

Cross relaxation mechanisms in fluorine-containing sample under conditions of DNP.

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1 Cross-relaxation in ^1H - ^{19}F system.

While searching for optimal DNP conditions for DNP in fluorinated samples, we stumbled upon a peculiar phenomenon, where after shutting down the MW irradiation, the ^{19}F magnetization builds up quickly to some higher than equilibrium value, then slowly reaches thermal equilibrium. These observations suggest that ^{19}F nuclei are in contact with some other reservoir of energy, which is colder than lattice. The most obvious candidate for such a reservoir are abundant protons. This fact however, is yet to be fully confirmed. In the following we'll try to take a closer look at the mechanisms, which might be responsible for this transfer process.

2 NOE.

The transfer process is very similar to NOE (nuclear Overhauser effect), which has been observed in solids quite a while ago. The latest reports published by the groups of Corzillius and Buntkowsky, describe NOE-like transfer from protons to carbons under conditions of MAS DNP (magnetic field of 9.4 T and temperature 100 K). Such transfer is induced by the motion of methyl groups, similarly other reports from the early days of NMR (Pines & Waugh etc.) also attribute the transfer to some molecular motion. Our experiments however were carried at much lower temperatures of 1.4 K amplitudes, where motions are quite suppressed. Our main hypothesis is that ^1H to ^{19}F transfer is caused by the presence of paramagnetic TEMPO

radical. When we inadvertently used TEMPOL instead, we discovered that the ^1H to ^{19}F transfer was either significantly suppressed, or disappeared altogether. In order to show that in more detail we need to demonstrate what happens to the cross-polarization effect, when TEMPO concentration varies.

3 Relaxation and Cross-relaxation.

In the following we estimate the rates of cross-relaxation based on the time-dependent perturbation theory (Ch. VIII.II.C on Abragam's book). Consider a two a levelsystem, with $|i\rangle$ - initial, and $|f\rangle$ - final states, and the time-dependent perturbation to a system Hamiltonian $V(t) = \hat{A}f(t)$, where $\overline{f(t)} = 0$. The transfer rate from the $|i\rangle$ - to $|f\rangle$ state can be calculated as:

$$W_{if} = \frac{|A_{if}|^2}{\hbar^2} J(\omega_{if}) \quad (1)$$

where $|A_{if}|$ is the matrix element between the two states, and $J(\omega_{if})$ is the spectral density at the transition frequency $\omega_{if} = \frac{E_f - E_i}{\hbar}$. For a stationary random process with exponentially decaying correlation function, the spectral density has the following form:

$$J(\omega) = \frac{\tau_c}{1 + \omega^2 \tau_c^2}, \quad (2)$$

where τ_c is the correlation time of the random process.

In a three spin system, consisting of an electron and two nuclei, the pairwise interactions between all of them should be taken into account. However, in order to simplify the calculation we split this into two separate cases.

4 One hyperfine and one dipolar coupling

In this section we will consider a cross-relaxation mechanism arising due to rapid changes of nuclear spin quantization axis due to random electron spin flips. Random tilting of quantization axis affects the dipolar interaction between nuclear spins, giving rise to rapidly changing flip-flop terms. In our calculation, we electron is coupled to a nearby proton via a hyperfine interaction, represented by a 2nd rank tensor A_{ij} ($i, j = x, y, z$); the proton in turn is coupled to a nearby

fluorine via a dipolar interaction with components D_{ij} as shown schematically in Figure 1A.

