THEORY OF COMPUTATION
ASSIGNMENT - 3

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1. (a) $L = \{a^n b^m : n \neq m,\ n \geq 0,\ m \geq 0\}$
$G = (\{S, A, B, X\}, \{a, b\}, S, P)$ where P is:
$S \to AX \mid XB \mid \Lambda$
$X \to aXb \mid \Lambda$
$A \to aA \mid a$
$B \to bB \mid b$

(b) $L = \{a^n b^m : 2n \leq m \leq 3n\}$
$G = (\{S\}, \{a, b\}, S, P)$ where P is given by
$S \to aSbb \mid aSbbb \mid \Lambda$

(c) $L = \{w \in \{a, b\}^* : n_a(w) = 2n_b(w) + 1\}$
$G = (\{S\}, \{a, b\}, P, S)$ where P is given by
$\cancel{S \Rightarrow} \cancel{Saab} \mid \cancel{Saba} \mid \cancel{Sbaa} \mid \cancel{a}$ $S \to XaX$
$X \to XaXaXbX \mid XaXbXaX \mid XbXaXaX \mid \Lambda$

2. (a) $S \to 1A \mid 0B$, $A \to 1AA \mid 0S \mid 0$, $B \to 0BB \mid 1S \mid 1$

Step 1 There are no null productions or unit productions

Step 2 Let $G_1 = (V_N', \{0, 1\}, P_1, S)$ where P_1 & V_N' are constructed as follows:
(i) $A \to 0$, $B \to 1$ is included in P_2
(ii) $S \to 1A$, $B \to 1S$ gives $S \to \cancel{1}C_1A$, $B \to C_1S$, $C_1 \to 1$

(iii) $S \to 0B$, $A \to 0S$ gives $S \to C_0B$, $A \to C_0S$, $C_0 \to 0$

(iv) $A \to 1AA$, $B \to 0BB$ gives $A \to C_1AA$, $B \to C_0BB$

$\therefore V_N' = \{S, A, B, C_0, C_1\}$

Step 3 $G_2 = (V_N'', \{0,1\}, P_2, S)$ where P_2 & V_N'' are constructed as follows:

(i) $A \to 0$, $B \to 1$, $S \to C_1A$, $B \to C_1S$, $C_1 \to 1$, $S \to C_0B$, $A \to C_0S$, $C_0 \to 0$ are included in P_2

(ii) $A \to C_1AA$ and $B \to C_0BB$ are replaced by $A \to C_1D_1$, $D_1 \to AA$ and $B \to C_0D_2$ $D_2 \to BB$

Thus $G_2 = (\{S, A, B, C_0, C_1, D_1, D_2\}, \{0,1\}, P_2, S)$ is in CNF where P_2 contains $S \to C_1A | C_0B$, $A \to 0 | C_0S | C_1D_1$, $B \to 1 | C_1S | C_0D_2$, $C_1 \to 1$, $C_0 \to 0$, $D_1 \to AA$, $D_2 \to BB$

(b) $S \to a | b | cSS$

Step 1 There are no null or unit productions.

Step 2 $G_1 = (V_N', \{a, b, c\}, P_1, S)$ where P_1 and V_N' are:

(i) $S \to a$ and $S \to b$ are included in P_1.

(ii) $S \to cSS$ is replaced by $S \to CDD$, $C \to c$

$\therefore V_N' = \{S, C\}$

Step 3 $G_2 = (V_N'', \{a, b, c\}, P_2, S)$ where P_2 is:

(i) $S \to a$, $S \to b$, $C \to c$ are included in P_2

(ii) $S \to CSS$ is replaced by $S \to CD$ and $D \to SS$

Thus the equivalent grammar in CNF is $G_2 = (\{S, C, D\}, \{a, b, c\}, P_2, S)$ where P_2 consists of $S \to a | b | CD$,

$C \to c$ and $D \to SS$.

