Generative Adversarial Nets

Ian J. Goodfellow,* Jean Pouget-Abadie,† Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair,‡ Aaron Courville, Yoshua Bengio§
Département d'informatique et de recherche opérationnelle
Université de Montréal
Montréal, QC H3C 3J7

Abstract

判别模型

We propose a new framework for estimating generative models via an adversarial process, in which we simultaneously train two models: a generative model G that captures the data distribution, and a discriminative model D that estimates the probability that a sample came from the training data rather than G. The training procedure for G is to maximize the probability of D making a mistake. This framework corresponds to a minimax two-player game. In the space of arbitrary functions G and D, a unique solution exists, with G recovering the training data distribution and D equal to $\frac{1}{2}$ everywhere. In the case where G and D are defined by multilayer perceptrons, the entire system can be trained with backpropagation. There is no need for any Markov chains or unrolled approximate inference networks during either training or generation of samples. Experiments demonstrate the potential of the framework through qualitative and quantitative evaluation of the generated samples.

这里生成模型是用来获取数据分布的。判别模型是用来估计数据样本来自训练数据而不是G的概率。G的训练过程就是最大化D判别错误的概率（也就是去生成尽可能真实的训练数据）。这个结构对应一个最小，最大化两个玩家的游戏。对于任意G和D的函数空间，存在一个唯一解，也就是G覆盖了整个训练集，而D的判别概率为1/2.（也就是G在模拟生成训练数据，而D从训练数据中取样并判别其来源）

多层感知机

1 Introduction

The promise of deep learning is to discover rich, hierarchical models [2] that represent probability distributions over the kinds of data encountered in artificial intelligence applications, such as natural images, audio waveforms containing speech, and symbols in natural language corpora. So far, the most striking successes in deep learning have involved discriminative models, usually those that map a high-dimensional, rich sensory input to a class label [14, 20]. These striking successes have primarily been based on the backpropagation and dropout algorithms, using piecewise linear units [17, 8, 9] which have a particularly well-behaved gradient . Deep *generative* models have had less of an impact, due to the difficulty of approximating many intractable probabilistic computations that arise in maximum likelihood estimation and related strategies, and due to difficulty of leveraging the benefits of piecewise linear units in the generative context. We propose a new generative model estimation procedure that sidesteps these difficulties. [1]

piecewise：分段
primarily：主要的

深度生成模型的影响较小，因为在极大似然估计和相关策略中难以逼近许多难以对付（计算）的或然计算，而且由于在生成上下文中难以利用分段线性单元的优点。

In the proposed *adversarial nets* framework, the generative model is pitted against an adversary: a discriminative model that learns to determine whether a sample is from the model distribution or the data distribution. The generative model can be thought of as analogous to a team of counterfeiters, trying to produce fake currency and use it without detection, while the discriminative model is analogous to the police, trying to detect the counterfeit currency. Competition in this game drives both teams to improve their methods until the counterfeits are indistiguishable from the genuine articles.

生成模型可以看成是类似于一批造假者 ，尝试生产假币并且不经检测使用，而判别模型就相当于警察，尝试去检测出这些假币。这个游戏中的竞争驱使生成器和判别器不断提升他们的性能（方法）直到假币不能与真品相区分。

*Ian Goodfellow is now a research scientist at Google, but did this work earlier as a UdeM student
†Jean Pouget-Abadie did this work while visiting Université de Montréal from Ecole Polytechnique.
‡Sherjil Ozair is visiting Université de Montréal from Indian Institute of Technology Delhi
§Yoshua Bengio is a CIFAR Senior Fellow.
[1] All code and hyperparameters available at `http://www.github.com/goodfeli/adversarial`

This framework can yield specific training algorithms for many kinds of model and optimization algorithm. In this article, we explore the special case when the generative model generates samples by passing random noise through a multilayer perceptron, and the discriminative model is also a multilayer perceptron. We refer to this special case as *adversarial nets*. In this case, we can train both models using only the highly successful backpropagation and dropout algorithms [16] and sample from the generative model using only forward propagation. No approximate inference or Markov chains are necessary.

推论，推断

2 Related work

Until recently, most work on deep generative models focused on models that provided a parametric specification of a probability distribution function. The model can then be trained by maximizing the log likelihood. In this family of model, perhaps the most succesful is the deep Boltzmann machine [25]. Such models generally have intractable likelihood functions and therefore require numerous approximations to the likelihood gradient. These difficulties motivated the development of "generative machines"–models that do not explicitly represent the likelihood, yet are able to generate samples from the desired distribution. Generative stochastic networks [4] are an example of a generative machine that can be trained with exact backpropagation rather than the numerous approximations required for Boltzmann machines. This work extends the idea of a generative machine by eliminating the Markov chains used in generative stochastic networks.

