Monte Carlo Simulation of Photon Transport in a Scattering Slab

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1 Introduction

In this work, I simulate photon transport in a homogeneous scattering slab using a Monte Carlo method. I compute the total remittance, angular distributions, and depth-resolved fluence.

2 Monte Carlo Model

I launch photon packets normally onto the slab with an initial weight of unity. Each photon undergoes repeated cycles of free propagation, absorption, and scattering until it exits the slab or its weight becomes negligible.

I sample the step length from the exponential distribution

$$s = -\frac{\ln \xi}{\mu_t}, \qquad \mu_t = \mu_a + \mu_s,$$

where ξ is a uniformly distributed random number. I model scattering using the Henyey–Greenstein phase function with anisotropy factor g. I treat absorption using the implicit capture method and terminate low-weight photons using Russian roulette.

3 Key Implementation Steps

In this section, I present the main algorithmic steps used in my Monte Carlo implementation.

3.1 Photon initialization

I initialize each photon at the top surface of the slab with unit weight and normal incidence.

```
p.x = 0.0;
p.y = 0.0;
p.z = 0.0;
p.ux = 0.0;
p.uy = 0.0;
p.uz = 1.0;
p.w  = 1.0;
p.scatters = 0;
```

3.2 Step size sampling

I sample the free path length using the total interaction coefficient.

```
double s = -log(rng_u01(&r)) / mu_t;
```

3.3 Absorption

I reduce the photon weight according to the absorption probability.

```
double dw = p.w * (MU_A / mu_t);
p.w -= dw;
```

3.4 Scattering

I sample the scattering angles using the Henyey–Greenstein phase function and update the photon direction.

```
spin(&p, &r, G);
p.scatters++;
```

3.5 Boundary handling

I detect boundary crossings and classify photons as reflected, transmitted, or ballistic.

```
if (z_new < 0.0) {
    Rd += p.w;
}
else if (z_new > D) {
    Tt += p.w;
    if (p.scatters == 0)
        Tball += p.w;
    else
        Tdiff_bin[angle_bin(p.uz)] += p.w;
}
```

3.6 Angular binning

I compute angular distributions using the exit angle and normalize them by the solid angle.

```
double alpha = acos(fabs(p.uz));
int bin = (int)(alpha / (0.5 * PI) * NANG);

double dOmega = 2.0 * PI * (cos(a0) - cos(a1));
Rd_sr[i]    = Rd_bin[i]    / (N * dOmega);
Tdiff_sr[i] = Tdiff_bin[i] / (N * dOmega);
```

3.7 Depth-resolved fluence

I accumulate absorbed energy in depth bins and compute the fluence profile.

```
int iz = (int)(p.z / dz);
A[iz] += dw;

Phi[i] = A[i] / (MU_A * dz * NPHOT);
```

4 Remittance Simulation

4.1 Simulation parameters

For the remittance benchmark, I use the following optical properties:

$$\mu_a = 10\,\mathrm{cm}^{-1}, \quad \mu_s = 90\,\mathrm{cm}^{-1}, \quad g = 0.75, \quad d = 0.02\,\mathrm{cm}, \quad n_{\mathrm{rel}} = 1.$$

The total interaction coefficient is $\mu_t = 100\,\text{cm}^{-1}$. For this configuration, I compute the ballistic transmittance analytically as

$$T_{\text{ballistic}} = \exp(-\mu_t d) = \exp(-2) = 0.1353.$$

I perform ten independent Monte Carlo simulations of 5×10^4 photons each.

