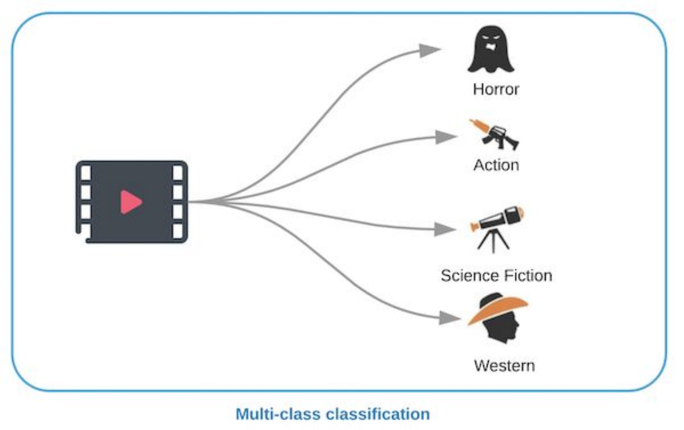
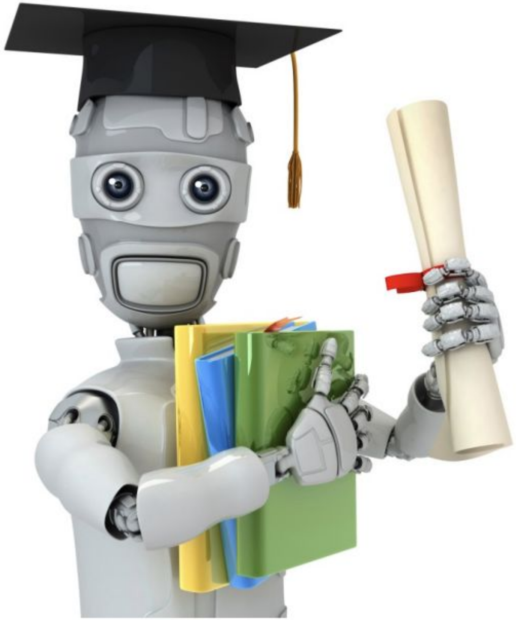
Logistic Regression

