft ice-cream stand**

First cost = Rs. 36,00,000

Net annual income = Annual income – Annual property tax

= Rs. 9,80,000 – Rs. 1,50,000

= Rs. 8,30,000

Life = 20 years

Interest rate = 12%, compounded annually

The cash flow diagram for this alternative is shown in Fig. 5.6.

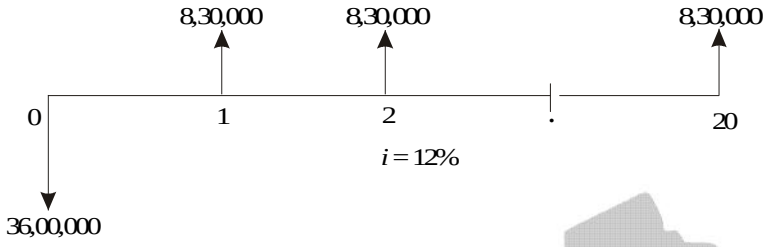


Fig. 5.6 Cash flow diagram for alternative 2.

The future worth of alternative 2 is calculated as

$$FW_2(12\%) = -36,00,000\,(F/P, 12\%, 20) + 8,30,000\,(F/A, 12\%, 20)$$

$$= -36,00,000\,(9.646) + 8,30,000\,(72.052)$$

$$= \text{Rs. } 2,50,77,560$$

The future worth of alternative 1 is greater than that of alternative 2. Thus, building the gas station is the best alternative.

EXAMPLE 5.3 The cash flow diagram of two mutually exclusive alternatives are given in Figs. 5.7 and 5.8.

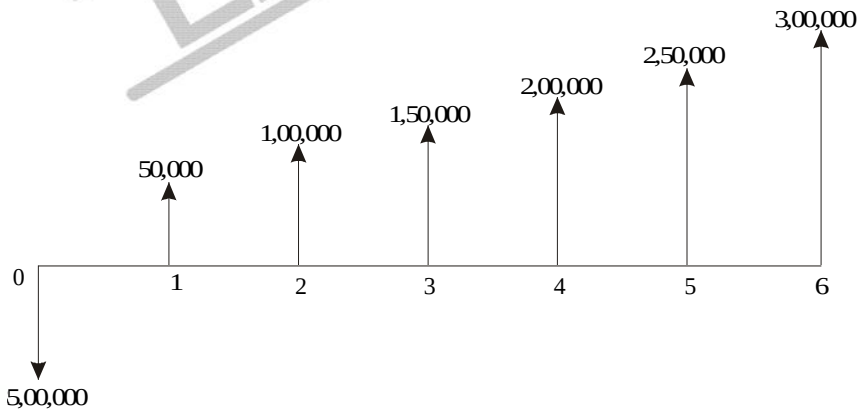


Fig. 5.7 Cash flow diagram for alternative 1.

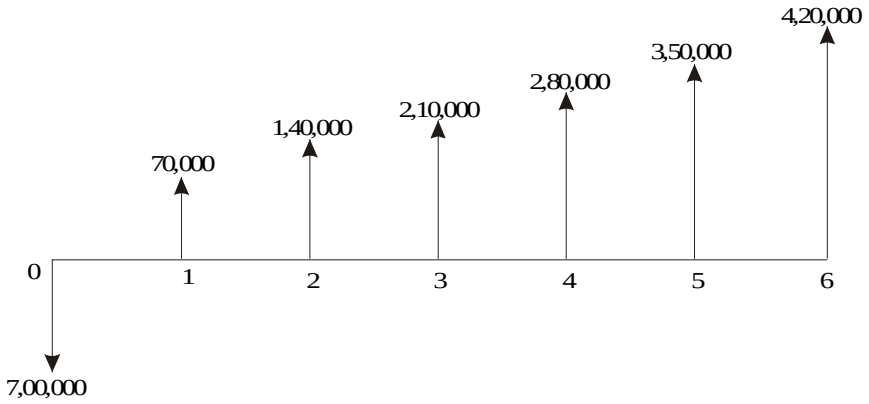


Fig. 5.8 Cash flow diagram for alternative 2.

(a) Select the best alternative based on future worth method at i = 8%.
(b) Rework part (a) with i = 9% and 20%

(a) Evaluation at *i = 8%*

***Alternative 1*—This comes under equal payment gradient series.**

P = Rs. 5,00,000
$A1$ = Rs. 50,000
G = Rs. 50,000
i = 8%
n = 6 years

The formula for the future worth of alternative 1 is

$$FW_1(8\%) = -P(F/P, 8\%, 6) + [A1 + G(A/G, 8\%, 6)] \quad (F/A, 8\%, 6)$$
$$= -5,00,000(1.587) + [50,000 + 50,000(2.2764)] \quad 7.336$$
$$= -79,35,000 + 1,63,820 \quad 7.336$$
$$= -79,35,000 + 12,01,784$$
$$= \text{Rs. } 4,08,283.52$$

***Alternative 2*—This comes under equal payment gradient series.**

P = Rs. 7,00,000
$A1$ = Rs. 70,000
G = Rs. 70,000
i = 8%
n = 6 years

The formula for the future worth of alternative 2 is

$FW_2(8\%) = -P(F/P, 8\%, 6) + [A1 + G(A/G, 8\%, 6)] \quad (F/A, 8\%, 6)$

$FW_2(8\%) = -7,00,000 \quad 1.587 + [70,000 + 70,000 \quad 2.2764] \quad 7.336$

$= -11,10,900 + 16,82,497$

$=$ Rs. 5,71,596.93

The future worth of alternative 2 is more than that of alternative 1. Therefore, alternative 2 must be selected.

(b) (i) Evaluation at $i = 9\%$: ***Alternative 1***

$P =$ Rs. 5,00,000

$A1 =$ Rs. 50,000

$G =$ Rs. 50,000

$n = 6$ years

The formula for the future worth of alternative 1 is as follows:

$FW_1(9\%) = -P(F/P, 9\%, 6) + [A1 + G(A/G, 9\%, 6)] \quad (F/A, 9\%, 6)$

$= -5,00,000\ (1.677) + [50,000 + 50,000\,(2.2498)] \quad 7.523$

$= -8,38,500 + 12,22,412.27$

$=$ Rs. 3,83,912.27

Alternative 2

$P =$ Rs. 7,00,000

$A1 =$ Rs. 70,000

$G =$ Rs. 70,000

$n = 6$ years

The formula for the future worth of the alternative 2 is

$FW_2(9\%) = -P(F/P, 9\%, 6) + [A1 + G(A/G, 9\%, 6)] \quad (F/A, 9\%, 6)$

$= -7,00,000 \quad 1.677 + [70,000 + 70,000 \quad 2.2498] \quad 7.523$

$= -11,73,900 + 17,11,377.18$

$=$ Rs. 5,37,477.18

The future worth of alternative 2 is more than that of alternative 1. Therefore, alternative 2 must be selected.

(ii) Evaluation at $i = 20\%$: ***Alternative 1***

$P =$ Rs. 5,00,000

$A1 =$ Rs. 50,000

$G =$ Rs. 50,000

$n = 6$ years

The formula for the future worth of alternative 1 is

$$FW_1(20\%) = -P(F/P, 20\%, 6) + [A1 + G(A/G, 20\%, 6)] \quad (F/A, 20\%, 6)$$
$$= -5,00,000(2.986) + [50,000 + 50,000(1.9788)] \quad 9.93$$
$$= -14,93,000 + 14,78,974.20$$
$$= \text{Rs. } -14,025.80$$

The negative sign of the future worth amount indicates that alternative 1 incurs loss.

Alternative 2

P = Rs. 7,00,000

$A1$ = Rs. 70,000

G = Rs. 70,000

n = 6 years

The formula for the future worth of alternative 2 is

$$FW_2(20\%) = -P(F/P, 20\%, 6) + [A1 + G(A/G, 20\%, 6)] \quad (F/A, 20\%, 6)$$
$$= -7,00,000 \quad 2.986 + [70,000 + 70,000 \quad 1.9788] \quad 9.93$$
$$= -20,90,200 + 20,70,563.88$$
$$= \text{Rs. } -19,636.12$$

The negative sign of the above future worth amount indicates that alternative 2 incurs loss. Thus, none of the two alternatives should be selected.

EXAMPLE 5.4 M/S Krishna Castings Ltd. is planning to replace its annealing furnace. It has received tenders from three different original manufacturers of annealing furnace. The details are as follows.

	Manufacturer		
	1	2	3
Initial cost (Rs.)	80,00,000	70,00,000	90,00,000
Life (years)	12	12	12
Annual operation and maintenance cost (Rs.)	8,00,000	9,00,000	8,50,000
Salvage value after 12 years	5,00,000	4,00,000	7,00,000

Which is the best alternative based on future worth method at i = 20%?

Solution Alternative 1—Manufacturer 1

First cost, P = Rs. 80,00,000

Life, n = 12 years

Annual operating and maintenance cost, A = Rs. 8,00,000
Salvage value at the end of furnace life = Rs. 5,00,000

The cash flow diagram for this alternative is shown in Fig. 5.9.

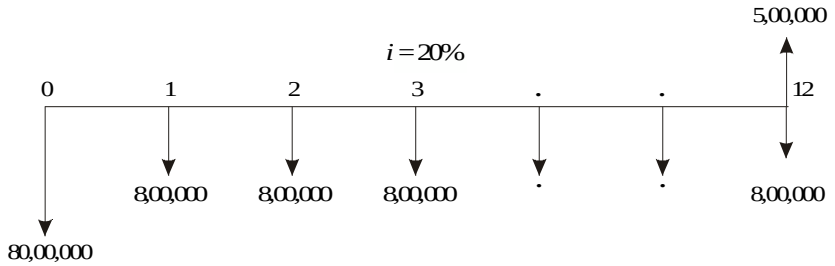


Fig. 5.9 Cash flow diagram for manufacturer 1.

The future worth amount of alternative 1 is computed as

$$FW_1(20\%) = 80,00,000\,(F/P, 20\%, 12) + 8,00,000\,(F/A, 20\%, 12) - 5,00,000$$
$$= 80,00,000(8.916) + 8,00,000\,(39.581) - 5,00,000$$
$$= \text{Rs. } 10,24,92,800$$

Alternative 2— Manufacturer 2

First cost, P = Rs. 70,00,000
Life, n = 12 years
Annual operating and maintenance cost, A = Rs. 9,00,000
Salvage value at the end of furnace life = Rs. 4,00,000

The cash flow diagram for this alternative is given in Fig. 5.10.

