

# Introduction to ML

Week 2



“aimed at getting you to  
kickass in AI”

# Agenda

1. Welcome
2. What is ML?
3. Supervised Learning
4. Model Representation
5. Cost function
6. Gradient Descent



Machine Learning

# Buzz Words



Features

1st order polynomial

Straight-line

Discrete value

Regression problem

Machine Learning

2nd order polynomial

Continuous values

# What is Machine Learning



According to Arthur Samuel in 1959, Machine Learning gives “**computers the ability to learn without being explicitly programmed.**”

- **Arthur Samuel**



Learning is any process by which a system improves performance from experience - “ML is concerned with computer programs that automatically improve their performance through experience”- **Herbert Simon**

# What is Machine Learning



A computer program is said to learn from experience  $E$  with respect to some class of task  $T$  and performance measure  $P$ , if its performance at tasks in  $T$ , as measured by  $P$ , improves with experience  $E$ . - **Tom Mitchell**

Learning = Improving with experience at some task.

- Improve over task,  $T$ (classification)
- With performance measure i.e. accuracy,  $P$ (correctly classify orange/apple)
- Based on experience ,  $E$ (watching you label fruits into orange/apple).

# Classification of Machine Learning

## Supervised Learning:

Predicting values. **Known** targets.

User inputs correct answers to learn from. Machine uses the information to guess new answers.

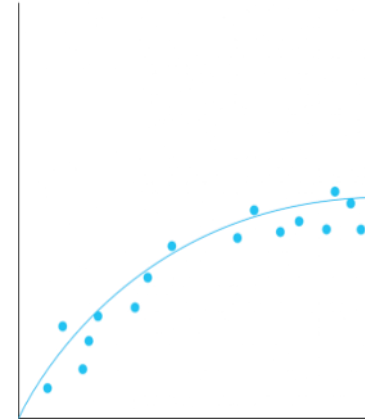
### **REGRESSION:**

Estimate continuous values  
(Real-valued output)

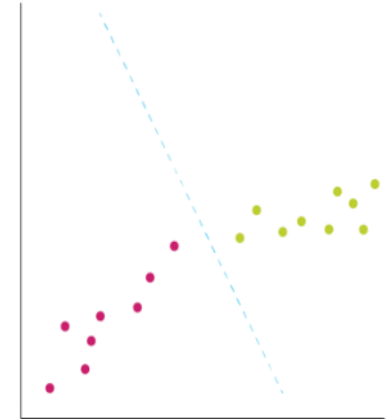
### **CLASSIFICATION:**

Identify a unique class  
(Discrete values, Boolean, Categories)

Regression



Classification



## Unsupervised Learning:

Search for structure in data. **Unknown** targets.

User inputs data with undefined answers. Machine finds useful information hidden in data.

### **Cluster Analysis**

Group into sets

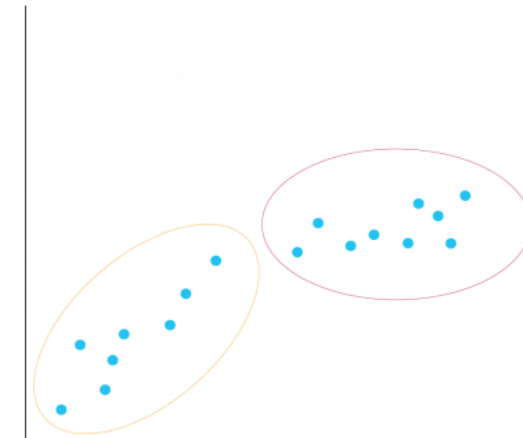
### **Density Estimation**

Approximate distributions

### **Dimension Reduction**

Select relevant variables

Clustering



Others: RL

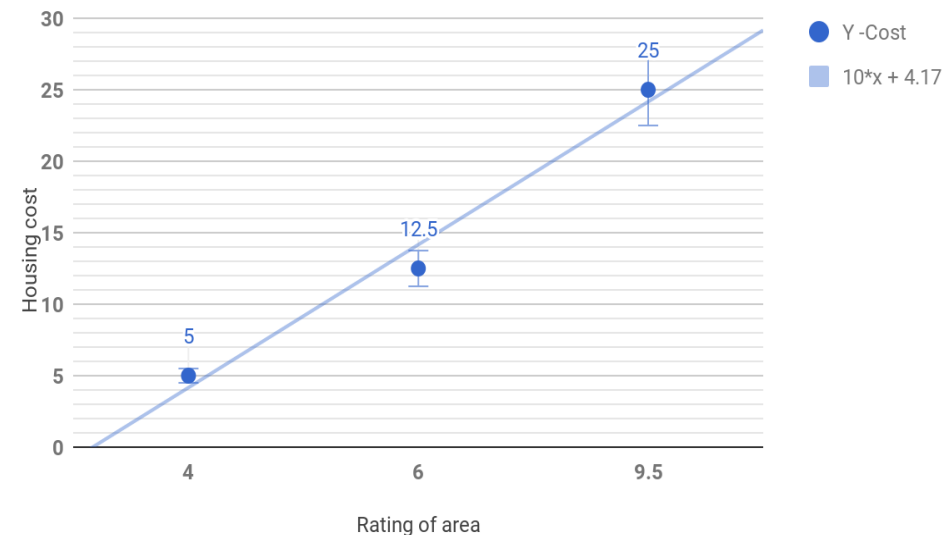
# Linear Regression Basic

- **Linear regression** allows us to understand relationship between two continuous variables.
- X: independent variable e.g. room size, security, road network(Lekki, Ipaja)
- Y: dependent variable e.g. cost of housing

$$Y = mX + b$$




X1: room size

Rating against Housing cost



Regression: predict continuous valued output (price)

# Linear Regression Basic - Supervised Learning (Housing Price)

Label	X(Housing state, security, light)	Y( Housing cost)
1.		N5,000,000
2.		???
3.		N25,000,000

Fun Illustration



# Linear Regression Basic - Supervised Learning (Housing Price)

Training set of housing prices (Portland, OR)	Size in feet <sup>2</sup> (x)	Price (\$) in 1000's (y)
	2104	460
	1416	232
	1534	315
	852	178
	...	...

