# Concatenation of Space-Time Block Codes and LDPC Codes

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***Abstract-*** **Space-Time coding is widely used in wireless communication systems, in order to provide diversity gain and overcome fading effects of multipath propagation, also by using channel codes, such as Convolutional, Turbo and LDPC coding, we can increase coding gain and obtain performance result near to Shannon limit. In this paper, we consider the concatenation of LDPC coding as outer code and Alamouti's $G_2$ space-time block codes as inner codes to achieve the benefits of both techniques; In addition the performance of proposed system is studied as a function of LDPC decoding parameters e. g. frame length, number of iteration, coding rate and constellation mapping over Rayleigh flat-fading and AWGN noisy channel.**

## I. INTRODUCTION

One of the most significant limitations to increase the data rate in wireless communication systems is multipath propagation that causes the time varying fading because of destructive superposition of delayed received signals; so finding a suitable technique to reduce this effect is very important. One of the best techniques to decrease fading effect is diversity. Space-time coding, including Space-Time Trellis Coding (STTC) [1] and Space-Time Block Coding (STBC) [2], combines spatial diversity and channel coding to overcome fading that results increasing the capacity of wireless channels. A model for orthogonal space-time code with two transmitter antenna and a very simple decoding algorithm was introduced by Alamouti in [3]. While space-time code is not designed to obtain coding gain, concatenating this code with an outer code can improve the performance of wireless communication systems [4],[5],[6],[7]. Since turbo [8] and LDPC codes [9] have very close performance to Shannon's channel capacity, so they are the best selection for this purpose.

In this paper, we examine the performance of serial concatenation of Alamouti's $G_2$ space-time block code, with the LDPC codes; also we investigate the Bit Error Rate (BER) and Frame Error Rate (FER) enhancement for various parameters such as frame length, and number of iteration for different modulation constellations. To achieve better performance we extracted soft output bits instead of using hard decision in STB Decoder and fed the soft output bits into LDPC decoder.

The rest of this paper is organized as follows. In section II, we show model of serial concatenation of space-time block code with LDPC code in $2\times2$ MIMO system. In next section, we describe the $G_2$ space-time block coding and the idea of symbol by symbol Maximum A-Posteriori (MAP) decoding algorithm for it. The soft output of this decoding algorithm delivers logarithmic probabilities which can be use as soft input for outer decoder based on LDPC. In section IV, we explain the applied LDPC coding and decoding algorithm. Section V, surveys the simulation results of introduced system for different parameters and channel conditions.

## II. SYSTEM MODEL

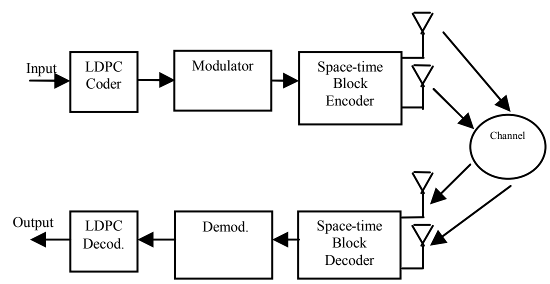


Fig. 1. Block Diagram of system for $2\times2$ MIMO channel

Fig. 1 shows our system model; at the first stage, randomly generated input bits are coded by LDPC encoder; this codes are mapped into complex symbols via mapping constellation for different modulations including BPSK, QPSK and 16QAM, then the complex symbols are passed to Alamouti's $G_2$ space-time block coder and output signal is transmitted via two transmitter antennas.

In the channel, transmitted signal is affected by AWGN noise and Rayleigh flat-fading. The fading assumed to be flat for the cases that the signal bandwidth is much smaller than coherence bandwidth of channel, i.e. $W << B_{coh}$ that is true for Applications with limited velocity for users [10]. However the study for high velocity user applications with frequency-selective fading could be a subject for further studies.

At the receiver, the signals of two receiver antennas are passed to STB decoder that is based on log-MAP algorithm; by taking into consideration of modulation mapping rule, the soft output results of STB decoder are converted to LLR of code bits, then they are fed to LDPC decoder. The details of STB decoder and LDPC decoder operation is described in next sections.