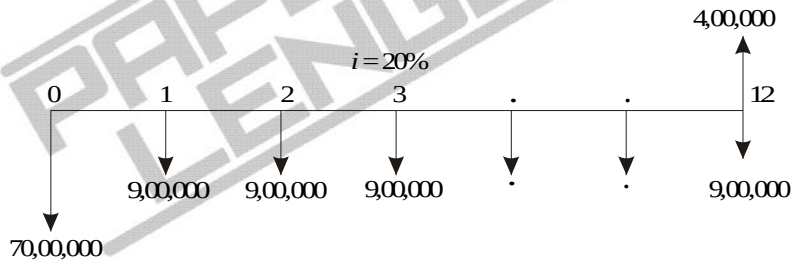


Fig. 5.10 Cash flow diagram for manufacturer 2.

The future worth amount of alternative 2 is computed as

$$FW_2(20\%) = 70,00,000(F/P, 20\%, 12) + 9,00,000(F/A, 20\%, 12) - 4,00,000$$
$$= 70,00,000(8.916) + 9,00,000\ (39.581) - 4,00,000$$
$$= \text{Rs. } 9,76,34,900$$

Alternative 3—Manufacturer 3

First cost, P = Rs. 90,00,000
Life, n = 12 years

Annual operating and maintenance cost, A = Rs. 8,50,000
Salvage value at the end of furnace life = Rs. 7,00,000

The cash flow diagram for this alternative is illustrated in Fig. 5.11.

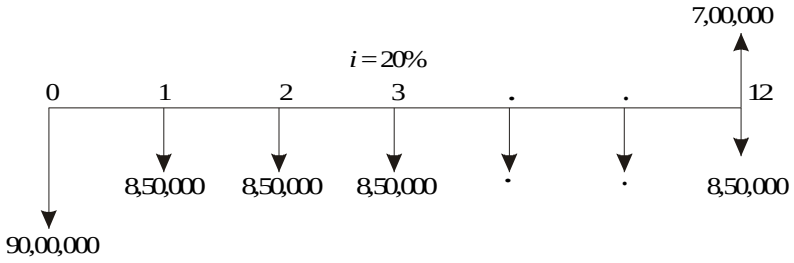


Fig. 5.11 Cash flow diagram for manufacturer 3.

The future worth amount of alternative 3 is calculated as

$$FW_3(20\%) = 90,00,000(F/P, 20\%, 12) + 8,50,000(F/A, 20\%, 12) - 7,00,000$$
$$= 90,00,000(8.916) + 8,50,000\,(39.581) - 7,00,000$$
$$= \text{Rs. } 11,31,87,850$$

The future worth cost of alternative 2 is less than that of the other two alternatives. Therefore, M/s. Krishna castings should buy the annealing furnace from manufacturer 2.

EXAMPLE 5.5 A company must decide whether to buy machine *A* or machine *B*:

	Machine A	*Machine B*
Initial cost	Rs. 4,00,000	Rs. 8,00,000
Useful life, in years	4	4
Salvage value at the end of machine life	Rs. 2,00,000	Rs. 5,50,000
Annual maintenance cost	Rs. 40,000	0

At 12% interest rate, which machine should be selected? (Use future worth method of comparison).

Solution Machine A

Initial cost of the machine, P = Rs. 4,00,000

Life, n = 4 years

Salvage value at the end of machine life, S = Rs. 2,00,000

Annual maintenance cost, A = Rs. 40,000

Interest rate, i = 12%, compounded annually.

The cash flow diagram of machine *A* is given in Fig. 5.12.

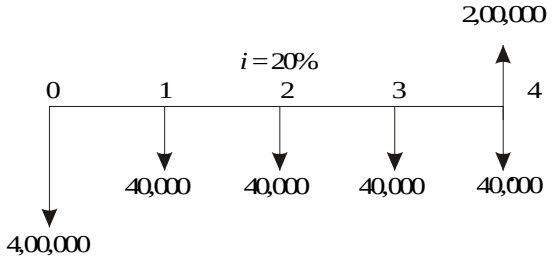


Fig. 5.12 Cash flow diagram for machine *A*.

The future worth function of Fig. 5.12 is

$$FW_A(12\%) = 4,00,000 \quad (F/P, 12\%, 4) + 40,000 \quad (F/A, 12\%, 4) - 2,00,000$$
$$= 4,00,000 \quad (1.574) + 40,000 \quad (4.779) - 2,00,000$$
$$= \text{Rs. } 6,20,760$$

Machine B

Initial cost of the machine, P = Rs. 8,00,000

Life, n = 4 years

Salvage value at the end of machine life, S = Rs. 5,50,000

Annual maintenance cost, A = zero.

Interest rate, i = 12%, compounded annually.

The cash flow diagram of the machine *B* is illustrated in Fig. 5.13.

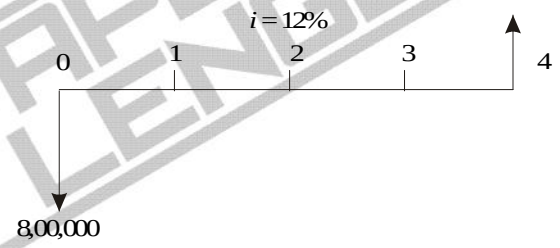


Fig. 5.13 Cash flow diagram for machine *B*.

The future worth function of Fig 5.13 is

$$FW_B(12\%) = 8,00,000 \quad (F/P, 12\%, 4) - 5,50,000$$
$$= 8,00,000 \quad (1.574) - 5,50,000$$
$$= \text{Rs. } 7,09,200$$

The future worth cost of machine *A* is less than that of machine *B*. Therefore, machine *A* should be selected.

QUESTIONS

1. A suburban taxi company is considering buying taxis with diesel engines instead of petrol engines. The cars average 50,000 km a year, with a useful life of three years for the taxi with the petrol engine and four years for the diesel taxi. Other comparative information are as follows:

	Diesel	*Petrol*
Vehicle cost	Rs. 5,00,000	Rs. 4,00,000
Fuel cost per litre	Rs. 9.00	Rs. 24.00
Mileage, in km/litre	30	20
Annual insurance premium	Rs. 500	Rs. 500
Salvage value at the end of vehicle life	Rs. 70,000	Rs. 1,00,000

Determine the more economical choice based on the future worth method of comparison if the interest rate is 15%, compounded annually.

2. A motorcycle is sold for Rs. 50,000. The motorcycle dealer is willing to sell it on the following terms:

(a) Make no down payment but pay Rs. 1,500 at the end of each of the first four months and Rs. 3,000 at the end of each month after that for 18 continuous months.

(b) Make no down payment but pay a total amount of Rs. 90,000 at the end of the 22nd month; till that time the buyer should mortgage property worth of Rs. 50,000, at present.

Based on these terms and a 12% annual interest rate compounded monthly, find the best alternative for the buyer based on the future worth method of comparison.

3. Consider the following two mutually exclusive alternatives.

	A	*B*
Cost	Rs. 4,000	Rs. 6,000
Uniform annual benefit	Rs. 640	Rs. 960
Useful life (years)	20	20

Using a 15% interest rate, determine which alternative should be selected based on the future worth method of comparison.

4. A company must decide whether to buy machine *A* or machine *B*:

	Machine A	*Machine B*
Initial cost	Rs. 4,00,000	Rs. 8,00,000
Useful life, (years)	5	5
Salvage value at the end of machine life	Rs. 2,00,000	Rs. 5,50,000
Annual maintenance cost	Rs. 40,000	0

At 15% interest rate, which machine should be selected? (Use the future worth method of comparison.)

5. Due to increasing awareness of customers, two different television manufacturing companies started a marketing war. The details of advertisements of the companies are as follows:

	Brand X	*Brand Y*
Selling price of a TV set	Rs. 15,000	Rs. 10,000
Amount returned to buyer after 5 years	Rs. 8,000	–

Select the most economical brand from the customer's point of view using the future worth method of comparison, assuming an interest rate of 15%, compounded annually.

6. Alpha Finance Company is coming with an option of accepting Rs. 10,000 now and paying a sum of Rs. 1,60,000 after 20 years. Beta Finance Company is coming with a similar option of accepting Rs. 10,000 now and paying a sum of Rs. 3,00,000 after 25 years. Compare and select the best alternative based on the future worth method of comparison with 15% interest rate, compounded annually.

7. An insurance company gives an endowment policy for a person aged 30 years. The yearly premium for an insured sum of Rs. 1,00,000 is Rs. 4,000. The policy will mature after 25 years. Also, the person is entitled for a bonus of Rs. 75 per thousand per year at the end of the policy. If a person survives till the end of the 25th year:

(a) What will be the total sum that he will get from the insurance company at that time?

(b) Instead of paying the premiums for the insurance policy, if the person invests an equal sum of Rs. 4,000 at the end of each year for the next 25 years in some other scheme which is having similar tax benefit, find the future worth of the investment at 15% interest rate, compounded annually.

(c) Rate the above alternatives assuming that the person is sure of living for the next 25 years.

ANNUAL EQUIVALENT METHOD

6.1 INTRODUCTION

In the annual equivalent method of comparison, first the annual equivalent cost or the revenue of each alternative will be computed. Then the alternative with the maximum annual equivalent revenue in the case of revenue-based comparison or with the minimum annual equivalent cost in the case of cost-based comparison will be selected as the best alternative.

6.2 REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the annual equivalent method of comparison is presented in Fig. 6.1.

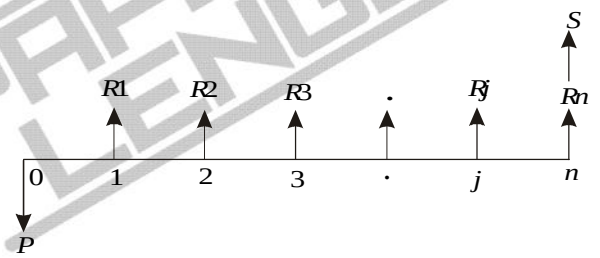


Fig. 6.1 Revenue-dominated cash flow diagram.

In Fig. 6.1, P represents an initial investment, Rj the net revenue at the end of the jth year, and S the salvage value at the end of the nth year.

