SCHOOL OF ENGINEERING
Faculty of Science and Engineering

MACQUARIE University

ELEC4844/8844 Practical Class – Week 1, 2024

Review of MATLAB

TASK 1

Write MATLAB functions to generate the following matrix with input N:

$$\begin{bmatrix} 0 & N & \cdots & N(N-1) \\ 1 & N+1 & \cdots & N(N-1)+1 \\ \vdots & \vdots & \ddots & \vdots \\ N-1 & 2N-1 & \cdots & N^2-1 \end{bmatrix}$$

a) using loop;

b) by reshaping a matrix;

c) based on matrix computation.

Verify your functions using $N = 6$.

TASK 2

a) Generate 1000 normally distributed random numbers ε_i, $i = 1, \ldots, 1000$, with mean of 0 and standard deviation of 2. Verify the mean and standard deviation.

b) Generate 1000 uniformly distributed random numbers x_i in the interval (0, 10), and compute

$y_i = 2.4x_i + \varepsilon_i$.

c) Plot the data points (x_i, y_i). Find the 10th largest number in y_i , $i = 1, \ldots, 1000$, and its corresponding x_i.

d) For an overdetermined system of equations

$$\beta \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \end{bmatrix}, \text{ or simply } \mathbf{x}^{\mathrm{T}}\beta = \mathbf{y} \text{ in the matrix notation,}$$

the method of least squares finds the best estimate of β as $\hat{\beta} = (\mathbf{x}\mathbf{x}^{\mathrm{T}})^{-1}\mathbf{x}\mathbf{y}$. Use this method to fit the data x_i and y_i generated above to the curve $y = \beta x$. Plot the fitted curve, and compare the estimate $\hat{\beta}$ with the true value β (= 2.4).

e) Make sure x_i and y_i are column vectors. Compute **x****y**. What result do you get?

f) Now, re-compute $y_i = 1.3 + 1.6x_i - 0.5x_i^2 + \varepsilon_i$. Use the method of least squares to fit the data x_i and y_i to the curve $y = \beta_1 + \beta_2 x + \beta_3 x^2$, and compare the obtained coefficients with their true values.

TASK 3

Recall that the roots of a real polynomial (whose coefficients are all real-valued) are either real-valued or complex-conjugated pairs. Therefore, any real polynomial can be represented in the factored form, of which all the factor polynomials are real polynomials of degree 1 or 2.

Use MATLAB functions roots and poly to convert the following polynomials to factored forms:

$$p_1(x) = 1 - x^4$$

$$p_2(x) = 1 - 6x + 11x^2 - 6x^3$$

$$p_3(x) = 1 + 2.8x + 1.7x^2 + 0.9x^3 + 2.3x^4 + 2.3x^5 + x^6$$

TASK 4

A point object at the coordinates (0, 2) is casted at speed v and angle θ (measured from positive x-axis), aiming at a target at (x_t, y_t). Assume no friction in the air, and gravity of Earth $g = 10$ m/s^2.

a) Draw the trajectory for $v = 5$ m/s and $\theta = 45°$;

b) If $x_t = 10$ m, $y_t = 3.5$ m, and $\theta = 45°$, what is the speed that the casted object can hit the target?

c) Find the minimum speed v_{min} that allow the object to hit the target, and the corresponding angle;

d) For the minimum speed and corresponding angle above, make a movie to show the cast in action. Refer to the following example on how to record and play movie using MATLAB.
https://au.mathworks.com/help/matlab/ref/movie.html

TASK 5

Consider the following difference equation

$$x[k+1] - x[k] = rx[k](1 - x[k]) \,, \qquad k = 0, 1, 2, \ldots$$

and $x[0] = 0.1$.

a) For the cases $r = 0.3$, 1.8, 2.2, 2.5 and 2.7, respectively, compute $x[k]$ up to $k = 50$.

b) Assume $x[k]$ reach equilibrium $\tilde{x}$ when k is large. One will have

$$0 = r\tilde{x}(1 - \tilde{x})$$

Since r is non-zero, either $\tilde{x} = 0$ or $\tilde{x} = 1$. Investigate whether these two equilibrium points are stable (i.e. convergent) or not (divergent), for the cases $r = 0.3$, 1.8 and 2.2, by using a small perturbation (e.g. for $\tilde{x} = 0$, starting with $x[0] = 0.001$ to see if $x[k]$ converges to 0 as k increases).

c) When $r = 2.5$ and 2.7, with $x[0] = 0.1$, $x[k]$ will never converge to any equilibrium point (this is mathematically related to "bifurcation" and "chaos"). Use MATLAB to visualise $x[k]$ for $k = 101$, 102, …, 200, with respect to r between 1.5 and 3.0. Think about how you are going to illustrate your results to clearly explain your findings.