ALIGARH INSTITUTE TECHNOLOGY

COMMITTED TO EXCELLENCE

SUBJECT: APPLIED MATHEMATICS (223)

CLASS: D.A.E 2^{ND} YEAR

CHAPTER # 1

TOPIC: FUNCTIONS AND LIMITS

TEACHER NAME: Ms. RIDA NAEEM

LECTURE # 1

Ch#1 FUNTIONS AND LIMITS

- What is function ?
- A function relates an input to an output.

Input → function → output

X=2 → X^2 → 4

Some Examples of Functions

- $3x + 2$ is a function
- x^3+1 is also a function
- Sine, Cosine and Tangent are functions used in trigonometry

Evaluation of a function :

Consider the function given by :

$$Y = f(x) = 3x^3 - 5x^2 - 7x + 14$$

The value of above function for $x = 1$ is given by.

$$f(1) = y = 3(1)^3 - 5(1)^2 - 7(1) + 14$$
$$= 3(1) - 5(1) - 7 + 14$$
$$= 3 - 5 - 7 + 14$$
$$y = 5$$

Or It can be written as,

$$f(1) = 5$$

Eg# 2 :-

Show that $f(x) = \frac{\sin x}{1+\cos x}$

is not defined if $x = \pi$

Solution :-

$$f(x) = \frac{\sin x}{1+\cos x} \rightarrow (1)$$

R.w

$\sin(\pi) = 0$

$\cos(\pi) = -1$

'x' replace by 'π'

eq (1) $\Rightarrow$ $f(\pi) = \frac{\sin\pi}{1+\cos\pi}$

$$= \frac{0}{1+(-1)}$$

$$= \frac{0}{1-1}$$

$$= \frac{0}{0} = \text{not defined.}$$

Eg# 4

If $f(x) = \frac{x+1}{x-1}$, Show that $f[f(x)] = x$

Solution :-

If, $X = f(x) = \frac{x+1}{x-1} \rightarrow$ eq ①

"X" replace by 'x'

eq ① $\Rightarrow$ $f(X) = \frac{X+1}{X-1}$

Now consider **L.H.S** :-

$f[f(x)] = f(x)$

$f[f(x)] = f(X)$

$= \frac{X+1}{X-1} \rightarrow$ eq ②

We know that $X = \frac{x+1}{x-1}$

Put value of "X" in eq (2)

$$\Rightarrow \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}$$

$$= \frac{\frac{(x+1)+(x-1)}{\cancel{(x-1)}}}{\frac{(x+1)-(x-1)}{\cancel{(x-1)}}}$$

$$= \frac{(x+1)+(x-1)}{(x+1)-(x-1)}$$

$$= \frac{\cancel{x}+\cancel{1}+x-\cancel{1}}{\cancel{x}+1-\cancel{x}+1}$$

$$= \frac{\cancel{2}x}{\cancel{2}}$$

$f[f(x)] = x$ hence proved.

TYPES OF FUNCTION

→ Even functions :-

"A function $f(x)$ of a variable x is said to be even if $f(-x) = f(x)$."

for example, $\cos x$, $\sin^2 x$ and x^2 are even functions of x.

→ Odd functions :-

A function $f(x)$ of a variable x is said to be odd for which $f(-x) = -f(x)$

for example, $\sin x$, $\cot x$, $\tan x$ and x^3 are all odd functions of x.

→ Neither odd nor Even functions :-

A function $f(x)$ of a variable x is said to be neither odd nor Even function for which $f(-x) \neq -f(x)$

Eg #1 :

If, $f(-x) = f(x)$

Solution ::

$$f(x) = x^4$$

"x" replace by "$-x$"

$$\Rightarrow \quad f(-x) = (-x)^4$$
$$= x^4$$

$$f(-x) = f(x)$$

$\therefore$ $f(x) = x^4$ is even function.

Eg #2 : $f(x) = \cos x$

$\because$ **Solution ::**

'x' replace by "$-x$"

$$\Rightarrow \quad f(-x) = \cos(-x)$$
$$= \cos x$$

$$f(-x) = f(x)$$

$\therefore$ $f(x) = \cos x$ is an even function.