The first step is to find the net present worth of the cash flow diagram using the following expression for a given interest rate, i:

$$PW(i) = -P + R1/(1 + i)^1 + R2/(1 + i)^2 + ...$$
$$+ Rj/(1 + i)^j + ... + Rn/(1 + i)^n + S/(1 + i)^n$$

In the above formula, the expenditure is assigned with a negative sign and the revenues are assigned with a positive sign.

In the second step, the annual equivalent revenue is computed using the following formula:

$$A = PW(i) \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= PW(i)\ (A/P, i, n)$$

where $(A/P, i, n)$ is called *equal payment series capital recovery factor*.

If we have some more alternatives which are to be compared with this alternative, then the corresponding annual equivalent revenues are to be computed and compared. Finally, the alternative with the maximum annual equivalent revenue should be selected as the best alternative.

6.3 COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the annual equivalent method of comparison is illustrated in Fig. 6.2.

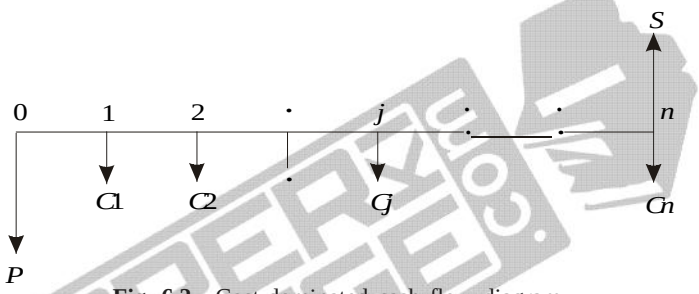


Fig. 6.2 Cost-dominated cash flow diagram.

In Fig. 6.2, P represents an initial investment, Cj the net cost of operation and maintenance at the end of the jth year, and S the salvage value at the end of the nth year.

The first step is to find the net present worth of the cash flow diagram using the following relation for a given interest rate, i.

$$PW(i) = P + C1/(1+i)^1 + C2/(1+i)^2 + \ldots$$
$$+ Cj/(1+i)^j + \ldots + Cn/(1+i)^n - S/(1+i)^n$$

In the above formula, each expenditure is assigned with positive sign and the salvage value with negative sign. Then, in the second step, the annual equivalent cost is computed using the following equation:

$$A = PW(i) \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= PW(i)\ (A/P, i, n)$$

where $(A/P, i, n)$ is called as equal-payment series capital recovery factor.

As in the previous case, if we have some more alternatives which are to be compared with this alternative, then the corresponding annual equivalent costs are to be computed and compared. Finally, the alternative with the minimum annual equivalent cost should be selected as the best alternative.

If we have some non-standard cash flow diagram, then we will have to follow the general procedure for converting each and every transaction to time zero and then convert the net present worth into an annual equivalent cost/ revenue depending on the type of the cash flow diagram. Such procedure is to be applied to all the alternatives and finally, the best alternative is to be selected.

6.4 ALTERNATE APPROACH

Instead of first finding the present worth and then figuring out the annual equivalent cost/revenue, an alternate method which is as explained below can be used. In each of the cases presented in Sections 6.2 and 6.3, in the first step, one can find the future worth of the cash flow diagram of each of the alternatives. Then, in the second step, the annual equivalent cost/revenue can be obtained by using the equation:

$$A = F\frac{i}{(1+i)^n - 1}$$

$$= F(A/F, i, n)$$

where $(A/F, i, n)$ is called *equal-payment series sinking fund factor.*

6.5 EXAMPLES

In this section, the application of the annual equivalent method is demonstrated with several numerical examples.

EXAMPLE 6.1 A company provides a car to its chief executive. The owner of the company is concerned about the increasing cost of petrol. The cost per litre of petrol for the first year of operation is Rs. 21. He feels that the cost of petrol will be increasing by Re.1 every year. His experience with his company car indicates that it averages 9 km per litre of petrol. The executive expects to drive an average of 20,000 km each year for the next four years. What is the annual equivalent cost of fuel over this period of time?. If he is offered similar service with the same quality on rental basis at Rs. 60,000 per year, should the owner continue to provide company car for his executive or alternatively provide a rental car to his executive? Assume i = 18%. If the rental car is preferred, then the company car will find some other use within the company.

Solution

Average number of km run/year = 20,000 km
Number of km/litre of petrol = 9 km

Therefore,

Petrol consumption/year = 20,000/9 = 2222.2 litre
Cost/litre of petrol for the 1st year = Rs. 21
Cost/litre of petrol for the 2nd year = Rs. 21.00 + Re. 1.00
= Rs. 22.00
Cost/litre of petrol for the 3rd year = Rs. 22.00 + Re. 1.00
= Rs. 23.00
Cost/litre of petrol for the 4th year = Rs. 23.00 + Re. 1.00
= Rs. 24.00
Fuel expenditure for 1st year = 2222.2 21 = Rs. 46,666.20
Fuel expenditure for 2nd year = 2222.2 22 = Rs. 48,888.40
Fuel expenditure for 3rd year = 2222.2 23 = Rs. 51,110.60
Fuel expenditure for 4th year = 2222.2 24 = Rs. 53,332.80

The annual equal increment of the above expenditures is Rs. 2,222.20 (G).

The cash flow diagram for this situation is depicted in Fig. 6.3.

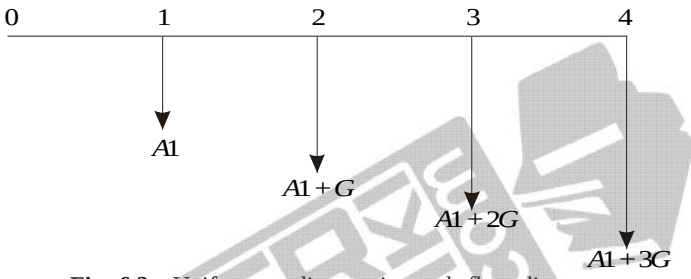


Fig. 6.3 Uniform gradient series cash flow diagram.

In Fig. 6.3, $A1$ = Rs. 46,666.20 and G = Rs. 2,222.20

$$A = A1 + G(A/G, 18\%, 4)$$
$$= 46,666.20 + 2222.2(1.2947)$$
$$= \text{Rs. } 49,543.28$$

The proposal of using the company car by spending for petrol by the company will cost an annual equivalent amount of Rs. 49,543.28 for four years. This amount is less than the annual rental value of Rs. 60,000. Therefore, the company should continue to provide its own car to its executive.

EXAMPLE 6.2 A company is planning to purchase an advanced machine centre. Three original manufacturers have responded to its tender whose particulars are tabulated as follows:

Manufacturer	*Down payment* (Rs.)	*Yearly equal installment* (Rs.)	*No. of installments*
1	5,00,000	2,00,000	15
2	4,00,000	3,00,000	15
3	6,00,000	1,50,000	15

Determine the best alternative based on the annual equivalent method by assuming i = 20%, compounded annually.

Solution Alternative 1

Down payment, P = Rs. 5,00,000
Yearly equal installment, A = Rs. 2,00,000
n = 15 years
i = 20%, compounded annually
The cash flow diagram for manufacturer 1 is shown in Fig. 6.4.

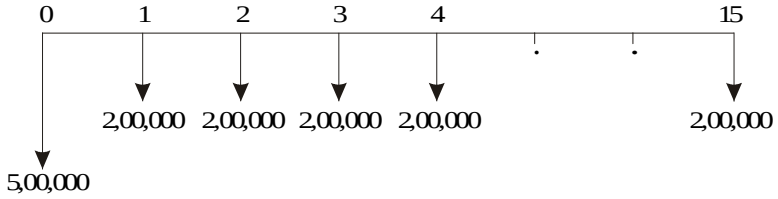


Fig. 6.4 Cash flow diagram for manufacturer 1.

The annual equivalent cost expression of the above cash flow diagram is

$$AE_1(20\%) = 5{,}00{,}000(A/P, 20\%, 15) + 2{,}00{,}000$$
$$= 5{,}00{,}000(0.2139) + 2{,}00{,}000$$
$$= 3{,}06{,}950$$

Alternative 2

Down payment, P = Rs. 4,00,000
Yearly equal installment, A = Rs. 3,00,000
n = 15 years
i = 20%, compounded annually

The cash flow diagram for the manufacturer 2 is shown in Fig. 6.5.

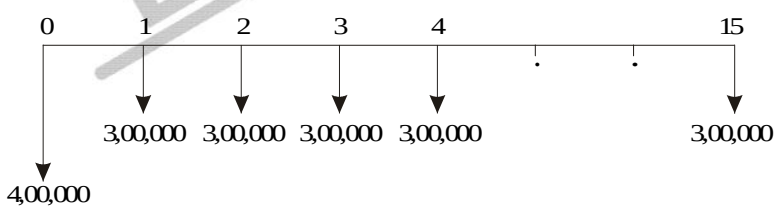


Fig. 6.5 Cash flow diagram for manufacturer 2.

The annual equivalent cost expression of the above cash flow diagram is

$$AE_2(20\%) = 4{,}00{,}000(A/P, 20\%, 15) + 3{,}00{,}000$$
$$= 4{,}00{,}000(0.2139) + 3{,}00{,}000$$
$$= \text{Rs. } 3{,}85{,}560.$$

Alternative 3

Down payment, P = Rs. 6,00,000
Yearly equal installment, A = Rs. 1,50,000
n = 15 years
i = 20%, compounded annually

The cash flow diagram for manufacturer 3 is shown in Fig. 6.6.

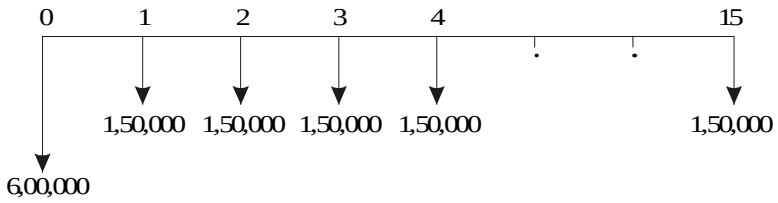


Fig. 6.6 Cash flow diagram for manufacturer 3.

The annual equivalent cost expression of the above cash flow diagram is

$$AE_3(20\%) = 6,00,000(A/P, 20\%, 15) + 1,50,000$$
$$= 6,00,000(0.2139) + 1,50,000$$
$$= \text{Rs. } 2,78,340.$$

The annual equivalent cost of manufacturer 3 is less than that of manufacturer 1 and manufacturer 2. Therefore, the company should buy the advanced machine centre from manufacturer 3.