(3.) (a) $S \to SS \mid 0S1 \mid 01$

Step 1 Rename S as A_1. Define G_1 as
$G_1 = (\{A_1\ A_2\ A_3\}, \{0, 1\}, P_1, A_1)$
where P_1 consists of
$A_1 \to A_1A_1 \mid A_2A_1A_3 \mid A_2A_3$, $A_2 \to 0$ and $A_3 \to 1$

Step 2 $A_1 \to A_1A_1$ is not in proper form.
Get a new variable Z_1 and productions $A_1 \to A_2A_1A_3Z_1 \mid A_2A_3Z_1$, $Z_1 \to A_1$, $Z_1 \to A_1Z_1$.
The new grammar is
$G_2 = (\{A_1, A_2, A_3, Z_1\}, \{a, b\}, P_2, A_1)$
where P_2 consists of
$A_1 \to A_2A_1A_3 \mid A_2A_3 \mid A_2A_1A_3Z_1 \mid A_2A_3Z_1$
$Z_1 \to A_1Z_1$, $Z_1 \to A_1$, $A_2 \to 0$, $A_3 \to 1$

Step 3 As A_3 productions & A_2 productions are in proper form we have to modify A_1 productions. So modified A_1 productions are
$A_1 \to 0A_1A_3 \mid 0A_3 \mid 0A_1A_3Z_1 \mid 0A_3Z_1$

Step 4 The productions $Z_1 \to A_1$ & $Z_1 \to A_1Z_1$ are modified. They are:
$Z_1 \to 0A_1A_3 \mid 0A_3 \mid 0A_1A_3Z_1 \mid 0A_3Z_1$
$Z_1 \to 0A_1A_3Z_1 \mid 0A_3Z_1 \mid 0A_1A_3Z_1Z_1 \mid 0A_3Z_1Z_1$

Thus the required grammar in GNF is

$G_3 = (\{A_1, A_2, A_3, Z_1\}, \{0,1\}, P_3, A_1)$ where

P_3 consists of

$A_1 \to 0A_1A_3 \mid 0A_3 \mid 0A_1A_3Z_1 \mid 0A_3Z_1$

$A_2 \to 0, \quad A_3 \to 1$

$Z_1 \to 0A_1A_3 \mid 0A_3 \mid 0A_1A_3Z_1 \mid 0A_3Z_1 \mid 0A_3Z_1Z_1 \mid 0A_1A_3Z_1Z_1$

(b) Step 1 Replacing $B \to aSb$ by $B \to aSC$ and $C \to b$

Renaming S, A, B, C by A_1, A_2, A_3, A_4

$\therefore G_1 = (\{A_1, A_2, A_3, A_4\}, \{a, b\}, P_1, A_1)$

where $P_1 \equiv$

$A_1 \to A_2A_3, \quad A_2 \to A_3A_3, \quad A_2 \to A_3A_1A_3, \quad A_2 \to b,$

$A_3 \to aA_2A_4, \quad A_3 \to a, \quad A_4 \to b$

Step 2, 3, 4 Step 2 & 3 are not necessary for G_1.

$\therefore$ we go to step 4. Modified A_2 productions are:

$A_1 \to aA_2A_4A_3A_3 \mid aA_3A_3 \mid aA_2A_4A_1A_3A_3 \mid aA_1A_3A_3 \mid bA_3$

Step 5 All are in GNF

$\therefore G_2 = (\{A_1, A_2, A_3, A_4\}, \{a, b\}, P_2, A_1)$

where P_2: $A_1 \to aA_2A_4A_3A_3 \mid aA_3A_3 \mid aA_2A_4A_1A_3A_3 \mid aA_1A_3A_3 \mid bA_3$

$A_2 \to aA_2A_4A_3 \mid aA_3 \mid aA_2A_4A_1A_3 \mid aA_1A_3 \mid b$

$A_3 \to aA_2A_4 \mid a, \quad A_4 \to b$

4. (a) $S \to a \mid abSb \mid aAb, \quad A \to bS \mid aAAb$

abab has 2 different derivations

$S \xrightarrow{\text{given}} abSb \xrightarrow{S \to a} abab$

$S \to aAb \xrightarrow{A \to bS} abSb \xrightarrow{S \to a} abab$

Hence its ambiguous.