使用观察通过生成过程反向传播导数

Our work backpropagates derivatives through generative processes by using the observation that

$$\lim_{\sigma\to 0}\nabla_{\boldsymbol{x}}\mathbb{E}_{\epsilon\sim\mathcal{N}(0,\sigma^2\boldsymbol{I})} f(\boldsymbol{x}+\epsilon) = \nabla_{\boldsymbol{x}} f(\boldsymbol{x}).$$

这里均值为0方差也趋向于零噪声是正态分布的

We were unaware at the time we developed this work that Kingma and Welling [18] and Rezende *et al.* [23] had developed more general stochastic backpropagation rules, allowing one to backpropagate through Gaussian distributions with finite variance, and to backpropagate to the covariance parameter as well as the mean. These backpropagation rules could allow one to learn the conditional variance of the generator, which we treated as a hyperparameter in this work. Kingma and Welling [18] and Rezende *et al.* [23] use stochastic backpropagation to train variational autoencoders (VAEs). Like generative adversarial networks, variational autoencoders pair a differentiable generator network with a second neural network. Unlike generative adversarial networks, the second network in a VAE is a recognition model that performs approximate inference. GANs require differentiation through the visible units, and thus cannot model discrete data, while VAEs require differentiation through the hidden units, and thus cannot have discrete latent variables. Other VAE-like approaches exist [12, 22] but are less closely related to our method.

已经开发了更一般的随机反向传播规则，允许一个通过有限方差的高斯分布反向传播，并且反向传播到协方差参数以及均值。这些反向传播规则可以允许学习生成器的条件方差，在这个文章中我们是以超参数来对待的。

变分自动编码器VAE，像GAN一样对于可微分的生成器网络配置了第二个神经网络。VAE中的第二网络是执行近似推理的识别模型。

GANS需要通过可见单位进行微分，因此不能模拟离散数据，而VAEs需要通过隐藏单元进行微分，因此不能具有离散潜变量。latent(潜在的)

难对付的，难加工的

Previous work has also taken the approach of using a discriminative criterion to train a generative model [29, 13]. These approaches use criteria that are intractable for deep generative models. These methods are difficult even to approximate for deep models because they involve ratios of probabilities which cannot be approximated using variational approximations that lower bound the probability. Noise-contrastive estimation (NCE) [13] involves training a generative model by learning the weights that make the model useful for discriminating data from a fixed noise distribution. Using a previously trained model as the noise distribution allows training a sequence of models of increasing quality. This can be seen as an informal competition mechanism similar in spirit to the formal competition used in the adversarial networks game. The key limitation of NCE is that its "discriminator" is defined by the ratio of the probability densities of the noise distribution and the model distribution, and thus requires the ability to evaluate and backpropagate through both densities.

这些方法对于深度模型来说是难以近似的，因为他们涉及到概率比，这个概率比不能通过变分近似来逼近，因为会降低概率界限。

使用一个先前训练好的模型来作为噪声分布，可以训练出更好的品质的模型。（使用先前训练的模型作为噪声分布，允许训练一系列模型以提高质量。）

NCE限制是它的判别器通过噪声分布和模型分布的概率密度比来定义，因此需要评估和通过密度反向传播的能力。

Some previous work has used the general concept of having two neural networks compete. The most relevant work is predictability minimization [26]. In predictability minimization, each hidden unit in a neural network is trained to be different from the output of a second network, which predicts the value of that hidden unit given the value of all of the other hidden units. This work differs from predictability minimization in three important ways: 1) in this work, the competition between the networks is the sole training criterion, and is sufficient on its own to train the network. Predictability minimization is only a regularizer that encourages the hidden units of a neural network to be statistically independent while they accomplish some other task; it is not a primary training criterion. 2) The nature of the competition is different. In predictability minimization, two networks' outputs are compared, with one network trying to make the outputs similar and the other trying to make the

可预测性最小化中，一个网络中的影藏单元训练来与第二个网络的输出单元不同。

不同：1.网络之间的竞争是独有的训练标准，并且对于自身网络的训练是充分的。而PM只是一个正则化（器）矩阵，鼓励网络的隐层单元在统计意义上独立，当他们完成某些其他任务时。2.竞争的性质是不同的。在PM中，两个网络的输出进行对比，其中一个网络要使得输出相似，而另外一个尝试使得输出不同。问题中的输出是单个标量。在GAN中，一个网络的输出作为另外一个网络的输入，并且尝试去选择一个输入使得另外一个网络不知道怎么处理。

3.具体的学习过程也是不同的。PM是一个最优化问题，有一个目标函数，学习实现目标函数的最小化。GAN是最小最大化游戏而不是最优化问题，并且有一个值函数，其中一个网络是最小化它，而另一个是是去最大化它。当达到鞍点时即关于一个玩家的策略的最小值，并且相对于另一个玩家的策略来说是最大的。

outputs different. The output in question is a single scalar. In GANs, one network produces a rich, high dimensional vector that is used as the input to another network, and attempts to choose an input that the other network does not know how to process. 3) The specification of the learning process is different. Predictability minimization is described as an optimization problem with an objective function to be minimized, and learning approaches the minimum of the objective function. GANs are based on a minimax game rather than an optimization problem, and have a value function that one agent seeks to maximize and the other seeks to minimize. The game terminates at a saddle point that is a minimum with respect to one player's strategy and a maximum with respect to the other player's strategy.

