Simple Noether thm Pf

- Consider particle w/ position q, velocity $\dot{q}$, Lagrangian $L(q,\dot{q})$ ← no time!
- Define momentum $p = \partial_{\dot{q}} L$

  Force $F = \partial_q L$

- Euler Lagrange $\frac{d}{dt}\partial_{\dot{q}} L = \partial_q L$

  or $\frac{d}{dt} p = F$

- Assume now $q = q(s)$ → a parameter that shifts q (e.g. changing the coordinates)
- Lagrangian is shift invariant if

$$\frac{d}{ds} L(q(s), \dot{q}(s)) = 0$$

Then, if $C = p \frac{d}{ds} q(s)$

$$\dot{C} = 0$$

Pf $\dot{C} = \frac{dp}{dt}\frac{dq}{ds} + p \frac{d}{dt}\frac{d}{ds} q$ (product rule)

$= \frac{\partial L}{\partial q}\frac{dq}{ds} + \frac{\partial L}{\partial \dot{q}}\frac{d^2 q}{dt\,ds}$ (def of p, $\frac{dp}{dt}$ from E-L)

$= \frac{d}{ds} L = 0$