Distributed Coding of Sources with Unknown Correlation Parameter

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Abstract— *Binary source estimation using a cluster of Wireless Sensor Network (WSN) is considered. The correlation between binary data source and received data at each sensor is modeled as an independent Binary Symmetric Channel (BSC). A novel distributed joint source channel coding (DJSCC) scheme is proposed to estimate source data using the intrinsic correlation in received data from sensors. Encoder is based on distributed structure of turbo codes and decoder is equipped with a self correlation estimation block to track data correlation adaptively.*

Low complexity encoding, fast decoding convergence, and self correlation parameter estimation capability makes the proposed system an appropriate choice for distributed source estimation in wireless sensor networks with unknown, unpredictable or even time-varying correlation parameter.

Keywords: Wireless sensor networks, Joint source channel coding, Distrbuted coding, Turbo codes

1. Introduction

In majority of wireless sensor networks, a sheer numbers of sensors collect data in an area and transmit it to a central base station. Dividing sensors into clusters with correlated data and employing distributed coding optimizes the overall system data flow efficiency. In this approach a group of tiny sensors inside a cluster encode the data in a distributed manner without communicating with each other and collectively transmit it to a sink node, where data is processed or relayed to a central base station using various relay techniques such as Amplify and Forward (AAF) and Decode and Forward (DAF) [1].

The idea behind that is a theorem introduced by Slepian and Wolf in 1973 [2]. They proved that it is possible to design source codes for a group of transmitters such that they can compress and transmit data at rate below joint entropy $H(X_1, X_2, ..., X_M)$ which is much less than independent data compression at rate $H(X_1) + ... + H(X_M)$. Optimum coding rate is achieved without need for inter transmitter communication. Wyner and Ziv later extended this result to continuous valued Gaussian distributed sources and defined rate distortion function for lossy coding [3].

The first realization of distributed source coding algorithm based on Slepian-Wolf theorem is introduced by Ramchandran, et. al. employing syndrom concept that is called DISCUSS [4]. Later, more powerful source codes are developed based on known coding schemes such as Low Density Parity Check Codes (LDPC) [5], Turbo Codes [6], [7], Irregular Repeat Accumulate (IRA) [8] , and Low Density Generator Matrix Codes (LDGM) [9]. Recent research works combine source and channel coding to propose distributed joint source-channel coding to improve system overall performane and reduce system complexity [10].

One special case is a scenario that a number of sensors around a common data source, transmit noisy version of data to a single receiver which is an example for Chief Executive Officer (CEO) problem [11]. The receiver aims to estimate the source data using data fusion and joint decoding of correlated sources [12]. In this paper, we address this scenario with a new method based on distributed turbo encoder structure at sensor side and a modified turbo decoder at the receiver side.

In the proposed system, the intrinsic correlation between sensors is used as redundancy information to implement a simple joint source channel coding scheme. Despite the low complexity structure of encoders, since they form a distributed turbo encoder, the proposed system performs very close to well known powerful channel codes in terms of BER performance.

Moreover, a new method is introduced to estimate the correlation among sensor's data at at the receiver to make the proposed system applicable in the case of unknown and time varying correlation parameters. Scalability is another benefit of the proposed system model, since one may add as many as required sensors to each cluster employing multiple turbo codes(MTC).

The rest of this paper is organized as follows. In section 2, CEO problem of a single WSN cluster with BSC based correlation model is investigated. In section 3, a new distributed coding scheme based on turbo encoder is proposed. Section 4 describes structure of employed decoder equipped with a self correlation estimation block. In secttion 5 the system is extended for arbitrary number of sensors. Section 6 includes simulation results followed by conclusion in section 7.

