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GRAPH THEORY
MC-405
CLASS TEST-2

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(1.) According to theorem,

$$\Sigma d(v_i) = 2 * e$$

where $d(v_i)$ = degree of vertex v_i
e = No. of edges.

Let us suppose there is a tree containing one vertex of deg 1, two vertices of deg 2, 3 vertices of deg 3 ... n vertices of deg n.

$$\Sigma d(v_i) = 1*1 + 2*2 + 3*3 + \cdots + n*n$$

$$\Sigma d(v_i) = \frac{n(n+1)(2n+1)}{6} \longrightarrow (i)$$

(Sum of squares of natural numbers)

According to property a tree containing n vertex contains (n-1) edges.

$$2*e = 2*(n-1) \longrightarrow (ii)$$

from (i) and (ii)

$$\Sigma d(v_i) \neq 2*e$$

Hence no such tree exists.

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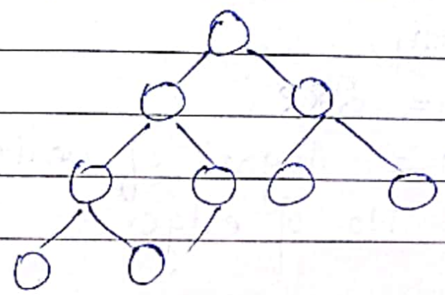
(2.) let us consider a tree containing n vertices.

for minimum height the binary tree should be a complete tree i.e every level should have be completely filled except possibly the last level.



Since there are 1 vertex in level 0, 2 vertex in level 1, 4 vertex in level 3 and so on (powers of 2)

When there are n vertices height will be:
$\lceil \log_2(n+1) - 1 \rceil$

Proof $1 + 2 + 2^2 + \dots 2^h = n$

$$\frac{2^{h+1} - 1}{2-1} = n$$

$$2^{h+1} - 1 = n$$

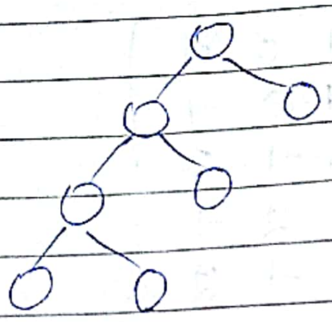
$$2^{h+1} = n+1$$

$$h+1 = \log_2(n+1)$$

$$h = \lceil \log_2(n+1) - 1 \rceil$$

For maximum height each child should have 2 children and only one of those should of further children

Tree can be constructed like



Hence height of tree $= \frac{(n-1)}{2}$

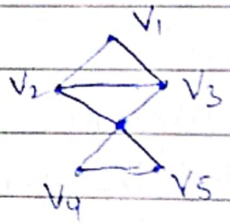
<u>Proof</u> for $n=1$ $h = \frac{1-1}{2} = 0$ O

Lets suppose $n=K$ has $\frac{K-1}{2}$ height

For $n = K+1$ we will add node in such a manner that height increases by 1

$$h = \frac{K-1}{2} + 1 = \frac{K-1+2}{2} = \frac{K+1}{2}$$

(3.)



Adj matrix

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Using Kirchoffs theorem

$$M = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ -1 & -1 & 3 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & -1 & 0 & -1 & 2 \end{bmatrix}$$

$$\text{cof}(a_{11}) = (-1)^{1+1} \begin{vmatrix} 3 & -1 & 0 & -1 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{vmatrix}$$

$$3 \begin{vmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{vmatrix} + 1 \begin{vmatrix} -1 & -1 & 0 \\ 0 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix} + 1 \begin{vmatrix} -1 & 3 & -1 \\ 0 & -1 & 2 \\ -1 & 0 & -1 \end{vmatrix}$$

$$3\left[3(4-1)+1(-2)\right] + \left[-1(4-1)+(-1)\right] + \left[-1(1)-3(2) -1(-1)\right]$$

$$= 3(9-2) + (-4) - 6 = 21 - 10 = 11$$

(4) If some edge of a connected graph G belongs to every spanning tree of G then that edge is a bridge of the Graph

An edge will belong to every spanning tree iff removing that edge will make the graph disconnected. Otherwise that edge can be safely removed from the Graph to form a spanning tree.