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3D reconstruction from a single image using geometric constraints

Frank A. van den Heuvel *

Delft University of Technology, Faculty of Civil Engineering and Geosciences, Thijsseweg 11, 2629 JA Delft, Netherlands

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Abstract

Photogrammetry has many advantages as a technique for the acquisition of three-dimensional models for virtual reality. But the traditional photogrammetric process to extract 3D geometry from multiple images is often considered too labour-intensive. In this paper a method is presented with which a polyhedral object model can be efficiently derived from measurements in a single image combined with geometric knowledge on the object. Man-made objects can often be described by a polyhedral model and usually many geometric constraints are valid. These constraints are inferred during image interpretation or may even be extracted automatically. In this paper different types of geometric constraints and their use for object reconstruction are discussed. Applying more constraints than needed for reconstruction will lead to redundancy and thereby to the need for an adjustment. The redundancy is the basis for reliability that is introduced by testing for possible measurement errors. The adjusted observations are used for object reconstruction in a separate step. Of course the model that is obtained from a single image will not be complete, for instance due to occlusion. An arbitrary number of models can be combined using similarity transformations based on the coordinates of common points. The information gathered allows for a bundle adjustment if highest accuracy is strived for. In virtual reality applications this is generally not the case, as quality is mainly determined by visual perception. A visual aspect of major importance is the photo-realistic texture mapped to the faces of the object. This texture is extracted from the same (single) image. In this paper the measurement process, the different types of constraints, their adjustment and the object model reconstruction are treated. A practical application of the proposed method is discussed in which a texture mapped model of a historic building is constructed and the repeatability of the method is assessed. The application shows the feasibility of the method and the potential of photogrammetry as an efficient tool for the production of 3D models for virtual reality applications.

Keywords: architecture; line photogrammetry; geometric constraints; adjustment; object reconstruction; virtual reality

1. Introduction

Many applications of virtual reality do not pose the highest accuracy demands. Because of this and the fact that photogrammetry is a specialised discipline that can be labour-intensive, other methods like a tacheometric survey are sometimes chosen for the acquisition of 3D models of existing structures. In this case photographs are taken only to obtain photo-realistic textures.

The goal of the research presented here is the development of an efficient method for the acquisition of 3D models that are suitable for virtual reality applications. In these applications aspects that relate to visual perception are often more important than the precision of the coordinates as such. Two major

* E-mail: F.A.vandenHeuvel@geo.tudelft.nl

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aspects of visual perception are: (1) the sensitivity of the eye to certain geometric properties of the model like parallelism, perpendicularity and symmetries; and (2) the amount of detail of the model; without increasing the amount of geometric detail, a model is considerably enhanced by adding detail in a radiometric way in the form of photo-realistic textures on the surface of the object model.

The need for efficiency improvement of photogrammetric model acquisition has stimulated research efforts directed towards automation (e.g. Streilein and Hirschberg, 1995; Hsieh, 1996; Lang and Förstner, 1996). But it has become clear that, at least for many years to come, we will not be able to reach full automation. Interpretation by an operator cannot be eliminated entirely. Research aims at a reduction of operator interaction by automation of parts of the photogrammetric process. Another opportunity for efficiency improvement lies in the reduction of the number of images to be manually processed. This approach for efficiency improvement is chosen here.

The method that is discussed in this paper is directed towards the application and user requirements listed above. It is a line-photogrammetric method (i.e. image lines are the basic measurements) suitable for objects that can be described by a polyhedral object model and for which a certain number of geometric constraints like parallelism of object lines hold. The number of constraints needed depends on the topology of the model and will be discussed in Sections 2 and 3. An application field in which these constraints are abundant is architecture. For most buildings a redundant number of constraints can be specified and therefore an adjustment can be applied (Section 4). Adjustment in combination with statistical testing and the elimination of erroneous measurements or constraints leads to a reliable object model even when only one image is used. Redundant constraints result in condition equations for the observations. In an adjustment based on only condition equations and no parameters, there is no need for approximate values. Another advantage is the absence of an object coordinate system and the related rank deficiency. From the adjusted image observations and the geometric object information available, the 3D coordinates of the object model can be computed (Section 5). The result of the proposed method is a boundary representation of the object with texture extracted from the image mapped to each face (Section 6).

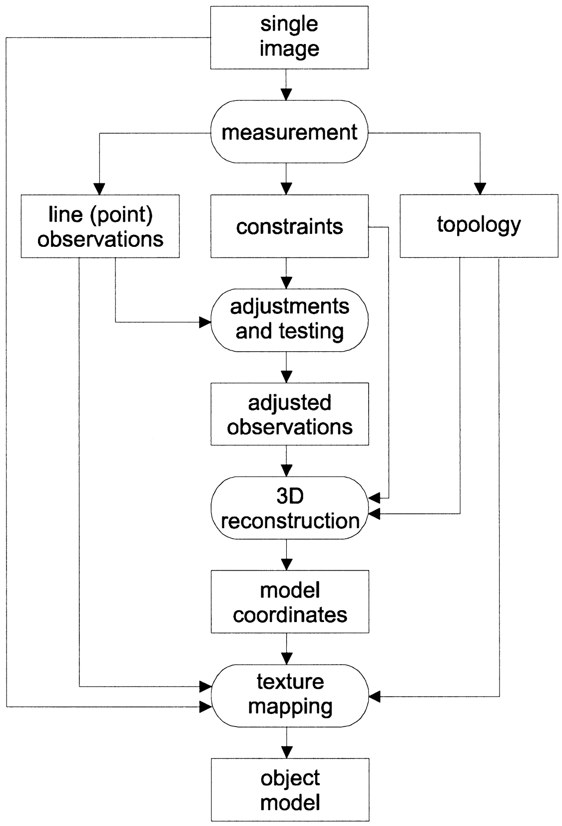

Fig. 1. The photogrammetric process for object modelling from a single image.

