On Minimum Number of Wireless Sensors Required for Reliable Binary Source Estimation

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***Abstract*—The CEO problem of estimating a single binary data source using multiple observations is considered. A closed form equation for maximum achievable information capacity of the system is derived for different system parameters including number of sensors, observation accuracy and channel quality. A new criterion for sensor clustering is provided based on the minimum number of sensors required to achieve arbitrarily low estimation error using different coding rates.**

A distributed joint source-channel coding (D-JSCC) scheme is proposed as an implementation example. The proposed coding scheme is based on the parallel concatenated convolutional codes (PCCC) that achieves high BER performance utilizing the number of sensors determined by the derived criterion[1].

Index Items- Distributed coding, wireless sensor networks, CEO problem, clustering algorithms, convolutional codes.

I. INTRODUCTION

In a clustered wireless sensor network (WSN), due to continuous nature of most environmental data sources, observed data by adjacent sensors are highly correlated. Spatial correlation for some practical continuous-valued data sources can be modeled with a closed form expression in terms of cluster size and sensors sepearation distances [1].

The overall data flow efficiency from transmitters to the destination can be improved considering this correlation in distributed source coding (DSC) design that relies on the Slepian-Wolf famous theorem [2]. Several distributed joint source channel codes (D-JSCC) are proposed to address this issue and to optimize data throughput from sensors with correlated data to a single receiver [3]–[6]. In most of these schemes, complete side information is assumed to be known at the destination.

One special case of distributed source coding is to estimate a single data source using different observations, that is called chief executive officer (CEO) problem. The distortion-rate function for discrete-valued data sources is calculated in [7]. The CEO problem for the case that source data and multiple observations are jointly Gaussian distributed is studied in [8]. Both these works consider source coding assuming error-free channels from sensors to the receiver.

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A practical scenario in which a single binary source is observed by a cluster of sensors is considered in this manuscript. Observation accuracy of sensors is modeled as a binary symmetric channel (BSC). This assumption is commonly used to model the correlation among different observations of a single binary source [9]. The sensors transmit the observed data collectively and independently through orthogonal AWGN channels to a sink node, which estimates the source data. The main contribution of this paper is to answer the question that how many sensors in each cluster is required to reliably estimate a binary source at the sink node considering both observation inaccuracy and channel quality.

A closed form expression is derived that relates system capacity to number of utilized sensors and system parameters. The result of this analysis can be used as a criterion in clustering algorithms in addition to different optimization criteria proposed in the literature such as power consumption, scalability, sensor life time and data flow efficiency [10], [11]. This dramatically increases the efficiency of clustering algorithm in terms of sensor deployment, power consumption and bandwidth usage.

The analytical results are examined with a D-JSCC scheme that is implemented based on the parallel concatenated convolutional codes (PCCC) with an iterative decoding algorithm. A similar coding scheme with complete side information available at destination is proposed in [5], while in our proposed scheme, the output of each sensor provides partial side information for the other sensors. The proposed decoder is equipped with a self correlation extraction block that makes it an appropriate option for the systems with time-varying and unknown correlation parameters.

The rest of this paper is organized as follows. In section II, the system model and assumptions are defined. In section III, information capacity of the system versus different system parameters is derived. Section IV describes a practical implementation of D-JSCC coding based on PCCC codes with an iterative decoding algorithm. Numerical results are analyzed in section V followed by conclusion in section VI.

II. SYSTEM MODEL

A cluster of sensors in proximity of a binary source is considered. The scenario is depicted in Figure 1, where source data $\{S(t)\}_{t=1}^{\infty}$ is an independent identically distributed (i.i.d)

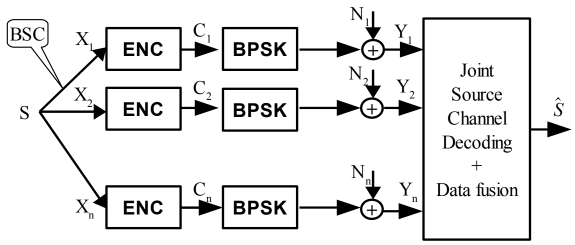

Fig. 1. System model for estimating a binary source data observed by multiple sensors.

