MC 405 (Graph Theory)

Academic Session : 2024-25

Assignment - 1

Q1. A graph G has adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(a) Is G a simple graph?

(b) What is the degree sequence of G?

(c) How many edges does G have?

Q2. If δ and Δ are respectively the minimum and maximum of the degrees of a graph G, show that $\delta \leq \frac{2m}{n} \leq \Delta$ where G is (n, m) graph.

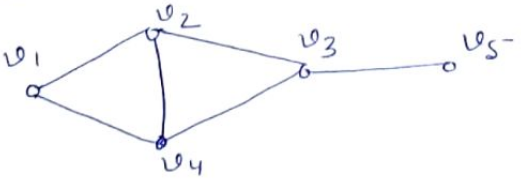
Q3. Show that every simple graph of order n is isomorphic to a subgraph of the complete graph with n vertices.

Q4. Show that the complement of a bipartite graph need not be a bipartite graph.

Q5. Prove that every graph G has a path of length $\delta(G)$.

Q6. Construct three non-isomorphic spanning subgraphs of G shown below:

G:



Q7. Prove that for any nontrivial connected graph G,

$$rad(G) \leq diam(G) \leq 2\, rad(G)$$

Q8. If G is a graph of order $n \geq 3$ s.t. $deg(v) \geq n/2 \quad \forall \; v \in V(G)$, Then prove that G is Hamiltonian.

Q9. Let u and v be nonadjacent vertices in a graph of order n s.t. $deg(u) + deg(v) \geq n$. Then $G + uv$ is Hamiltonian iff G is Hamiltonian.

Q10. Apply Dijkstra's algorithm to find the shortest path from a to f for the graph given below: