Long-term modelling of Kangerlussuaq Glacier, East Greenland

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Overview

Background

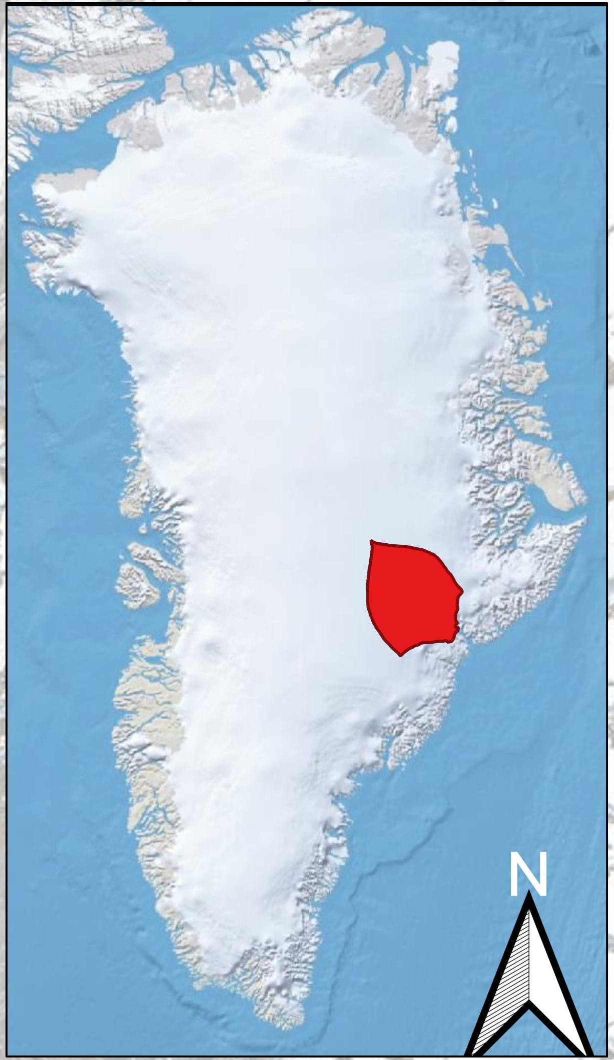
- Data and initial idea
- Modelling basics

