Page No.
Date :

AIMAN SIDDIQUA - 2K18/MC/008

GRAPH THEORY

MC-405

ASSIGNMENT-3

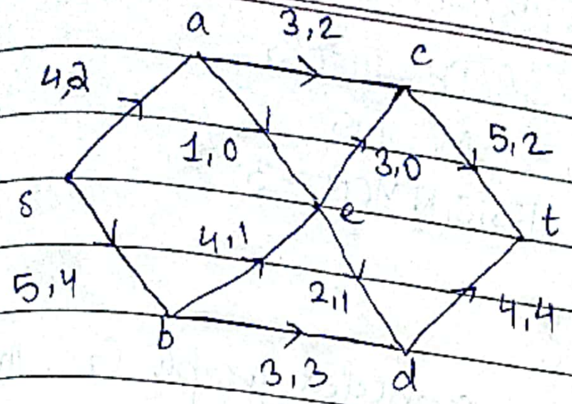
1. If v is a cut vertex of a connected graph G, then G will be disconnected by removing v. Let U and V be the two components on removing v. Let $x \in U$ and $y \in V$. Since U and V are separate components on removing v, there exist no edge between x and y.

Now $\bar{G}$ will contain an edge between x and y since its a component of G. Hence removing v wont disconnect the graph $\bar{G}$. Therefore v wont be a cut vertex of $\bar{G}$. Hence Proved.

2. Let us suppose we color a vertex v with color 1. As each vertex is adjacent to vertices colored with only one of the three colors, we can color all the adjacent vertices of v with a color 2. In a proper K-coloring of a graph each vertex is adjacent to at most (K-1) colors. Hence its possible to color all vertices of G using only 2 colors such that no adjacent vertices have the same color. Therefore $\chi(G) = 2$.

3. Law of conservation of flow for any intermediate vertex x states total flow into x equal to total flow out of x.

$$\sum_{w \in V} f(w,x) = \sum_{v \in V} f(x,v)$$



(i) The flow into a $= \sum_{v \in V} f(v,a) = f(s,a) = 2$

The flow out of a $= \sum_{v \in V} f(a,v) = f(a,c) + f(a,e) = 2+0 = 2$

$$\sum_{v \in V} f(v,a) = \sum_{v \in V} f(a,v)$$

The flow into d $= \sum_{v \in V} f(v,d) = f(b,d) + f(e,d) = 3+1 = 4$

The flow out of d $= \sum_{v \in V} f(d,v) = f(d,t) = 4$

$$\sum_{v \in V} f(v,d) = \sum_{v \in V} f(d,v)$$

The flow into e $= \sum_{v \in V} f(v,e) = f(a,e) + f(b,e) = 0+1 = 1$

The flow out of e $= \sum_{v \in V} f(e,v) = f(e,c) + f(e,d) = 0+1 = 1$

$$\sum_{v \in V} f(v,e) = \sum_{v \in V} f(e,v)$$

(ii) $\operatorname{val}(f) = \sum_{v \in V} f(s,v) = f(s,a) + f(s,b)$

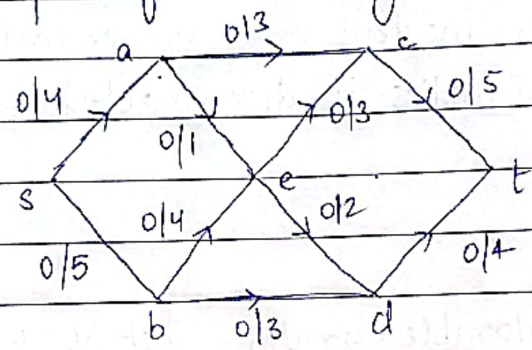
$= 2+4 = 6.$

$= \sum_{v \in V} f(v,t) = f(c,t) + f(d,t) = 2+4 = 6.$

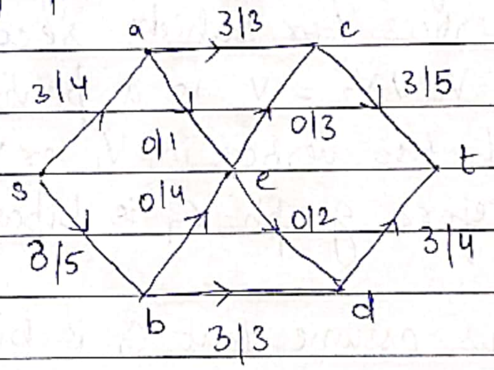
(iii) Capacity of (s-t) cut defined by $S = \{s, a, b\}$ and $T = \{c, d, e, t\}$:

$$c(a,c) + c(a,e) + c(b,e) + c(b,d)$$
$$= 3 + 1 + 4 + 3 = 11$$

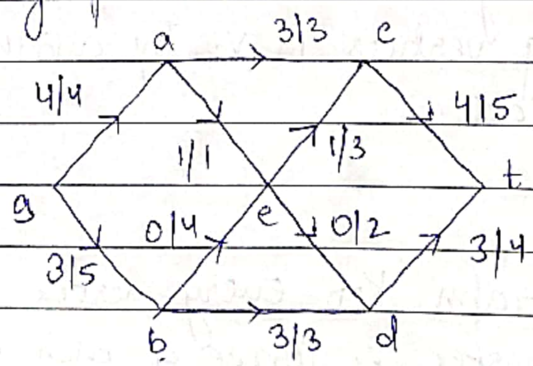
(iv) Using ford fulkerson algorithm on the following graph:



Taking path s-a-c-t and s-b-d-t



Taking path s-a-e-c-t



Flow to t is 4+3 = 7.

Given flow to t is 6. Hence its not maximum.

Page No.
Date :

4. A graph is called non-separable if it does not contain cut vertex and subset E' of E is called cut set if deletion of all the edges from E' makes the graph disconnected.

Let $u_i, u_{i+1} \dots u_n$ be the vertices adjacent to a vertex v. Let us ~~also~~ remove edges $u_iv, u_{i+1}v \dots u_nv$. The graph becomes disconnected into two components. Hence removal of edges incident on a vertex makes it disconnected. These set of edges makes it disconnected which is known as cut-set.

5. Let G be a bicolorable graph. Let V_1 be the set of vertices for which first color is assigned and V_2 be the set of all vertices for which second color is assigned. Then $V_1 \cup V_2 = V$ is a partition of V in G. Otherwise at least two vertices in V_1 or V_2 have the same color. Therefore graph G is bipartite.

Conversely, let us assume that G is bipartite. Let (V_1, V_2) be the partition of V in G. Then a 2-coloring for G can be given by coloring the vertex in V_1 by one color and the remaining vertices in V_2 by another color. Hence G is bicolorable.

6. In a complete graph K_{2n} every vertex is connected to every other vertex. $\therefore$ degree of each vertex is $2n-1$.

For a_1 i.e K_2 the answer is 1. (•—•)

Now in K_{2n} we can pick a vertex v and match it with (2n-1) vertices. After matching v we are left with (2n-2) vertices.

Therefore
$$a_n = (2n-1)\,a_{n-1}$$
$$= (2n-1)(2n-3)\,a_{n-2}$$
$$\vdots$$
$$= (2n-1)(2n-3)\ldots 5.3.1$$

$$= \frac{((2n-1)(2n-3)\ldots 5.3.1) \times (2n \times (2n-2) \times \ldots 4 \times 2)}{2n \times (2n-2) \times \ldots \times 4 \times 2}$$

$$= \frac{(2n)!}{2^n \; n \times (n-1) \ldots 2 \times 1}$$

$$\boxed{a_n = \frac{(2n)!}{2^n \times n!}}$$

7. Using theorem :- If every vertex of a Graph G with n vertices has degree $d \geq n/2$ then G is a Hamiltonian Graph. Hence G has a Hamitonian cycle. (Cycle containing all vertices of G)

Taking every second edge of this cycle yields a perfect matching.

Hence a graph G with |V| even and each vertex with degree $d \geq |V|/2$ has a perfect matching.