2. Correlation modeling

A cluster of sensors in proximity of a source is considered. The scenario is depicted in fig. 1, where binary source data $S = \{s^1, s^2, ..., s^N\}$ is an independent identically distributed $(i.i.d)$ Bernoulli sequence with length N and $P(s^k = 0) = P(s^k = 1) = 1/2$. Each sensor measures a noisy version of data denoted by $X_i = \{x_i^1, x_i^2, ..., x_i^N\}$. The channel error between source data S and data sensed by i^{th} senor is modeled as a BSC channel with crossover probability of P_i, therefore the k^{th} bit in X_i is

$$x_i^k = s^k \oplus e_i^k \tag{1}$$

where e_i^k is the error of bit k in the i^{th} sensor with probability of $P(e_i^k = 1) = P_i,\ P(e_i^k = 0) = 1 - P_i$. Hereafter, without loss of generality it is assumed that $0 \leq P_1 \leq P_2 \leq ... \leq P_M 1/2$, where P_i is the correlation parameter between source data and observation of the i^{th} sensor.

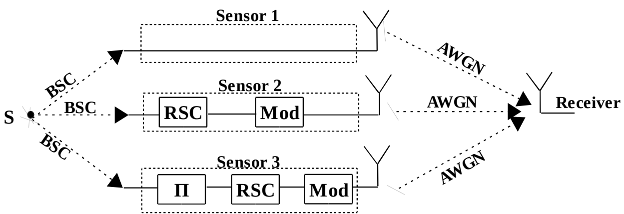

Fig. 1: A cluster of the three sensors observe a single binary data source, and form a distributed turbo encoder structure

The BSC channels are assumed to be memoryless, hence the sequence X_i is also an $i.i.d$ sequence, i.e.

$$P(X_1, X_2, ..., X_M|S) = \prod_{k=1}^{N} P(x_1^k, x_2^k, ..., x_M^k|s^k) \tag{2}$$

Since the BSC channels between source and sensors are assumed to be independent, the correlation parameter between each set of two sources can be modeled as follows:

$$\begin{aligned} P(x_i^k = 0|x_j^k = 0) &= \frac{P(x_i^k = 0, x_j^k = 0)}{P(x_j^k = 0)} \\ &= \frac{\sum_{m=0}^{1} P(x_i^k = 0, x_j^k = 0|s^k = m)P(s^k = m)}{\sum_{n=0}^{1} P(x_j^k = 0|s^k = n)P(s^k = n)} \\ &= \frac{\sum_{m=0}^{1} P(x_i^k = 0|s^k = m)P(x_j^k = 0|s^k = m)P(s^k = m)}{\sum_{n=0}^{1} P(x_j^k = 0|s^k = n)P(s^k = n)} \\ &= \frac{(1/2)[(1-P_i)(1-P_j) + P_iP_j]}{1/2} = 1 + 2P_iP_j - P_i - P_j \end{aligned} \tag{3}$$

Similarly it can be shown that:

$$\begin{aligned} P(x_i^k = 1|x_j^k = 1) &= 1 + 2P_iP_j - P_i - P_j \\ P(x_i^k = 1|x_j^k = 0) &= P_i + P_j - 2P_iP_j \\ P(x_i^k = 0|x_j^k = 1) &= P_i + P_j - 2P_iP_j \end{aligned} \tag{4}$$

for small values of P_i and P_j, the term P_iP_j is negligible and equation set (4) can be approximated by

$$\begin{aligned} P(x_i^k = 0|x_j^k = 0) = P(x_i^k = 1|x_j^k = 1) = 1 - P_i - P_j \\ P(x_i^k = 1|x_j^k = 0) = P(x_i^k = 0|x_j^k = 1) = P_i + P_j \end{aligned} \tag{5}$$

This means that, sensors are pairwise correlated and correlation between the i^{th} and the j^{th} sensors can be modeled as a BSC channel with parameter $P_{ij} = P_i + P_j - 2P_iP_j$. This result can also be demonstrated with another intuitive approach. A BSC channel with equiprobable inputs is bidirectional and we have $P(X = x|Y = y) = P(Y = y|X = x)$, therefore the channel between two sensors can be modeled as a cascade of two consecutive BSC channels with parameters P_i and P_j that results in a BSC channel with parameter $(1-P_i)P_j + P_i(1-P_j) = P_i + P_j - 2P_iP_j \approx P_i + P_j$. These results are used in decoder implementation of the proposed method to estimate the correlation parameter at the receiver.

