Intro to Linear Algebra For Artificial Intelligence

By Orevaoghene Ahia



Objective and Outline

OBJECTIVE

 To attain a clear understanding of Linear Algebra that is applicable in Data Science and Artificial Intelligence.

OUTLINE

- Tensors, Scalars, Vector and Matrices
- Relevance in Data Science and Machine Learning or Artificial Intelligence

Class Discussion

Differentiate between
Elementary Algebra and Linear
Algebra

INTRODUCTION

Linear algebra is the branch of mathematics concerning linear equation such as and their representations through matrices and theology spaces. (wikipedia.com)

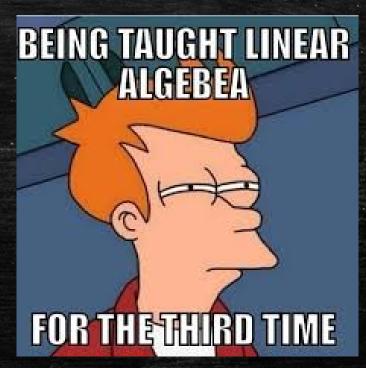
Introduction

- Linear algebra is used in many fields of mathematics, natural sciences, computer science, and social science. Such as
- ☐ Geometry
- □ Chemistry
- □Coding theory
- □ Economics
- **□**Genetics
- **MACHINE LEARNING**

Introduction

Linear Algebra is a fundamental and important backbone of any machine learning algorithm. Hence it is important that we attain an in-depth understanding Linear Algebra as Data Science and Artificial Intelligence experts.

But why should we learn Linear Algebra?



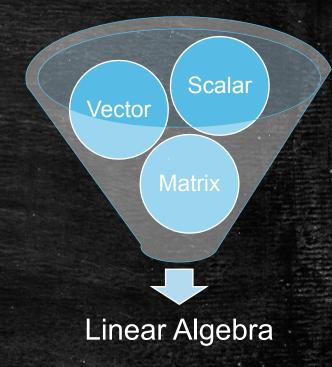
Introduction



- Linear Algebra is a fundamental and important backbone of machine learning algorithms.
- Its concepts are a crucial prerequisite for understanding the theory behind Machine Learning, especially if you are working with Deep Learning Algorithms
- In order to be able to read and interpret statistics, you must learn the notation and operations of linear algebra.
- Algorithms written in standard 'for-loop' notation can be reformulated as matrix equations providing significant gains in computational efficiency.

Tensor Scalar Vector Matrix

- The two primary mathematical entities that are of interest in linear algebra are the vector and the matrix.
- They are examples of a more general entity known as a tensor. Tensors possess an order (or rank), which determines the number of dimensions in an array required to represent it. Scalar Vector Matrix Tensor



 1
 1
 2
 1
 2
 3
 2

 2
 3
 4
 4
 5
 4

Tensor Scalar Vector Matrix

(11)

SCALAR

5 3 7

Row Vector

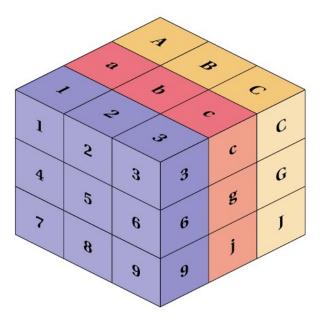
(shape 1x3)

1.5

Column Vector (shape 3x1)

4 19 8 16 3 5

MATRIX



TENSOR

Scalars

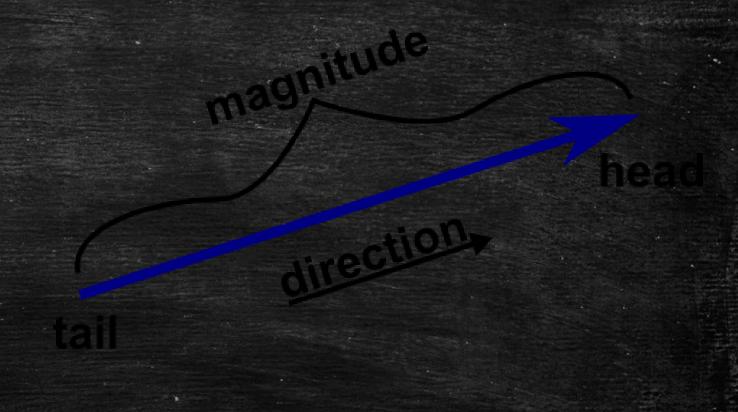
Any real number, or any quantity that can be measured using a single real number. (http://www.mathwords.com)