Eg#3 : $f(x) = x^5$

Solution :-

'x' replace by "-x"

$$f(-x) = (-x)^5$$

$$f(-x) = -x^5$$

$\therefore$ $f(x) = x^5$ is an odd function.

Eg#4 :-

If $f(x) = \sin x$

Solution :-

'x' replace by '-x'

$$\Rightarrow f(-x) = \sin(-x)$$

$$= -\sin x$$

$$\Rightarrow f(-x) = -f(x)$$

$\therefore$ $f(x) = \sin x$ is an odd function.

Ex : 1.1

Q.1 Given a polynomial function.

$P: \mathbb{R} \rightarrow \mathbb{R}$ with $P(x) = x^2 + 2x + 3$

find.

(i) $p(-3)$

$P(x) = (-3)^2 + 2(-3) + 3$

$p(-3) = 9 - 6 + 3$

$3 + 3$

6

(iii) $p(-1)$

$p(x) = x^2 + 2x + 3$

$p(-1) = (-1)^2 + 2(-1) + 3$

$= 1 - 2 + 3$

$= -1 + 3$

$= 2$

(iv) $p(0)$

$p(x) = x^2 + 2x + 3$

$p(0) = (0)^2 + 2(0) + 3$

$= 0 + 0 + 3$

$p(0) = 3$

Q2:- If $f(x) = 2x^3 - 3x^2 + 4x - 3$ Show that

$$20 f(2) = 5 f(3)$$

Solution :-

$$f(x) = 2x^3 - 3x^2 + 4x - 3 \longrightarrow (1)$$

'x' replace by 2

$$eq\ (1) \Rightarrow f(2) = 2(2)^3 - 3(2)^2 + 4(2) - 3$$

$$= 2(8) - 3(4) + 8 - 3$$

$$= 16 - 12 + 8 - 3$$

$$= 16 - 12 + 5$$

$$= 4 + 5$$

$$= 9$$

'x' replace by 3

eq ① $\Rightarrow$ $f(3) = 2(3)^3 - 3(3)^2 + 4(3) - 3$

$= 2(27) - 3(9) + 12 - 3$

$= 54 - 27 + 9$

$= 27 + 9$

$f(3) = 36$

Now consider

$20f(2) = 5f(3)$

Put values of $f(2)$ and $f(3)$.

$20(9) = 5(36)$

$180 = 180$

Hence proved

L.H.S = R.H.S.

Q10 Show that following $f(x)$ is an odd function of 'x'

(i) $\dfrac{e^x - 1}{e^x + 1}$

Solution :-

Let $f(x) = \dfrac{e^x - 1}{e^x + 1} \longrightarrow$ eq ①

'x' replace by '$-x$'

eq ① $\Rightarrow$ $f(-x) = \dfrac{e^{-x} - 1}{e^{-x} + 1}$ $\qquad \because e^{-x} = \dfrac{1}{e^x}$

$$f(-x) = \dfrac{\dfrac{1}{e^x} - 1}{\dfrac{1}{e^x} + 1}$$

$\dfrac{1}{e^x} - 1$

$\dfrac{(1 \times 1) - (1 \times e^x)}{e^x}$

Taking l.c.m in numinator and denominator

$\dfrac{1 - e^x}{e^x}$

$$f(-x) = \frac{\frac{1-e^x}{e^x}}{\frac{1+e^x}{e^x}}$$

$$= \frac{1-e^x}{1+e^x}$$

Taking minus sign common.

$$= -\left(\frac{e^x-1}{e^x+1}\right)$$

$$= -f(x)$$

Hence the given function is odd.

Ex: 1.1 Assignment Questions

1. Given a polynomial function

$P: \mathbb{R} \to \mathbb{R}$ with $p(x) = x^2 + 2x + 3$

find:

(a) $P(-2)$ (b) $P(3)$

2. If $f(x) = 3x^3 + 2x^2 - x + 4$

Prove that $2f(3) = 2f(1)$

3. Show that $f(x)$ following is an odd function of 'x'

(i) $f(x) = \dfrac{a^x + 1}{a^x - 1}$