ELEC4844/8844 Signal Processing for Software Defined Radio

Assignment 2, Semester 1, 2024

Assignment 2 consists of 3 parts, accounting for 10 marks in total. For questions requiring MATLAB/Simulink simulation, you must include your MATLAB codes and/or Simulink programs, as well as appropriate numerical and graphical results in your answers.

Assignment 2 must be completed individually. In Defence 2, you will present your Assignment 2 results and will be asked questions.

PART 1. COMPLEX ENVELOPE OF SIGNAL AND NOISE (2 MARKS)

An RF signal, $s(t)$, is received with additive white Gaussian noise, $\eta(t)$, such that $r(t) = s(t) + \eta(t)$.

$$r(t) = \mathrm{Re}\left[\tilde{r}(t)e^{j2\pi f_c t}\right]$$
$$s(t) = \mathrm{Re}\left[\tilde{s}(t)e^{j2\pi f_c t}\right]$$
$$\eta(t) = \mathrm{Re}\left[\tilde{\eta}(t)e^{j2\pi f_c t}\right]$$

where $\tilde{r}(t)$, $\tilde{s}(t)$, and $\tilde{\eta}(t)$ are called the complex envelopes of $r(t)$, $s(t)$, and $\eta(t)$, respectively, and f_c (>>1) is the centre frequency of the complex envelope (in Hz).

Suppose

$$s(t) = A\cos(2\pi f_1 t + \theta)$$

and the noise $\eta(t)$ has mean value of 0 and standard deviation of σ.

The signal-to-noise ratio (SNR) is defined as

$$SNR = P_s/P_\eta$$

where P_s is the average power of $s(t)$ and P_η is the average power of $\eta(t)$. And

$$SNR_{dB} = 10\log_{10} SNR$$

1a) For $P_s = 2$ (W), what is the value of A (V)?

Further, for $SNR_{dB} = 0$, 3, 6, 10, 20, 30, 40 and 50 dB, what are the respective values of P_η (W)? Plot P_η (on the y-axis in log scale) as a function of SNR_{dB} (on the x-axis in linear scale).

1b) What is the analytical expression of the corresponding $\tilde{s}(t)$?

Using the parameters $f_1 = 2$ kHz, $\theta = 0$, $f_c = 1$ kHz, and sampling rate $f_s = 1/T_s = 10$ kHz, plot

$$|\tilde{r}[n]| = |\tilde{r}([n-1]T_s)| = |\tilde{s}([n-1]T_s) + \tilde{\eta}([n-1]T_s)|$$

as a function of time $t = [n-1]T_s$, $1 \le n \le 1000$, for the cases of $SNR_{dB} = 0$, 3, 6, 10, 20, 30, 40 and 50 dB as separate curves. Set the x-axis to $0 \le t \le 0.1$ and the y-axis to $0 \le |\tilde{r}[n]| \le 8$ for all the plots, and comment on the effect of increasing SNR_{dB} on the appearance of $|\tilde{r}[n]|$.

PART 2. AM-SSB MODULATION AND DEMODULATION (4 MARKS)

The file "speech.wav" contains an audio signal sampled at f_s = 176.4 kHz for a total period of 10 seconds (i.e. N = 1,764,000). Use MATLAB or Simulink to:

2a) Extract the audio signal $m[n]$, $n = 0, 1, \ldots, N-1$, and create its upper-sideband analytic signal $m_{\mathrm{USB}}[n]$, which has power only for positive frequencies and no power for negative frequencies. Plot m_{USB} as a function of time in the range of [0, 0.01], and the power spectral density of m_{USB}.

2b) Upconvert the analytic signal $m_{\mathrm{USB}}[n]$ to a passband signal, $s_{\mathrm{USB}}[n]$, where

$$s_{\mathrm{USB}}[n] = \frac{1}{2}\mathrm{Re}\{m_{\mathrm{USB}}[n]e^{j2\pi f_c n/f_s}\}$$

and f_c = 50 kHz. Plot the power spectral density of s_{USB}.

2c) Demodulate $s_{\mathrm{USB}}[n]$ to obtain $r[n]$. Explain your method. Plot the demodulated signal r as a function of time in the range of [0, 0.01] and its power spectral density, and compare to those of the original audio signal.

2d) Downsample the demodulated signal $r[n]$ by a factor of 4 to obtain $r_d[m]$ at sampling frequency of 44.1 kHz. Plot the downsampled signal r_d as a function of time in the range of [0, 0.01] and its power spectral density. Play the original audio signal and the downsampled demodulated audio signal. Describe how the two signals compare.

PART 3. FM TRANSMISSION OVER ADDITIVE NOISE CHANNEL (4 MARKS)

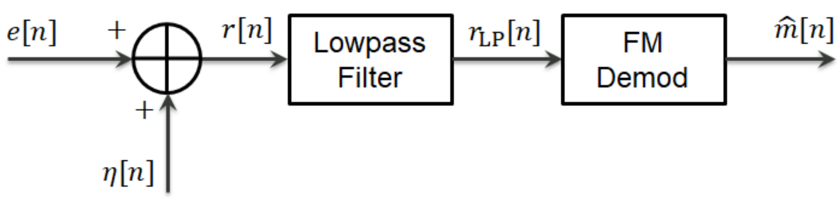
Given a baseband message signal $m(t)$, its FM modulation is expressed as

$$s_{\mathrm{FM}}(t) = \mathrm{Re}\left[e(t)e^{j2\pi f_c t}\right]$$

in which

$$e(t) = A\exp\left\{j2\pi f_\Delta \int_0^t m(\tau)d\tau\right\}$$

is the complex envelope of the modulated FM signal, and f_Δ is the frequency deviation. Use MATLAB or Simulink to simulate the FM transmission over an additive noise channel, as illustrated by the following diagram.



3a) Derive the expression of the discrete-time complex envelope $e[n]$ with respect to the sampling frequency f_s. Then, for a test message signal $m(t) = \cos(2\pi f_m t)$ with $f_m = 1$ kHz, generate the discrete-time signal $m[n]$ and its discrete-time complex envelope $e[n]$ ($n = 0, \ldots, N - 1$), with $f_s = 500$ kHz, $N = 5{,}000{,}000$, $f_\Delta = 75$ kHz, and $A = 1$. Plot the power spectral density of $e[n]$.

3b) The discrete-time channel noise is

$$\eta[n] = \frac{\sigma}{\sqrt{2}}\{\alpha_r[n] + j\alpha_i[n]\}$$

where $\alpha_r[n]$ and $\alpha_i[n]$ are two independent, Gaussian, zero mean random sequences with standard deviation equal to 1. The received signal is

$$r[n] = e[n] + \eta[n]$$

with signal-to-noise ratio $SNR = P_e/P_\eta$.

For $SNR_{dB} = 10$, 20, 30, and 40 dB, generate $r[n]$ and plot its power spectral density.

3c) The received signal $r[n]$ is filtered through a lowpass filter with bandwidth $BW = f_\Delta + f_m$. Create a digital lowpass filter to obtain $r_{\mathrm{LP}}[n]$ from the $r[n]$ generated above. Explain what the filter does to the power spectral density.

3d) The demodulated FM signal $\hat{m}[n]$ can be obtained based on phase differentiation of the received signal. Find this method in engineering literature, and implement the algorithm in MATLAB or Simulink. Plot demodulated signal in the time domain in comparison to the test message signal with proper zooming, and comment on the effect of the channel noise.