We neglect the hyperfine interactions of ^{19}F nuclei at this stage to simplify the calculation and evaluate the effect of hyperfine and dipolar interaction on relaxation exclusively. The spin Hamiltonian for such coupled three spin system can be written as:

$$\mathcal{H} = \omega_S \hat{S}_z + \sum_{i,j=x,y,z} A_{i,j} \hat{S}_i \hat{H}_j + \omega_H \hat{H}_z + \omega_F \hat{F}_z + \sum_{i,j=x,y,z} D_{ij} \hat{H}_i \hat{F}_j, \quad (3)$$

where $\hat{H}_i, \hat{F}_i, \hat{S}_i$ - are the angular momenta of proton, fluorine and electron spins respectively. Taking into account that electron Zeeman interaction is greater than any other terms, the Hamiltonian in the electron spin rotating frame would take a form:

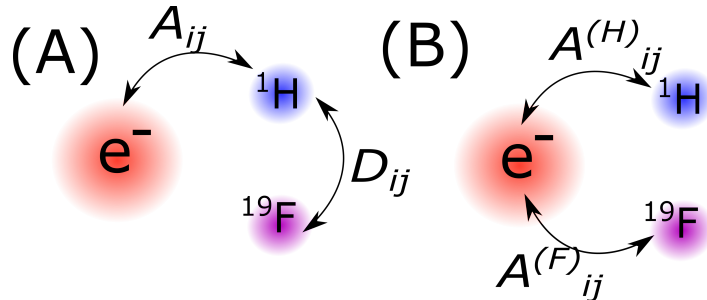
$$\mathcal{H} = A \hat{S}_z \hat{H}_z + B \hat{S}_z \hat{H}_x + D_{i,j} \hat{H}_i \hat{F}_j + \omega_H \hat{H}_z + \omega_F \hat{F}_z + \sum D_{i,j} \hat{H}_i \hat{F}_j, \quad (4)$$

where A and B are the secular and non-secular terms of the hyperfine interaction (term proportional to $\hat{S}_z \hat{H}_y$ was eliminated by a rotation in the spin space). The time-dependence in this Hamiltonian arises due to random flips of an electron spin, having a projection $m_S(t) = \pm \frac{1}{2}$ projection on the z-axis:

$$\mathcal{H} = (\omega_H + A m_S(t)) H_z + B H_x m_S(t) + \omega_F \hat{F}_z + \sum D_{i,j} \hat{H}_i \hat{F}_j \quad (5)$$

The first two terms would collectively correspond to quantization of a nuclear spin along a new effective magnetic field $\tilde{\mathbf{B}} = \frac{1}{\gamma}(\omega_H + A m_S(t), \gamma B, 0)$. By rotating the spin operator around H_y by a small angle $\phi(t) = \arctan(\frac{m_S(t)B}{\omega_H + m_S(t)A}) \approx \frac{m_S B}{\omega_H}$ one can rewrite this part of Hamiltonian along the new axis: $B_z(t) \hat{H}_z + B_x(t) \hat{H}_x = \tilde{\omega}_H(t) \hat{\tilde{H}}_z$. The \hat{H}_y operator generates a rotation in

Figure 1: Interactions used to calculation. (A) Electron spin is hyperfine coupled to a proton, which is dipolar coupled to nearby fluorine. (B) Electron spin is hyperfine coupled to both a proton and a fluorine nuclei.



the spin space, the dipolar part of the Hamiltonian can will then be transformed as:

$$\begin{aligned}\hat{H}_{dip}(t) &= \exp(-i\phi(t)\hat{H}_y) \sum_{i,j=x,y,z} D_{i,j} \hat{H}_i \hat{F}_j \exp(i\phi(t)\hat{H}_y) \approx \\ &\approx \phi(t) \left[\sum_{i,j=x,y,z} D_{i,j} \hat{H}_i \hat{F}_j, \hat{H}_y \right] = \frac{m_S(t)B}{\omega_H} \left[\sum_{i,j=x,y,z} D_{i,j} \hat{H}_i \hat{F}_j, \hat{H}_y \right]\end{aligned}\quad (6)$$