Data at the i^{th} sensor is coded to form $C_i = c_i^1, c_i^2, ..., c_i^{N/R},\ i = 0, 1, ...M$, where R is coding rate at each sensor. The coded data is modulated with Binary Phase Shift Keying Scheme $(BPSK)$ to form output symbol frame $Y_i = y_i^1, y_i^2, ..., y_i^{N/R}$.

$$y_i^k = 2c_i^k - 1 \tag{6}$$

The output frames of sensors are transmitted through noisy orthogonal channels to the destination. The received signal is the combination of all $R_i = Y_i + N_i$, where N_i is Gaussian channel noise between the i^{th} sensor and the receiver. The k^{th} bit of R_i is

$$r_i^k = y_i^k + n_i^k \tag{7}$$

The goal is to estimate the common data which is shown by $\hat{S}$ based on the received signals, hence the problem is to minimize $E[(S - \hat{S})^2|R_1, R_2, ..., R_M]$.

3. Distributed coding for three sensors with correlated data

In this section, a coding method for the simple case of three sensors $M = 3$ with equal correlation parameters $P_1 = P_2 = P_3 = P$ is introduced. In the proposed approach as depicted in fig. 1, the first sensor transmits its data without coding so that the output stream of the first sensor contains only systematic bits $C_1 = \{c_1^1, c_1^2, ..., c_1^N\} =$

$\{i_1^1, i_1^2, ..., i_1^N\}$. The second sensor encodes data using an RSC encoder with polynomial $f(D) = 1 + D^2 + D^3$ and $g(D) = 1 + D + D^3$, where $f(.)$ and $g(.)$ are forward and feedback polynomials, respectively. The resulting parity bits $C_2 = \{c_2^1, c_2^2, ..., c_2^N\} = \{p_1^1, p_1^2, ..., p_1^N\}$ are transmitted to the receiver. Finally, the third sensor encodes its data with the same RSC encoder after random interleaving to form the output sequence $C_3 = \{c_3^1, c_3^2, ..., c_3^N\} = \{p_2^1, p_2^2, ..., p_2^N\}$. These three sensors together form a distributed structure of a generic turbo encoder. Data in three branches of encoder are not the same, while they are correlated versions of a single data source and consequently may differ slightly. This property which makes the encoder structure different from ordinary turbo codes, will be considered in design of decoder structure that is described in the next section.

4. Decoding structure

All the frames transmitted by sensors are gathered at the receiver to form one systematic bit set and two parity bit sets. Inspired by this fact, a turbo-based decoder with some modifications is implemented at the receiver. The Max-Log-MAP algorithm is used in RSC decoders to decrease decoding complexity utilizing $log(e^x + e^y) \approx max(x, y)$ approximation [13].

As shown in fig. 2, in the designed turbo decoder, the two parity sets correspond to correlated versions of the systematic bits. Therefore applying the systematic bits to one of the decoders as apriori information similar to a standard turbo decoder, biases the output of this particular decoder to systematic bits. This effect degrades the performance, especially for large values of correlation crossover probability. To remove this undesired effect, the Log likelihood Ratio (LLR) of symbols corresponding to systematic bits are applied as apriori information to both decoders in the first decoding iteration. This modification, makes a balance between these two decoders to increases the BER performance.

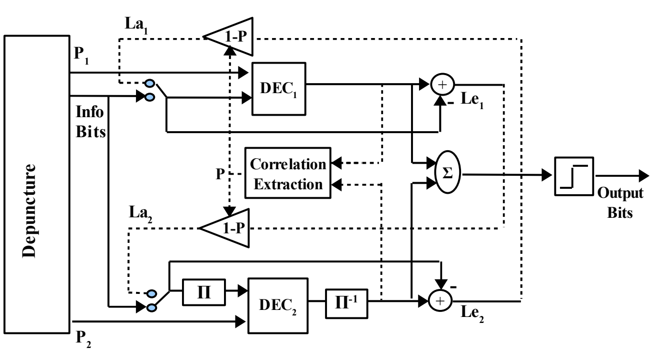

Fig. 2: Decoder structure based on turbo decoder with self correlation estimation block

The second modification is based on considering the relation between the inputs of three encoders according to (5) at transmitter side. The similar relation, $L_{a_2} = (1 - 2\hat{P})L_{e_1}$ is applied to the extrinsic LLRs of each decoder before using them as apriori information for the other decoder, where $\hat{P}$ is the estimated value of BSC crossover probability using a new method presented in next section.