EXAMPLE 6.3 A company invests in one of the two mutually exclusive alternatives. The life of both alternatives is estimated to be 5 years with the following investments, annual returns and salvage values.

	Alternative	
	A	B
Investment (Rs.)	– 1,50,000	– 1,75,000
Annual equal return (Rs.)	+ 60,000	+ 70,000
Salvage value (Rs.)	+ 15,000	+ 35,000

Determine the best alternative based on the annual equivalent method by assuming i = 25%.

Solution Alternative A

Initial investment, P = Rs. 1,50,000
Annual equal return, A = Rs. 60,000
Salvage value at the end of machine life, S = Rs. 15,000
Life = 5 years
Interest rate, i = 25%, compounded annually

The cash flow diagram for alternative A is shown in Fig. 6.7.

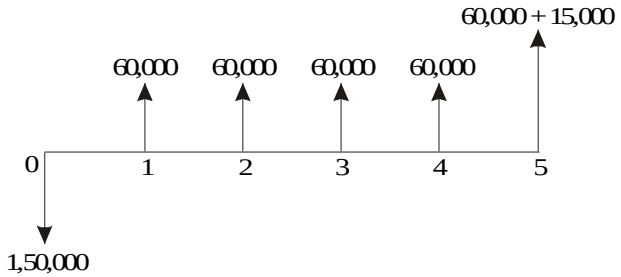


Fig. 6.7 Cash flow diagram for alternative A.

The annual equivalent revenue expression of the above cash flow diagram is as follows:

$$AE_A(25\%) = -1,50,000(A/P, 25\%, 5) + 60,000 + 15,000(A/F, 25\%, 5)$$
$$= -1,50,000(0.3718) + 60,000 + 15,000(0.1218)$$
$$= \text{Rs. } 6,057$$

Alternative B

Initial investment, P = Rs. 1,75,000
Annual equal return, A = Rs. 70,000
Salvage value at the end of machine life, S = Rs. 35,000
Life = 5 years
Interest rate, i = 25%, compounded annually

The cash flow diagram for alternative B is shown in Fig. 6.8.

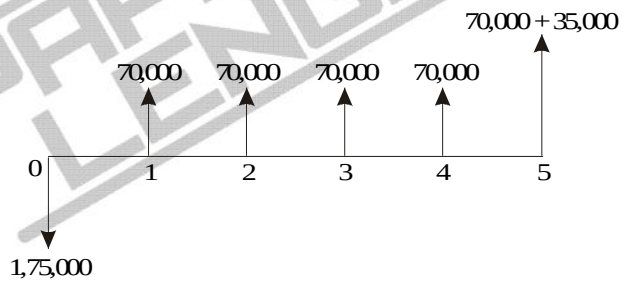


Fig. 6.8 Cash flow diagram for alternative B.

The annual equivalent revenue expression of the above cash flow diagram is

$$AE_B(25\%) = -1,75,000(A/P, 25\%, 5) + 70,000 + 35,000(A/F, 25\%, 5)$$
$$= -1,75,000(0.3718) + 70,000 + 35,000(0.1218)$$
$$= \text{Rs. } 9,198$$

The annual equivalent net return of alternative B is more than that of alternative A. Thus, the company should select alternative B.

EXAMPLE 6.4 A certain individual firm desires an economic analysis to determine which of the two machines is attractive in a given interval of time. The minimum attractive rate of return for the firm is 15%. The following data are to be used in the analysis:

	Machine X	*Machine Y*
First cost	Rs. 1,50,000	Rs. 2,40,000
Estimated life	12 years	12 years
Salvage value	Rs. 0	Rs. 6,000
Annual maintenance cost	Rs. 0	Rs. 4,500

Which machine would you choose? Base your answer on annual equivalent cost.

Solution Machine X

First cost, P = Rs. 1,50,000
Life, n = 12 years
Estimated salvage value at the end of machine life, S = Rs. 0.
Annual maintenance cost, A = Rs. 0.
Interest rate, i = 15%, compounded annually.

The cash flow diagram of machine X is illustrated in Fig. 6.9.

Fig. 6.9 Cash flow diagram for machine X.

The annual equivalent cost expression of the above cash flow diagram is

$$AE_X(15\%) = 1,50,000(A/P, 15\%, 12)$$
$$= 1,50,000(0.1845)$$
$$= \text{Rs. } 27,675$$

Machine Y

First cost, P = Rs. 2,40,000

Life, n = 12 years

Estimated salvage value at the end of machine life, S = Rs. 60,000

Annual maintenance cost, A = Rs. 4,500

Interest rate, i = 15%, compounded annually.

The cash flow diagram of machine Y is depicted in Fig. 6.10.

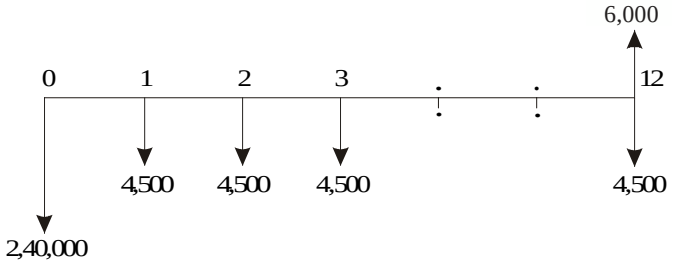


Fig. 6.10 Cash flow diagram for machine Y.

The annual equivalent cost expression of the above cash flow diagram is

$$AE_Y(15\%) = 2,40,000(A/P, 15\%, 12) + 4,500 - 6,000(A/F, 15\%, 12)$$
$$= 2,40,000(0.1845) + 4,500 - 6,000(0.0345)$$
$$= \text{Rs. } 48,573$$

The annual equivalent cost of machine X is less than that of machine Y. So, machine X is the more cost effective machine.

EXAMPLE 6.5 Two possible routes for laying a power line are under study. Data on the routes are as follows:

		Around the lake	*Under the lake*
Length		15 km	5 km
First cost	(Rs.)	1,50,000/km	7,50,000/km
Useful life	(years)	15	15
Maintenance cost	(Rs.)	6,000/km/yr	12,000/km/yr
Salvage value	(Rs.)	90,000/km	1,50,000/km
Yearly power loss	(Rs.)	15,000/km	15,000/km

If 15% interest is used, should the power line be routed around the lake or under the lake?

Solution Alternative 1—Around the lake

First cost = 1,50,000 15 = Rs. 22,50,000

Maintenance cost/yr = 6,000 15 = Rs. 90,000

Power loss/yr = 15,000 15 = Rs. 2,25,000

Maintenance cost and power loss/yr = Rs. 90,000 + Rs. 2,25,000
= Rs. 3,15,000

Salvage value = 90,000 15 = Rs. 13,50,000

The cash flow diagram for this alternative is shown in Fig. 6.11.

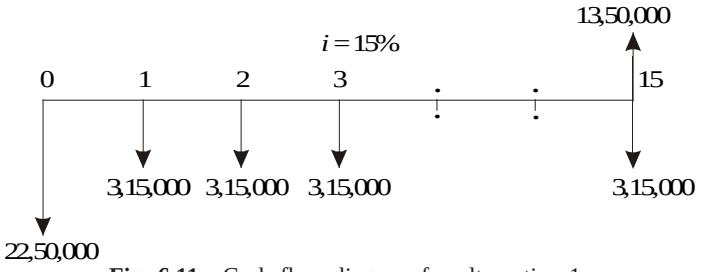


Fig. 6.11 Cash flow diagram for alternative 1.

The annual equivalent cost expression of the above cash flow diagram is

$$AE_1(15\%) = 22,50,000(A/P, 15\%, 15) + 3,15,000 - 13,50,000(A/F, 15\%, 15)$$
$$= 22,50,000(0.1710) + 3,15,000 - 13,50,000(0.0210)$$
$$= \text{Rs. } 6,71,400$$

Alternative 2—Under the lake

First cost = 7,50,000 5 = Rs. 37,50,000
Maintenance cost/yr = 12,000 5 = Rs. 60,000
Power loss/yr = 15,000 5 = Rs. 75,000
Maintenance cost and power loss/yr = Rs. 60,000 + Rs. 75,000
= Rs. 1,35,000
Salvage value = 1,50,000 5 = Rs. 7,50,000

The cash flow diagram for this alternative is shown in Fig. 6.12.

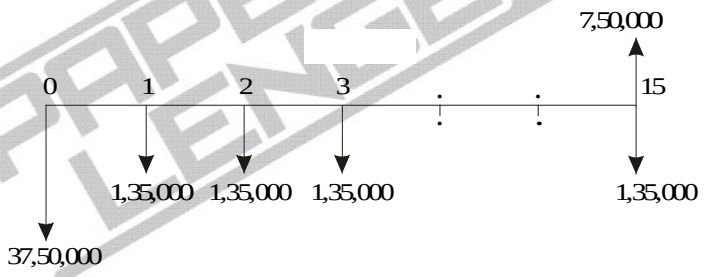


Fig. 6.12 Cash flow diagram for alternative 2.

The annual equivalent cost expression of the above cash flow diagram is

$$AE_2(15\%) = 37,50,000(A/P, 15\%, 15) + 1,35,000 - 7,50,000(A/F, 15\%, 15)$$
$$= 37,50,000(0.1710) + 1,35,000 - 7,50,000(0.0210)$$
$$= \text{Rs. } 7,60,500$$

The annual equivalent cost of alternative 1 is less than that of alternative 2. Therefore, select the route around the lake for laying the power line.

EXAMPLE 6.6 A suburban taxi company is analyzing the proposal of buying

cars with diesel engines instead of petrol engines. The cars average 60,000 km a year with a useful life of three years for the petrol taxi and four years for the diesel taxi. Other comparative details are as follows:

	Diesel	*Petrol*
Vehicle cost (Rs.)	3,90,000	3,60,000
Fuel cost per litre (Rs.)	8	20
Mileage in km/litre	30	20
Annual repairs (Rs.)	9,000	6,000
Annual insurance premium (Rs.)	15,000	15,000
Resale value at the end of vehicle life (Rs.)	60,000	90,000

Determine the more economical choice if interest rate is 20%, compounded annually.

Solution Alternative 1— Purchase of diesel taxi

Vehicle cost = Rs. 3,90,000
Life = 4 years
Number of litres/year 60,000/30 = 2,000 litres
Fuel cost/yr = 2,000 8 = Rs. 16,000
Fuel cost, annual repairs and insurance premium/yr
= Rs. 16,000 + Rs. 9,000 + Rs. 15,000 = Rs. 40,000
Salvage value at the end of vehicle life = Rs. 60,000

The cash flow diagram for alternative 1 is shown in Fig. 6.13.

