A Reputation-based Stackelberg Game Approach for Spectrum Sharing with Cognitive Cooperation

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Abstract— **In this paper, we consider the problem of spectrum sharing with cooperation in a wireless system with two pairs of source-destination links, which have different priorities to access the channel. The cognitive low priority user (LPU) cooperates with the high priority user (HPU), acting as a relay, in exchange for an opportunity to access the channel. We study the interactions between the HPU and the LPU using a game theoretical approach to search for the optimal time allocation for individual transmissions and cooperation among users. We propose a new Stackelberg game formulation, in which the reputation of both users is considered to encourage cooperation and prevent misbehavior. The proposed utility functions are designed so that fairness and energy efficiency are taken into account.**

I. Introduction

Cooperation and cognition techniques have been identified as means to improve throughput and efficiency of spectrum usage in wireless networks. Cognitive cooperation, a combination of the above two techniques, can improve the spectrum utilization by allowing a cognitive user to access the channel with lower priority, as long as it cooperates with the high priority user. Additionally, cognitive cooperation may increase the throughput of the high priority user (HPU), since the low priority user (LPU) is required to act as a relay.

The cooperation between nodes based on the traditional model of cognitive networks, with primary (licensed) and secondary (unlicensed) users, has been studied in numerous recent work. In [1]–[3], cooperative relays are enabled in idle time slots, taking advantage of the bursty nature of traffic of the primary source, and the stable throughput regions are analyzed. It was shown that, in such cognitive cooperative systems, having packets relayed by the secondary would help to empty the primary queue, thus creating better transmitting opportunities for the secondary, and increasing the stable throughput of both the primary as well as the secondary node as compared to the non-cooperative case. The occurrence of retransmissions by primary users is also reduced, resulting in power savings.

The study of cognitive cooperation has been further enhanced by using game-theoretic tools to assist with the resource allocation among users [4]–[7]. A scenario with multiple primary networks is considered in [4], where the authors compare three pricing models for the spectrum to be sold to secondary users. A game model for spectrum access is presented in [5], with secondary users searching for transmission opportunities in portions of the spectrum unused by primary users.

A spectrum leasing scheme is proposed in [6], where a primary user allocates the channel to a secondary ad hoc network for a fraction of the time, and the secondary network helps forward primary's packets using distributed space-time coding technique. The proposed scheme uses a hierarchical Stackelberg game model, where the primary user is the game leader and selects the fractions of time to be used for cooperative and individual transmissions, aiming to maximize its own rate. Following the decision made by the leader user, the group of follower users optimizes their own power to obtain higher transmission rates. A priced-based game model for spectrum leasing is proposed in [7], where the time allocation and also the price of spectrum are set by the primary, while the selected secondary user may increase its transmission rate by optimizing its transmission power.

The inter-network fairness has not been addressed in the aforementioned game theoretical approaches for resource allocation, nor has the energy cost involved in the transmissions. These are important issues in spectrum sharing problems, given the increasing concern with energy efficiency in wireless networks, and the benefits of a fair resource allocation that attends the needs of multiple users. We note that it may not be interesting for the secondary network to cooperate if the time allocated for its transmissions is very small, or if the amount of energy spent with cooperation is very large, not compensated by the transmission opportunity received as a reward. In this sense, incorporating fairness into game definition is also a mechanism to encourage cooperation among users.

Cooperation is not an inherent characteristic of multi-user communication networks, as users contend for resources, and often present selfish and rational behavior. Therefore, different incentive-based approaches have been studied in the literature to encourage cooperative packet forwarding. These schemes can be categorized into pricing-based and reputation-based schemes [8]. In pricing-based schemes, the relay node earns credits when it forwards other users' packets. The credit is usually in the form of virtual currency. Thus, a central controller is required to ensure the payment among the users [9]–[11]. In reputation-based schemes, the nodes adjust their strategies based on the reputation of other nodes. Hence, each user tries to maintain a good reputation to

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benefit from cooperation in following interactions [12]–[14].