Baseline model

- Connecting Little Ice Age to present day

Short introduction to dJUICE

- Challenge it solves
- Tutorial



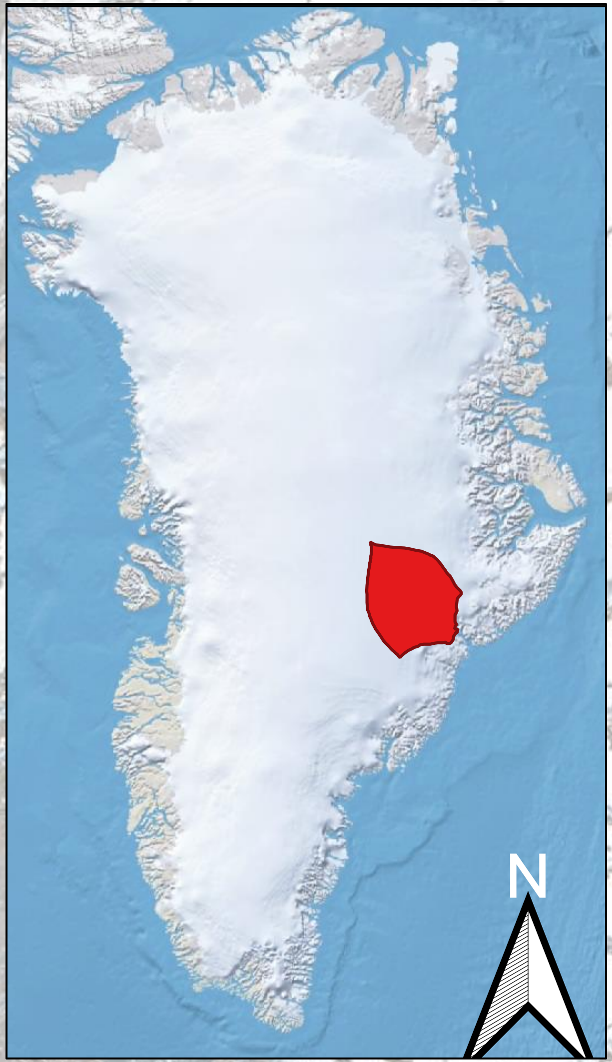
Introduction

Interest area

- Ice sheet and glacier modelling
- Little Ice Age approx. 1300-1850

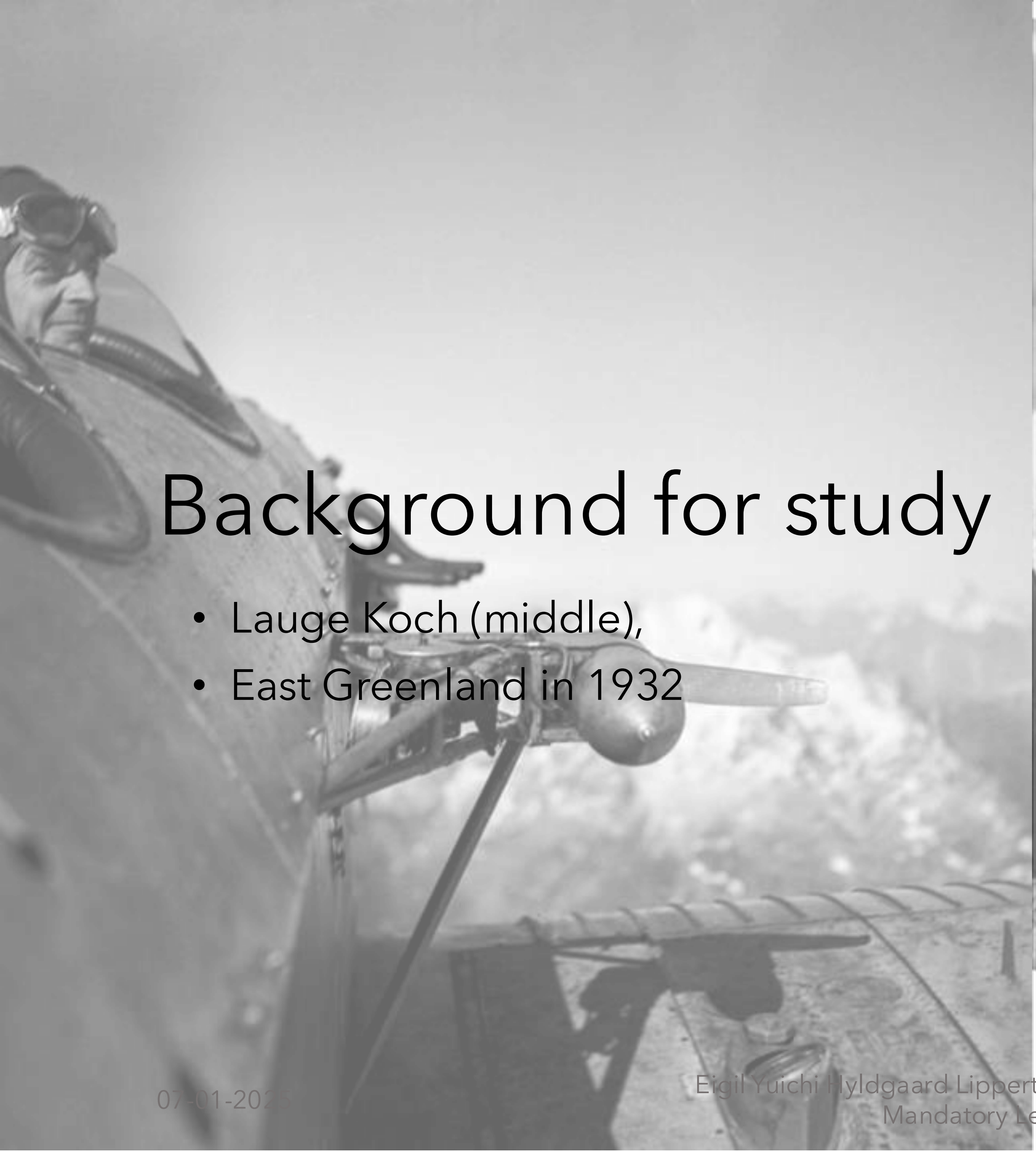
Research

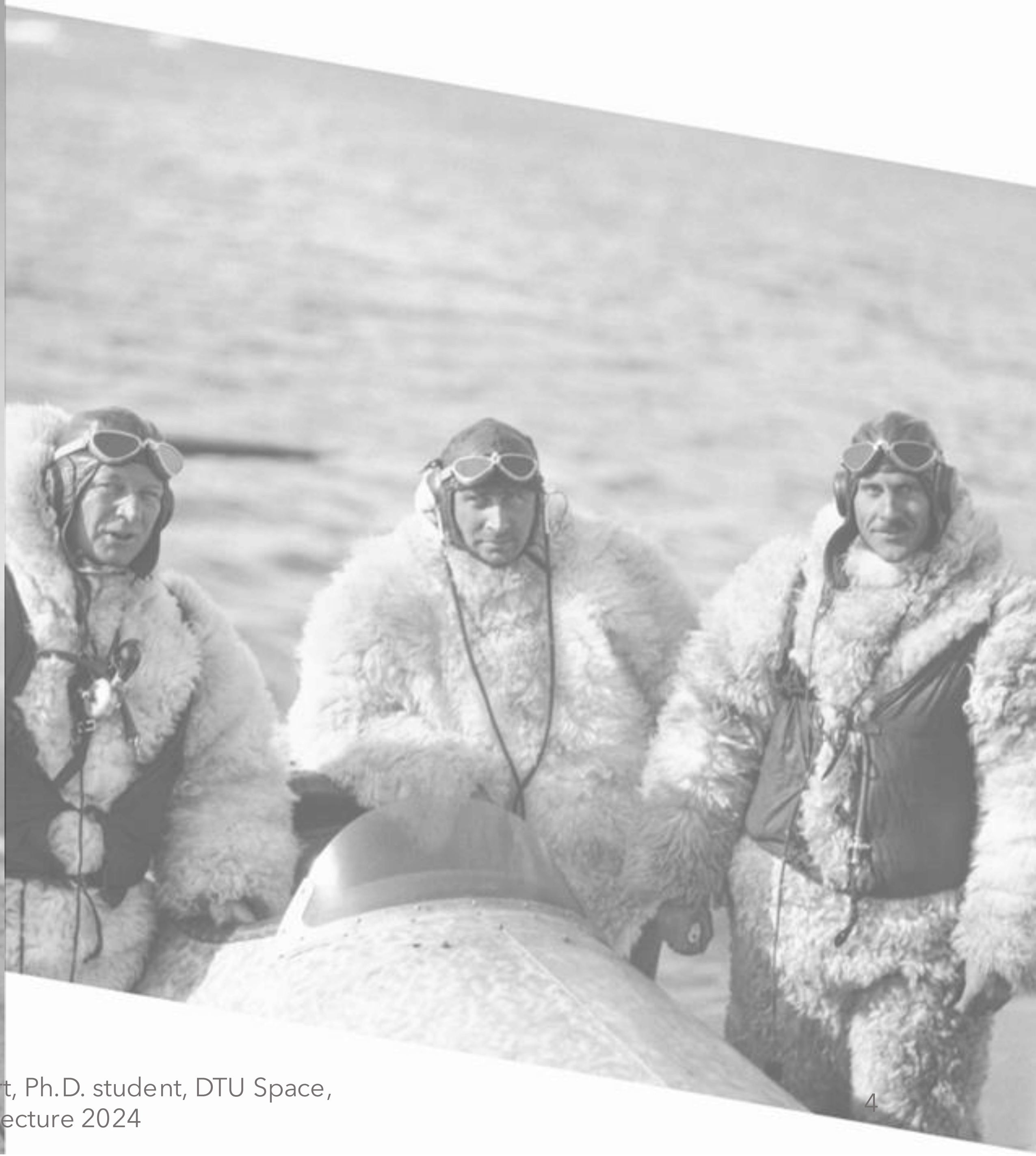
- Questions:
  - Next 100 years?
  - Centennial dynamics and processes
  - Present-day retreat in a larger picture