Bernoulli sequence with $p(S(t) = 0) = p(S(t) = 1) = 1/2$. Each sensor observes a noisy version of data denoted by X_i. The observation error at sensor X_i is modeled as a BSC channel with crossover probability P_i, such that $X_i(t) = S(t) \oplus E_i(t)$ where $E_i(t)$ is a binary random variable (RV) with probability,

$$p(E_i(t) = 1) = 1 - p(E_i(t) = 0) = P_i \tag{1}$$

P_i is the probability of bit flipping due to observation inaccuracy in the i^{th} sensor. The observation error probability is a small value and for simplicity equal crossover probability is assumed hereafter,

$$P_i = P_b \ll \frac{1}{2} \qquad i = 1, 2, ..., n \tag{2}$$

where n is the number of sensors. Since the BSC channels between the source and sensors are assumed to be independent, using the Bayes' theorem and considering independent observation errors, the correlation parameter between each set of two sensors can be modeled as follows

$$\begin{aligned}
&p(X_i = 0|X_j = 0) = \frac{p(X_i = 0, X_j = 0)}{p(X_j = 0)} \\
&= \frac{\sum_{m=0,1} p(X_i = 0, X_j = 0|S = m)p(S = m)}{\sum_{k=0,1} p(X_j = 0|S = k)p(S = k)} \\
&= \frac{\sum_{m=0,1} p(X_i = 0|S = m)p(X_j = 0|S = m)p(S = m)}{\sum_{k=0,1} p(X_j = 0|S = k)p(S = k)} \\
&= \frac{(1/2)[(1 - P_i)(1 - P_j) + P_i P_j]}{1/2} = 1 + 2P_i P_j - P_i - P_j \\
&= 1 + 2P_b^2 - 2P_b \approx 1 - 2P_b \quad (\text{for} \quad P_b \ll \frac{1}{2})
\end{aligned} \tag{3}$$

where X_i represents $X_i(t)$ at any time t = 0,1,2,... . Similarly it can be shown that

$$\begin{aligned}
&p(X_i = 1|X_j = 1) \approx 1 - 2P_b \\
&p(X_i = 1|X_j = 0) = p(X_i = 0|X_j = 1) \approx 2P_b
\end{aligned} \tag{4}$$

This means that all sensors are pairwise correlated. One can model this with a BSC channel with parameter of $2P_b$. This result is used in decoder design for the proposed coding scheme in section IV.

III. CHANNEL CAPACITY

The channel between source and destination in the system consists of serial concatenation of broadcast BSC channels and parallel AWGN channels. In this section, capacity of such a hybrid channel is calculated to define the maximum possible information exchange per transmission between source data S and its estimation $\hat{S}$ for a given system parameter set, using the following definition for channel capacity

$$C = \max_{<p(S)>}\{I(S; \hat{S})\} \tag{5}$$

where $I(.)$ is mutual information function. By symmetry, the capacity is maximized for equal probabilities of '0' and '1', $p(S = 0) = p(S = 1) = \frac{1}{2}$. Also, since $\hat{S}$ is calculated based on the received symbols $Y_1, Y_2, ..., Y_n$, hence (5) can be rewritten as

$$C = I(S; Y_1, Y_2, ..., Y_n) \quad \text{for } p(S = 0) = p(S = 1) = \frac{1}{2} \tag{6}$$

Hereafter for simplicity, RV sets $\{X_1, X_2, ..., X_n\}$ and $\{Y_1, Y_2, ..., Y_n\}$ are shown with X^n and Y^n, respectively. χ^n and $\mathcal{Y}^n$ are the support set of X^n and Y^n, respectively. The mutual information is calculated as

$$I(S; Y^n) = H(Y^n) - H(Y^n|S) \tag{7}$$

The absolute and conditional entropy of RV set Y^n with density function $f(y^n)$ are calculated as follows

$$H(Y^n) = \int_{\mathcal{Y}^n} f(y^n) log(f(y^n)) dy^n \tag{8}$$

$$H(Y^n|S) = \sum_{s=0,1} \int_{\mathcal{Y}^n} f(s, y^n) log(f(y^n|s)) dy^n \tag{9}$$