(b) $S \to aB \mid ab$, $A \to aAB \mid a$, $B \to ABb \mid b$

ab has 2 different derivations

$S \to ab$

$S \to aB \to ab$

Hence its ambiguous.

5. (a) $L = \{a^{n^2} \mid n \geq 1\}$

Step 1 Assuming $L = \{a^{n^2} \mid n \geq 1\}$ is context free. Let n be the natural no. which can be obtained by pumping lemma.

Step 2 Let $z = a^{n^2}$

and $z = uvwxy$ where $1 \leq |vx| \leq n$

($\because |vwx| \leq n$ by definition of pumping lemma.)

Let $|vx| = m$ where $m \leq n$. By pumping lemma uv^2wx^2y is in L. Also $|uv^2wx^2y| > n^2$

Let $|uv^2wx^2y| = k^2$ where $k \geq n+1$

Also $|uv^2wx^2y| = n^2 + m < n^2 + 2n + 1$

$\therefore |uv^2wx^2y|$ lies btw n^2 & $(n+1)^2$

Step 3 As shown above $|uv^2wx^2y|$ lies strictly btw squares of two consecutive numbers. Hence our assumption is a contradiction.

$\therefore L = \{a^{n^2} \mid n \geq 1\}$ is not a context free grammar by pumping lemma.

(b) $L = \{a^m b^m c^n \mid m \leq n \leq 2m\}$

Step 1. Assuming $L = \{a^m b^m c^n \mid m \leq n \leq 2m\}$ is a context free grammar. Let n be the natural no. which can be obtained by pumping lemma.

Step 2 : Let $z = a^n b^n c^{2n}$
Then $z = uvwxy$ where $1 \le |vx| \le n$. So vx cannot contain all the 3 symbols a, b and c.
If vx contains only a's & b's then we can choose i such that uv^iwx^iy has more than $2n$ occurences of a (or b) and exactly $2n$ occurences of c. Hence $uv^iwx^iy \notin L$.

Step 3 : Hence our assumption is contradiction.
$\therefore$ L is not a context free grammar by pumping lemma.

(6.) (a) $M = (\{q_0, q_1, q_2\}, \Sigma, \{a, b, z\}, \delta, q_0, z, \{q_2\})$

$\delta(q_0, A, z) = \{(q_2, z)\}$, $\delta(q_0, c, z) = \{(q_0, z)\}$,
$\delta(q_0, c, a) = \{(q_0, a)\}$, $\delta(q_0, c, b) = \{(q_0, b)\}$,
$\delta(q_0, a, z) = \{(q_0, az)\}$, $\delta(q_0, b, z) = \{(q_0, bbz)\}$,
$\delta(q_0, a, a) = \{(q_0, aa)\}$, $\delta(q_0, b, b) = \{(q_0, bbb)\}$,
$\delta(q_0, b, b) = \{(q_0, \Lambda)\}$, $\delta(q_0, b, b) = \{(q_0, b)\}$,
$\delta(q_0, b, a) = \{(q_1, \Lambda)\}$, $\delta(q_1, \Lambda, a) = \{(q_0, \Lambda)\}$

(b) $L = \{wcw^R : w \in \{a, b\}\}$

$M = (\{q_0, q_1, q_2\}, \Sigma, \{a, b, z\}, \delta, q_0, z, \{q_2\})$

$\delta(q_0, a, a) = \{(q_0, aa)\}$, $\delta(q_0, b, a) = \{(q_0, ba)\}$,
$\delta(q_0, a, b) = \{(q_0, ab)\}$, $\delta(q_0, b, b) = \{(q_0, bb)\}$,
$\delta(q_0, a, z) = \{(q_0, az)\}$, $\delta(q_0, b, z) = \{(q_0, bz)\}$,