Scalar

SCALAR

- Asealar isavaarialelenthatedesseibasi agsinglanbumber
- Integers, rational numbers, ratal unumbers cetc.
- Ittis denoted with antitalic fant

i.e.a,n,x



Vectors

A vector is an object that has both a magnitude and a direction. A column of numbers

Vector

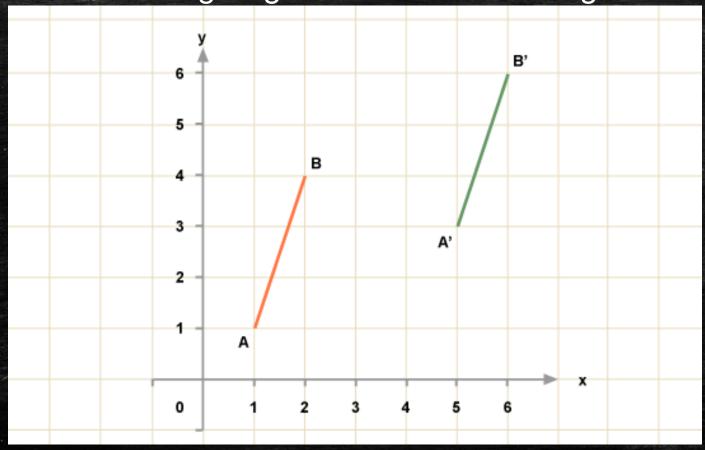
- A vector is a 1-d array of numbers.
- It can be real, binary or integers.
- A Vector has just a single index, which can point to a specific value within the Vector. For example, V2 refers to the second value within the Vector, which is X2 in the graph below.



Vector

 We can also think of vectors as identifying points in space, with each element giving the coordinate along a different

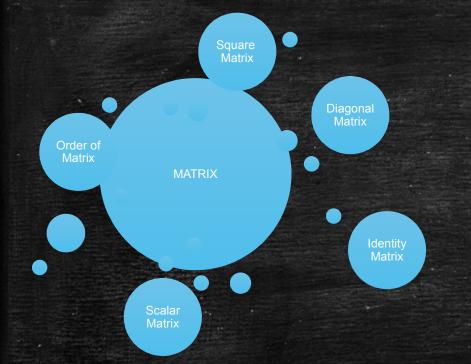
axis.



Vector

VECTOR

- In machine learning vectors often represent feature vectors, with their individual components specifying how important a particular feature is.
- Such features could include :
 - Relative importance of words in a text document
 - The intensity of a set of pixels in a two-dimensional image.
 - Historical price values for a cross-section of financial instruments
 - Classes to be re



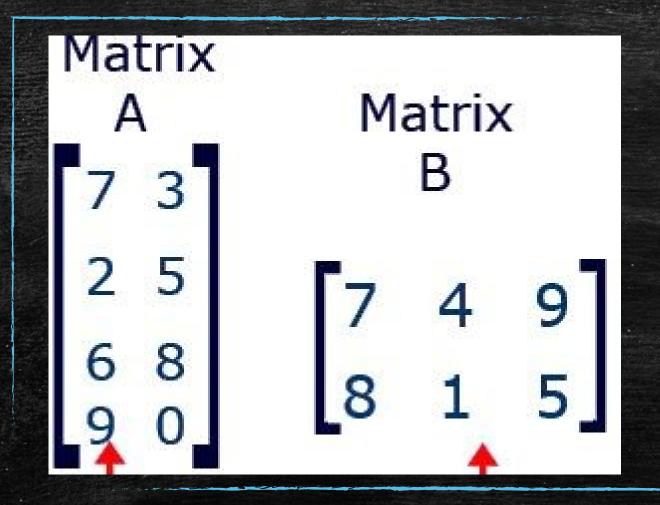
Class Discussion

m x n is a conventional way of saying ____ and ___ in matrix.

Matrix

A matrix (plural: matrices) is a rectangular *array* of numbers, symbols, or expressions, arranged in *rows* and *columns*.