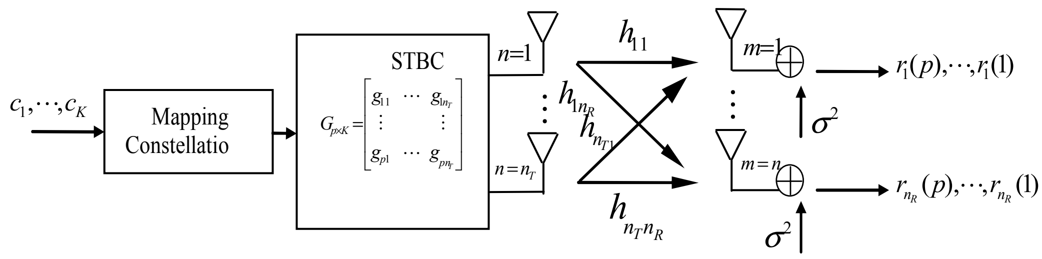


Fig. 2. Space– time block coder over Rayleigh flat fading channel with $n_T$ transmitter and $n_R$ receiver antennas.

## III. Space-Time Block Codes

In this section, we describe the orthogonal space-time block coding for $n_T$ transmitter antennas and $n_R$ receiver antennas, and also explain symbol by symbol MAP decoding algorithm equations for space-time block decoder of this assumed system that results soft output. Then we focus on the Alamouti's coding scheme for $2\times 2$ MIMO channels [3].

If $b_i$, $i=1,\cdots,n$ are the frame number of output bits with length of n, in the first step, they are mapped to codes denoted by $c_1,\cdots,c_K$, where $c_i \in [0,1,\cdots,M-1]$, and $M$ is the number of modulation symbols, and $K = n/\log_2 M$, then each symbol is mapped to complex symbols denote by $x_1,\cdots,x_k$ according to constellation mapping rule.

Then these complex symbols are fed to Alamouti's $G_2$ space-time coder, according to Alamouti's scheme, each two subsequent symbols $s_1, s_2$ are transmitted via two antennas based on the following matrix with orthogonal columns:

$$G_2 = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \qquad (1)$$

The row number defines time of transmission and the column number defines transmitter antenna. The symbols of each row are transmitted simultaneously from two antennas, and then passed through $2\times 2$ MIMO channel with Rayleigh flat fading, and AWGN noise with a two-sided spectral power density of $\sigma^2 = N_0/2$.

In the receiver, the received signal $r$ is composed from the transmitted signals as following:

$$r_m(t) = \sum_{i=1}^{N_T} g_{tm} h_{im} + n_m(t) \qquad (2)$$

Where $r_m(t)$ is the received signal from $m_{th}$ receiver antenna in instant $(t)$, and $H$ is channel matrix with $h_{ij}$ entries, $h_{ij}$ denotes fading channel coefficient between $i_{th}$ transmitter antenna and $j_{th}$ receiver antenna, and $n_m(t)$ is the Gaussian distributed noise in time $(t)$ at receiver $m$.

The following part describes the soft output decoder for space-time block codes and present its equation in general situation for $n_T \times n_R$ MIMO channel. The most appropriate decoder which can be used in this receiver is MAP decoder that maximizes the $P(c_1,\cdots,c_K \mid r_1,\cdots,r_p)$, where $c_i$ is the transmitted symbol before mapping into complex symbol $x_i$ with constellation rule.

For calculating a-posteriori probability $P(c_1,\cdots,c_K \mid r_1,\cdots,r_p)$, we can use Bayes' rule and derive the equation (3).

$$\begin{aligned} &P(c_1,\cdots,c_K \mid r_1,\cdots,r_p) = \\ &conts\ .\ p(r_1,\cdots,r_p \mid c_1,\cdots,c_K).P(c_1,\cdots,c_K) \end{aligned} \qquad (3)$$

By taking into account that the noise is additive white Gaussian, formula (3) can be rewritten as:

$$\begin{aligned} &p(r_1,\cdots,r_p \mid c_1,\cdots,c_K) = \\ &const.\exp\left(-\frac{1}{2\sigma^2}\sum_{m=1}^{n_R}\sum_{t=1}^{p}\left| r_m(t) - \sum_{n=1}^{n_T} g_{tm} h_{nm} \right|^2\right. \end{aligned} \qquad (4)$$