The last modification is that the average of output LLRs of both decoders, $L_{av}^{out} = \frac{L_1^{out} + L_2^{out}}{2}$ is used, instead of using output LLRs of one decoder in a standard turbo decoder, to estimate data in the last iteration. The BER improvement due to these modifications are presented in section 6.

4.1 Correlation extraction by post processing of estimated data

In this section, the correlation estimation scheme is demonstrated. $Y_i = \{y_i^1, y_i^2, ..., y_i^N\}$ is the transmitted BPSK symbols by the i^{th} sensor. If $\hat{y}_i^k$ be the estimation of these symbols at the i^{th} RSC decoder, after each iteration of decoding algorithm, it intends to get more and more close to transmitted systematic symbols and cancels the noise effect. Therefore we have

$$\hat{y}_i^k \approx y_i^k = 2x_i^k - 1 \tag{8}$$

Considering the correlation model in (1), it converts to

$$\hat{y}_i^k = 2(s^k \oplus e_i^k) - 1 \tag{9}$$

To estimate the BSC crossover probability between sensors as correlation parameter sensors, a new random variable ρ_{ij}^k is defined as

$$\rho_{ij}^k = \frac{| \hat{y}_i^k - \hat{y}_j^k |}{2} \tag{10}$$

substituting (9) in (10) results in

$$\rho_{ij}^k = | (s^k \oplus e_i^k) - (s^k \oplus e_j^k) | \tag{11}$$

There are two possible cases for bit error occurrence at the two sensors. First, we consider a case when no error occurs or it occurs in both sensors, $e_i^k = e_j^k$. In this case, the two terms cancel each other. The probability of this event is

$$\begin{aligned} P(\rho_{ij}^k = 0) &= P(e_i^k = e_j^k) \\ &= (1 - P_i)(1 - P_j) + P_i P_j \\ &= 1 + 2P_i P_j - P_i - P_j \end{aligned} \tag{12}$$

The other case is when the error occurs just in one sensor. Without loss of generality, bit flipping due to error is assumed to be at sensor i, $e_i^k = \bar{e}_j^k = 1 - e_j^k = 1$. In this case the two terms correspond to i^{th} and j^{th} differ,

$$\begin{aligned} \rho_{ij}^k &= | (s^k \oplus 1) - (s^k \oplus 0) | \\ &= | \bar{s}^k - s^k | = 1 \qquad (if\ e_i^k \neq e_j^k) \end{aligned} \tag{13}$$

we used the fact that, s^k is either 0 or 1, therefore $\bar{s}^k - s^k = \pm 1$. The probability of this case is

$$
\begin{aligned}
P(\rho_{ij}^k = 1) &= P(e_i^k \neq e_j^k) \\
&= P_i(1-P_j) + (1-P_i)P_j \\
&= P_i + P_j - 2P_iP_j
\end{aligned} \tag{14}
$$

Expected value of ρ_{ij}^k is derived using (12) and (14)

$$
\begin{aligned}
E(\rho_{ij}^k) &= 0.P(\rho_{ij}^k = 0) + 1.P(\rho_{ij}^k = 1) \\
&= (1-P_i)P_j + (1-P_j)P_i = P_{ij}
\end{aligned} \tag{15}
$$

Since ρ_{ij}^k is either 0 or 1, we have $(\rho_{ij}^k)^2 = \rho_{ij}^k$, and consequently

$$
E[(\rho_{ij}^k)^2] = E(\rho_{ij}^k) = P_{ij} \tag{16}
$$

ρ_{ij}^k and ρ_{ij}^l are independent RVs, and their joint expected value can be calculated as

$$
E(\rho_{ij}^k \rho_{ij}^l) = E(\rho_{ij}^k)E(\rho_{ij}^l) = P_{ij}^2 \tag{17}
$$

