ELEC4844/8844 Practical Class – Week 3, 2024

DFT & Digital Filtering

TASK 1

a) Recall the formula of DFT:

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{N-1} x[n]W_N^{kn}$$

For a test signal

$x[n] = u[n] - u[n-8]$, $n = 0, \ldots, 255$

use matrix-based computation to obtain its DFT:

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ X[N-1] \end{bmatrix} = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & \cdots & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & \cdots & W_N^{N-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ W_N^0 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \\ x[N-1] \end{bmatrix}$$

Compare the result with that computed by the MATLAB function fft.

b) Given a continuous-time signal:

$$x(t) = \cos 2\pi f_1 t + \cos 2\pi f_2 t + \cos 2\pi f_3 t$$

in which f_1 = 6.5 kHz, f_2 = 7 kHz and f_3 = 9 kHz. Use sampling frequency f_S = 32 kHz, and explore the following 3 situations:

1. Sampling 16 points from the continuous signal, and take 16-point DFT.
2. Sampling 16 points from the continuous signal, and take 256-point DFT.
3. Sampling 256 points from the continuous signal, and take 256-point DFT.

Plot and compare the magnitude of the DFT sequences.

c)* What is the minimum length of the sampled data in order to distinguish the three frequency components above in b? Use MATLAB to demonstrate your answer.

TASK 2

a) MATLAB has a number of default window functions, such as:

- rectwin – rectangular window
- bartlett – triangular window
- hann – Hann window
- hamming – Hamming window
- blackman – Blackman window

Using MATLAB function wvtool to investigate these window functions of 16-, 32-, and 64-point length, and compare their main lobe widths and side lobe amplitudes.

b) Given a continuous-time signal

$$x(t) = 10\cos 2\pi f_1 t + \cos 2\pi f_2 t + 4\cos 2\pi f_3 t$$

in which f_1 = 6 kHz, f_2 = 6.5 kHz and f_3 = 8 kHz. The sampling frequency is f_S = 32 kHz, and the signal last for 0.1 second. Use MATLAB filterDesigner to design a 100-order FIR filter based on Hamming window, aiming to remove the f_2 frequency component.

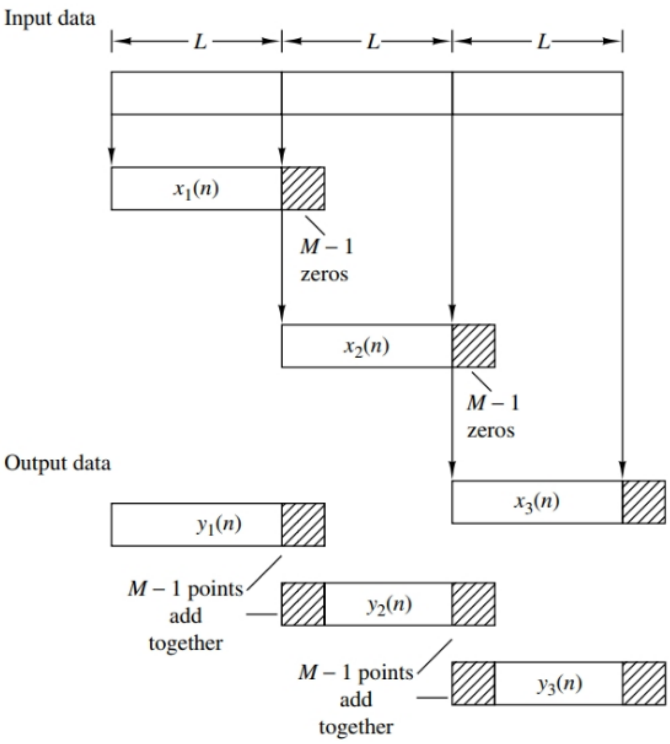
c) Plot the filter coefficients (i.e. impulse response), magnitude and phase response.

d) Apply the filter to the signal to generate the output, and compare it with the input in both time and frequency domain.

e)* Can you achieve ≥80 dB attenuation at f_2 with the filter above? If not, what can you do to meet this specification?

TASK 3

a) Implement frequency-domain block processing based on DFT and IDFT (MATLAB functions fft and ifft) to filter the signal in Task 2 (lasting for 0.1 second), using the overlap-add method as illustrated below:



Set the length of each signal block as 512. Draw the filtered output of each signal block with proper time shift, and the overall output.

b)* Alternatively, block processing can be performed by the **overlap-save** method, using the MATLAB filter function and taking advantage of the initial and final conditions. Inspect the syntax of the filter function, and use it to perform block processing for filtering the signal above.