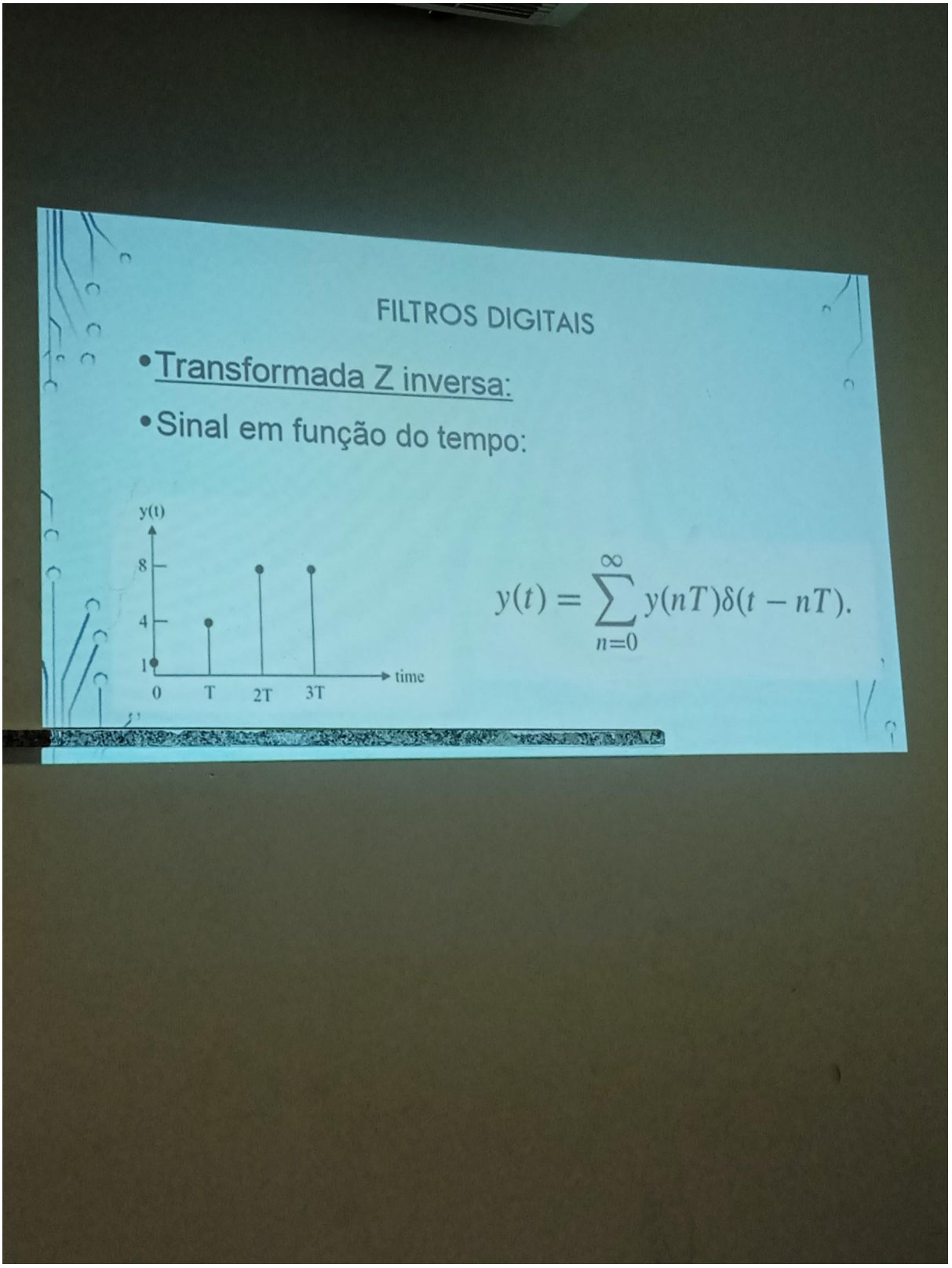
