Can AI be Creative in Mathematics?

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We need to be creative to generate new ideas. By dwelling in the gap between what is known and unknown, we place ourselves in an environment that fosters creativity and may enable us to achieve results beyond the reach of AI models.

1 Why and How to be Creative

We know that our creativity and discoveries originate in a part of our heads called the brain. So, if we want to conduct research, we need to leverage the brain's full potential. Why are some people more creative? Why was Einstein so creative?

Not all of us have identical brains; even though our brain structures are the same and we are born with approximately 86 billion neurons, our brains are wired differently, resulting in unique connections (synapses). Some people are more cautious, while others are more adventurous—traits that can be observed just weeks after birth.

Genetics also influences our potential; some of us are genetically predisposed to slightly faster processing speeds, more efficient memory circuitry, and being more prone to anxiety or having a more resilient mood.

So even though we have the same brain with the same structure, they are wired differently.

Environment and experience can have a crucial impact on our brains. Our brains have a mechanics that allow our brain to adapt when we gain knowledge or skills, encounter something new, or experience profound emotions like awe or grief. This mechanism of the brain is called *neuroplasticity*[1].

Some important facts about neuroplasticity are:

- The brain strengthens the connections that are used frequently, and the reverse of this is also true, that is, the brain prunes away connections that are not used, so "use it or lose it".
- A child raised in an environment of love and rich language will develop a network of connections, while a child who is neglected, malnourished, or experienced trauma has a different network of connections.
- Learning also changes your brain. When you learn mathematics, your prefrontal cortex builds stronger connections.

Thus, if we have a great environment, opportunities to learn new things, good genetics, and a brain rich in neurons and synapses, it increases the likelihood of being creative. However, these are not necessary conditions. Many creative individuals, such as Einstein, lacked some of these factors, yet their creativity led to groundbreaking discoveries that changed human understanding.

We cannot change the past, our genetics, or many other factors that influence creativity. However, we will discuss how to position ourselves intellectually—like Einstein—so that as our brain creates new neurons and neural pathways, these changes enhance our creativity and lead to groundbreaking discoveries, and so we may reach results that AI cannot reach.

In the following, we first define creativity. Then, as we consider AI as a probabilistic decision maker, we present anecdotes from the history of mathematics to illustrate how certain ideas—born from human creativity—emerge in ways that AI might not be able to obtain.

2 What is creativity?

The first question is: what constitutes research or creative work? There is no unanimous definition of creative or research work; however, many refer to it as "the production of something novel and useful." If creativity is the production of something novel, what does novel mean? According to the Oxford Dictionary, "novel" means "different from anything known before; new, interesting, and often seeming slightly strange." Therefore, *creativity* means we should seek out what is unknown and bring it into existence. Since we already understand known ideas, creativity involves discovering facts or creating something previously unknown.

[1]We leave it to the readers to explore how practices such as meditation, exercise, and proper nutrition can help the brain develop new connections and neurons, ultimately increasing their potential for creativity.