where y^n is realization of Y^n, and dy^n is $dy_1 dy_2 ... dy_n$. Using conditional probability definition, we have

$$f(s, y^n) = p(s) f(y^n|s) \tag{10}$$

$$f(y^n) = \sum_{s=0,1} p(s) f(y^n|s) \tag{11}$$

Therefore (8) and (9) can be rewritten as

$$H(Y^n) = \int_{\mathcal{Y}^n} \sum_{s=0,1} p(s) f(y^n|s) log(\sum_{s=0,1} p(s) f(y^n|s)) dy^n \tag{12}$$

$$H(Y^n|S) = \sum_{s=0,1} \int_{\mathcal{Y}^n} p(s) f(y^n|s) log(f(y^n|s)) dy^n \tag{13}$$

In order to calculate $f(y^n|S)$, note that the received symbol Y_i depends on the i^{th} observation X_i, and X_i depends on the value of S. In other words, if the observation X_i is known, Y_i does not depend on S. One can assume three RV sets $\{S\}$, $\{X^n\}$ and $\{Y^n\}$ form a Markovian Chain [12]. Consequently, it follows that

$$f(y^n|s) = \sum_{x^n \in \chi^n} f(y^n|x^n)p(x^n|s) \tag{14}$$

Noting that the BSC channels between the source and different observations are independent, $p(x^n|s)$ can be rewritten as

$$\begin{aligned} p(x^n|s) &= p(x_1|s)p(x_2|s)...p(x_n|s) \\ &= P_b^k(1-P_b)^{(n-k)} \end{aligned} \tag{15}$$

where k is the number of flipped bits in the observation set realization x^n with respect to source realization value s.

To calculate $f(y^n|x^n)$, noting that the channel between sensors X_i and the destination are uncorrelated AWGN channels with $SNR = P/\sigma_N^2$, where P is the signal power and σ_N^2 is the noise variance. It is assumed that $-\sqrt{P}$ and $\sqrt{P}$ are sent by the i^{th} sensor for $X_i = 0$ and $X_i = 1$, respectively. This results in

$$f(y^n|x^n) = f(y_1|x_1)f(y_2|x_2)...f(y_n|x_n) \tag{16}$$

$$f(y_i|x_i) = \frac{1}{\sqrt{2\pi\sigma_N^2}} e^{-\frac{(y_i-(2x_i-1)\sqrt{P})^2}{2\sigma_N^2}} \tag{17}$$

This means that RV set Y^n is jointly Gaussian with mean equal to x^n and variance matrix $\sigma_N^2.I_{n\times n}$ where $I_{n\times n}$ is the $n \times n$ unitary Matrix. Therefore, (16) is rewritten as

$$f(y^n|x^n) = \frac{1}{(2\pi\sigma_N^2)^{n/2}} e^{-\frac{\sum_{i=1}^{n}(y_i-(2x_i-1)\sqrt{P})^2}{2\sigma_N^2}} \tag{18}$$

Using (15) and (18), the equation (14) can be converted to

$$f(y^n|s) = \frac{1}{(2\pi\sigma_N^2)^{n/2}} \sum_{x^n \in \chi^n} P_b^k(1-P_b)^{(n-k)} . e^{-\frac{\sum_{i=1}^{n}(y_i-(2x_i-1)\sqrt{P})^2}{2\sigma_N^2}} \tag{19}$$

where k is the number of ones in RV set x^n. Replacing (19) in (12) and (13) and noting that by symmetry the value of integral does not depend on the positions of 1s and 0s in the RV set x^n and considering the fact that the number of RV sets x^n including k number of ones is $C_n^k = \frac{n!}{k!(n-k)!}$, the entropy equations (12) and (13) are reduced to

$$H(Y^n) = \int_{\mathcal{Y}^n} \sum_{s=0,1} p(s)\alpha log(\sum_{s=0,1} p(s)\alpha) dy^n \tag{20}$$

$$H(Y^n|S) = \sum_{s=0,1} \int_{\mathcal{Y}^n} p(s)\alpha log(\alpha) dy^n \tag{21}$$

where α is defined as

$$\alpha = \frac{1}{(2\pi\sigma_N^2)^{n/2}} \sum_{k=0}^{n} C_n^k P_b^k (1-P_b)^{(n-k)} . e^{-\frac{\sum_{i=1}^{k}(y_i-\sqrt{P})^2 + \sum_{i=k+1}^{n}(y_i+\sqrt{P})^2}{2\sigma_N^2}} \tag{22}$$