4.2 Results

From the simulations, I obtain the following mean values:

$$R_d = 0.0979 \pm 0.0003,$$

$$T_t = 0.6607 \pm 0.0004.$$

I compute the diffuse component of the transmittance by subtracting the ballistic contribution:

$$T_{\text{diffuse}} = T_t - T_{\text{ballistic}} = 0.5254.$$

```
ham7zait@H-A-Tservices:~/MontecarloSimulation$ ./mc_remittance
Run  1/10: Rd=0.097977, Tt=0.662230 (Tball=0.136980)
Run  2/10: Rd=0.098127, Tt=0.660345 (Tball=0.135560)
Run  3/10: Rd=0.098254, Tt=0.658993 (Tball=0.133540)
Run  4/10: Rd=0.097405, Tt=0.661323 (Tball=0.139380)
Run  5/10: Rd=0.098633, Tt=0.661625 (Tball=0.135820)
Run  6/10: Rd=0.097779, Tt=0.660056 (Tball=0.136220)
Run  7/10: Rd=0.097560, Tt=0.661383 (Tball=0.137040)
Run  8/10: Rd=0.096053, Tt=0.662123 (Tball=0.135400)
Run  9/10: Rd=0.098057, Tt=0.660343 (Tball=0.134940)
Run 10/10: Rd=0.098946, Tt=0.658908 (Tball=0.132400)

=== Benchmark (Slide p.32 case) ===
mu_a=10.00 cm^-1, mu_s=90.00 cm^-1, g=0.75, d=0.0200 cm, n_rel=1.00
Expected ballistic T = exp(-(mu_a+mu_s)*d) = exp(-2.000000) = 0.135335

Results over 10 runs of 50000 photons each:
Rd mean = 0.097879,  SE = 0.000250
Tt mean = 0.660733,  SE = 0.000378
Diffuse T (using expected ballistic) = 0.525398
Measured ballistic mean (from scatters==0) = 0.135728
```

4.3 Interpretation

I observe a relatively high transmittance due to the small slab thickness. A significant fraction of photons cross the slab without interaction, forming the ballistic component, while scattered photons contribute to the diffuse reflectance and transmittance. The numerical values I obtain are consistent with established Monte Carlo benchmark results for this configuration.

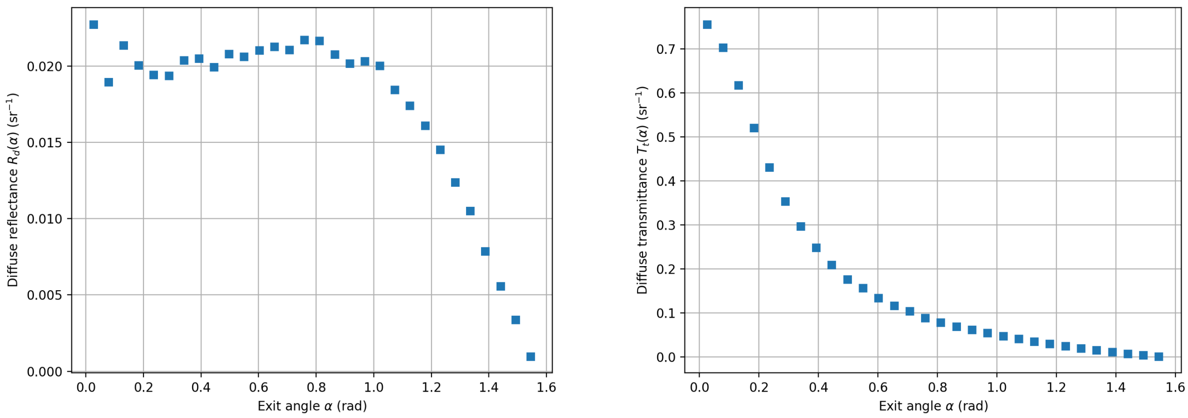
5 Angular Dependence of Remittance

I record the exit angle of each photon leaving the slab as

$$\alpha = \arccos(|u_z|),$$

where u_z is the direction cosine with respect to the slab normal. I compute angular distributions by binning photons in $\alpha \in [0, \pi/2]$ and normalizing by the corresponding solid angle.

When computing angular transmittance, I exclude ballistic photons and retain only the diffuse contribution.