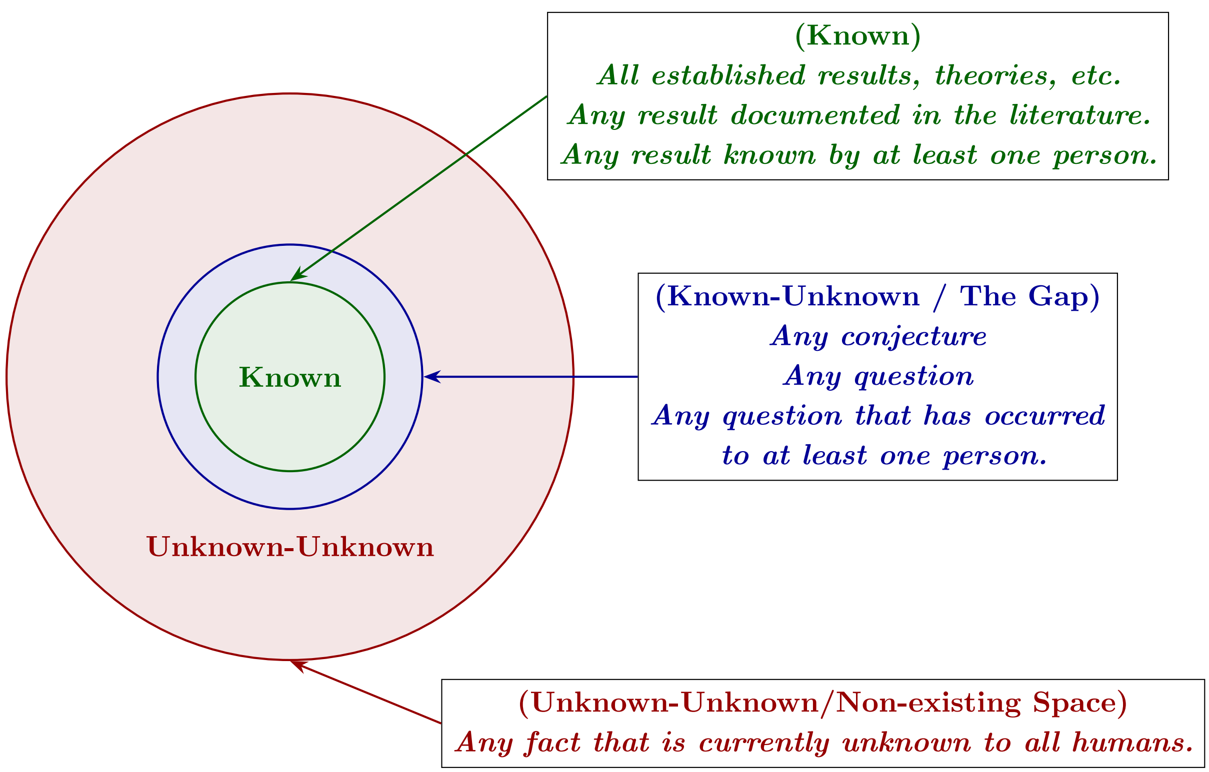
Thus, in mathematics, creativity involves solving problems without predetermined solutions or generating new ideas and theories. To address a problem, we must first understand the existing concepts related to it. Solving a problem allows us to discover something previously unknown. Similarly, developing a new idea or theory in mathematics requires familiarity with established knowledge. Therefore, to introduce original ideas and theories, we must locate ourselves in the gap between what is already known and what remains unknown. This leads us to define the main terminologies used in this note: known, unknown, known-unknown (or the gap), unknown-unknown, as well as existing and non-existing spaces.

The *known* space is the space of anything known before, any results, ideas, theories, methods, etc.

The *known-unknown* or the *gap* is mainly the space of the problems that we know we don't know.

The *unknown-unknown* space includes something that we don't know that we don't know. We call this space the *non-existing* space. So the existence is subject to human knowledge. Anything not in the non-existing space is in the *existing* space.

The *unknown* space includes both known-unknown and unknown-unknown.



So if we want to be creative, we should first learn to stay in the gap, which requires us to know the known ideas and find a question in the gap[2].

In the following section, we share three anecdotes to further clarify our terminology.

3 The gap between the known and the unknown

If we want to be creative, we have to locate ourselves in the gap[3]. Let's look back at history and share anecdotes to better understand this concept.

3.1 Anecdote: What is a set?

At the beginning, we had naive set theory, that is, everything was defined in a somewhat informal way, and they used natural language to describe sets and their operations. For example, in naive set theory, a set can be defined by the assumption of any property without restriction.

Russell learned this idea, and when he learned this idea, and all the ideas about what a set is at that time. So he captured first all the ideas in the known space. Then he locates himself in the gap by asking the question Does this definition lead to a contradiction? Does it lead us to some statement that is both true and false? At that time, the answer was unknown, and it was in the non-existent space. Russell came up with the solution, and he found a paradox in 1901, which is known as Russell's paradox. Thus, Russell's paradox came into existence.

Russell's Paradox

Let $R = \{x | x \notin x\}$. *Then*

$$R \in R \iff R \notin R.$$

Discovering something that was unknown is preceded by a question that did not exist earlier! In Russell's paradox and naive set theory story, the question was whether the naive set theory was consistent, and the answer to this brought something from the unknown to the known.

After 1901, a new question that did not exist came into existence! How to define set theory so it will be consistent such that we don't face something like Russell's paradox. The answer to this question again brings something from the unknown to the known and needs creativity.

[2] All of these processes are done by our brain, so we should learn how to make our brain work optimally so we can learn the known space faster and better, and when we are in the gap, then we could probably find something novel.

[3] In mathematics, when you do a master's or PhD in a subtopic, your supervisor guides you to read the known ideas and gives you a question that locates you in the gap between known and unknown.

3.2 Anecdote: ZFC

After Russell's paradox, the boundary of the known and the unknown changed. At that moment, the known boundary now included that the naive set theory is not consistent, and a question now came into existence that can we give a set theory that is consistent and doesn't face some issues like Russell's paradox? Zermelo and Frankel, from 1908 to 1928, designed a set theory called Zermelo-Frankel set theory that doesn't face some problems like Russell's paradox. That is creativity to take something that is in the unknown space and bring it to existence and make it known.