In Fig. 1 a flow chart of the photogrammetric process for object modelling from a single image is depicted. The interpretation of a single image by an operator results in photogrammetric measurements, the topology of the object model and geometric constraints. This information is processed in two steps: adjustment of redundant constraints followed by 3D reconstruction using adjusted observations.

Of course, only a part of the three-dimensional object can be modelled from a single image. If a complete object model is desired, additional images have to be taken and processed in the same way as the first image. With the identification of corresponding object points in overlapping parts of the models, the parameters of a 3D similarity transformation can be computed. In this way the relative position and orientation of the partial models are computed and thereby the relative orientation of the images associated with these models. The partial models can thus be transferred to a common coordinate system. The number of images needed to generate a complete object model mainly depends on the geometric complexity of the object involved. For some objects not more than two images are needed for a full model.

The approach outlined above is not only suitable for virtual reality applications, but also for the reconstruction of destroyed historic buildings of which only a few (single) images are available. If parameters of the camera model like the focal length are unknown, it is possible to estimate them with the use of geometric object information (Williamson and Brill, 1989; Kraus et al., 1996; van den Heuvel, 1997). The method described in this paper assumes the camera to be calibrated, i.e. the interior orientation parameters are known.

The proposed method of object reconstruction from a single image is demonstrated in an example application in which a texture mapped model of a historic building is obtained (Section 7).

2. The measurement process

The measurement process is the process of gathering all information that is needed for the 3D reconstruction from a single image. Two types of information are distinguished: measured image lines and object information. In Section 2.1 the photogrammetric observations are introduced, i.e. the position and orientation of image lines or image points being the intersections of the lines. Together with the measurement of image lines the topology of the object is specified. In Section 2.2 the gathering of different types of additional, mainly geometric object information is discussed.

2.1. The image lines and object topology

In this research a line-photogrammetric approach is chosen. The advantages of this approach over conventional (point-based) photogrammetry can be found in Patias et al. (1995). The main advantages are the improved possibilities for automatic feature extraction and the fact that only a part of a line needs to be visible in the image. The advantages apply especially to applications where many linear features are present like in architectural photogrammetry.

The image line observations can be derived by manual measurement or by semi-automatic line extraction. The automatic (straight) line extraction has been extensively studied (e.g. Burns et al., 1986). Even in the case that lines are extracted automatically, operator interaction cannot be avoided. First the line segments corresponding to relevant edges of the object have to be selected from the extracted straight lines. Possibly some lines that were not extracted automatically have to be measured manually. Second, topological relations between the lines have to be specified by the operator, i.e. line segments are connected to closed polylines that border a face of the object. An example of the result of this process is depicted in Fig. 11.

Independent of the way in which the image lines are derived each line is defined by the image coordinates (x, y) of its end points. In the mathematical model, however, the normal vector ($\boldsymbol{n}$) to the interpretation plane is introduced as an observation vector instead of the coordinates of the end points i and j (see Fig. 2):

$$\boldsymbol{n} = \frac{\boldsymbol{x}^i \times \boldsymbol{x}^j}{\left|\boldsymbol{x}^i \times \boldsymbol{x}^j\right|} \tag{1}$$

with:

$$\boldsymbol{x} = \frac{(x, y, -c)}{|x, y, -c|}$$

and c = camera constant.

Interior orientation parameters have to be known to compute directions in object space ($\boldsymbol{x}$) from points in the image. In the sequel image coordinates are assumed to be corrected for deviations from the pin-hole camera model.

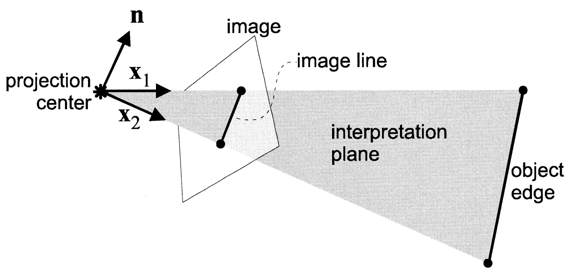

Fig. 2. The interpretation plane relates image line and object edge.

2.2. The additional object information

The object reconstruction from image lines of a single image in combination with the object topology is only possible when additional information is available. The additional information consists of object information in the form of topological and geometric constraints.

In van den Heuvel and Vosselman (1997) an overview of different types of constraints is given. The most common constraints are: (1) coplanarity; (2) parallelism; (3) perpendicularity; (4) symmetry; (5) distance (ratio). Below, the specification of these constraints during the measurement process is treated. The subject of the next section is the mathematical formulation of the constraints and their contribution to the object reconstruction.

(1) The specification of *coplanarity* constraints is performed in one step with the line measurement and topology specification. Image lines are measured as a part of a closed polyline. In object space the polyline is assumed to surround a planar face of the object so all the individual lines of the polyline are coplanar. Furthermore, different polylines (i.e. faces) can be specified to be coplanar.

(2) Each edge of the object model is assigned a code for its presumed orientation in object space. *Parallelism* is assumed to be present between lines that have the same code. The three major object orientations are distinguished from other orientations.

(3) *Perpendicularity* can be assigned to any combination of orientations in object space. This constraint is applied to combinations of the major object orientations by default.

(4) *Symmetry* can be assigned to an edge of a polyline. The symmetry applies to the angles between this edge and the two connected edges (see Fig. 3). Of course many more types of symmetry constraints could be defined, for instance if a door has to be centred within a façade. This and other symmetry constraints might also be incorporated in the form of distance ratio constraints.

