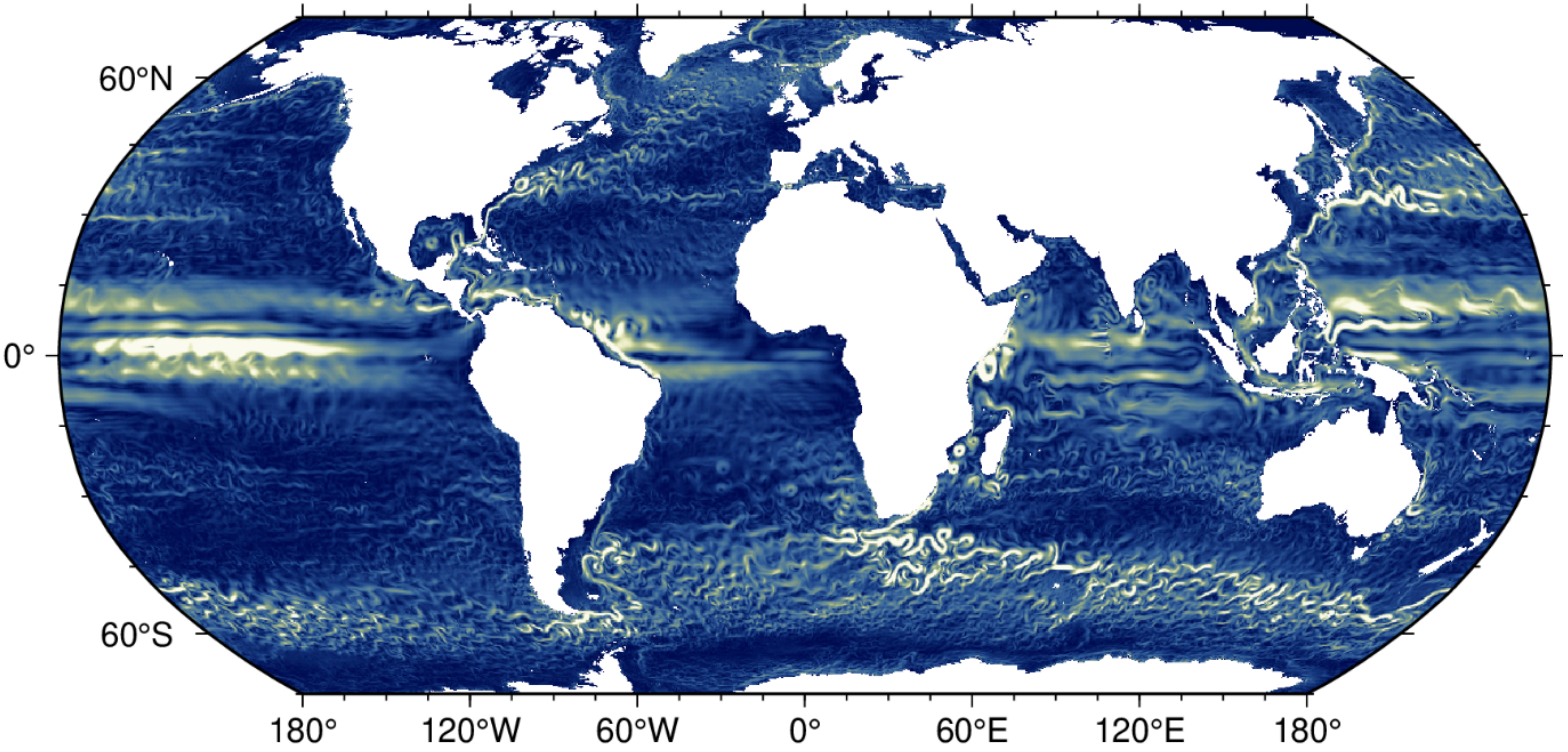
Inferring the ocean subsurface using generative AI

Andre Souza

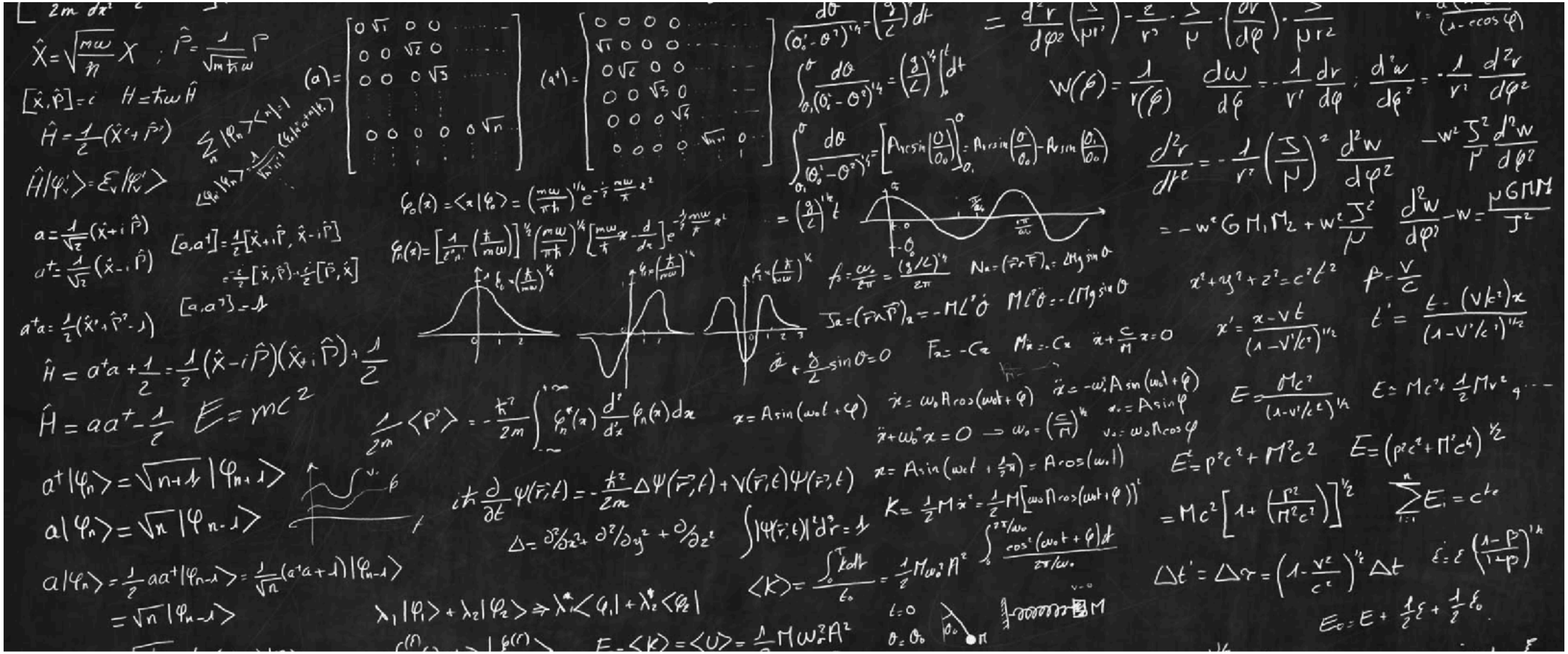
Recap

We’ve seen Satellites

We’ve seen Ocean



We’ve seen Models



We’ve seen various ways of combining models with data

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This talk will discuss (yet) another way

Combining Models with Data

- Physical theories let us make inferences using less information (Maxwell's equations)
- For complex systems (atmosphere, ocean, chaotic systems) the best we can do are numerical models (computational irreducibility)
- Using computational models with data-driven approaches allow us to extend the theory + data inferences

Combining Models with Data

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- For complex systems (atmosphere, ocean, chaotic systems) the best we can do are numerical models (computational irreducibility)
- Using computational models with data-driven approaches allow us to extend the theory + data inferences

What kind of questions do we ask?

Information we want

$$y = f(x)$$

A mapping

Information we have

What kind of questions do we ask?

Information we want

$$\rho(y|x)$$

A Statistical Inference

Information we have

What kind of questions do we ask?

$$y = f(x)$$

$$\rho(y|x)$$

$$\rho(y|x) = \delta(y - f(x))$$

What kind of questions do we ask?

$$y = x^2$$

$$\rho(y|x)$$

$$\rho(y|x) = \delta(y - x^2)$$

What kind of questions do we ask?

$$y = x^2$$

$$\rho(x|y)$$

$$\rho(x|y) = \frac{\delta(x - \sqrt{y}\,) + \delta(x + \sqrt{y}\,)}{2}$$

What kind of questions do we ask?

Useful Information

$$\rho(y|x) = \delta(y - f(x))$$

"Useless" Information

$$\rho(y|x) = \rho(y)$$

What kind of questions do we ask?

ill-posed

$$y = f(x)$$

well-posed

$$\boxed{\rho(y|x)}$$

What kind of questions do we ask?

ill-posed

$$y = f(x)$$

well-posed

$$\boxed{\rho(y|x)}$$

connection

$$f(x) = \mathbb{E}_y \rho(y|x)$$

What can we do now that we couldn't do before?

$$\rho(y|x)$$

high-dimensional

What can we do now that we couldn't do before?

$$\rho(y|x)$$

Method: Score-Based Diffusion Models

Architecture: Modified U-Net

Applications

$$\rho(y|x)$$

Information we want

Information we have

Application: Ocean Inference

$$\rho(y|x)$$

Ocean Interior

Ocean Surface

What we have

What we want

Application: Ocean Inference

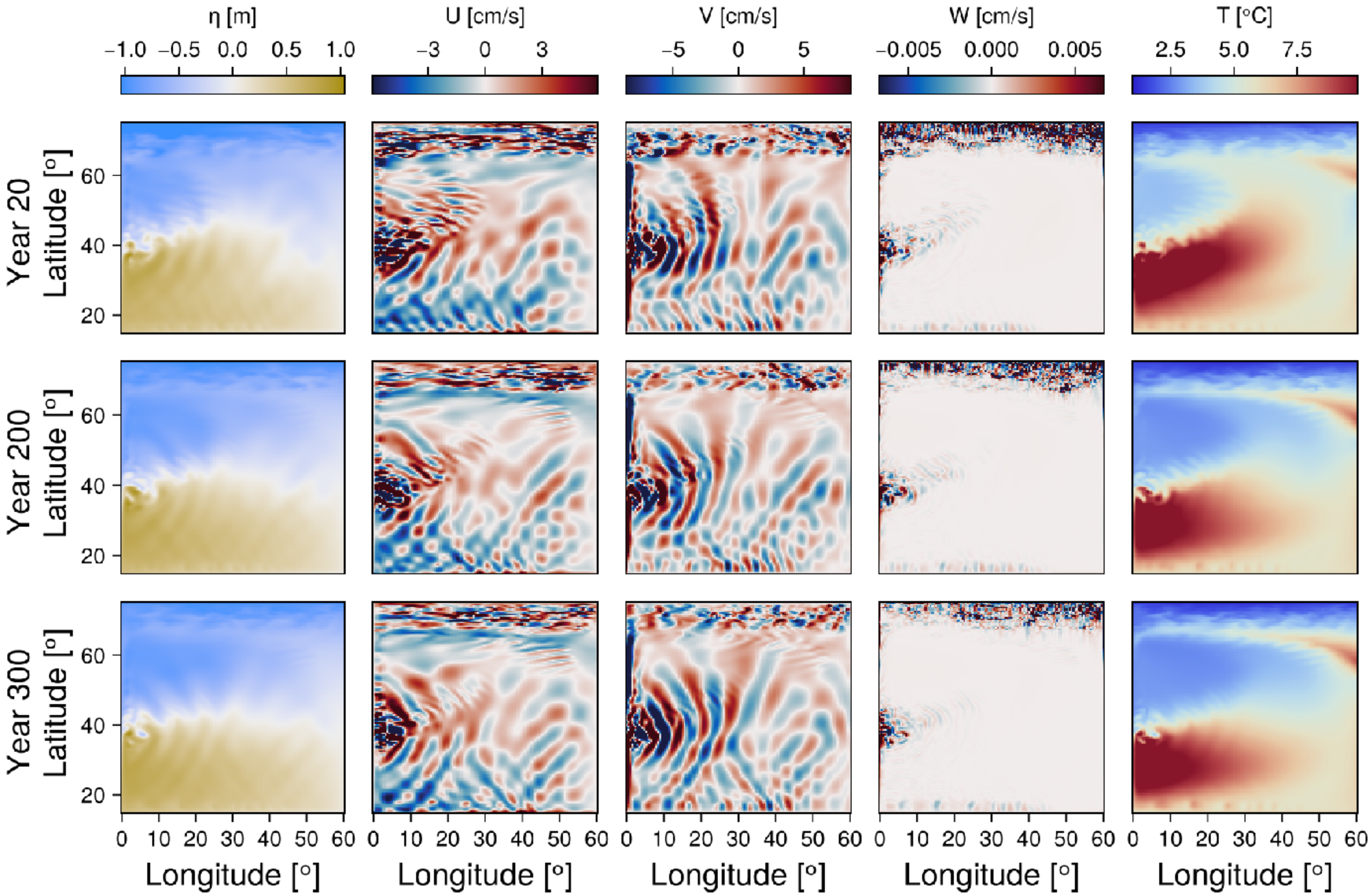
$$\rho(y|x)$$

Model: 3D Navier-Stokes with 256x256x15x3 degrees of freedom (non-stationary)

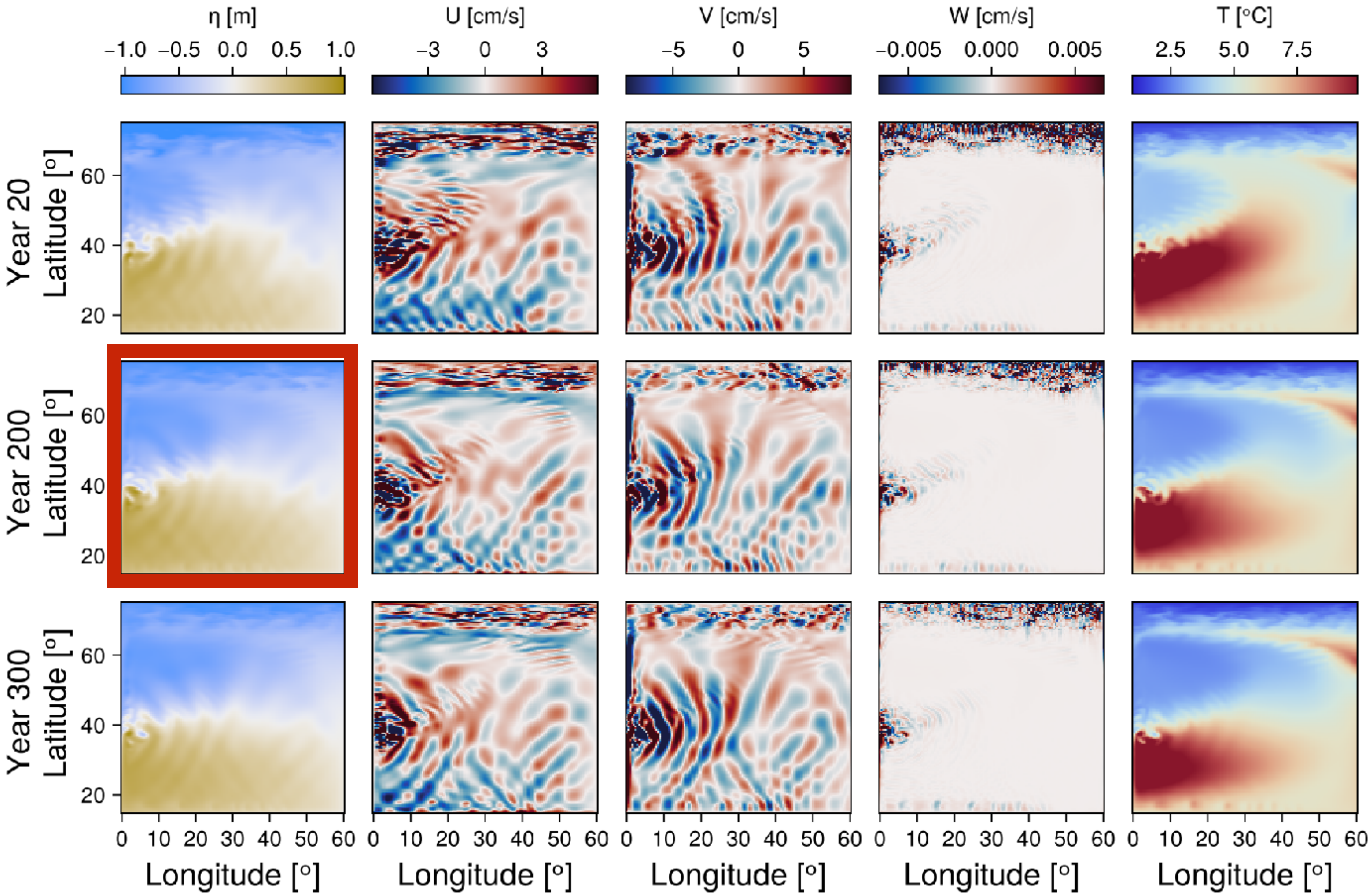
Given information "x": Free Surface Height (different levels of coarse-graining)