Since $P(c_1,\cdots,c_k)$ which is a-priori information for transmitting symbols, is statistically independent, so a-priori term can be written as:

$$P(c_1,c_2,\cdots,c_K) = \prod_{i=1}^{K} P(c_i) \qquad (5)$$

In most cases $P(c_i)$, $i=1,\cdots,K$ are the same and equal to $1/\log_2 M$, and by using the orthogonallity property of $G$ columns, equation (4) can lead to relation (6):

$$\ln p(r_1,\cdots,r_p \mid c_1,\cdots,c_K) = const \quad - \\ \frac{1}{2\sigma^2}\sum_{m=1}^{n_R}\sum_{t=1}^{p}[-r_m(t)\sum_{n=1}^{n_T} g_{tn}^* h_{nm}^* - r_m(t)^* \sum_{n=1}^{n_T} g_{tn} h_{nm} \\ + \sum_{t=1}^{p}\sum_{n=1}^{n} |g_{tn}|^2 |h_{nm}|^2)\} \tag{6}$$

In the above equations, each $g_{tm}$ corresponds to a certain symbol $c_i$. Consequently, we can decouple the above a-posteriori probabilities into separate equations for each transmitted symbol $c_i$, and obtain them independently by using the following equation in (7).

$$\ln P(c_i \mid r_1,\cdots,r_P) = const \quad + \\ \ln p(r_1,\cdots,r_p) + \ln P(c_i) \tag{7}$$

In this contribution, we study $2\times 2$ MIMO system ($n_T = 2$ & $n_R = 2$), so the two transmitted symbols' a-posteriori probabilities can be obtained from (8) and (9):

$$\ln P(c_1 \mid r(1), r(2)) = const \quad - \\ -\frac{1}{2\sigma^2}\{\left|[\sum_{m=1}^{n_R=2}(r_m(1)h_{1m}^* + r_m(2)^* h_{2m})] - s_1\right|^2 + \\ + (-1 + \sum_{m=1}^{n_R=2}\sum_{n=1}^{n_T=2}|h_{nm}|^2)|s_1|^2\} + \ln P(c_1) \tag{8}$$

$$\ln P(c_2 \mid r(1), r(2)) = const \quad - \\ -\frac{1}{2\sigma^2}\{\left|[\sum_{m=1}^{n_R=2}(r_m(1)h_{2m}^* + r_m(2)^* h_{1m})] - s_2\right|^2 + \\ + (-1 + \sum_{m=1}^{n_R=2}\sum_{n=1}^{n_T=2}|h_{nm}|^2)|s_2|^2\} + \ln P(c_2) \tag{9}$$

Since we used the space-time block code as inner code and concatenated it with the outer code, which is based on LDPC coding, we skip hard decision in STC decoder and couple the soft a-posteriori outputs in format of LLR into LDPC decoder.

## IV. Low Density Parity Check(LDPC) codes

LDPC codes are one of the most common channel coding schemes that gives near performance to Shannon capacity in AWGN and Rayleigh fading channels, these codes are first discovered by Gallager in 1962 [8] and reexamined by Mackay and Neal in 1996 [11]. Also they are implemented by many other researches; e.g. in [12]. Chung and et al. in [13] showed that an irregular LDPC with ½ rate and $10^7$ block length can achieve to $10^{-6}$ bit error rate at $0.04\ dB$, close to Shannon capacity of AWGN channel.

LDPC code is a Linear block code by a very sparse parity check matrix, in LDPC Coder the vector of input bits with $k$ bit length is multiplied by the generator matrix $(G_{k\times n})$ and composes the vector of codes of length $n$, so the rate is $R = k/n$. We applied LDPC matrix that is used in Wimax system for frame size of 576 and 2304.

In order to check received codes via channel, they are multiplied to parity check matrix $(H_{(n-k)\times n})$, which is presented by $(n,k,t,j)$, and so $(n-k)$ parity-check equations are resulted. In Regular LDPC codes, $t$ is number of ones in each column, $j$ is number of ones in each row, and j > t, where both of these numbers are very small compared to block length $(n)$.