Multi-class classification: One-vs-all

WEEK 4, COHORT 4





Machine Learning

Logistic Regression

Multi-class classification: One-vs-all

Multiclass classification

Email foldering/tagging: Work, Friends, Family, Hobby

y=1 y=2 y=3 y=4

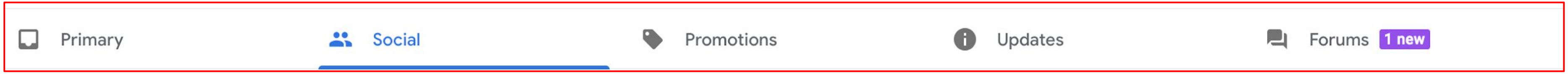
Medical diagrams: Not ill, Cold, Flu

y=1 2 3

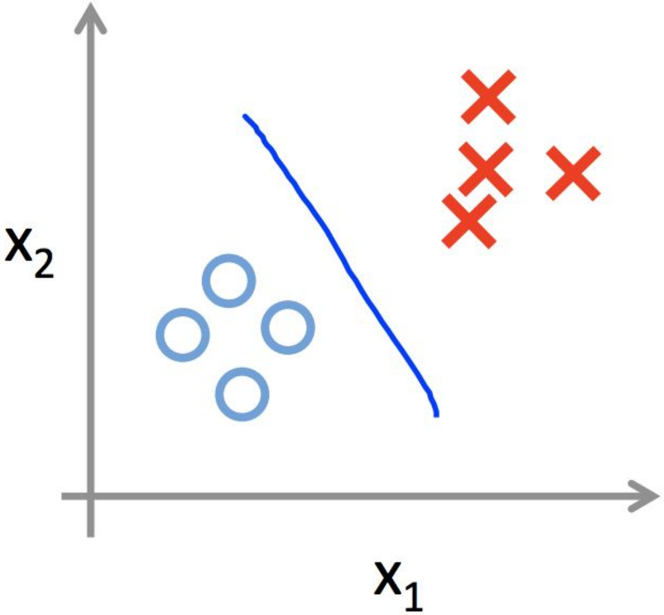
Weather: Sunny, Cloudy, Rain, Snow

y=1 2 3 4 ←

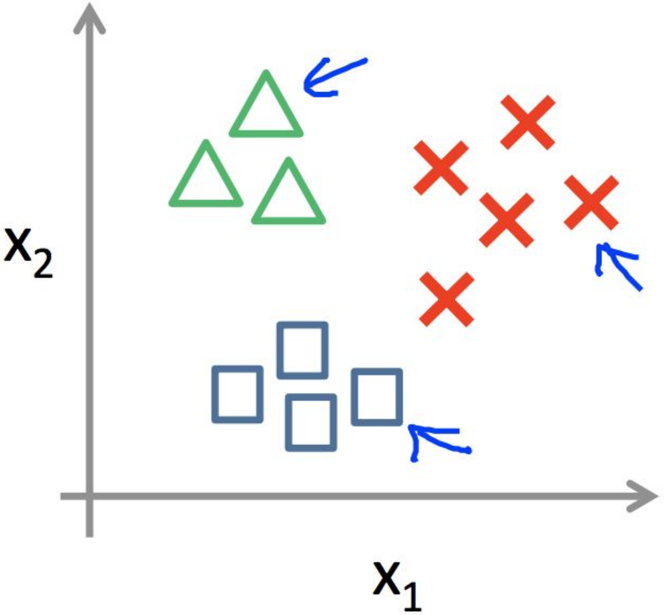
0 1 2 3



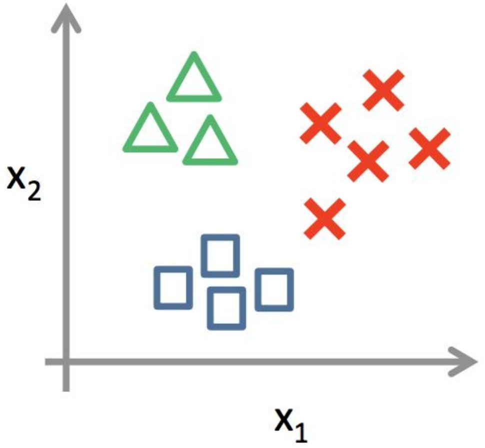
Binary classification:



Multi-class classification:



One-vs-all (one-vs-rest):