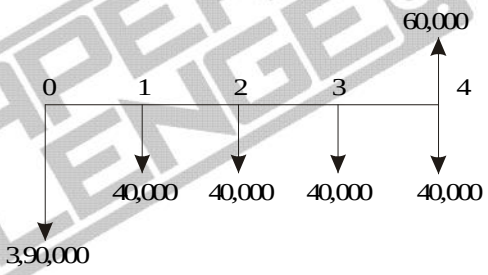


Fig. 6.13 Cash flow diagram for alternative 1.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(20\%) = 3{,}90{,}000(A/P, 20\%, 4) + 40{,}000 - 60{,}000(A/F, 20\%, 4)$$
$$= 3{,}90{,}000(0.3863) + 40{,}000 - 60{,}000(0.1863)$$
$$= \text{Rs. } 1{,}79{,}479$$

Alternative 2—Purchase of petrol taxi

Vehicle cost = Rs. 3,60,000
Life = 3 years
Number of litres/year 60,000/20 = 3,000 litres
Fuel cost/yr = 3,000 20 = Rs. 60,000

Fuel cost, annual repairs and insurance premium/yr
= Rs. 60,000 + Rs. 6,000 + Rs. 15,000 = Rs. 81,000
Salvage value at the end of vehicle life = Rs. 90,000

The cash flow diagram for alternative 2 is shown in Fig. 6.14.

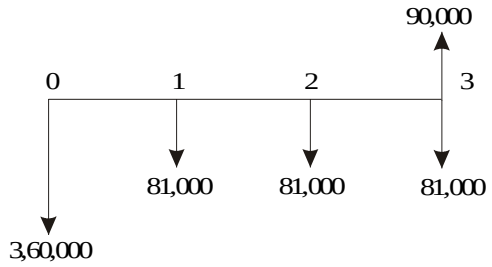


Fig. 6.14 Cash flow diagram for alternative 2.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(20\%) = 3{,}60{,}000(A/P, 20\%, 3) + 81{,}000 - 90{,}000(A/F, 20\%, 3)$$
$$= 3{,}60{,}000(0.4747) + 81{,}000 - 90{,}000(0.2747)$$
$$= \text{Rs. } 2{,}27{,}169$$

The annual equivalent cost of purchase and operation of the cars with diesel engine is less than that of the cars with petrol engine. Therefore, the taxi company should buy cars with diesel engine. (*Note:* Comparison is done on common multiple lives of 12 years.)

EXAMPLE 6.7 Ramu, a salesman, needs a new car for use in his business. He expects that he will be promoted to a supervisory job at the end of third year and so his concern now is to have a car for the three years he expects to be "on the road". The company will reimburse their salesman each month the fuel cost and maintenance cost. Ramu has decided to drive a low-priced automobile. He finds, however, that there are two different ways of obtaining the automobile. In either case, the fuel cost and maintenance cost are borne by the company.

(a) Purchase for cash at Rs. 3,90,000.

(b) Lease a car. The monthly charge is Rs. 10,500 on a 36-month lease payable at the end of each month. At the end of the three-year period, the car is returned to the leasing company.

Ramu believes that he should use a 12% interest rate compounded monthly in determining which alternative to select. If the car could be sold for Rs. 1,20,000 at the end of the third year, which option should he use to obtain it?

Alternative 1—Purchase car for cash
Purchase price of the car = Rs. 3,90,000
Life = 3 years = 36 months
Salvage value after 3 years = Rs. 1,20,000

Interest rate = 12% (nominal rate, compounded annually)
= 1% compounded monthly

The cash flow diagram for alternative 1 is shown in Fig. 6.15.

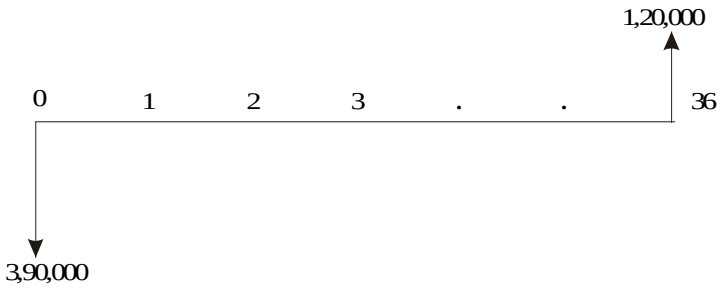


Fig. 6.15 Cash flow diagram for alternative 1.

The monthly equivalent cost expression [*ME*(1%)] of the above cash flow diagram is

$$ME(1\%) = 3{,}90{,}000(A/P, 1\%, 36) - 1{,}20{,}000(A/F, 1\%, 36)$$
$$= 3{,}90{,}000(0.0332) - 1{,}20{,}000(0.0232)$$
$$= \text{Rs. } 10{,}164$$

Alternative 2—Use of car under lease

Monthly lease amount for 36 months = Rs. 10,500

The cash flow diagram for alternative 2 is illustrated in Fig. 6.16.

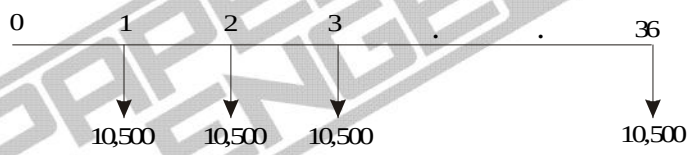


Fig. 6.16 Cash flow diagram for alternative 2.

Monthly equivalent cost = Rs.10,500.

The monthly equivalent cost of alternative 1 is less than that of alternative 2. Hence, the salesman should purchase the car for cash.

EXAMPLE 6.8 A company must decide whether to buy machine A or machine B.

	Machine A	*Machine B*
Initial cost (Rs.)	3,00,000	6,00,000
Useful life (years)	4	4
Salvage value at the end of machine life (Rs.)	2,00,000	3,00,000
Annual maintenance (Rs.)	30,000	0

At 15% interest rate, which machine should be purchased?

Solution Machine A

Initial cost = Rs. 3,00,000
Useful life (years) = 4
Salvage value at the end of machine life = Rs. 2,00,000
Annual maintenance = Rs. 30,000
Interest rate = 15%, compounded annually

The cash flow diagram of machine A is depicted in Fig. 6.17.

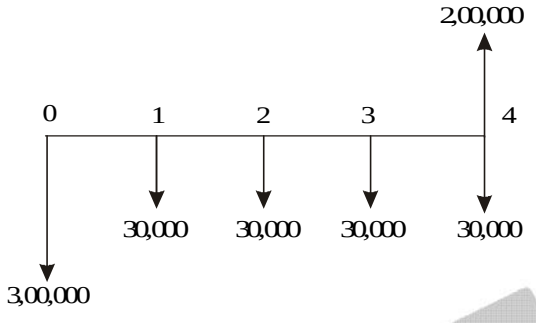


Fig. 6.17 Cash flow diagram for machine A.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(15\%) = 3{,}00{,}000(A/P, 15\%, 4) + 30{,}000 - 2{,}00{,}000(A/F, 15\%, 4)$$
$$= 3{,}00{,}000(0.3503) + 30{,}000 - 2{,}00{,}000(0.2003)$$
$$= \text{Rs. } 95{,}030$$

Machine B

Initial cost = Rs. 6,00,000
Useful life (years) = 4
Salvage value at the end of machine life = Rs. 3,00,000
Annual maintenance = Rs. 0.
Interest rate = 15%, compounded annually

The cash flow diagram of machine B is illustrated in Fig. 6.18.

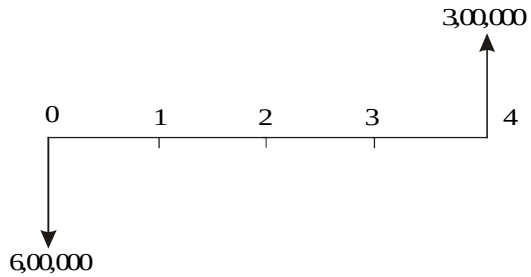


Fig. 6.18 Cash flow diagram for machine B.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(15\%) = 6,00,000(A/P, 15\%, 4) - 3,00,000(A/F, 15\%, 4)$$
$$= 6,00,000(0.3503) - 3,00,000(0.2003)$$
$$= \text{Rs. } 1,50,090$$

Since the annual equivalent cost of machine A is less than that of machine B, it is advisable to buy machine A.

EXAMPLE 6.9 Jothi Lakshimi has arranged to buy some home recording equipment. She estimates that it will have a five year useful life and no salvage value at the end of equipment life. The dealer, who is a friend has offered Jothi Lakshimi two alternative ways to pay for the equipment.

(a) Pay Rs. 60,000 immediately and Rs. 15,000 at the end of one year.

(b) Pay nothing until the end of fourth year when a single payment of Rs. 90,000 must be made.

If Jothi Lakshimi believes 12% is a suitable interest rate, which alternative is the best for her?

Solution Alternative 1

Down payment = Rs. 60,000
Payment after one year = Rs. 15,000

The cash flow diagram for alternative 1 is shown in Fig. 6.19.

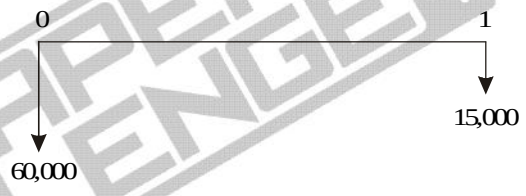


Fig. 6.19 Cash flow diagram for alternative 1.

The present worth equation of the above cash flow diagram is

$$PW(12\%) = 60,000 + 15,000(P/F, 12\%, 1)$$
$$= 60,000 + 15,000(0.8929)$$
$$= 73,393.50$$

The above present worth is represented in Fig. 6.20.

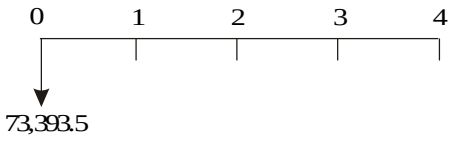


Fig. 6.20 Resultant cash flow diagram.

The annual equivalent expression of the above cash flow diagram is

$$AE(12\%) = 73{,}393.5(A/P, 12\%, 4)$$
$$= 73{,}393.5(0.3292)$$
$$= \text{Rs. } 24{,}161.14$$

Alternative 2

Payment after four years = Rs. 90,000

The cash flow diagram for alternative 2 is shown in Fig. 6.21.