In this paper, we consider a simple system model with two users sharing the same channel, where one user has higher priority to use the channel resources than the other. We model the interaction of the users aiming to obtain the optimum time allocation for the individual transmissions and the cooperative transmission. Given the different priorities in using the resources, a natural approach is to use a Stackelberg game model, in which the HPU is the leader, while the LPU is the follower player in the game. Our game model considers both concepts of pricing and reputation, and can be implemented in a distributed manner. The pricing mechanism is implemented using the spectrum as a real currency to be exchanged between users, instead of a virtual one. The reputation mechanism is based on cooperation credits, and is used to monitor the behavior of both users over the course of time. The players sequentially make their decisions, observing the reputation of their opponent, and take the best actions considering how cooperative the other player was in the previous rounds. As a result, the HPU is encouraged to allow the LPU to access the spectrum, and selfish misbehavior of the LPU in packet forwarding is discouraged.

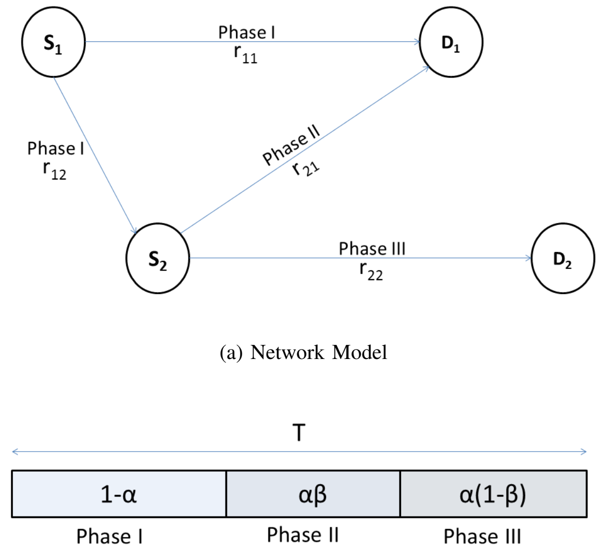
In comparison to other models presented in the literature, in particular in [6] and [7], the main contributions of our model include (i) the use of a reputation-based game to perform spectrum allocation, providing the framework to analyze the response of the users in the long run, (ii) the use of utility functions that account for fairness and energy efficiency in resource allocation, (iii) the use of a more complete formulation for the achievable rates, which considers not only the information exchange through a decode and forward multi-hop channel, but also the information flow through the direct link from source to destination, and (iv) the definition of a Stackelberg game in which the resource allocation is defined not only by the leader, but by the follower as well.

II. System Model

The system model consists of two point-to-point wireless links sharing the same channel. One source-destination pair has priority in using the channel resources, and is referred to as high priority user (HPU). The other pair has lower priority in using the resources, and its nodes are called low priority user (LPU). A cooperative spectrum sharing scenario is analyzed, in which the low priority source node relays packets from the high priority source node, in exchange for an interval to transmit its own packets.

Each time frame is allocated between three activities: the high and low priority users individual transmissions, and the cooperative relaying. Each time frame is associated with a cooperation cycle, characterized by two variables, α and β. In each frame, the channel is allocated as follows [7]:

- Phase I: only the HPU transmits its data for $(1-\alpha)T$ seconds, $(0 \leq \alpha \leq 1)$;
- Phase II: the LPU relays high priority user's data to D_1 for $\alpha\beta T$ seconds, $(0 \leq \beta \leq 1)$;



(a) Network Model

(b) Time Frame Model

Fig. 1. System Model

- Phase III: the LPU transmits its own data for $\alpha(1-\beta)T$ seconds.

Figure 1(a) illustrates the network model with the four nodes. The HPU and LPU are identified with priority index $i = 1,\ 2$, respectively. The source nodes are denoted with S_1 and S_2, and the destination nodes with D_1 and D_2. The picture indicates the communication links that are active in each phase, and the transmission rates in each link. Figure 1(b) presents the channel allocation model for one time frame of duration T seconds, defined by the parameters α and β, to be determined by the game model.