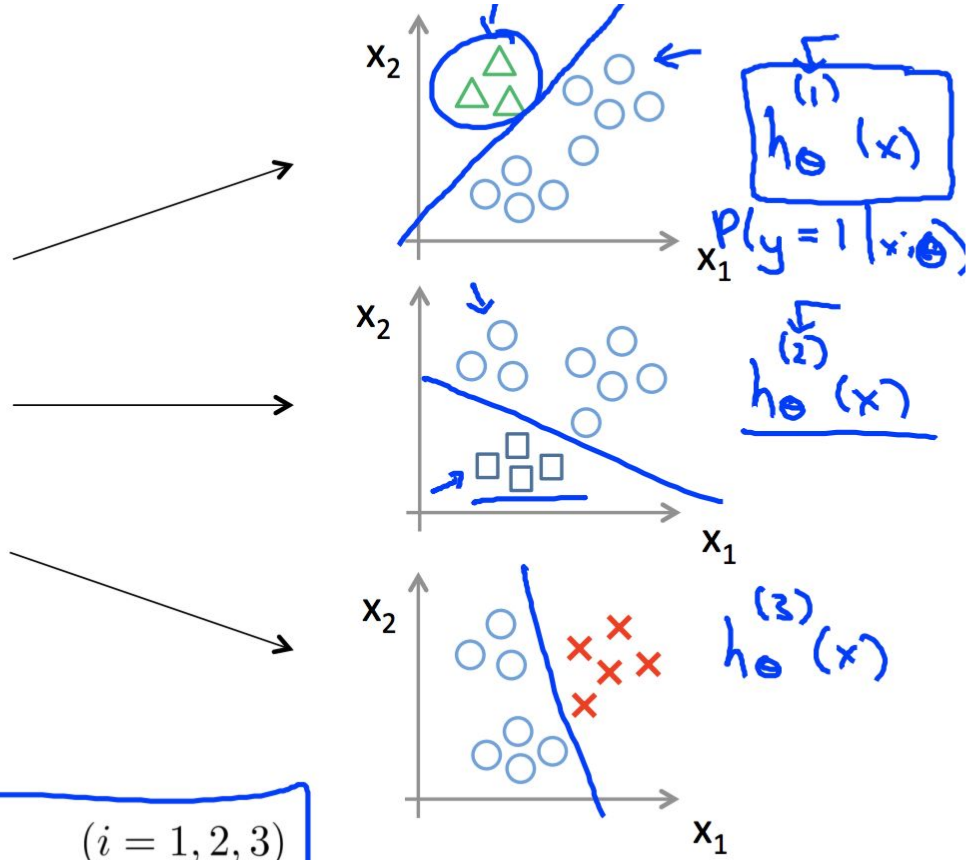


Class 1: △

Class 2: □

Class 3: ✕

$$\boxed{h_\theta^{(i)}(x) = P(y=i|x;\theta) \qquad (i=1,2,3)}$$



Multiclass Classification: One-vs-all

y ∈ {0,1, … , n}

$$y \in \{0, 1 \ldots n\}$$

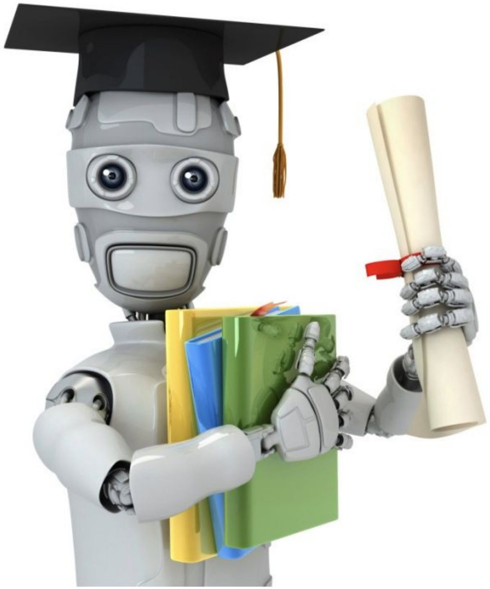
$$h_\theta^{(0)}(x) = P(y = 0|x;\theta)$$

$$h_\theta^{(1)}(x) = P(y = 1|x;\theta)$$

$$\cdots$$

$$h_\theta^{(n)}(x) = P(y = n|x;\theta)$$

$$\text{prediction} = \max_i (h_\theta^{(i)}(x))$$

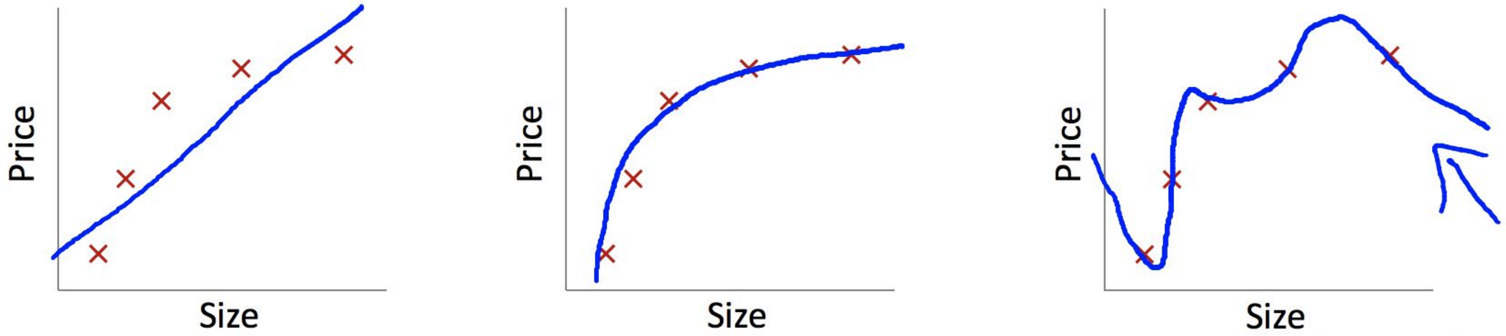


Machine Learning

Regularization

The problem of overfitting

Example: Linear regression (housing prices)



$\rightarrow \theta_0 + \theta_1 x$

"Underfit" "High bias"