(5) Geometry from a single image lacks scale information. Scale can be supplied with the length of an edge of the object model. In virtual reality applications scale plays a minor role as the model can generally be viewed at any scale. *Distance ratios* are more important because they allow determination of the relative scale between objects in the scene. This is especially important for unconnected objects where relative scale cannot be determined. This is illustrated in Fig. 4 where relative scale (and therefore relative position) can only be determined when a distance ratio between two edges is supplied, or when a coplanarity constraint is used stating that the two quadrangles are in one plane. The distance ratio constraint is not implemented.

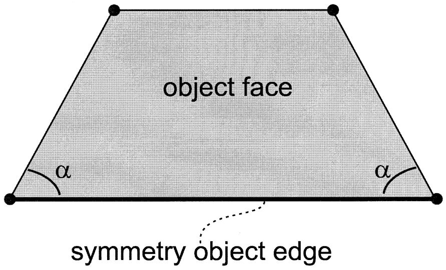

Fig. 3. Symmetry constraint: equal angles.

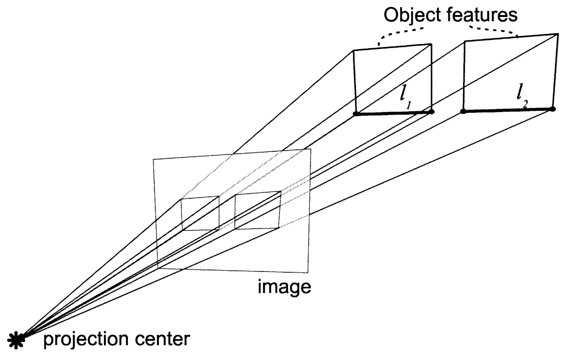

Fig. 4. The distance ratio $l_1 : l_2$ determines the relative scale and position of the two quadrangles.

For an efficient handling of geometric constraints a dedicated user-interface is a must. This user-interface should facilitate image line measurement in combination with the specification of geometric constraints. So far such a user-interface has only been partly implemented.

3. Mathematical formulation of geometric constraints

In this section two aspects of the geometric constraints introduced in the previous section will be dealt with. The first aspect is the way in which the constraint can be applied in the object reconstruction. The second aspect is the formulation of the constraint in the form of a condition equation in case of redundancy. This formulation is needed for the mathematical model of the adjustment discussed in Section 4. Combinations of different types of constraints will frequently result in redundancy and thereby in the formulation of condition equations. The condition equations resulting from parallelism and perpendicularity constraints have been implemented and will be discussed in detail.

The geometric constraints can be split into two groups. The first group contains the constraints that result from topological relations between image and object features and constraints resulting from the object topology itself. These constraints are called the topology constraints (van den Heuvel and Vosselman, 1997). The second group of constraints contains additional information on the geometry of the object and therefore they are called geometric object constraints or internal constraints.

3.1. Topology constraints

The topology constraints ensure a valid boundary representation and a valid image–object topology. The topology constraints are all of the coplanarity type. Three types of topology constraints can be distinguished: object point, line and plane constraints.

3.1.1. Object point constraint

The intersection of two object lines i and j results in an object point. The two interpretation planes associated with the projections of the two lines in the image have to intersect at a line through the object point (see Fig. 5). This is the object point constraint and the way it is used for object reconstruction. In formula:

$$\boldsymbol{x} = \frac{\boldsymbol{n}^i \times \boldsymbol{n}^j}{|\boldsymbol{n}^i \times \boldsymbol{n}^j|} \tag{2}$$

Redundancy arises when more than two lines intersect at an object point. The condition equation involved is the coplanarity of the normals to the interpretation planes:

$$\det(\boldsymbol{n}^i, \boldsymbol{n}^j \boldsymbol{n}^k) = \left[\boldsymbol{n}^i, \boldsymbol{n}^j, \boldsymbol{n}^k\right] = 0 \tag{3}$$

In other words: the image lines associated with the same object point have to intersect at a single point in the image. This constraint is superfluous if image lines are specified by their end points. Then a single point is stored as the end point of several lines.

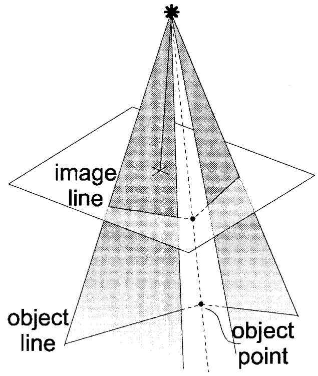

Fig. 5. The intersection of two interpretation planes.

3.1.2. Object line constraint

An object line has to be in the interpretation plane associated with an image line. In other words the object line i (direction vector $\boldsymbol{d}$) and the interpretation plane j have to be coplanar. This is the object line constraint that can be written as:

$$\boldsymbol{d}^i \cdot \boldsymbol{n}^j = 0 \tag{4}$$

For a single image no condition equation can arise from this constraint alone, as there exists a one-to-one relation between an image line and an object line.

3.1.3. Object plane constraint or coplanarity

A polyline measured in the image specifies the relation between a set of image lines being the projection of object edges bordering a single planar face of the object. The object plane constraint states that all the object lines have to be in one plane and therefore the intersections of these lines (i.e. the object points) have to be in the same plane. For object reconstruction this means that all the object lines can be determined by intersection of interpretation planes and object plane (Figs. 5 and 6 and Section 5). Of course the object plane then has to be known.

The combination of the coplanarity constraint with other types of constraints leads to redundancy. The general form of the condition equations is very

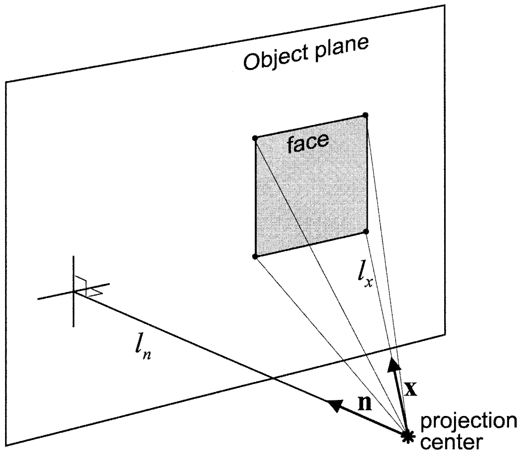

Fig. 6. Reconstruction of an object face.

similar to Eq. 4 where the normal to the interpretation plane has to be replaced by the normal to the object plane. The condition equation resulting from the combination of coplanarity and parallelism is presented in the next section.