Background for study

- Lauge Koch (middle),
- East Greenland in 1932

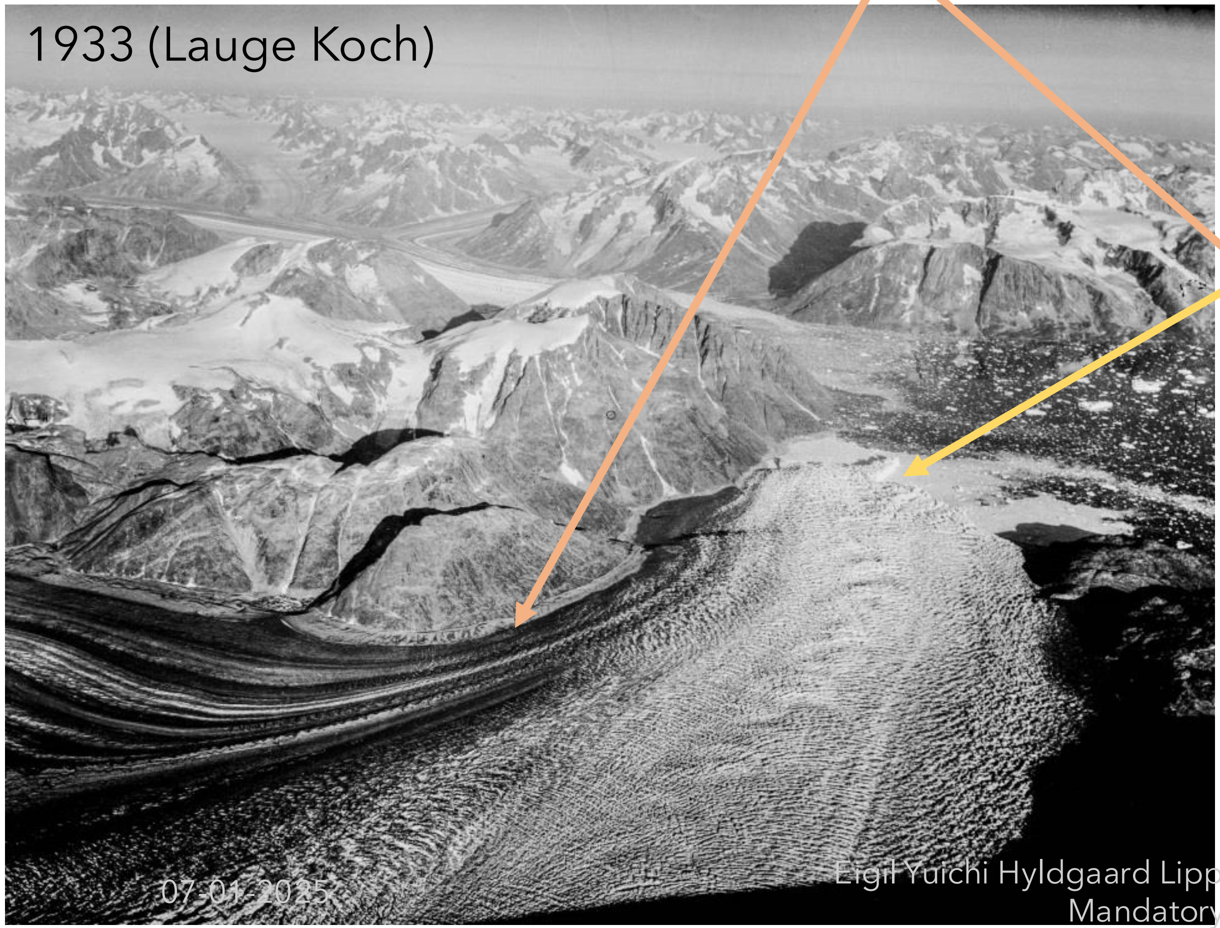


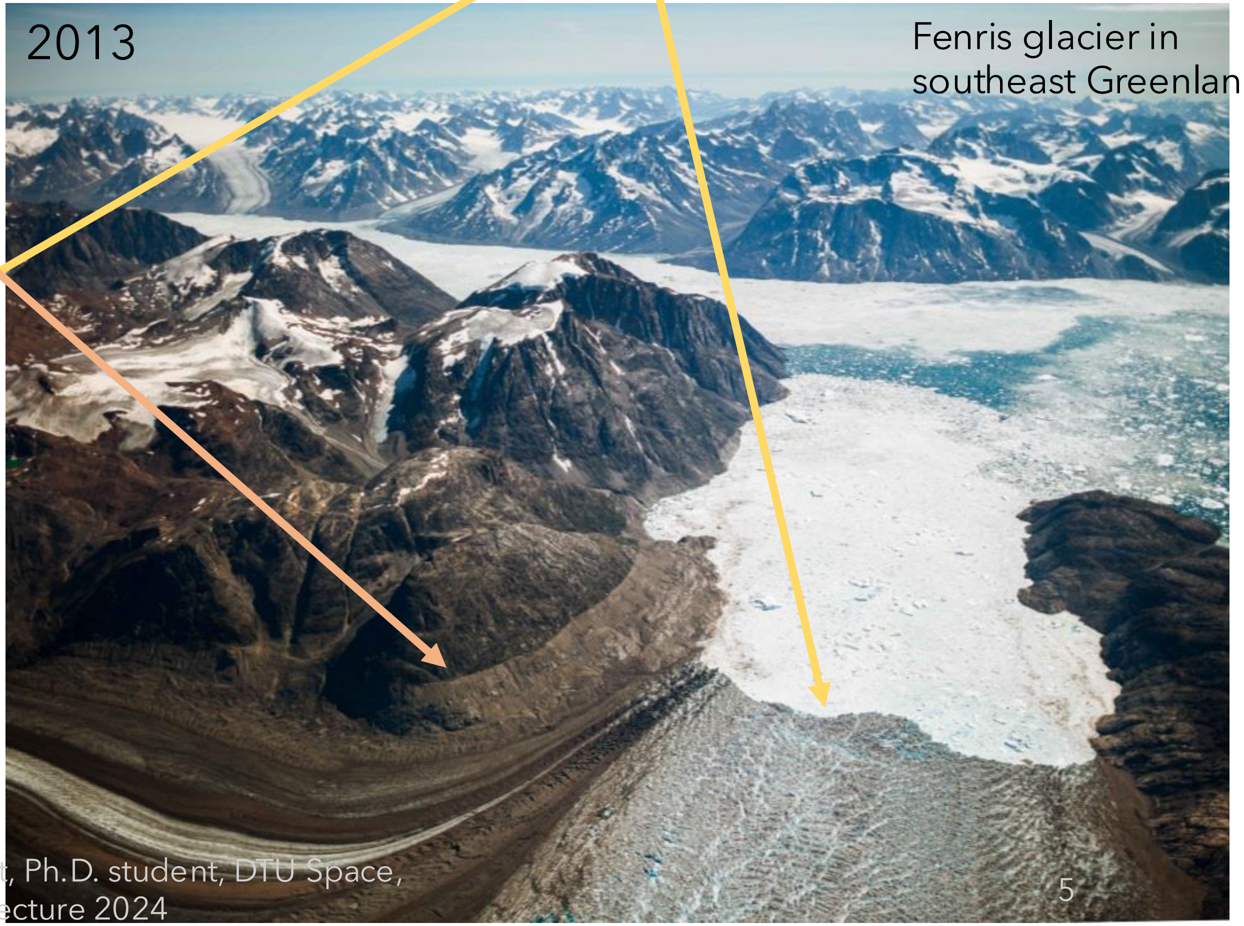


Mapping changes

Trimline – Little Ice Age Maximum

Front migration



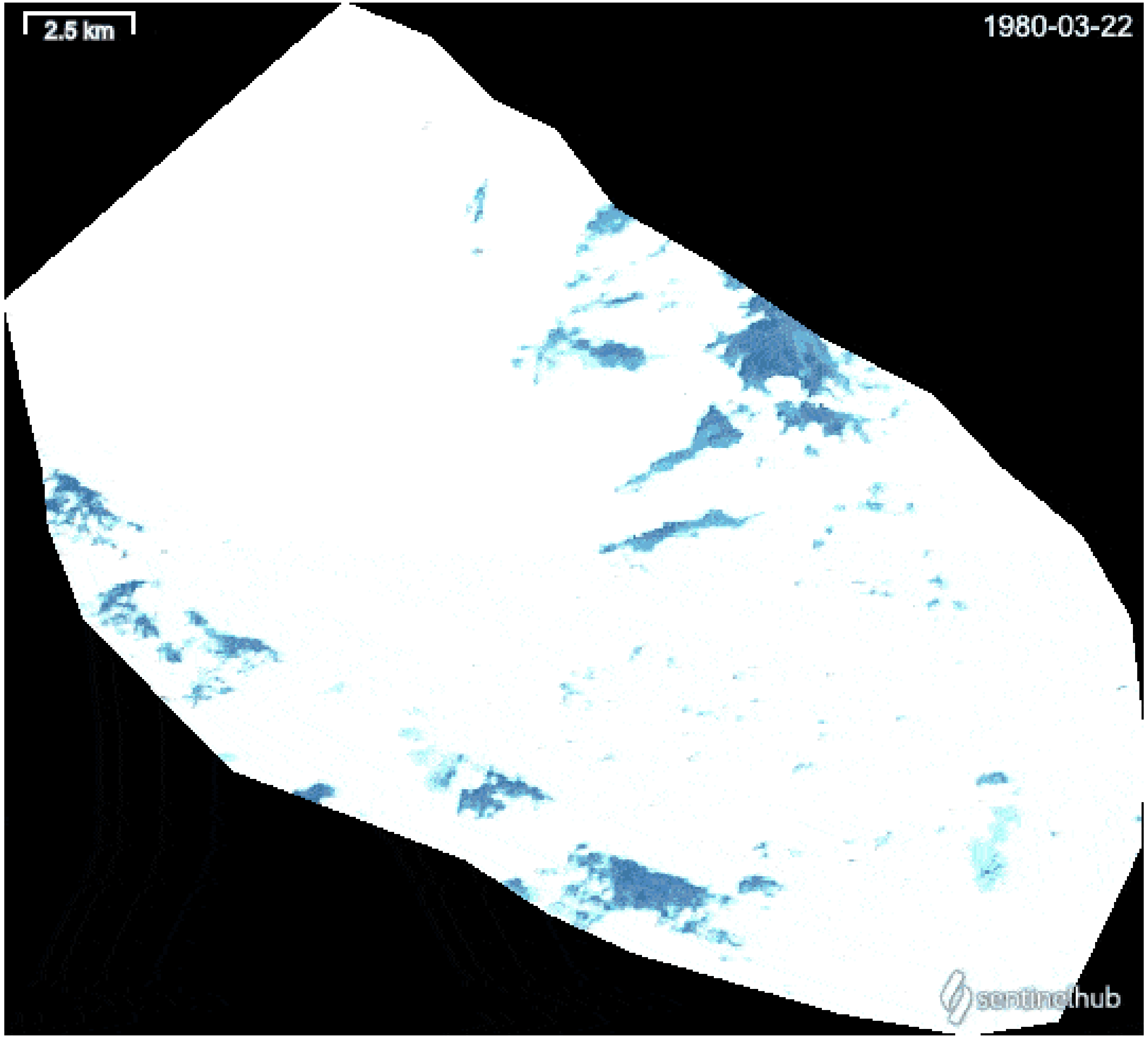


Fenris glacier in southeast Greenland

Method

Ice flow

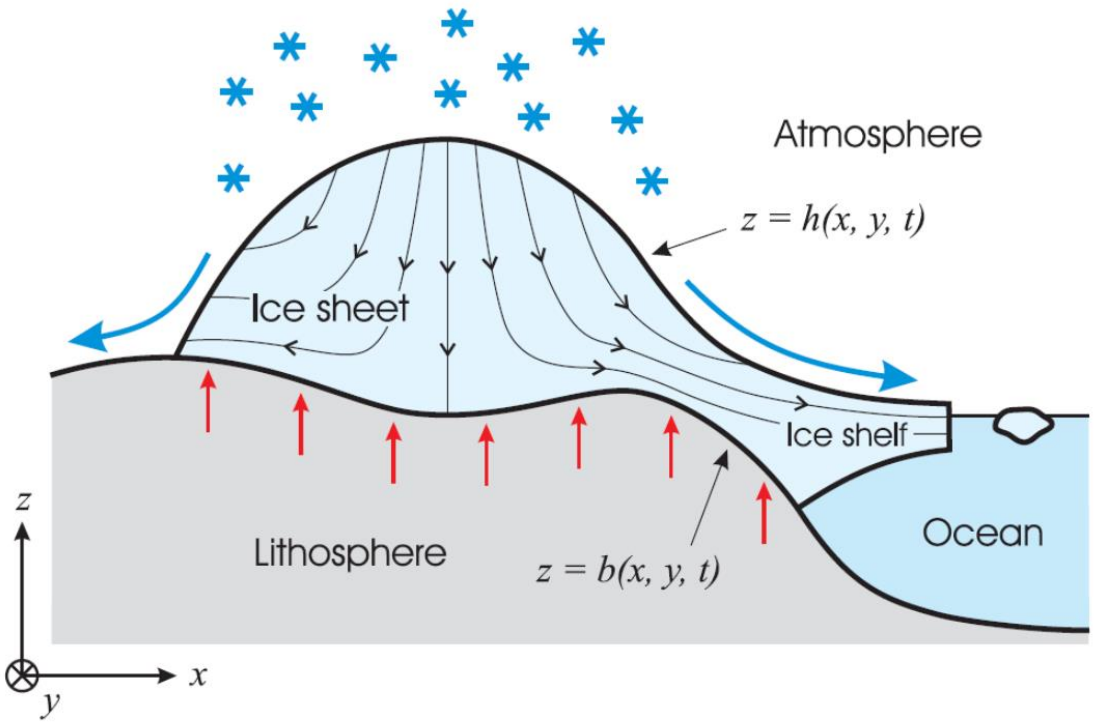
- Landsat imagery
- Kangerlussuaq Glacier
- Notice ice flow



Method

Modelling basics

- Navier-stokes – Stokes
  - *neglect momentum advection and inertia*
- Ice is non-Newtonian
  - *Power law relation between stress and strain*
  - *Glen's flow law*
- Exists in complex climatic system



$$
\begin{aligned}
\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} &= \frac{\partial p}{\partial x} \\
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} &= \frac{\partial p}{\partial y} \\
\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} &= \frac{\partial p}{\partial z} + \rho_i g
\end{aligned}
$$

