THEORY OF COMPUTATION
CLASS TEST-2

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(1.) $P = \{ S \rightarrow aAa,\ A \rightarrow bBB,\ B \rightarrow ab,\ C \rightarrow aB \}$

There are no null and unit productions here.

STEP 1

$W_1 = \{B\}$ as $B \rightarrow ab$ is terminal

$W_2 = \{B\} \cup \{A, C\}$ $\quad (A_1 \mid A_1 \rightarrow \alpha$ with $\alpha \in (\Sigma \cup W_1)^* \}$

$W_3 = \{B, A, C\} \cup \{S\}$

$\therefore G_1 = (\{S, A, B, C\}, \{a, b\}, \{ S \rightarrow aAa,\ A \rightarrow bBB,\ B \rightarrow ab,\ C \rightarrow aB \}, S)$

STEP 2

$W_1 = \{S\}$
$W_2 = \{S\} \cup \{a, A\}$ $\quad (S \rightarrow aAa)$
$W_3 = \{S, a, A\} \cup \{b, B\}$ $\quad (A \rightarrow bBB)$
$W_4 = \{S, a, A, b, B\} \cup \phi$

$W = \{S, A, B, a, b\}$

$V_N' = \{V_{N_1} \cap W\} = \{S, A, B\}$

$\Sigma' = \{W \cap \Sigma_1\} = \{a, b\}$

$P' = \{ S \rightarrow aAa , A \rightarrow bBB, B \rightarrow ab\}$

$\therefore$ Reduced Grammar $G_1 = \{V_N', \Sigma', P', S\}$

(2.) $S \rightarrow 1A \mid 0B$
$A \rightarrow 1AA \mid 0S \mid 0$
$B \rightarrow 0BB \mid 1S \mid 1$

STEP 1 There are no null or unit productions here.

STEP-2

Elimination of terminals on R.H.S.

Let $G_1 = (V_N', \Sigma_1, P_1, S)$

(i) $A \rightarrow 0, B \rightarrow 1$ are included in P_1

(ii) $S \rightarrow 1A$ gives $S \rightarrow C_1A$ and $C_1 \rightarrow 1$
$S \rightarrow 0B$ gives $S \rightarrow C_0B$ and $C_0 \rightarrow 0$

(iii) $A \rightarrow 1AA$ gives $A \rightarrow C_1AA$ and $C_1 \rightarrow 1$
$A \rightarrow 0S$ gives $A \rightarrow C_0S$ and $C_0 \rightarrow 0$

(iv) $B \rightarrow 0BB$ gives $B \rightarrow C_0BB$ and $C_0 \rightarrow 0$
$B \rightarrow 1S$ gives $B \rightarrow C_1S$ and $C_1 \rightarrow 1$

Hence $V_N' = \{S, A, B, C_0, C_1\}$

$P_1 = \{S \to C_1A \mid C_0B,\ A \to C_1AA \mid C_0S \mid 0$
$B \to C_0BB \mid C_1S \mid 1,\ C_0 \to 0,\ C_1 \to 1\}$

STEP-3

Restricting the number of variables on R.H.S

Let $G_2 = (V_N'', \Sigma, P_2, S)$

i) $S \to C_1A \mid C_0B,\ A \to C_0S \mid 0,\ B \to C_1S, 1,\ C_0 \to 0,$ $C_1 \to 1$ are added to P_2 as they are in required form.

ii) $A \to C_1AA$ gives $A \to C_1C'$ $\quad C' \to AA$
$B \to C_0BB$ gives $B \to C_0C''$ $\quad C'' \to BB$

Hence $V_N'' = \{S, A, B, C_0, C_1, C', C''\}$

$P_2 = \{S \to C_1A \mid C_0B,\ A \to C_1C' \mid C_0S \mid 0,$
$B \to C_0C'' \mid C_1S \mid 1,\ C_0 \to 0,\ C_1 \to 1,$
$C' \to AA,\ C'' \to BB\}$

G_2 is the required grammar.

3. $S \to SbS \mid a$ $\quad$ String : ababababa

Leftmost Derivation.

$S \to SbS \to abS \to abSbS \to ababS$
$\to ababSbS \to abababS \to abababa$

Scanned with CamScanner

Right most Derivation:

$S \to SbS \to Sba \to SbSba \to Sbaba$
$\to SbSbaba \to Sbababa \to abababa$

Another right most Derivation for abababa:

$S \to SbS \to SbSbS \to SbSbSbS \to SbSbSba$
$\to SbSbaba \to Sbababa \to abababa$

Since we have more than one derivation for abababa, the grammar is ambiguous.

4. $L = \{ a^{n^2} \mid n \geq 1 \}$

To show it is not context free

Step 1 Assume L is context free. Let n be the natural numbers obtained by using the pumping lemma.

Step 2
Let $z = a^{n^2}$.
We write $z = uvwxy$ where $1 \leq |vx| \leq n$
(This is possible because $|vwx| \leq n$ by pumping lemma)

Let $|vx| = m$, $m \leq n$
By pumping lemma, uv^2wx^2y is in L.

As $|uv^2wx^2y| > n^2$, $|uv^2wx^2y| = K^2$ where
$K \geq n+1 \longrightarrow$ (1.)

But $|uv^2wx^2y| = n^2+m < n^2+2n+1 \longrightarrow$ (2.)

From (1.) & (2.)
$|uv^2wx^2y|$ strictly lies between n^2 and $(n+1)^2$ which means $uv^2wx^2y \notin L$.

This is a contradiction.
Hence $\{a^{n^2} : n \geq 1\}$ is not context free.

(PUMPING LEMMA FOR CFL :
Let L be a CFL. Then we can find a natural number n such that :
(i) Every $z \in L$ with $|z| \geq n$ can be written as $uvwxy$ for some string u, v, w, x, y
(ii) $|vx| \geq 1$
(iii) $|vwx| \leq n$
(iv) $uv^kwx^ky \in L \;\forall\; k \geq 0$)