Taking the average of ρ_{ij}^k over all bits of the frame, a new variable $\hat{P}$ is defined as

$$
\hat{P} = \frac{\sum_{k=1}^{N} \rho_{ij}^k}{N} \tag{18}
$$

Using the above calculation it concludes that

$$
E(\hat{P}) = \frac{\sum\limits_{k=1}^{N} E(\rho_{ij}^k)}{N} = E(\rho_{ij}^k) = P_{ij} \tag{19}
$$

$$
\begin{aligned}
E(\hat{P}^2) &= \frac{1}{N^2} E(\sum_{k=1}^{N}\sum_{l=1}^{N} \rho_{ij}^k \rho_{ij}^l) \\
&= \frac{1}{N^2}\sum_{k=1}^{N} E[(\rho_{ij}^k)^2] + \frac{1}{N^2}\sum_{k=1}^{N}\sum_{l=1, l\neq k}^{N} E(\rho_{ij}^k \rho_{ij}^l) \\
&= \frac{1}{N}(P_{ij}) + \frac{N(N-1)}{N^2}(P_{ij})^2
\end{aligned} \tag{20}
$$

$$
\sigma_{\hat{P}}^2 = E(\hat{P}^2) - E^2(\hat{P}) = \frac{1}{N}(P_{ij} - P_{ij}^2) \tag{21}
$$

Therefore $\hat{P}$ is a RV with mean P_{ij} and variance $\sigma_{\hat{P}}^2$ which approaches zero if the frame length N is selected large enough. In the other words, $\hat{P}$ is a good approximation of pairwise BSC crossover probability P_{ij} that models the correlation among sensors. The above justification is true for arbitrary number of sensors and for especial case of three sensors with equal crossover probability, we have $P_1 = P_2 = P_3 = P$, and consequently $\hat{P} \approx P_{ij} \approx P_i + P_j = 2P$.

5. Extenstion to arbitrary number of sensors

Simulation results presented in section 6 demonstrate considerable improvement of BER performance for the proposed decoding scheme for three sensors, but lower error floor may be required for some applications. To achieve higher BER performance, the proposed system model is extended to more sensors using distributed structure of Multiple Turbo Codes (MTC).

In this approach, all the sensors consist of a random interleaver followed by a RSC encoder. Hence these parallel RSC encoders form a distributed multiple turbo encoder. Despite the basic proposed system for three sensor case where each sensor performs at rate 1, in this case the output of each sensor can include both systematic and parity bits, and consequently any arbitrary coding rate per sensor can be obtained using different puncturing patterns.

A multiple turbo decoder is employed at the receiver. There are different cooperation methods among RSC decoders inside a multiple turbo decoder including serial, master-slave, and parallel structure [14]. The parallel structure is chosen in the proposed system, because of its superior BER performance.

All the modifications stated in section 4 is developed in this decoder too. To calculate apriori information for each RSC decoder in this scheme, the output LLRs of all the other RSC decoders are averaged, to enhance the performance of decoding algorithm. Then, the calculated apriori information is scaled by correlation parameter between encoders.

$$
L_{a_i}^k = (1-2\hat{P})\frac{\sum\limits_{j=1, j\neq i}^{N} L_{e_j}^k}{N-1} \tag{22}
$$

where $\hat{P}$ is the estimated correlation factor at the receiver. The pairwise correlation parameter is extracted from received data using method proposed in section4.1, and its average over all pairs of sensors is used as the best estimation of the pairwise correlation parameter.

6. Simulation Results

In this section the simulation results for proposed systems is presented. In all simulations unless otherwise explicitly specified, data frames with 256 bits length, BPSK modulation, i.i.d equiprobable Bernoulli bit stream, orthogonal AWGN channels and BSC correlation model with parameter P=0.01 are used.