In the LDPC decoder, we use iterative sum product with log-MAP algorithm that is based on belief propagation theory. Some modification has been done on sum product decoder and max-log-MAP and also min-sum algorithm is proposed with a little degradation in performance and reducing decoder complexity, for this reason, we used max-log-MAP approach for our simulation.

## V. Simulation Results

In fig. 3, the BER result for LDPC code with coding rate (R =5/6 and R=3/4) is depicted for Rayleigh flat -fading channel and is compared to the proposed system (concatenation of LDPC with space-time block code). The figure demonstrates about 8 dB improvement in the performance in low BER. In other words, we can obtain the same error rate in proposed system with 8 dB lower transmission power compared to LDPC code without STB code in the same channel situations.

Since we use space-time block code for producing diversity gain, fading effect is considerably decreased which resulted in an enormous enhancement. This improve is also fairly in conformance with results of Alamouti's simulation for uncoded systems presented in [3].

In fig. 4, we plot BER for LDPC coding concatenated with STBC for different rates and frame lengths, this figure shows that low coding rates results better performance or less BER at the same SNR, So choosing appropriate coding rate is trade off between bit rate and bit error rate in the predefined transmission conditions.

Also this figure shows better performance in bit and frame error rate for bigger frame size in cost of decoding complexity.

Fig. 5 illustrates the FER for the mentioned cases and demonstrates the similar improvement for FER.

Since in practical systems, due to bandwidth limitation, M-array modulations usually are used instead of BPSK; so our system is simulated for QPSK and 16QAM modulation. Fig. 6 shows about 8dB improvement for proposed system in comparison to LDPC coding with QPSK and 16QAM modulation over Rayleigh fading channel that leads to suitable performance for these systems.

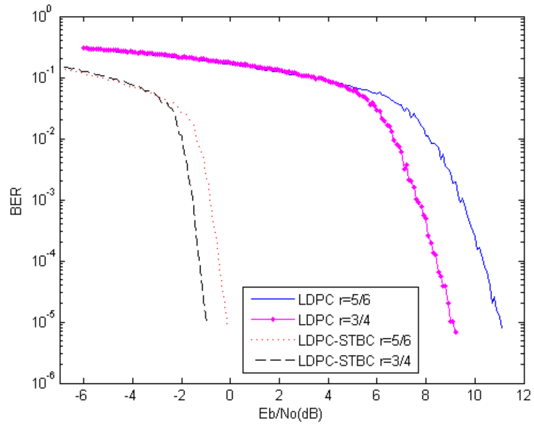


Fig. 3. BER comparison between LDPC and LDPC-STBC system for frame size: 576 and BPSK Modulation

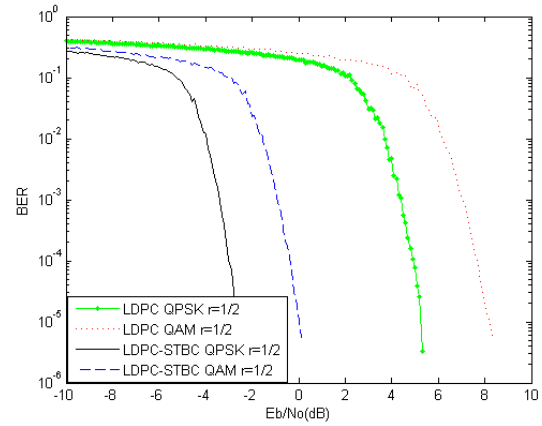


Fig. 6. BER for proposed LDPC-STBC system for different symbol constellation with frame size: 576 and rate:1/2 (Rayleigh Fast Fading)

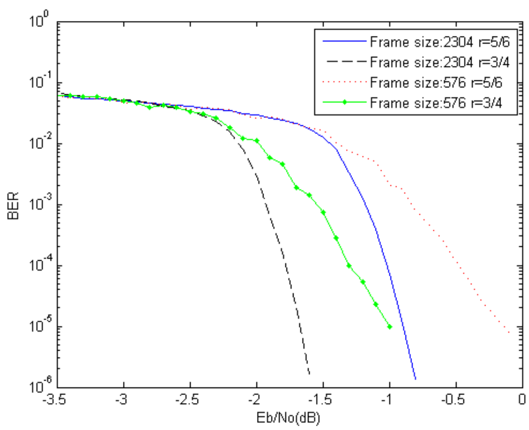


Fig. 4. BER for LDPC-STBC system with BPSK Modulation for different frame sizes and coding rates