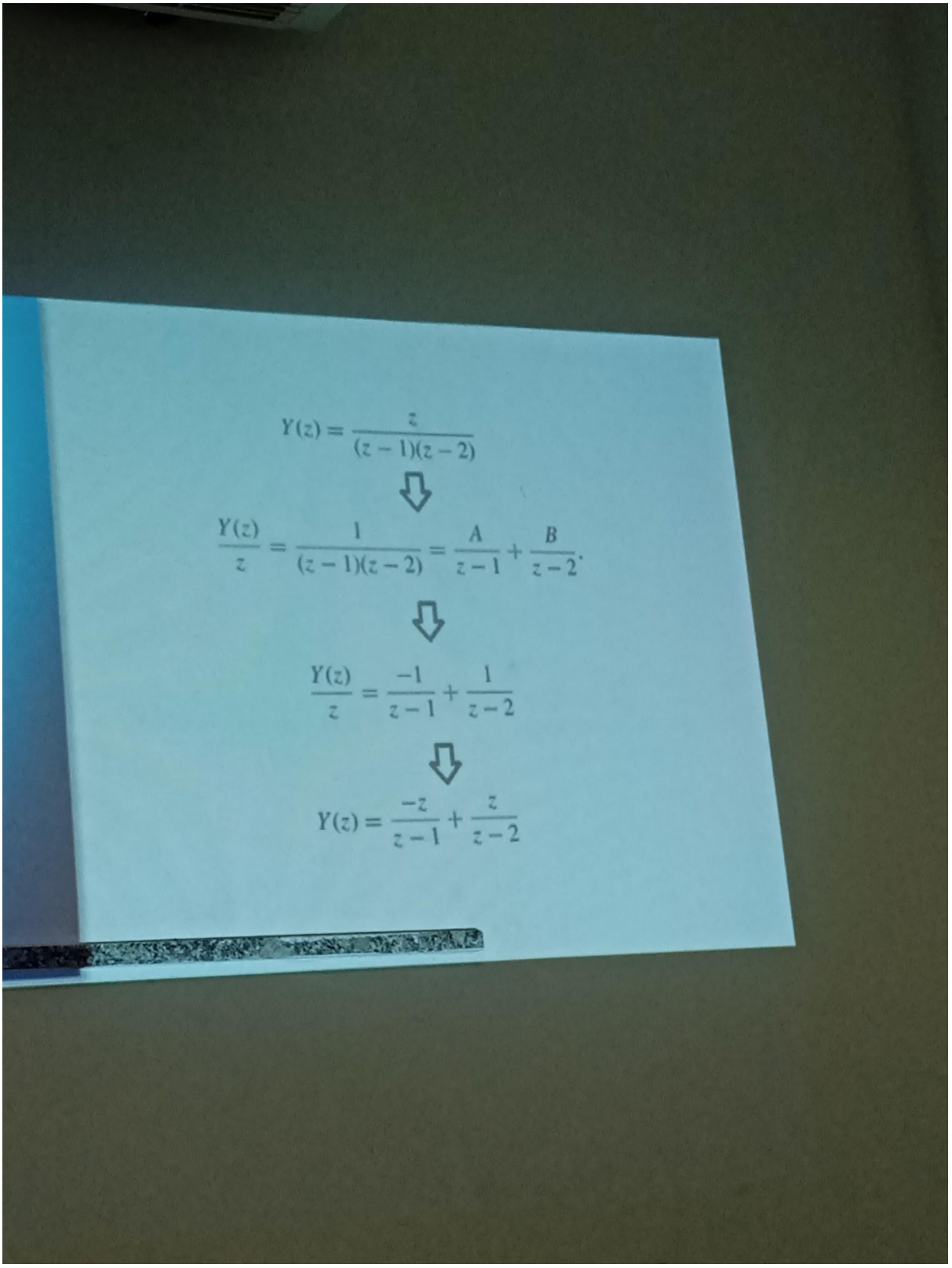
Aula 06-09 finalizando Transformada Z