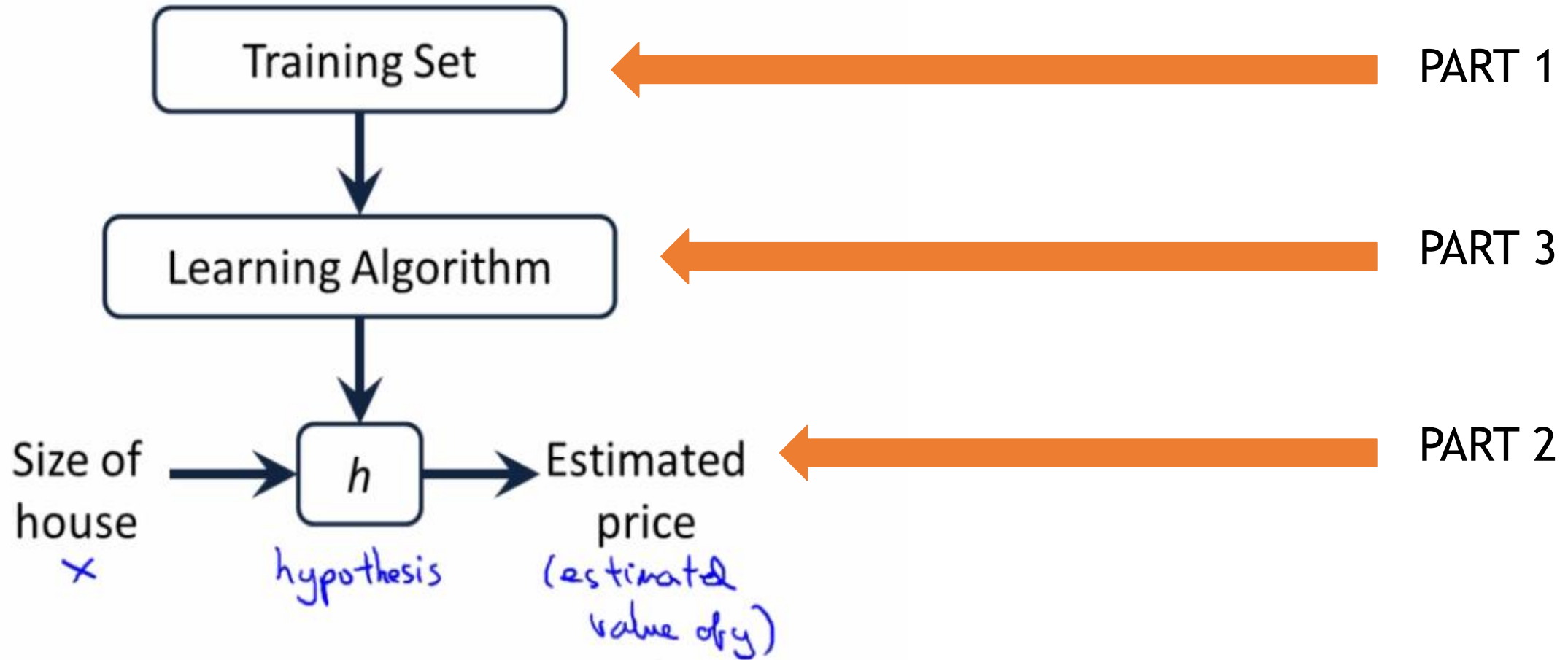
Notation:

**m** = Number of training examples

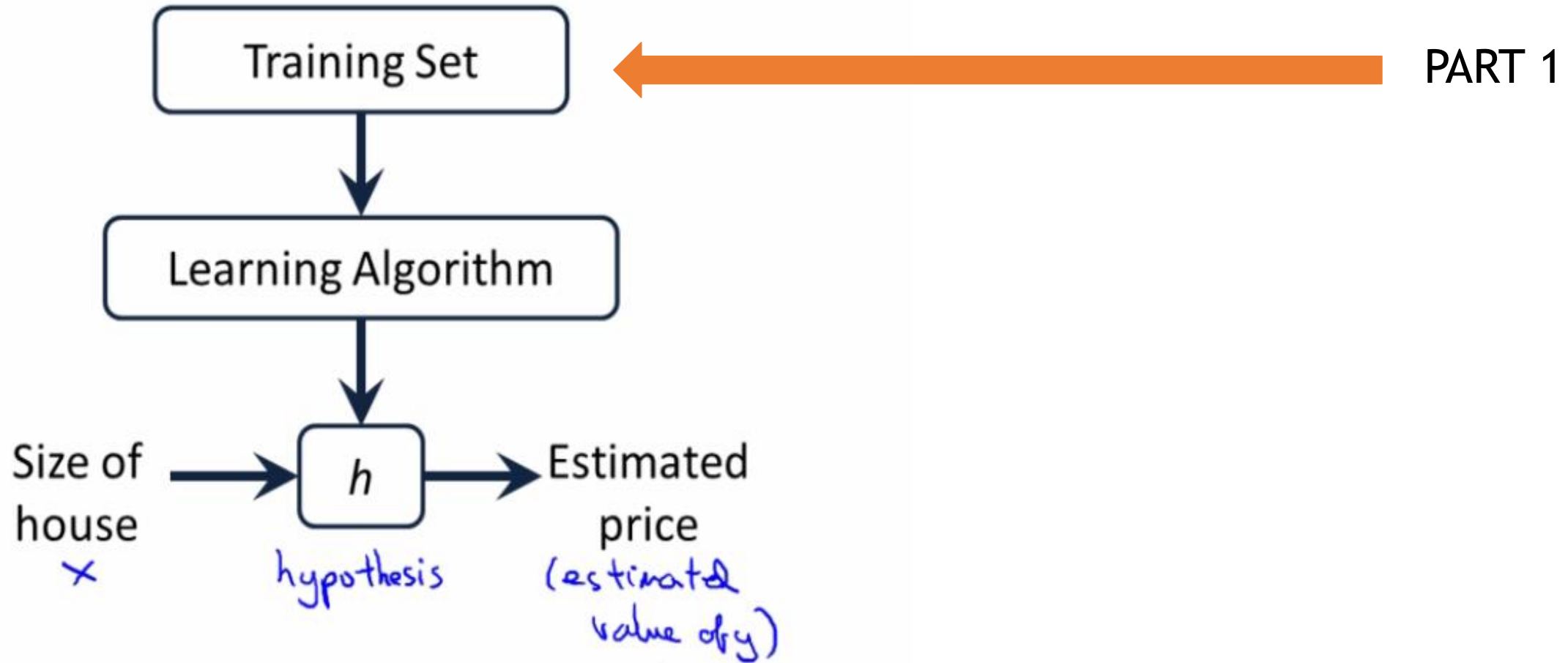
**x**'s = "input" variable / features

**y**'s = "output" variable / "target" variable

# Linear Regression Basic - Model Representation



# Linear Regression Basic - Model Representation



# Linear Regression Basic - Model Representation

Training set of housing prices (Portland, OR)

Size in feet <sup>2</sup> ( $x$ )	Price (\$) in 1000's ( $y$ )
→ 2104	460
1416	232
→ 1534	315
852	178
...	...