3.2. Object constraints

The object constraints apply to the geometry of the object. They represent (a priori) geometric object information of object lines or planes. Three types of object constraints are discussed: parallelism, perpendicularity and symmetry. Coplanarity of lines of a single object face was treated in the previous section. Coplanarity between different object faces is not implemented.

3.2.1. Parallelism

Parallelism of object lines is one of the major geometric object constraints. First because it is used as the basis for object reconstruction and second because man-made objects often show many parallel edges. For most buildings the assumption of parallelism is easily inferred during the interpretation process because of our knowledge of the way in which they are constructed. Parallelism of object lines can be detected in a semi-automatic way by so-called vanishing point detection (van den Heuvel, 1998).

Parallelism is a powerful constraint for object reconstruction due to the fact that the orientation of

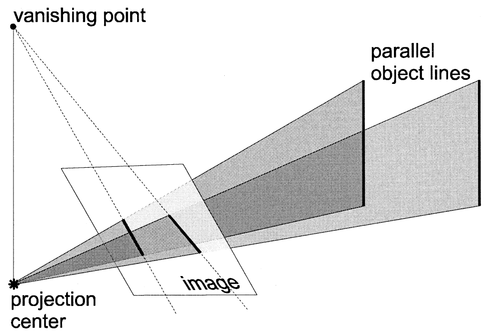

Fig. 7. Vanishing point from parallel object lines.

the parallel lines in object space can be derived from the intersection of the interpretation planes related to these lines (Fig. 7). The object space direction $\boldsymbol{d}$ is computed in the same way as the direction to an object point (Eq. 2) from the normals to the interpretation planes i and j as follows:

$$\boldsymbol{d} = \frac{\boldsymbol{n}^i \times \boldsymbol{n}^j}{|\boldsymbol{n}^i \times \boldsymbol{n}^j|} \tag{5}$$

If there are more than two parallel object lines present in the image, a condition equation independent of previous ones can be established for each additional line.

This condition equation is identical to the object point condition Eq. 3, which is not surprising knowing that all image lines of parallel object lines have to intersect at a point in the image plane: the vanishing point.

Parallelism in combination with other object constraints gives rise to other types of condition equations. The condition equation from the combination of three parallelism constraints on pairs of coplanar object lines with three different directions $\boldsymbol{d}$ can be written as:

$$\left[\boldsymbol{d}^i, \boldsymbol{d}^j, \boldsymbol{d}^k\right] = \left[\boldsymbol{n}^{i1} \times \boldsymbol{n}^{i2}, \boldsymbol{n}^{j1} \times \boldsymbol{n}^{j2}, \boldsymbol{n}^{k1} \times \boldsymbol{n}^{k2}\right] = 0 \tag{6}$$

This condition suffices if the lines border one face due to the object point constraints (Section 3.1.1). An additional condition equation is needed for coplanarity between object lines in different faces.

3.2.2. Perpendicularity

As many objects are constructed along three perpendicular axes, perpendicularity is an important

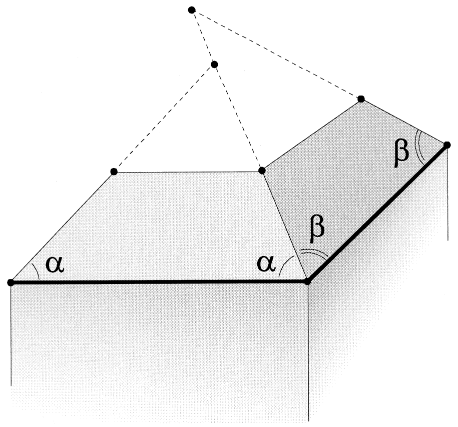

Fig. 8. Two redundant symmetry constraints for a house roof.

constraint. For object reconstruction this constraint is used to construct the third direction $\boldsymbol{d}^k$ from two available object directions ($\boldsymbol{d}^i, \boldsymbol{d}^j$). The object directions needed can be computed from parallelism constraints using Eq. 5:

$$\boldsymbol{d}^k = \boldsymbol{d}^i \times \boldsymbol{d}^j \tag{7}$$

Redundancy due to perpendicularity constraints will usually originate from combinations with other types of constraints. Independent of the way in which the object orientations are derived, the condition equation for perpendicularity can be written as:

$$\boldsymbol{d}^i \cdot \boldsymbol{d}^j = 0 \tag{8}$$

If the three major object orientations are assumed to be perpendicular, 'only' three independent condition equations of this type can be formulated when parallelism conditions are applied to all lines of each object orientation.

3.2.3. Symmetry

The symmetry constraint is defined in Section 2.2 (Fig. 3). To be able to apply this constraint the object coordinates of the end points of the symmetry edge have to be known. The procedure for object reconstruction using the symmetry constraint is explained in Section 5.2.

Redundancy can arise from combinations of symmetry constraints and other constraints. The resulting condition equations have not been implemented. A combination of two symmetry constraints that leads to redundancy occurs for instance for the type of roof of the house visualised in Fig. 8.