Predicted Information "y": The 3D Ocean Interior State

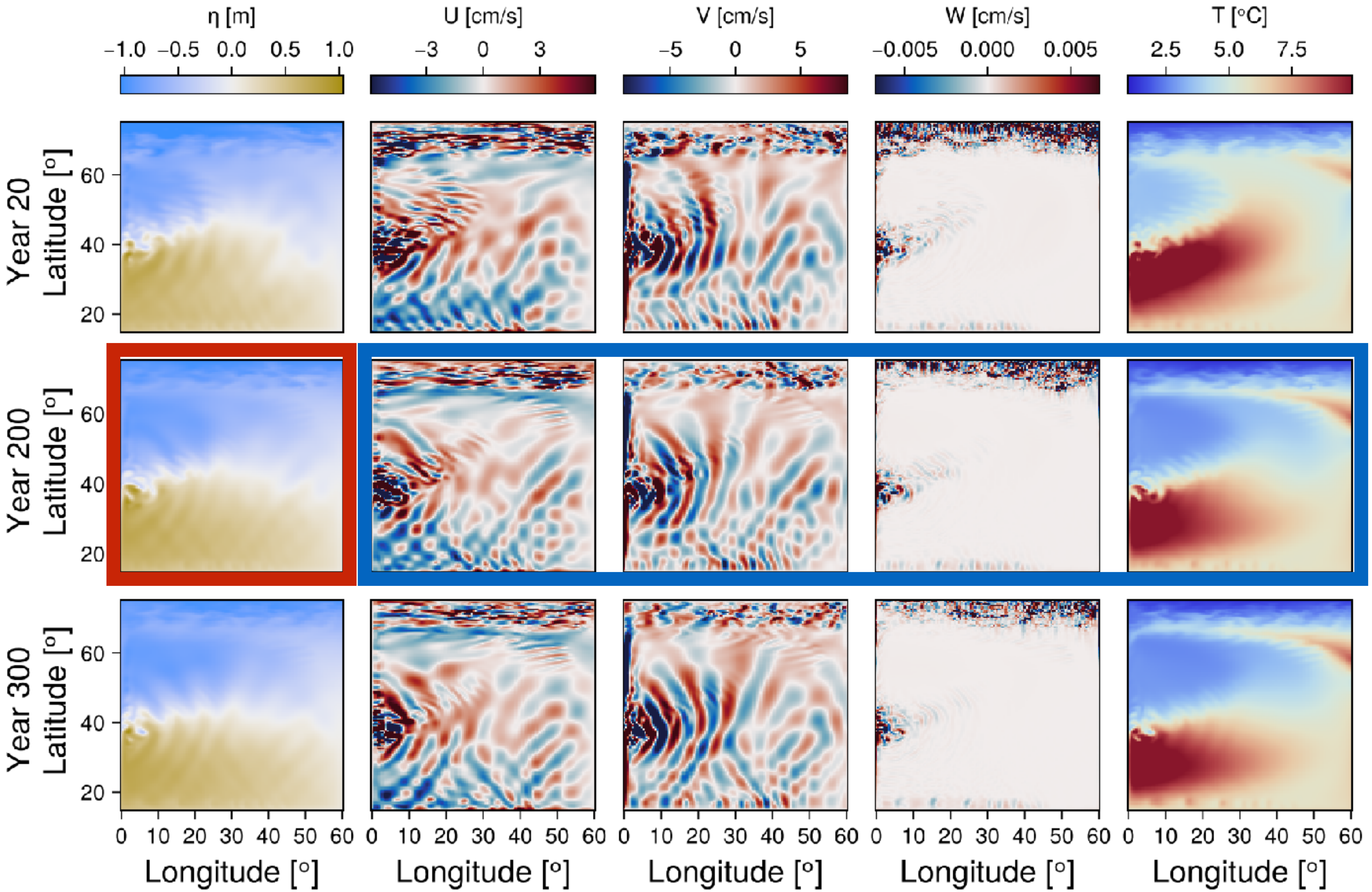
Application: Ocean Inference Task (400m depth)



Application: Ocean Inference Task (400m depth)

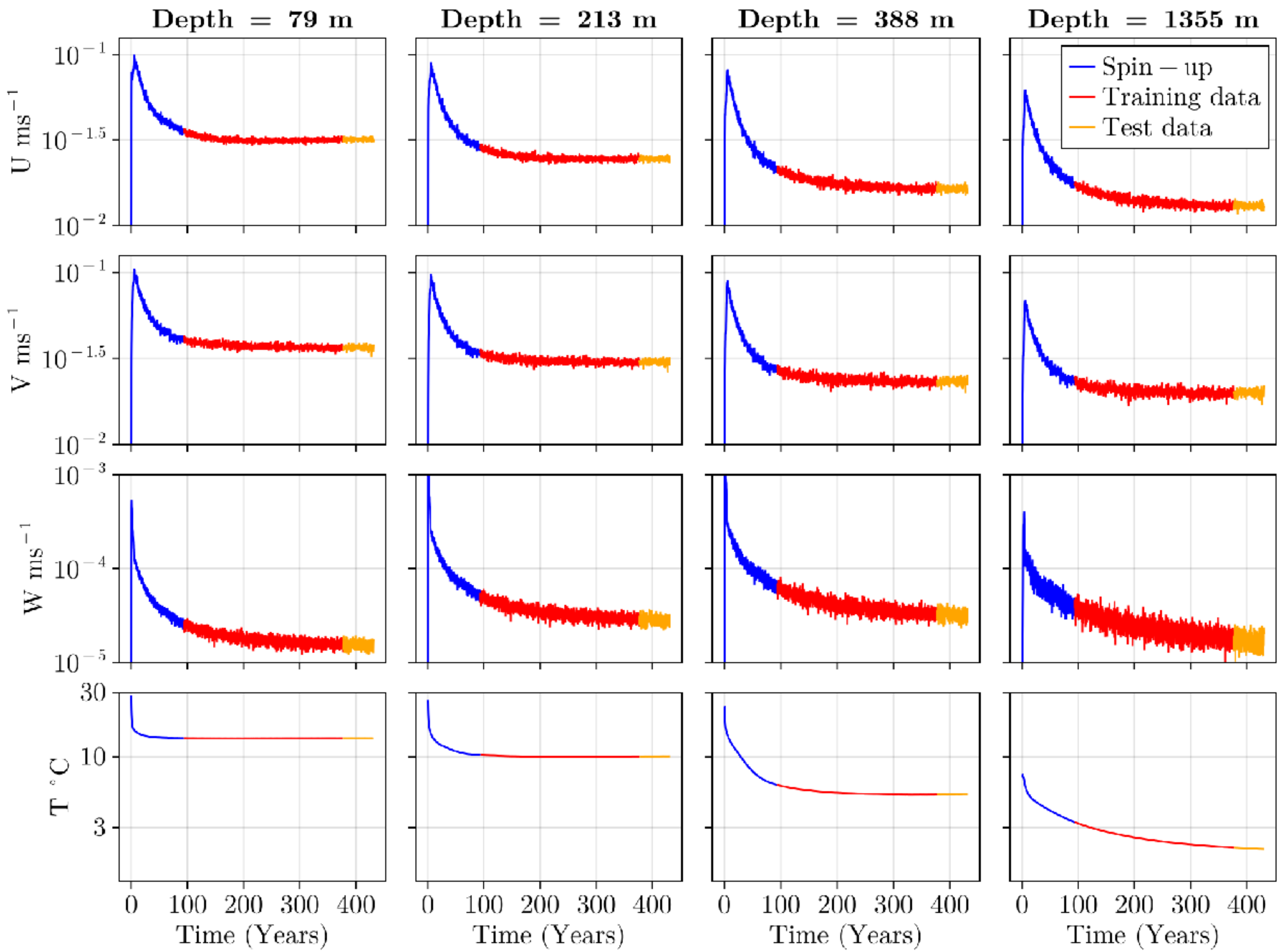


Application: Ocean Inference Task (400m depth)



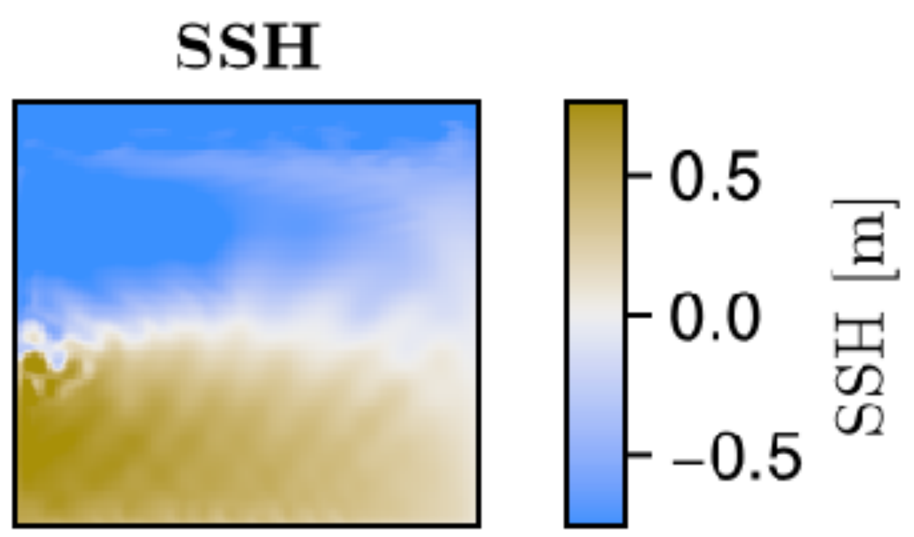
Training and Test Set: Non-stationarity

Application: Ocean Inference

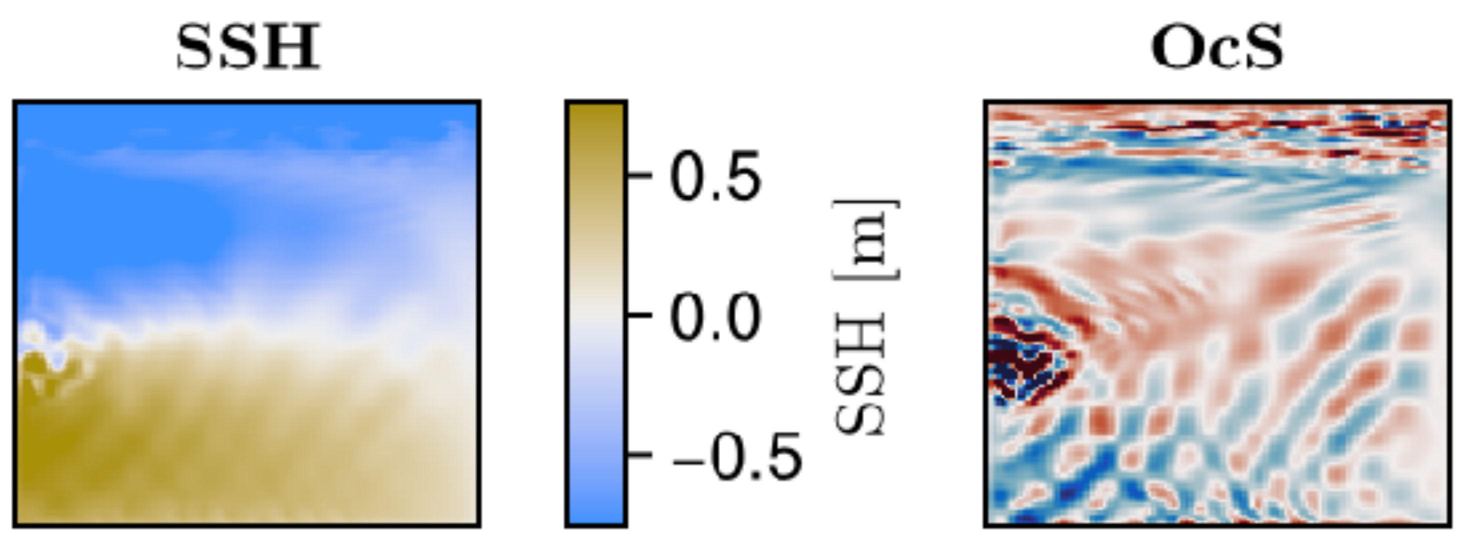


Results

Application: Ocean Inference Result (400m depth)



Application: Ocean Inference Result (400m depth)



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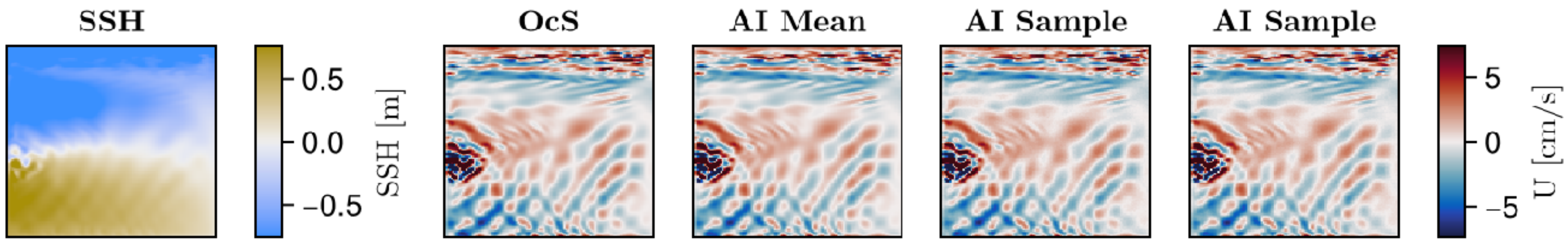