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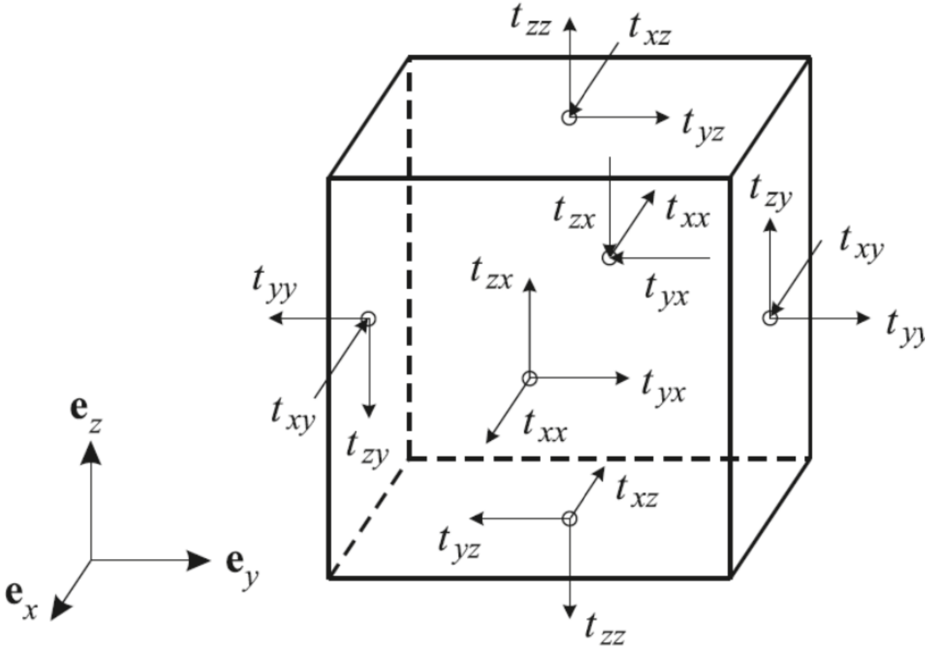
- Shallow Shelf Approximation (SSA)
  - *Horizontal dimension are much larger than vertical*



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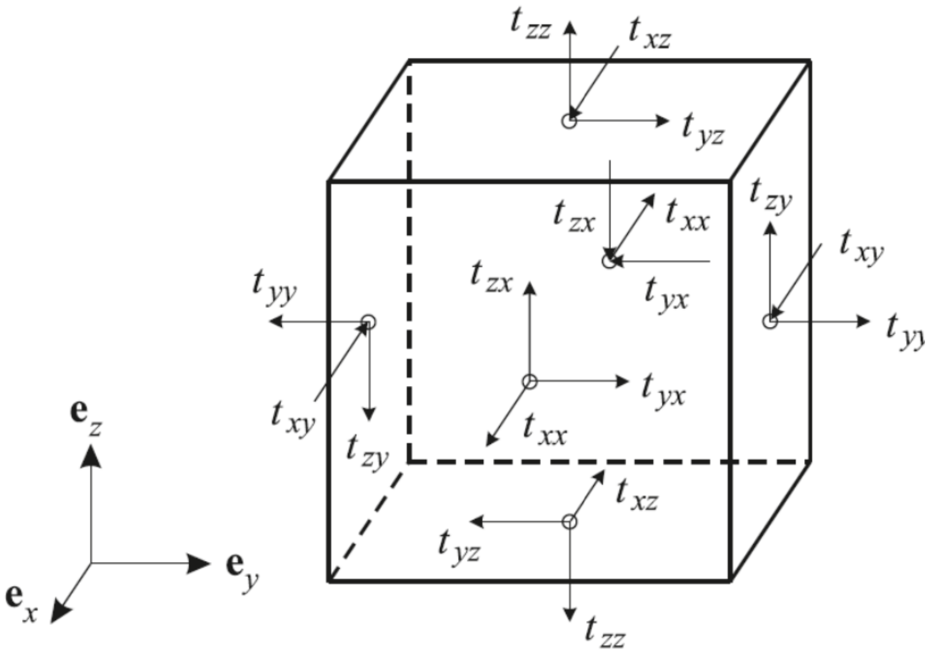

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\begin{aligned}
2\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} &= \rho_i g \frac{\partial s}{\partial x} \\
2\frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{xx}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} &= \rho_i g \frac{\partial s}{\partial y}
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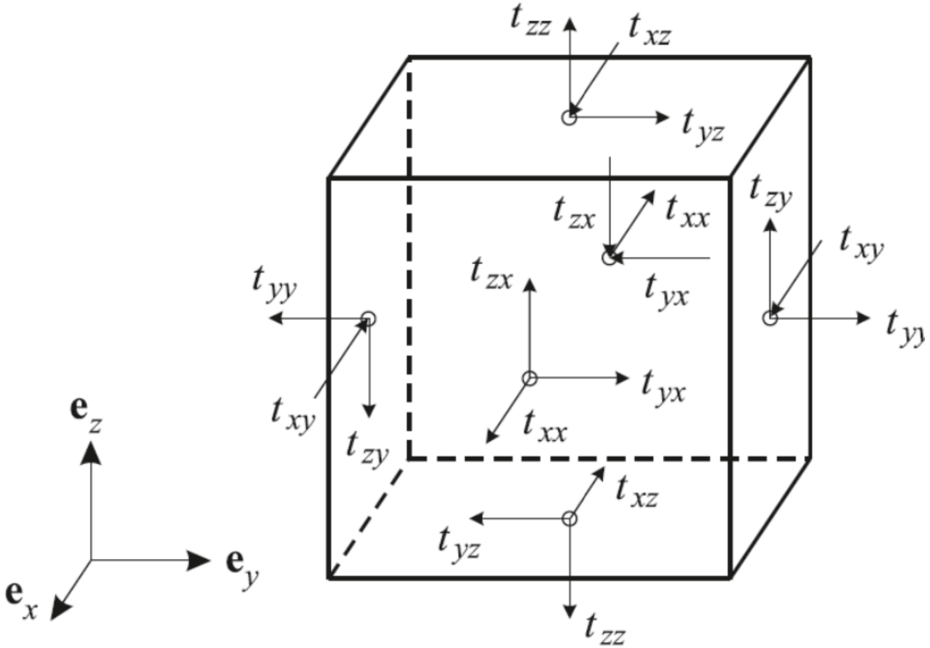
  - *Horizontal dimension are much larger than vertical*
  - *Assumptions: membrane*



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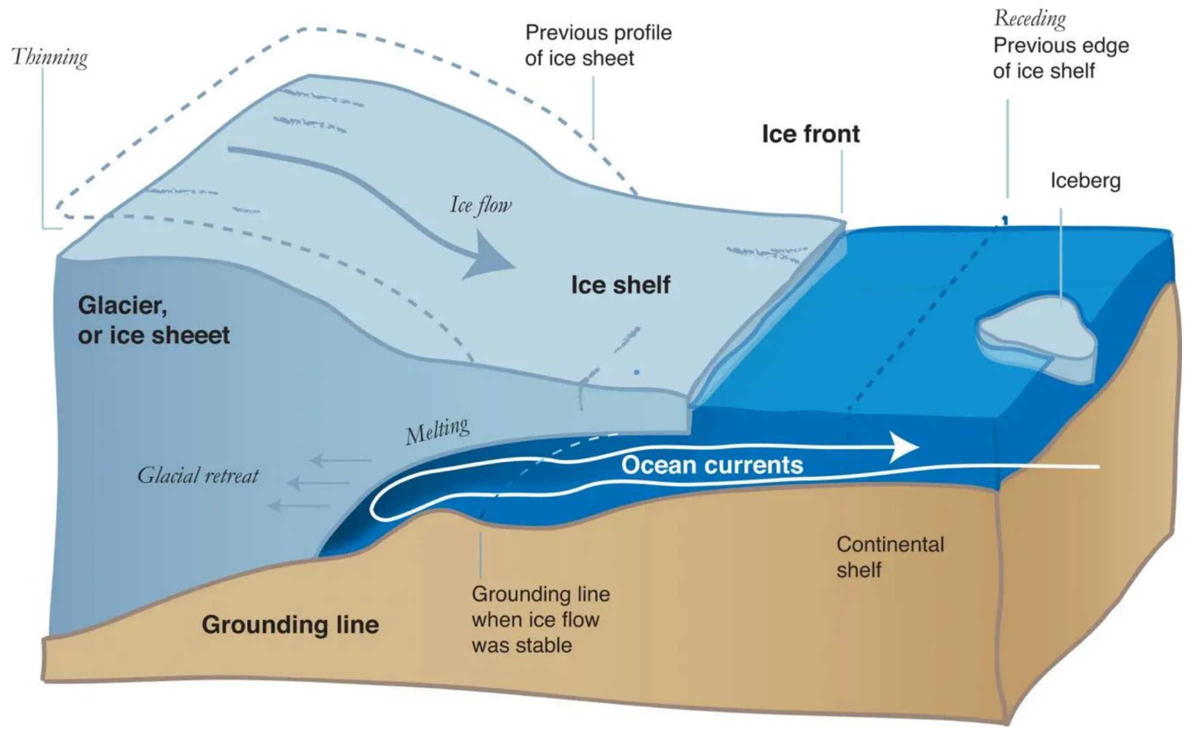
Method