4. Adjustment and testing

A redundant number of geometric constraints leads to condition equations that build the functional model for a least-squares adjustment. In contrast to the mathematical model adopted by Zielinski (1993), this model contains no parameters and therefore requires no approximate values. Zielinski solves for the parameters of the object lines while in this approach the object parameters are derived in a separate step (see Section 5). After linearisation this model can be written as:

$$\boldsymbol{B}^{\mathrm{T}} E[\boldsymbol{y}] = 0;\ \boldsymbol{Q}_y \tag{9}$$

with $E\{\}$ mathematical expectation, $\boldsymbol{y}$ vector of observations (image lines), $\boldsymbol{B}$ design matrix (partial derivatives), $\boldsymbol{Q}_y$ covariance matrix of the observations.

The conditions can be processed in a sequential manner (van den Heuvel and Vosselman, 1997). This has advantages for an interactive measurement system because a condition can be tested as soon as it is specified by the operator. Rejection of such a statistical test forces the operator to a re-evaluation of the constraints or the measurements involved.

Condition equations resulting from combinations of parallelism and perpendicularity constraints have been implemented. Although dependency between condition equations can be detected numerically, it is avoided by automatic selection of independent constraints. Independence of parallelism equations is guaranteed by the selection of $n - 2$ condition equations, if n lines have been specified to be parallel. Each condition equation involves three lines (see Section 3.2). After application of parallelism constraints perpendicularity constraints can be applied to combinations of orientations in object space. Each perpendicularity condition involves two pairs of lines. Each pair of two parallel lines can be used to compute an orientation in object space. In the current system the three major object orientations (X, Y and Z) can be made to be perpendicular by applying three perpendicularity constraints (combinations X–Y, Y–Z and Z–X). These constraints are adjusted in a sequential manner also.

The adjusted observations that are input to the 3D reconstruction are computed as follows (Teunissen, 1994):

$$\hat{\boldsymbol{y}} = \mathbf{P}_b^{\perp} \boldsymbol{y} \tag{10}$$

with:

$$\mathbf{P}_B^{\perp} = \mathbf{I} - \mathbf{Q}_y \mathbf{B} (\mathbf{B}^{\mathrm{T}} \mathbf{Q}_y \mathbf{B})^{-1} \mathbf{B}^{\mathrm{T}}$$

In addition to the adjusted observations, the covariance matrix of adjusted observations is needed for the 3D reconstruction in order to perform error propagation and assess the precision of the resulting coordinates. The covariance matrix of the adjusted observations is computed with:

$$\mathbf{Q}_{\hat{y}} = \mathbf{P}_B^{\perp} \mathbf{Q}_y \tag{11}$$

The major types of condition equations — parallelism and perpendicularity — have been implemented. Condition equations that result from combinations with other types of constraints are missing. As a consequence the model computed in the 3D reconstruction will not fulfil these constraints and minor discrepancies show up during object reconstruction.

5. 3D reconstruction

The 3D reconstruction is the step of the photogrammetric process in which the object coordinates are computed. As no object coordinate system is introduced it is appropriate to name the 3D coordinates of the object *model coordinates*, especially because a seven-parameter similarity transformation is needed to arrive at coordinates in a world coordinate system. This transformation is called *absolute orientation* of a model in stereo photogrammetry. Absolute orientation can be performed if the world coordinates of at least three (suitable) object points are available. To avoid confusion, the 3D coordinates computed during reconstruction are named *object coordinates*, although the coordinate system is related to the system of the camera.

The approach for object reconstruction presented here is similar to the one presented in Braun (1994). The procedure of Braun is an iterative one in which object information is added in each iteration step until the (visible part of the) object is fully reconstructed. Another difference from the procedure adopted here is the fact that the adjustment step is performed after the object reconstruction, while in our approach the adjustment is executed first and does not involve object parameters.

5.1. The procedure for reconstruction

In the object reconstruction first the orientations of the lines and planes in object space are determined followed by the calculation of the positions of the object features.

The object coordinate computation is performed in four steps: (1) orientation of the object lines; (2) orientation of the object planes; (3) positioning of the object planes; (4) computation of the coordinates of object points.

Step 1: orientation of the object lines. The spatial orientation of the object lines is computed as the intersection of two interpretation planes of lines known to be parallel in object space (Eq. 5)

Step 2: orientation of the object planes. The orientation ($\boldsymbol{n}$) of each plane of the object is derived from two non-parallel lines in the plane. The orientation of these lines ($\boldsymbol{d}^i$, $\boldsymbol{d}^j$) is known from step 1.

$$\boldsymbol{n} = \frac{\boldsymbol{d}^i \times \boldsymbol{d}^j}{|\boldsymbol{d}^i \times \boldsymbol{d}^j|} \tag{12}$$

Note that more than one face of the object can be located in an object plane due to coplanarity constraints.

Step 3: positioning of the object planes. The position of the plane in space is derived from the known position of an object point in the plane. The position of a plane is specified with the distance to the projection centre along its normal (l_n, Fig. 6). The position of the plane is computed as a projection of the distance from the point to the projection centre (l_x):

$$l_n = (\boldsymbol{n} \cdot \boldsymbol{x}) l_x \tag{13}$$

If there is no known point available in the plane (which is the case for the first plane to be computed) the distance to the projection centre is set to 1. Thereby the choice for the scale of the model is made.

Step 4: computation of the coordinates of object points. The positions of all object lines in the plane are determined by the intersection of the interpretation plane of the line and the object plane. The intersection of the object lines leads to the coordinates of the object points. In practice image points are computed from the intersections of image lines and rays corresponding to image points are intersected with the object plane.

The planes are processed in an order in which they are connected by common points. This implies that the topology of the object model has to be such that all faces are connected. However, the connection can be obtained from object constraints in the form of distance ratios between unconnected parts of the model or by coplanarity between different object faces (Fig. 4).