$\delta(q_0, c, z) = \{(q_1, z)\}$, $\delta(q_0, c, a) = \{(q_1, a)\}$,
$\delta(q_0, c, b) = \{(q_1, b)\}$, $\delta(q_1, a, a) = \{(q_1, \Lambda)\}$,
$\delta(q_1, b, b) = \{(q_1, \Lambda)\}$, $\delta(q_1, \Lambda, z) = \{(q_2, z)\}$

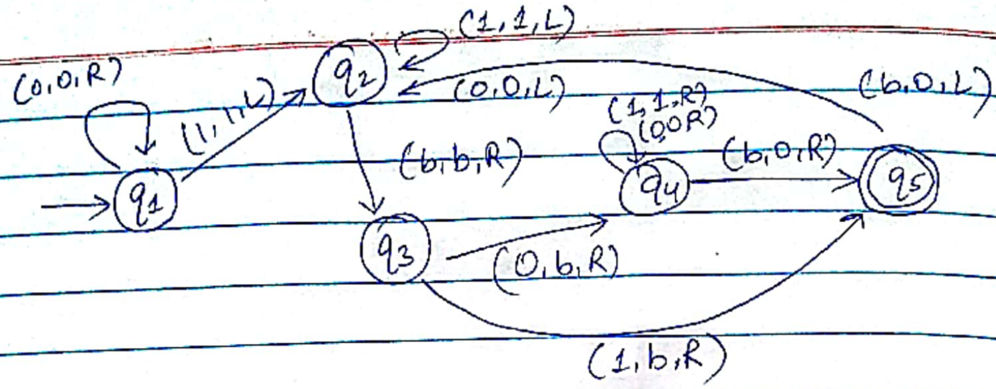
(c) $M = (\{q_0, q_1, q_2, q_3, q_4\}, \Sigma, \{a, b, z\}, \delta, q_0, z, \{q_4\})$

$\delta(q_0, \Lambda, z) = \{(q_1, z)\}$, $\delta(q_0, a, z) = \{(q_0, az)\}$,
$\delta(q_0, a, a) = \{(q_0, aa)\}$, $\delta(q_0, b, a) = \{(q_0, \Lambda)\}$,
$\delta(q_0, b, z) = \{(q_1, bz)\}$, $\delta(q_1, b, a) = \{(q_1, \Lambda)\}$,
$\delta(q_1, b, z) = \{(q_2, bz)\}$, $\delta(q_2, b, b) = \{(q_2, bb)\}$,
$\delta(q_2, c, b) = \{(q_3, \Lambda)\}$, $\delta(q_3, c, b) = \{(q_3, \Lambda)\}$
$\delta(q_3, \Lambda, z) = \{(q_4, \Lambda)\}$

7\. $(q_0, aacaa, z_0) \vdash (q_0, acaa, az_0) \vdash (q_0, caa, aaz_0)$

$\vdash (q_1, a, aaz_0) \vdash (q_1, a, az_0) \vdash (q_1, \Lambda, z_0) \vdash (q_f, \Lambda, z_0)$

(i) Yes. Final ID is (q_f, Λ, z_0)
(ii) Yes. Final ID is (q_1, Λ, az_0)
(iii) No. pda halts at (q_1, ba, az_0)
(iv) Yes final ID is $(q_1, \Lambda, abaz_0)$
(v) Yes. Final ID is $(q_0, \Lambda, babaz_0)$

8. 

10. For 1213: $q_1 1213 \vdash b q_2 213 \vdash bb q_3 13$
But $\delta(q_3, 1)$ is not defined, Turing Machine halts.
For 2133 and 312, the turing machine does not start.