$m = 47$

Notation:

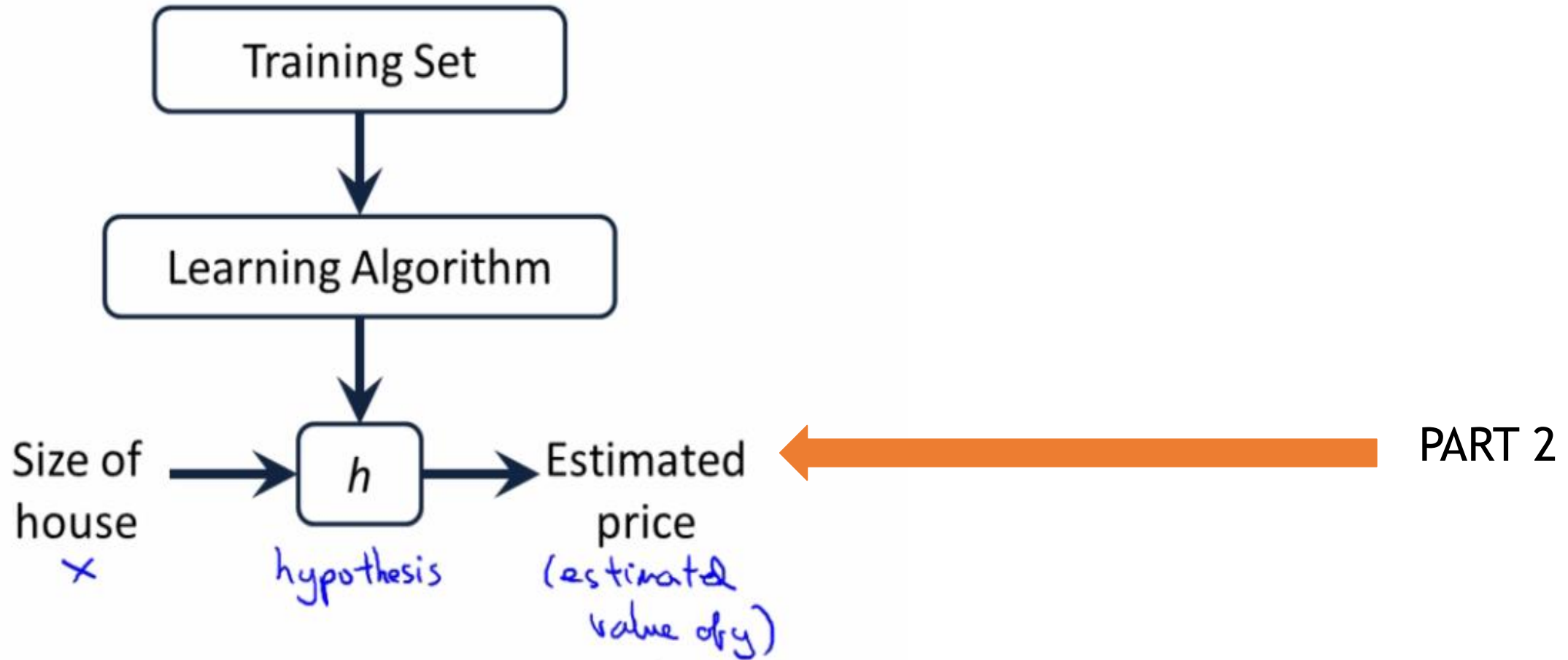
- $m$  = Number of training examples
- $x$ 's = "input" variable / features
- $y$ 's = "output" variable / "target" variable

$(x, y)$  - one training example

$(x^{(i)}, y^{(i)})$  -  $i^{\text{th}}$  training example

$$\begin{cases} x^{(1)} = 2104 \\ x^{(2)} = 1416 \\ y^{(1)} = 460 \end{cases}$$

# Linear Regression Basic - Model Representation



# Linear Regression Basic - Model Representation

Training Set	Size in feet <sup>2</sup> (x)	Price (\$) in 1000's (y)
	2104	460
	1416	232
	1534	315
	852	178
	...	...

}  $n = 47$

Hypothesis: 
$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

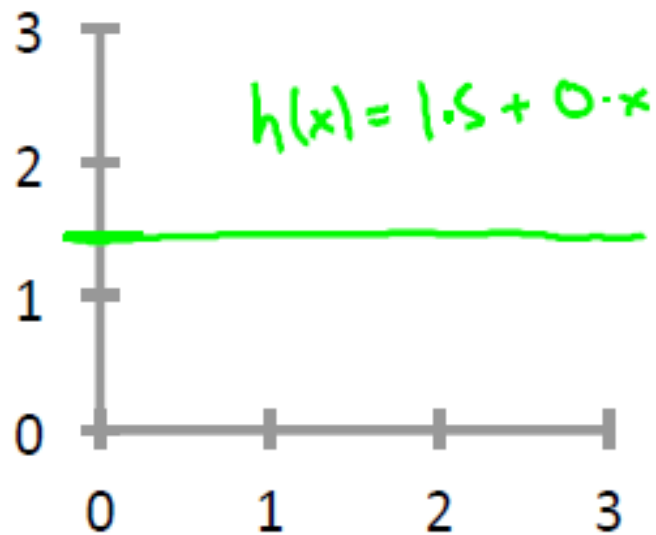
$\theta_i$ 's: Parameters

↑      ↑

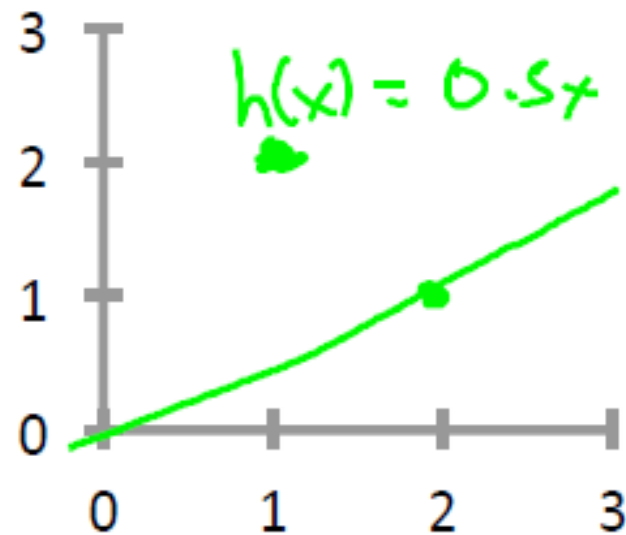
How to choose  $\theta_i$ 's ?

# Linear Regression Basic - Model Representation

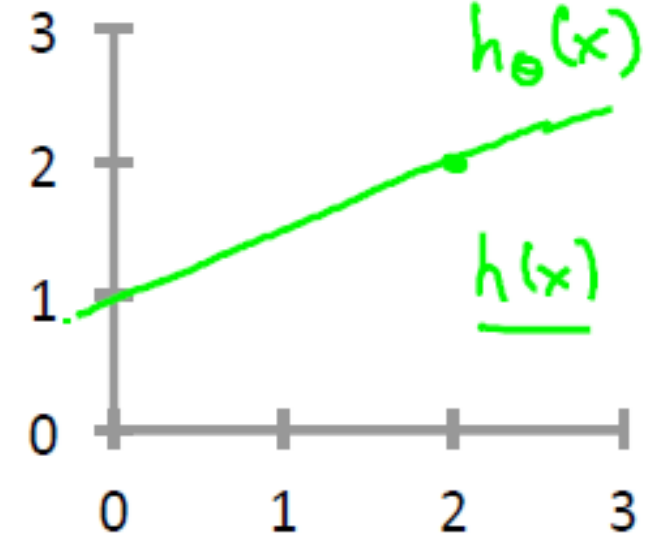
$$\underline{h_{\theta}(x)} = \theta_0 + \theta_1 x$$