Matrix - Quick Exercise

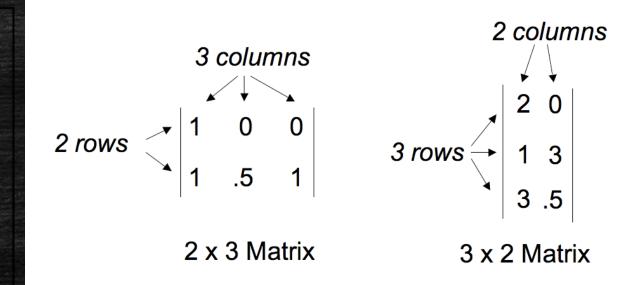


Based on the previous class Exercise, can you identify the value of m and n in this matrix A and B?

Matrix



n columns J changes



Matrix

- Some of you might wonder the relevance of matrix operations will be useful in the context of deep learning.
- But general and more broadly, linear algebra is the 'language in which machine learning was written'. If we can understand the basics of the language, we'll be in a much better position to grasp the more complex ideas that form the backbone of neural network models in lectures.
- We will begin by looking at matrix addition and then consider matrix multiplication. These operations will eventually allow us to discuss a topic known as matrix inversion, which will form the foundation we need.

Matrix

Matrix Addition

- Matrices can be added to scalars, vectors and other matrices.
- Each of these operations has a precise definition.
- These techniques are used frequently in machine learning and deep learning so it is worth familiarising yourself with them.

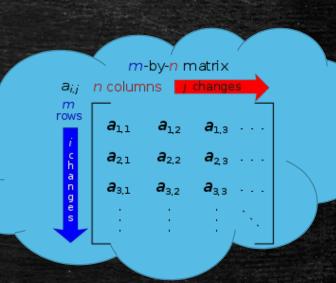
Watrix-Watrix Addition

- Given two matricesoffisize mnxAn [@ppland, bt is plassible to soptain be solution a invalish list a matrix sum which is a matrix sum
- i.e. where , c i.e. A+B=C where , $\left[a_{ij}\right]+\left[b_{ij}\right]=\left[c_{ij}\right]$

Note:

- The specific elements of Aanobs.
- The two matrix to be added must have equal size (bexcept in the coase for badadstasting)

•• if ite.
$$(m \times n)_a + (m \times n)_b = (m \times n)_c$$



Matrix-Matrix Addition

- Watrix-Watrix addition has a COMUNIATATE Manature.

i.e.
$$A + B = B + A$$

- Watrix-Watrix addition has a ASSOCIATE Mature.

i.e.
$$A + (B + C) = (A + B) + C$$

Matrix-Matrix Addition

$\mathbf{A} + \mathbf{B} = egin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \ a_{21} & a_{22} & \cdots & a_{2n} \ dots & dots & \ddots & dots \ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} + egin{bmatrix} b_{11} & b_{12} & \cdots & b_{12} \ b_{21} & b_{22} & \cdots & b_{22} \ dots & dots & \ddots & dots \ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix} \ egin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \end{bmatrix}$

Whenever I try to understand Math



GIES AT MEMECENTER COM

Matrix-Matrix Addition

ADD $A + B = \begin{bmatrix} a_1 & a_2 \\ a_2 & a_3 \end{bmatrix} + \begin{bmatrix} b_1 & b_2 \\ b_2 & b_3 \end{bmatrix}$

SUBTRACT

$$A - B = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} - \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 - b_1 & a_2 - b_2 \\ \\ a_3 - b_3 & a_4 - b_4 \end{bmatrix}$$

Matrix-Matrix Addition

$$egin{bmatrix} 1 & 3 \ 1 & 0 \ 1 & 2 \end{bmatrix} + egin{bmatrix} 0 & 0 \ 7 & 5 \ 2 & 1 \end{bmatrix} =$$

$$X + Y = \begin{bmatrix} 3 & 5 & 2 & 4 \\ 7 & 6 & 8 & 4 \\ 5 & 1 & 3 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 6 & 3 & 7 \\ 9 & 3 & 8 & 1 \\ 4 & 6 & 9 & 7 \end{bmatrix}$$
• Try solving with some python code

Class Exercise

- Try solving this 2 matrix problems with your hand.