Modelling basics

- Using Glen’s flow law
  - *Non linear stress-strain relation*
  - *Strain can be related to velocity*
- Shallow Shelf Approximation (SSA)
  - *We assumed ice shelf!*
  - *Grounded ice?*



$$
\begin{aligned}
\frac{\partial}{\partial x}\left(2\eta h\left(2\frac{\partial u}{\partial x}+\eta\frac{\partial v}{\partial y}\right)\right)+\frac{\partial}{\partial y}\left(\eta h\left(\frac{\partial u}{\partial y}+\eta\frac{\partial v}{\partial x}\right)\right) &= \rho_i g h\frac{\partial s}{\partial x}\\
\frac{\partial}{\partial y}\left(2\eta h\left(2\frac{\partial v}{\partial y}+\eta\frac{\partial u}{\partial x}\right)\right)+\frac{\partial}{\partial x}\left(\eta h\left(\frac{\partial v}{\partial x}+\eta\frac{\partial u}{\partial y}\right)\right) &= \rho_i g h\frac{\partial s}{\partial y}
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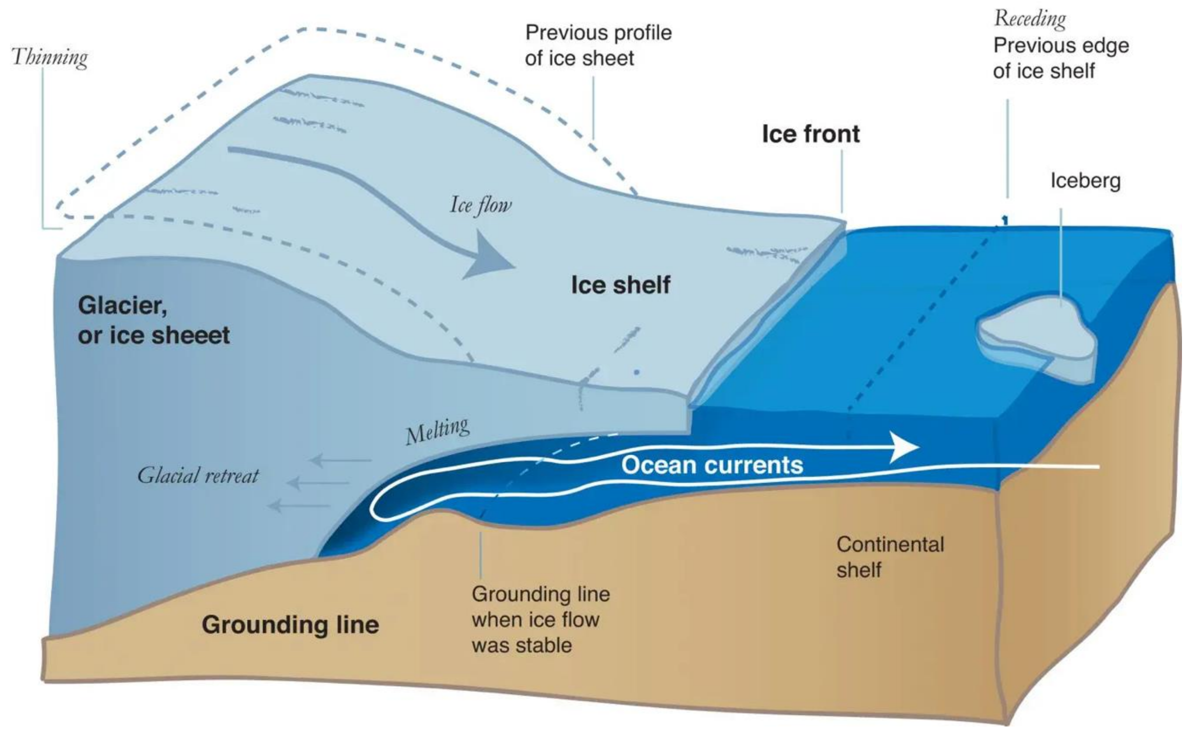
  - *Friction term*
  - *Need to know friction coeffiecient*



$$\frac{\partial}{\partial x}\left(2\eta h\left(2\frac{\partial u}{\partial x}+\eta\frac{\partial v}{\partial y}\right)\right)+\frac{\partial}{\partial y}\left(\eta h\left(\frac{\partial u}{\partial y}+\eta\frac{\partial v}{\partial x}\right)\right)-\tau_x = \rho_i g h\frac{\partial s}{\partial x}$$
$$\frac{\partial}{\partial y}\left(2\eta h\left(2\frac{\partial v}{\partial y}+\eta\frac{\partial u}{\partial x}\right)\right)+\frac{\partial}{\partial x}\left(\eta h\left(\frac{\partial v}{\partial x}+\eta\frac{\partial u}{\partial y}\right)\right)-\tau_y = \rho_i g h\frac{\partial s}{\partial y}$$

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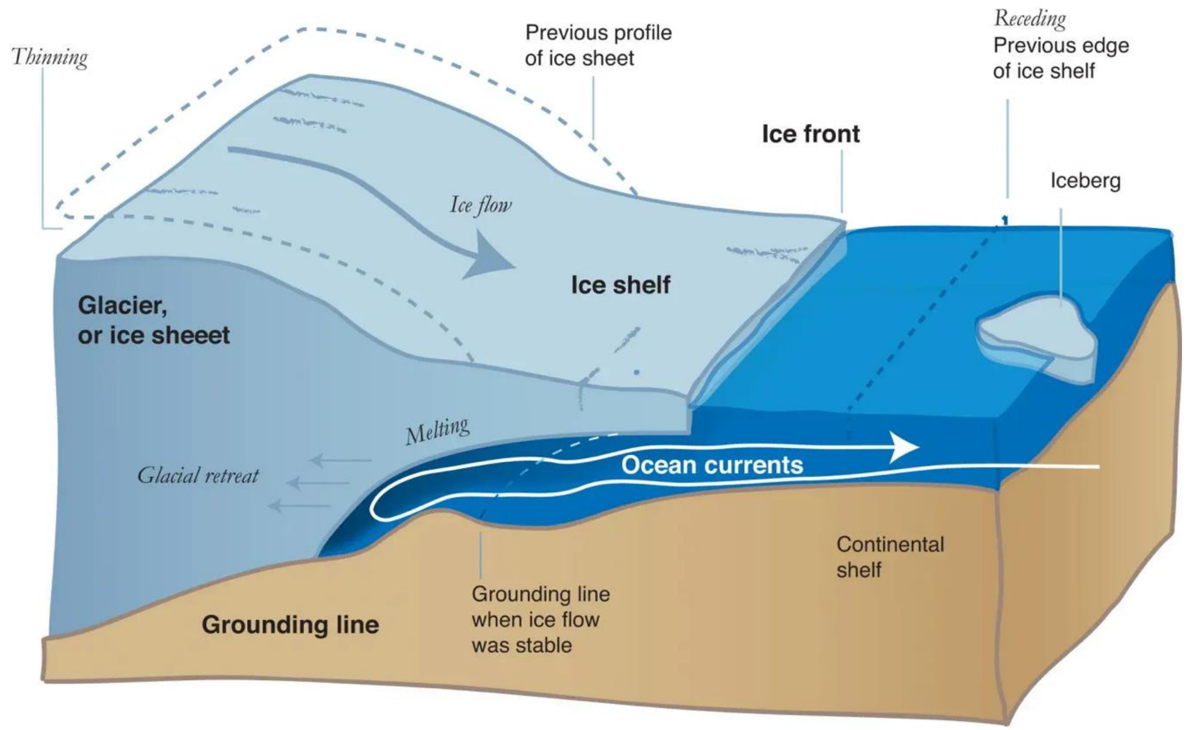

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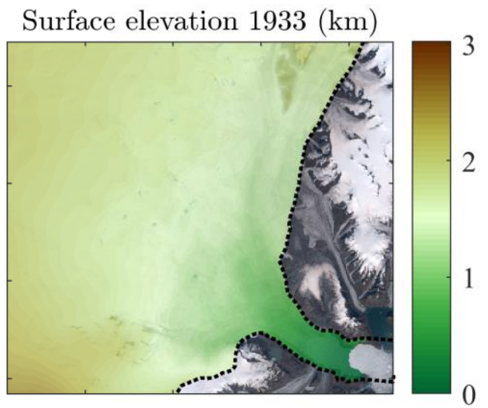
\end{aligned}
$$

