GRAPH THEORY
MC-405
CLASS TEST-1

AIMAN SIDDIQUA - 2K18/MC/008.

(1) Let there be K vertices in first component. Therefore the second component will have n-K vertices.

The number of edges in a complete graph with n vertices are $\frac{n(n-1)}{2}$.

So these two components will have $\frac{K(K-1)}{2}$ and $\frac{(n-K)(n-K-1)}{2}$ edges respectively.

$$\text{Total edges} = \frac{K(K-1)}{2} + \frac{(n-K)(n-K-1)}{2}$$

$$m = K^2 - nK + \frac{n}{2}(n-1)$$

Using differentiation to obtain minimum value.

$m' = 2K - n$

$m'' = 2$

Since $m'' > 0$ minimum value of m is at

$2K - n = 0$

$K = \frac{n}{2}$

Putting the value in m.

$$m = \left(\frac{n}{2}\right)^2 - n\left(\frac{n}{2}\right) + \frac{n}{2}(n-1)$$

$$= \frac{n^2}{4} - \cancel{\frac{n^2}{2}} + \cancel{\frac{n^2}{2}} - \frac{n}{2}$$

$$= \frac{n}{2}\left(\frac{n}{2} - 1\right) = \frac{n(n-2)}{4}$$

Hence Proved.

(2.) Havel Hakimi Theorem states that a given sequence is graphical if the following steps dont lead to a negative number:

(i) Sort the given sequence in decreasing order.

(ii) If the first term is k remove it and subtract ~~k~~ 1 from k terms.

Given sequence : 6, 5, 5, 4, 3, 3, 2, 2, 2

Removing 6 : 4, 4, 3, 2, 2, 1, 2, 2

4, 4, 3, 2, 2, 2, 2, 1

Removing 4 : 3, 3, 2, 2, 2, 1, 1, 1

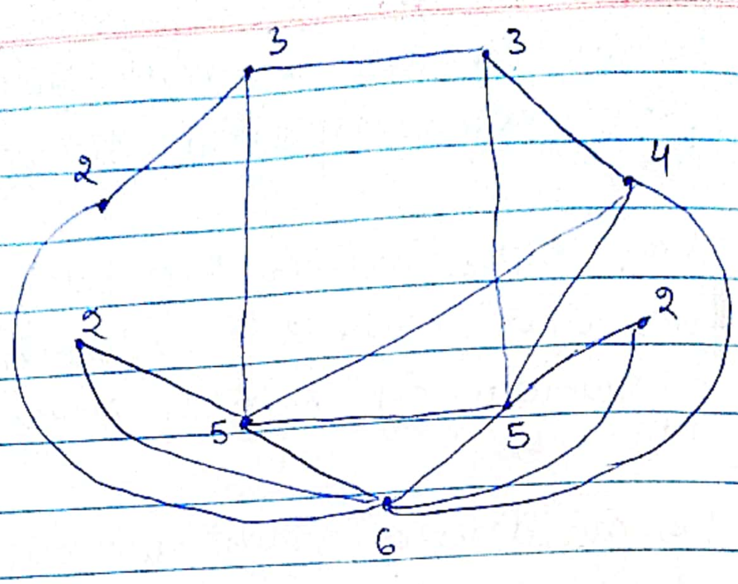
~~Removing 3 : 2, 2, 2, 1, 1, 1, 1~~

Removing 3 : 1, 1, 1, 1, 1, 1, 1

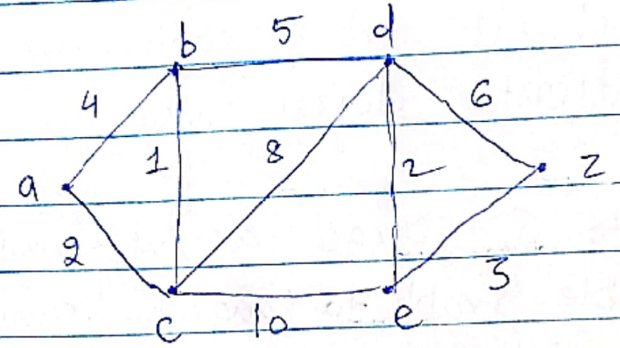
Continuing to remove 1 we get 0 at the end.

It corresponds to a graph with isolated vertex.

Graph :



3.



Starting with a all other vertex at ∞.

	a	b	c	d	e	z	
	0	∞	∞	∞	∞	∞	(Selecting a)
a	-	4	2	∞	∞	∞	(Selecting c)
c	-	3	-	10	12	∞	(Selecting b)
b	-	-	-	8	12	∞	(Selecting d)
d	-	-	-	-	10	14	(Selecting e)
e	-	-	-	-	-	13	(Selecting z)

Path : a - c - b - d - e - z   Cost : 13

4. Let us consider a graph with 10 vertices. These vertices represent 10 people of the committee.

Since every member has sat next to atmost four other members, each member has not previously sat next to atleast five members.

We can draw a graph with edges connecting members who have not sat next to each other. Each vertex will have atleast a degree five.

According to G.A. Dirac a sufficient condition for a simple graph to have a Hamiltonian circuit is that the degree of every vertex be atleast $n/2$.

Here there are 10 vertices and each has atleast degree 5. $\therefore$ It is a hamiltonian circuit. So it can provide a seating arrangement that is required.