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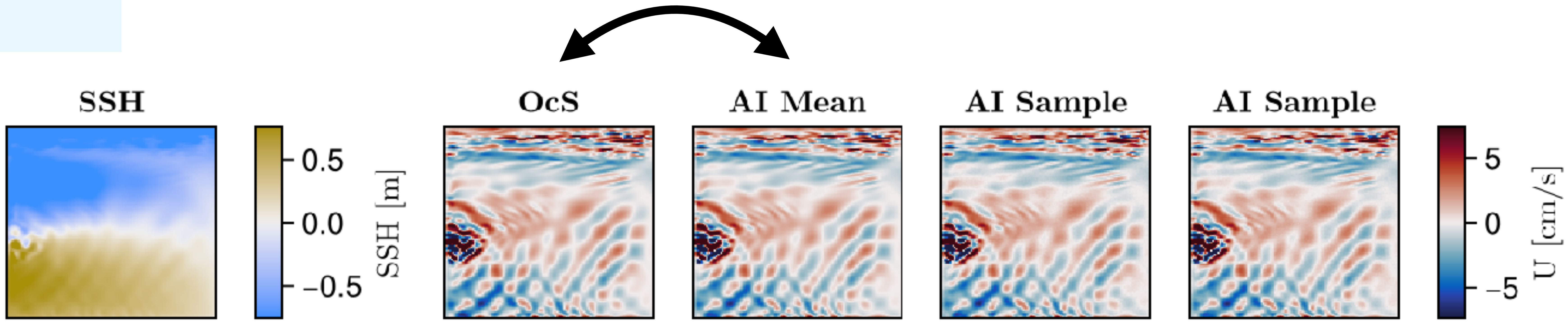

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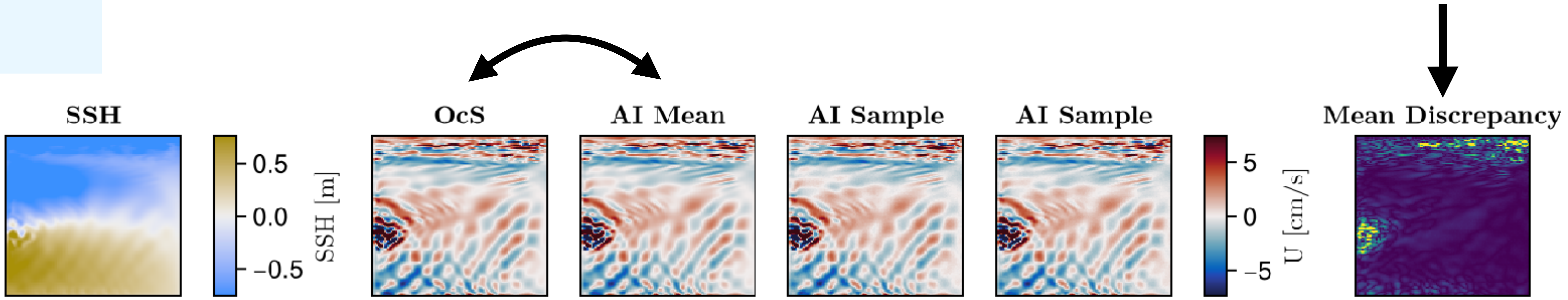

Application: Ocean Inference Result (400m depth)

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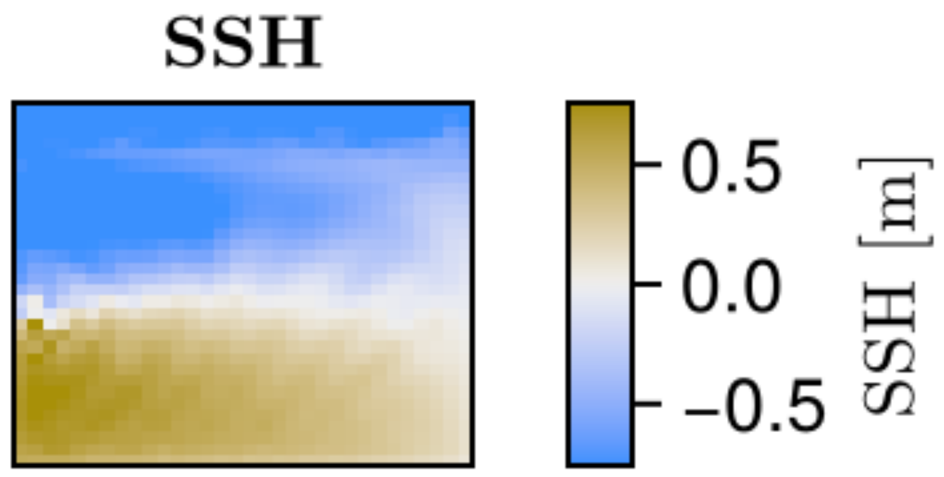
Application: Ocean Inference Result (400m depth)

(with Coarse-Graining)

Application: Ocean Inference Result (400m depth)



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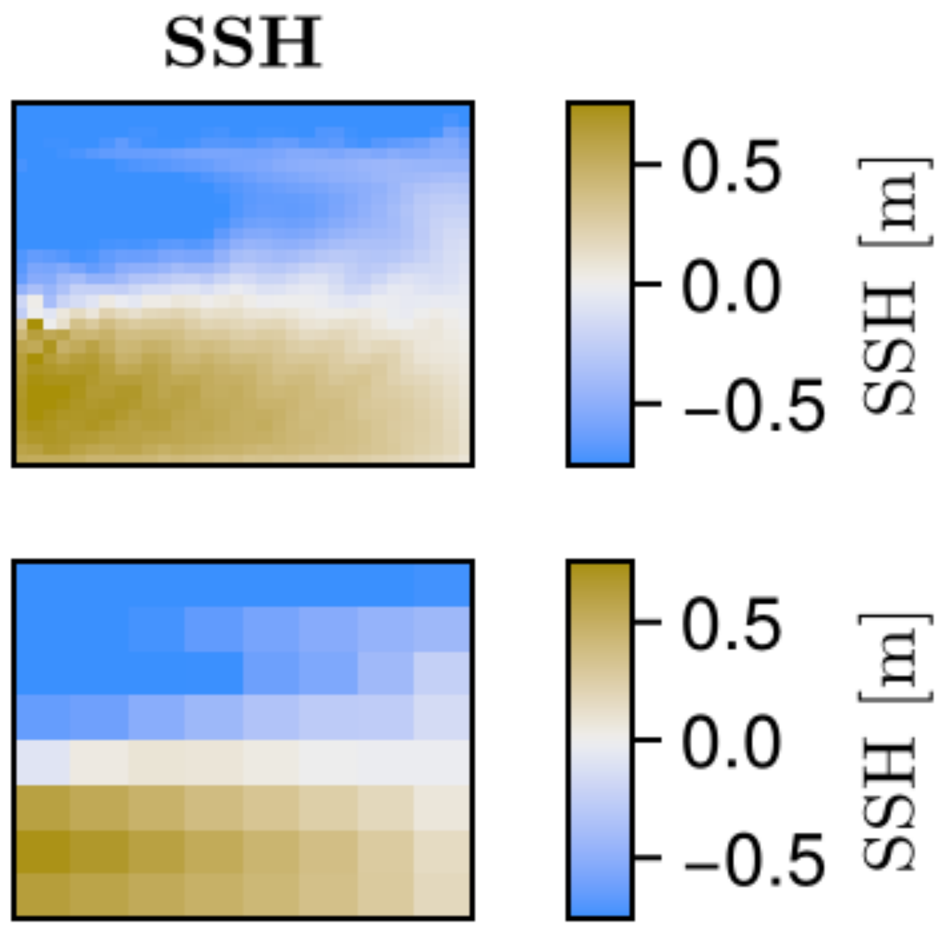
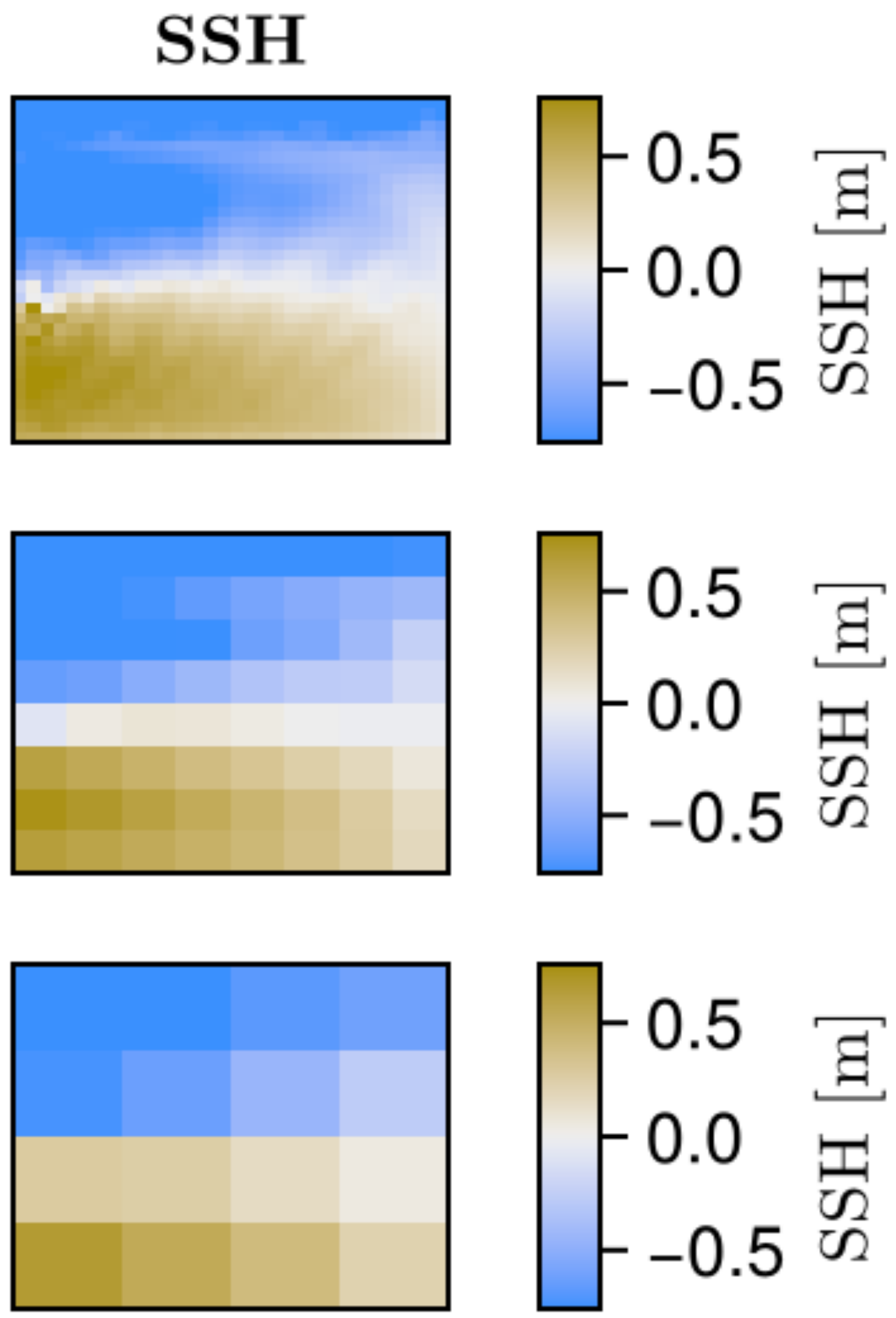


Application: Ocean Inference Result (400m depth)



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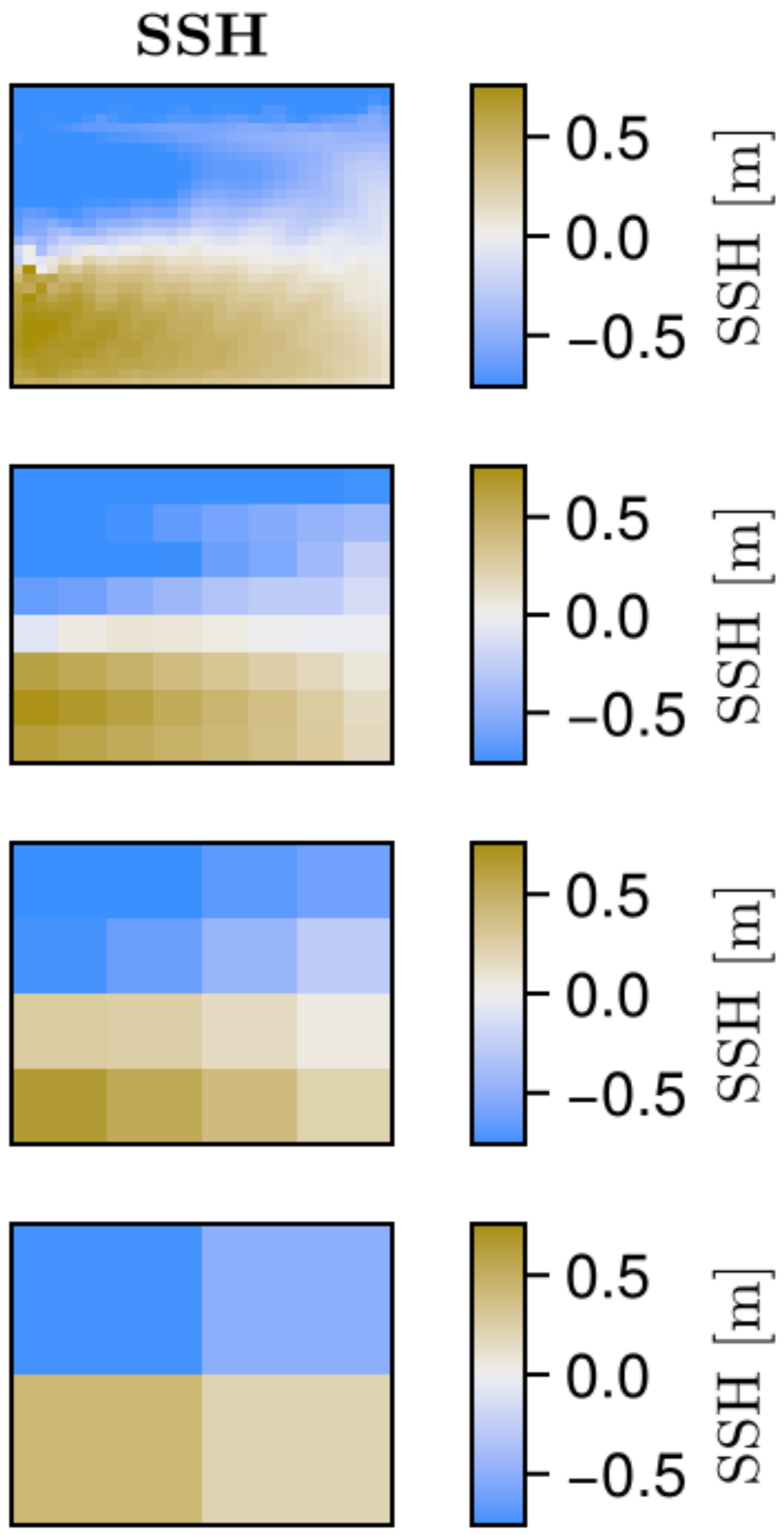

