FINANCIAL ENGINEERING
CLASS TEST-2

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2K18/MC/008

(1) $S(0) = \text{Rs } 100$

$K = \text{Rs } 80$

$T = 5$

$r = 0.05$

$\sigma = 0.3$

div = Rs 20 for first 2 years and Rs 30 for next 3 years

$$S_a = S(0) - (div)\, e^{-r\, t_{div}}$$

$$= 100 - 20 e^{-0.05 \times 2} - 30 e^{-(0.05) \times 3}$$

$$= 56.082$$

$$d_1 = \frac{\ln(S_a/K) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln(56.082/80) + (0.05 + 0.5 \times (0.3)^2) \times 5}{0.3 \times \sqrt{5}}$$

$$= \frac{-0.3552 + 0.475}{0.6708}$$

$$= 0.1786$$

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$$d_2 = d_1 - \sigma\sqrt{T}$$
$$= 0.1786 - 0.3 \times \sqrt{5}$$
$$= -0.4922$$

$$\phi(d_1) = \phi(0.1786) = 0.5714$$
$$\phi(d_2) = \phi(-0.4922) = 0.3121$$

$$C(0) = S_0\,\phi(d_1) - Ke^{-rT}\,\phi(d_2)$$

$$C(0) = (56.082) \times (0.5714) - (62.304) \times (0.3121)$$
$$= \text{Rs } 12.60$$

(2) Given $\{W(t), t \geq 0\}$ is a Brownian Motion.

Let $0 \leq s < t$. Then

$$W^2(t) = \left((W(t) - W(s)) + W(s)\right)^2$$

$$= (W(t) - W(s))^2 + 2W(s)(W(t) - W(s)) + W^2(s)$$

$$E\left((W(t) - W(s))^2 \mid F(s)\right) = E(W(t) - W(s))^2 = t - s$$

Since $W(t)$ is a Brownian Motion the increment $W(t) - W(s)$ is independent of $F(s)$ and distributed as $N(0, t-s)$.

$$E\left(W(s)\{W(t) - W(s)\} \mid F(s)\right) = W(s)\, E(W(t) - W(s))$$
$$= 0$$

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$$E\left(W^2(t) \mid F(s)\right) = E\left(\left(W(t) - W(s) + W(s)\right)^2 \mid F(s)\right)$$

$$= E\left(\left(W(t) - W(s)\right)^2 \mid F(s)\right) + 2E\left(\left(W(t) - W(s)\right) \cdot W(s) \mid F(s)\right) + E\left(W^2(s) \mid F(s)\right)$$

$$= t - s + 0 + W^2(s)$$

$$E\left(W^2(t) \mid F(s)\right) = t - s + W^2(s)$$

$$E\left(W^2(t) - t \mid F(s)\right) = W^2(s) - s$$

Hence it satisfies the condition of Martingale $E(W^2(t) - t)$ exists and $W(t) - t$ is F_t measurable.

So it is a Martingale.

3. Discrete Time Filtration:

Let Ω be the sample space and $F_0 = \{\phi, \Omega\}$

Then a filtration in discrete time is an increasing sequence of $F_0 \subset F_1 \subset F_2 \dots$ of σ-field one past time instant.

Eg Let $\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

E_H = Head in fist toss

E_T = Tail in first toss

$F_0 = \{\phi, \Omega\}$ $F_1 = \{\phi, \{E_H\}, \{E_T\}, \Omega\}$

$F_2 = \{\phi, \{E_H\}, \{E_T\}, \{E_{HH}\}, \{E_{TT}\}, \{E_{HH}^c\}, \{E_{TT}^c\}, \{E_{HH} \cup E_{TT}\}, \Omega\}$

Then $F_0 \subset F_1 \subset F_2$.

$\{S_n, n=0,1,2\}$, $S_0=0$ is symmetric random walk.

$$\Omega = \{\{0,1,2\}\ \{0,1,0\}, \{0,-1,0\}, \{0,-1,-2\}\}$$

$$F_0 = \{\phi, \Omega\}$$

E_1 = Positive for $n=1$
E_{-1} = Negative for $n=1$

$$F_1 = \{\phi, \{E_1\}, E_{-1}, \Omega\}$$

$$F_2 = \{\phi, \{E_1\}, \{E_{-1}\}, \{E_{11}\}, \{E_{-1-1}\}, \{E_{11}^c\}, \{E_{-1-1}^c\}, \{E_{11} \cup E_{-1-1}\}, \Omega\}$$

Hence $F_0 \subset F_1 \subset F_2$

Where E_{11} = Positive for $n=1,2$
E_{-1-1} = Negative for $n=1,2$

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