Fig. 3 shows the BER performance for the proposed system, in which three sensors performing as a distributed turbo encoder. The coding rate of each sensor for 'TC' and 'Modified-TC' methods is one, because each sensor in these methods transmit either systematic bits or parity bits. 'TC' represents a basic scenario with three sensors perform

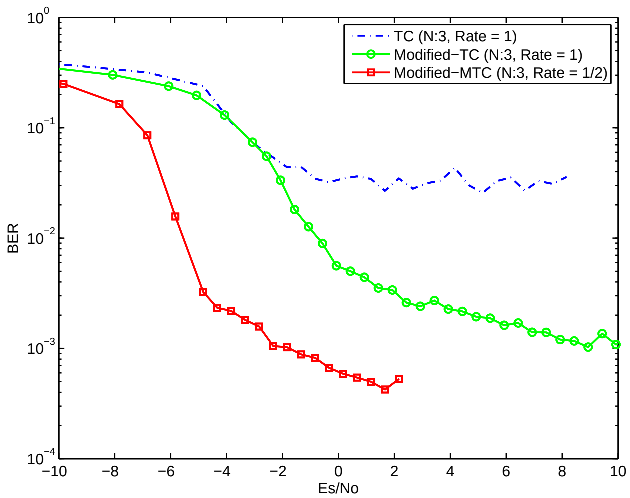

Fig. 3: Comparison of different decoding schemes for 3 sensors with correlated data, correlation parameter: 0.01

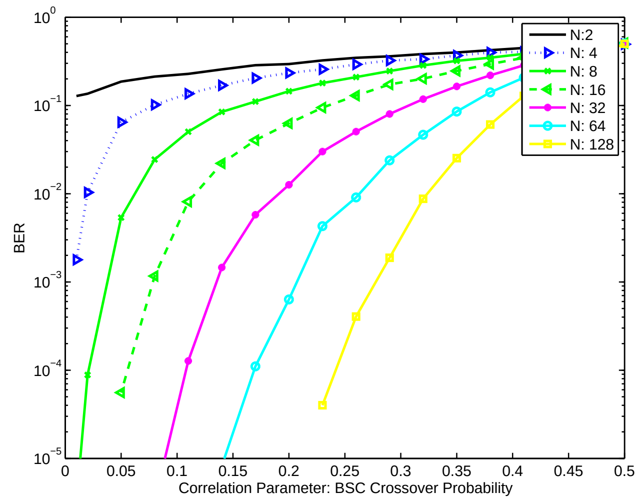

Fig. 4: BER performance of modified Multiple Turbo Decoder versus BSC crossover probability for different number of sensors at SNR=-6dB

as distributed turbo encoder and a basic turbo decoder applied at the receiver. Likewise, 'Modified-TC' represents the proposed modified turbo decoder. The simulation results show considerable improvement in BER floor, because BER floor of the proposed scheme for BSC crossover probability of 0.01 is in the orders of $10{-}3$, while the lowest achievable BER floor using a generic turbo decoder is in the orders of $10{-}2$. 'Modified-MTC' represents the extended system for 3 sensors perform at rate=$1/2$, that shows higher BER performance and results in BER floor less than 10^{-3} at the same SNR values. This result for this scheme is achieved in the cost of lower channel coding rate, as in this scenario each sensor transmits both systematic and parity bits.

Fig. 4 shows BER performance of extended system for different number of sensors versus BSC crossover probability as the correlation parameter. The results show that as the number of RSC encoders is increased, BER performance of the system is improved in the cost of more complexity at the receiver. In addition the system BER performance is proportional to correlation among sensors, which is improved as the BSC crosspver probability is decreased. The optimum value for the number of sensors may be determined based on desired BER performance, available bandwidth and channel condition.

Convergence of the proposed system is analyzed in fig. 5. The decoder almost reaches its final BER performance after 5 iterations for different SNR values, while at least 10 iteration is required for a generic turbo decoder in most cases. This is due to the fact that the output LLR's of RSC decoders represent the correlated versions of the source data. Therefore the system performance can not be improved by more iterations and decoder algorithm saturates very fast.