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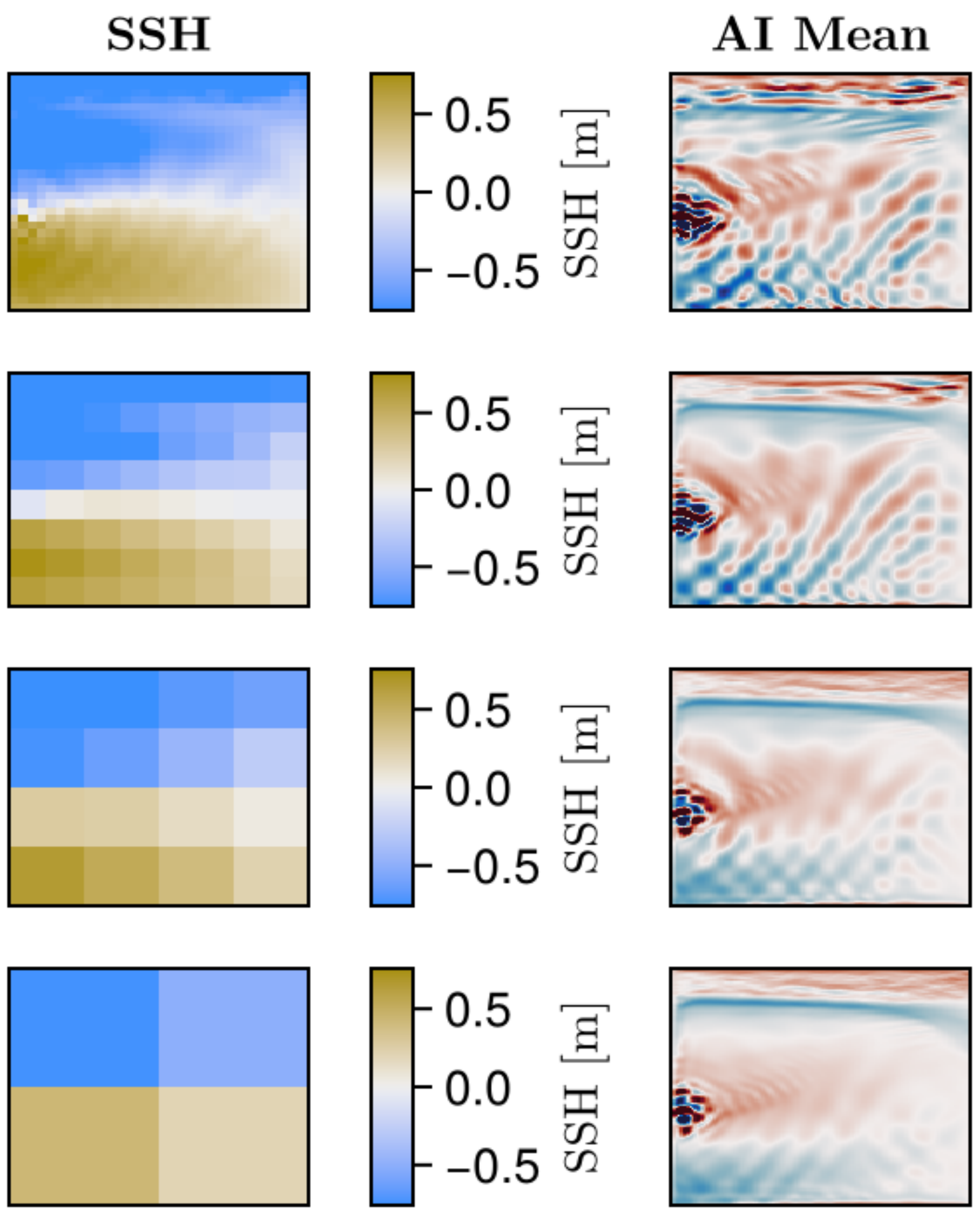

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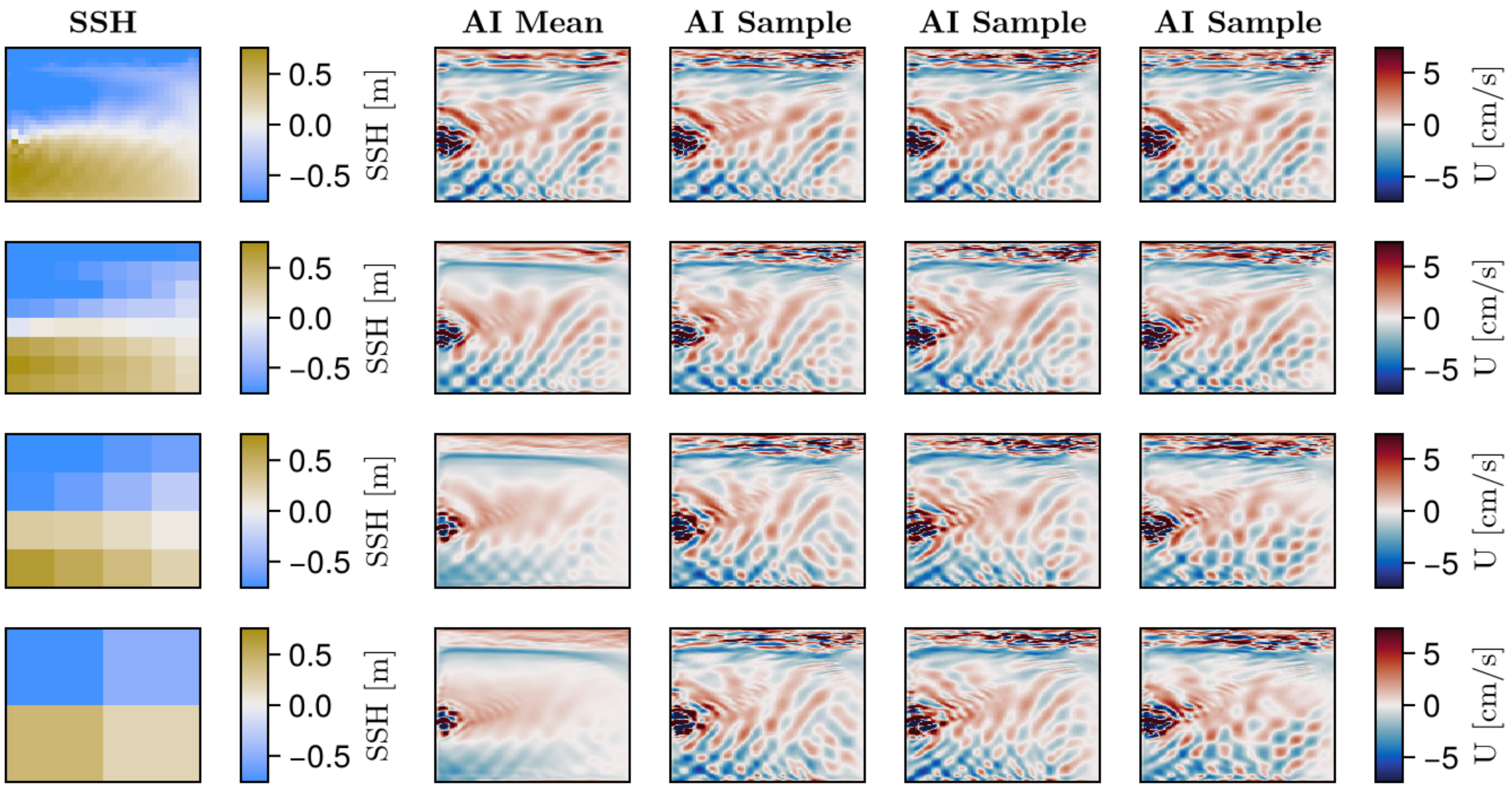

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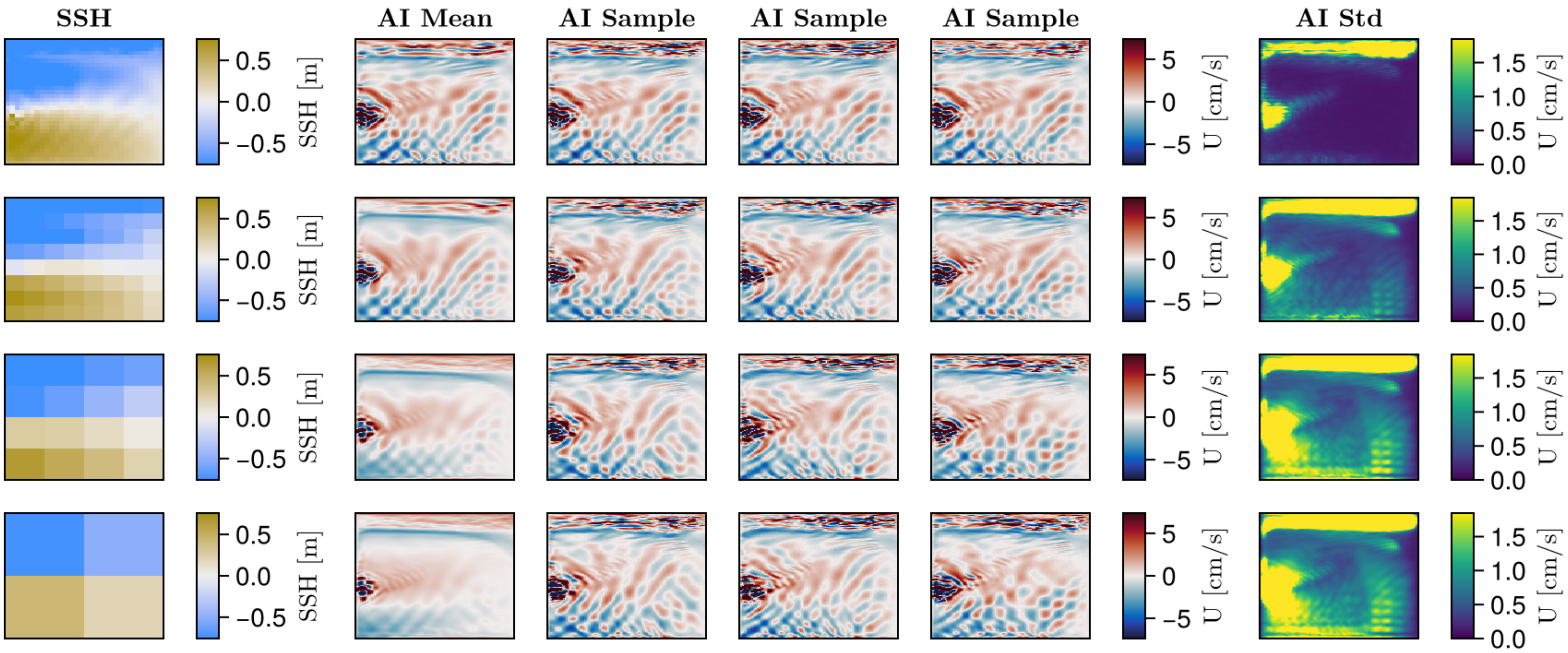
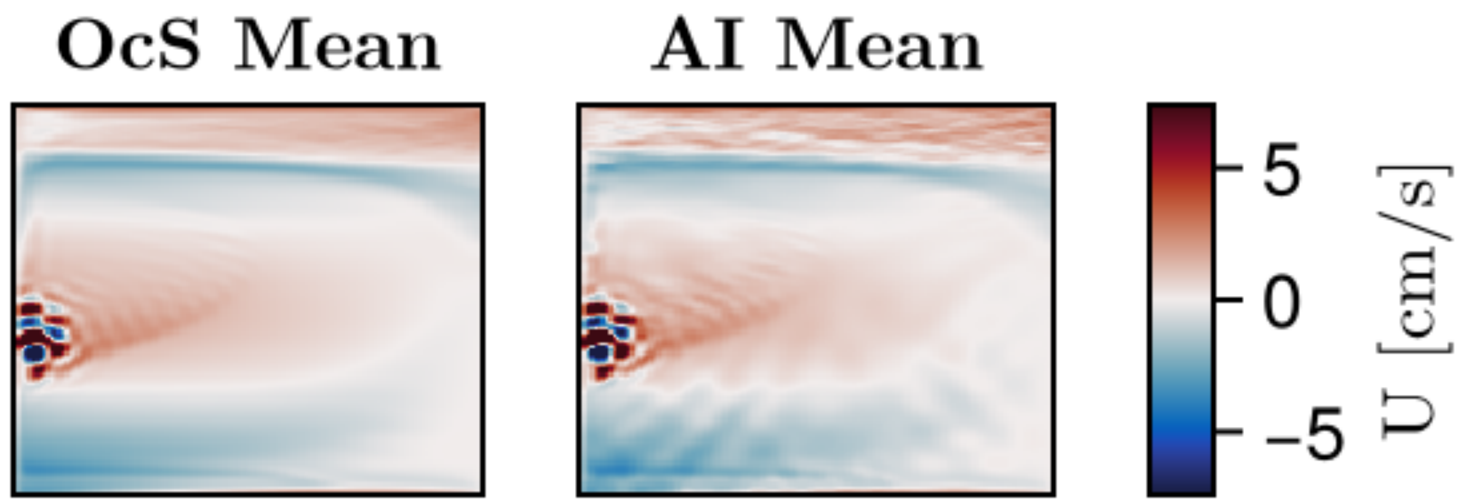


Application: Ocean Inference Result (400m depth)

(with Extreme Coarse-Graining)

Application: Regression to Ensemble Mean



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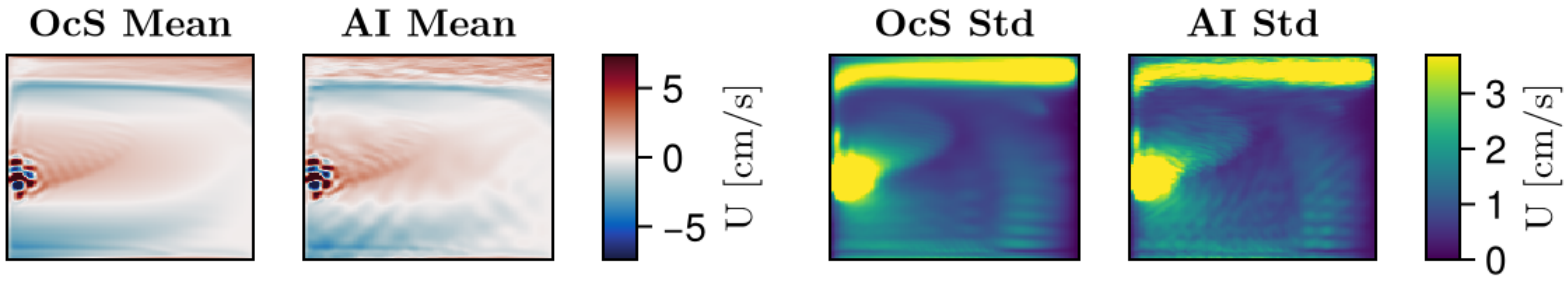