$\rightarrow \theta_0 = 1.5$   
 $\rightarrow \theta_1 = 0$



$\rightarrow \theta_0 = 0$   
 $\rightarrow \theta_1 = 0.5$



$\rightarrow \theta_0 = 1$   
 $\rightarrow \theta_1 = 0.5$

# Linear Regression Basic - Cost Function

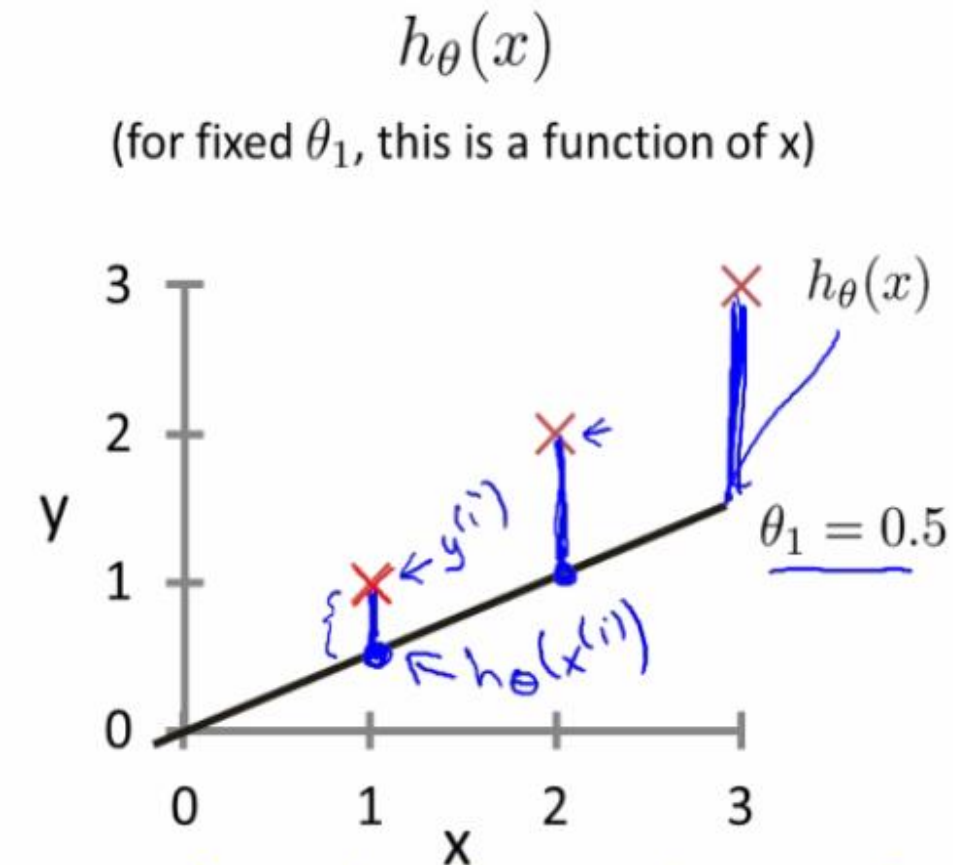
A **cost function** tells us “how good” our model is at making predictions for a given set of parameters. The cost function has its own curve and its own gradients. The slope of this curve tells us how to update our parameters to make the model more accurate.

Cost Function:

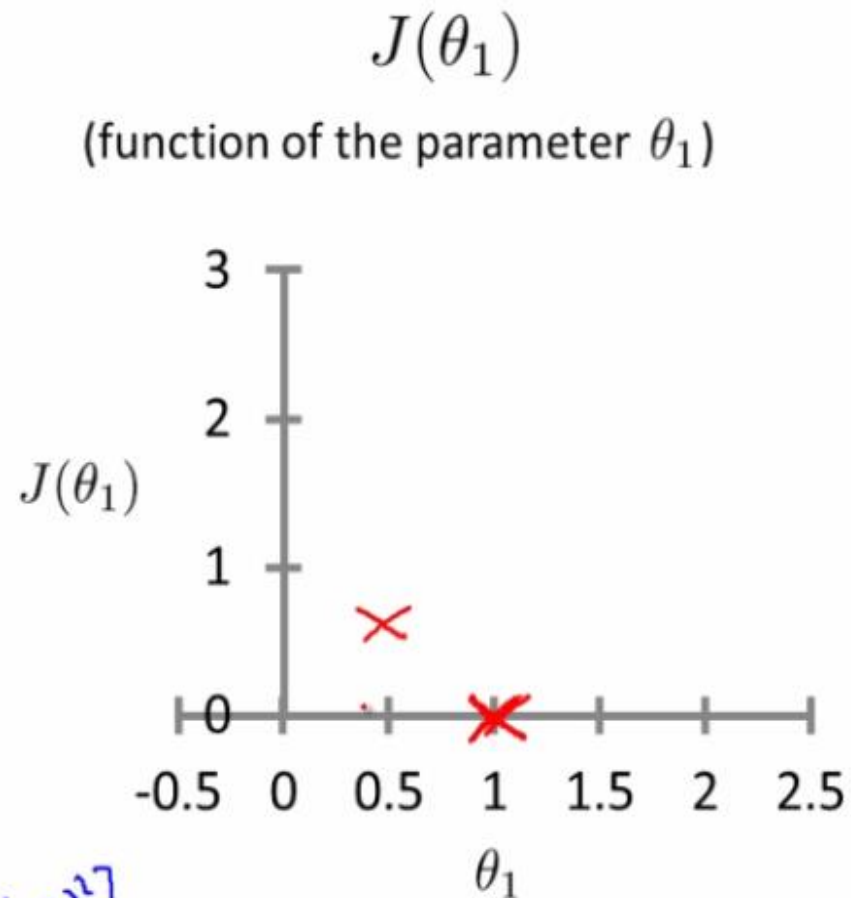
$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$



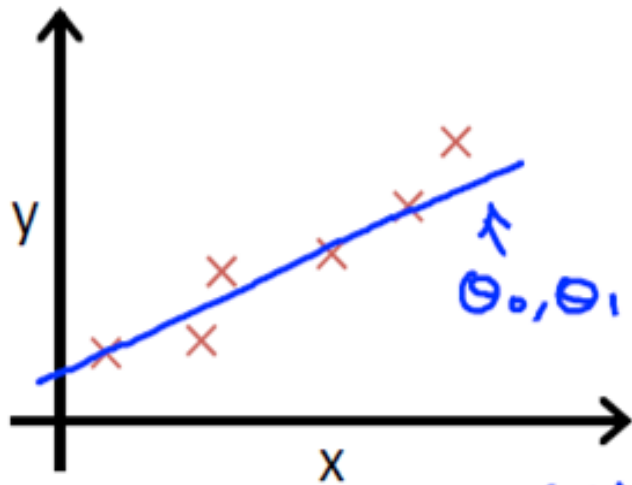
# Linear Regression Basic - Cost Function



$$J(0.5) = \frac{1}{2m} [(0.5-1)^2 + (1-2)^2 + (1.5-3)^2]$$
$$= \frac{1}{2 \times 3} (3.5) = \frac{3.5}{6} \approx 0.58$$



# Linear Regression Basic - Cost Function



$(x^{(i)}, y^{(i)})$

Idea: Choose  $\theta_0, \theta_1$  so that  $h_{\theta}(x)$  is close to  $y$  for our training examples  $(x, y)$