Substituting (20) and (21) in (7), system capacity is defined as a closed form function of the number of sensors, observation crossover probability and the SNR value at the receiver.

$$C = max\{I(S;Y^n)\} = f(n, SNR, P_b) \tag{23}$$

Based on Shannon's well-known channel capacity theorem, reliable communication with arbitrary low probability of error is possible, provided that the coding rate remains below the channel capacity [13]. For a given number of sensors in a particular channel noise level and observation accuracy, the maximum coding rate is defined. On the other hand, if the coding rate is fixed, which is the case in most applications, the minimum number of sensors can be determined to set up a reliable communication system.

IV. PROPOSED CODING/DECODING SCHEME FOR CORRELATED SOURCES

A. Coding for three sensors with correlated data

In this section, a coding scheme based on the PCCC coding is proposed for the introduced system model. In the proposed scheme, each sensor consists of a random interleaver followed by a recursive systematic convolutional (RSC) encoder with feedforward polynomial $f(D) = 1 + D^2 + D^3$ and feedback polynomial $g(D) = 1 + D + D^3$, where $f(D)$ and $g(D)$ are irreducable primitive polynomials widely used in RSC based code design [14].

The sensors arrange the observed binary sequence into frames with length M to form frames $\{X_i(t)\}_{t=1}^{M}$. Each frame is passed through an RSC encoder to generate both systematic and parity bit sets. Random interleaving is performed prior to encoding in order to increase the minimum distance of the output codewords. The output codewords are punctured with an appropriate puncturing method to achieve desired coding rates. Consequently, the output frame of each sensor $\{C_i(t)\}_{t=1}^{M/R}$ may include systematic bits, parity bits, or both. Coding rates $R = 1$ and $R = \frac{1}{2}$ used in the simulation results. For $R = 1$, the redundancy is provided by the intrinsic similarity among the sensors' observed data.

The punctured output frames are BPSK modulated with different carrier frequencies and are transmitted through independent AWGN channels to the sink node. Since the input of sensors are corrupted versions of the same data source, the combination of the resulting codewords form a codeword similar to that of a PCCC encoder. The encoder structure is distributed among sensors. Intuitively, a multiple turbo decoder (MTD) can be used at the sink node to decode the input data sequence.

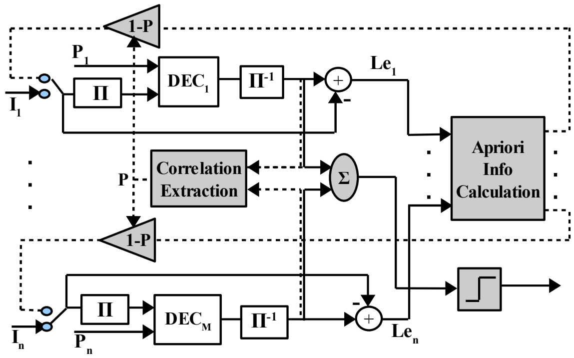

Fig. 2. Proposed decoder based on multiple turbo decoder with self correlation extraction block.

B. Decoder structure

The proposed MTD based decoder structure is depicted in Figure 2. Multiple turbo decoders may have different structures to exchange information between RSC decoder blocks including serial, master-slave, and parallel structures. Parallel structure is chosen in the proposed method due to its superior BER performance [15]. The following modifications are applied to the decoder structure in order to improve the BER performance, considering the similarity of sensors data.

