Homework 4: Miniproject - learning nonlinear hyperbolic wave equations

ENM5320 Spring 2025, UPenn

Due date: Mar. 17^{th} by midnight (11:59pm)

This is a **group assignment**. You should collaborate with your peers to assemble a team that fills gaps in your own background. Broadly, you will need a good coder, someone familiar with ML, and someone who can wrestle with math.

This assignment is the culmination of what we've learned in the finite difference section of the course. It is intentionally open-ended - it is up to you to decide how you want to approach this. Broadly, the less of what we've learn so far you employ, the more you'll have to suffer through hyperparameter tuning to get something that works. The group who performs best on the final question will receive +2pts on their final grade, and the respect and admiration of their peers. If this stresses you out don't worry - completing Q1 is sufficient to get complete credit.

Instructions:

1. **Q1. Recover a discrete model for the linear wave equation.** Generate a dataset consisting of several solutions to the linear wave equation, using the exact solution provided in class. Demonstrate convincingly that your model can generalize to unseen initial conditions and make predictions at times longer than those used in the training dataset.

   Remark: Treat this like a peer-reviewed paper; when you submit to a conference or journal it is up to you to clearly articulate your approach and your result, and I'm anticipating that you can similarly articulate what you've done in a page or so of text and figures (you can take more if you need to).

2. **Q2. Learn a mystery model.** Having validated that your approach works for the simple linear system in Q1, run it now on the data provided in the repo: the jupyter notebook (`https://github.com/natrask/ENM5320/blob/main/Assignments/Dataset_HW4/mysteryDatasetLoader.ipynb`) will load the accompanying pickle file (`https://github.com/natrask/ENM5320/blob/main/Assignments/Dataset_HW4/solutions.pkl`). To visualize, a solution for the initial condition $f(x) = \sin 2\pi x/L$ is given below. Using this trained model, make a prediction over the same space and time window using the initial condition $f(x) = \cos \frac{3}{2}\pi x/L$ and submit your solution at the final time as a

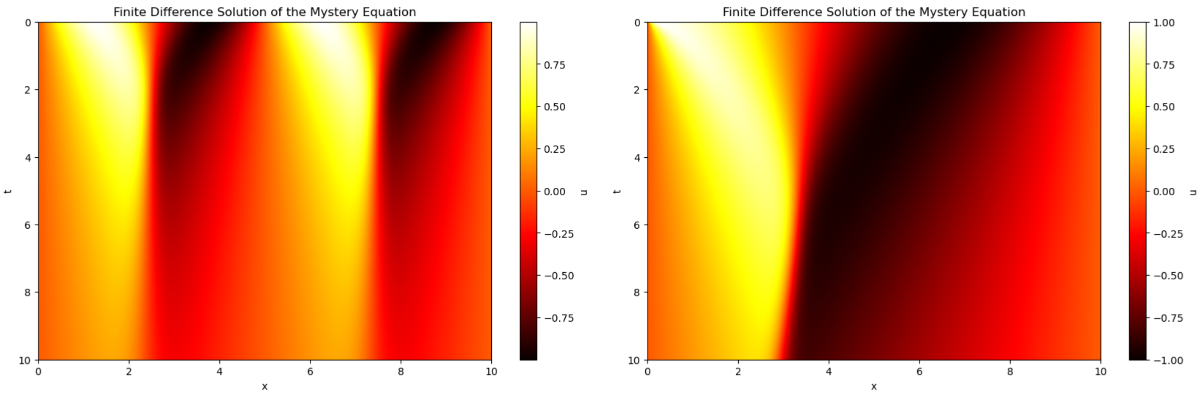

Figure 1: Solution to nonlinear mystery PDE in space and time. Left - initial condition. Right - solution for held out dataset.

numpy array in canvas - we will compare who is closest to the true model to assess the winner of the challenge. We have also provided code below as well as an image of the final solution so you can tell if you're in the ballpark or not.

The data is generated on a grid specified as follows:

```
L = 10.0  # Length of the domain
T = 5.0  # Total time
Nx = 400  # Number of spatial points
Nt = 2000  # Number of time points
dx = L / (Nx - 1)  # Spatial step size
dt = T / Nt  # Time step size
```