1) Solve the following recurrence relation.

a) $x(n) = x(n-1) + 5$ for $n > 1$ $x(1) = 0$

at $n=1$; $x(1) = 0$ (Given)

at $n=2$; $x(2) = x(2-1)+5$

$= x(1)+5$

$= 0+5$

$x(2) = 5$

at $n=3$; $x(3) = x(3-1)+5$

$= x(2)+5$

$= 5+5$

$x(3) = 10$

at $n=4$; $x(4) = x(4-1)+5$

$= x(3)+5$

$= 10+5$

$x(4) = 15$

$\because$ $x(n)$ Increases by 5 for each

increment of 5

difference $(d) = 5$

$x(n) = x(1) + (n-1)\cdot d$

Here, $x(1) = 0$, $d = 5$

$x(n) = 0 + (n-1)5$

$x(n) = 5(n-1)$

Ans: $x(n) = 5(n-1)$

b). $x(n) = 3x(n-1)$ for $n > 1$ $\quad x(1) = 4$

$n = 1$; $x(1) = 4$ (Given)

$n = 2$; $x(2) = 3x(2-1)$

$= 3x(1)$

$= 3 \times 4$

$x(2) = 12$

$n = 3$; $x(3) = 3x(3-1)$

$= 3x(2)$

$= 3 \times 12$

$x(3) = 36$

$n = 4$; $x(4) = 3x(4-1)$

$= 3x(3)$

$= 3 \times 36$

$x(4) = 108$

$\therefore$ $x(n)$ obtained by Multiplying the previous term by 3

Ratio = 3

$x(n) = x(1) \cdot r^{n-1}$

Here. $x(1) = 4$, $r = 3$

$x(n) = 4 \cdot 3^{n-1}$

Ans:- $x(n) = 4 \times 3^{n-1}$

i). $x(n) = x(n/2) + n$ for $n>1$ $x(1) = 1$ (solve for $n = 2k$)

$n = 2k$

$n = 1; \; x(1) = 1$

$n = 2; \; x(2) = x(2/2) + n = x(1) + 2$

$= 1+2$

$= 3$

$x(2) = 3$

$n = 4: \; x(4) = x(4/2) + 4 = x(2) + 4$

$= 3 + 4$

$x(4) = 7$

$n = 8; \; x(8) = x(8/2) + 8 = x(4) + 8$

$x(8) = 7 + 8 = 15$

$n = 16; \; x(16) = x(16/2) + 16 = x(8) + 16$

$= 15 + 16$

$x(16) = 31$

$x(2^k) = x(2^{k-1}) + x^{2k}$

$x(2^k) = 2^{k+1} - 1$

$\because \; 2^k = n$

$x(n) = x(2^k) = 2^{(\log_2 n)+1} - 1$

$= 2 \cdot 2^{\log_2 n} - 1$

$= 2n - 1$

Ans :- $x(n) = 2n - 1$

d). $x(n) = x(n/3)+1$ for $n>1$ $x(1)=1$ Solve for $n=3k$

$x(1)=1$ (Given)

$n=3$ $\quad x(3/3)+1 = x(1)+1$

$= 1+1 = 2$

$x(3) = 2$

$n=9$ $\quad x(9) = x(9/3)+1$

$= x(3)+1$

$x(9) = 2+1 = 3$

$n=27$ $\quad x(27) = x(27/3)+1$

$= x(9)+1$

$= 3+1$

$= 4$

$x(n) = 1+\log_3^n$ hold true for $n=3^k$

$x(n) = \cancel{1}+\log_3^n$

$x(n) = 1+\log_3^n$

2) Evaluate the following recurrences completely

i) $T(n) = T(n/2)+1$, where $n=2k$ for all $k \geq 0$.

Assume $n=2k$ i.e. $k = \log n$

$T(2^k) = T\left(\frac{2^k}{2}\right)+1$

$= T(2^{k-1})+1$

$= (T(2^{k-2}) + 1 + 1)$

$= T(2^{k-2}) + 2$

$= [T(2^{k-3}) + 1] + 2$

$= T(2^{k-3}) + 3$

$T(2^k) = T(2^{k-k}) + k$

$= T(2^0) + k$

$= T(1) + k$

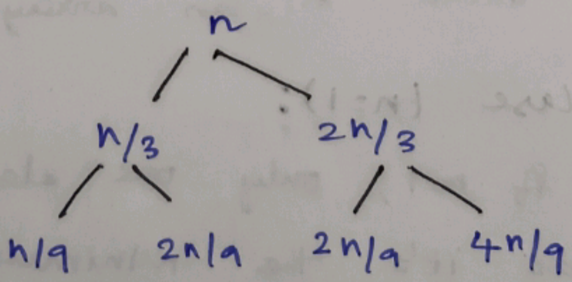
If $T(1) = 1$, we get

$T(2^k) = 1 + k$

i.e $T(n) = \log n + 1$

Thus, we get $T(n) = \Theta(\log n)$

(ii) $T(n) = T(n/3) + T(2n/3) + cn$, where 'c' is a constant and 'n' is the input size.



$T(n) =$ "$+n$" $=$ Sum of the all no in this tree

Length $= \log_3 n$

$T(n) \geq n \log_3 n$

$\therefore$ T is $\Omega(n \log n)$

depth $= \log_{3/2} n$

$T(n) \leq n \log_{3/2} n$

T is $\Theta(n \log n)$

3). Consider the following recursion algorithm.

```
Min [A[0 .... n-1]]
  if n-1 return A[0]
  else  temp = min (A[0.. ... n-2])
        if temp ≤ A[n-1] return temp
  else
        Return A[n-1].
```

a). what does this algorithm computer?

This algorithm computer the minimum value in an array A.

1. Best case $(n=1)$:

if $n=1$, only one element. It returns the A[0] as it's the minimum value in a single element array.

2. Recursive case $(n>1)$:

-> if $n>1$, create the temporary variable (temp).

→ Call recursively (A [0 to n-2)] = first n-1 element .

→ comparing temp with Last element (A[n-1])

If temp ≤ A[n-1]

return temp

else

return A[n-1].

b). Setup a recurrance relation for the algorith basic operation count and solve it

Base case = $T(1) = C_1$ [C_1 is constant → return single element]

recursive case = $T(n) = T(n-1) + C_2$ [C_2 constant representing the basic operating for comparison and assignment].

Final Solution:-

$$T(n) = C_2 * n^2 + (C_1 - C_2)$$

$$T(n) = O(n^2).$$

4). Analyse the order of growth.

i) $F(n) = 2n^2+5$ and $g(n) = 7n$ use the $\Omega(g(n))$ th

notation

As n grows. $2n^2$ grows much faster than 7n

$F(n) = 2n^2 + 5 >= c * 7n$

if $n=1$ $7 = 7$

$n = 2$ $13 = 14$

$n = 3$ $23 = 14$

$n = 4$ $37 = 28$

$n = 5$ $55 = 35$

$n \geq 4$. $F(n) = 2n^2 > 7n$

F(n) is always greater than or equal to c^* g(n).

$F(n) = \Omega(g(n))$

$\therefore$ F(n) is at least as fast as the order of growth of g(n). F(n) grows at least as fast as 7n as n apparoarnee positive infinity.