5.2. Use of perpendicularity and symmetry

The 3D reconstruction described uses parallelism and coplanarity. Perpendicularity is only used for reconstruction of an object orientation that is specified to be perpendicular to two orientations known from parallelism constraints. Due to the adjustment of perpendicularity conditions the object orientations reconstructed using adjusted observations are perpendicular.

In case of symmetry the first step of the procedure described in the previous section has to be replaced. The directions of the two edges connected to the so-called symmetry edge are computed from the symmetry constraint (see Fig. 3). The procedure continues with the steps 2 to 4 described in the previous section.

In Fig. 9 the object reconstruction from the symmetry constraint is depicted. A direction resulting from the intersection of interpretation planes is intersected with a plane of symmetry instead of a plane of the object itself. The following steps result in the object directions of the two lines connected to the symmetry edge:

(1) The plane of symmetry is constructed from the position vectors of the end points of the symmetry edge.

(2) The direction from the projection centre to the intersection point is derived from the intersection of the two interpretation planes.

(3) The intersection of this direction with the plane of symmetry results in the object coordinates of the intersection of the lines and thereby to the directions of the two lines.

The symmetry constraint can be used for object reconstruction if the projection centre is not near the plane of symmetry nor near the plane to be reconstructed because then the intersection of the three planes involved is not well defined (step 2).

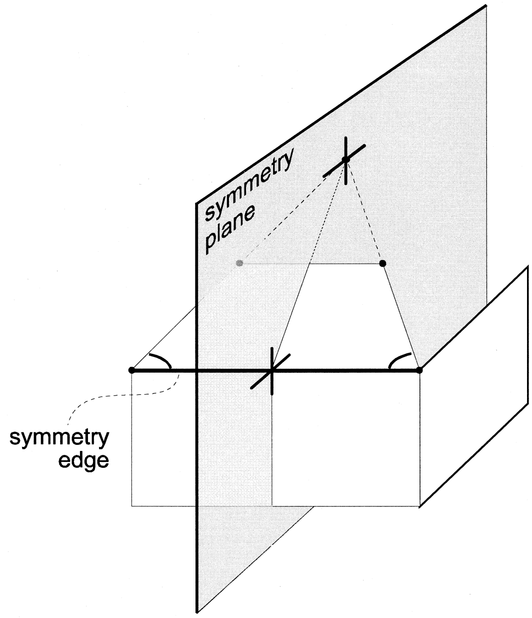

Fig. 9. Object reconstruction with the symmetry constraint.

5.3. Combining models from different images

A single image will show only a part of a 3D object. If a complete model of an object is desired, the method described here has to be applied to several partly overlapping images. This does not mean we are back at a traditional multi-image approach because the number of images needed here is reduced considerably by the use of geometric constraints for object reconstruction. In fact the images are treated sequentially and only have to have a minimum of three points in common. These object points should not be on (or near to) one line as the seven parameters of a 3D similarity transformation have to be computed from their coordinates. These parameters can then be applied in the transformation of the coordinates of one model to the coordinate system of the other model.

Although this procedure of combining overlapping models is quite effective, there are some drawbacks. A complete set of condition equations has to be applied in order to guarantee a unique solution for the object coordinates. To ensure unique coordinates when combining two models, condition equations have to be formulated that involve observations of both images. Without these condition equations the two models are not identical with respect to their shape. Because the applied constraints are all directional constraints like parallelism and perpendicularity, the major type of conditions to be applied are conditions that guarantee that distance ratios in the two models are identical. These types of condition equations that involve observations of more than one image are not implemented.

The approach of combining models by transformation is very suitable for determining approximate values for the relative orientation of two images or the exterior orientation of an image when coordinates of control points are available. This approach for exterior orientation is described in detail in van den Heuvel (1997). The derived approximate values can be used for a rigorous multi-image bundle adjustment. This adjustment will eliminate the discrepancies mentioned above. However, the geometric constraints have to be applied in this adjustment in order not to loose this information, i.e. in order to ensure parallelism of object lines, perpendicularity and other constraints.

5.4. Image acquisition

It is important to carefully select the images to be used because in the approach chosen here a minimum number of images is processed. Furthermore, the usefulness for reconstruction of some constraints depends on the orientation of the image relative to the object. Some rules of thumb for image acquisition for reconstruction from a single image are listed here.

(1) What is not visible in the image can not appear in the model constructed from it. This is an obvious rule and a clear limitation of the procedure. However, it can be overcome by combining several models (Section 5.3) and some occluded object points can be reconstructed with the assumption of intersection of object lines.

(2) The more pixels the projection of a face occupies, the better the precision of the orientation of the plane of the face derived from parallelism constraints applied to its edges. Besides, the subsequent texture mapping is improved.

(3) For a complete model of an object an angle between the optical axis and the three major object orientations of 45 degrees is optimal. Then faces with different orientation can be reconstructed facilitating the connection with overlapping models.

(4) For a model of an (almost) planar face the optical axis has to be as perpendicular to the face as possible. But in that case the symmetry constraint can not be applied.

(5) To apply the symmetry constraint the optical axis has to have an angle as close as possible to 45 degrees with the plane of symmetry and with the plane to be reconstructed (see Section 5.2).

(6) For a visualisation as realistic as possible the textures should be free of shadows. This is the reason why images should preferably be taken not with direct point-like light sources like the sun but with a clouded sky.

6. Texture mapping and visualisation

To visualise the derived model the photogrammetric data are converted to VRML (Virtual Reality Modelling Language). VRML is a format for 3D data with features like hierarchical transformations, light sources, viewpoints, geometry, animation, fog, material properties and texture mapping (Carey and Bell, 1997). VRML is an open format that has become popular because of its suitability for publishing 3D data on the World Wide Web. For this reason there is a lot of software available that can handle VRML. This software allows a user-friendly interactive examination and visualisation of the data.

