AIMAN SIDDIQUA - 2K18/MC/008

FINANCIAL ENGINEERING

ASSIGNMENT-4

1. (a) $E(R_{K_1}) = 0.2 \times (-10.0) + 0 \times 0.5 + 20 \times 0.3$
$= -2 + 0 + 6 = 4\%$

$E(R_{K_2}) = 0.2 \times (-30) + 20 \times 0.5 + 15 \times 0.3$
$= -6 + 10 + 4.5 = 8.5\%$

(b) 60% of available fund is invested in K_1, then weight
$K_1 = 0.6$ & $K_2 = 0.4$

$E(\text{Portfolio}) = W_{R_1} E(R_{K_1}) + W_{R_2} E(R_{K_2})$
$= 0.6 \times 4 + 0.4 \times 8.5$
$= 5.8\%$

(c) let $w_{K_1} = n$, then $w_{K_2} = 1-n$
$n(4) + (1-n)(8.5) = 20$
$4n + 8.5 - 8.5n = 20$
$-4.5n = 11.5$
$n = -2.56$

As $n \in [0,1]$, the given expected return is not possible.

2. $E(R_{K_1}) = 0.4 \times (-10) + 0.2 \times (0) + 0.4 \times 20$
$= -4 + 8 = 4\%$

$E(R_{K_2}) = 0.4 \times 20 + 0.2 \times 20 + 0.4 \times 10$
$= 8 + 4 + 4 = 16\%$

$\sigma^2(K_1) = \frac{1}{3}(10^{-4})\left[(14)^2 + 4^2 + (16)^2\right] = 0.0156$

$$\sigma^2(K_2) = \frac{1}{3}(10^{-4})\left[4^2+4^2+6^2\right] = 0.02267$$

$$\rho_{12} = -0.96309$$

$$\sigma_V^2 = (0.4)^2 \times (0.0156) + (0.6)^2 \times (0.02267)$$
$$= 0.00023$$

σ_V^2 is smaller than σ_1^2 & σ_2^2

If 80% is invested in stock 1 & 20% in stock 2

$$\sigma_V^2 = (0.8)^2 \times (0.0156) + (0.2)^2 \times (0.002267)$$
$$= 0.00829$$

3. To prove: $\sigma_V^2 \leq \max(\sigma_1^2 \,\&\, \sigma_2^2)$ if short selling is not allowed.

Let us assume that $\sigma_1^2 \leq \sigma_2^2$. If short sales are not allowed then w_1 & $w_2 > 0$ and

$$w_1\sigma_1 + w_2\sigma_2 \leq (w_1+w_2)\sigma_2 = \sigma_2$$
$$(\text{As } w_1 + w_2 = 1)$$

Since the correlation coefficient satisfies $-1 \leq \rho \leq 1$. Then

$$\sigma^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2$$
$$\leq (w_1\sigma_1 + w_2\sigma_2)^2 \leq \sigma_2^2$$

Hence Proved.

4. $m = \begin{bmatrix} 0.2 & 0.13 & 0.04 \end{bmatrix}$

$n = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

$\sigma_1 = 0.25$, $\sigma_2 = 0.28$, $\sigma_3 = 0.2$

$C_{12} = 0.3$, $C_{13} = 0.15$, $C_{23} = 0.4$

$$C = \begin{bmatrix} 0.0625 & 0.021 & 0.0075 \\ 0.023 & 0.754 & 0.0024 \\ 0.0075 & 0.0224 & 0.04 \end{bmatrix}$$

$$\mu C^{-1} = \begin{bmatrix} 12.33 & 3.54 & 20.723 \end{bmatrix}$$

$$\mu C^{-1} v^T = 36.59$$

$$w \times \frac{vC^{-1}}{\mu C^{-1} v^T} = \begin{bmatrix} 0.337 & 0.097 & 0.568 \end{bmatrix}$$

The expected return & standard deviation of this portfolio are 0.10265

standard deviation $= \sigma_v$

$= \sqrt{w_C w_T}$

$= 0.1653$

5. for given correlation $\rho = -1$

So $\mu_{min} = \dfrac{\sigma_1 \mu_1 + \sigma_2 \mu_2}{\mu_1 + \mu_2}$

$= \dfrac{0.05 \times 0.08 + 0.02 \times 0.1}{0.05 + 0.02}$

$= 0.085 \leq 1$

$\mu_{min} = 8.5\%$

$W_1 = 1 - S_{min}$ & $W_2 = S_{min}$

where $S_{min} = \dfrac{\sigma_1}{\sigma_1 + \sigma_2} = \dfrac{0.05}{0.05 + 0.02} = 0.714.$

$\rho = -1 \Rightarrow w_1 = 0.286$ & $w_2 = 0.714$

$S_{min} = 71.4\%$ $H_m = 8.317\%$

For $\rho = 0.5$

$w_1 = 1 - S_{min}$ & $w_2 = S_{min}$

$S_{min} = 0.7894$

$w_1 = 0.2106$

$H_{min} = (H_2 - H_1) S_{min} + H_1 = 8.421\%$

$\sigma_{min} = 1.86\%$

$\rho = 0.5$ $w_1 = 21.06\%$ $w_2 = 78.04\%$

$H_{min} = 8.421\%$ $\sigma_{min} = 1.986\%$

Hence Proved.