有趣迷人的

Generative adversarial networks has been sometimes confused with the related concept of "adversarial examples" [28]. Adversarial examples are examples found by using gradient-based optimization directly on the input to a classification network, in order to find examples that are similar to the data yet misclassified. This is different from the present work because adversarial examples are not a mechanism for training a generative model. Instead, adversarial examples are primarily an analysis tool for showing that neural networks behave in intriguing ways, often confidently classifying two images differently with high confidence even though the difference between them is imperceptible to a human observer. The existence of such adversarial examples does suggest that generative adversarial network training could be inefficient, because they show that it is possible to make modern discriminative networks confidently recognize a class without emulating any of the human-perceptible attributes of that class.

细微的，感觉不到的

在数据x上学习生成分布Pg，我们定义输入噪声变量上的一个分布Pz(z),然后表示一个到数据空间的映射G(z;thetag),这里G是多层感知机参数是thetag，输出一个向量。我们同样定义另外一个多层感知机D，它输出的是单个标量。D(x)表示x来自样本数据x而不是Pg的概率。我们对D的训练是最大化概率将分别正确的标签放入训练样本和生成样本。

3 Adversarial nets

Pz是生成器的输入的分布
Pg是生成器输出的分布，Pg的目标是逼近x的分布

The adversarial modeling framework is most straightforward to apply when the models are both multilayer perceptrons. To learn the generator's distribution p_g over data $\boldsymbol{x}$, we define a prior on input noise variables $p_{\boldsymbol{z}}(\boldsymbol{z})$, then represent a mapping to data space as $G(\boldsymbol{z};\theta_g)$, where G is a differentiable function represented by a multilayer perceptron with parameters θ_g. We also define a second multilayer perceptron $D(\boldsymbol{x};\theta_d)$ that outputs a single scalar. $D(\boldsymbol{x})$ represents the probability that $\boldsymbol{x}$ came from the data rather than p_g. We train D to maximize the probability of assigning the correct label to both training examples and samples from G. We simultaneously train G to minimize $\log(1-D(G(\boldsymbol{z})))$. In other words, D and G play the following two-player minimax game with value function $V(G,D)$:

同时

$$\min_G \max_D V(D,G) = \mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}(\boldsymbol{x})}[\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z}\sim p_{\boldsymbol{z}}(\boldsymbol{z})}[\log(1-D(G(\boldsymbol{z})))]. \tag{1}$$

D(G(z))就是G(z)被判定为训练样本的概率，而G的目标是最大化它，也就是最小化（1-D（G（z）））

下节，我们做一个竞争网的理论分析。说明训练规则允许网络来覆盖数据并生成分布，当G和D没有参数限制时。

In the next section, we present a theoretical analysis of adversarial nets, essentially showing that the training criterion allows one to recover the data generating distribution as G and D are given enough capacity, i.e., in the non-parametric limit. See Figure 1 for a less formal, more pedagogical explanation of the approach. In practice, we must implement the game using an iterative, numerical approach. Optimizing D to completion in the inner loop of training is computationally prohibitive, and on finite datasets would result in overfitting. Instead, we alternate between k steps of optimizing D and one step of optimizing G. This results in D being maintained near its optimal solution, so long as G changes slowly enough. The procedure is formally presented in Algorithm 1.

不那么正式，教学的，示范的

在实践中，我们必须使用迭代的数值方法来实现游戏。在内循环中完成D的优化是计算上受限制的（禁止的，抑制性的），并且在有限数据上会造成过拟合。相反我们在k步优化D和一步优化G中替换。这让D保持其近似最优解，只要G的变化足够慢。如算法一所示。

饱和

In practice, equation 1 may not provide sufficient gradient for G to learn well. Early in learning, when G is poor, D can reject samples with high confidence because they are clearly different from the training data. In this case, $\log(1-D(G(\boldsymbol{z})))$ saturates. Rather than training G to minimize $\log(1-D(G(\boldsymbol{z})))$ we can train G to maximize $\log D(G(\boldsymbol{z}))$. This objective function results in the same fixed point of the dynamics of G and D but provides much stronger gradients early in learning.

在实际中，等式1不会为G提供充足的梯度来学习（是因为更新的很少吗？）。在学习初期，G是很差的，D可以很容易的拒绝G产生的数据，因为它们和训练数据之间差别太大了。这种情况下log（1-D（G(z)））就饱和了。（这就说明梯度对G不充分，因为刚开始D（G（z））很小，所以log就为0，梯度消失了）

4 Theoretical Results

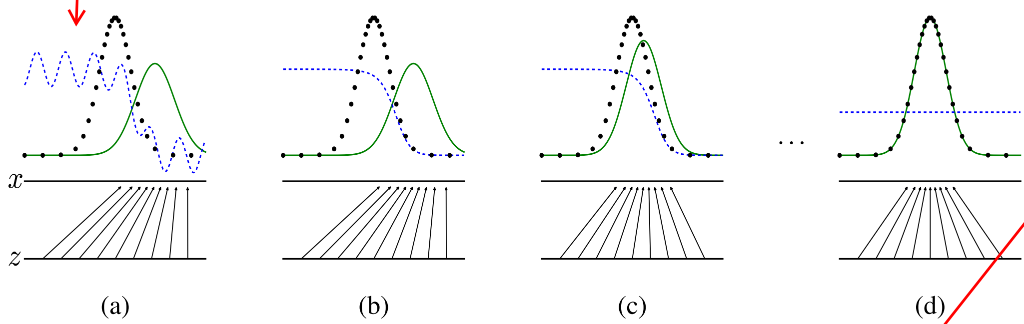
生成器定义一个概率分布Pg作为样本分布，z服从分布Pz。因此给定充足时间和能力，我们希望算法1收敛到一个好的估计器Pdata。