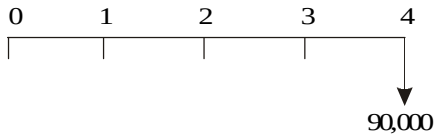


Fig. 6.21 Cash flow diagram of alternative 2.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(12\%) = 90{,}000(A/F, 12\%, 4)$$
$$= 90{,}000(0.2092)$$
$$= \text{Rs. } 18{,}828$$

The annual equivalent cost of alternative 2 is less than that of alternative 1. Hence, Jothi Lakshimi should select alternative 2 for purchasing the home equipment.

EXAMPLE 6.10 A transport company has been looking for a new tyre for its truck and has located the following alternatives:

Brand	*Tyre warranty* (months)	*Price per tyre* (Rs.)
A	12	1,200
B	24	1,800
C	36	2,100
D	48	2,700

If the company feels that the warranty period is a good estimate of the tyre life and that a nominal interest rate (compounded annually) of 12% is appropriate, which tyre should it buy?

Solution In all the cases, the interest rate is 12%. This is equivalent to 1% per month.

Brand A

Tyre warranty = 12 months

Price/tyre = Rs. 1,200

The cash flow diagram for brand A is shown in Fig. 6.22.

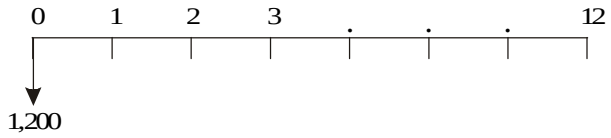


Fig. 6.22 Cash flow diagram of brand A.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(1\%) = 1{,}200(A/P, 1\%, 12)$$
$$= 1{,}200(0.0888)$$
$$= \text{Rs. } 106.56$$

Brand B

Tyre warranty = 24 months

Price/tyre = Rs. 1,800

The cash flow diagram for brand B is shown in Fig. 6.23.

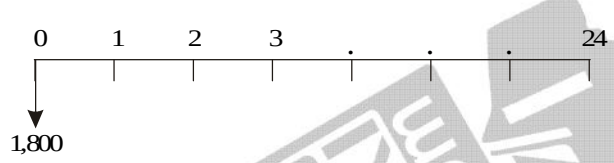


Fig. 6.23 Cash flow diagram of brand B.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(1\%) = 1{,}800(A/P, 1\%, 24)$$
$$= 1{,}800(0.0471)$$
$$= \text{Rs. } 84.78$$

Brand C

Tyre warranty = 36 months

Price/tyre = Rs. 2,100

The cash flow diagram for brand C is shown in Fig. 6.24.

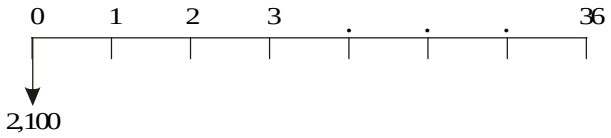


Fig. 6.24 Cash flow diagram of brand C.

The annual equivalent expression of the above cash flow diagram is

$$AE(1\%) = 2{,}100(A/P, 1\%, 36)$$
$$= 2{,}100(0.0332)$$
$$= \text{Rs. } 69.72$$

Brand D

Tyre warranty = 48 months

Price/tyre = Rs. 2,700

The cash flow diagram for brand D is shown in Fig. 6.25.

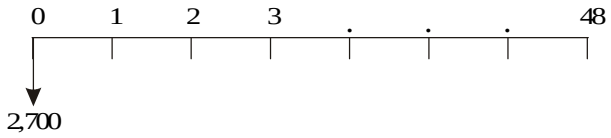


Fig. 6.25 Cash flow diagram of brand D.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(1\%) = 2{,}700\,(A/P,\ 1\%,\ 48)$$
$$= 2{,}700\,(0.0263)$$
$$= \text{Rs. } 71.01$$

Here, minimum common multiple lives of tyres is considered. This is 144 months. Therefore, the comparison is made on 144 month's basis.

The annual equivalent cost of brand C is less than that of other brands. Hence, it should be used in the vehicles of the trucking company. It should be replaced four times during the 144-month period.

QUESTIONS

1. A company has three proposals for expanding its business operations. The details are as follows:

Alternative	*Initial cost* (Rs.)	*Annual revenue* (Rs.)	*Life* (years)
*A*1	25,00,000	8,00,000	10
*A*2	20,00,000	6,00,000	10
*A*3	30,00,000	10,00,000	10

Each alternative has insignificant salvage value at the end of its life. Assuming an interest rate of 15%, compounded annually, find the best alternative for expanding the business operations of the company using the annual equivalent method.

2. An automobile dealer has recently advertised for its new car. There are three alternatives of purchasing the car which are explained below.

Alternative 1 The customer can take delivery of a car after making a down payment of Rs. 25,000. The remaining money should be paid in 36 equal monthly installments of Rs. 10,000 each.

Alternative 2 The customer can take delivery of the car after making a down payment of Rs. 1,00,000. The remaining money should be paid in 36 equal monthly installments of Rs. 7,000 each.

Alternative 3 The customer can take delivery of the car by making full payment of Rs. 3,00,000.

Suggest the best alternative of buying the cars for the customers by assuming an interest rate of 20% compounded annually. Use the annual equivalent method.

3. A small-scale industry is in the process of buying a milling machine. The purchase value of the milling machine is Rs. 60,000. It has identified two banks for loan to purchase the milling machine. The banks can give only 80% of the purchase value of the milling machine as loan. In Urban Bank, the loan is to be repaid in 60 equal monthly installments of Rs. 2,500 each. In State Bank, the loan is to be repaid in 40 equal monthly installments of Rs. 4,500 each. Suggest the most economical loan scheme for the company, based on the annual equivalent method of comparison. Assume a nominal rate of 24%, compounded monthly.

4. There are two alternatives of replacing a machine. The details of the alternatives are as follows:

Alternative 1

Purchase value of the new machine	: Rs. 2,00,000
Life of the machine	: 10 years
Salvage value of the new machine at the end of its life	: Rs. 20,000
Annual operation and maintenance cost	: Rs. 40,000
Buyback price of the existing machine	: Rs. 25,000

Alternative 2

Purchase value of the new machine	: Rs. 3,00,000
Life of the machine	: 10 years
Salvage value of the new machine at the end of its life	: Rs. 15,000
Annual operation and maintenance cost	: Rs. 35,000
Buyback price of the existing machine	: Rs. 5,000

Suggest the best replacement option for the company using the annual equivalent cost method of comparison by assuming 20% interest rate, compounded annually.

5. A company receives two options for purchasing a copier machine for its office.

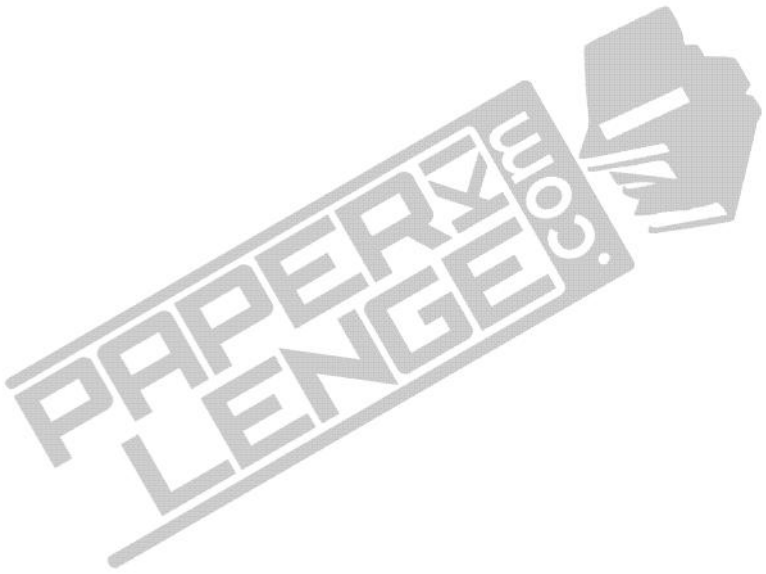
Option 1 Make a down payment of Rs. 30,000 and take delivery of the copier machine. The remaining money is to be paid in 24 equal monthly installments of Rs. 4,500 each.

Option 2 Make a full payment of Rs. 1,00,000 and take delivery of the copier machine.

Suggest the best option for the company to buy the copier machine based on the annual equivalent method of comparison by assuming 15% interest rate, compounded annually.

6. Find the best alternative using the annual equivalent method of comparison. Assume an interest rate of 15% compounded annually.

Alternative	*A*	*B*	*C*
Initial cost (Rs.)	5,00,000	8,00,000	6,00,000
Annual receipt (Rs.)	2,00,000	1,50,000	1,20,000
Life (years)	10	10	10
Salvage value (Rs.)	1,00,000	50,000	30,000



RATE OF RETURN METHOD

7.1 INTRODUCTION

The rate of return of a cash flow pattern is the interest rate at which the present worth of that cash flow pattern reduces to zero. In this method of comparison, the rate of return for each alternative is computed. Then the alternative which has the highest rate of return is selected as the best alternative.

In this type of analysis, the expenditures are always assigned with a negative sign and the revenues/inflows are assigned with a positive sign.

A generalized cash flow diagram to demonstrate the rate of return method of comparison is presented in Fig. 7.1.

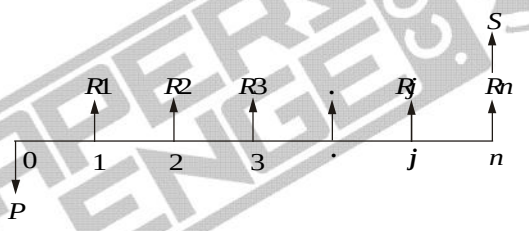


Fig. 7.1 Generalized cash flow diagram.

In the above cash flow diagram, P represents an initial investment, Rj the net revenue at the end of the jth year, and S the salvage value at the end of the nth year.

The first step is to find the net present worth of the cash flow diagram using the following expression at a given interest rate, i.

$$PW(i) = -P + R1/(1 + i)^1 + R2/(1 + i)^2 + ...$$
$$+ Rj/(1 + i)^j + ... + Rn/(1 + i)^n + S/(1 + i)^n$$

Now, the above function is to be evaluated for different values of i until the present worth function reduces to zero, as shown in Fig. 7.2.