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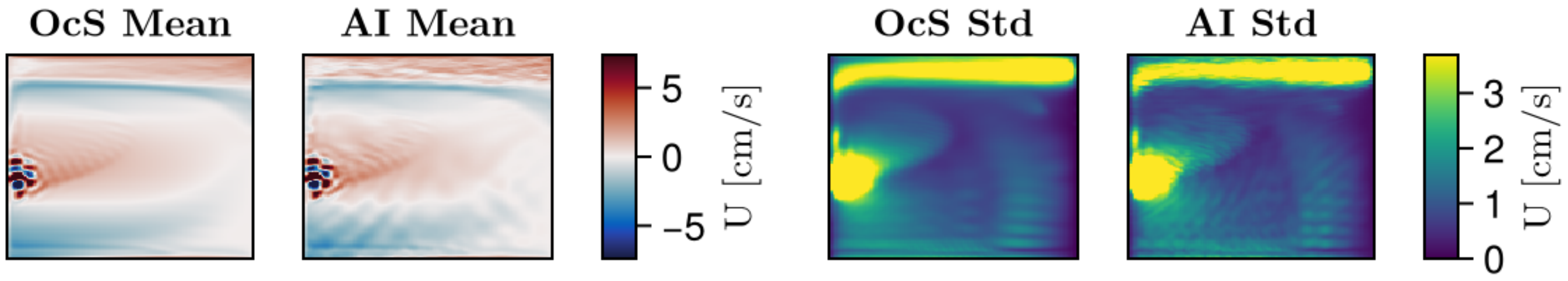
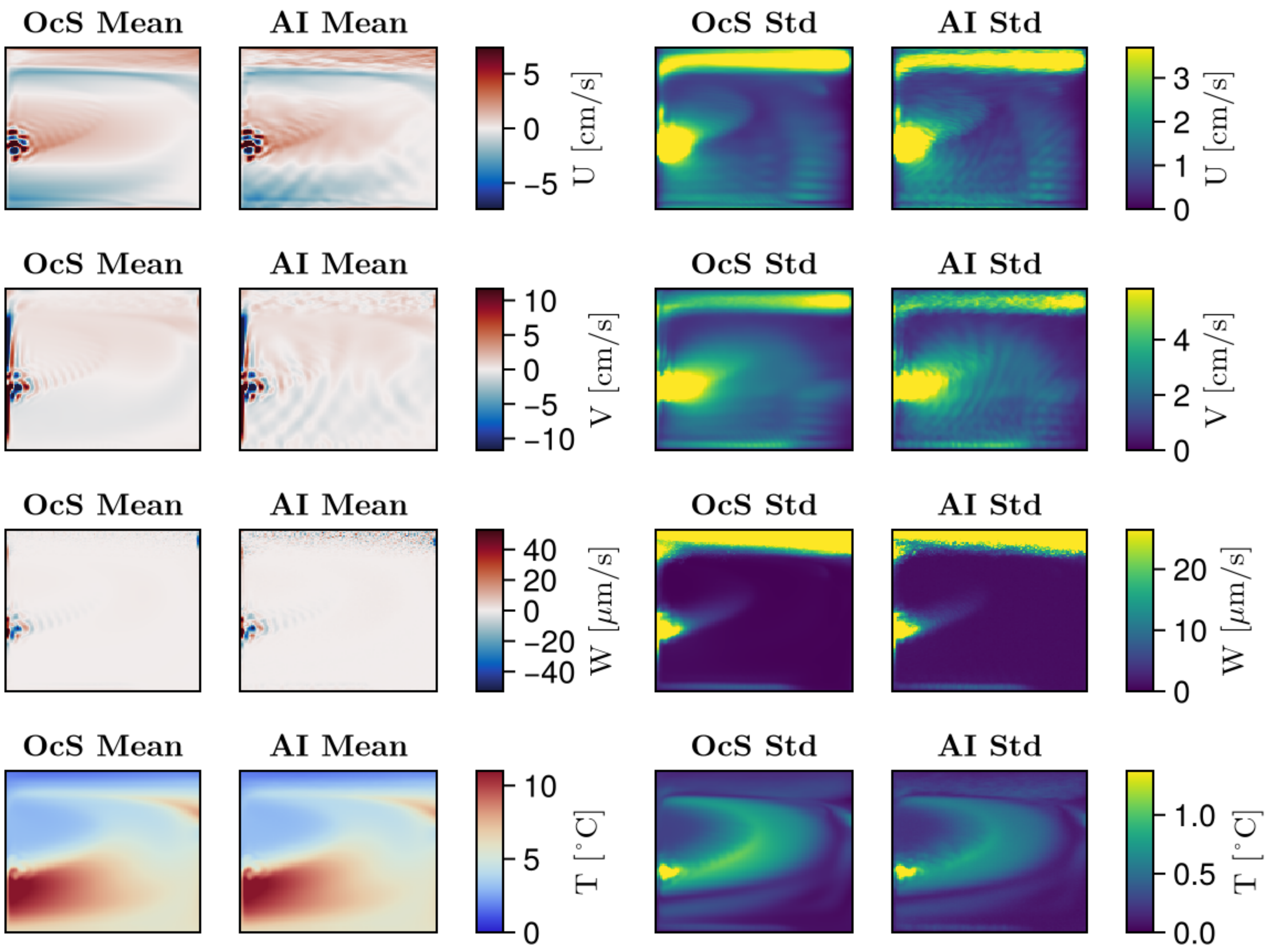


$$\rho(y|x) = \rho(y)$$

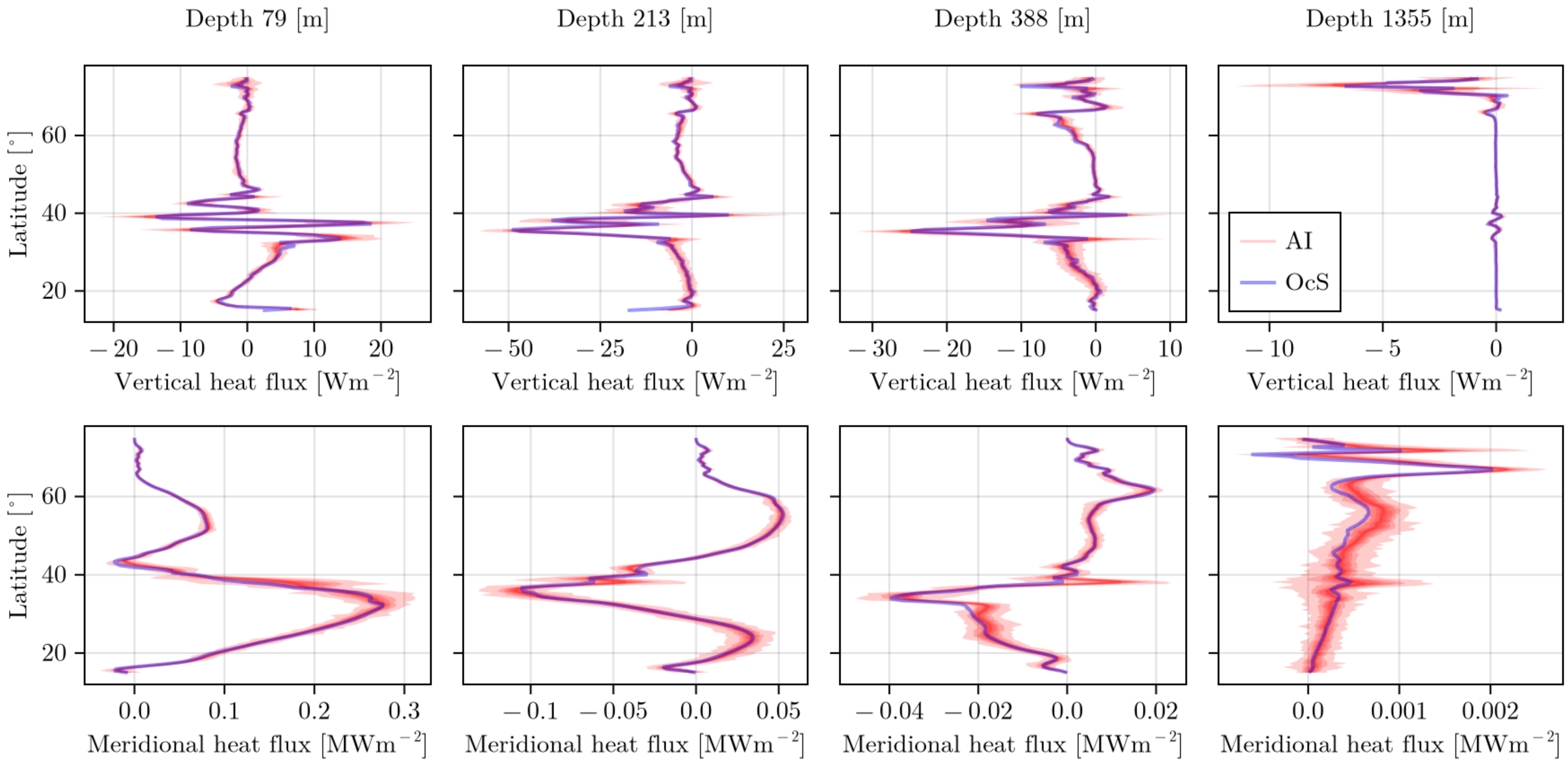
Application: Regression to Ensemble Mean



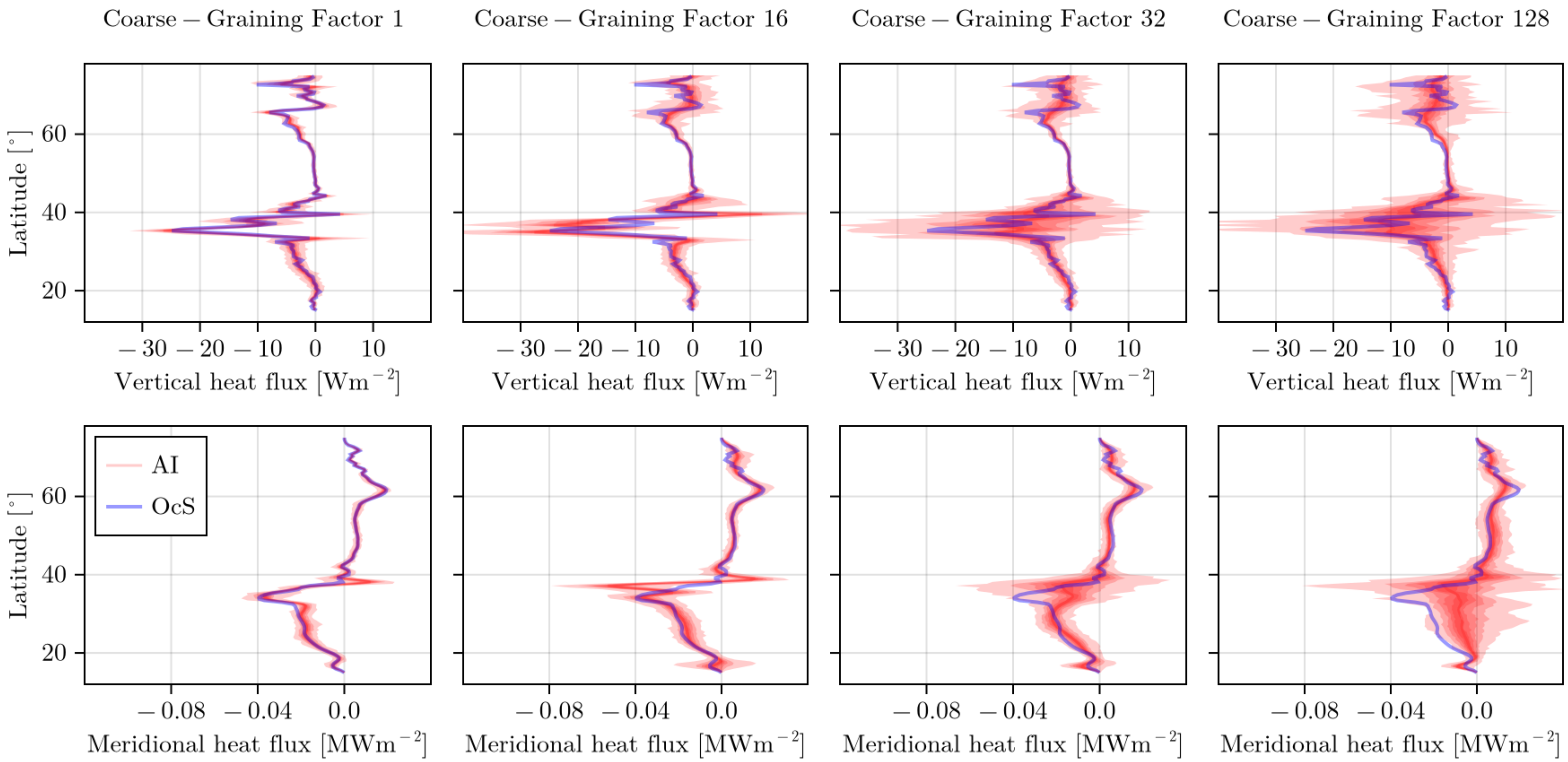
$$\rho(y|x) = \rho(y)$$

Results: Covariances

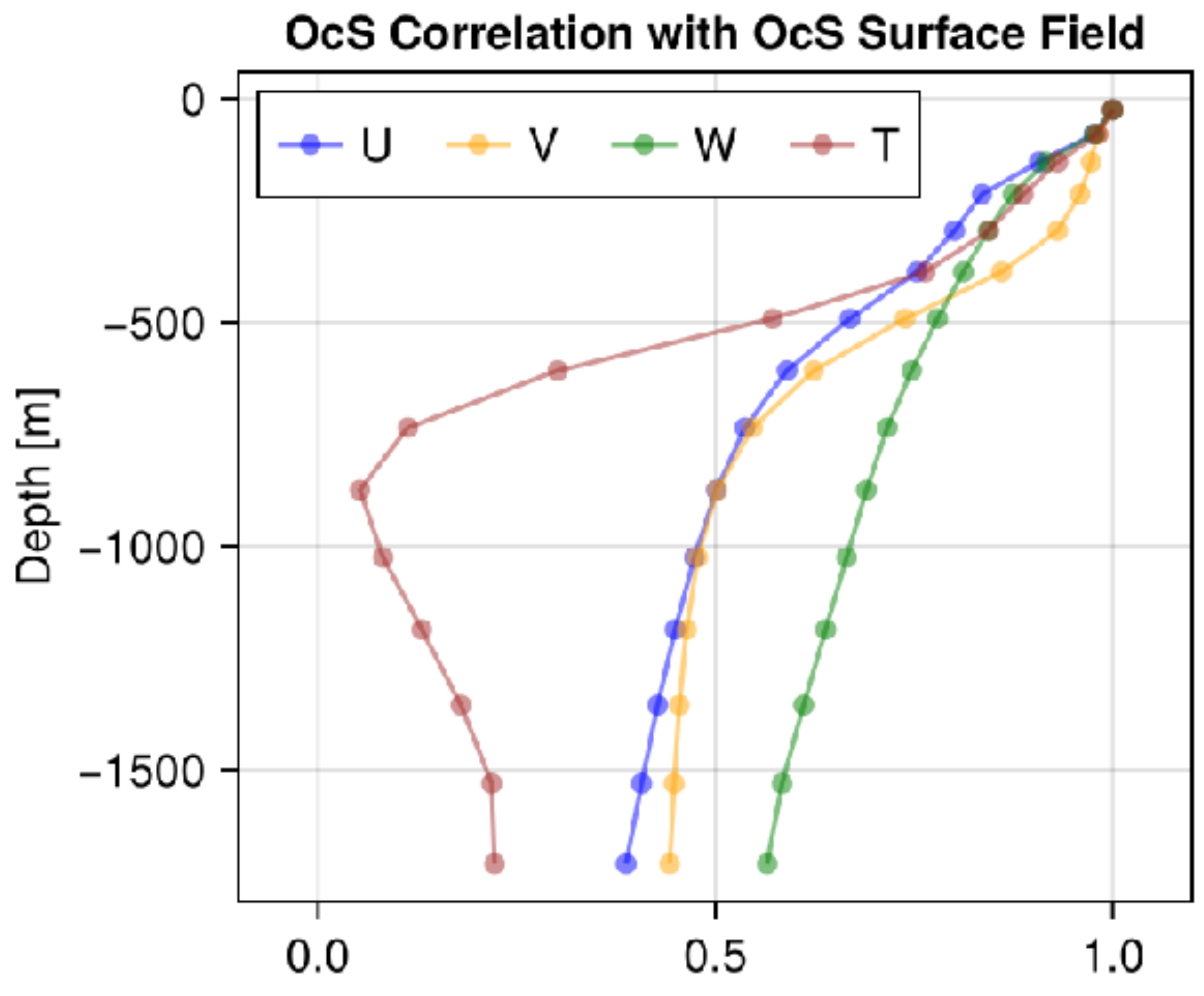
Application: Ocean Inference Covariances

