AIMAN SIDDIQUA - 2K18/MC/008

GRAPH THEORY

MC-405

ASSIGNMENT-2

1. Given: Tree of order n containing vertices of only deg 1 and 3.

Let the number of vertices of deg 1 be x.

$\therefore$ number of vertices of deg 3 will be $n-x$

Using the theorem

$$\sum d(v_i) = 2*e \quad (e = \text{No. of edges})$$

$$x + 3*(n-x) = 2*(n-1)$$

{ A tree has $n-1$ edges }

$$x + 3n - 3x = 2n-2$$

$$3n - 2x = 2n-2$$

$$n = 2x-2$$

$$x = \frac{n+2}{2}$$

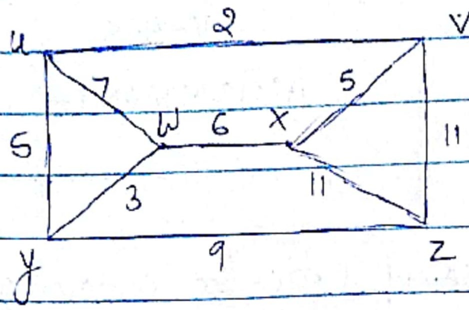
Number of vertices of deg 3 $= n-x$

$$= n - \frac{n+2}{2}$$

$$= \frac{2n-n-2}{2}$$

Vertices of deg 3 $= \frac{n-2}{2}$

(2) Given graph:



KRUSKALS ALGORITHM

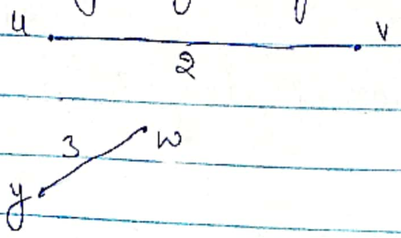
(i) Sorting edges in non-decreasing order of weight.

2	(u-v)
3	(w-y)
5	(u-y)
5	(v-x)
6	(w-x)
7	(u-w)
9	(y-z)
11	(x-z)
11	(v-z)

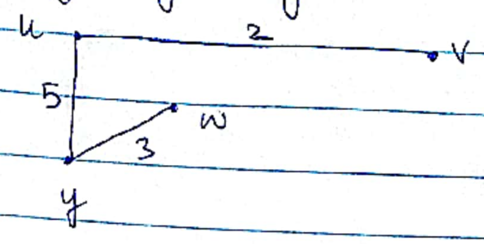
(ii) Picking edges one by one so no cycle is formed.

- Picking edge u-v (2)

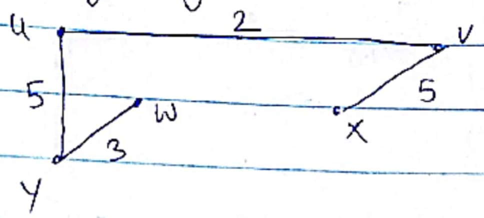
- Picking edge w-y (3)



- Picking edge u-y (5)



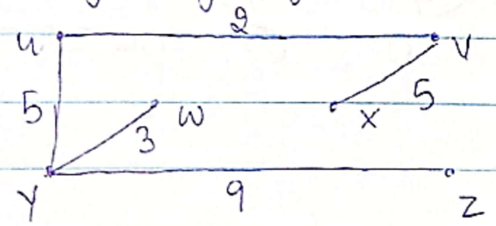
- Picking edge v-x (5)



- Discarding edge w-x (6) because it forms a cycle.

- Discarding edge u-w (7) because it forms a cycle

- Picking edge y-z (9)



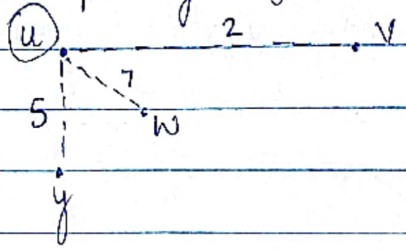
The number of edges in the above graph is one less than the number of vertices.
Hence it is the minimum spanning tree of the given graph.

PRIMS ALGORITHM.

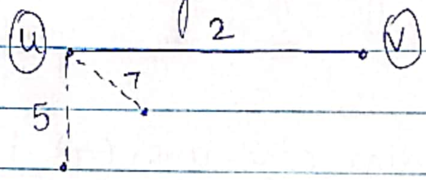
Step 1 : Choosing u

u.

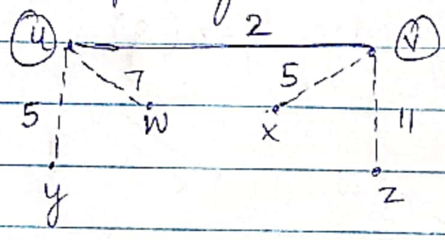
Step 2 : Updating adjacent vertices of u.



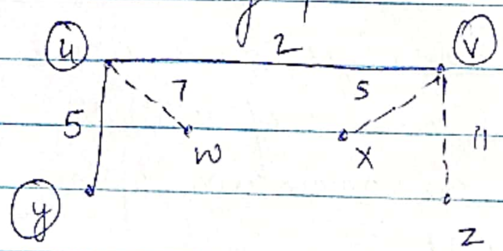
Step 3 : Choosing v-u



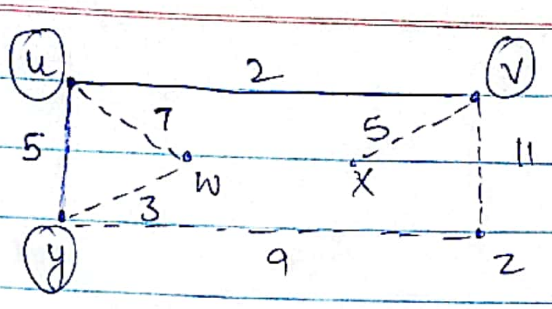
Step 4 : Updating vertices adjacent to v.