In the figure, the present worth goes on decreasing when the interest rate is increased. The value of i at which the present worth curve cuts the X-axis is the rate of return of the given proposal/project. It will be very difficult to find the exact value of i at which the present worth function reduces to zero.

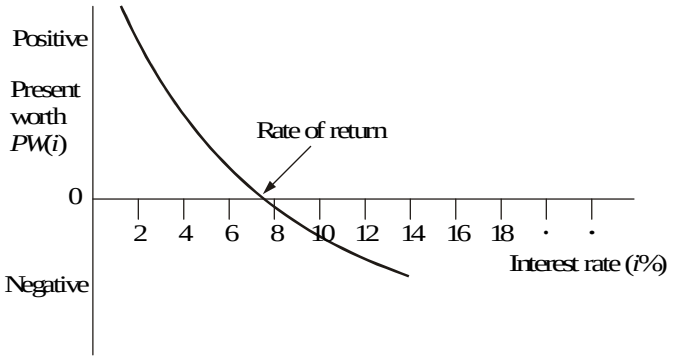


Fig. 7.2 Present worth function graph.

So, one has to start with an intuitive value of i and check whether the present worth function is positive. If so, increase the value of i until $PW(i)$ becomes negative. Then, the rate of return is determined by interpolation method in the range of values of i for which the sign of the present worth function changes from positive to negative.

7.2 EXAMPLES

In this section, the concept of rate of return calculation is demonstrated with suitable examples.

EXAMPLE 7.1 A person is planning a new business. The initial outlay and cash flow pattern for the new business are as listed below. The expected life of the business is five years. Find the rate of return for the new business.

Period	0	1	2	3	4	5
Cash flow (Rs.)	–1,00,000	30,000	30,000	30,000	30,000	30,000

Solution

Initial investment = Rs. 1,00,000
Annual equal revenue = Rs. 30,000
Life = 5 years
The cash flow diagram for this situation is illustrated in Fig. 7.3.

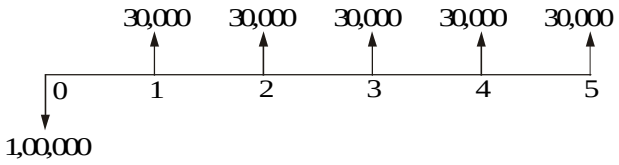


Fig. 7.3 Cash flow diagram.

The present worth function for the business is

$PW(i) = -1,00,000 + 30,000(P/A, i, 5)$

When $i = 10\%$,

$PW(10\%) = -1,00,000 + 30,000(P/A, 10\%, 5)$

$= -1,00,000 + 30,000(3.7908)$

$=$ Rs. 13,724.

When $i = 15\%$,

$PW(15\%) = -1,00,000 + 30,000(P/A, 15\%, 5)$

$= -1,00,000 + 30,000(3.3522)$

$=$ Rs. 566.

When $i = 18\%$,

$PW(18\%) = -1,00,000 + 30,000(P/A, 18\%, 5)$

$= -1,00,000 + 30,000(3.1272)$

$=$ Rs. $-6,184$

$$i = 15\% + \frac{566 - 0}{566 - (-6184)} \quad (3\%)$$

$$= 15\% + 0.252\%$$

$$= 15.252\%$$

Therefore, the rate of return for the new business is 15.252%.

EXAMPLE 7.2 A company is trying to diversify its business in a new product line. The life of the project is 10 years with no salvage value at the end of its life. The initial outlay of the project is Rs. 20,00,000. The annual net profit is Rs. 3,50,000. Find the rate of return for the new business.

Solution

Life of the product line (n) = 10 years
Initial outlay = Rs. 20,00,000
Annual net profit = Rs. 3,50,000
Scrap value after 10 years = 0

The cash flow diagram for this situation is shown in Fig. 7.4.

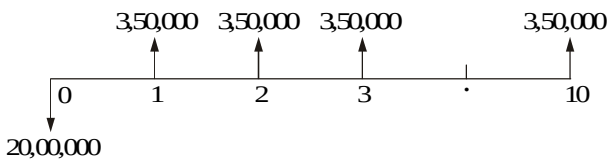


Fig. 7.4 Cash flow diagram.

The formula for the net present worth function of the situation is

$$PW(i) = -20,00,000 + 3,50,000(P/A, i, 10)$$

When $i = 10\%$,

$$PW(10\%) = -20,00,000 + 3,50,000(P/A, 10\%, 10)$$
$$= -20,00,000 + 3,50,000(6.1446)$$
$$= \text{Rs. } 1,50,610.$$

When $i = 12\%$,

$$PW(12\%) = -20,00,000 + 3,50,000(P/A, 12\%, 10)$$
$$= -20,00,000 + 3,50,000(5.6502)$$
$$= \text{Rs. } -22,430.$$

$$i = 10\% + \frac{1,50,610 - 0}{1,50,610 - (-22,430)} \quad (2\%)$$

$$= 11.74\ \%$$

Therefore, the rate of return of the new product line is 11.74%

EXAMPLE 7.3 A firm has identified three mutually exclusive investment proposals whose details are given below. The life of all the three alternatives is estimated to be five years with negligible salvage value. The minimum attractive rate of return for the firm is 12%.

	Alternative		
	*A*1	*A*2	*A*3
Investment	Rs. 1,50,000	Rs. 2,10,000	Rs. 2,55,000
Annual net income	Rs. 45,570	Rs. 58,260	Rs. 69,000

Find the best alternative based on the rate of return method of comparison.

Solution ***Calculation of rate of return for alternative*** *A*1

Initial outlay = Rs. 1,50,000

Annual profit = Rs. 45,570

Life = 5 years

The cash flow diagram for alternative *A*1 is shown in Fig. 7.5.

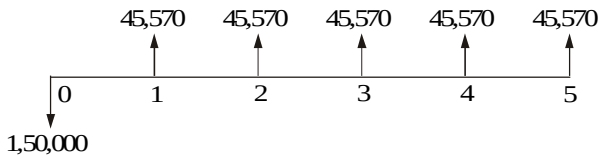


Fig. 7.5 Cash flow diagram for alternative *A*1.

The formula for the net present worth of alternative $A1$ is given as

$$PW(i) = -1,50,000 + 45,570(P/A, i, 5)$$

When i = 10%,

$$PW(10\%) = -1,50,000 + 45,570(P/A, 10\%, 5)$$
$$= -1,50,000 + 45,570(3.7908)$$
$$= \text{Rs. } 22,746.76$$

When i = 12%,

$$PW(12\%) = -1,50,000 + 45,570(P/A, 12\%, 5)$$
$$= -1,50,000 + 45,570(3.6048)$$
$$= \text{Rs. } 14,270.74$$

When i = 15%,

$$PW(15\%) = -1,50,000 + 45,570(P/A, 15\%, 5)$$
$$= -1,50,000 + 45,570(3.3522)$$
$$= \text{Rs. } 2,759.75$$

When i = 18%,

$$PW(18\%) = -1,50,000 + 45,570(P/A, 18\%, 5)$$
$$= -1,50,000 + 45,570(3.1272)$$
$$= \text{Rs. } -7,493.50$$

Therefore, the rate of return of the alternative $A1$ is

$$i = 15\% + \frac{2,759.75 - 0}{2,759.75 - (-7,493.50)} \quad (3\%)$$
$$= 15\% + 0.81\%$$
$$= 15.81\%$$

Calculation of rate of return for alternative $A2$

Initial outlay = Rs. 2,10,000
Annual profit = Rs. 58,260
Life of alternative $A2$ = 5 years

The cash flow diagram for alternative $A2$ is shown in Fig. 7.6.

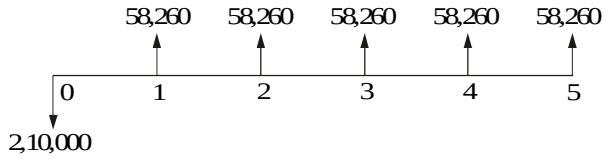


Fig. 7.6 Cash flow diagram for alternative $A2$.

The formula for the net present worth of this alternative is

$$PW(i) = -2,10,000 + 58,260(P/A, i, 5)$$

When $i = 12\%$,

$$PW(12\%) = -2,10,000 + 58,260(P/A, 12\%, 5)$$
$$= -2,10,000 + 58,260(3.6048)$$
$$= \text{Rs. } 15.65$$

When $i = 13\%$,

$$PW(13\%) = -2,10,000 + 58,260(P/A, 13\%, 5)$$
$$= -2,10,000 + 58,260(3.5172)$$
$$= \text{Rs. } -5,087.93$$

Therefore, the rate of return of alternative $A2$ is

$$i = 12\% + \frac{15.65 - 0}{15.65 - (-5,087.93)} \quad (1\%)$$
$$= 12\% + 0\%$$
$$= 12\%$$

Calculation of rate of return for alternative $A3$

Initial outlay = Rs. 2,55,000
Annual profit = Rs. 69,000
Life of alternative $A3$ = 5 years

The cash flow diagram for alternative $A3$ is depicted in Fig. 7.7.

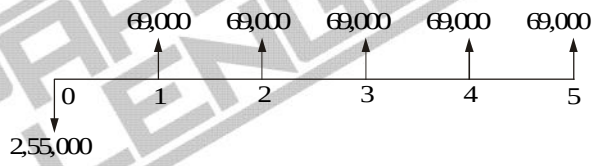


Fig. 7.7 Cash flow diagram for alternative $A3$.

The formula for the net present worth of this alternative $A3$ is

$$PW(i) = -2,55,000 + 69,000(P/A, i, 5)$$

When $i = 11\%$,

$$PW(11\%) = -2,55,000 + 69,000(P/A, 11\%, 5)$$
$$= -2,55,000 + 69,000(3.6959)$$
$$= \text{Rs. } 17.1$$

When $i = 12\%$,

$$PW(12\%) = -2,55,000 + 69,000(P/A, 12\%, 5)$$
$$= -2,55,000 + 69,000(3.6048)$$
$$= \text{Rs. } -6,268.80$$

Therefore, the rate of return for alternative $A3$ is

$$i = 11\% + \frac{17.1 - 0}{17.1 - (-6{,}268.80)} \quad 1\%$$

$$= 11\%$$

The rates of return for the three alternatives are now tabulated.