The generator G implicitly defines a probability distribution p_g as the distribution of the samples $G(\boldsymbol{z})$ obtained when $\boldsymbol{z}\sim p_{\boldsymbol{z}}$. Therefore, we would like Algorithm 1 to converge to a good estimator of p_{data}, if given enough capacity and training time. The results of this section are done in a non-parametric setting, e.g. we represent a model with infinite capacity by studying convergence in the space of probability density functions.

这结果是可以实现的，在参数不加设置（限制）时，即我们给模型以无限能力，通过在概率密度函数空间中学习收敛。

We will show in section 4.1 that this minimax game has a global optimum for $p_g = p_{\text{data}}$. We will then show in section 4.2 that Algorithm 1 optimizes Eq 1, thus obtaining the desired result.

从刚开始好区分，后面就不好区分了



GAN被训练，通过同时更新判别分布D（蓝色），以致他能判别样本到底来自哪里。

Figure 1: Generative adversarial nets are trained by simultaneously updating the **d**iscriminative distribution (D, blue, dashed line) so that it discriminates between samples from the data generating distribution (black, dotted line) $p_{\boldsymbol{x}}$ from those of the **g**enerative distribution p_g (G) (green, solid line). The lower horizontal line is the domain from which $\boldsymbol{z}$ is sampled, in this case uniformly. The horizontal line above is part of the domain of $\boldsymbol{x}$. The upward arrows show how the mapping $\boldsymbol{x} = G(\boldsymbol{z})$ imposes the non-uniform distribution p_g on transformed samples. G contracts in regions of high density and expands in regions of low density of p_g. (a) Consider an adversarial pair near convergence: p_g is similar to p_{data} and D is a partially accurate classifier. (b) In the inner loop of the algorithm D is trained to discriminate samples from data, converging to $D^*(\boldsymbol{x}) = \frac{p_{\text{data}}(\boldsymbol{x})}{p_{\text{data}}(\boldsymbol{x})+p_g(\boldsymbol{x})}$. (c) After an update to G, gradient of D has guided $G(\boldsymbol{z})$ to flow to regions that are more likely to be classified as data. (d) After several steps of training, if G and D have enough capacity, they will reach a point at which both cannot improve because $p_g = p_{\text{data}}$. The discriminator is unable to differentiate between the two distributions, i.e. $D(\boldsymbol{x}) = \frac{1}{2}$.

向上的箭头展示了映射x-G(z)怎样将非均匀的分布Pg强迫在转换样本上。

下面的水平线（z），表示z抽样范围，在这个案例中是均匀抽样的。上面的水平线是样本x的领域。

G在高密度区域中收缩，在PG的低密度区域扩展。

Algorithm 1 Minibatch stochastic gradient descent training of generative adversarial nets. The number of steps to apply to the discriminator, k, is a hyperparameter. We used $k = 1$, the least expensive option, in our experiments.

---

for number of training iterations **do**

  for k steps **do**

  • Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \dots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.

  • Sample minibatch of m examples $\{\boldsymbol{x}^{(1)}, \dots, \boldsymbol{x}^{(m)}\}$ from data generating distribution $p_{\text{data}}(\boldsymbol{x})$.

  • Update the discriminator by ascending its stochastic gradient:

梯度上升方向最大化

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[\log D\left(\boldsymbol{x}^{(i)}\right) + \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right)\right].$$

  end for

  • Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \dots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.

  • Update the generator by descending its stochastic gradient:

梯度下降方向，最小化

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right).$$

end for

The gradient-based updates can use any standard gradient-based learning rule. We used momentum in our experiments.

4.1 Global Optimality of $p_g = p_{\text{data}}$

We first consider the optimal discriminator D for any given generator G.

Proposition 1. *For G fixed, the optimal discriminator D is*

$$D^*_G(\boldsymbol{x}) = \frac{p_{data}(\boldsymbol{x})}{p_{data}(\boldsymbol{x}) + p_g(\boldsymbol{x})} \tag{2}$$

Proof. The training criterion for the discriminator D, given any generator G, is to maximize the quantity $V(G, D)$

D是想最大化D(x)，最小化D(G(z))，也就是判别训练样本为训练样本的概率要最大，而判别生成样本为训练样本的概率要最小。所以1-D，变为都是最大化

$$\begin{aligned}V(G, D) &= \int_{\boldsymbol{x}} p_{\text{data}}(\boldsymbol{x}) \log(D(\boldsymbol{x}))dx + \int_{z} p_{\boldsymbol{z}}(\boldsymbol{z}) \log(1 - D(g(\boldsymbol{z})))dz \\ &= \int_{\boldsymbol{x}} p_{\text{data}}(\boldsymbol{x}) \log(D(\boldsymbol{x})) + p_g(\boldsymbol{x}) \log(1 - D(\boldsymbol{x}))dx\end{aligned}$$