In a generic parallel structure MTD decoder, since all the RSC decoders correspond to the RSC encoded version of the same data, the average extrinsic bit log likelihood ratios (LLR) of all decoders except one decoder is applied as apriori information to that RSC decoder. The crossover probability between sensors is $2P_b$ according to (24), which is considered in LLR exchange process. First, $\hat{P}_b$ is calculated as the estimation of P_b at the end of each iteration using an algorithm proposed in [16]. Then, average of the resulting extrinsic LLRs in the next iteration are scaled down with similarity factor $(1-2\hat{P}_b)$ before applying as apriori information to the RSC decoders,

$$L_{a_i}(k) = (1-2\hat{P}_b)\frac{\sum\limits_{j=1,j\neq i}^{n} L_{e_j}(k)}{n-1} \qquad ,n>1 \qquad (24)$$

where $L_a(k)$ and $L_e(k)$ are apriori and extrinsic LLRs of the k^{th} systematic bit, respectively.

Another modification is performed in hard decision block, where the average of the output LLRs of all RSC decoders $LLR_{av} = \frac{\sum_{i=1}^{n} L_{o_i}}{n}$ is used instead of using output LLRs of a particular RSC decoder to estimate the output data. This causes the final output bits to converge to the original data source rather than converging to the observation of that particular sensor. The BER improvement due to these modifications is considerable and is presented in the simulation results.

V. SIMULATION RESULTS

In this section, results of both the analytical expression derived for the system capacity in section III and the PCCC based distributed coding scheme proposed in section IV are analyzed.

Equations (7), (20), and (21) in section III, provide a closed form expression to calculate the achievable capacity as a function of observation accuracy, channel quality and number of employed sensors. This capacity curve is depicted in Figure 3.

The observation accuracy is modeled as a BSC channel with crossover probability (P_b) where lower crossover probability means higher accuracy. The channel quality is presented by the SNR value of each of the orthogonal channels at the receiver. Figure 3 shows that capacity is directly proportional with the observation accuracy and channel quality as expected.

Figures 4 and 5 provide two different slices of Figure 3 to demonstrate the effect of observation accuracy and channel quality on the capacity curve more clearly. Figure 4 shows that the capacity changes considerably with observation accuracy. As an extreme case of totaly unobservable source, when the crossover probability is $\frac{1}{2}$, no information is detectable at the destination regardless of the number of employed sensors and the sensor-sink channel quality.

In Figure 5, the channel capacity is plotted for a fixed observation accuracy $P_b = 0.01$. The capacity is an increasing function of the channel quality. If a certain value of capacity is desired, it can be achieved at lower SNR values, if more sensors are utilized in the system. For instance, to achieve capacity of $\frac{1}{2}$ bit/transmission, the required SNR values for $N = 2,3,4,5$ sensors are $-2.570dB$, $-4.366dB$, $-5.628dB$, and $-6.621dB$, respectively. A different perspective reveals that in power constrainted sensors, the presented graphs may be used to determine the minimum number of sensors to achieve a certain level of capacity. For instance, at least 4 sensors are needed to achieve a capacity of $\frac{1}{2}$ for SNR=-5dB.

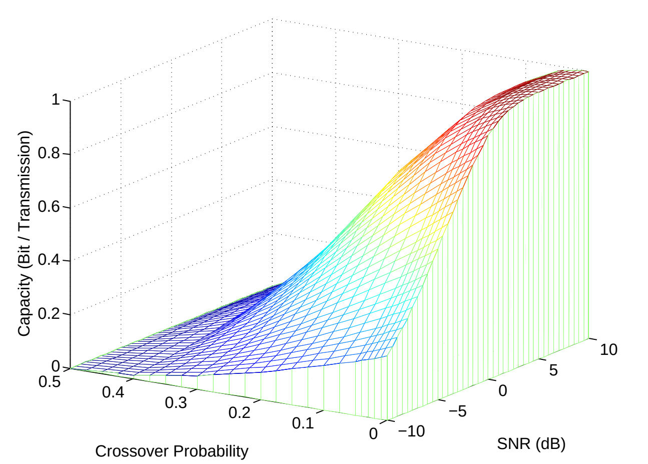

Fig. 3. Information capacity of system versus observation accuracy (BSC crossover probability) and channel quality (SNR) for 4 sensors.