This time dependent Hamiltonian contains terms such as $\hat{H}_\pm \hat{F}_\mp$ and $\hat{H}_\pm \hat{F}_\pm$ is capable of inducing zero quantum (flip-flop) or double-quantum (flop-flop) transitions respectively. We will skip obtaining the exact equation for the rate, but rather focus in evaluating it by the order of magnitude. Using equation 1 we can estimate the zero-quantum transfer rate as:

$$\begin{aligned}W_{ZQ} &\sim \left| \frac{BD}{\omega_H} \right|^2 J(\omega_{ZQ}) \\ J(\omega) &= \frac{\tau_c}{1 + \omega^2 \tau_c^2}\end{aligned}\quad (7)$$

where B - is non-secular proton hyperfine interaction, D - dipolar coupling, ZQ stands for zero-quantum ($\omega_{ZQ} = \omega_H - \omega_F$), τ_c - electron correlation time, which is case is determined by electron spin-diffusion (τ_c should be smaller than 100 μs based on my CPMG measurements outlined in 2012 JMR paper - **this requires more explanations**). At the same time, the electron-induced $W_n = \frac{1}{T_{1n}}$ relaxation of ^{19}F nuclei can be estimated as (Ch. IX.II.A Abragam):

$$W_n \sim |B|^2 J(\omega_F) \quad (8)$$

Since $\omega_F \tau_c \gg 1$ and $\omega_{ZQ} \tau_c \gg 1$, the spectral densities are equal to $J(\omega_F) \approx \frac{1}{\omega_F^2 \tau_c}$ and $J(\omega_{ZQ}) \approx \frac{1}{\omega_{ZQ}^2 \tau_c}$. We can then look that the ratio of two rates, assuming that $\omega_H \approx \omega_F$:

$$\frac{W_{ZQ}}{W_n} \sim \left| \frac{D}{\omega_H} \right|^2 \left(\frac{\omega_F}{\omega_{ZQ}} \right)^2 \sim \left(\frac{D}{\omega_{ZQ}} \right)^2 \quad (9)$$

Since a typical dipolar coupling is $D \sim 100$ kHz and $\omega_{ZQ} \sim 8$ MHz, then $W_{ZQ} \ll W_n$ i.e. the ZQ process is several orders of magnitude slower than nuclear relaxation rate $\frac{1}{T_1}$ due to presence of electrons. Similar argument is true for for a double-quantum process.

5 Electron-induced proton-fluorine flip-flops

Now we will include the hyperfine interactions for both nuclei, while neglecting the dipolar interaction between them as shown schematically in Figure 1B. The Hamiltonian for such a system would therefore be in the following form:

$$\mathcal{H} = \omega_S \hat{S}_z + \sum_{i,j=x,y,z} A_{ij}^{(H)} \hat{S}_i \hat{H}_j + \sum_{i,j=x,y,z} A_{ij}^{(F)} \hat{S}_i \hat{F}_j - \omega_H \hat{H}_z - \omega_F \hat{F}_z, \quad (10)$$

where $A_{ij}^{(H)}, A_{ij}^{(F)}$ are the hyperfine interaction tensors of protons and fluorines respectively. In the electron spin rotating frame, using $A_{H,F}$ and $B_{H,F}$ for secular and non-secular terms of a hyperfine interaction the Hamiltonian can be rewritten as:

$$\mathcal{H} = A_H \hat{S}_z \hat{H}_z + B_H \hat{S}_z \hat{H}_x - \omega_H \hat{H}_z + A_F \hat{S}_z \hat{F}_z + B_F \hat{S}_z \hat{F}_x - \omega_F \hat{F}_z \quad (11)$$