Matrix-Matrix Addition

$$\begin{bmatrix} 1 & 3 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 7 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 3+0 \\ 1+7 & 0+5 \\ 1+2 & 2+1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 8 & 5 \\ 3 & 3 \end{bmatrix}$$

$$X + Y = \begin{bmatrix} 3 & 5 & 2 & 4 \\ 7 & 6 & 8 & 4 \\ 5 & 1 & 3 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 6 & 3 & 7 \\ 9 & 3 & 8 & 1 \\ 4 & 6 & 9 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 3 + 2 & 5 + 6 & 2 + 3 & 4 + 7 \\ 7 + 9 & 6 + 3 & 8 + 8 & 4 + 1 \\ 5 + 4 & 1 + 6 & 3 + 9 & 8 + 7 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 11 & 5 & 11 \\ 16 & 9 & 16 & 5 \\ 9 & 7 & 12 & 15 \end{bmatrix}$$

Solution

- If you got the answer, then you deserve some accolades.
- Why don't you try subtracting.
 Compare your answer with that of your neighbour

Matrix-Sealar Addition

• Itis also possible to add stalara values of a matrix of size $x_n, x_n = t \rho_n p_j p_j \phi$ of $a = [b_{ij}]$

i.e.
$$B \in x + A$$

- Watrix-Sealar additioniss also commutative yello and a some sealar additioniss also commutative yello and a some sealar additioniss also commutative yello and a some sealar additioniss also commutative.

i.e.
$$A + (x + C) = (A + x) + C$$

 $A + x = x + A$

Broadcasting

- For certain applications in machine learning it is possible to define a shorthand notation known as broadcasting. But what is Broadcasting?
- Consider

Matrix Multiplication

- Writix addition might have been simply for most soft we levely waver when it somest to include the supply and it is the supply and it is the supply to the supply the supply to the su complex.

One very important operation we should consider before hitting at
 Charkery important operation we should consider before hitting at matrix multiplication is Matrix Transpose

Matrix Transpose Matrix Transpose

The transpose of a matrix $A = \begin{bmatrix} a_{ij} \\ a_{ij} \end{bmatrix}$ of size $m \times n$ is denoted by A^T . The transpose of a matrix of size misndenoted by of size and it is given element-wise. $A^T = \begin{bmatrix} a_{ij} \\ a_{ij} \end{bmatrix}_{m \times n}$ $A^T = \begin{bmatrix} a_{ji} \\ a_{min} \end{bmatrix}_{m \times n}$

$$A^T = \left[a_{ji}\right]_{n \times m}$$

Change you spot what changed in A^T and A?
Change you spot what changed in ?

Matrix Transpose - Example

$$m{A} = egin{bmatrix} a_{11} & a_{12} & a_{13} \ a_{21} & a_{22} & a_{23} \end{bmatrix}, \quad m{A}^T = egin{bmatrix} a_{11} & a_{21} \ a_{12} & a_{22} \ a_{13} & a_{23} \end{bmatrix}$$

$$oldsymbol{x} = egin{bmatrix} x_1 \ x_2 \ x_2 \end{bmatrix}, \quad oldsymbol{x}^T = [\ x_1 \ x_2 \ x_3]$$

Matrix Transpose of Guiz

- ATTrueApr false or false
- $x^T = x$ orwhere 'x' is a scalar true or false
- Is there are flecting effect on the matrix alguethed in eding on al diagonals elements? nitetry (trimts position as position of the control of the control

Matrix-Matrix Multiplication

- Tithe reason why matrix multiplication is a complex operation compared too matrix diadotifio because economic tith volves multiply of very compared to matrix diadotifio because the compared to matrix diadotifio because the complex operation. rthutipatrices element-wise.
- Instead, a more complex procedure is utilised, for each element, involving an entire row of one matrix and an entire column of the involving an entire row of one matrix and an entire column of the THEY other.

- Given a matrix $A = \begin{bmatrix} a_{ij} \end{bmatrix}_{\substack{m \times p}}$ and a matrix $B = \begin{bmatrix} b_{ij} \end{bmatrix}_{\substack{n \times p}}$ the matrix G is a matrix G and G is G and G is a matrix G is a matrix G and G is a matrix G and G is a matrix G and G is a matrix G is a matrix

Matrix-Matrix Multiplication

From the previous discussion, can you determine the dimension of the product of AB?

$$oldsymbol{A} = egin{bmatrix} 2 & 5 & 1 \ 7 & 3 & 6 \end{bmatrix}, \quad oldsymbol{B} = egin{bmatrix} 1 & 8 \ 9 & 4 \ 3 & 5 \end{bmatrix}$$

Matrix-Matrix Multiplication

How about the product BA?