A Rayleigh fading channel is assumed, and represented with the channel coefficients h_{ij} for transmission between nodes i and j. The coefficients are assumed to be constant during one time frame of duration T. The nodes transmit with power P, and White Gaussian Noise is added at the receiver. Denote with γ_{ij} the signal-to-noise ratio (SNR) of signal from source node i received at node j, with $i,\ j = 1,\ 2$. The transmission rate is approximated using the Shannon capacity formulation. That is, the transmission rate between nodes i and j is given by $r_{ij} = log_2(1 + \gamma_{ij})$.

The Decode-and-Forward (DF) relaying method is employed at the LPU, assuming that at phase II the LPU forwards the fully decoded message received from the HPU's source. Amplify-and-Forward relaying method is not adequate for this scenario, as it is not flexible regarding the time sharing between direct transmission and cooperation.

III. Game Definition

A reputation-based Stackelberg game is proposed to model the interactions between the HPU and the LPU in a cooperative spectrum sharing scenario. A two-player game is defined, with HPU as a leader and LPU as a follower, to obtain the optimum time allocation of spectrum in a fair and energy-efficient manner.

In what follows, the round k of the game will be identified with the superscript k, $k \in \{1, 2, \ldots\}$. Recall that the users' parameters are identified by the priority index i, $i \in \{1, 2\}$. The game is run for each time slot. In each round of the game, the players interact to define the variables α^k and β^k, which determine the time allocation for HPU transmission, cooperation, and LPU transmission for one time slot, as described in Section II. The two source nodes are assumed to communicate with each other using a low rate dedicated control channel.

The decisions of the users take into account the reputation of their opponent, represented by a *cooperation credit*. The cooperation credit encapsulates the history of the cooperative behavior of the user, reflecting its willingness to cooperate in consecutive game rounds up to, but not including, round k. The use of a single parameter avoids the need to keep track of all the actions of each user, saving memory space and simplifying the game model.

In order to keep track of the users' reputation through a cooperation credit, the users' strategies in each round are defined as intermediate variables, s_i^k, which have impact on both the cooperation credit, and the values of α^k and β^k. The leader (HPU) chooses the value of α^k by selecting the value of the intermediate variable s_1^k, which is the HPU's strategy, and taking into account the LPU's cooperation credit C_2^k. In the sequence, the follower (LPU) chooses its strategy as the best response to the HPU's action, also taking into account the credit C_1^k.

The remaining of this Section describes in more detail the four elements that define our game model, namely the strategies, the cooperation credits, the recursion rules for α and β, and the utility functions.

A. Users' Strategies

The strategy of user i in round k is defined as an intermediate variable s_i^k, taking values in a set $\mathcal{S}$. The set $\mathcal{S}$ is assumed to be a closed interval on the real line, $\mathcal{S} = [s_{min}, s_{max}]$, where $0 \leq s_{min} < s_{max} \leq 1$. The intermediate variable s_i^k is used in the game to update both the cooperation credit and the values assigned to α^k and β^k. The implications of the value selected for s_i^k will be made clear when we describe the recursion rules used to update these variables.

In general, the strategy s_i^k incurs variation of user i's cooperation during game round k. If the opponent has positive credit history, s_i^k will result in an increase of the i-th user cooperation time, while for an opponent with negative credit history, s_i^k results in a reduction of the i-th user cooperation time.

In a Stackelberg game, the strategies are presented sequentially. In each round, the leader (HPU) selects s_1^k which optimizes its own utility, denoted with $U_1^k(s_1^k, s_2^k)$, assuming that the LPU is a rational player and it will respond to HPU's action with its best strategy. In other words, the HPU maximizes its own utility anticipating the reaction of the LPU. The best response of the LPU is the strategy that maximizes its own utility, $U_2^k(s_1^k, s_2^k)$, and it depends not only on the action of the HPU, but also on the cooperation credits and other parameters. The detailed description of the optimization problems is provided in Section IV.