Since $B_H, B_F \ll \omega_H, \omega_F$, the non-secular terms can be regarded as a perturbation to a secular Hamiltonian. Let's denote the eigenstates of unperturbed Hamiltonian as $|k\rangle$: $|1\rangle = |\alpha_H \alpha_F\rangle$, $|2\rangle = |\alpha_H \beta_F\rangle$, $|3\rangle = |\beta_H \alpha_F\rangle$, $|4\rangle = |\beta_H \beta_F\rangle$. In this basis the matrix of perturbation $\hat{V} = m_S B_H \hat{H}_x + m_S B_F \hat{F}_x = \frac{m_S}{2} (B_H (H_+ + H_-) + B_F (F_+ + F_-))$ would have the following elements:

$$V_{ik} = \frac{m_S}{2} \begin{bmatrix} 0 & B_F & B_H & 0 \\ B_F & 0 & 0 & B_H \\ B_H & 0 & 0 & B_F \\ 0 & B_H & B_F & 0 \end{bmatrix} \quad (12)$$

The unperturbed energies will be:

$$\begin{aligned} E_1^0 &= -\frac{\omega_H + \omega_F}{2} + \frac{m_S}{2} (A_H + A_F) \\ E_2^0 &= -\frac{\omega_H - \omega_F}{2} + \frac{m_S}{2} (A_H - A_F) \\ E_3^0 &= \frac{\omega_H - \omega_F}{2} - \frac{m_S}{2} (A_H - A_F) \\ E_4^0 &= \frac{\omega_H + \omega_F}{2} - \frac{m_S}{2} (A_H + A_F) \end{aligned} \quad (13)$$

The perturbation theory would produce the corrections to the eigenfunctions $|\Psi\rangle = |\phi^{(0)}\rangle + |\phi^{(1)}\rangle + |\phi^{(2)}\rangle \dots$, where $|\phi^{(l)}\rangle$ is the l th order correction. The first order perturbation to the

eigenfunction $|k^{(1)}\rangle$ can be written as:

$$|k^{(1)}\rangle = \sum_{l \neq k} \frac{|l^{(0)}\rangle \langle l^{(0)} | \hat{V} | k^{(0)} \rangle}{E_k^{(0)} - E_l^{(0)}} = \sum_{l \neq k} \frac{|l^{(0)}\rangle V_{lk}}{E_k^{(0)} - E_l^{(0)}} \quad (14)$$

Such first order perturbation only produce mixing with functions differing by $m_H = \pm 1, \Delta m_F = 0$, or $m_H = 0, \Delta m_F = \pm 1$ because $V_{23} = V_{14} = 0$. The second order perturbation contains more additional terms:

$$|k^{(2)}\rangle = \sum_{l \neq k, m \neq l} \frac{|l^{(0)}\rangle \langle l^{(0)} | \hat{V} | m^{(0)} \rangle \langle m^{(0)} | \hat{V} | k^{(0)} \rangle}{(E_k^{(0)} - E_l^{(0)})(E_k^{(0)} - E_m^{(0)})} = \sum_{l \neq k, m \neq l} \frac{|l^{(0)}\rangle V_{lm} V_{mk}}{(E_k^{(0)} - E_l^{(0)})(E_k^{(0)} - E_m^{(0)})} \quad (15)$$

This interaction mixes states different by $m_H = \pm 1, \Delta m_F = \mp 1$ and $m_H = \pm 1, \Delta m_F = \pm 1$.

For $|2\rangle = |\alpha_H \beta_F\rangle$, the mixing coefficient for $|3\rangle = |\beta_H \alpha_F\rangle$ can be calculated as:

$$\begin{aligned} c_{23}^{(2)} &= \sum_{m \neq 3} \frac{|l^{(0)}\rangle V_{2m} V_{m3}}{(E_2^{(0)} - E_m^{(0)})(E_3^{(0)} - E_m^{(0)})} = \frac{V_{21} V_{13}}{(E_2^{(0)} - E_1^{(0)})(E_3^{(0)} - E_1^{(0)})} + \frac{V_{24} V_{43}}{(E_2^{(0)} - E_4^{(0)})(E_3^{(0)} - E_4^{(0)})} \approx \\ &\approx \frac{m_S^2}{2} \frac{B_F B_H}{\omega_H \omega_H} \end{aligned}$$