(a) Diffuse reflectance $R_d(\alpha)$ (b) Diffuse transmittance $T_t(\alpha)$

Figure 1: Angular dependence of diffuse reflectance and transmittance.

5.1 Interpretation

I observe that the diffuse reflectance is nearly isotropic for small exit angles and decreases toward grazing angles. The diffuse transmittance decreases monotonically with increasing exit angle, indicating that photons leaving the slab at larger angles undergo more scattering events. These trends are characteristic of multiple scattering in thin slabs.

6 Depth-Resolved Fluence

6.1 Simulation parameters

For the fluence study, I use the following optical properties:

$$\mu_a = 0.1\,\mathrm{cm}^{-1}, \quad \mu_s = 100\,\mathrm{cm}^{-1}, \quad g = 0.9, \quad d = 1\,\mathrm{cm}.$$

I consider two refractive indices: $n_{\mathrm{rel}} = 1$ and $n_{\mathrm{rel}} = 1.37$. I compute the transport mean free path as

$$\ell_t' = \frac{1}{\mu_a + \mu_s(1-g)} \approx 0.099\,\mathrm{cm}.$$

6.2 Results and interpretation

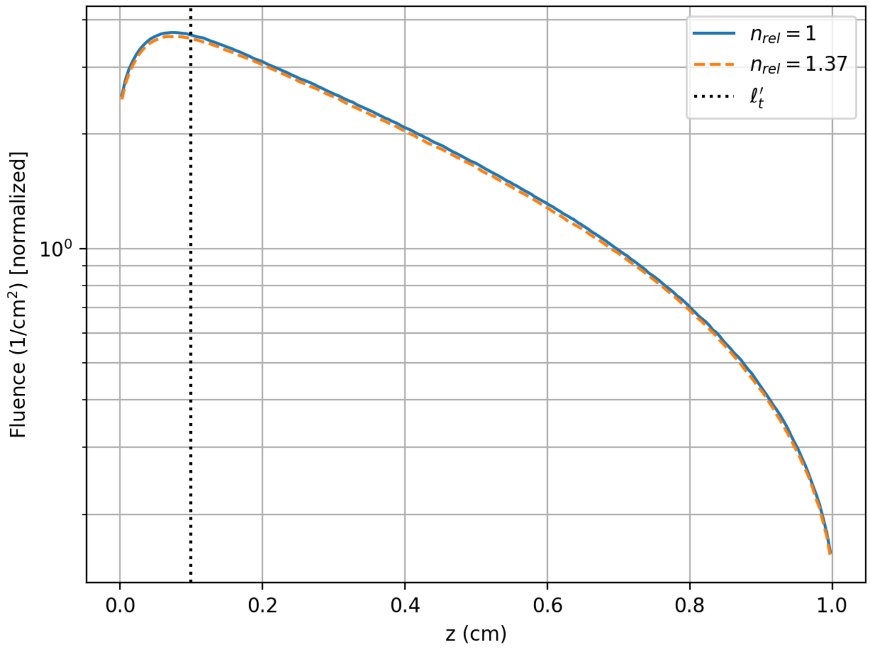


Figure 2: Depth dependence of fluence for two refractive indices. The vertical line indicates the transport mean free path ℓ_t'.

I observe that the fluence increases just beneath the surface due to strong backscattering, reaches a maximum, and then decays approximately exponentially with depth. Beyond one transport mean free path, the fluence follows diffusion theory. When I increase the refractive index, I observe stronger photon confinement inside the slab and a higher fluence.

7 Conclusion

In this work, I implemented a Monte Carlo simulation of photon transport in a scattering slab. I computed the remittance, angular distributions, and depth-resolved fluence for benchmark configurations. The results I obtained are physically consistent and agree with analytical expectations and reference Monte Carlo results. The simulations illustrate the transition from ballistic to diffusive photon transport and the influence of refractive index on photon confinement.