Transformada Z inversa

- Utilizada para converter uma função no domínio Z para o domínio do tempo
- A técnica mais simples é convertendo a função,no domínio Z, para a soma de frações parciais

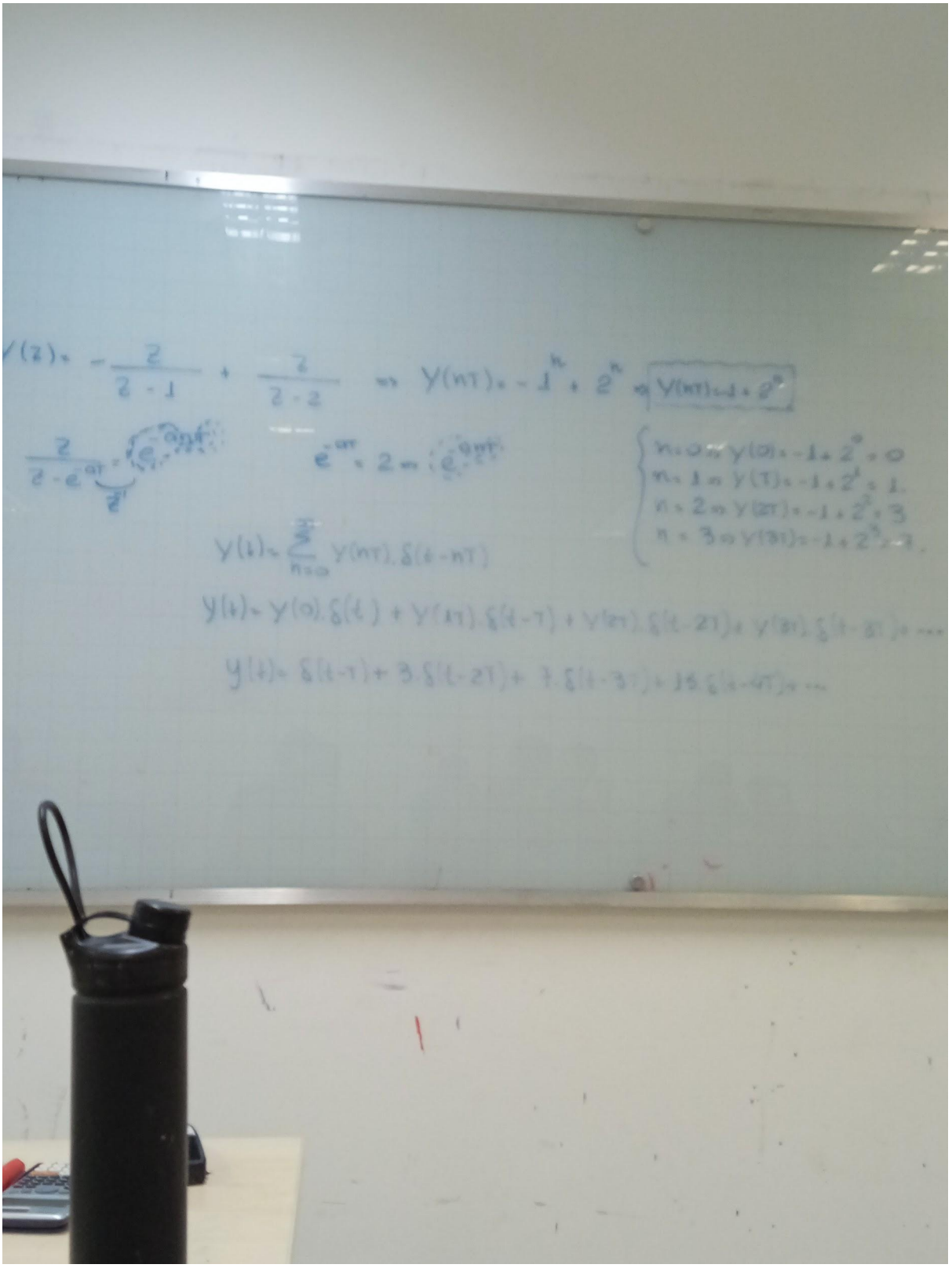


Passos para realizar a transformada Z inversa

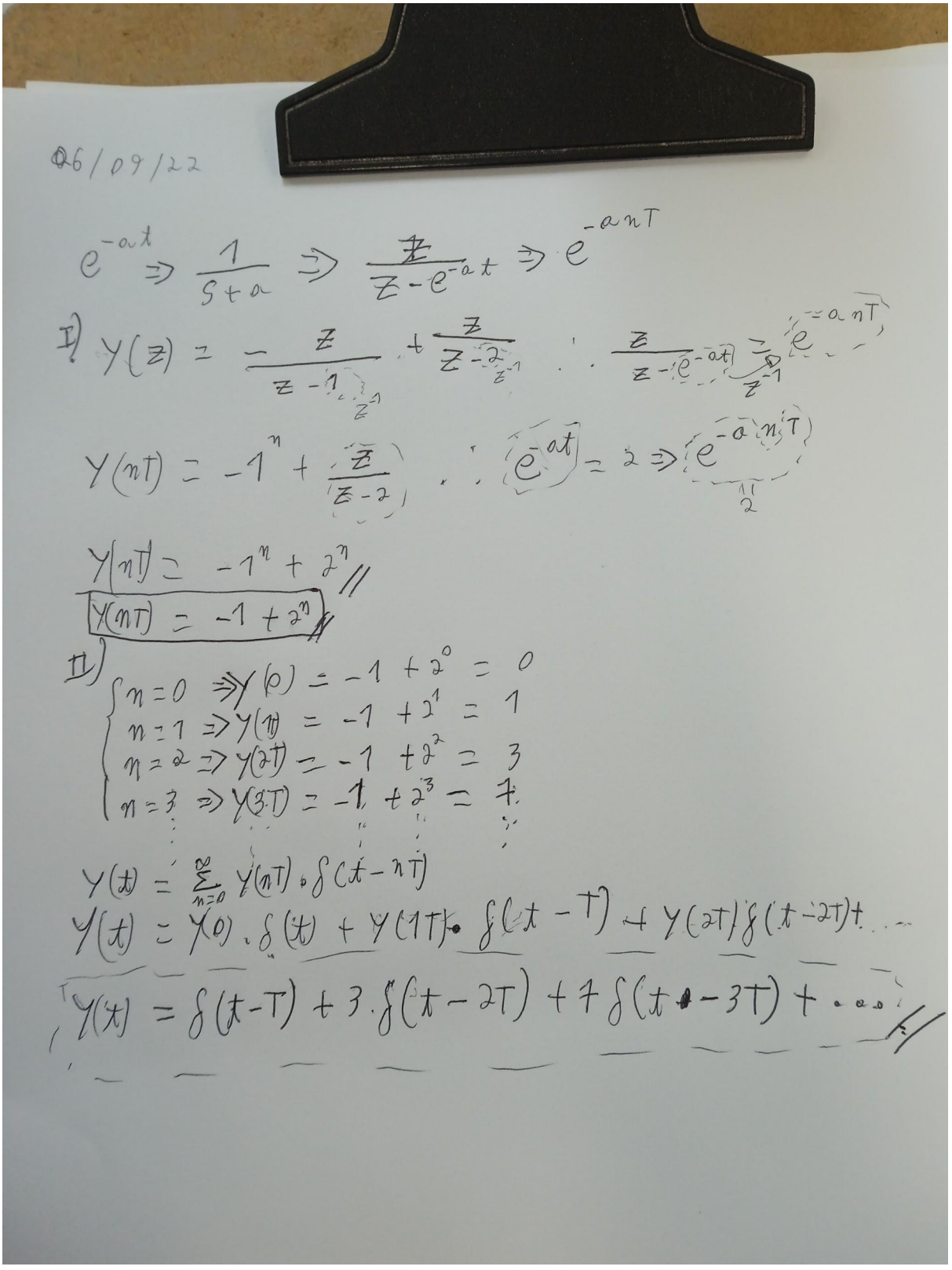


Pegando o resultado…

Anotação do professor

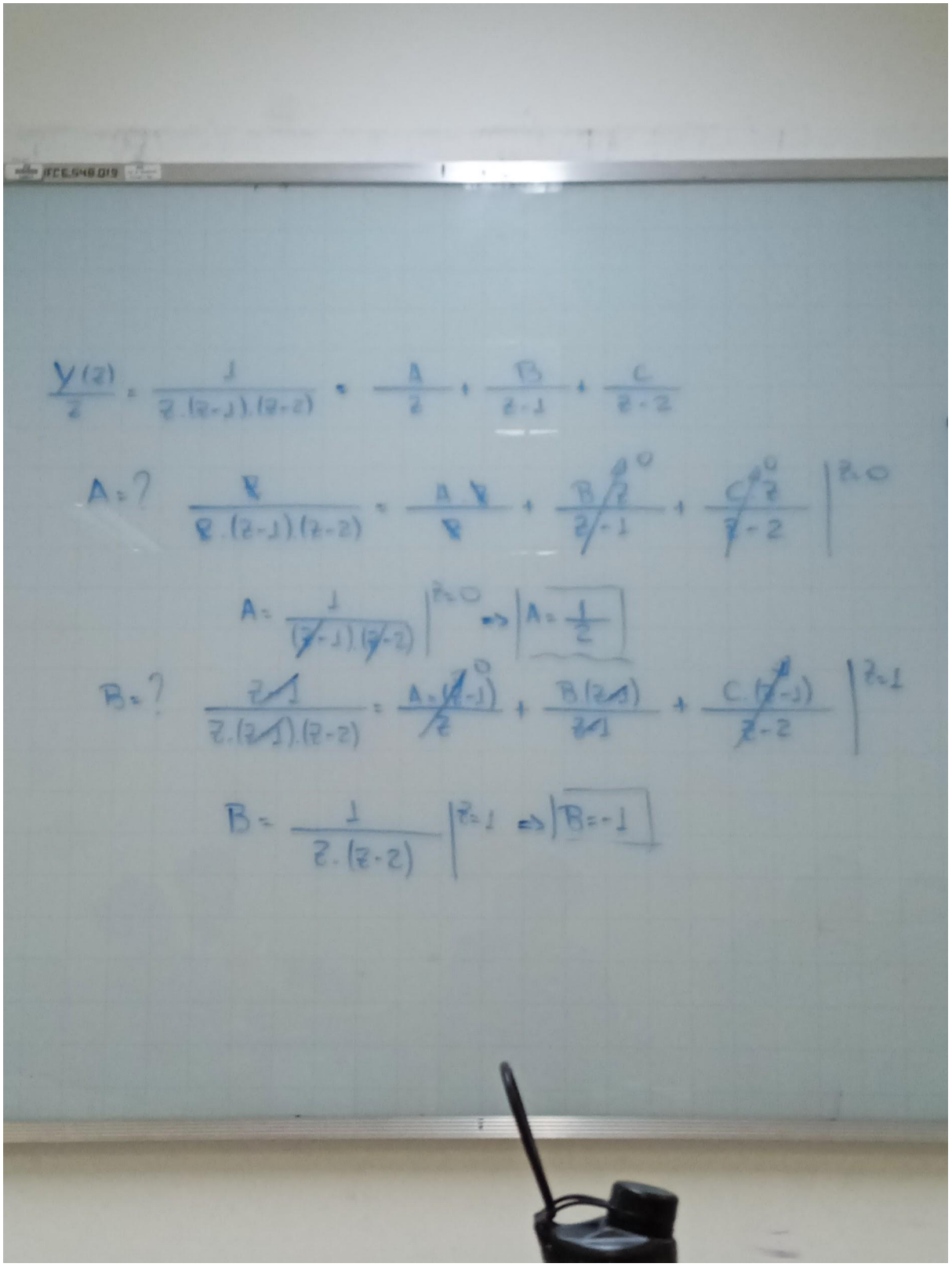


Anotação do Cristiano



