

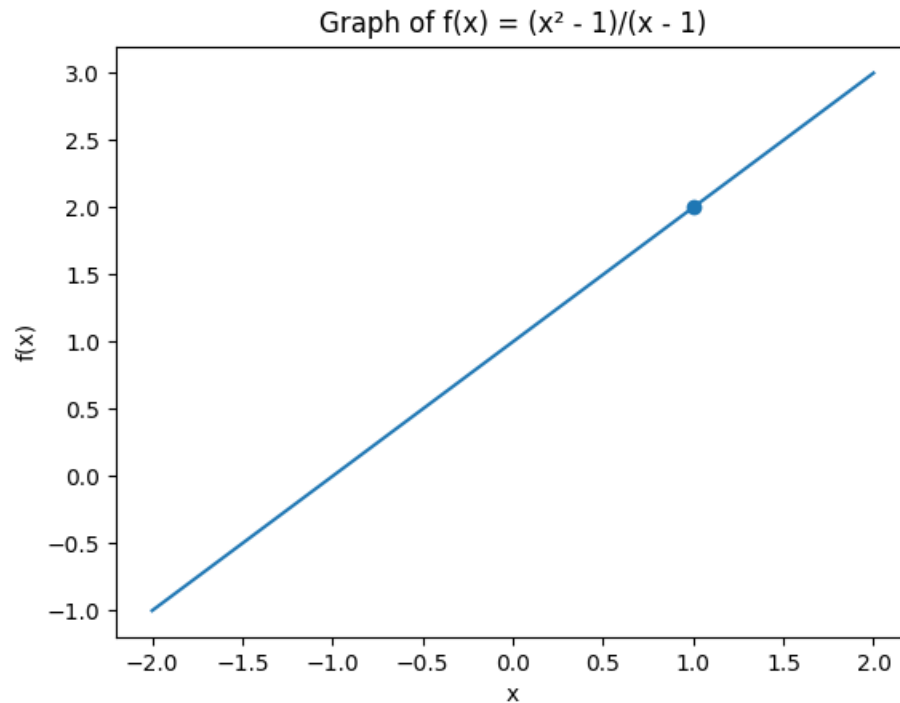
# Limits of the Function

Introduction, Definition, Examples &  
Practice Questions

# Introduction to Limits

- • The concept of limit is fundamental in calculus.
- • It describes the behavior of a function as the input approaches a certain value.
- • Limits help define continuity, derivatives, and integrals.
- • It tells us what value a function approaches even if it does not reach it.

# Graphical Representation of Limit



# Definition of Limit

- If  $f(x)$  approaches  $L$  as  $x$  approaches  $a$ , then:
- $\lim_{x \rightarrow a} f(x) = L$
- This means the value of  $f(x)$  becomes arbitrarily close to  $L$
- as  $x$  gets closer to  $a$ .

# Limits of Algebraic Functions

- Example:
- Find  $\lim_{x \rightarrow 2} (x^2 - 4)/(x - 2)$
- Solution:
- Factor:  $(x-2)(x+2)/(x-2)$
- $= x + 2$
- Substitute  $x = 2$
- Answer = 4

# Limits of Trigonometric Functions

- Example:
- Find  $\lim_{x \rightarrow 0} (\sin x)/x$
- Standard Result:
- $\lim_{x \rightarrow 0} (\sin x)/x = 1$

# Limits of Logarithmic Functions

- Example:
- Find  $\lim_{x \rightarrow 1} \ln(x)$
- Since  $\ln(x)$  is continuous at  $x = 1$ ,
- Substitute directly:
- $\ln(1) = 0$

# Limits of Exponential Functions

- Example:
- Find  $\lim_{x \rightarrow 0} (e^x - 1)/x$
- Standard Result:
- $\lim_{x \rightarrow 0} (e^x - 1)/x = 1$

# Practice Questions

- 1. Find  $\lim_{x \rightarrow 3} (x^2 - 9)/(x - 3)$
- 2. Find  $\lim_{x \rightarrow 0} (\tan x)/x$
- 3. Find  $\lim_{x \rightarrow 2} \log(x)$
- 4. Find  $\lim_{x \rightarrow \infty} (1 + 1/x)^x$
- 5. Find  $\lim_{x \rightarrow 0} (1 - \cos x)/x^2$

# Learning Objectives

- After studying this topic, students will be able to:
  - Understand the concept of limits.
  - Evaluate limits algebraically.
  - Apply standard trigonometric limits.
  - Solve limits involving logarithmic and exponential functions.
  - Solve real-world problems using limits.