Alternative	*A*1	*A*2	*A*3
Rate of return	15.81%	12%	11%

From the above data, it is clear that the rate of return for alternative $A3$ is less than the minimum attractive rate of return of 12%. So, it should not be considered for comparison. The remaining two alternatives are qualified for consideration. Among the alternatives $A1$ and $A2$, the rate of return of alternative $A1$ is greater than that of alternative $A2$. Hence, alternative $A1$ should be selected.

EXAMPLE 7.4 For the cash flow diagram shown in Fig. 7.8, compute the rate of return. The amounts are in rupees.

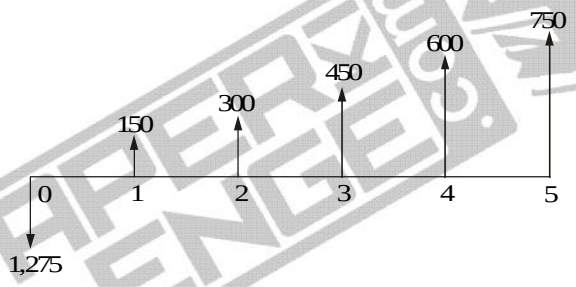


Fig. 7.8 Cash flow diagram.

Solution For the positive cash flows of the problem,

$$A1 = \text{Rs. } 150, \qquad G = \text{Rs. } 150$$

The annual equivalent of the positive cash flows of the uniform gradient series is given by

$$A = A1 + G(A/G, i, n)$$
$$= 150 + 150(A/G, i, 5)$$

The formula for the present worth of the whole diagram

$$= -1{,}275 + [150 + 150(A/G, i, 5)] \quad (P/A, i, 5)$$

$$PW(10\%) = -1{,}275 + [150 + 150(A/G, 10\%, 5)] \quad (P/A, 10\%, 5)$$
$$= -1{,}275 + [150 + 150(1.8101)] \quad (3.7908)$$
$$= \text{Rs. } 322.88$$

$$PW(12\%) = -1{,}275 + [150 + 150(A/G, 12\%, 5)] \quad (P/A, 12\%, 5)$$
$$= -1{,}275 + [150 + 150(1.7746)] \quad (3.6048)$$
$$= \text{Rs. } 225.28$$
$$PW(15\%) = -1{,}275 + [150 + 150(A/G, 15\%, 5)] \quad (P/A, 15\%, 5)$$
$$= -1{,}275 + [150 + 150(1.7228)] \quad (3.3522)$$
$$= \text{Rs. } 94.11$$
$$PW(18\%) = -1{,}275 + [150 + 150(A/G, 18\%, 5)] \quad (P/A, 18\%, 5)$$
$$= -1{,}275 + [150 + 150(1.6728)] \quad (3.1272)$$
$$= \text{Rs. } -21.24$$

Therefore, the rate of return for the cash flow diagram is

$$i = 15\% + \frac{94.11 - 0}{94.11 - (-21.24)} \quad 3\%$$

$$= 15\% + 2.45\% = 17.45\%$$

EXAMPLE 7.5 A company is planning to expand its present business activity. It has two alternatives for the expansion programme and the corresponding cash flows are tabulated below. Each alternative has a life of five years and a negligible salvage value. The minimum attractive rate of return for the company is 12%. Suggest the best alternative to the company.

	Initial investment (Rs.)	*Yearly revenue* (Rs.)
Alternative 1	5,00,000	1,70,000
Alternative 2	8,00,000	2,70,000

Solution Alternative 1

Initial outlay = Rs. 5,00,000
Annual revenue = Rs. 1,70,000
Life of alternative 1 = 5 years

The cash flow diagram for alternative 1 is illustrated in Fig. 7.9.

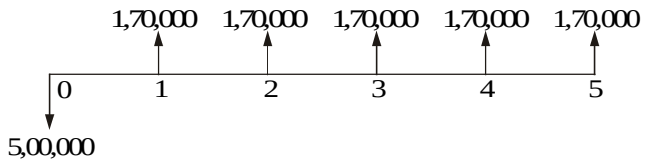


Fig. 7.9 Cash flow diagram for alternative 1.

The formulae for the net present worth of alternative 1 are as follows:

$$PW_1(i) = -5,00,000 + 1,70,000(P/A, i, 5)$$

$$PW_1(15\%) = -5,00,000 + 1,70,000(P/A, 15\%, 5)$$
$$= -5,00,000 + 1,70,000(3.3522)$$
$$= \text{Rs. } 69,874$$

$$PW_1(17\%) = -5,00,000 + 1,70,000(P/A, 17\%, 5)$$
$$= -5,00,000 + 1,70,000(3.1993)$$
$$= \text{Rs. } 43,881$$

$$PW_1(20\%) = -5,00,000 + 1,70,000(P/A, 20\%, 5)$$
$$= -5,00,000 + 1,70,000(2.9906)$$
$$= \text{Rs. } 8,402$$

$$PW_1(22\%) = -5,00,000 + 1,70,000(P/A, 22\%, 5)$$
$$= -5,00,000 + 1,70,000(2.8636)$$
$$= \text{Rs. } -13,188$$

Therefore, the rate of return of alternative 1 is

$$i = 20\% + \frac{8,402 - 0}{8,402 - (-13,188)} 2\%$$
$$= 20.78\%$$

Alternative 2

Initial outlay = Rs. 8,00,000
Annual revenue = Rs. 2,70,000
Life = 5 years

The cash flow diagram for alternative 2 is depicted in Fig. 7.10.

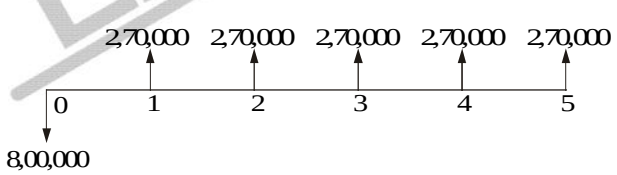


Fig. 7.10 Cash flow diagram for alternative 2.

The formula for the net present worth of alternative 2 is:

$$PW_2(i) = -8,00,000 + 2,70,000(P/A, i, 5)$$

$$PW_2(20\%) = -8,00,000 + 2,70,000(P/A, 20\%, 5)$$
$$= -8,00,000 + 2,70,000(2.9906)$$
$$= \text{Rs. } 7,462$$

$$PW_2(22\%) = -8,00,000 + 2,70,000\,(P/A,\ 22\%,\ 5)$$
$$= -8,00,000 + 2,70,000\,(2.8636)$$
$$= \text{Rs. } -26,828$$

Thus, the rate of return of alternative 2 is

$$i = 20\% + \frac{7,462 - 0}{7,462 - (-26,828)} \quad 2\%$$
$$= 20.435\%$$

Since the rate of return of alternative 1 is greater than that of the alternative 2, select alternative 1.

QUESTIONS

1. Consider the following cash flow of a project:

Year	0	1	2	3	4	5
Cash flow	–10,000	4,000	4,500	5,000	5,500	6,000

Find the rate of return of the project.

2. A person invests a sum of Rs. 2,00,000 in a business and receives equal net revenue of Rs. 50,000 for the next 10 years. At the end of the 10th year, the salvage value of the business is Rs. 25,000. Find the rate of return of the business.

3. A company is in the process of selecting the best alternative among the following three mutually exclusive alternatives:

Alternative	*Initial investment*	*Annual revenue* (Rs.)	*Life* (years)
*A*1	Rs. 5,00,000	1,00,000	10
*A*2	Rs. 8,00,000	1,40,000	10
*A*3	Rs. 3,00,000	70,000	10

Find the best alternative based on the rate of return method of comparison.

4. A shipping firm is considering the purchase of a materials handling system for unloading ships at a dock. The firm has reduced their choice to three different systems, all of which are expected to provide the same unloading speed. The initial costs and the operating costs estimated for each system are now tabulated.

System	*Initial cost*	*Annual operating expenses*
*S*1	Rs. 6,50,000	Rs. 91,810
*S*2	Rs. 7,80,000	Rs. 52,600
*S*3	Rs. 7,50,000	Rs. 68,417

The life of each system is estimated to be five years and the firm's minimum attractive rate of return is 15%. If the firm must select one of the materials handling systems, which one is the most desirable?.

5. A firm has identified three mutually exclusive alternatives. The life of all three alternatives is estimated to be five years. The minimum attractive rate of return is 12%. Find the best alternative based on the rate of return method.

Alternative	*A*1	*A*2	*A*3
Initial investment (Rs.)	2,00,000	2,80,000	3,60,000
Annual income (Rs.)	52,000	72,000	1,00,000

6. An automobile company is planning to buy a robot for its forging unit. It has identified two different companies for the supply of the robot. The details of cost and incremental revenue of using robots are summarized in the following table:

	Brand	
	Speedex	*Giant*
Initial cost (Rs.)	5,00,000	9,00,000
Annual incremental revenue (Rs.)	80,000	2,50,000
Life (years)	8	8
Life-end salvage value (Rs.)	40,000	60,000

The minimum attractive return for the company is 12%. Suggest the best brand of robot to the company based on the rate of return method.

7. A bank introduces two different investment schemes whose details are as follows: Find the best investment alternative from the investor's point of view.

	Alpha Bank	*Beta Bank*
Deposit amount (Rs.)	1,00,000	2,00,000
Period of deposit (years)	5 years	3 years
Maturity amount (Rs.)	3,00,000	4,50,000

8. A company is planning for its expansion programme which will take place after five years. The expansion requires an equal sum of Rs. 5,00,000 for consecutive three years. Gamma Bank has recently introduced a scheme in this line. If the company invests Rs. 7,00,000 now with this bank, it will make equal repayments of Rs. 5,00,000 for three consecutive years starting from the end of the fifth year from now. The minimum attractive rate of return for the company is 12%. Suggest whether the company should invest with the Gamma Bank for its expansion programme.

9. Consider the following table which summarizes data of two alternatives.

	First cost	*Annual return*	*Life*
Alternative 1	Rs. 5,00,000	Rs. 1,50,000	10 yrs
Alternative 2	Rs. 8,00,000	Rs. 2,50,000	10 yrs

Find the best alternative based on the rate of return method of comparison.

10. A company is planning to expand its present business activity. It has two alternatives for the expansion programme and the corresponding cash flows are given in the following table. Each alternative has a life of five years and a negligible salvage value. The minimum attractive rate of return for the company is 15%. Suggest the best alternative to the company.

	Initial investment (Rs.)	*Yearly revenue* (Rs.)
Alternative 1	4,50,000	1,50,000
Alternative 2	7,50,000	2,50,000