Lets treat matrix multiplication before answering this questions.

$$oldsymbol{A} = egin{bmatrix} 2 & 5 & 1 \ 7 & 3 & 6 \end{bmatrix}, \quad oldsymbol{B} = egin{bmatrix} 1 & 8 \ 9 & 4 \ 3 & 5 \end{bmatrix}$$

Matrix-Matrix Multiplication

How do we do multiply two 2 x 2 matrix ?

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \bullet \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

As a smart G, let me just jump to the answer.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \bullet \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Do you
Understand?

Matrix-Matrix Multiplication

How do we do multiply two 2 x 2 matrix ?

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \bullet \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

As a smart G, let me just jump to the answer.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \bullet \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

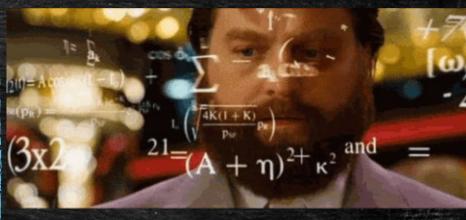
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \bullet \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \bullet \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \bullet \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \bullet \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \bullet \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$



Matrix-Matrix Multiplication

Following shows you how each of the elements in resulting matrix are calculated.

Matrix-Matrix Multiplication

 If it is not clear to you yet on how each of the elements in the resulting matrix came, take a look at the following arrows. I hope it got clearer to you.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \bullet \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Welcome Back
Lets solve the
matrix
multiplication

Matrix-Matrix Multiplication

From the previous discussion, can you determine the dimension of the product of AB?

$$oldsymbol{A} = egin{bmatrix} 2 & 5 & 1 \ 7 & 3 & 6 \end{bmatrix}, \quad oldsymbol{B} = egin{bmatrix} 1 & 8 \ 9 & 4 \ 3 & 5 \end{bmatrix}$$

Matrix-Matrix Multiplication

How about the product BA?

How about this one?

$$m{A} = egin{bmatrix} 2 & 5 & 1 \ 7 & 3 & 6 \end{bmatrix}, \quad m{B} = egin{bmatrix} 1 & 8 \ 9 & 4 \ 3 & 5 \end{bmatrix}$$

SOLUTION

Matrix-Matrix Multiplication

$$m{AB} = egin{bmatrix} 2 \cdot 1 + 5 \cdot 9 + 1 \cdot 3 & 2 \cdot 8 + 5 \cdot 4 + 1 \cdot 5 \ 7 \cdot 1 + 3 \cdot 9 + 6 \cdot 3 & 7 \cdot 8 + 3 \cdot 4 + 6 \cdot 5 \end{bmatrix} = egin{bmatrix} 50 & 41 \ 52 & 98 \end{bmatrix}$$

$$\boldsymbol{B}\boldsymbol{A} = \begin{bmatrix} 1 \cdot 2 + 8 \cdot 7 & 1 \cdot 5 + 8 \cdot 3 & 1 \cdot 1 + 8 \cdot 6 \\ 9 \cdot 2 + 4 \cdot 7 & 9 \cdot 5 + 4 \cdot 3 & 9 \cdot 1 + 4 \cdot 6 \\ 3 \cdot 2 + 5 \cdot 7 & 3 \cdot 5 + 5 \cdot 3 & 3 \cdot 1 + 5 \cdot 6 \end{bmatrix} = \begin{bmatrix} 58 & 29 & 49 \\ 46 & 57 & 33 \\ 41 & 30 & 33 \end{bmatrix}$$

Matrix-Sealar Multiplication

Scalar-matrix multiplication is simple than matrix matrix multiplication. Given a matrix and a_i scalar as A_i the is calar matrix product A_i is calculated by multiplying the plane of A_i and A_i is calculated by multiplying the plane of A_i and A_i is a constant A_i and A_i is such that A_i is A_i and A_i and A_i is A_i and A_i and A_i is A_i and A_i

Class Exercise?

What if we have scalars calars calcinopyly by involving A? $(\lambda + \mu)A = ?$ $\lambda(\mu A) = ?$

Matrix-Scalar Multiplication

- The second result states that if you add two scalars together and then multiply the result by a matrix it gives the same answer as if you individually multiplied the matrix separately by each scalar and added the result.
- The third result states that the order of scalar multiplication does not matter. If you multiply the matrix by one scalar, and then multiply the result by another scalar it gives the same answer as if you first multiplied both scalars together and then by the matrix.
- All of these results follow from the simple rules of scalar multiplication and addition.