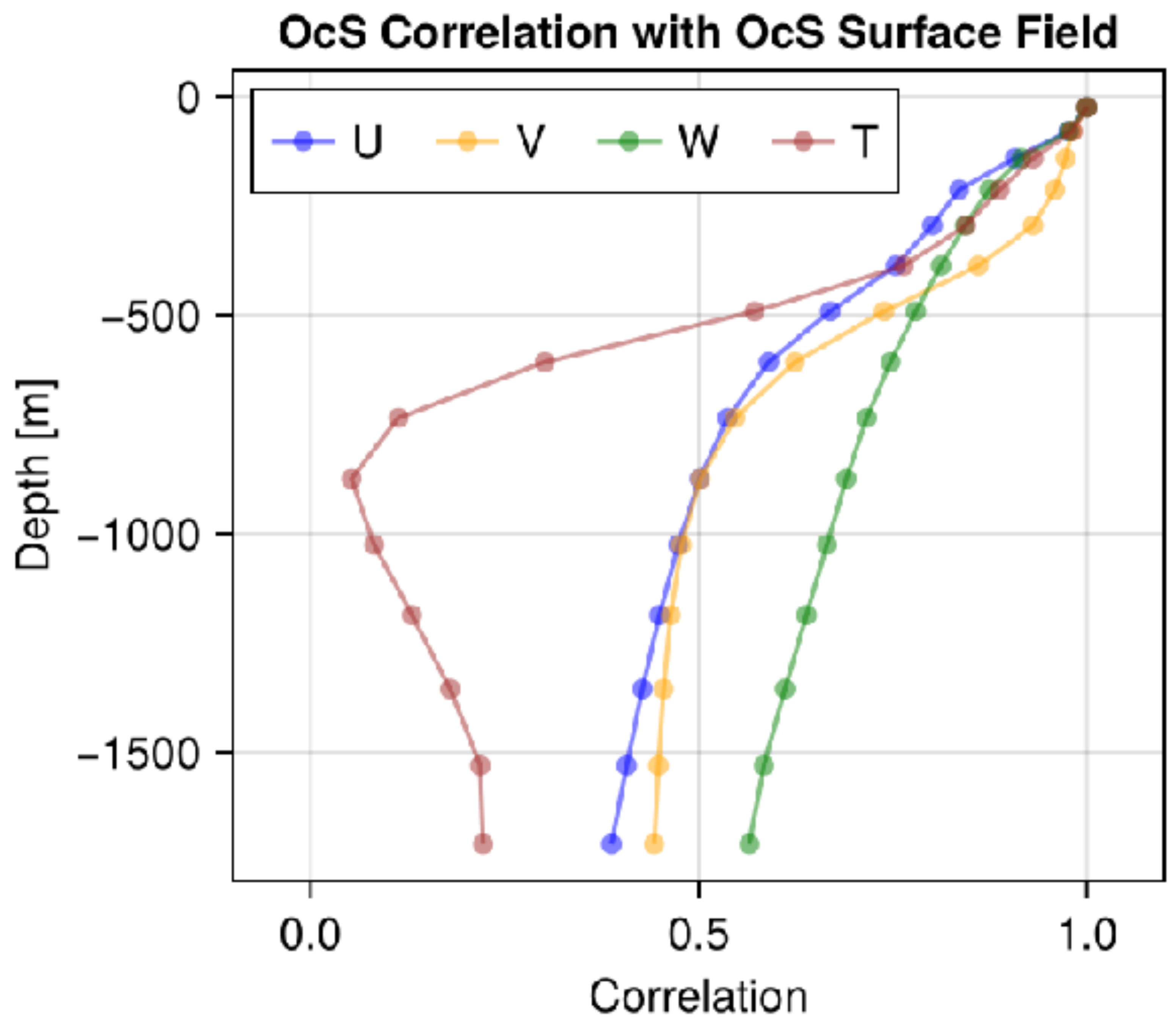


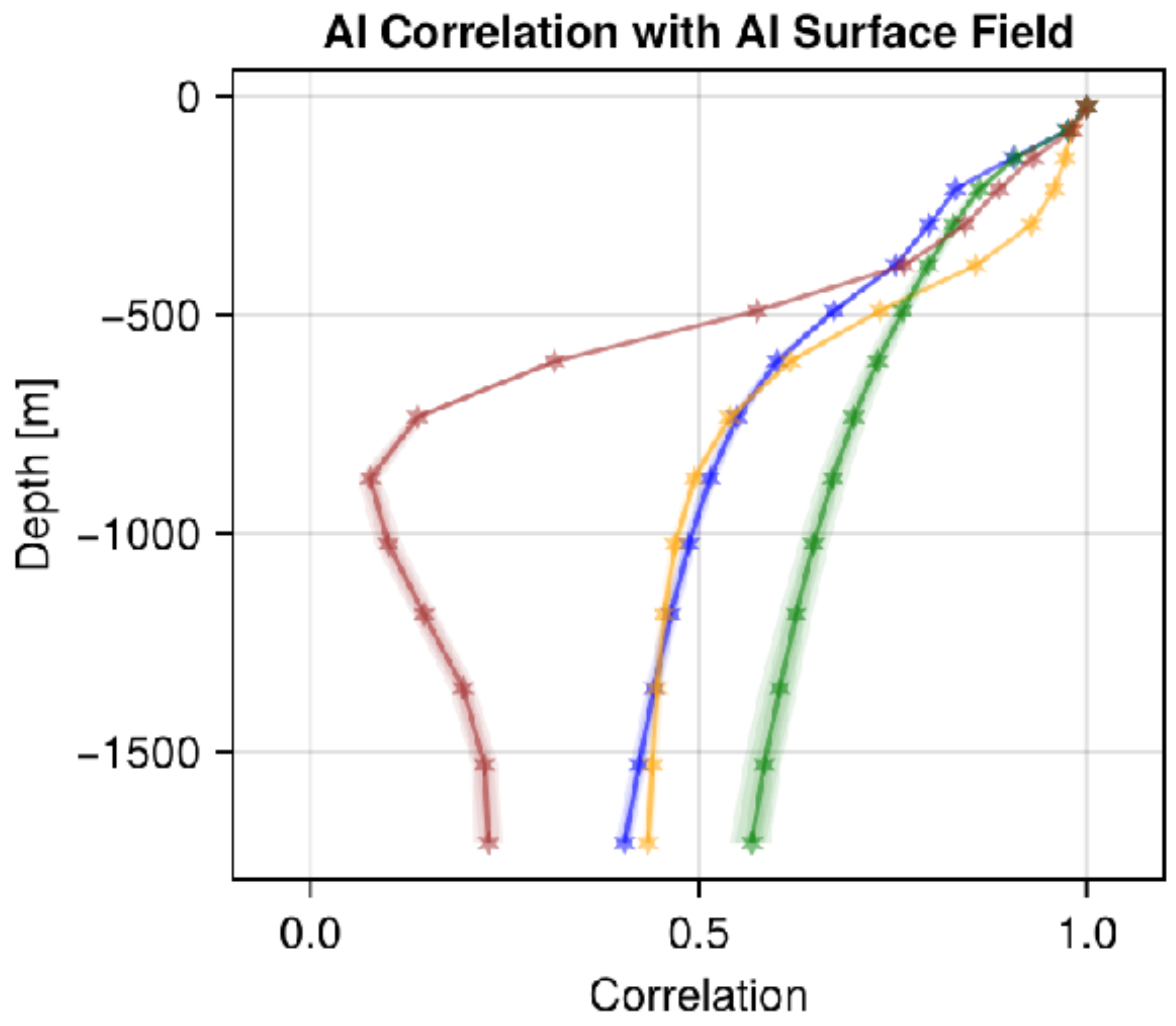
Application: Ocean Inference Covariances

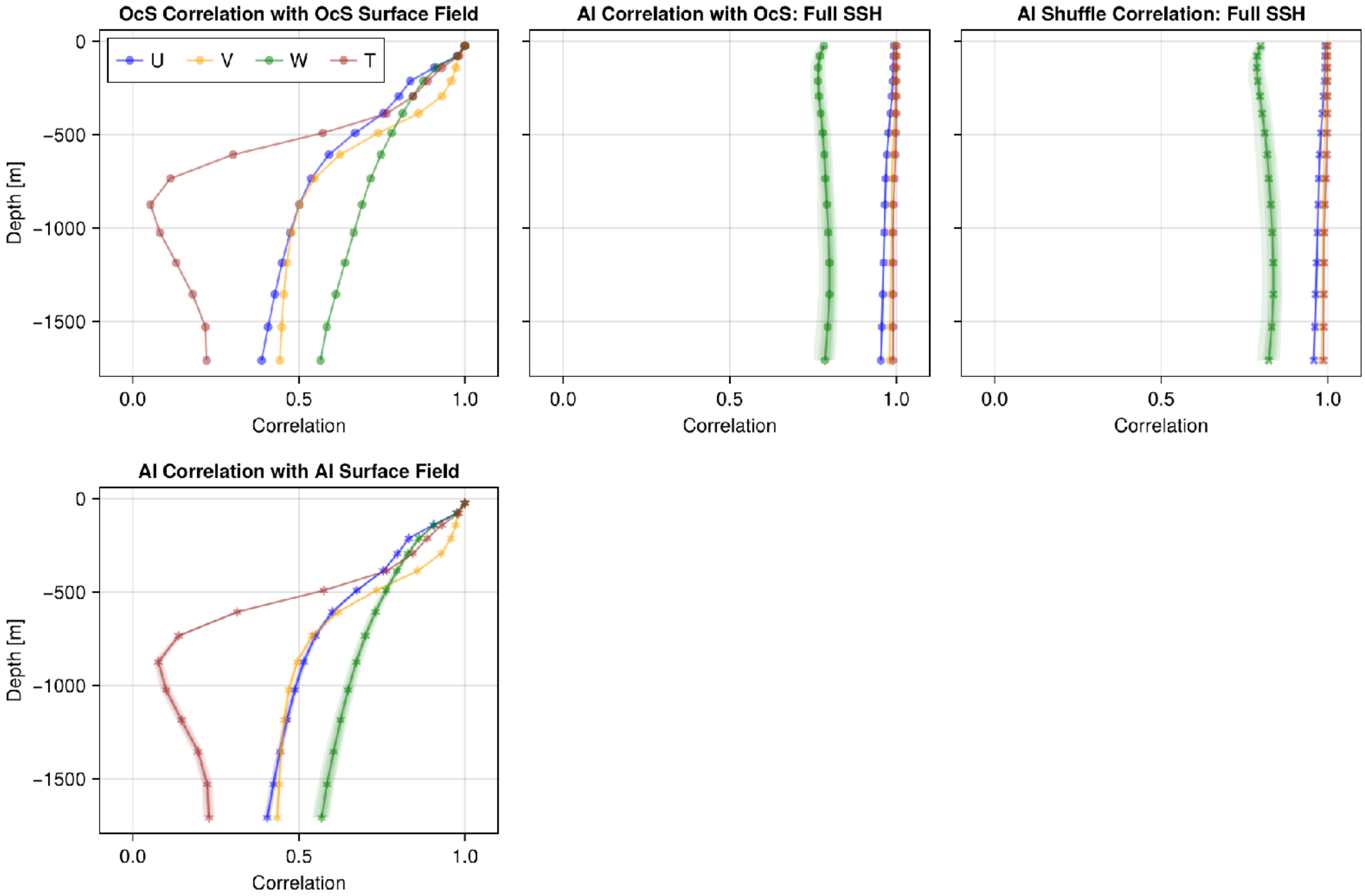


Results: Depth Correlations

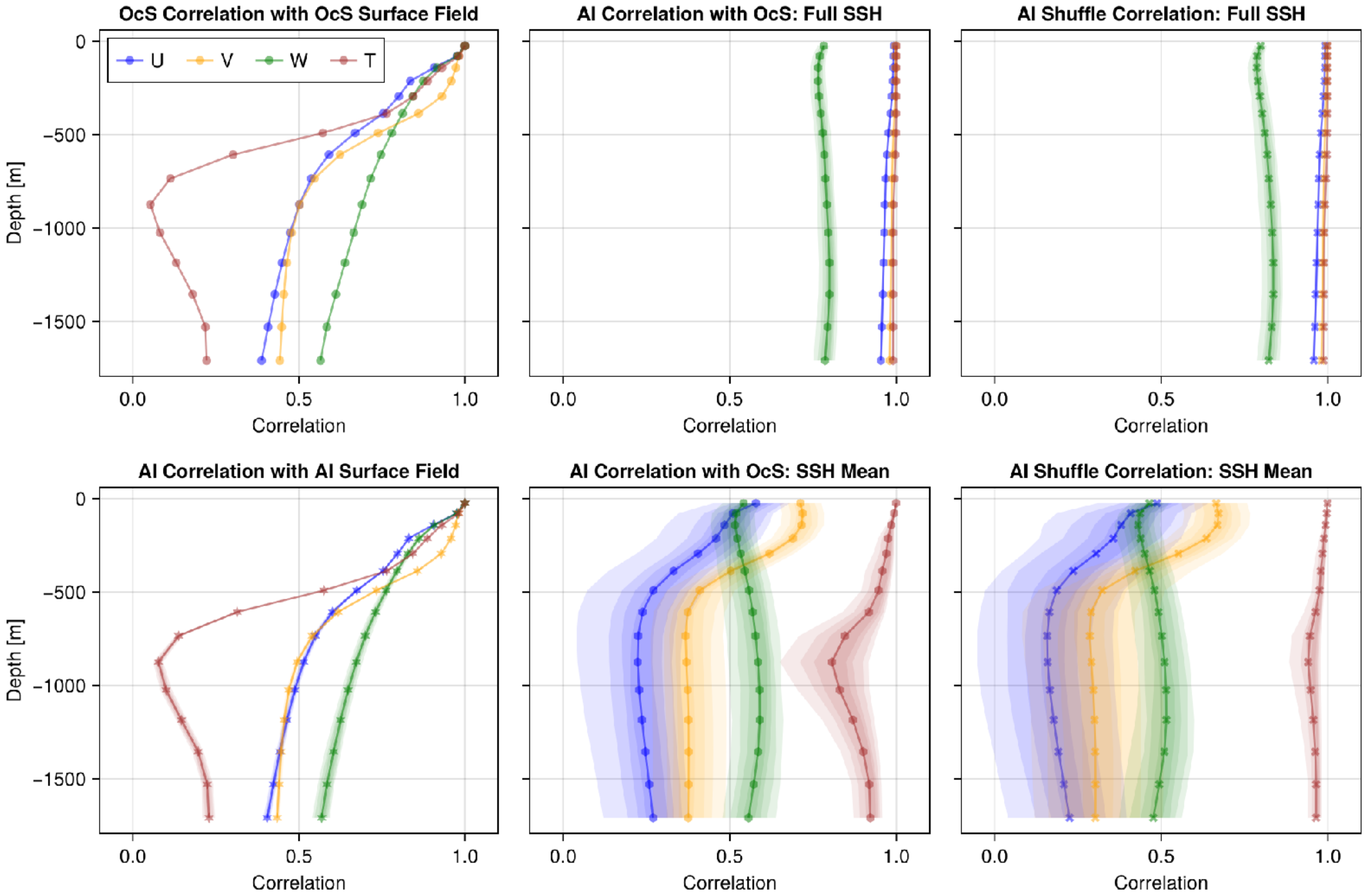

OcS Correlation with OcS Surface Field
U
V
W
T
Depth [m]
0
−500
−1000
−1500
0.0
0.5
1.0


OcS Correlation with OcS Surface Field
U
V
W
T
Depth [m]
0
−500
−1000
−1500
0.0
0.5
1.0
Correlation


AI Correlation with AI Surface Field
Depth [m]
0
−500
−1000
−1500
0.0
0.5
1.0
Correlation