B. Cooperation Credits

The cooperation credits are defined as a mechanism to encourage cooperation by using the reputation of the users. The reputation of the HPU is based on its willingness to lease the spectrum to the LPU. For the LPU, the reputation is based on how reliable it is in forwarding the relayed packets from the HPU. Both users are encouraged to sustain a good reputation, so that they can benefit from cooperation in subsequent periods. In the common Stackelberg game models, the solution of each round of the game is obtained by the one-shot backward-induction process [15]. In our proposed Stackelberg game model, the reputation of the users is included in this process, encouraging the cooperation among the users.

The credit in each round, denoted with C_i^k, contains accumulated information about the user's willingness to cooperate. To represent this concept of willingness, C_i^k assumes values on a symmetric interval $[-C, C]$, with negative values representing lack of cooperation, and positive values representing willingness to cooperate.

The cooperation credit is calculated using a recursion rule, with initial value C_i^0, updated based on the user's selected strategy and on the opponent's willingness to cooperate. The credit should be reduced if the opponent's credit is positive, but the selected strategy is not cooperative. The credit should be increased if the opponent's credit is positive and the selected strategy is cooperative. With this reasoning, we define the credit change as

$$\Delta C_i^k = (2s_i^k - 1)sgn(C_{-i}^k),\ k \geq 0, \tag{1}$$

where C_{-i}^k is the credit of the opponent, and $sgn(x)$, $x \in \mathbb{R}$ is the Sign function, defined as $+1$ for non-negative values, and -1 for negative values of x.

To accumulate credit history, the recursion rule is defined with initial value C_i^0 and updated as

$$C_i^{k+1} = C_i^k + \Delta C_i^k,\ k \geq 0. \tag{2}$$

The effect of the initial values C_1^0 and C_2^0 is studied in the numerical analysis presented in Section V.

C. Recursion Rule for α and β

The ultimate goal in each round of the game is to define $\alpha^k \in [0, 1]$ and $\beta^k \in [0, 1]$. The game is initiated with values $\alpha^0 = 0.5$ and $\beta^0 = 0.5$. The recursion rules that update these variables are as follows:

$$\alpha^k = \max\left(0, \min(\alpha^{k-1} + \alpha_s s_1^k C_2^k, 1)\right),\ k \geq 1, \tag{3}$$

$$\beta^k = \max\left(0, \min(\beta^{k-1} + \beta_s s_2^k C_1^k, 1)\right),\ k \geq 1, \tag{4}$$

where the functions max and min are used to bound the values in the interval $[0, 1]$, and α_s and β_s are nonzero quantization constant steps to modify α and β, respectively.

D. Utility Functions

To complete the game definition, the utility functions of the players are defined in this Section. The proposed utility functions account for energy efficiency and fairness in cooperative spectrum sharing, and consist of two parts, namely the throughput utility $U_{i,t}$ and the energy utility $U_{i,e}$, where subindex i identifies the HPU ($i = 1$) and the LPU ($i = 2$). In round k of the game the users select their strategies s_i^k with the objective of maximizing the throughput and minimizing energy. Therefore, the utility functions are of the form

$$U_1^k(s_1^k, s_2^k) = U_{1,t}^k - U_{1,e}^k, \tag{5}$$

$$U_2^k(s_1^k, s_2^k) = U_{2,t}^k - U_{2,e}^k. \tag{6}$$

The energy utilities $U_{i,e}$ are functions of the energy spent with transmission during the time slot k, and they introduce a cost of transmission to improve energy efficiency. The energy utilities are defined as

$$U_{1,e}^k = \delta_1(1 - \alpha^k)P, \tag{7}$$

$$U_{2,e}^k = \delta_2\alpha^k(1 - \beta^k)P + \delta_3\alpha^k\beta^k P, \tag{8}$$

where δ_1, δ_2 and δ_3 are normalizing coefficients, necessary to make rate and energy comparable. We will refer to the coefficients δ_1 and δ_2 as *transmission costs* for HPU and LPU, respectively. We will refer to δ_3 as the *cooperation cost coefficient*.