$\rightarrow \theta_0 + \theta_1 x + \theta_2 x^2$

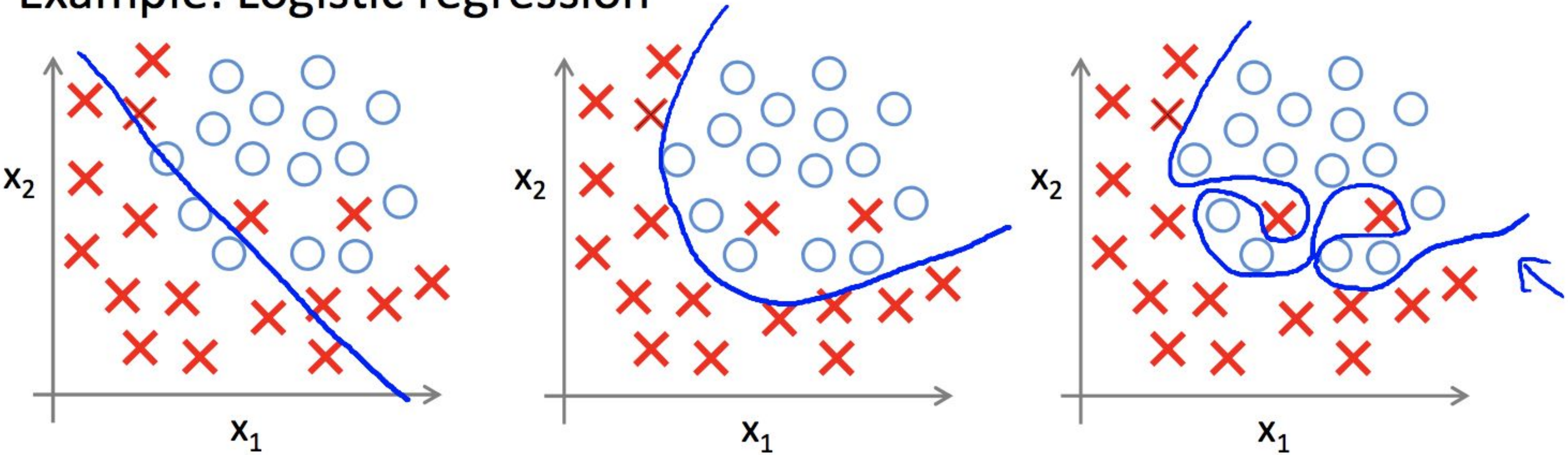
"Just right"

$\rightarrow \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

"Overfit" "High variance"

Overfitting: If we have too many features, the learned hypothesis may fit the training set very well ($J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2 \approx 0$), but fail to generalize to new examples (predict prices on new examples).

Example: Logistic regression



$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

(g = sigmoid function)

"Underfit"

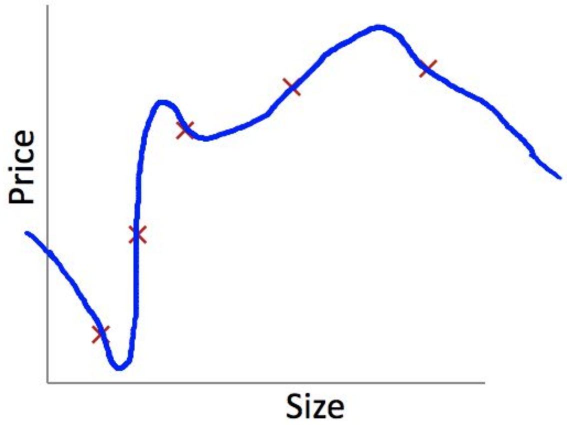
$$g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2 + \theta_5 x_1 x_2)$$

$$g(\theta_0 + \theta_1 x_1 + \theta_2 x_1^2 + \theta_3 x_1^2 x_2 + \theta_4 x_1^2 x_2^2 + \theta_5 x_1^2 x_2^3 + \theta_6 x_1^3 x_2 + \dots)$$

"Overfit"

Addressing overfitting:

x_1 = size of house
x_2 = no. of bedrooms
x_3 = no. of floors
x_4 = age of house
x_5 = average income in neighborhood
x_6 = kitchen size
$\vdots$
x_{100}