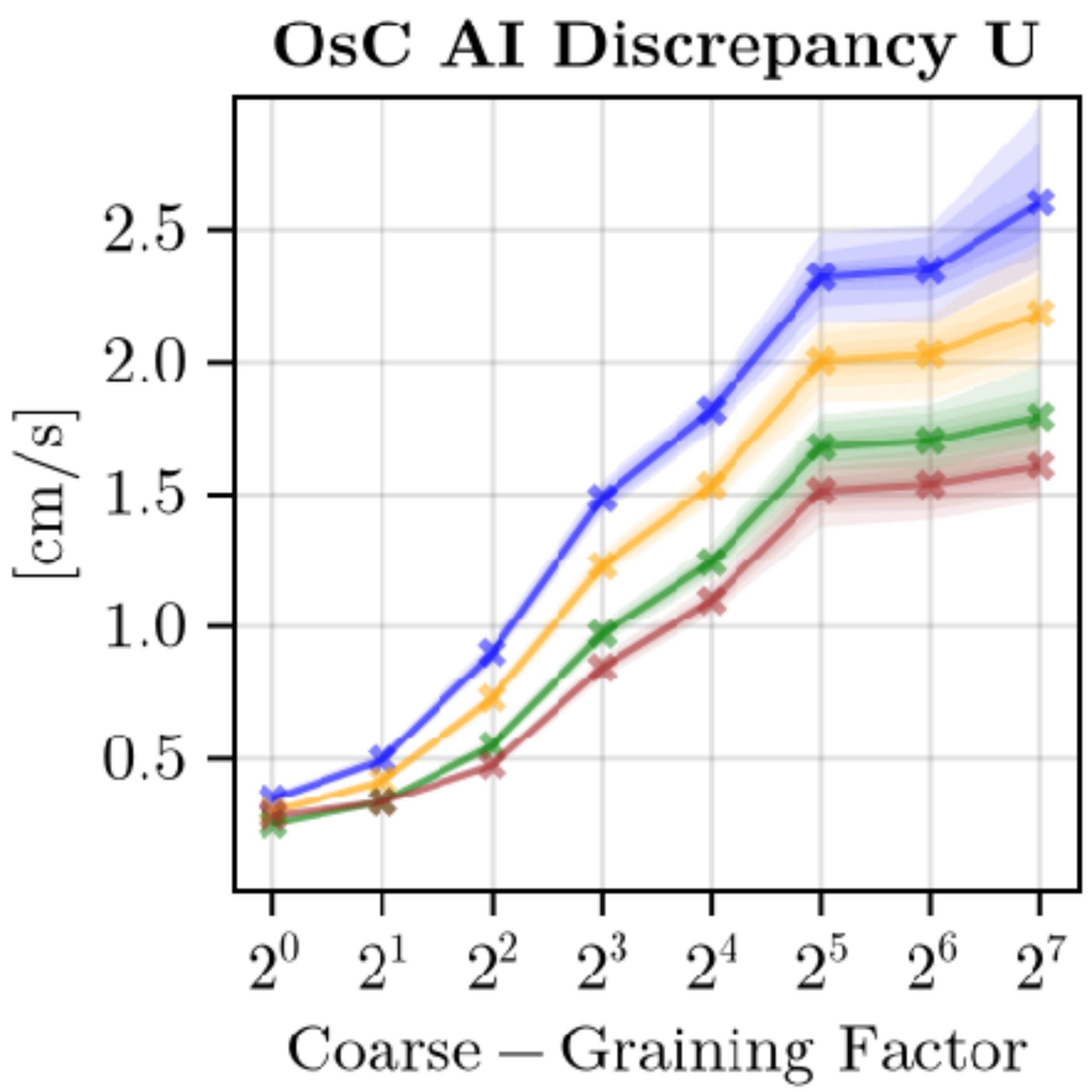


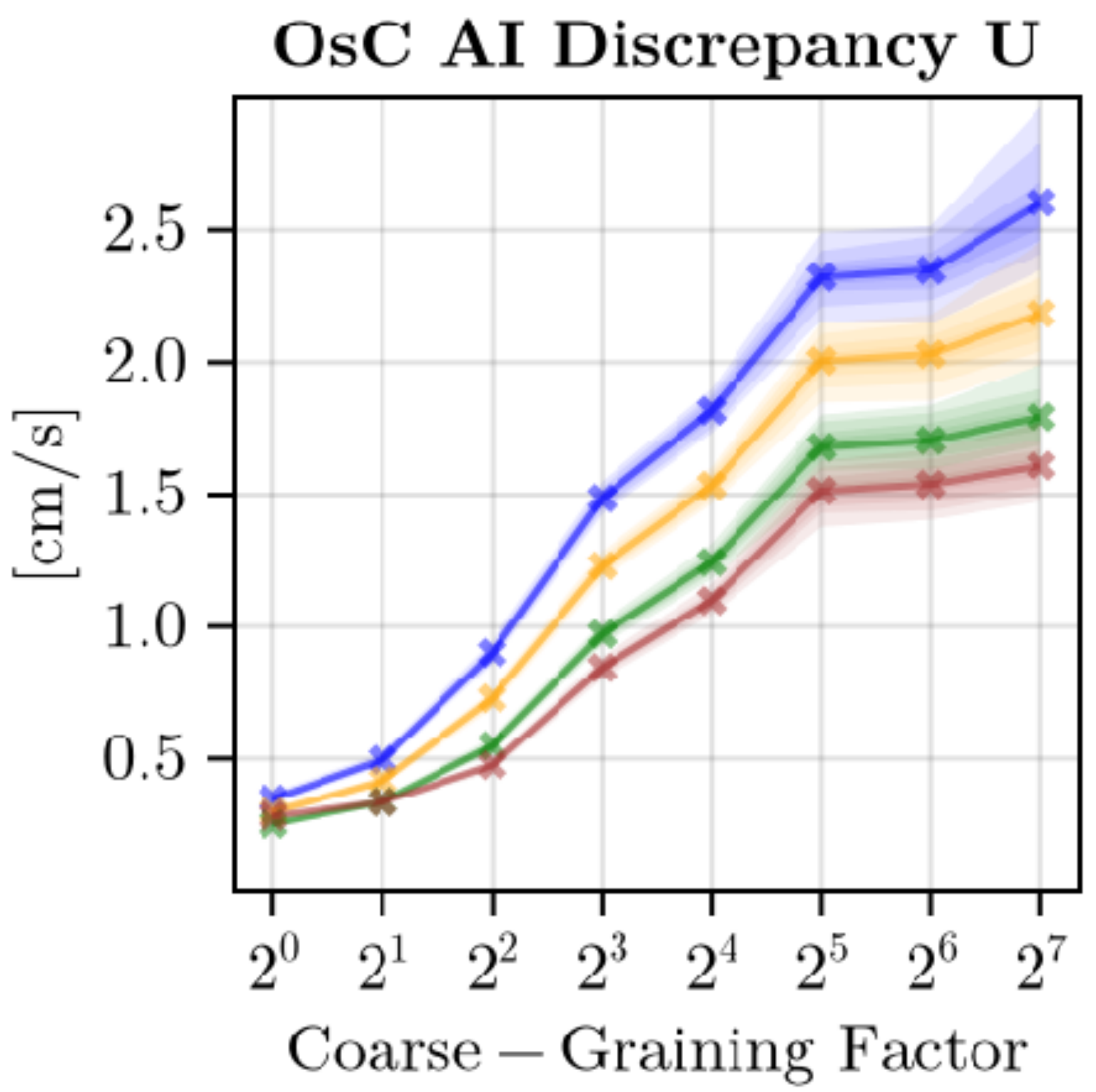
OcS Correlation with OcS Surface Field
AI Correlation with OcS: Full SSH
AI Shuffle Correlation: Full SSH
AI Correlation with AI Surface Field
U
V
W
T
Depth [m]
Correlation
0
−500
−1000
−1500
0.0
0.5
1.0



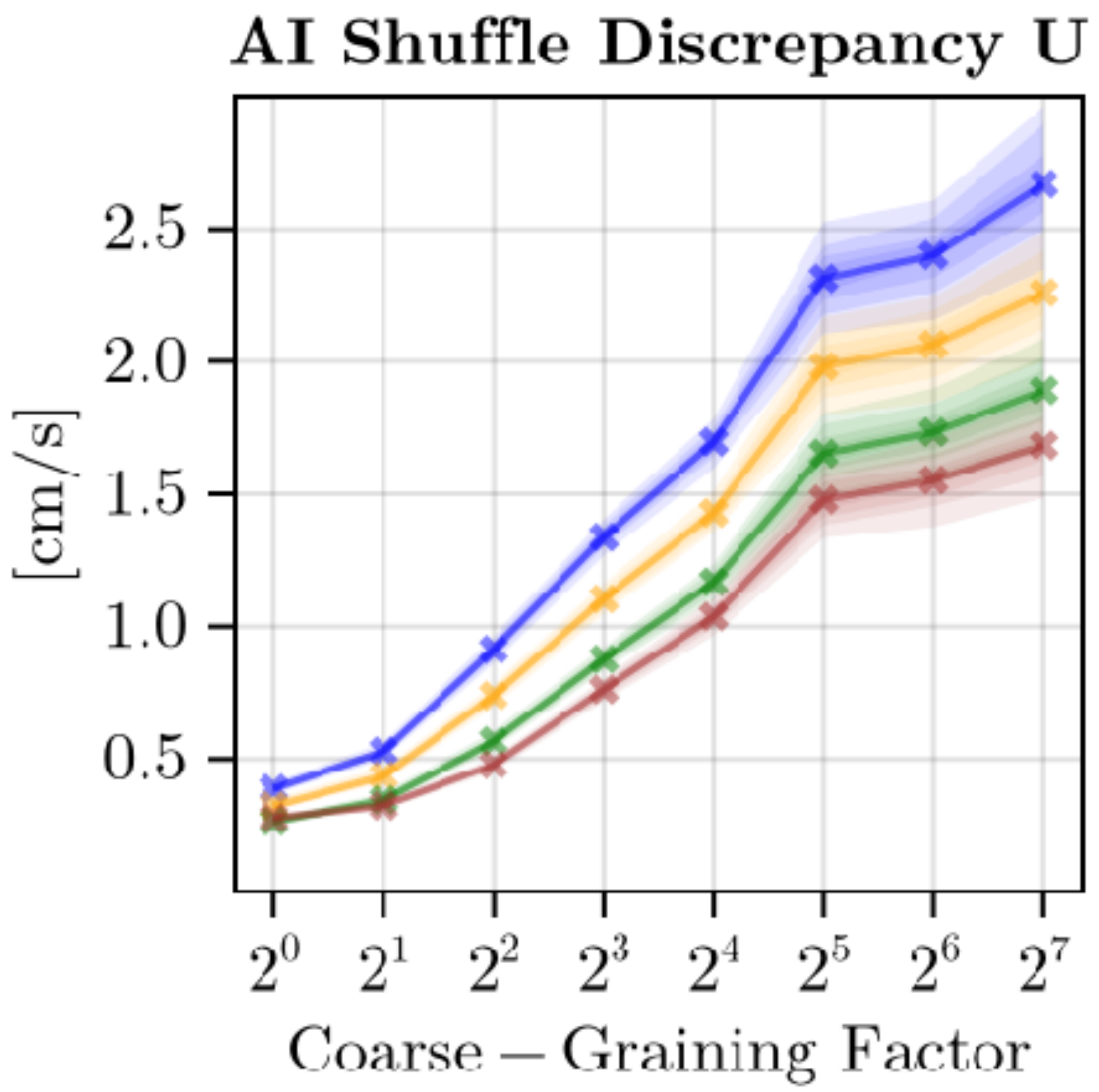
OcS Correlation with OcS Surface Field
AI Correlation with OcS: Full SSH
AI Shuffle Correlation: Full SSH
AI Correlation with AI Surface Field
AI Correlation with OcS: SSH Mean
AI Shuffle Correlation: SSH Mean
U
V
W
T
Depth [m]
0
−500
−1000
−1500
0.0
0.5
1.0
Correlation

Results: Discrepancies

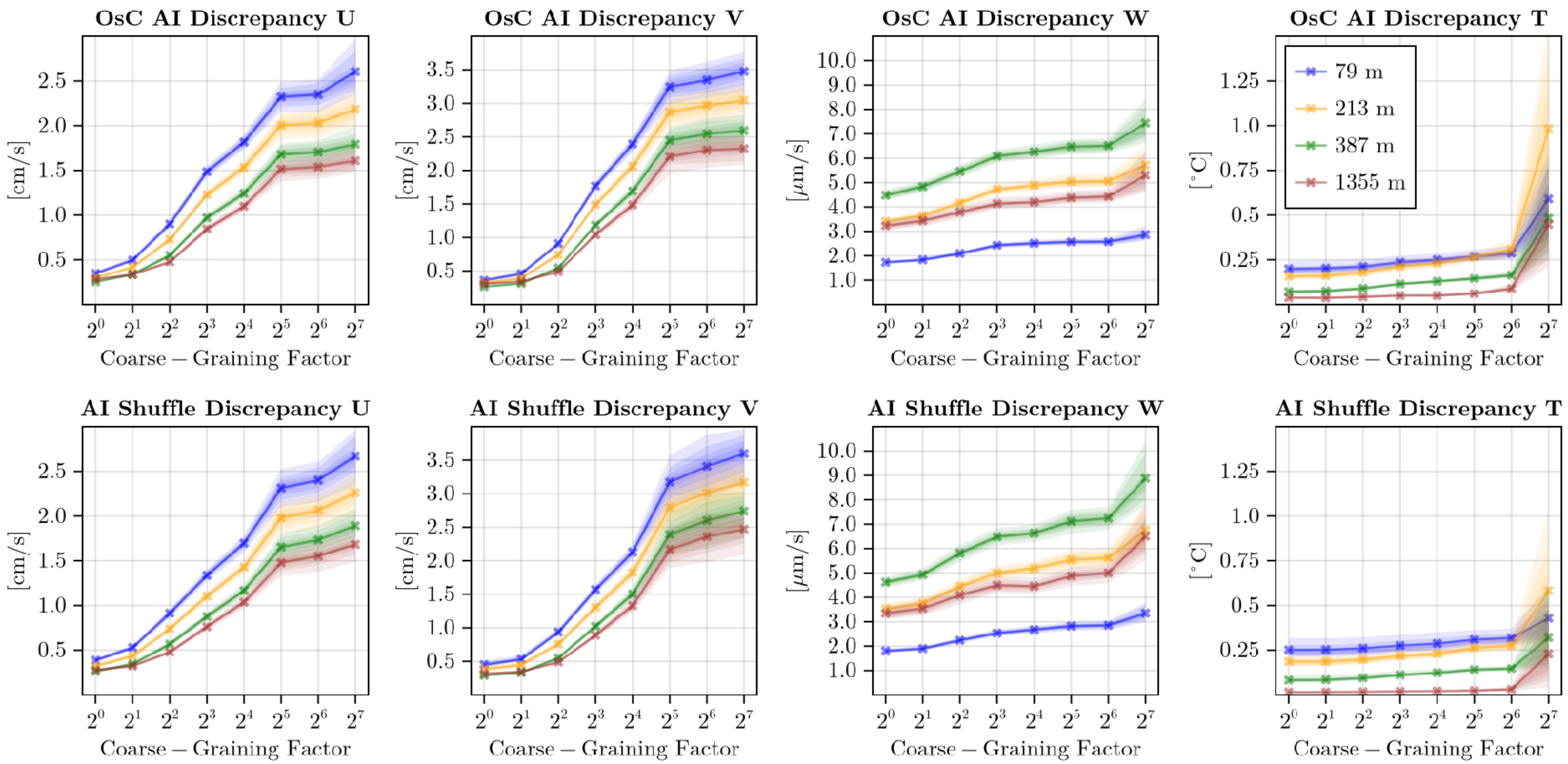

OsC AI Discrepancy U
[cm/s]
2.5
2.0
1.5
1.0
0.5
2^0 2^1 2^2 2^3 2^4 2^5 2^6 2^7
Coarse – Graining Factor