Pegando outro exemplo

Anotação do professor


$\frac{Y(z)}{z} = \frac{1}{z.(z-1).(z-2)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-2}$
$A = ?$ $\frac{1}{(z-1).(z-2)} = A + \frac{Bz}{z-1} + \frac{Cz}{z-2} \Big|_{z=0}$
$A = \frac{1}{(z-1).(z-2)} \Big|_{z=0} \Rightarrow A = \frac{1}{2}$
$B = ?$ $\frac{1}{z.(z-2)} = \frac{A.(z-1)}{z} + B + \frac{C.(z-1)}{z-2} \Big|_{z=1}$
$B = \frac{1}{z.(z-2)} \Big|_{z=1} \Rightarrow B = -1$

$C = ? \Rightarrow \quad \frac{z}{z\cdot(z-1)\cdot(z-2)} = \frac{A\cdot(z-2)}{z} + \frac{B\cdot(z-2)}{z-1} + \frac{C\cdot(z-2)}{(z-2)}\Big|_{z=2}$

$$C = \frac{1}{z\cdot(z-1)}\Big|^{z=2} \Rightarrow \boxed{C = \frac{1}{2\cdot(1)} = \frac{1}{2}}$$

$$\frac{Y(z)}{z} = \frac{1/2}{z} - \frac{1}{z-1} + \frac{1/2}{z-2}$$

$$Y(z) = \frac{1/2\cdot z}{z} - \frac{z}{z-1} + \frac{1}{2}\cdot\frac{z}{z-2}$$

$$Y(z) = \frac{1}{2} - \frac{z}{z-1} + \frac{1}{2}\cdot\frac{z}{z-2}$$

$$\frac{C\cdot(z-z)}{(z-z)}\Big|_{z=z}$$

$$e^{-at} \Rightarrow \frac{1}{s+a} \Rightarrow \frac{z}{z-e^{-aT}} \Rightarrow e^{-anT} = 1^n$$

$$\frac{z}{z-1} \Rightarrow 1 = e^{-aT}$$

$$= \frac{1}{z}$$

$$\frac{Y}{X} \quad e^{-aT}$$

$$Y(nT) = a - 1^n + \frac{z^n}{z}$$

$$Y(nT) = a - 1^n + z^{n-1} \Rightarrow Y(nT) = a - 1 + z^{n-1}.$$

$$n=0 \Rightarrow Y(0) = \frac{1}{2} - 1 + z^{-1} \Rightarrow Y(0) = \frac{1}{2} - 1 + \frac{1}{2}$$