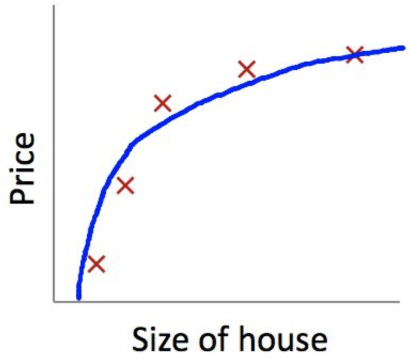


Addressing overfitting:

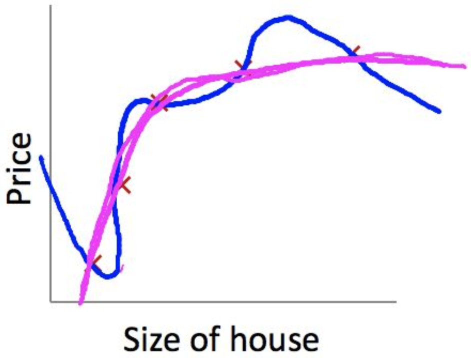
Options:

1. Reduce number of features.
   - Manually select which features to keep.
   - Model selection algorithm (later in course).
2. Regularization.
   - Keep all the features, but reduce magnitude/values of parameters θ_j.
   - Works well when we have a lot of features, each of which contributes a bit to predicting y.

Intuition



$$\theta_0 + \theta_1 x + \theta_2 x^2$$



$$\theta_0 + \theta_1 x + \theta_2 x^2 + \cancel{\theta_3 x^3} + \cancel{\theta_4 x^4}$$

Suppose we penalize and make θ_3, θ_4 really small.

$$\rightarrow \quad \min_{\theta} \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2 + 1000\,\theta_3^2 + 1000\,\theta_4^2$$

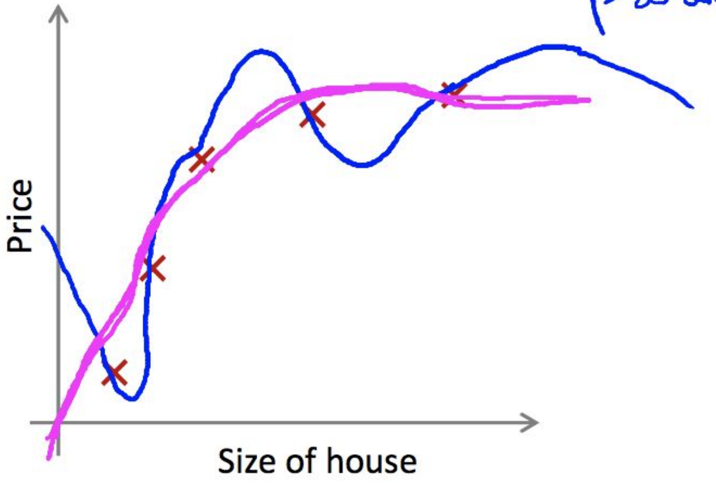
$\theta_3 \approx 0 \qquad \theta_4 \approx 0$

Regularization.

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2 + \lambda \sum_{j=1}^{n} \theta_j^2\right]$$

regularization parameter

$$\min_\theta J(\theta)$$



In regularized linear regression, we choose θ to minimize

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda\sum_{j=1}^{n}\theta_j^2\right]$$

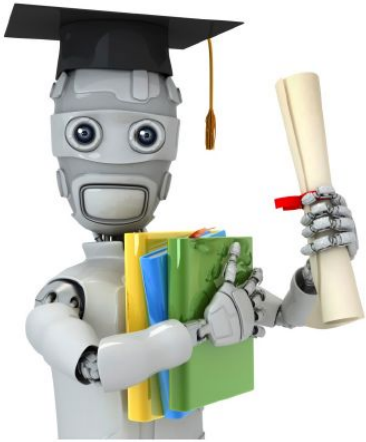
What if λ is set to an extremely large value (perhaps for too large for our problem, say $\lambda = 10^{10}$)?

- Algorithm works fine; setting λ to be very large can't hurt it
- Algortihm fails to eliminate overfitting.
- Algorithm results in underfitting. (Fails to fit even training data well).
- Gradient descent will fail to converge.

Addressing Overfitting

1. Reduce the number of features
   a. Manually select which features to keep
   b. Use a model selection algorithm(studied later in course)

2. Regularization
   a. Keep all the features, but reduce the parameters θ_j.



Machine Learning