Assignment 2

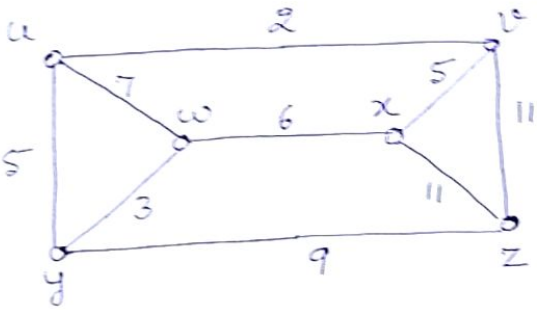
Acad. Session (2021 – 2022)

MC 403

Graph Theory

Q1. A certain tree T of order n contains only vertices of deg. 1 and 3. Show that T contains $(n-2)/2$ vertices of degree 3.

Q2. Apply both Kruskal's and Prim's algorithms to find a minimum spanning tree in the weighted graph given below:



Q3. The tree graph of a connected graph G is the graph whose vertices are the spanning trees $T_1, T_2, \cdots, T_i, \cdots$ of G with T_i and T_j joined iff they have exactly $(n-2)$ edges in common, where $n = |V(G)|$. Show that the tree graph of any connected graph is connected.

Q4. A graph without cycles is a "forest". Show that the size of a forest of order n having k components is $n-k$.

Q5. Let G be a connected weighted graph and T a minimum spanning tree of G. Show that T is unique iff the weight of each edge e of G that is not in T exceeds the weight of every other edge on the cycle in $T+e$.

Q6. Show that for every two integers 2 and t with $2 \leq 2 \leq t$, there exists a connected graph G s.t. 2 is the minimum number of end vertices in a spanning tree and t is the maximum number of end vertices in a spanning tree of G.