The conversion to VRML is fully automatic and consists of two parts: geometry conversion and texture mapping (see Fig. 1). In the geometric part the object coordinates and the topology information are converted. The texture mapping is performed with an eight-parameter image rectification for each face of the object. When there are more than four points bordering a face the eight parameters are determined through a least-squares adjustment. For only three points in a face the projective transformation is approximated by a six-parameter affine transformation.

For the image plane to object plane rectification no exterior orientation data are needed because the rectification implies a direct transformation from image to object plane. Exterior orientation data has to be available for the method used in Debevec et al. (1996), which is based on a projection of image data onto the surface of the object. The advantage of their approach is that faces that are partly visible can be handled.

Image rectification involves resampling. The size of a pixel of the texture (in units of the object coordinate system) can be freely selected. The resampling allows a choice of three interpolation methods for resampling: nearest neighbour, bilinear and cubic convolution. Besides, a number of image enhancement options are available in order to obtain texture images that are homogeneous with respect to brightness and contrast. The texture of each face is written to a separate file in JPEG format. In VRML the texture file name is stored, the object face it belongs to and the positions in the texture image of the corner points of the face.

7. Application example

Two images of a historic building in Delft were used to test the method and assess the precision that can be obtained (Fig. 10). The images were taken on a clouded day with a digital camera Kodak DCS420 with a 20-mm lens. The camera had been calibrated beforehand. No artificial targets, control points or scale bars were used.

Fig. 10. Original images.

Fig. 11. Line and topology measurements (faces are not indicated).

The lines were measured manually (Fig. 11). Together with the line measurement the object topology was specified and thereby the coplanarity of the lines bordering a face. Parallelism of object lines was manually registered in a separate file with a code for each line. The code corresponds to the orientation of the line in object space. For the three major object orientations a total of 107 and 96 condition equations (first and second image, respectively) resulting from parallelism have been sequentially adjusted. Three perpendicularity constraints were used to ensure perpendicularity between the three major object orientations. For the lines of the oblique roof edges no parallelism constraints were applied because the orientation computed from parallelism would not be accurate due to the small distance between the parallel edges.

In the first image 38 faces and 94 points were measured. In the second image 4 of these faces were not visible and the number of points reduces to 82. The topologies of the models that result from the two images are identical, apart from one face that was only partly visible in the second image. The two models have 56 points in common. The rest of the object points are not identical due to occlusion.

If point measurements in the two images had been used for coordinate computation by forward intersection, the redundancy would have been equal to the number of object points. In the single image example presented here the redundancy is higher. Important for the visual perception of the model is the fact that with this approach faces are perfectly planar and parallel or perpendicular as defined during the measurement process, if a complete set of condition equations is applied. In the models that resulted from this experiment some minor discrepancies remain because only the condition equations related to parallelism and perpendicularity have been implemented.

The result of the two reconstructions were converted to VRML for visualisation. Two views from different angles are pictured in Figs. 12 and 13 (the models can be accessed at www.geo.tudelft.nl/frs/architec/single.html). These models directly result from the measurements and are not edited in any way. The first view in Figs. 12 and 13 is shown from the same position as the original image was taken from (compare to Fig. 10). The quality of the texture mapping decreases rapidly when the angle between the optical axis and the object face is reduced. This is visible in some of the faces of the roofs. It has to be stressed that the two images were processed fully independently. They only share the same topology information. The quality of the textures can be improved by using more favourable images for texture mapping. In this way the disturbing effects in the textures due to unmodelled parts can also be minimised. This is clearly illustrated by the comparison of the two models with respect to the effects of the unmodelled ornaments on the roof. If only one im-

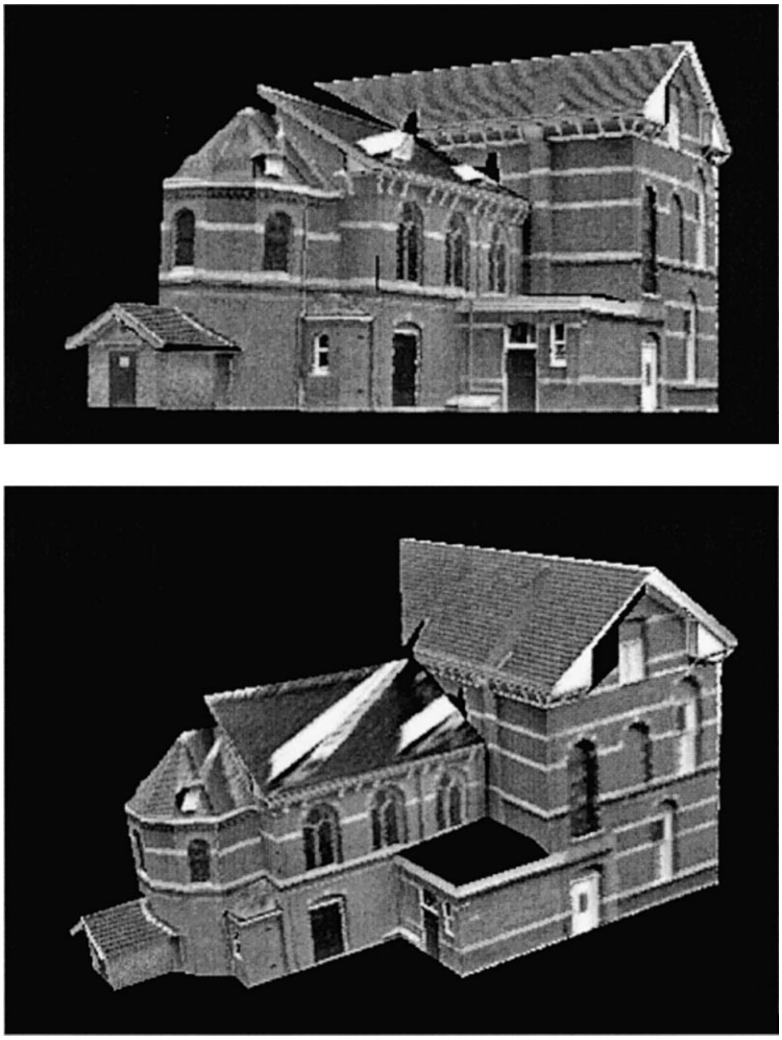

Fig. 12. Two views of the model reconstructed from the first image (the model can be accessed at www.geo.tudelft.nl/frs/architec/single.html).

age is used for texture mapping, the model is best viewed from a position close to where the image was taken.