For any $(a, b) \in \mathbb{R}^2 \setminus \{0, 0\}$, the function $y \to a \log(y) + b \log(1 - y)$ achieves its maximum in $[0, 1]$ at $\frac{a}{a+b}$. The discriminator does not need to be defined outside of $Supp(p_{\text{data}}) \cup Supp(p_g)$, concluding the proof. □

Note that the training objective for D can be interpreted as maximizing the log-likelihood for estimating the conditional probability $P(Y = y|\boldsymbol{x})$, where Y indicates whether $\boldsymbol{x}$ comes from p_{data} (with $y = 1$) or from p_g (with $y = 0$). The minimax game in Eq. 1 can now be reformulated as:

$$\begin{aligned}C(G) =& \max_D V(G, D) \\ =& \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}}[\log D^*_G(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z} \sim p_{\boldsymbol{z}}}[\log(1 - D^*_G(G(\boldsymbol{z})))] \\ =& \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}}[\log D^*_G(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{x} \sim p_g}[\log(1 - D^*_G(\boldsymbol{x}))] \\ =& \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}}\left[\log \frac{p_{\text{data}}(\boldsymbol{x})}{P_{\text{data}}(\boldsymbol{x}) + p_g(\boldsymbol{x})}\right] + \mathbb{E}_{\boldsymbol{x} \sim p_g}\left[\log \frac{p_g(\boldsymbol{x})}{p_{\text{data}}(\boldsymbol{x}) + p_g(\boldsymbol{x})}\right]\end{aligned} \tag{4}$$

Theorem 1. *The global minimum of the virtual training criterion $C(G)$ is achieved if and only if $p_g = p_{data}$. At that point, $C(G)$ achieves the value $-\log 4$.*

Proof. For $p_g = p_{\text{data}}$, $D^*_G(\boldsymbol{x}) = \frac{1}{2}$, (consider Eq. 2). Hence, by inspecting Eq. 4 at $D^*_G(\boldsymbol{x}) = \frac{1}{2}$, we find $C(G) = \log \frac{1}{2} + \log \frac{1}{2} = -\log 4$. To see that this is the best possible value of $C(G)$, reached only for $p_g = p_{\text{data}}$, observe that

$$\mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}}[-\log 2] + \mathbb{E}_{\boldsymbol{x} \sim p_g}[-\log 2] = -\log 4$$

两个分布越接近，其KL散度就越小。

and that by subtracting this expression from $C(G) = V(D^*_G, G)$, we obtain:

$$C(G) = -\log(4) + KL\left(p_{\text{data}} \middle\| \frac{p_{\text{data}} + p_g}{2}\right) + KL\left(p_g \middle\| \frac{p_{\text{data}} + p_g}{2}\right) \tag{5}$$

where KL is the Kullback–Leibler divergence. We recognize in the previous expression the Jensen–Shannon divergence between the model's distribution and the data generating process:

$$C(G) = -\log(4) + 2 \cdot JSD\left(p_{\text{data}} \| p_g\right) \tag{6}$$

Since the Jensen–Shannon divergence between two distributions is always non-negative, and zero iff they are equal, we have shown that $C^* = -\log(4)$ is the global minimum of $C(G)$ and that the only solution is $p_g = p_{\text{data}}$, i.e., the generative model perfectly replicating the data distribution. □

4.2 Convergence of Algorithm 1

Proposition 2. *If G and D have enough capacity, and at each step of Algorithm 1, the discriminator is allowed to reach its optimum given G, and p_g is updated so as to improve the criterion*

$$\mathbb{E}_{\boldsymbol{x} \sim p_{data}}[\log D^*_G(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{x} \sim p_g}[\log(1 - D^*_G(\boldsymbol{x}))]$$

then p_g converges to p_{data}

Proof. Consider $V(G, D) = U(p_g, D)$ as a function of p_g as done in the above criterion. Note that $U(p_g, D)$ is convex in p_g. The subderivatives of a supremum of convex functions include the derivative of the function at the point where the maximum is attained. In other words, if $f(x) = \sup_{\alpha \in \mathcal{A}} f_\alpha(x)$ and $f_\alpha(x)$ is convex in x for every α, then $\partial f_\beta(x) \in \partial f$ if $\beta = \arg\sup_{\alpha \in \mathcal{A}} f_\alpha(x)$. This is equivalent to computing a gradient descent update for p_g at the optimal D given the corresponding G. $\sup_D U(p_g, D)$ is convex in p_g with a unique global optima as proven in Thm 1, therefore with sufficiently small updates of p_g, p_g converges to p_x, concluding the proof. □

In practice, adversarial nets represent a limited family of p_g distributions via the function $G(\boldsymbol{z}; \theta_g)$, and we optimize θ_g rather than p_g itself, so the proofs do not apply. However, the excellent performance of multilayer perceptrons in practice suggests that they are a reasonable model to use despite their lack of theoretical guarantees.