3.3 Anecdote: Continuum hypothesis, be careful if you stay in the gap for a long time

What is infinity? Infinity is usually considered a quantity bigger than any real number. Then how many numbers are in $\mathbb{Z}$? How many numbers are in $\mathbb{R}$? Are they the same? How mathematicians look at the number of elements in $\mathbb{Z}$ and $\mathbb{R}$?

The number of elements in a set S, denoted by $|S|$, is called the cardinality of that set. In mathematics, instead of looking at how many numbers are in a set and comparing the number of elements, we say two sets have the same cardinality if there is a bijection between those two sets. Georg Cantor proved that there is no such bijection between the set of real numbers $\mathbb{R}$ and the set of integers $\mathbb{Z}$. The cardinality of the set of real numbers is $\aleph_1$, and the cardinality of the set of integers is $\aleph_0$. There are even infinitely many infinities (an infinite hierarchy of aleph numbers: $\aleph_0, \aleph_1, \aleph_2, \dots$).

Now, after Cantor showed that the cardinality of reals and integers are different, that was something unknown that became known. He then asked a question to further push the boundary of the known world. He proposed the following.

Continuum Hypothesis

Any subset of the real numbers is either finite, countably infinite, or has the cardinality of the real numbers. That means there is not a set A such that $\mathbb{Z} \subset A \subset \mathbb{R}$ with $\aleph_0 < |A| < \aleph_1$.

The above proposed problem put the mathematicians at the time in the gap between the known and the unknown.

The continuum hypothesis was proposed by Cantor in 1878, and it was one of Hilbert's 23 problems presented at the International Congress of Mathematics in 1900 in Paris[4].

The work of Cohen and Gödel showed that the continuum hypothesis cannot be solved

[4] Cantor believed that the continuum hypothesis was true, and he tried to prove it. Cantor suffered a lot of mental problems, and he died of a heart attack in a sanatorium on January 6, 1918.

by using ZFC. Gödel also showed that mathematics is incomplete, that is, choosing a set of axioms cannot capture all of the true statements in mathematics! So these are the facts that, with novel methods, came into existence[5].

4 Types of creativity and which one can be captured by AI

We have so far considered being creative if we can find a question that previously didn't exist, and then that helped us expand the boundary of our known space if we can solve that question. Thus, the first step of being creative is having a question. Problems are in two ways in mathematics, either they try to find a relation between known ideas, for example, if a and b are even, then their product is even, or the second type is something that the problem is going to bring our knowledge and ideas to one step further, like if the continuum hypothesis is independent of ZFC.

If the question that we have can be answered with the methods that already exist, then probably AI will capture this area soon; however, if we bring new ideas to existence, make new theories, or make new methods that didn't exist earlier, that is we bring something from unknown-unknown space to the known space, then AI probably cannot do it becasue so far machines are probabilistic decision makers who feed from the known ideas. If our question cannot be answered by known methods and needs a new method that previously didn't exist and was located in the unknown-unknown space, then this would be a great question to consider: Can machines, as probabilistic decision makers fed from known space, solve such a problem? Perhaps this question is as profound as asking whether machines could ever become conscious. Do we even have a clear definition of "consciousness"?

In the future, probably whenever a probelm comes into existence, there will be a computer system that we give it to and see if, in an efficient amount of time, it can answer it or not, and if not, then we can work on it. Or we consider computers as mathematicians that work on that question, and we will compete to see who will be the first to answer it, mathematicians or computers, because we really don't know at the beginning if the answer to a problem lies in the unknown space or the known space. The advantage that we have over computers is that if the method needed to solve the problem is in the unknown space, we may be the only ones who can answer the question, not the computers?

[5] Gödel ultimately died as a result of self-imposed starvation. So be careful if you stay for a long time in the gap between the known and the unknown spaces, especially when a notion of infinity is involved.

5 The brain that evolve in the gap

What made Einstein more creative than others is that he lingered in the gap between known ideas and the unknown. As he said, he isn't smarter than others; he just doesn't give up on the questions. Since different parts of Einstein's brain were connected, perhaps staying in the gap for longer periods leads to more connections and synapses in the brain, which could help solve problems.

References

[1] W. Suzuki, *Healthy Brain, Happy Life: A Personal Program to Activate Your Brain and Do Everything Better*, HarperCollins, (2015).

[2] P. R. Halmos, *Naive Set Theory*, Springer-Verlag, (1974).

[3] H. B. Enderton, *Elements of Set Theory*, Academic Press, New York, (1977).