The two sets of coordinates allow for a repeatability test to assess the geometric quality of the models assuming the geometric constraints are valid. The coordinates of one of the models have been transformed with a seven-parameter similarity transformation to the coordinate system of the other model in order to compare the two sets of coordinates. The results are summarised in Table 1. All numbers in the table are relative to the longest distance between two points of the model. The Y-axis is roughly in the vertical direction. The depth direction is in the $X-Z$ plane and the differences between the two sets

Table 1
Difference between the two sets of coordinates (fraction of object size)

Fraction $\times 10^{-3}$	X	Y	Z
RMSE	7.7	4.3	10.6
Minimum	−20.6	−9.9	−24.1
Maximum	17.7	7.8	14.8

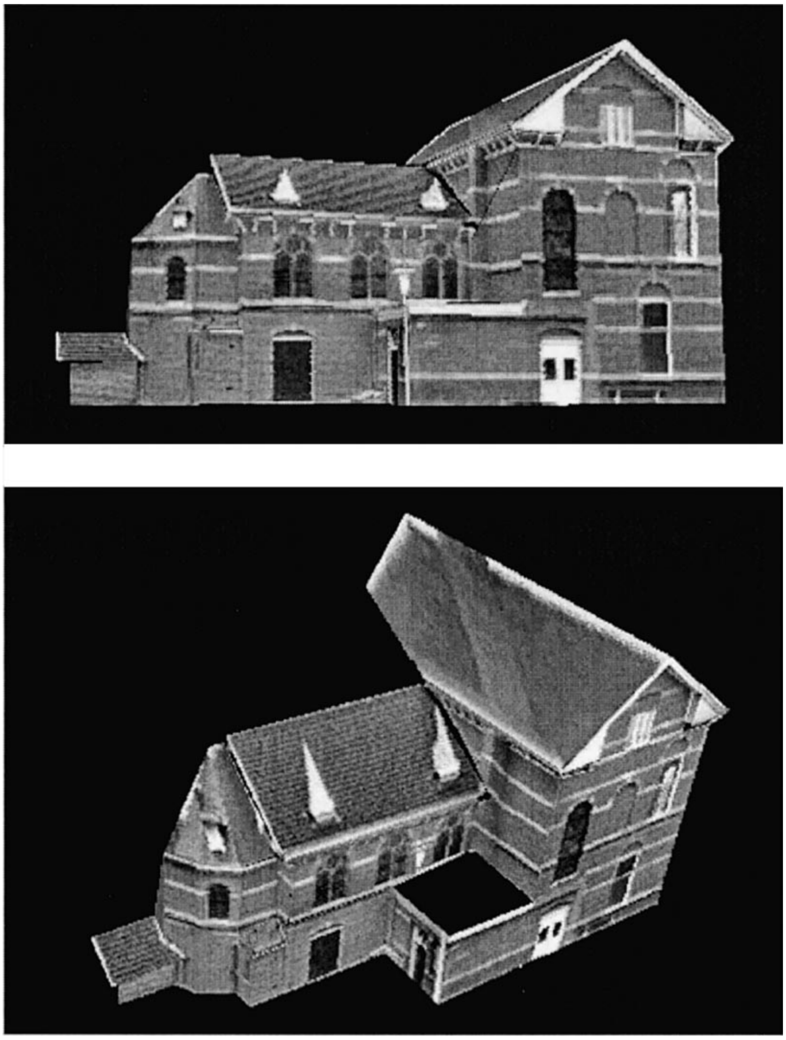

Fig. 13. Two views of the model reconstructed from the second image.

are 2 to 2.5 times larger in this plane. For a rigorous assessment of the precision that can be obtained with the presented method a comparison with reference data of a higher order of precision has to be undertaken.

8. Conclusions

A method is presented that aims at making photogrammetry a more efficient tool for the acquisition of models of man-made objects meeting the demands of many virtual reality applications. The method is based on a combination of line-photogrammetric measurement of a single image and geometric object information that is derived through image interpretation. The object information needed for reconstruction are constraints such as parallelism of object lines, perpendicularity or symmetries. In man-made objects such as buildings these constraints appear frequently. An adjustment using a mathematical model of condition equations is applied in case of redundancy due to combinations of parallelism and perpendicularity constraints. This redundancy allows for the statistical testing of the constraints and the observations involved. The adjusted observations are input to the object reconstruction procedure by which object coordinates are computed from image measurements and geometric constraints.

The proposed method is applied for modelling a part of a historic building. This example application showed the feasibility of the proposed method for object reconstruction from measurements in a single image. Comparison with the results of the reconstruction of the same part of the building from a second image resulted in an assessment of the repeatability. After similarity transformation the RMS of the coordinate differences was 1% of the object size or less.

Visual inspection of the acquired models led to the conclusion that the models show a high degree of realism in which texture mapping plays a major role. Texture mapping can be improved if images taken from different angles are available. Then disturbing effects in the textures due to unmodelled object features can be minimised. Missing parts of the model can be added by using additional images. Even for a complete model a considerable reduction in the number of images to be processed is achieved in comparison to conventional photogrammetric techniques. In combination with semi-automatic line extraction and generation of constraint hypotheses there is a great potential for efficiency improvement.

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