Matrix

Terms related to Matrix

- Order of matrix If a matrix has 3 rows and 4 columns, order of the matrix is 3*4 i.e. row*column.
- Square matrix The matrix in which the number of rows is equal to the number of columns.
- Diagonal matrix A matrix with all the non-diagonal elements equal to 0 is called a diagonal matrix.
- Upper triangular matrix Square matrix with all the elements below diagonal equal to 0.

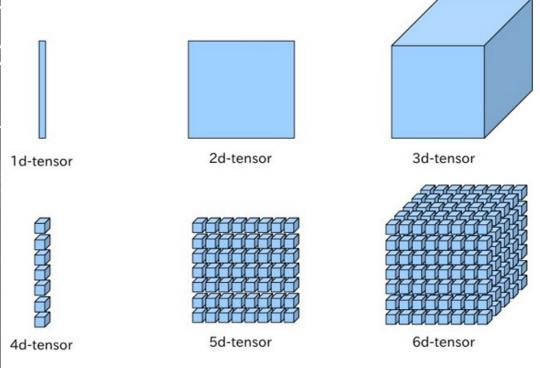
Matrix

- Lower triangular matrix Square matrix with all the elements above the diagonal equal to 0.
- Scalar matrix Square matrix with all the diagonal elements equal to some constant k.
- Identity matrix Square matrix with all the diagonal elements equal to 1 and all the non-diagonal elements equal to 0.
- Column matrix The matrix which consists of only 1 column.
 Sometimes, it is used to represent a vector.
- Row matrix A matrix consisting only of row.
- Trace It is the sum of all the diagonal elements of a square matrix

Tensors

Tensor

- **Tensors** possess an order (or rank), which determines the number of dimensions in an array required to represent it.
- In machine learning, and deep learning in particular, a 3rd-order tensor can be used describe the intensity values of multiple channels (red, green and blue) from a two dimensional image.



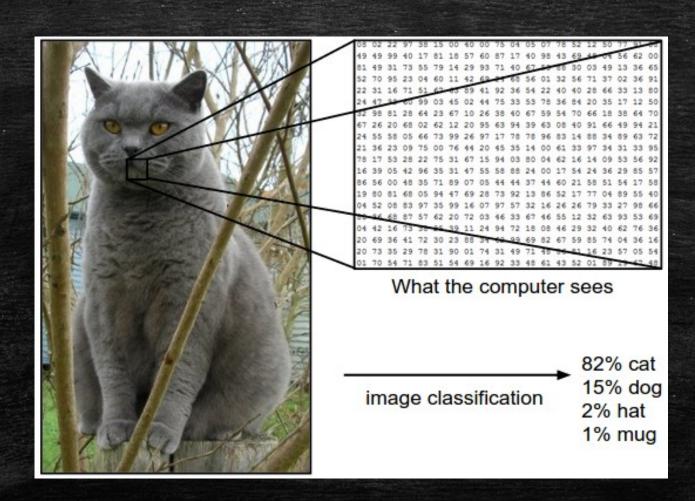
Relevance of Matrices to Machine Learning

Why would you do this to me?

Image Processing

- It is very easy for our brains to recognize images.
- But making a computer recognize images is not an easy task, and is an active area of research in Machine Learning and Computer Science in general.
- So how can an image such as above with multiple attributes like colour be stored in a computer?
- This is achieved by storing the pixel intensities in a construct called Matrix.
 Then, this matrix can be processed to identify colours etc.
- So any operation which you want to perform on this image would likely use Linear Algebra and matrices at the back end.

Image Classification



XGBOOST

- If you are somewhat familiar with the Data Science domain, you might have heard about the word "XGBOOST"
- It is an algorithm employed most frequently by winners of Data Science Competitions.
- It stores the numeric data in the form of **Matrix** to give predictions.
- This enables XGBOOST to process data faster and provide more accurate results. Moreover, not just XGBOOST but various other algorithms use Matrices to store and process data.

Natural Language Processing

- Another active area of research in Machine Learning is dealing with text and the most common techniques employed are Bag of Words, Term Document Matrix etc.
- All these techniques in a very similar manner store counts(or something similar) of words in documents and store this frequency count in a Matrix form to perform tasks like Semantic analysis, Language translation, Language generation etc.



- So, now we understand the importance of Linear Algebra in machine learning.
- We have seen image, text or any data, in general, employing matrices to store and process data. This should be motivation enough to familiarize ourselves with it.



