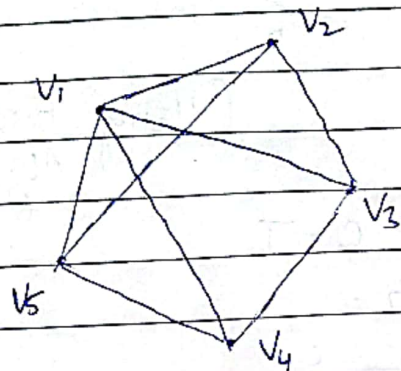


GRAPH THEORY

MC-405

CLASS TEST-3

①



- (i) There are λ choices for v_1 .
- (ii) v_2 and v_4 must be assigned a different color from v_1 .
- (iii) If v_2 and v_4 are assigned the same color then v_3 and v_5 cannot be assigned that color. v_3 and v_5 can have same or diff. colors.
- (iv) If v_2 and v_4 are assigned different colors then v_3 and v_5 cannot be assigned those two colors. v_3 and v_5 can have same or diff colors

$$\begin{array}{ccccccccc} v_1 & v_2 & & v_4 & & v_3 & & v_5 \\ \lambda & (\lambda-1) & & (\lambda-1) & & (\lambda-2) & & (\lambda-2) & + \\ & \lambda & & (\lambda-1) & & (\lambda-1) & & (\lambda-2) & (\lambda-3) & + \end{array}$$

$\begin{array}{ccccc} v_1 & v_2 & & v_4 & & v_3 & & v_5 \end{array}$

v_2 and v_4 different
 v_3 & v_5 same

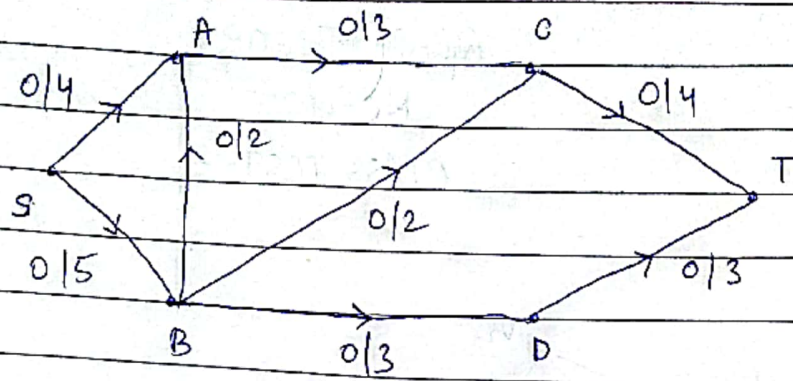
$$\lambda (\lambda-1) (\lambda-2) (\lambda-3) (\lambda-3) +$$

$$\lambda (\lambda-1) (\lambda-2) (\lambda-3) (\lambda-4) \quad \text{All different}$$

$$\lambda (\lambda-1) (\lambda-2) \left[\lambda^2 - 3\lambda + 2 + \lambda^2 - 4\lambda + 3 + \lambda^2 - 6\lambda + 9 + \lambda^2 - 7\lambda + 12 \right]$$

$$\lambda (\lambda-1) (\lambda-2) [4\lambda^2 - 20\lambda + 26]$$

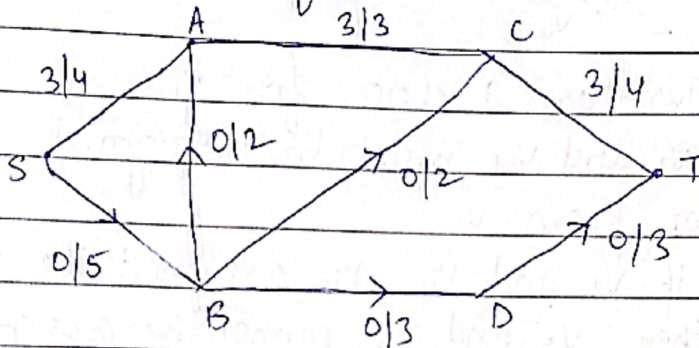
(2)



(Using Ford-Fulkerson Algorithm)

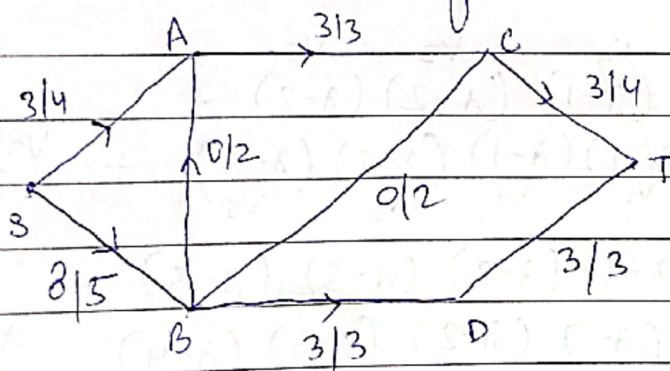
Step 1

Using path S - A - C - T
Bottleneck flow: 3.



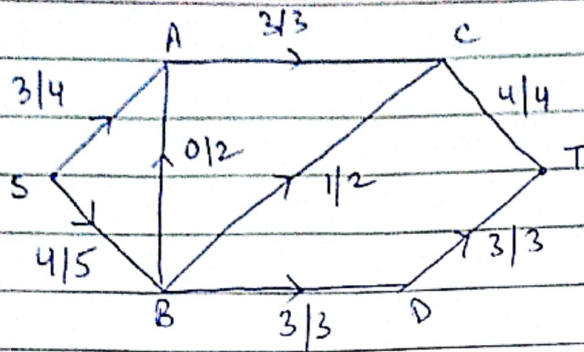
Step 2

Using path S - B - D - T
Bottleneck flow: 3



Step 3

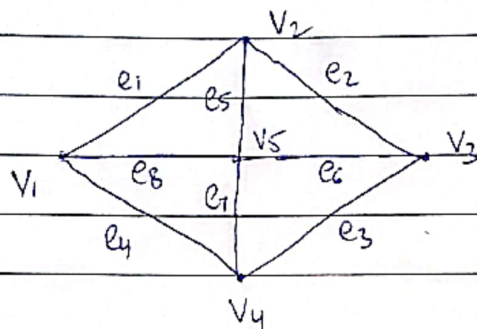
Using path S - B - C - T
Bottleneck flow: 1



Since flow at T is maximum now, the maximum flow is $4 + 3 = 7$.

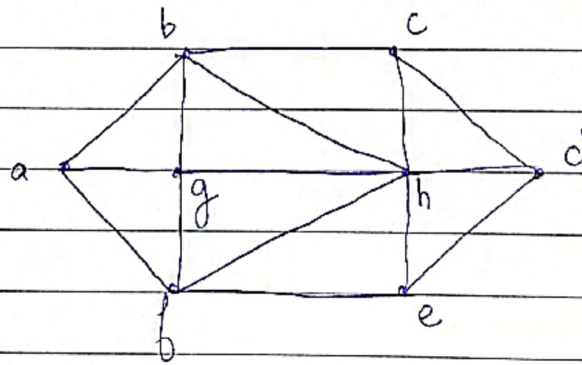
(3)

(a)



Out set : $\{e_2, e_6, e_3\}$

(b)



(i) Matching with 6 vertices : $\{ab, gh, cd\}$

(ii) Perfect matching : $\{ab, gh, cd, fe\}$

(iii) set of edge not a matching : $\{ab, bc, ed\}$

(iv) Maximum matching : $\{ab, gh, cd, fe\}$