$x, y$

minimize  $\theta_0, \theta_1$

$$\frac{1}{2m} \sum_{i=1}^m \left( h_{\theta}(x^{(i)}) - y^{(i)} \right)^2$$

#training examples

$$h_{\theta}(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$$

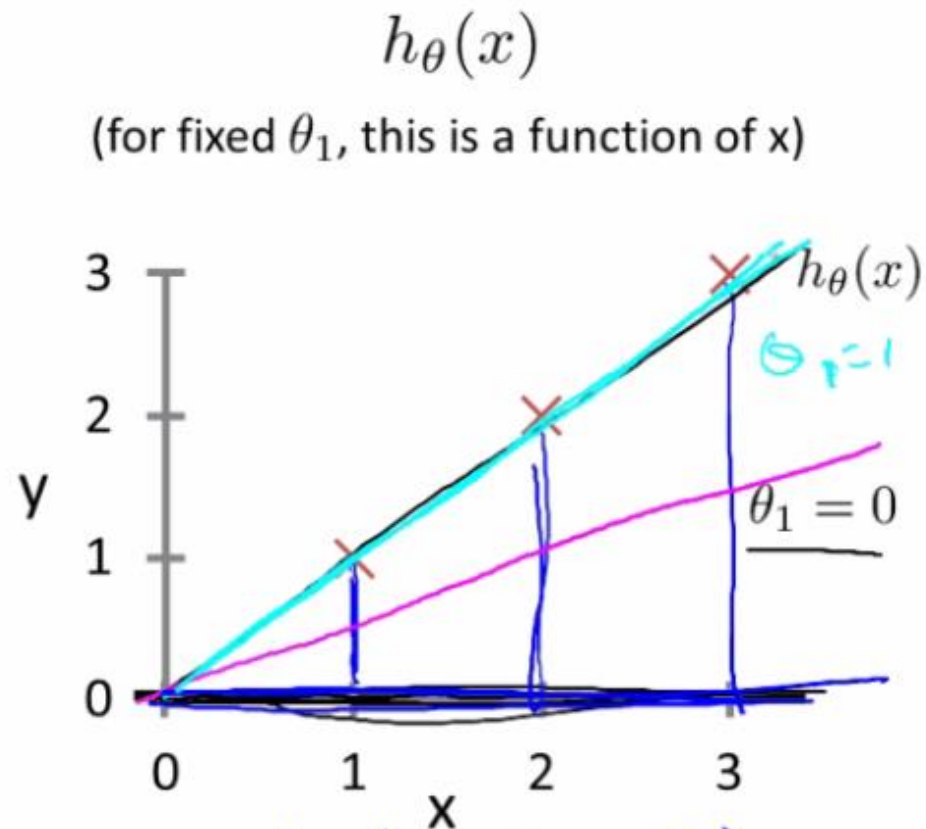
$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

minimize  $\theta_0, \theta_1$   $J(\theta_0, \theta_1)$

Cost function

Squared error function

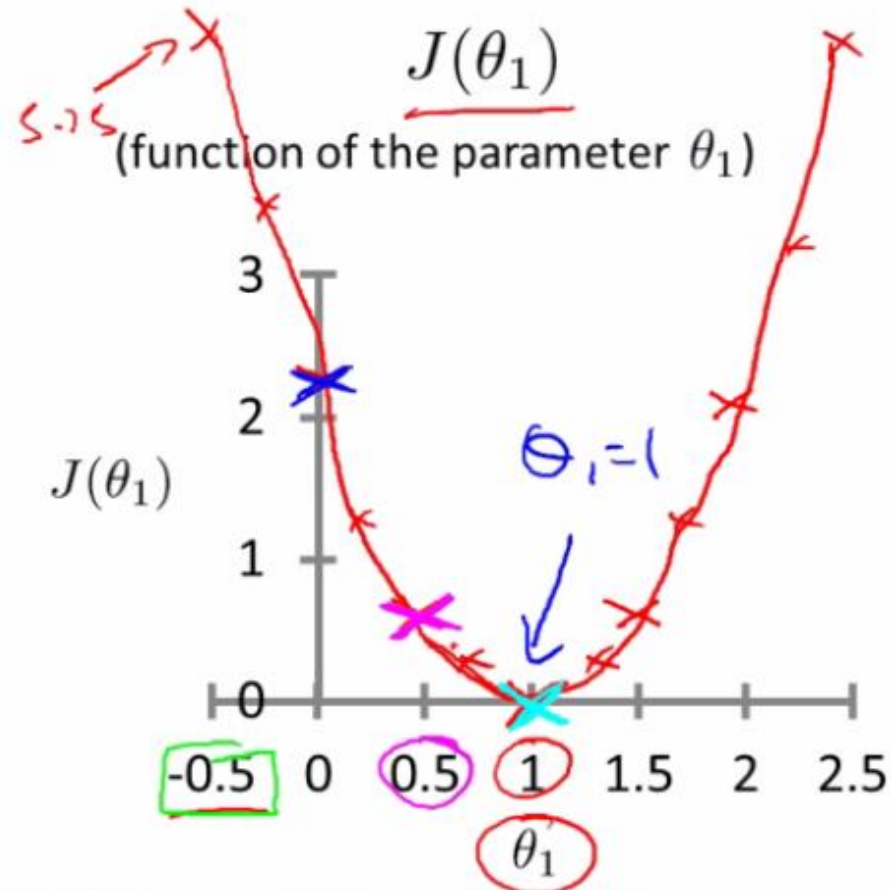
# Linear Regression Basic - Cost Function



