ELEC4844/8844 Practical Class – Week 4, 2024

Multirate Signal Processing

TASK 1

a) For the discrete-time signal

$$x[n] = \cos\left(\frac{\pi}{36}n\right) + \cos\left(\frac{1.5\pi}{36}n\right)$$

with $n = 0, \ldots, 127$. Plot $x[n]$ and its spectrum (magnitude only). Apply zero-padding if appropriate.

b) The following signal is related to $x[n]$ as

$$x_1[n] = \begin{cases} x[n], & n = 0, 4, 8, \ldots \\ 0, & \text{otherwise} \end{cases}$$

with $n = 0, \ldots, 127$. Plot $x_1[n]$ and its spectrum (magnitude only) in comparison to $x[n]$.

c) The following two signals are related to $x[n]$ as

$$x_d[m] = x[4m],\ m = 0, \ldots, 31$$

$$x_u[m] = \begin{cases} x[m/4], & m = 0, 4, 8, \ldots, 508 \\ 0, & \text{otherwise for } m \in [0, 511] \end{cases}$$

Plot both in the time domain and their spectra (magnitude only) in comparison to $x[n]$.

TASK 2

A signal $s(t)$ is given by

$$s(t) = Ae^{j(2\pi f_c t + \theta)}$$

$s(t)$ is sampled at frequency $f_S = 1.2$ MHz for a duration of 0.1 s to create $s[n]$.
$x[n]$ is created by downsampling $s[n]$ by a factor of 2, to obtain a signal that is sampled at 600 kHz.
$y[n]$ is created by upsampling $x[n]$ by a factor of 10, to obtain a signal that is sampled at 6 MHz.
$z[n]$ is created by filtering $y[n]$ with a 128-order lowpass filter, which is designed using Kaiser window with $\beta = 6$, cutoff frequency of 300 kHz, and gain of 5.

a) Construct the lowpass filter above, and plot its frequency response.

b) Set $A = 1$, $\theta = 0$, $f_c = 200$ kHz. Plot the time-domain sampled signals (in real and imaginary parts) of $s[n]$, $x[n]$, $y[n]$, and $z[n]$ with respect to time t in the range from 0 to 0.1 ms, and their spectra (in real and imaginary parts) with respect to f between (-1/2, 1/2] times the respective sampling rate.

c) Set $f_c = 500$ kHz instead, and repeat b).

TASK 3

Consider the CD→DAT example in the lecture slide (*p*.23)

a) Use MATLAB filterDesigner to design the lowpass filter, with target passband ripple of 0.05 and stopband attenuation of 40 dB. Assume the passband and stopband frequencies are related to cutoff frequency as

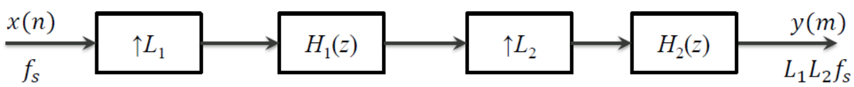
$$f_{\text{pass}} = f_c - 1 \text{ kHz}$$

$$f_{\text{stop}} = f_c + 1 \text{ kHz}$$

How many coefficients does the FIR filter need in order to achieve such performance?

b) Plot the spectrum (magnitude only, in dB) of the FIR filter, and zoom in to inspect the normalised angular frequency between $[0, \pi/8]$.

c)* Use multistage design to reduce the computation requirement, for example, by letting $L = L_1L_2 = 10\times16$.



Design your system. Assume passband ripple of 0.025 and stopband attenuation of 40 dB for each filter. Plot the spectra (magnitude only, in dB) of the two FIR filters. How many coefficients have the two filters got in total?