The throughput utility of each user is defined as the logarithm of its achievable rate to incorporate the fairness in spectrum sharing. Although we consider a non-cooperative game, the use of logarithmic utility functions still assigns higher priority to the user with lower effective rate, approximating the proportional fair resource allocation introduced in [16].

For the LPU, the transmission using a single link between S_2 and D_2, with rate r_{22}, results in the following expression for the throughput utility:

$$U_{2,t}^k = \log(1 + \alpha^k(1 - \beta^k)r_{22}). \tag{9}$$

Remark 1: For the LPU, we sum one unit to the effective rate, to avoid the utility function convergence problem at $\alpha^k = 0$ and assign a zero value to it for no cooperation between the users.

For the HPU, the transmission using both the direct path and the relay path requires a more elaborate expression for the throughput utility. A single-relay system with decode-and-forward (DF) and Time Division (TD) was considered in [17], where one cooperation cycle is divided into two phases: the relay is either in receive (RX) or in transmit (TX) mode. The achievable rate is presented in [17], accounting for the possibility that the source node also transmits together with the relay node (in Phase II). For our proposed model, the achievable transmission rate for the HPU is obtained from the results in [17], according to our assumption that the source node is silent during Phase II. As a result, the throughput utility for the HPU is

$$U_{1,t}^k = \log\big(\min(\frac{1-\alpha}{2}r_{12}, \frac{1-\alpha}{2}r_{11} + \frac{\alpha\beta}{2}r_{21})\big). \tag{10}$$

The final expressions for the utility functions are

$$\begin{aligned} U_1^k(s_1^k, s_2^k) = \log\big(&\min(\frac{1-\alpha}{2}r_{12}, \frac{1-\alpha}{2}r_{11} + \frac{\alpha\beta}{2}r_{21})\big) \\ &-\delta_1(1 - \alpha^k)P, \end{aligned} \tag{11}$$

$$\begin{aligned} U_2^k(s_1^k, s_2^k) = &\log(1 + \alpha^k(1 - \beta^k)r_{22}) \\ &-\delta_2\alpha^k(1 - \beta^k)P - \delta_3\alpha^k\beta^k P. \end{aligned} \tag{12}$$

IV. Equilibrium Analysis

In this Section, the equilibrium of the proposed Stackelberg game model is analyzed. In each round of the game, the players take their actions sequentially, in order of priority, noticing the credit of each other. At the first stage, the HPU (leader) optimizes its utility, under the assumption that the LPU is rational and it selects its best strategy in response to the HPU's strategy. At the second stage, the LPU (follower) selects its optimal strategy observing the strategy of the HPU. Hence, the equilibrium solution can be found by a backward-induction process.

Theorem 1: The Stackelberg equilibrium of the proposed model exists in each round of the game, and it is unique if $\alpha^k \neq 0$, $\beta^k \neq 0, 1$ and $C_1^k \neq 0$.

Proof: The leader of game (HPU) maximizes its own utility with respect to its own strategy s_1^k, assuming the LPU's rational reaction s_2^k. This is a single parameter maximization problem, for which the solution always exists, and is unique if for any s_1^k, there exists only one possible LPU's reaction, s_2^k. For any given strategy of the HPU, the optimal response (rational reaction) of the LPU, denoted with $\bar{s}_2^k$, is obtained by solving the following optimization problem:

$$\begin{aligned} \max_{s_2^k} \quad & U_2^k(s_1^k, s_2^k) \\ \text{s.t.} \quad & s_{min} \leq s_2^k \leq s_{max}. \end{aligned} \tag{13}$$

The inequality constraints in optimization problem (13) are affine functions, and the objective function is strictly concave if $\alpha^k \neq 0$ and $C_1^k \neq 0$, since the second derivative is

$$\frac{\partial^2 U_2^k}{\partial {s_2^k}^2} = -\big(\frac{\alpha^k\beta_s C_1^k r_{22}}{1 + \alpha^k(1 - \beta^k)r_{22}}\big)^2 \leq 0. \tag{14}$$