$$J(0) = \frac{1}{2m} (1^2 + 2^2 + 3^2) \\ = \frac{1}{6} \cdot 14 \approx 2.3$$

$$h(x) = -0.5x$$

minimize  $J(\theta_1)$   
 $\theta_1$



# Linear Regression Basic - Cost Function



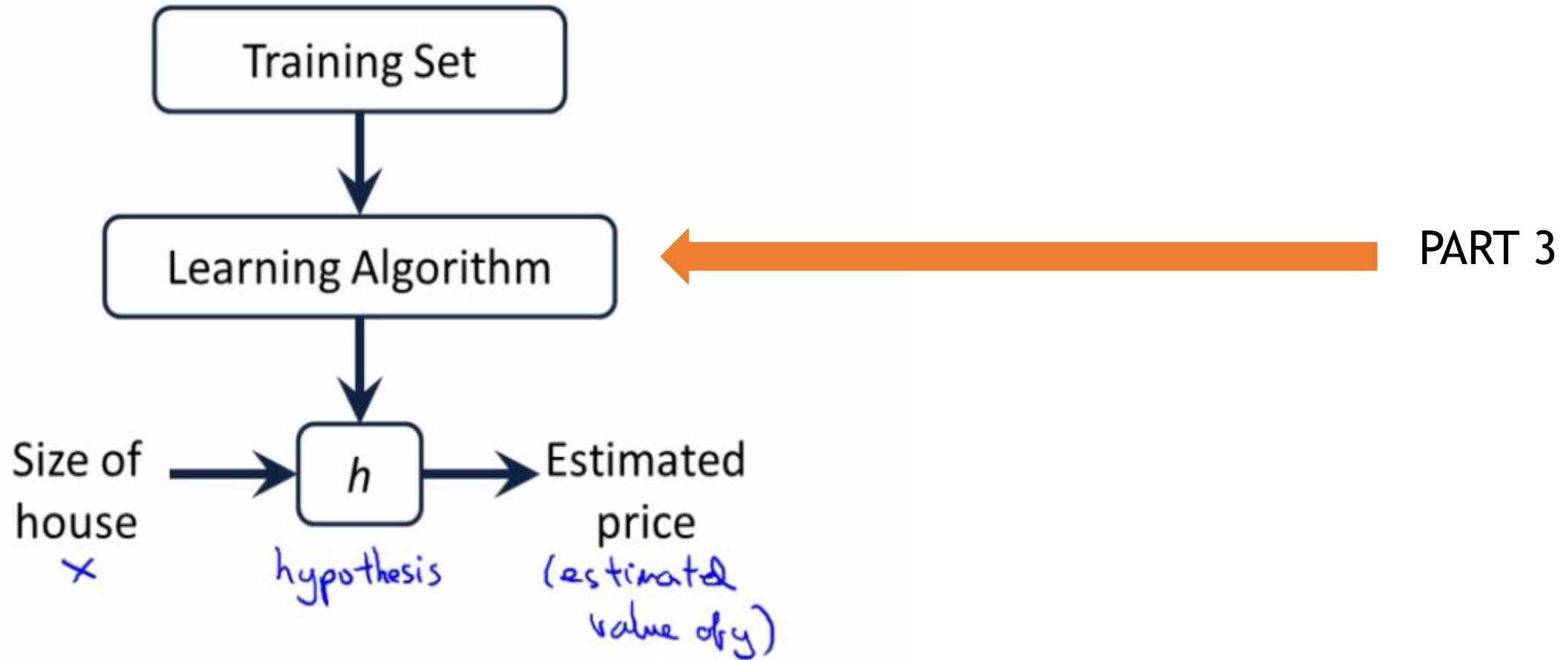
Hypothesis:  $h_{\theta}(x) = \theta_0 + \theta_1 x$

Parameters:  $\theta_0, \theta_1$

Cost Function:  $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$

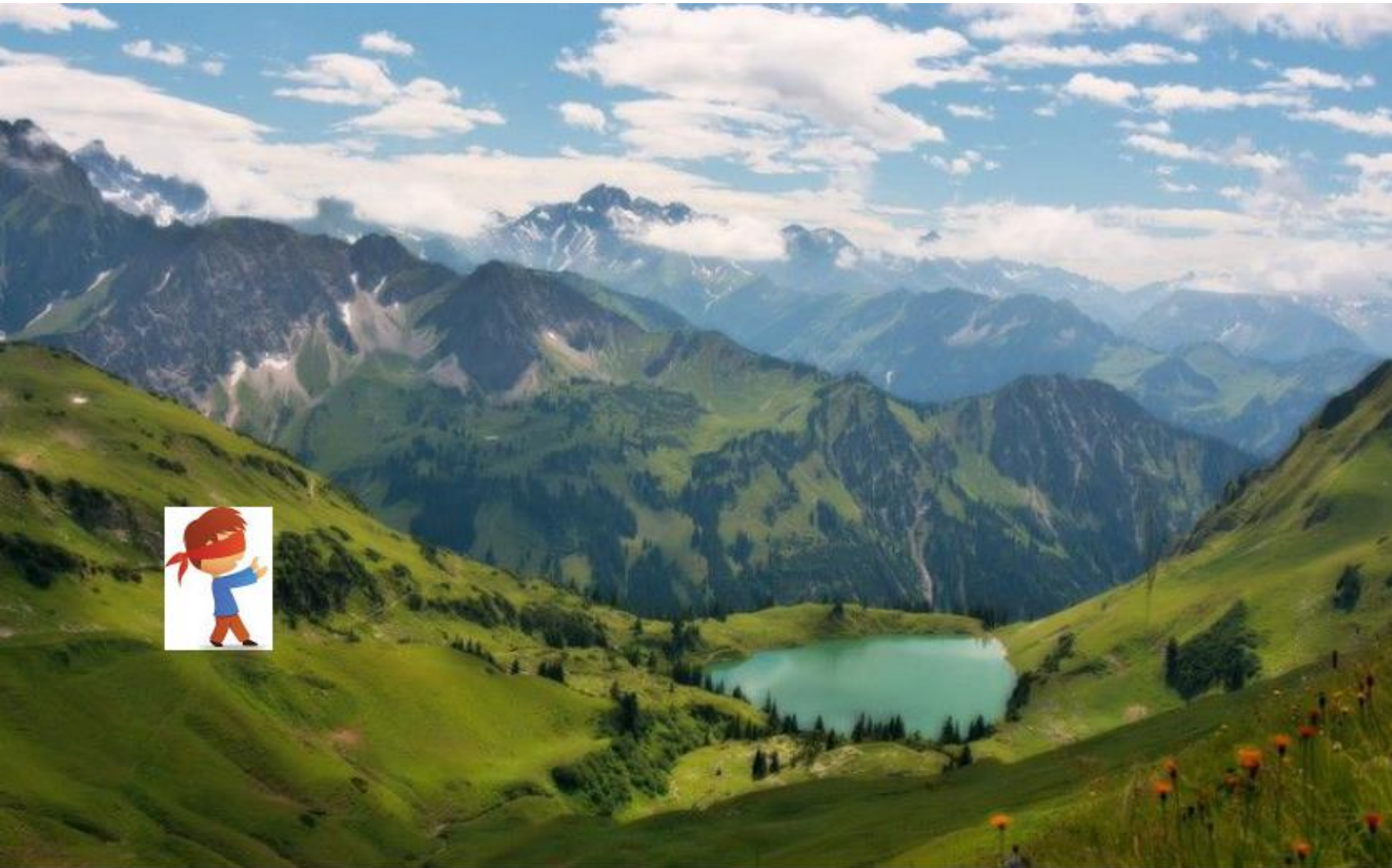
Goal:  $\underset{\theta_0, \theta_1}{\text{minimize}} J(\theta_0, \theta_1)$

# Linear Regression Basic - Model Representation





# Linear Regression Basic - Optimization



“The core of ML is optimization”- Siraj Raval

# Linear Regression Basic - Optimization

**Gradient Descent** is the most common optimization algorithm in *machine learning* and *deep learning*.

It is a **first-order optimization algorithm**. This means it only takes into account the first derivative when performing the updates on the parameters. **On each iteration**, we update the parameters in the opposite direction of the gradient of the **objective function**(aka **Cost function**)  $J(w)$  w.r.t the **parameters** where the gradient gives the direction of the steepest ascent.

The size of the step we take on each iteration to reach the **local minimum** is determined by the **learning rate  $\alpha$** . Therefore, we follow the direction of the slope downhill until we reach a **local minimum**.