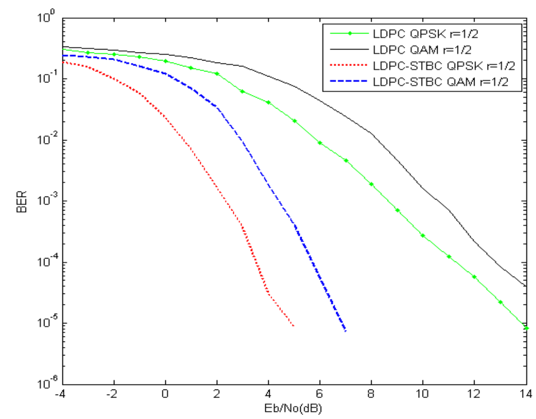


Fig. 7. BER for proposed LDPC-STBC system for different symbol constellation with frame size: 576 and rate:1/2 (Rayleigh Block Fading of length 16 symbols)

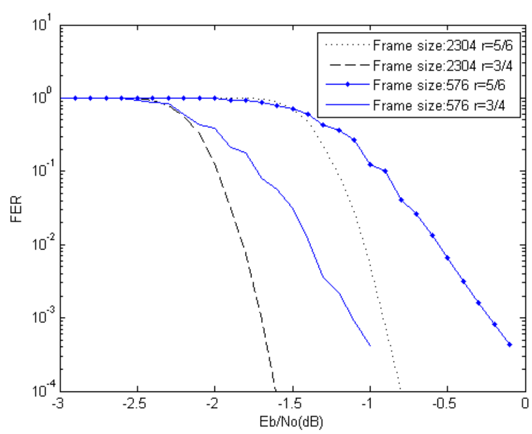


Fig. 5. FER for LDPC-STBC system with BPSK Modulation for different frame sizes and coding rates

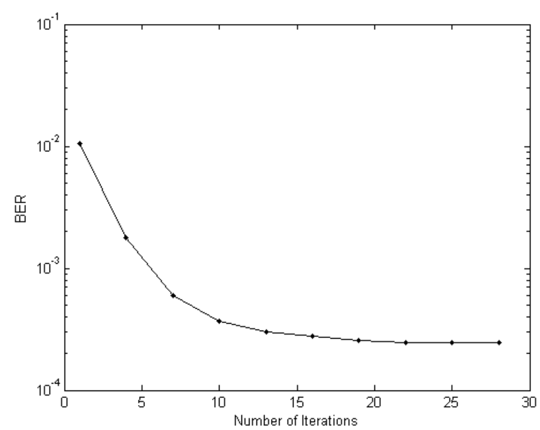


Fig. 8. BER for proposed LDPC-STBC system versus number of iterations

Fig. 7, shows the same comparison between LDPC and proposed system for different constellations. But in this case, we assumed that the fading coefficients are slow changing and are constant for 16 consequent symbols which is named as Block Rayleigh fading. The simulation results shows that although the performance of system is lower than the case with fast fading, but it could be enhanced by concatenating LDPC with STC codes in this scenario too.

In fig. 8, BER is plotted versus iteration number of LDPC decoding for this system and shows that the BER decreases while the number of iterations increases; but the mentioned improvement is negligible for more than 30 iterations.

## VI. CONCLUSION

In this paper, we proposed concatenation of LDPC with STB coding and examined it in Rayleigh fading channel. The simulation result shows that we can obtain improved performance in comparison to LDPC coding without diversity technique; so this system is very convenient to use in applications with limited power.

Moreover, by using QPSK & QAM modulation, we can achieve acceptable performance in Rayleigh fading channels in case of bandwidth limitation.

## ACKNOWLEDGEMENT

The authors would like to thank Mobile Company of IRAN (MCI) for financial and technical support of this project.

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