Hence, under these conditions the convex optimization problem (13) admits a unique solution $\bar{s}_2^k$. Following the KKT conditions, the solution $\bar{s}_2^k$ assumes one of the three values in (15), based on the HPU's strategy and system parameters. The corresponding conditions are not mentioned here due to the limited space.

$$\bar{s}_2^k = \begin{cases} s_{min} \\ \dfrac{as_1^k + b}{cs_1^k + d} \\ s_{max} \end{cases} \tag{15}$$

where, a,b,c and d are constant values, defined as follows:

$$a = \alpha_s C_2^k (1-\beta^{k-1}),$$
$$b = \frac{1}{r_{22}} + \alpha^{k-1}(1-\beta^{k-1}) - \frac{1}{(\delta_2-\delta_3)P},\ \delta_2 \neq \delta_3$$
$$c = \alpha_s \beta_s C_1^k C_2^k,$$
$$d = \alpha^{k-1}\beta_s C_1^k.$$

Therefore, for any given s_1^k, the LPU would response with a unique $\bar{s}_2^k$. The best response of the HPU, $\bar{s}_1^k$ is then obtained as a solution to $\max_{s_1^k} U_1^k(s_1^k, \bar{s}_2^k)$ subject to the constraint $s_{min} \leq s_1^k \leq s_{max}$.

The special cases of $\alpha^k = 0$, $\beta^k = 0$ and $\beta^k = 1$ result in trivial time allocations: (i) individual transmission for HPU during whole time frame if $\alpha = 0$, (ii) no cooperation if $\beta = 0$, and (iii) no individual transmission for LPU if $\beta = 1$. $C_1^k = 0$ refers to the case that the LPU has no ground to judge whether the HPU is cooperative or selfish, hence it takes the strategy of the previous round. ■

Theorem 2: The solution of the proposed game in each round, $(\bar{s}_1^k, \bar{s}_2^k)$, is a Nash equilibrium.

Proof: A strategy set $(s_1^{k\star}, s_2^{k\star})$ achieves Nash equilibrium if, and only if

$$\forall i \in \{1,2\},\ \forall s_i^k \in \mathcal{S},$$
$$U_i^k(s_i^{k\star}, s_{-i}^{k\star}) \geq U_i^k(s_i^k, s_{-i}^{k\star}), \tag{16}$$

where s_{-i}^k, denotes the strategy of the opponent of player i in round k of the game.

The Stackelberg equilibrium solution $(\bar{s}_1^k, \bar{s}_2^k)$ is obtained with the backward-induction process, as described in Theorem 1. First the leader (HPU) selects its strategy to maximize its utility $U_1^k(s_1^k, s_2^k(s_1^k))$ with respect to s_1^k, where $s_2^k(s_1^k)$ is the corresponding optimal response of the LPU. Once the strategy $\bar{s}_1^k$ of the HPU is announced, the LPU selects its best response $\bar{s}_2^k = s_2^k(\bar{s}_1^k)$, which maximizes the utility $U_2^k(\bar{s}_1^k, s_2^k)$ with respect to s_2^k. Under the conditions in Theorem 1, this optimal strategy is unique for any choice of HPU strategy. The solution $(\bar{s}_1^k, \bar{s}_2^k)$ is achieved upon observing the best strategy of the HPU, $\bar{s}_1$ by the LPU. Therefore, at both stages of the backward-induction process, the players set their strategy as the best possible response to the other one, which follows the definition of Nash equilibrium. In other words, none of $U_i^k(\bar{s}_1^k, \bar{s}_2^k)$ is improvable by varying s_i^k, since it violates the above-mentioned Stackelberg optimization procedure. Hence, it is concluded that Stackelberg solution of the game is a Nash equilibrium. ■

V. Numerical Results

In this section, the effectiveness of the proposed model is illustrated with numerical results. We assume normalized power $P = 1$. Recall that γ_{ij} denotes the SNR of signal from source node i received at node j, with $i, j \in \{1, 2\}$. The cooperative credits assume values in the interval $[-C, C] = [-20, 20]$, and the initial value, $C_i^0, i \in \{1, 2\}$ is set to one. We set $\alpha^0 = \beta^0 = 0.5$ and $\alpha_s = \beta_s = 1$. The cost coefficients of individual transmissions are set to $\delta_1 = 0.5$, $\delta_2 = 2$, unless otherwise stated. With these values for the transmission costs, the HPU is less concerned about the energy than the LPU. The cooperation cost coefficient is set to $\delta_3 = 0.1$, unless otherwise stated, so that cooperation is encouraged by assigning a low coefficient to the energy spent in relay activity.

