

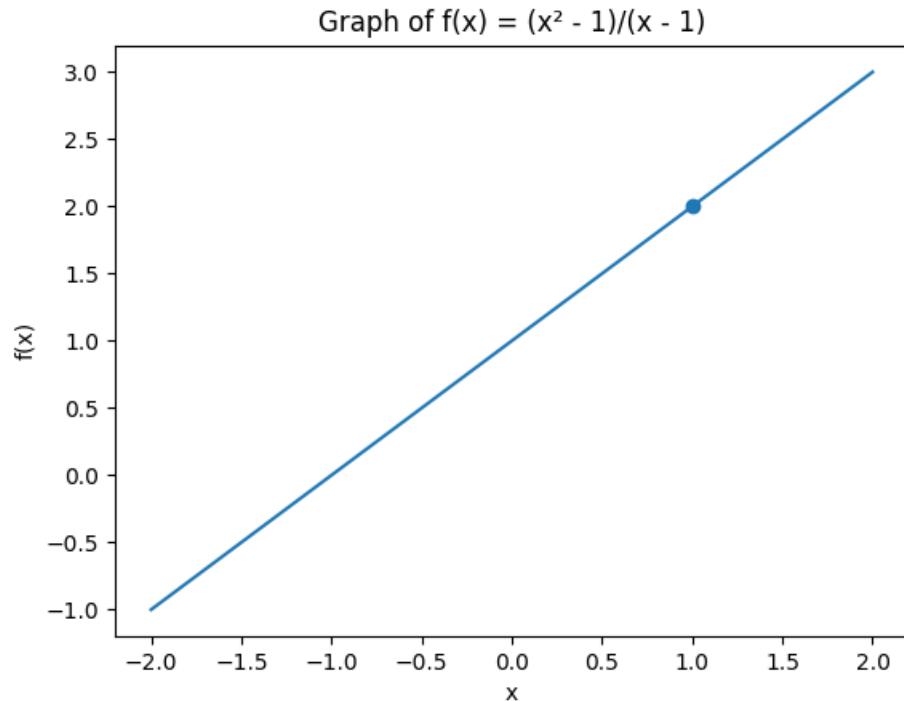
Limits of the Function

Introduction, Definition, Examples &
Practice Questions

Introduction to Limits

- The concept of limit is fundamental in calculus.
- It describes the behavior of a function as the input approaches a certain value.
- Limits help define continuity, derivatives, and integrals.
- It tells us what value a function approaches even if it does not reach it.

Graphical Representation of Limit



Definition of Limit

- If $f(x)$ approaches L as x approaches a , then:
- $\lim (x \rightarrow a) f(x) = L$
- This means the value of $f(x)$ becomes arbitrarily close to L
- as x gets closer to a .

Limits of Algebraic Functions

- Example:
- Find $\lim_{x \rightarrow 2} (x^2 - 4)/(x - 2)$
- Solution:
- Factor: $(x-2)(x+2)/(x-2)$
- $= x + 2$
- Substitute $x = 2$
- Answer = 4

Limits of Trigonometric Functions

- Example:
- Find $\lim_{x \rightarrow 0} (\sin x)/x$
- Standard Result:
- $\lim_{x \rightarrow 0} (\sin x)/x = 1$

Limits of Logarithmic Functions

- Example:
- Find $\lim_{x \rightarrow 1} \ln(x)$
- Since $\ln(x)$ is continuous at $x = 1$,
- Substitute directly:
- $\ln(1) = 0$

Limits of Exponential Functions

- Example:
- Find $\lim_{x \rightarrow 0} (e^x - 1)/x$
- Standard Result:
- $\lim_{x \rightarrow 0} (e^x - 1)/x = 1$

Practice Questions

- 1. Find $\lim_{x \rightarrow 3} (x^2 - 9)/(x - 3)$
- 2. Find $\lim_{x \rightarrow 0} (\tan x)/x$
- 3. Find $\lim_{x \rightarrow 2} \log(x)$
- 4. Find $\lim_{x \rightarrow \infty} (1 + 1/x)^x$
- 5. Find $\lim_{x \rightarrow 0} (1 - \cos x)/x^2$

Learning Objectives

- After studying this topic, students will be able to:
 - Understand the concept of limits.
 - Evaluate limits algebraically.
 - Apply standard trigonometric limits.
 - Solve limits involving logarithmic and exponential functions.
 - Solve real-world problems using limits.