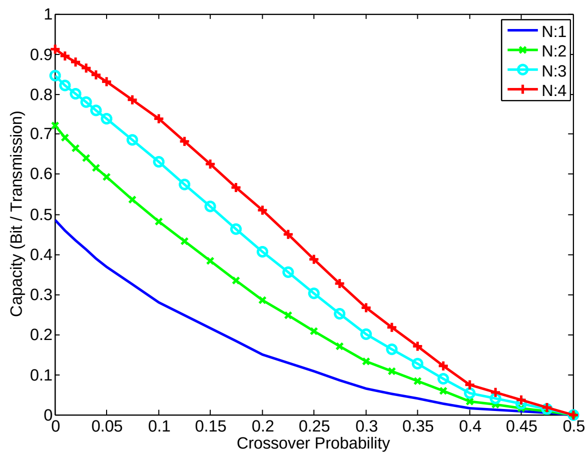

Fig. 4. Capacity of system versus observation accuracy, $SNR = 0dB$.

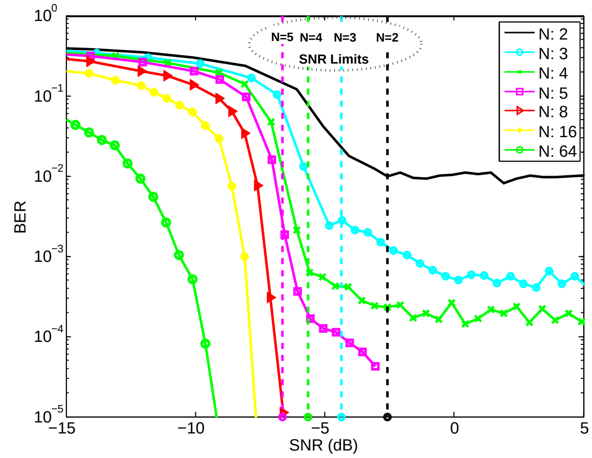

Fig. 6. BER Performance of proposed coding scheme based on PCCC codes with modified multiple turbo decoder. Observation accuracy is modeled with a BSC channel with crossover probability of 0.01.

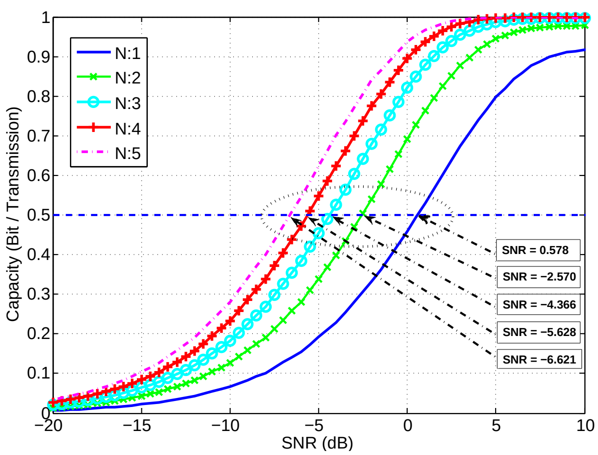

Fig. 5. Information capacity of system versus channel SNR. Observation accuracy is modeled with a BSC channel with crossover probability P_b = 0.01.

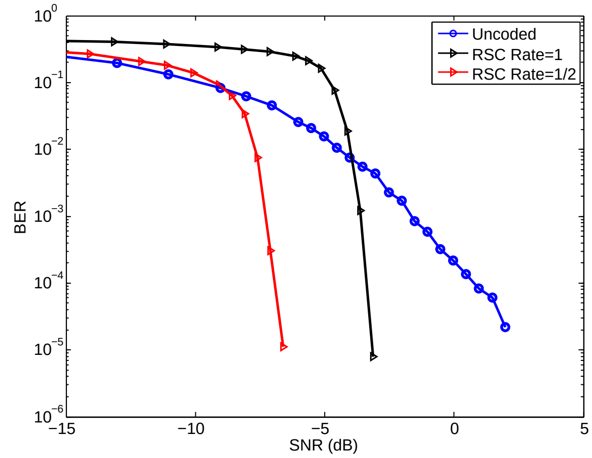

Fig. 7. Comparison of system BER Performance for sensors composed of RSC encoders with coding rates of 1 and 1/2 with uncoded sensors.

However, the capacity curve saturates at high SNR values and the maximum capacity is limited by the number of sensors.