OsC AI Discrepancy U
[cm/s]
2.5
2.0
1.5
1.0
0.5
2^0 2^1 2^2 2^3 2^4 2^5 2^6 2^7
Coarse − Graining Factor

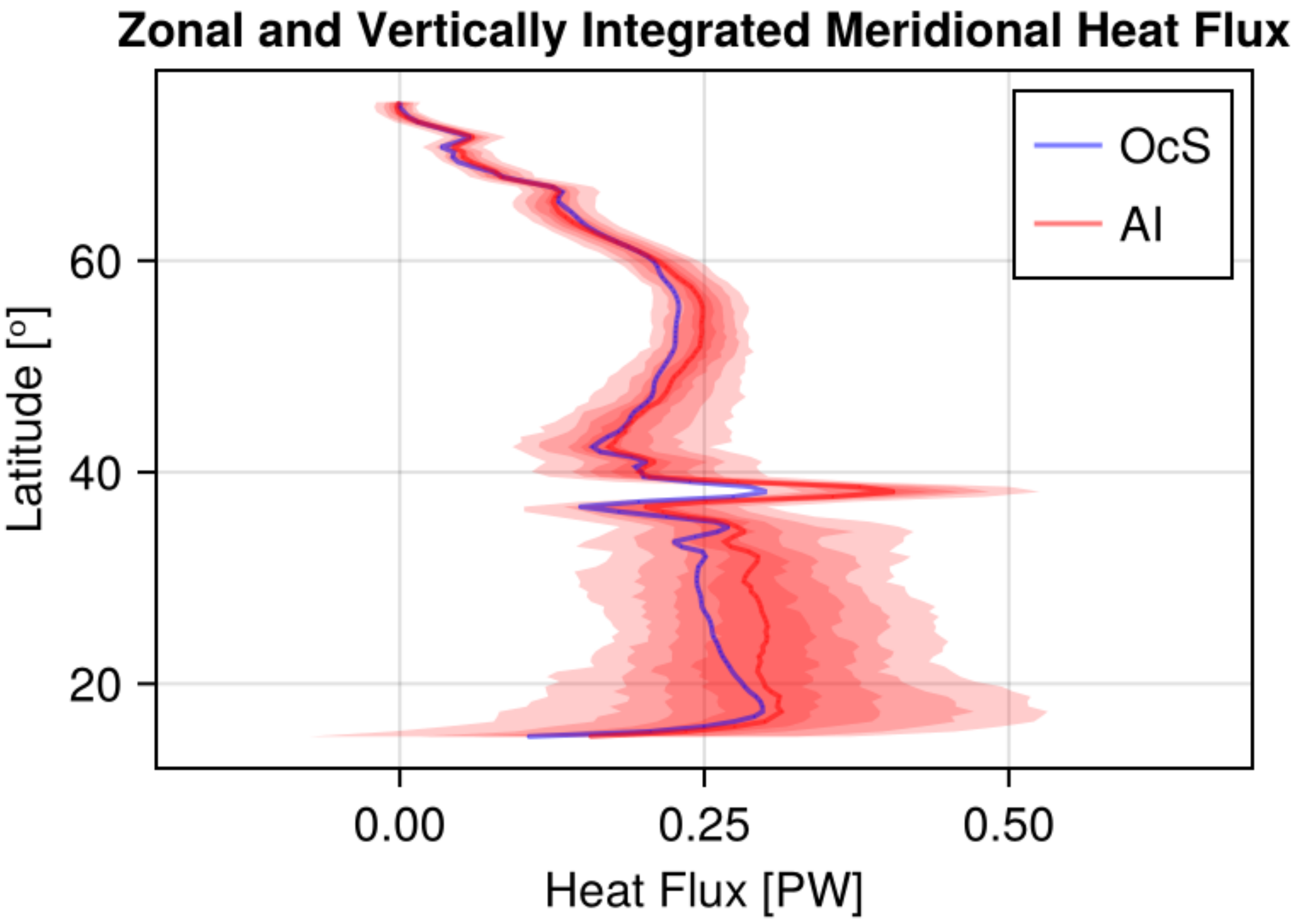


AI Shuffle Discrepancy U
[cm/s]
2.5
2.0
1.5
1.0
0.5
2^0 2^1 2^2 2^3 2^4 2^5 2^6 2^7
Coarse − Graining Factor


OsC AI Discrepancy U
OsC AI Discrepancy V
OsC AI Discrepancy W
OsC AI Discrepancy T
AI Shuffle Discrepancy U
AI Shuffle Discrepancy V
AI Shuffle Discrepancy W
AI Shuffle Discrepancy T
[cm/s]
[μm/s]
[°C]
Coarse – Graining Factor
79 m
213 m
387 m
1355 m

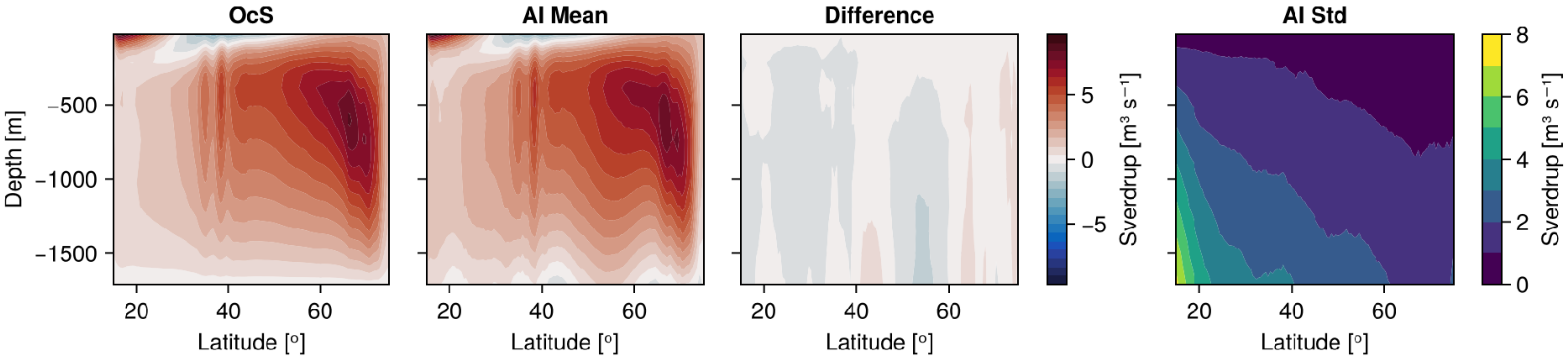
Results: Heat Transport

Application: Ocean Inference Combining Depths



Results: Overturning

Application: Ocean Inference Combining Depths



Take aways

- Questions in Geophysics should be reformulated as probabilistic inference questions

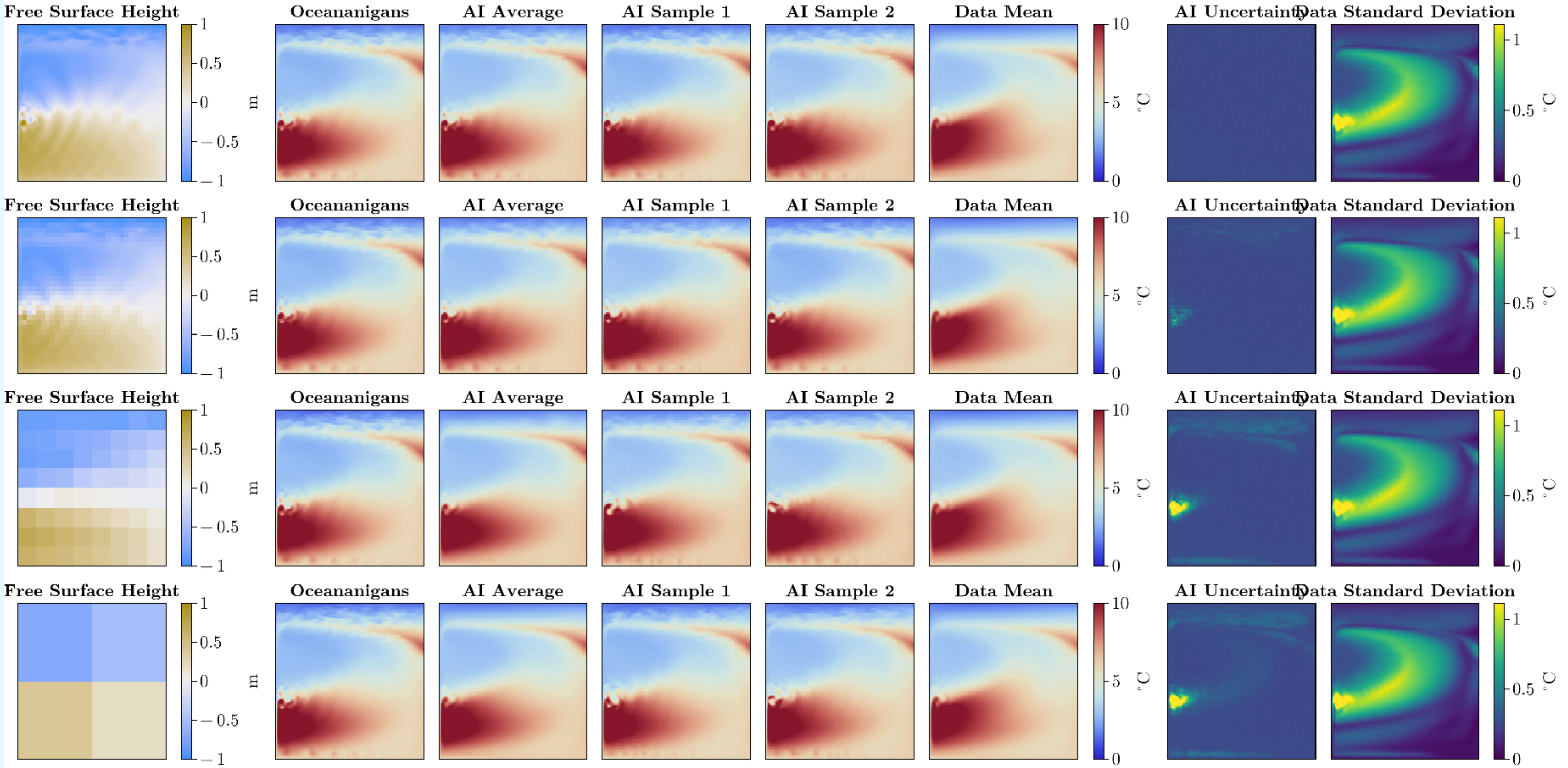
- Gen AI Provides Meaningful Uncertainty Estimates

Questions?

Drawbacks

- Expensive to Evaluate
- Expensive to Train
- Harder to understand the theory of why it works
- Harder for interpretability
- Harder to evaluate
- Requires modifying old / designing new architectures

Application: Ocean Inference Result (400m depth)



Brief Overview of Score-Matching

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Empirical Distribution

$$\rho(x) = \frac{1}{N}\sum_{n=1}^{N}\delta(x - x^n)$$

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Regularize

$$\rho_\sigma(x) = \frac{1}{N\sqrt{2\pi\sigma^2}} \sum_{n=1}^{N} \exp(-(x - x^n)^2/(2\sigma^2))$$

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Derivative of Regularized

$$\nabla \ln \rho_\sigma(x) = \frac{\sum_{n=1}^{N} (x^n - x) \exp(-(x - x^n)^2/(2\sigma^2))}{\sum_{m=1}^{N} \exp(-(x - x^m)^2/(2\sigma^2))}$$

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Neural-Network implicit regularization

$$\mathbb{E}|\nabla \ln \rho_\sigma(x) - N_\theta(x)|^2$$

Brief Overview of Score-Matching

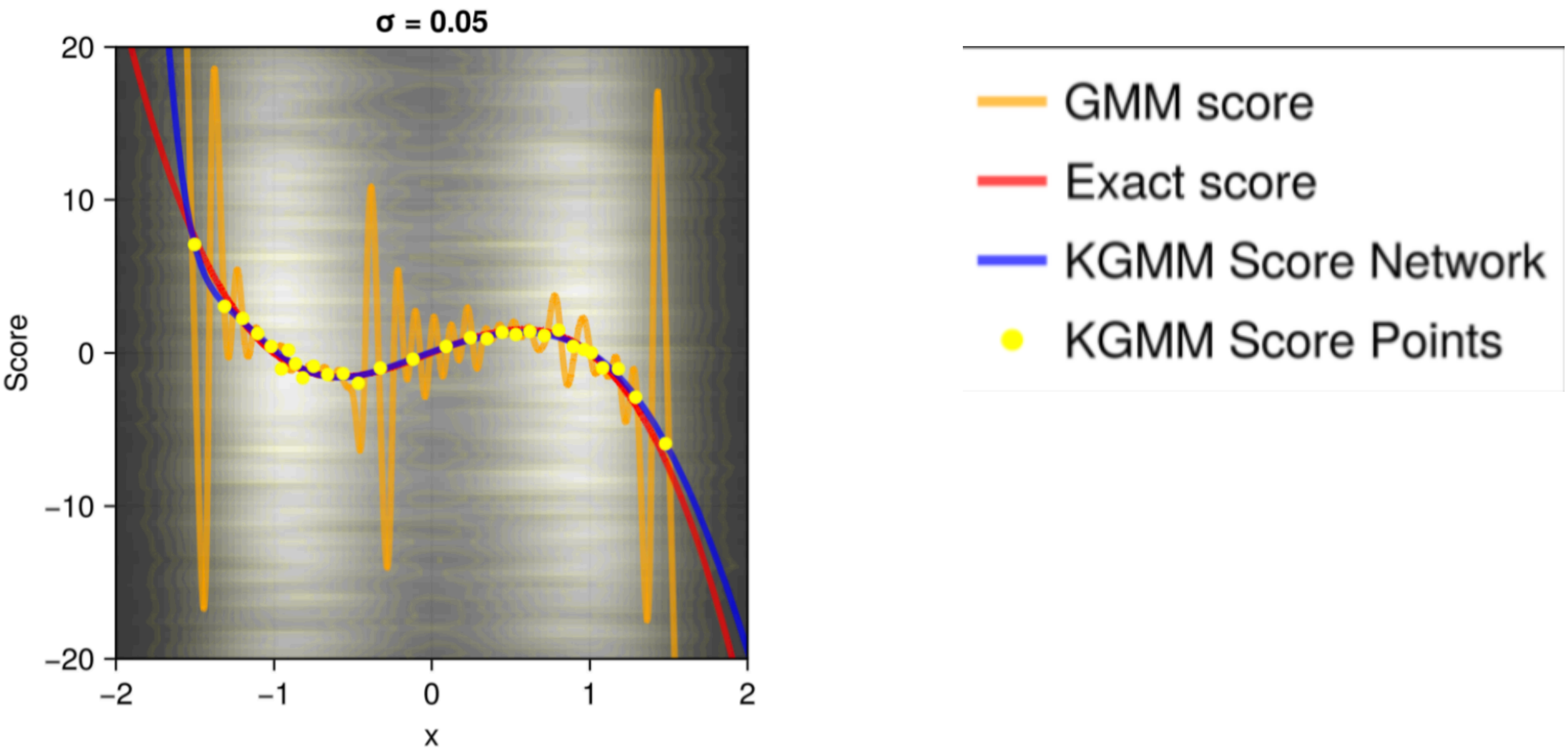
Instead of minimizing

$$\mathbb{E}|\nabla \ln \rho_\sigma(x) - N_\theta(x)|^2$$

We minimize

$$\mathbb{E}_x \mathbb{E}_Z |Z/\sigma + N_\theta(x + \sigma Z)|^2$$

Brief Overview of Score-Matching



Arxiv, Giorgini, Bischoff, Souza:
KGMM: A K-means Clustering Approach to GaussianMixture Modeling for Score Function Estimation