Also we can calculate the second order corrections to energies:

$$E_2^{(2)} = \sum_{k \neq 2} \frac{V_{2k}^2}{E_2^{(0)} - E_k^{(0)}} \approx \left(\frac{m_S}{2}\right)^2 \left(\frac{B_H^2}{\omega_H} - \frac{B_F^2}{\omega_F}\right) \quad (16)$$

$$E_3^{(2)} = \sum_{k \neq 3} \frac{V_{3k}^2}{E_3^{(0)} - E_k^{(0)}} \approx \left(\frac{m_S}{2}\right)^2 \left(\frac{B_H^2}{\omega_H} - \frac{B_F^2}{\omega_F}\right) \quad (17)$$

Although \hat{V} does produce mixing of states 2 and 3, the random jumps of an electron between states with $m_S = \pm \frac{1}{2}$ will not produce relaxation, because $m_S^2 = \frac{1}{4}$ for both. Nevertheless, the presence of such mixing implies that nuclei in the vicinity of an electron can be coupled even if they are spatially remote from one another.

Hyperfine couplings of protons (and fluorines similarly) can be estimated as $B \sim \frac{78}{r^3} \frac{\text{MHz}}{\text{\AA}^3}$, which for a distance of 2 \AA would produce a hyperfine coupling of about 10 MHz. In addition, the protons of nitroxide methyl groups have large anisotropic hyperfine couplings of about 10

MHz (J. Phys. Chem. 1982, 86, 4011). Altogether it means that proton nuclei with coupling on the order of 10 MHz are always present in the vicinity of TEMPO. This implies that the mixing strength could achieve up to $D = \frac{B_H B_F}{8\omega_H \omega_F} \approx 0.1\text{MHz}$ at 3.3 T field, while the dipolar couplings among methylene or methyl protons are $\approx 50\text{ kHz}$ ¹.

6 Cross-relaxation due to level-crossings.

As appears from Eq.1 the energy levels E_2 and E_3 could be rather close in energy, and could momentarily cross during the electron flip. The effective Hamiltonian for a system of two nuclei ignoring non-secular hyperfine terms is:

$$\mathcal{H} = A_H m_S \hat{H}_z + B_H m_S \hat{H}_x - \omega_H \hat{H}_z + A_F m_S \hat{F}_z + B_F m_S \hat{F}_x - \omega_F \hat{F}_z + D(\hat{H}_+ \hat{F}_- + \hat{H}_- \hat{F}_+), \quad (18)$$

where D is either a dipolar coupling, or a coupling emerging due to second order effects of the hyperfine interaction. The Hamiltonian matrix in basis $|1\rangle, |2\rangle, |3\rangle, |4\rangle$ is:

$$\hat{H}_{ik} = \begin{bmatrix} -\frac{\omega_H + \omega_F}{2} + \frac{m_S}{2}(A_H + A_F) & 0 & 0 & 0 \\ 0 & -\frac{\omega_H - \omega_F}{2} + \frac{m_S}{2}(A_H - A_F) & D & 0 \\ 0 & D & \frac{\omega_H - \omega_F}{2} - \frac{m_S}{2}(A_H - A_F) & 0 \\ 0 & 0 & 0 & \frac{\omega_H + \omega_F}{2} - \frac{m_S}{2}(A_H + A_F) \end{bmatrix} \quad (19)$$

Let's focus on the evolution among states $|2\rangle$ and $|3\rangle$. Then we could introduce some effective spin $\hat{I}^{(23)}$. The effective Hamiltonian driving the evolution of such spin is:

$$\hat{H}_{eff} = \left(-\frac{\omega_H - \omega_F}{2} + \frac{m_S}{2}(A_H - A_F)\right)\hat{I}_z^{(23)} + D\hat{I}_x^{(23)} \quad (20)$$