Model	MNIST	TFD
DBN [3]	138 ± 2	1909 ± 66
Stacked CAE [3]	121 ± 1.6	$\mathbf{2110 \pm 50}$
Deep GSN [5]	214 ± 1.1	1890 ± 29
Adversarial nets	$\mathbf{225 \pm 2}$	$\mathbf{2057 \pm 26}$

Table 1: Parzen window-based log-likelihood estimates. The reported numbers on MNIST are the mean log-likelihood of samples on test set, with the standard error of the mean computed across examples. On TFD, we computed the standard error across folds of the dataset, with a different σ chosen using the validation set of each fold. On TFD, σ was cross validated on each fold and mean log-likelihood on each fold were computed. For MNIST we compare against other models of the real-valued (rather than binary) version of dataset.

5 Experiments

We trained adversarial nets an a range of datasets including MNIST[21], the Toronto Face Database (TFD) [27], and CIFAR-10 [19]. The generator nets used a mixture of rectifier linear activations [17, 8] and sigmoid activations, while the discriminator net used maxout [9] activations. Dropout [16] was applied in training the discriminator net. While our theoretical framework permits the use of dropout and other noise at intermediate layers of the generator, we used noise as the input to only the bottommost layer of the generator network.

We estimate probability of the test set data under p_g by fitting a Gaussian Parzen window to the samples generated with G and reporting the log-likelihood under this distribution. The σ parameter of the Gaussians was obtained by cross validation on the validation set. This procedure was introduced in Breuleux *et al.* [7] and used for various generative models for which the exact likelihood is not tractable [24, 3, 4]. Results are reported in Table 1. This method of estimating the likelihood has somewhat high variance and does not perform well in high dimensional spaces but it is the best method available to our knowledge. Advances in generative models that can sample but not estimate likelihood directly motivate further research into how to evaluate such models. In Figures 2 and 3 we show samples drawn from the generator net after training. While we make no claim that these samples are better than samples generated by existing methods, we believe that these samples are at least competitive with the better generative models in the literature and highlight the potential of the adversarial framework.

6 Advantages and disadvantages

黑体场景

This new framework comes with advantages and disadvantages relative to previous modeling frameworks. The disadvantages are primarily that there is no explicit representation of $p_g(\boldsymbol{x})$, and that D must be synchronized well with G during training (in particular, G must not be trained too much without updating D, in order to avoid "the Helvetica scenario" in which G collapses too many values of $\mathbf{z}$ to the same value of $\mathbf{x}$ to have enough diversity to model p_{data}), much as the negative chains of a Boltzmann machine must be kept up to date between learning steps. The advantages are that Markov chains are never needed, only backprop is used to obtain gradients, no inference is needed during learning, and a wide variety of functions can be incorporated into the model. Table 2 summarizes the comparison of generative adversarial nets with other generative modeling approaches.

The aforementioned advantages are primarily computational. Adversarial models may also gain some statistical advantage from the generator network not being updated directly with data examples, but only with gradients flowing through the discriminator. This means that components of the input are not copied directly into the generator's parameters. Another advantage of adversarial networks is that they can represent very sharp, even degenerate distributions, while methods based on Markov chains require that the distribution be somewhat blurry in order for the chains to be able to mix between modes.

7 Conclusions and future work

This framework admits many straightforward extensions:

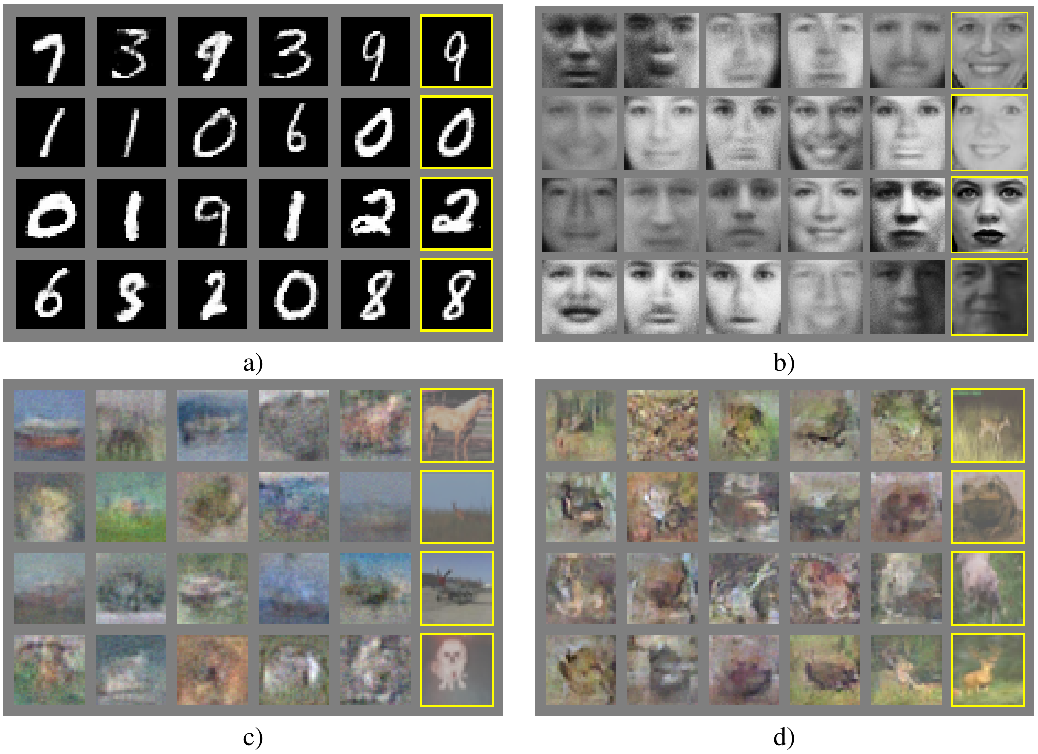


Figure 2: Visualization of samples from the model. Rightmost column shows the nearest training example of the neighboring sample, in order to demonstrate that the model has not memorized the training set. Samples are fair random draws, not cherry-picked. Unlike most other visualizations of deep generative models, these images show actual samples from the model distributions, not conditional means given samples of hidden units. Moreover, these samples are uncorrelated because the sampling process does not depend on Markov chain mixing. a) MNIST b) TFD c) CIFAR-10 (fully connected model) d) CIFAR-10 (convolutional discriminator and "deconvolutional" generator)

Figure 3: Digits obtained by linearly interpolating between coordinates in $\boldsymbol{z}$ space of the full model.

1. A *conditional generative* model $p(\boldsymbol{x} \mid \boldsymbol{c})$ can be obtained by adding $\boldsymbol{c}$ as input to both G and D.
2. *Learned approximate inference* can be performed by training an auxiliary network to predict $\boldsymbol{z}$ given $\boldsymbol{x}$. This is similar to the inference net trained by the wake-sleep algorithm [15] but with the advantage that the inference net may be trained for a fixed generator net after the generator net has finished training.
3. One can approximately model all conditionals $p(\boldsymbol{x}_S \mid \boldsymbol{x}_{\not S})$ where S is a subset of the indices of $\boldsymbol{x}$ by training a family of conditional models that share parameters. Essentially, one can use adversarial nets to implement a stochastic extension of the deterministic MP-DBM [10].
4. *Semi-supervised learning*: features from the discriminator or inference net could improve performance of classifiers when limited labeled data is available.
5. *Efficiency improvements:* training could be accelerated greatly by devising better methods for coordinating G and D or determining better distributions to sample $\mathbf{z}$ from during training.

This paper has demonstrated the viability of the adversarial modeling framework, suggesting that these research directions could prove useful.

	Deep directed graphical models	Deep undirected graphical models	Generative autoencoders	Adversarial models
Training	Inference needed during training.	Inference needed during training. MCMC needed to approximate partition function gradient.	Enforced tradeoff between mixing and power of reconstruction generation	Synchronizing the discriminator with the generator. Helvetica.
Inference	Learned approximate inference	Variational inference	MCMC-based inference	Learned approximate inference
Sampling	No difficulties	Requires Markov chain	Requires Markov chain	No difficulties
Evaluating $p(x)$	Intractable, may be approximated with AIS	Intractable, may be approximated with AIS	Not explicitly represented, may be approximated with Parzen density estimation	Not explicitly represented, may be approximated with Parzen density estimation
Model design	Models need to be designed to work with the desired inference scheme — some inference schemes support similar model families as GANs	Careful design needed to ensure multiple properties	Any differentiable function is theoretically permitted	Any differentiable function is theoretically permitted

Table 2: Challenges in generative modeling: a summary of the difficulties encountered by different approaches to deep generative modeling for each of the major operations involving a model.

Acknowledgments

We would like to acknowledge Patrice Marcotte, Olivier Delalleau, Kyunghyun Cho, Guillaume Alain and Jason Yosinski for helpful discussions. Yann Dauphin shared his Parzen window evaluation code with us. We would like to thank the developers of Pylearn2 [11] and Theano [6, 1], particularly Frédéric Bastien who rushed a Theano feature specifically to benefit this project. Arnaud Bergeron provided much-needed support with LaTeX typesetting. We would also like to thank CIFAR, and Canada Research Chairs for funding, and Compute Canada, and Calcul Québec for providing computational resources. Ian Goodfellow is supported by the 2013 Google Fellowship in Deep Learning. Finally, we would like to thank Les Trois Brasseurs for stimulating our creativity.

References

[1] Bastien, F., Lamblin, P., Pascanu, R., Bergstra, J., Goodfellow, I. J., Bergeron, A., Bouchard, N., and Bengio, Y. (2012). Theano: new features and speed improvements. Deep Learning and Unsupervised Feature Learning NIPS 2012 Workshop.

[2] Bengio, Y. (2009). *Learning deep architectures for AI*. Now Publishers.

[3] Bengio, Y., Mesnil, G., Dauphin, Y., and Rifai, S. (2013). Better mixing via deep representations. In *ICML'13*.

[4] Bengio, Y., Thibodeau-Laufer, E., and Yosinski, J. (2014a). Deep generative stochastic networks trainable by backprop. In *ICML'14*.