Figure 6 depicts BER performance of the proposed coding scheme. In this simulation, the source data is an i.i.d equiprobable Bernoulli binary sequence. Observation accuracy is set to $P_b = 0.01$ and data frames with 256 bits length and BPSK modulation are considered.

The results show that as the number of sensors in each cluster is increased, the system BER performance is improved at the cost of higher receiver complexity. Since RSC encoders operate at coding rate $\frac{1}{2}$, based on Shannon's theorem they achieve arbitrary low error rates at the SNR values higher than the limits derived from the capacity curve. The results confirm the sub optimality of the proposed coding scheme, since it achieves relatively low bit error rate at the derived SNR limits. However, it is noticeable that the error floor can not be improved considerably by increasing the SNR value, since it is limited by the number of sensors and observation accuracy. This is due to the fact that encoding is done at the sensors and not at the source data location. Therefore, the observation accuracy defines the error floor. To achieve lower error floors, a large number of sensors must be employed at each cluster.

In Figure 7, performance of the proposed system for RSC encoders with rates $= \frac{1}{2}$ and 1 are compared to the system employing uncoded sensors and maximum likelihood detector. The results demonstrate considerable improvement in system performance even for RSC encoders with rate $= 1$ compared to the uncoded system. It is obvious that using lower coding rates improves the system performance. The proposed system outperforms an uncoded system if the SNR value is relatively high. For very low SNR values the coding is not helpful. The coding rate ranges between $\frac{1}{2}$ and 1 based on the puncturing method. When rate $= 1$ sensors are used, the redundancy is provided by the correlation among sensors' data.

Convergence properties of the proposed system is analyzed

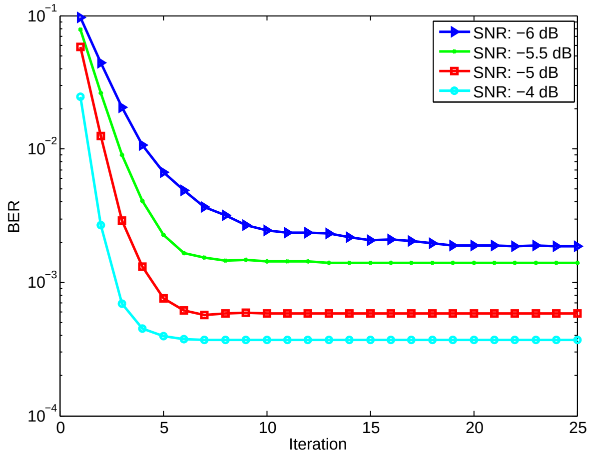

Fig. 8. BER performance of modified multiple turbo decoder for 4 sensors versus different decoder iterations, BSC crossover probability is 0.01.

in Figure 8. The decoder converges very fast and almost reaches its asymptotic performance at iterations between 5 and 10 for different SNR values, while typically more than 10 iterations is required for generic turbo decoders to converge [17]. The fast convergence of the system is desirable in complexity constrainted sink nodes or in application with strict latency requirements. The fast convergence property is based on the fact that the output LLRs of RSC decoders represent the correlated versions of the source data, therefore the system performance may not be further improved after a few iterations.

VI. Conclusions

In this article, a CEO problem of estimating a binary source is considered. System information capacity is derived as a function of observation accuracy, channel quality and the number of sensors. Observation accuracy is mostly defined by the application and is relatively low for the applications that sensors can not be placed at the exact data source location. Also, the channel SNR is limited by the power consumption constrained tiny sensors. The derived expression for the capacity defines the achievable capacity for different number of sensors. Inversely, if the coding rate is fixed which is the case in most practical applications, the minimum required number of sensors can be determined in order to achieve arbitrary low error rates for any given set of system parameters.

A practical PCCC based distributed coding scheme is proposed that provides acceptable performance using the intrinsic correlation among sensors data. The SNR values required to achieve low error rate using different number of sensors is very close to the limits derived from the system capacity curve. The fast convergence property and simple encoder structures of the proposed system makes it an appropriate choice for wireless sensor networks. Both analytical and simulation results show that the performance of the system is saturated by the SNR value. Consequently, the number of sensors are mainly imposed by the desired performance level and the observation accuracy.

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