TABLE I

Effect of initial cooperative credits (C_1^0, C_2^0) on convergence of α

(C_1^0,C_2^0)	(-1,-1)	(-1,1)	(1,1)	(1,-1)
k	15	11	14	11

(C_1^0,C_2^0)	(-15,15)	(15,-15)	(-15,10)	(-10,15)
k	5	5	5	5

In Table I, we study the effect of the initial cooperative credits, C_1^0 and C_2^0, on the convergence of α. By convergence we mean that the consecutive values are within a convergence interval, and the simulation is terminated if $\left|\alpha^k - \alpha^{k-1}\right| < \epsilon$. We set $\epsilon = 0.0005$, and present the results for the number game iterations k, necessary for convergence. The results show that when there is a large difference between the user's initial credits, the game settles in few rounds, before the users have the chance to interact with each other to obtain the optimal time allocation. However when the initial cooperation credits are close, the users iterate for more time slots, and the reputation mechanism can be more effective to encourage the cooperation. Small values of C_i^0 promote more iterations, and the pair $(C_1^0, C_2^0) = (1, 1)$ is verified to be an adequate choice. Noting the results of Table I, the number of game rounds is set to $k = 12$ to ensure the results are settled down in their final values.

In Figure 2, we show the behavior of the parameter α when varying the quality of the relay channel, represented by the SNR in the link (S_1-S_2), denoted with γ_{12}. We set $\gamma_{21} = 20$ dB, $\gamma_{22} = 10\ dB$. As showed in Fig. 2 for small values of γ_{12}, a small value is obtained for parameter α, meaning that the HPU prefers to use the direct link noting the bad quality

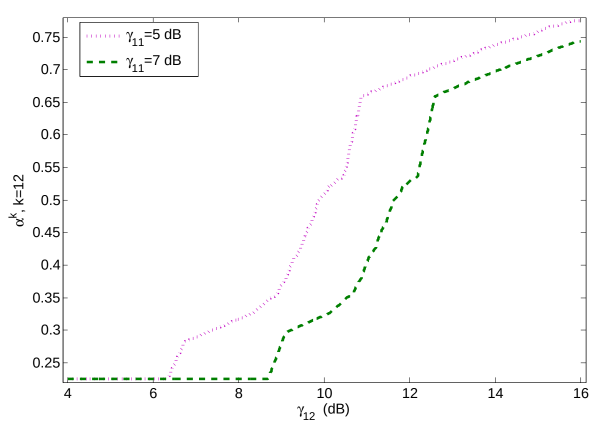


Fig. 2. α^k versus γ_{12} for different values of γ_{11}, $k = 12$.

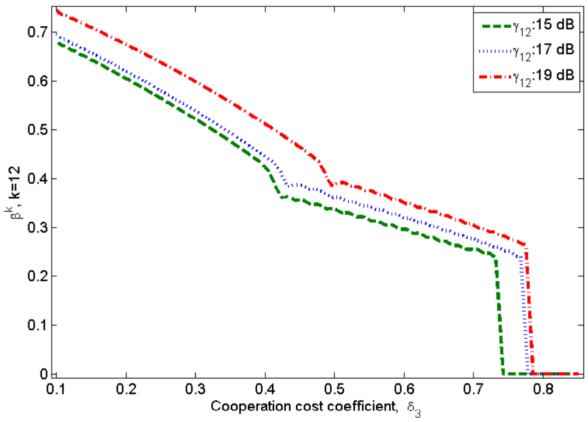


Fig. 3. β^k versus cooperation cost coefficient δ_3, for different values of γ_{12}, $k = 12$

of the relay channel. When γ_{12} becomes larger than γ_{11}, the HPU is encouraged to cooperate, since the achievable rate can be improved by using the relay. Therefore, we observe that α increases with increasing γ_{12}, resulting in a larger interval allocated for cooperation. We also observe that for a smaller value of γ_{11}, α assumes larger values. This is because a smaller γ_{11} represents worse conditions in the direct channel, which also encourages cooperation.