[5] Bengio, Y., Thibodeau-Laufer, E., Alain, G., and Yosinski, J. (2014b). Deep generative stochastic networks trainable by backprop. In *Proceedings of the 30th International Conference on Machine Learning (ICML'14)*.

[6] Bergstra, J., Breuleux, O., Bastien, F., Lamblin, P., Pascanu, R., Desjardins, G., Turian, J., Warde-Farley, D., and Bengio, Y. (2010). Theano: a CPU and GPU math expression compiler. In *Proceedings of the Python for Scientific Computing Conference (SciPy)*. Oral Presentation.

[7] Breuleux, O., Bengio, Y., and Vincent, P. (2011). Quickly generating representative samples from an RBM-derived process. *Neural Computation*, **23**(8), 2053–2073.

[8] Glorot, X., Bordes, A., and Bengio, Y. (2011). Deep sparse rectifier neural networks. In *AISTATS'2011*.

[9] Goodfellow, I. J., Warde-Farley, D., Mirza, M., Courville, A., and Bengio, Y. (2013a). Maxout networks. In *ICML'2013*.

[10] Goodfellow, I. J., Mirza, M., Courville, A., and Bengio, Y. (2013b). Multi-prediction deep Boltzmann machines. In *NIPS'2013*.

[11] Goodfellow, I. J., Warde-Farley, D., Lamblin, P., Dumoulin, V., Mirza, M., Pascanu, R., Bergstra, J., Bastien, F., and Bengio, Y. (2013c). Pylearn2: a machine learning research library. *arXiv preprint arXiv:1308.4214*.

[12] Gregor, K., Danihelka, I., Mnih, A., Blundell, C., and Wierstra, D. (2014). Deep autoregressive networks. In *ICML'2014*.

[13] Gutmann, M. and Hyvarinen, A. (2010). Noise-contrastive estimation: A new estimation principle for unnormalized statistical models. In *Proceedings of The Thirteenth International Conference on Artificial Intelligence and Statistics (AISTATS'10)*.

[14] Hinton, G., Deng, L., Dahl, G. E., Mohamed, A., Jaitly, N., Senior, A., Vanhoucke, V., Nguyen, P., Sainath, T., and Kingsbury, B. (2012a). Deep neural networks for acoustic modeling in speech recognition. *IEEE Signal Processing Magazine*, **29**(6), 82–97.

[15] Hinton, G. E., Dayan, P., Frey, B. J., and Neal, R. M. (1995). The wake-sleep algorithm for unsupervised neural networks. *Science*, **268**, 1558–1161.

[16] Hinton, G. E., Srivastava, N., Krizhevsky, A., Sutskever, I., and Salakhutdinov, R. (2012b). Improving neural networks by preventing co-adaptation of feature detectors. Technical report, arXiv:1207.0580.

[17] Jarrett, K., Kavukcuoglu, K., Ranzato, M., and LeCun, Y. (2009). What is the best multi-stage architecture for object recognition? In *Proc. International Conference on Computer Vision (ICCV'09)*, pages 2146–2153. IEEE.

[18] Kingma, D. P. and Welling, M. (2014). Auto-encoding variational bayes. In *Proceedings of the International Conference on Learning Representations (ICLR)*.

[19] Krizhevsky, A. and Hinton, G. (2009). Learning multiple layers of features from tiny images. Technical report, University of Toronto.

[20] Krizhevsky, A., Sutskever, I., and Hinton, G. (2012). ImageNet classification with deep convolutional neural networks. In *NIPS'2012*.

[21] LeCun, Y., Bottou, L., Bengio, Y., and Haffner, P. (1998). Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, **86**(11), 2278–2324.

[22] Mnih, A. and Gregor, K. (2014). Neural variational inference and learning in belief networks. Technical report, arXiv preprint arXiv:1402.0030.

[23] Rezende, D. J., Mohamed, S., and Wierstra, D. (2014). Stochastic backpropagation and approximate inference in deep generative models. Technical report, arXiv:1401.4082.

[24] Rifai, S., Bengio, Y., Dauphin, Y., and Vincent, P. (2012). A generative process for sampling contractive auto-encoders. In *ICML'12*.

[25] Salakhutdinov, R. and Hinton, G. E. (2009). Deep Boltzmann machines. In *AISTATS'2009*, pages 448–455.

[26] Schmidhuber, J. (1992). Learning factorial codes by predictability minimization. *Neural Computation*, **4**(6), 863–879.

[27] Susskind, J., Anderson, A., and Hinton, G. E. (2010). The Toronto face dataset. Technical Report UTML TR 2010-001, U. Toronto.

[28] Szegedy, C., Zaremba, W., Sutskever, I., Bruna, J., Erhan, D., Goodfellow, I. J., and Fergus, R. (2014). Intriguing properties of neural networks. *ICLR*, **abs/1312.6199**.

[29] Tu, Z. (2007). Learning generative models via discriminative approaches. In *Computer Vision and Pattern Recognition, 2007. CVPR'07. IEEE Conference on*, pages 1–8. IEEE.