6. For given condition $\rho = -1$

So, $H_{min} = \dfrac{\sigma_1 V_1 + \sigma_2 V_2}{\sigma_1 + \sigma_2}$

$= 0.0857 < 1$

$H_{min} = 8.57\%$

$w_1 = 1 - S_{min}$ & $w_2 = S_{min}$

where $S_{min} = \dfrac{0.05}{0.05 + 0.02} = 0.784$

$w_1 = 0.286$ & $w_2 = 0.714$

$S_{min} = 71.4\%$ $H_{min} = 8.75\%$

For $\rho = 0.5$

$w_1 = 1 - S_{min}$

$S_{min} = 0.7894$

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$w_1 = 1 - S_{min} = 0.2106$

$\mu_{min} = (\mu_2 - \mu_1) S_{min} + \mu_1 = 8.421\%$

$\sigma_{min} = 1.986\%$

For $\rho = -0.5$

$w_1 = 1 - S_{min} = 0.4187$

$w_2 = S_{min} = 0.5813$

$\mu_{min} = 8.83\%$ $\sigma_{min} = 1.4\%$

For $\rho = 0$

$S_{min} = 86.2\%$ $w_1 = 13.8\%$ $w_2 = 86.2\%$..

$\mu_{min} = (\mu_2 - \mu_1) S_{min} + \mu_1 = 8.276\%$

$\sigma_{min} = 1.850\%$

For $\rho = 1$

$$S_{min} = \frac{\sigma_1}{\sigma_1 - \sigma_2} = \frac{0.05}{0.05 - 0.02} > 0$$

Hence $w_1 = 1 - S_{min} < 0$

$\therefore$ Investor takes short forward position on asset

$\mu_{min} = 66.67\%$ $\sigma_{min} = 0$

(7.) Taking $\alpha = 0$, $\beta = 1$

$$10 v_1^{(1)} + 4 v_2^{(1)} = 1$$

$$4 v_1^{(1)} + 12 v_2^{(1)} + 6 v_3^{(1)} = 1$$

$$6 v_2^{(1)} + 10 v_3^{(1)} = 1$$

The Solution $v^{(1)} = (x_0, 0, x_0)$

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$10v_1^{(2)} + 4v_2^{(2)} = 5$

$4v_1^{(2)} + 12v_2^{(2)} + 6v_3^{(2)} = 6$

$6v_2^{(2)} + 10v_3^{(2)} = 1$

Solving we get $v(2) = \left\{ \frac{3}{10}, \frac{1}{2}, \frac{-1}{5} \right\}$

Now normalizing $v(1)$

$w^{(1)} = v(1)\,\text{norm} = (1/2, 0, 1/2)$

Normalizing $v^{(3)}$, $w^{(2)} = \text{Norm}(w^{(2)})$

$y_1^{(-1)} = m^T w^{(1)} = [5\ 6\ 1]^T [1/2, 0, 1/2]$

$y^{-(-1)} = 3010$

$v^{-2} = m^T w^{(2)} = [5\ 6\ 1][1/2\ 5/6\ -1/3]$

$\lambda v^{-(1)} + (1-\lambda) y^{(-2)} = 2.8$

$\lambda = \dfrac{2.8 - y^{(-2)}}{y^{2(1)} - y^{-(-2)}} = \dfrac{2.8 - 2.16}{3.716}$

$\lambda = 1.048$

$y = \lambda w(1) + (1-\lambda) w^{(2)}$

$= 1 - \frac{1}{25} \cdot \frac{27}{60}$

This is not the most efficient portfolio.

(9.) We have

$0.06 - r_f = 0.5(r_m - r_f)$

$0.12 - r_f = 1.5(r_m - r_f)$

Solving we get $r_m = 0.09$ & $r_f = 0.03$

∴ Security market line is

$$r_i = 0.03 + \beta\,(0.06)$$

Hence when $\beta = 2$

$$r_i = 0.03 + 2 \times 0.06 = 0.03 + 0.12 = 0.15$$

Therefore expected return on asset is 15%

10. Given

$r_1 = 9.5\%$ $\qquad \beta_1 = 0.8$

$r_2 = 13.3\%$ $\qquad \beta_2 = 1.8$

Now

$$0.095 - r_f = 0.8\,(r_m - r_f)$$
$$0.135 - r_f = 1.8\,(r_m - r_f)$$

By solving we get $r_f = 0.031$ $\quad r_m = 0.111$

Risk free return = 3.1%

Return on market portforlio = 11.1%