This Hamiltonian is very simple, which is why we could replace its action by the vector model. Effectively, the rotation of magnetization in this model takes place around effective fields $B_+ = (D, 0, \Omega + a)$ and $B_- = (D, 0, \Omega - a)$ depending on the electron state m_S . Consider initial magnetization $M_z = 1$ at $t = 0$ aligned along z-axis, it rotates around B_+ field, followed by a an electron flip at time τ_1 , rotation around B_- field for interval τ_2 and a second electron flip. Since the electron flip is random, it is possible to show that the final magnetization would have

¹Interestingly, the effect of mixing among nuclei in the vicinity of electron would be more pronounced for nuclei with lower γ , because the mixing coefficient does not scale with γ , while the dipolar couplings will scale as γ^2

be consist of two components, a defined projection along B_+ :

$$q = \frac{D^2 + \Omega^2 - a^2}{\sqrt{D^2 + (\Omega + a)^2} \sqrt{D^2 + (\Omega - a)^2}} \quad (21)$$

and some vector p which orientation is completely random due to the nature of electron flips. Realization of random process in different three spin systems is different, which is why p averages to 0. **[This part may need to be elaborated more, in effect it says that nuclear coherences decay very fast]**

Lets consider several scenarios.

Scenario 1. If $D \ll \Omega, a, |\Omega - a|$ the equation for q simplifies to:

$$q \approx 1 - \frac{D^2 a^2}{(\Omega^2 - a^2)^2} \quad (22)$$

After n step of random electron jumps taking place during interval $t = 2n\tau_c$, the magnetization will decrease by a factor:

$$q^2 = (1 - \frac{D^2 a^2}{(\Omega^2 - a^2)^2})^n = \exp\left(-\ln(1 - \frac{D^2 a^2}{(\Omega^2 - a^2)^2})n\right) \approx \exp\left[-\frac{D^2 a^2}{(\Omega^2 - a^2)^2} \frac{t}{2\tau_c}\right] \quad (23)$$

In other words the rate of nuclear flip-flop is $W_{ff} = -\frac{D^2 a^2}{(\Omega^2 - a^2)^2} \frac{1}{2\tau_c}$. Taking the ratio of a flip-flop rate and nuclear relaxation rate and assuming $B \sim a$ one obtains:

$$\frac{W_{ff}}{W_n} \approx (\frac{D\omega_H}{\Omega^2 - a^2})^2 \quad (24)$$

For the smallest $a \ll \Omega$ under our experimental conditions is $\frac{W_{ff}}{W_n} < 1$. For nuclei located closer to the electron and thus having larger hyperfine couplings $\frac{W_{ff}}{W_n} \approx 1$.

Scenario 2.

However, there is chance that some nuclear pairs have frequency difference $a \approx \Omega$. For those the value of $q = \frac{D}{\sqrt{D^2 + 4\Omega^2}} \ll 1$. In practice it means that for one of the electron m_S states the levels of nuclear subsystem become degenerate and large D mixes them very rapidly.

Scenario 3. Another interesting case is of $\Omega = 0$, i.e. when both nuclei have the same

Larmor frequency. The q then is:

$$q = \frac{D^2 - a^2}{D^2 + a^2} \quad (25)$$

For small $D \ll a$ this can be simplified as $q \approx 1 - \frac{D^2}{a^2}$, and the ratio of a flip-flop rate to nuclear relaxation time is:

$$\frac{W_{ff}}{W_n} \approx \left(\frac{D\omega_H}{a^2}\right)^2 \quad (26)$$

For $a \approx 3$ this ratio becomes ≈ 1 and for smaller a it becomes even bigger. In practice it means that **flip-flops among nuclei of one kind are very likely to occur with nuclear magnetization lifetime.**

7 Conclusions.

We have shown that the flip-flip rate may be rather significant for fluorine-proton and proton-proton transfer.

8 Conclusions.

In this sketchy calculations we have compared two potential mechanisms for proton to fluorine magnetization transfer induced by the electron spin flips. When electron is strongly hyperfine coupled to both fluorine and proton nuclei, its fluctuations induce nuclei flip-flop transitions.