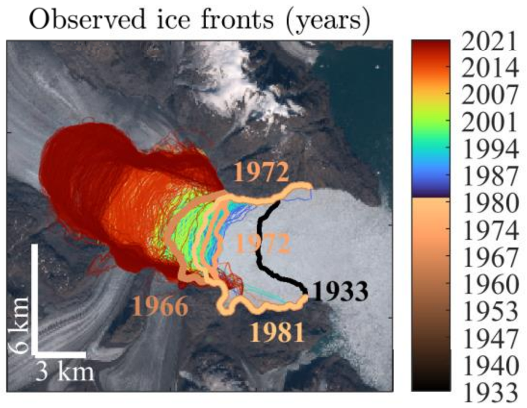
Method

Modelling overview

- Shelfy-Stream Approximation (SSA)
- Finite-element method: Ice-sheet and Sea-level System Model (ISSM)
- Anisotropic 2D-mesh adapted to observed velocities
- Budd friction law: $\tau_b = -C_B^2 N^{\frac{q}{p}} |v_b|^{\frac{1}{p}-1} v_b$

Method





Front observation

Surface elevation

Initialize transient

Forcings: SMB & front observations

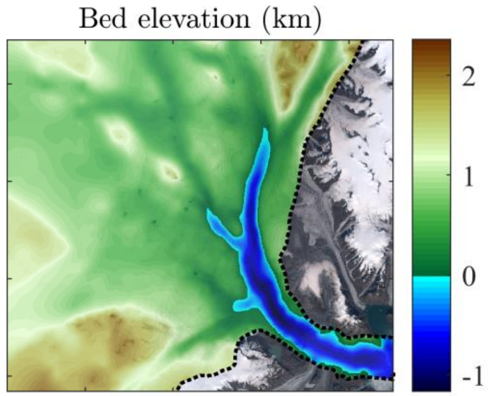
Compute misfit

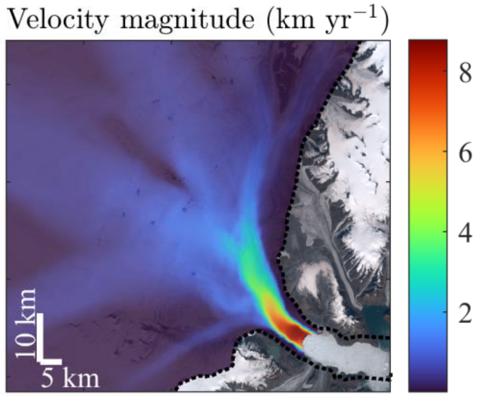
Solve inverse problem

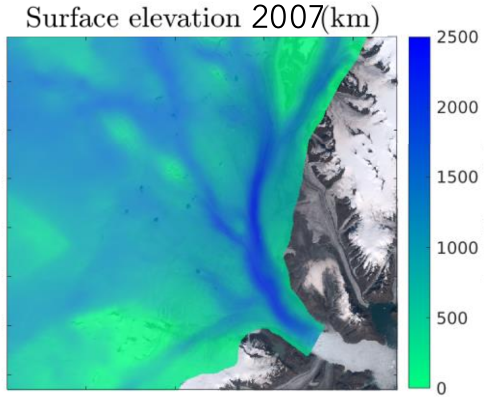
Ice geometry

Surface velocity

1933 2007 2021







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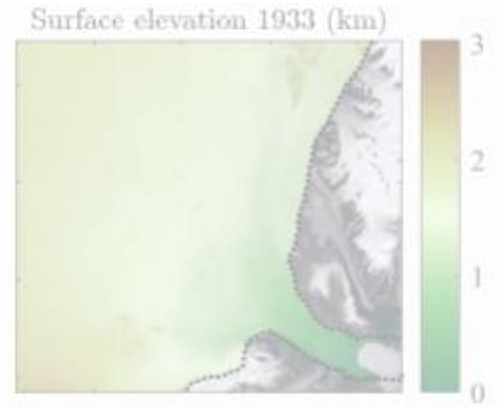

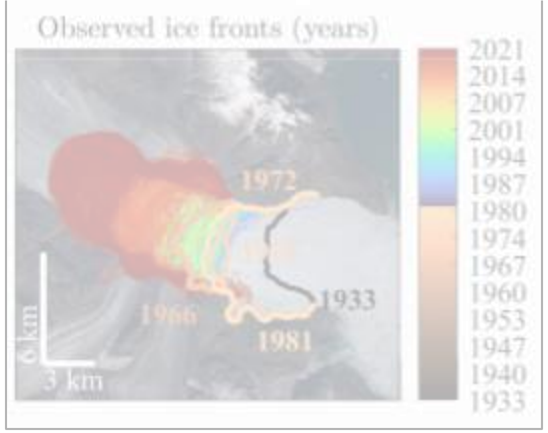

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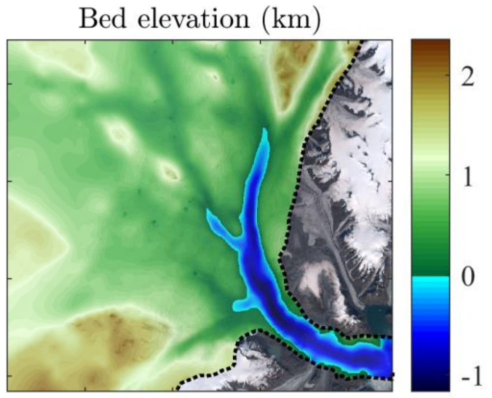
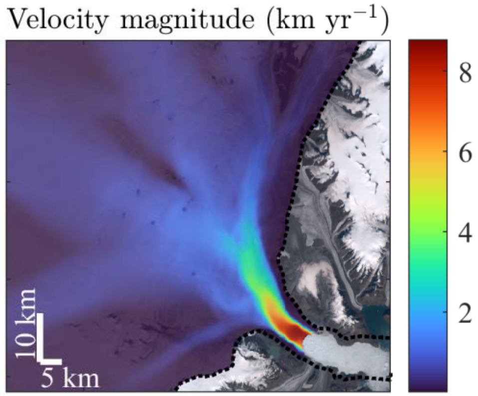
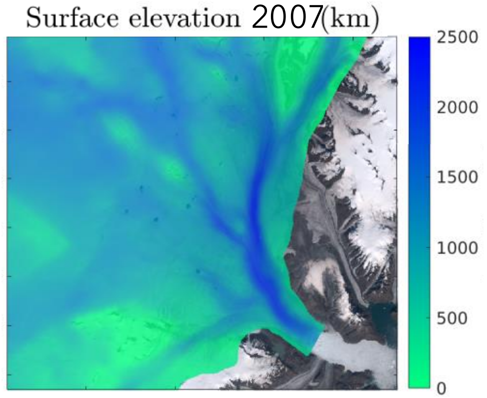
1933 | 2007 | 2021







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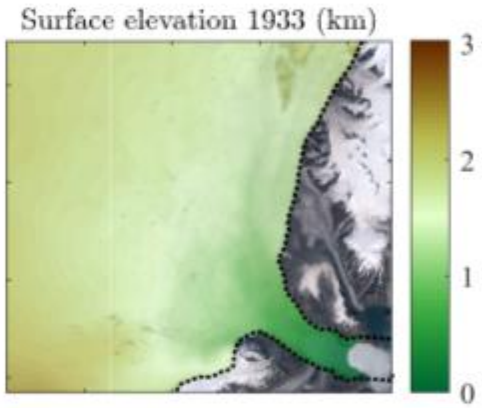

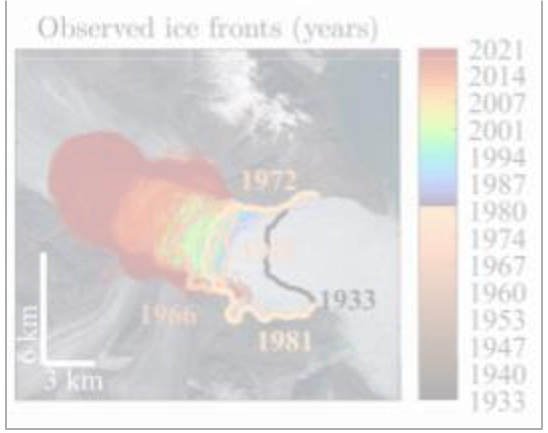

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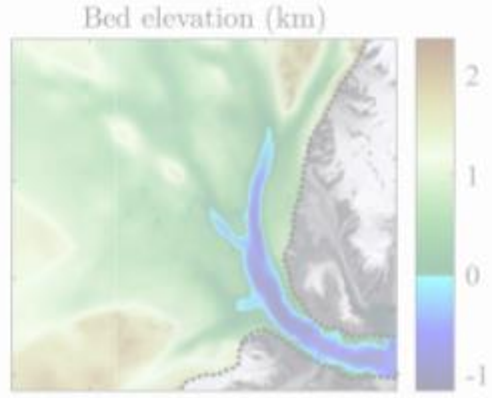