Parameters:

$$\theta_0, \theta_1$$

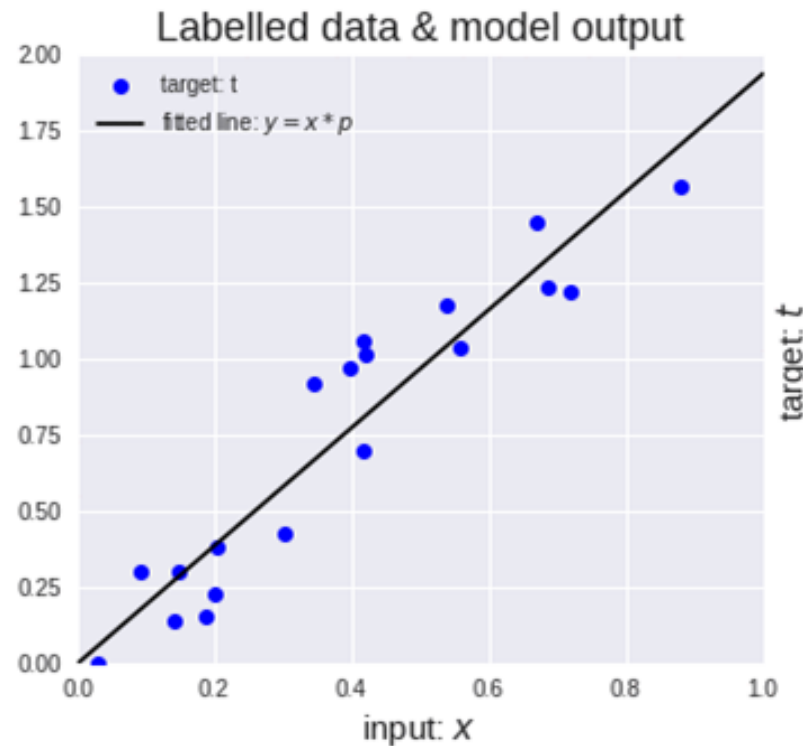
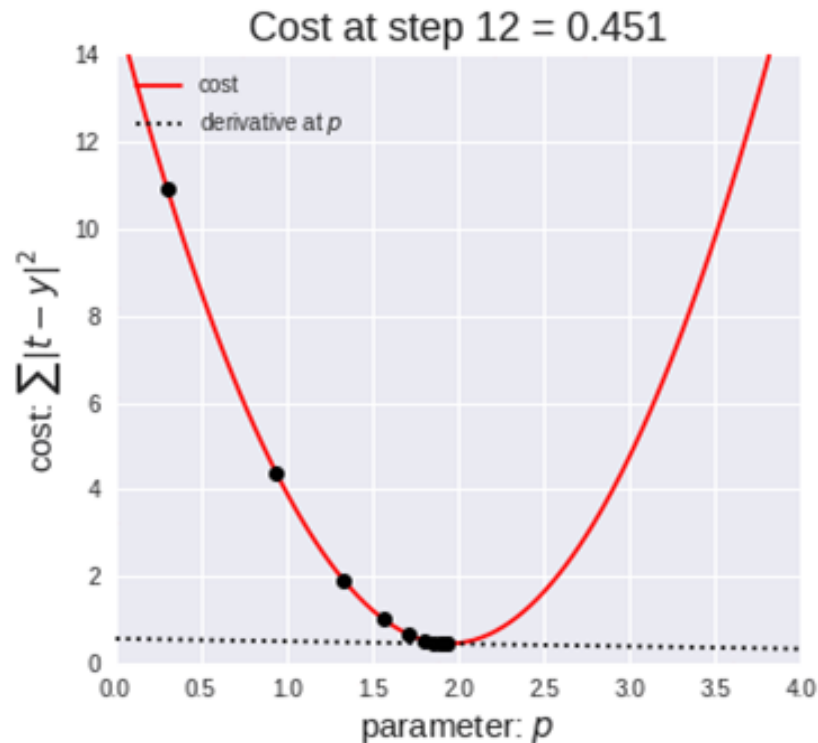
Goal: minimize  $J(\theta_0, \theta_1)$   
 $\theta_0, \theta_1$

Cost Function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

# Linear Regression Basic - Optimization

Gradient Descent: It is an **iterative** optimization algorithm used in machine learning to find the best result (minima of a curve).



- Start with some  $m, b$  value(0,0)
- Keep changing  $m, b$  to reduce  $J(m,b)$  until we get to minimum

Hyperparameters:  
 $\alpha$  - learning rate

- Iterative means we need to get the results multiple times to get the most optimal result.



# Linear Regression Basic - Optimization



$\theta_0$  : Reading time

Parameters:

$\theta_0, \theta_1$



$\theta_1$

Repeat until convergence {

$$\theta_j \leftarrow \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

}



:Reading Technique

# Linear Regression Basic - Optimization

Repeat until convergence {

$$\theta_j \leftarrow \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

}

$$\theta_0: \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})$$

$$\theta_1: \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x^{(i)}$$

$$\frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_0} \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

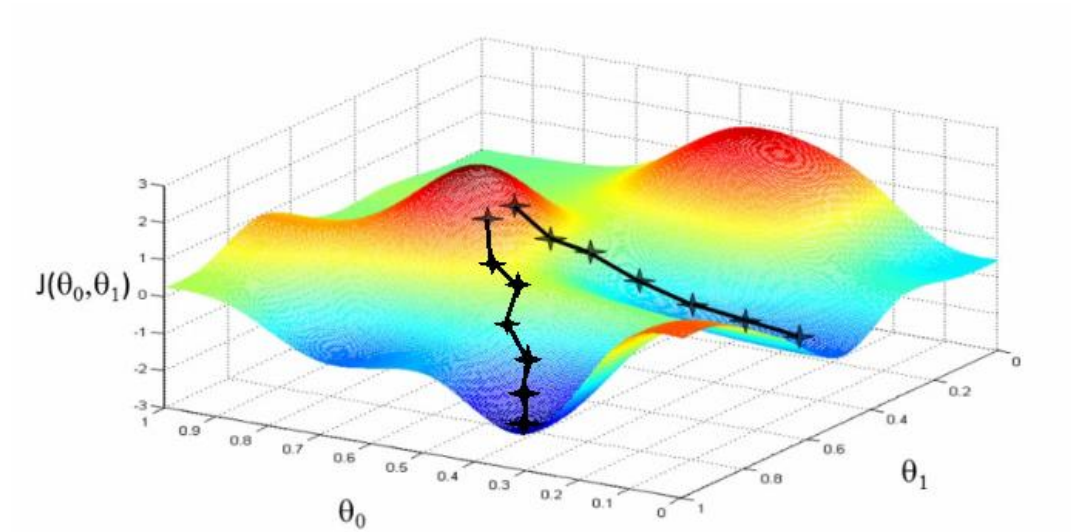
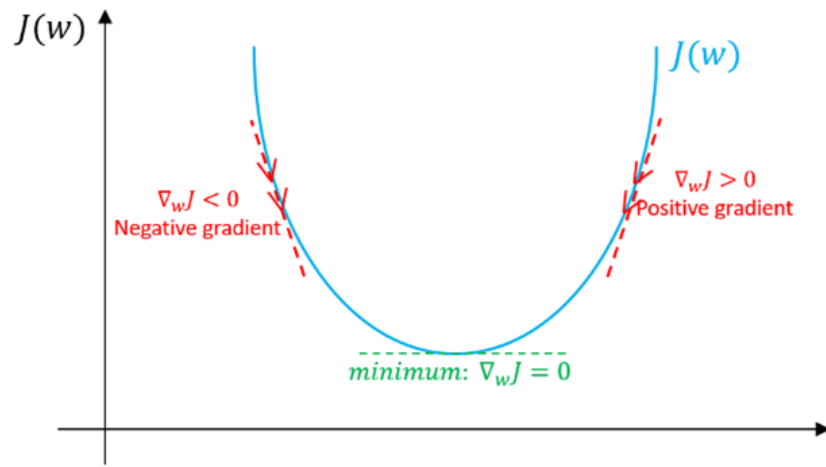
repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x^{(i)}$$

}

# Linear Regression Basic - Optimization



- A **convex function** is a function that is local optima free. In our linear regression problem, there was only one minimum.