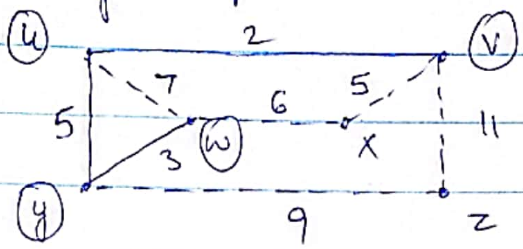
Step 5 : Choosing y-u



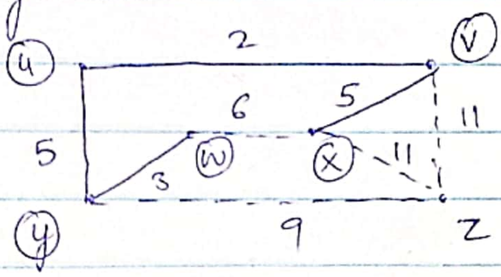
Step 6 : Updating vertices adjacent to y



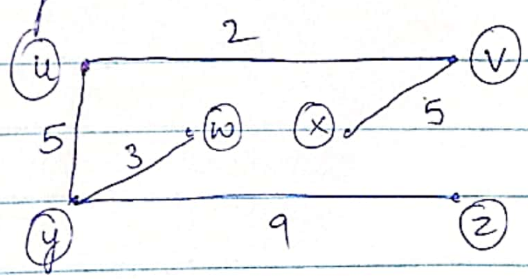
- Choosing w-y



- Choosing x-v



- Choosing z-x



The above is the MST of the given graph.

Cost = 2+ 5+ 3 + 5 + 9 = 24

(3.) Let us consider a Graph G with n vertices. Construct a random spannig tree of the graph. It will have $(n-1)$ edges. We can construct another spanning tree of the graph by removing an edge from the previous spannig tree and adding a different edge. These two spanning tree will have $(n-2)$ edges common and 1 edge different. We can construct multiple spanning trees in this manner and each will have $(n-2)$ edges common with the previous ones.

The given condition is that spanning trees are connected if they have $(n-2)$ edges in common. All trees formed in this manner will be connected.

Let us now consider that there are 2 disconnected sets of spanning trees which are connected among themselves. Lets suppose one ST from first set has 2 edges different from a ST from second set. We can remove an edge from the first ST and add the uncommon edge from the second ST to form a new ST that will have $(n-1)$ edges common with both the ST (one from first set and one from second set). Hence this new spanning tree will be connected with both sets and therefore the sets can be connected using it.

In a similar fashion we can reduce 3 different edges to 2 different edges and then to one.

Hence tree graph of a connected graph is connected.

(4) Let us suppose the k components have order $n_1, n_2 \ldots n_k$.
Here $n_1 + n_2 + \cdots + n_k = n$

Since each component is a tree, it will have $n_i - 1$ edges. Thus the total number of edges is

$$\sum_{i=1}^{k} (n_i - 1) = \sum_{i=1}^{k} n_i - \sum_{i=1}^{k} 1$$

$$= (n_1 + n_2 + \cdots n_k) - k$$

$$= n - k$$

Hence size of forest of order n having k components is $n-k$.

(5.) Let us assume the minimum spanning tree is not unique and there exists another MST T'.
Since T and T' differ there is atleast one edge that belongs to one but not the other. Let e_1 be this edge with the least weight among all. Lets suppose e_1 is in T.
As T' is a MST, $\{e_1\} \cup T'$ must contain a cycle C.
As a tree, T contains no cycle therefore C must have an edge e_2 that is not in T.
Since e_1 was chosen as the lowest weight edge among those belonging to exactly one of T and T', the weight of e_2 must be greater than the weight of edge e_1.
Replacing e_2 with e_1 in T' therefore yields a spanning tree with a smaller weight.
This contradicts the assumption that T' is a MST.

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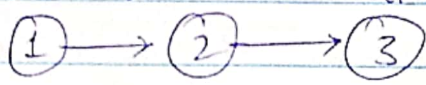
If e_2 was equal to or less than e_1 than T' would be a MST and ~~it~~ wont be unique.
Hence no edge should be greater in weight than edges in T for it to be unique.

(6.) For any 2 integers r and t, with $2 \le r \le t$, we can construct a graph (connected) such that r is the minimum pendant vertices in spanning tree of G and t is the maximum pendant vertices.

To show this we begin with the base case, $2 = r = t$, we have G, a graph with three vertices:



In the spanning tree of this the min and max no. of end vertices will always be 2.

Now we will show this for any $2 \le r \le t$.
We begin with a k connected graph with $t - r + 2$ vertices. The minimum number of end vertices in a spanning tree will be 2 and maximum end vertices will be $t - r + 1$.
Now we can connect $r - 1$ pendant vertices to any vertex in this graph.
The resulting graph G is the required graph.