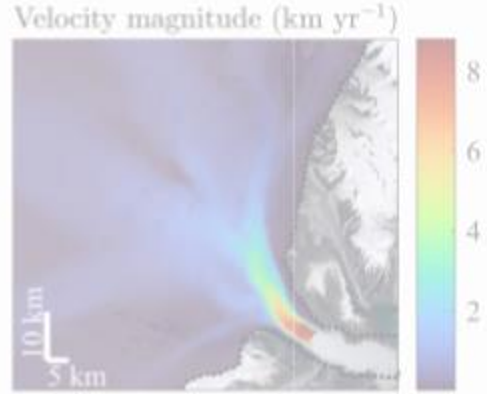

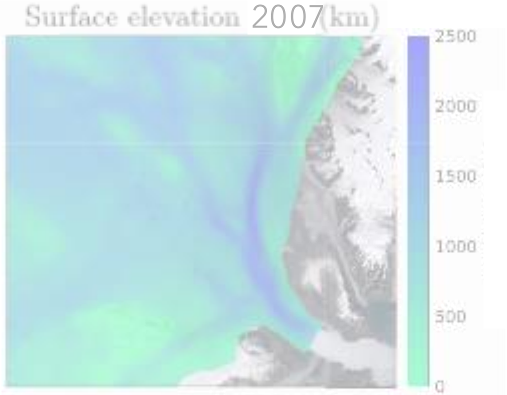

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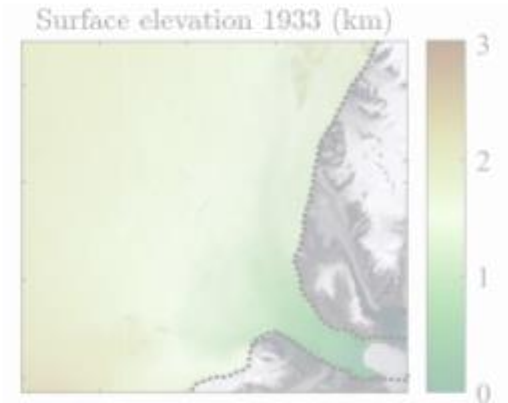

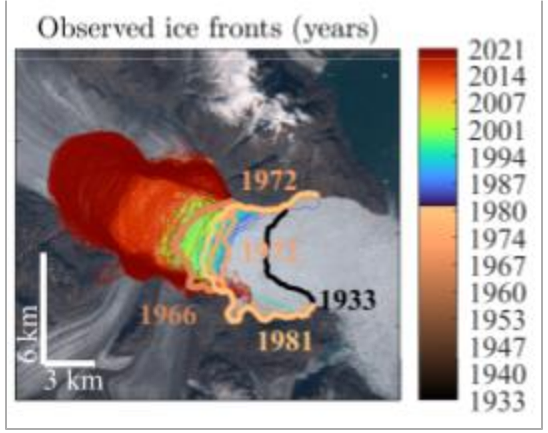

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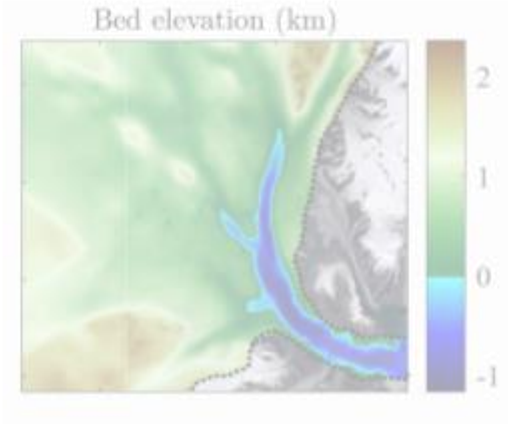

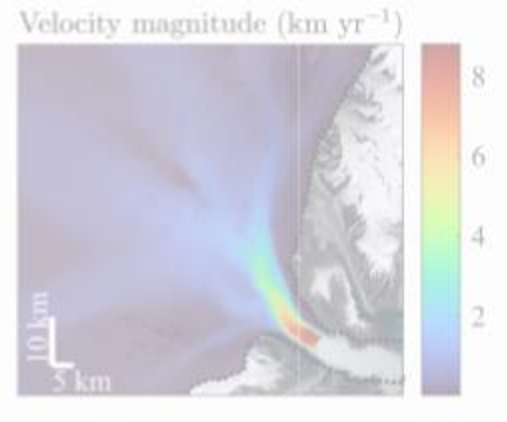

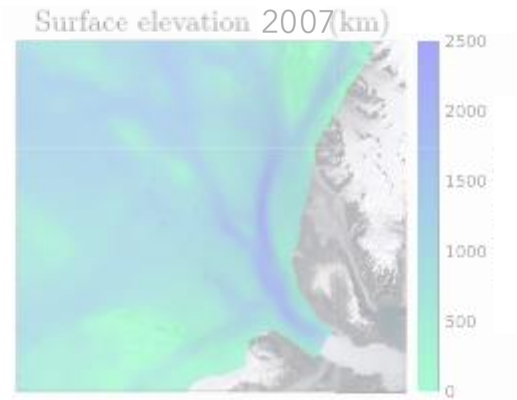

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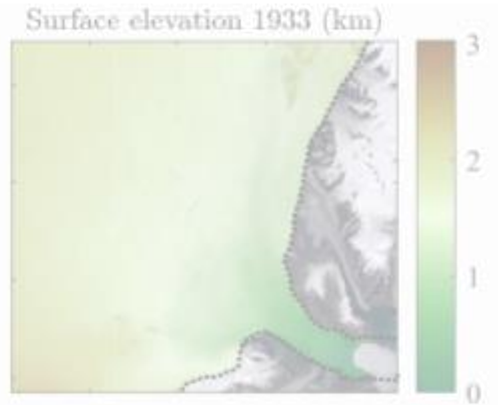

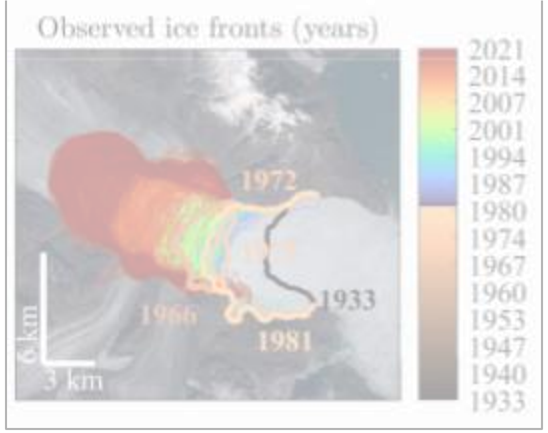

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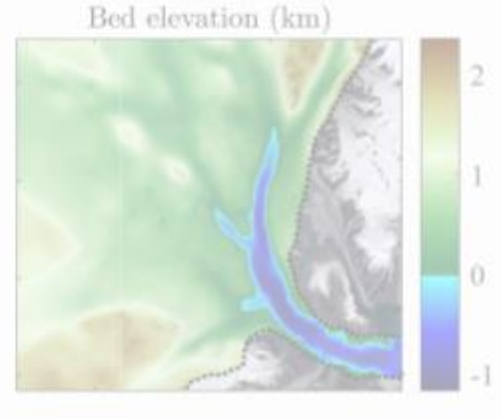
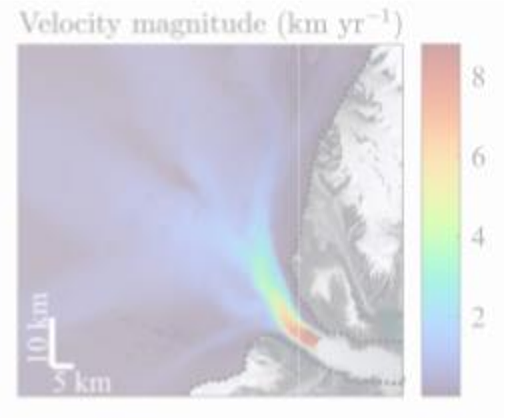
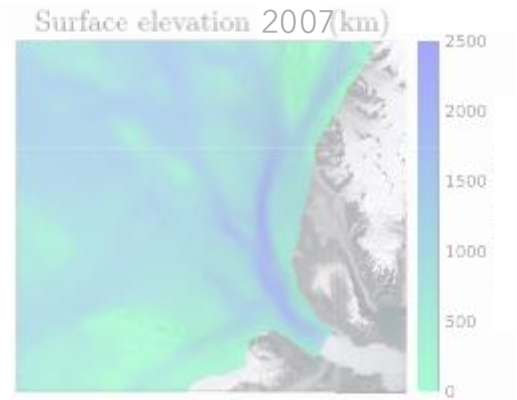