Note: a non convex function has **multiple optima** and is not appropriate for machine learning (although we can still use the algorithms, but they may get stuck in **non-optimal solutions**).

# Linear Regression Basic - Optimization

Choosing a proper learning rate can be difficult...

- Learning rate is too small
  - => This is painfully slow convergence
- Learning rate is too large
  - => This can hinder convergence and cause the loss function to fluctuate around the minimum or even to diverge.



$\alpha$  :Reading Technique

The learning rate is an hyperparameter

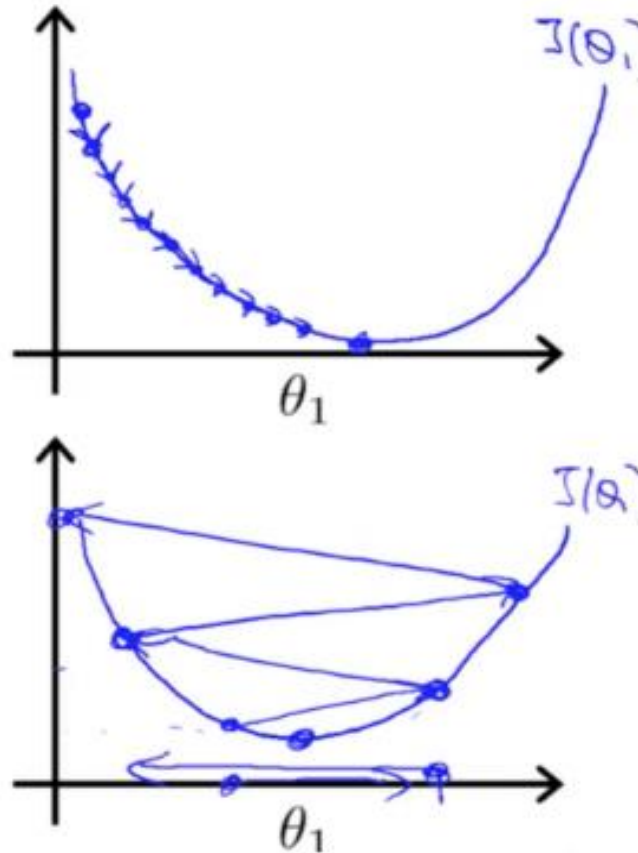
# Linear Regression Basic - Optimization

Choosing a proper learning rate can be difficult...

$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If  $\alpha$  is too small, gradient descent can be slow.

If  $\alpha$  is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.



- The most commonly used rates are : 0.001, 0.003, 0.01, 0.03, 0.1, 0.3.

# Linear Regression Basic - Optimization

## Choosing a proper learning rate can be difficult...

A model parameter is a configuration variable that is **internal** to the model and whose value can be estimated from data.

- They are required by the model when making predictions.
- They are often not set manually by the practitioner.
- Their values define the skill of the model on your problem.

$\theta_0$  : Reading time

$\theta_1$  : Sleeping time

A model hyperparameter is a configuration that is **external** to the model and whose value cannot be estimated from data.

- They are often used in processes to help estimate model parameters.
- They are often specified by the practitioner.
- They are often tuned for a given predictive modeling problem.

$\alpha$  : Reading Technique



# Gradient Descent for Normal Guys

```
1 def gradient_descent(x, y, m_current=6, b_current=5, epochs=1000, learning_rate=0.0001):
2     N = float(len(y))
3     list_cost = []
4     for i in range(epochs):
5         y_current = (m_current * x) + b_current
6         cost = sum([data**2 for data in (y-y_current)]) / N
7         list_cost.append(cost)
8         m_gradient = -(2/N) * sum(x * (y - y_current))
9         b_gradient = -(2/N) * sum(y - y_current)
10        m_current = m_current - (learning_rate * m_gradient)
11        b_current = b_current - (learning_rate * b_gradient)
12    return m_current, b_current, list_cost
```

$$\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) = \frac{2}{2\theta_j} \cdot \frac{1}{2m} \sum_{i=1}^m (\underbrace{h_{\theta}(x^{(i)})}_{\theta_0 + \theta_1 x^{(i)}} - y^{(i)})^2$$
$$= \frac{2}{2\theta_j} \frac{1}{2m} \sum_{i=1}^m (\theta_0 + \theta_1 x^{(i)} - y^{(i)})^2$$

$$\theta_0, j=0: \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})$$

$$\theta_1, j=1: \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x^{(i)}$$

Correct: Simultaneous update

- $\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$
- $\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$
- $\theta_0 := \text{temp0}$
- $\theta_1 := \text{temp1}$

# Gradient Descent for PhD... vectorization

```
1 def computeCost(X, y, theta=[[0],[0]]):  
2     m = y.size  
3     J = 0  
4     h = x.dot(theta)  
5     J = 1/(2*m)*np.sum(np.square(h-y))  
6     return(J)
```

## Correct: Simultaneous update

→  $\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$   
→  $\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$   
→  $\theta_0 := \text{temp0}$   
→  $\theta_1 := \text{temp1}$

```
1 def gradientDescent(X, y, theta=[[0],[0]], alpha=0.0001, num_iters=1500):  
2     m = y.size  
3     J_history = np.zeros(num_iters)  
4     for iter in np.arange(num_iters):  
5         h = X.dot(theta)  
6         theta = theta - alpha*(1/m)*np.dot(X.T, h-y)  
7         J_history[iter] = computeCost(X, y, theta)  
8     return(theta, J_history)
```

The gradient can be calculated as:

$$f'(m, b) = \begin{bmatrix} \frac{df}{dm} \\ \frac{df}{db} \end{bmatrix} = \begin{bmatrix} \frac{1}{N} \sum -2x_i(y_i - (mx_i + b)) \\ \frac{1}{N} \sum -2(y_i - (mx_i + b)) \end{bmatrix}$$



# Hilton's Closing Prayer

Our father who art in  $n$ -dimensions  
hallowed by the backprop,  
thy loss be minimized,  
thy gradients unvarnished,  
on earth as it is in Euclidean space.  
Give us this day our daily hyperparameters,  
and forgive us our large learning rates,

as we forgive those whose parameters diverge,  
and lead us not into discrete optimization,  
but deliver us from local optima.  
For thine are dimensions,  
and the GPUs, and the glory,  
forever and ever. Dropout.



AIO