Fig. 6 demonstrates the effect of the correlation parameter

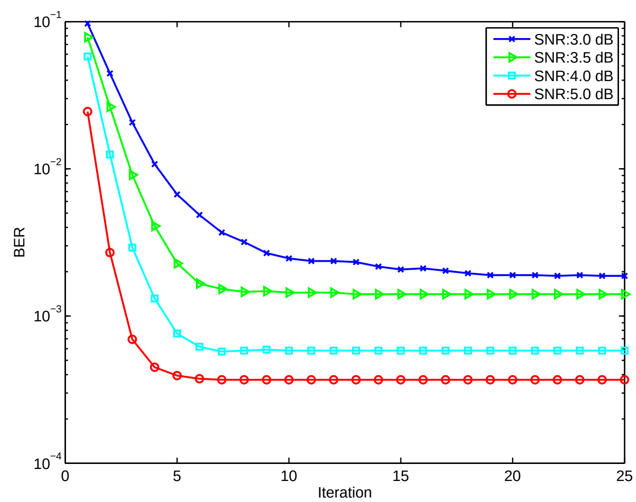

Fig. 5: BER performance of modified Multiple Turbo Decoder for 4 sensors versus different decoder iterations, Correlation parameter is 0.01

estimation in the proposed method. In the first method, the correlation parameter is assumed known at the receiver, while in the second method, it is estimated by post processing of the received symbols. Simulation results prove the accuracy of the developed estimation method, since the system BER performance for two cases are almost the same and there is no considerable BER performance degradation due to estimation of correlation parameter at the receiver.

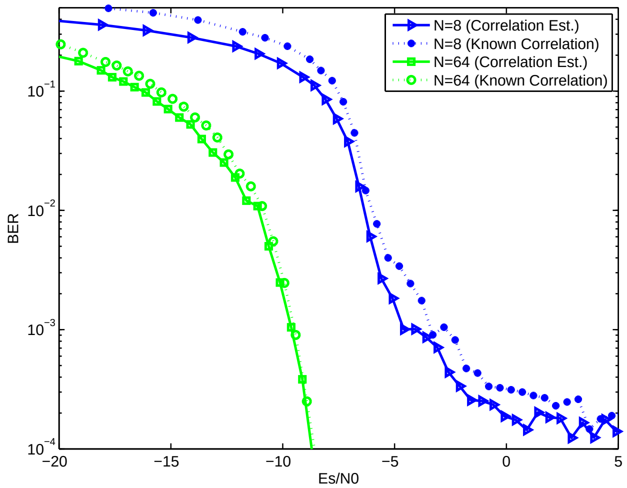

Fig. 6: Comparison of BER performance of modified MTC Decoder with known correlation and MTC Decoder with self estimation of correlation at the receiver. Correlation parameter is 0.05

7. Conclusion

In this article, a distributed joint source-channel coding technique based on turbo codes for CEO problem in a WSN is proposed. The simulation results show considerable improvement in BER floor for the proposed system compared to the same scenario and using a standard turbo decoder. The proposed system is extended to utilize as many sensors as required to further improve the BER performance of the system in the cost of more complexity at the receiver.

Independent BSC channels are used to model the correlation among sensors data in the system model. The BSC crossover probability is considered as correlation parameter and is used in the proposed decoder to improve BER performance and convergence of decoding scheme. A new method is developed to estimate the correlation parameter at the decoder that improves the performance of decoder when the correlation parameter is not known at the receiver. The decoding performance approaches the case when the correlation is assumed known at the receiver. It makes the proposed system an appropriate option to be employed in applications with unknown, unpredictable or even time-varying correlation factor without considerable loss in the system BER performance.

The system converges even faster than an standard turbo decoder. Data rate of sensors and consequently the system data rate can be managed using different puncturing methods.

Another benefit of the proposed system is scalability, employing multiple turbo codes. Therefore arbitrary number of sensors in each cluster can be utilized based on the average correlation parameter, sensors' power constraint, noise power, coding rate and available bandwidth. The system can be extended for other channel models such as fading and interference channels.

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