In Figure 3, we present the behavior of the parameter β while varying the cooperation cost coefficient δ_3. We set $(\delta_1, \delta_2) = (0.7, 1.8)$, $\gamma_{11} = 5$ dB, $\gamma_{21} = 20$ dB, $\gamma_{22} = 10$ dB. We observe that β decreases as the cooperation cost coefficient increases. This is because for large values of δ_3 the cooperation is discouraged, and the portion of time allocated to cooperation ($\alpha\beta$) is reduced. If δ_3 is significantly large, providing relay service would be very costly for the LPU, and it may not be compensated by the transmission opportunity received as a reward. In that case, the LPU prefers to not cooperate with the HPU. Additionally, we observe that when the relay channel has better quality, represented by larger values of γ_{12}, the users are more encouraged to cooperate, increasing the resulting values of β.

Next, we evaluate the performance of the proposed model in terms of fairness. Once the parameters α^k and β^k are obtained as a result of one round of the game, we have defined the time allocation as described in Section II. The corresponding achievable rates are calculated as shown in the utility functions (11) and (12). Let $\mathbf{r}^k = (r_1^k, r_2^k)$ be the vector with the achievable rates corresponding to the time allocation in round k. We will compare two rate vectors using the Jain's Fairness Index [18], defined below.

Definition 1 (Jain's Index): For a given allocation vector $\mathbf{r} \in \mathbb{R}^2$, the Jain's index is calculated as

$$\mathcal{J}(\mathbf{r}) = \frac{\left(\sum_{i=1}^{2} r_i\right)^2}{2\sum_{i=1}^{2} r_i^2}. \tag{17}$$

In Table II, we present values of the Jain's index while changing the quality of the direct channel, represented by the SNR value γ_{11}, when $\gamma_{12} = 16$ dB, $\gamma_{21} = 20$ dB and $\gamma_{22} = 10$ dB. We denote with $\mathbf{r}$ the rate vector obtained with considering fairness (with the logarithm), and $\hat{\mathbf{r}}$ the vector without it. We observe that incorporating fairness in the utility functions definition by replacing the rates with the logarithm of the rates is effective in promoting fairness with respect to the Jain's Index, and $\mathcal{J}(\mathbf{r}) > \mathcal{J}(\hat{\mathbf{r}})$.

TABLE II
JAIN'S INDEX VERSUS γ_{11} (IN DB), WITH AND WITHOUT LOGARITHMS IN THE UTILITY FUNCTIONS.

$\gamma_{11} =$	2	4	6	8
$\mathcal{J}(\mathbf{r})$	0.9578	0.9457	0.9392	0.9258
$\mathcal{J}(\hat{\mathbf{r}})$	0.5458	0.5374	0.5233	0.5219

VI. CONCLUSION

A new Stackelberg game model to study spectrum sharing among the low and high priority users in a cooperative scenario is proposed. The solution of the game contains two parameters, α and β, which define the time allocation to individual transmissions and cooperative transmission. Our model considers the reputation of each user, represented by the cooperation credits, to encourage the cooperation and prevent misbehavior. By this we keep track of the users' cooperative behavior without needing to save the entire users' action history. Furthermore, in the proposed model, the utility functions account for fairness and energy efficiency. The effect of the initial values for users' cooperation credits in the game results is studied, and the conditions under which the players will interact for a longer time before reaching the optimal values of α and β are discussed. Furthermore, the variation of these parameters versus the channel quality and the cooperation transmission cost is investigated. Finally, the performance of the proposed game model is compared to the case with no fairness concerns to verify that this model results in more fair transmission rates for the users.

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