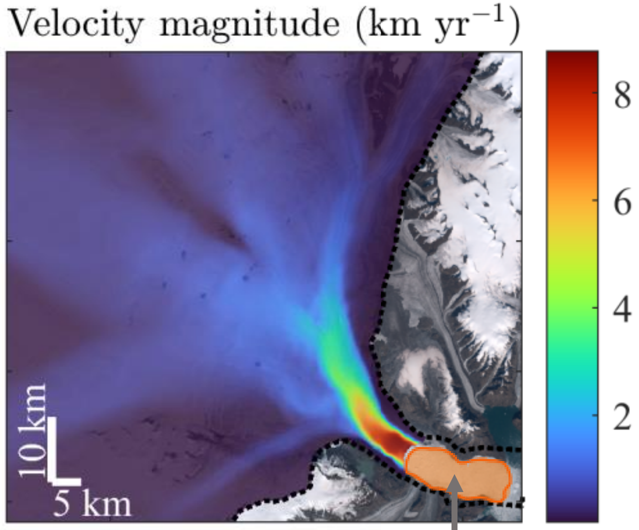


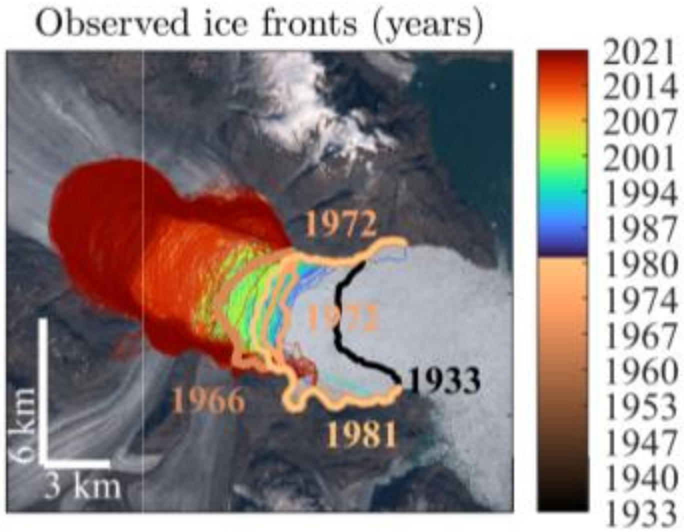


Method

Inferring friction coeffficent

- No observed velocities in 1933



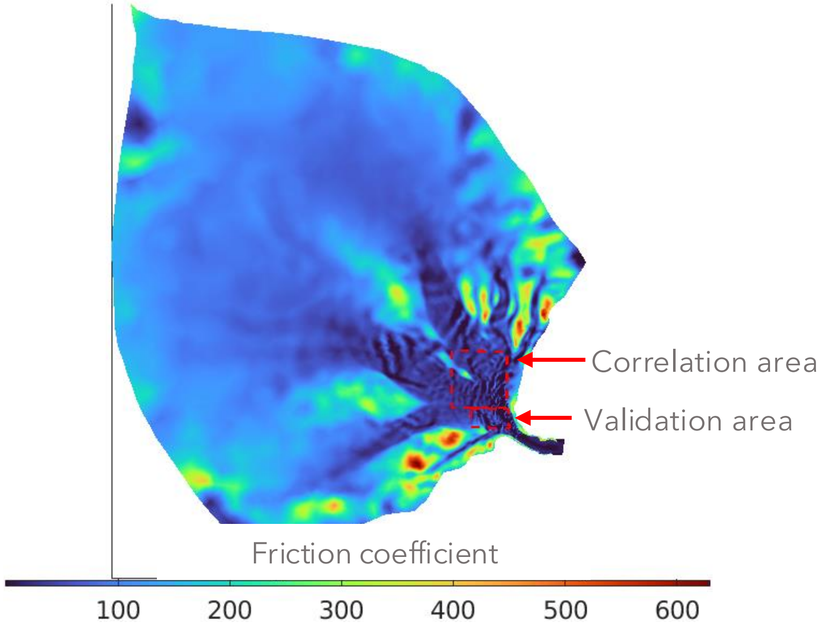


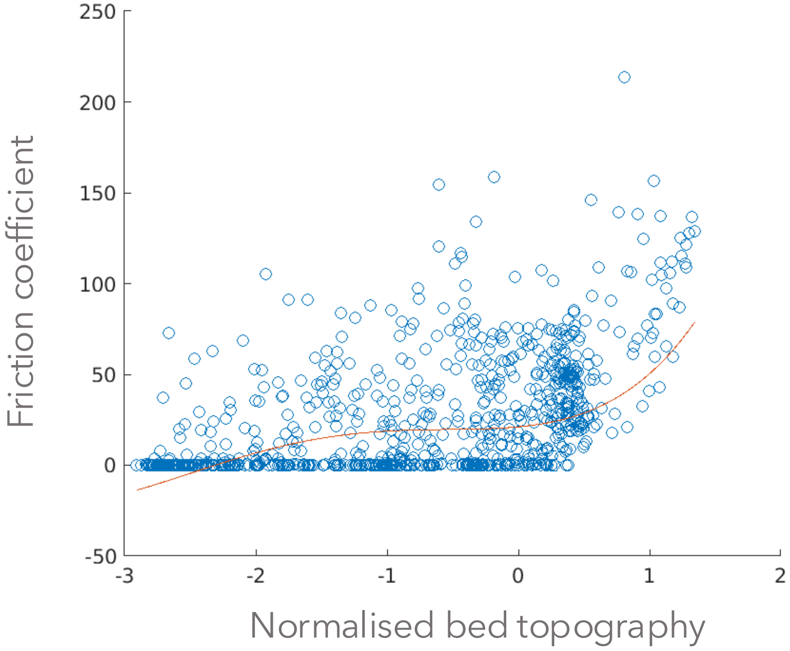
How to infer friction for this area?

Method

Statistical correlation extrapolation

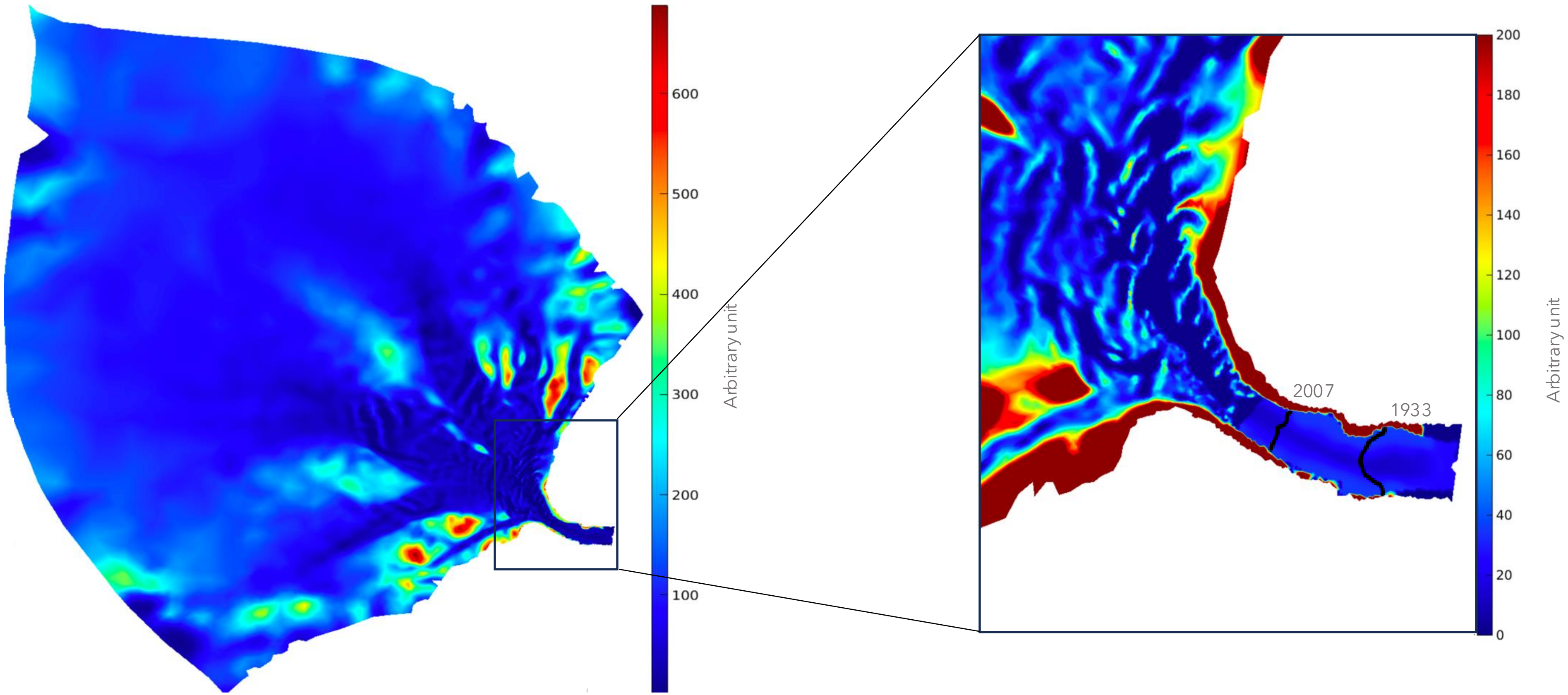
- Correlation + offset
  - *Offset = tuning knob*
- Realistic initial velocities
- Avoid ice "built-up"
- Affects everything
  - *A "plug" or "slide"*





Method

Resulting friction coefficient



Method

Friction coefficient

- Tried various friction laws
- Correlation and tuning
- Tried spinning-up and correct initial geometry
- Auto differentiation (AD)?

dJUICE.jl

ISSM equivalent in Julia

- MATLAB like syntax
  - *Ease transition*
- Early stage:
  - *Stress balance and transient solutions*
- Enzyme for AD
- 2 simple tutorials

Basal stress model

- Physics informed "inversion" of basal friction

$$
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- Flow chart