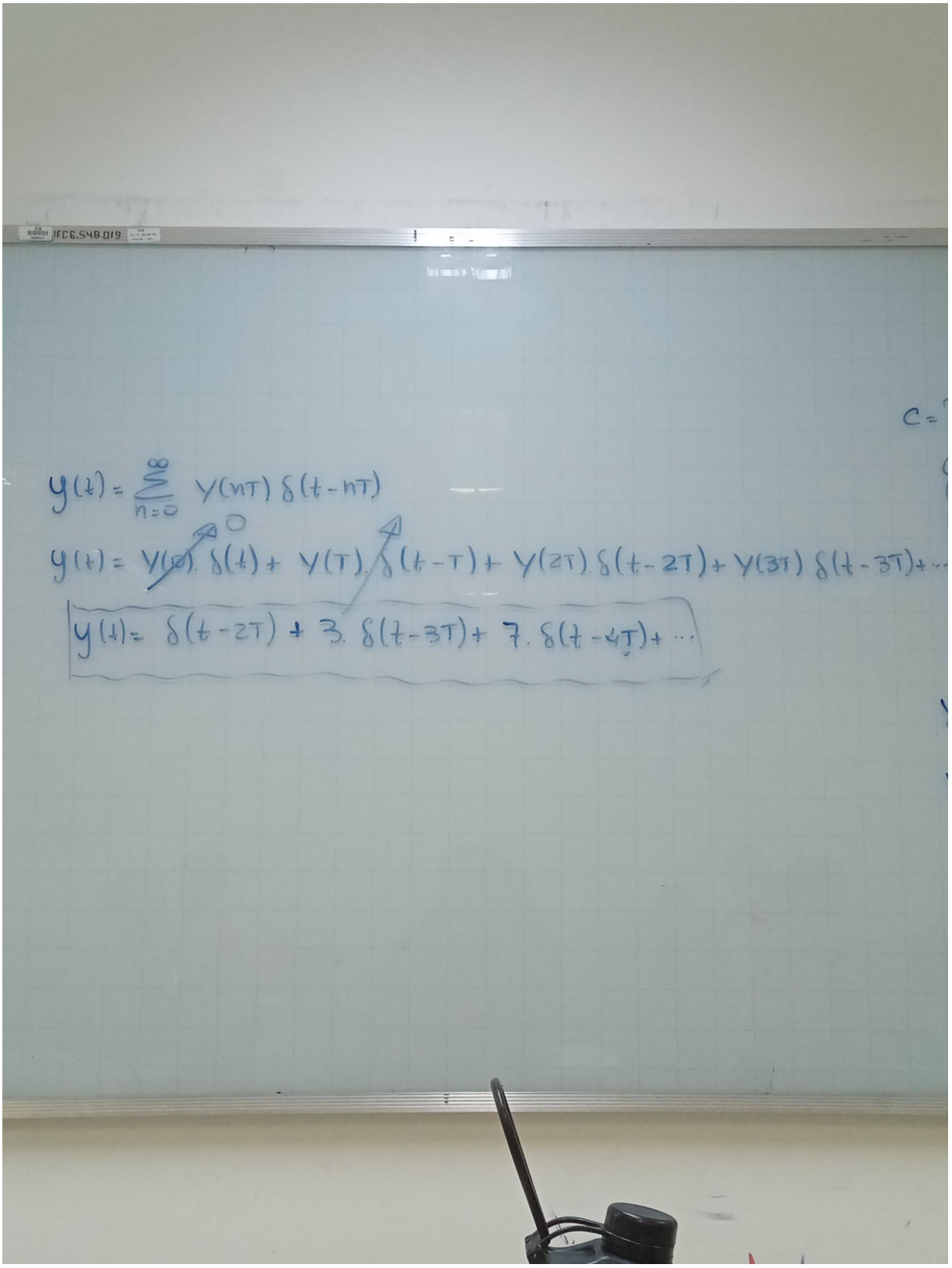
$$Y(0) = 0;$$

$$Y(T) = -1 + z^0 \Rightarrow \boxed{Y(T) = 0}$$

$$Y(2T) = -1 + z^1 \Rightarrow \boxed{Y(2T) = 1}$$

$$Y(3T) = -1 + z^2 \Rightarrow \boxed{Y(3T) = 3}$$

$$Y(4T) = -1 + z^3 \Rightarrow \boxed{Y(4T) = 7}$$



Anotação do Cristiano

26/09/22

$e^{-at} \Rightarrow \frac{1}{s+a} \Rightarrow \frac{z}{z-e^{-aT}} \Rightarrow e^{-anT}$

I) $Y(z) = \frac{z}{z-1} + \frac{z}{z-2}$ ∴ $\frac{z}{z-e^{-aT}} \overset{z^{-1}}{=} e^{-anT}$

$Y(nT) = -1^n + \frac{z}{(z-2)}$ ∴ $e^{-at} = 2 \Rightarrow e^{-a(nT)}$, $\frac{1}{2}$

$Y(nT) = -1^n + 2^n$ //

$\boxed{Y(nT) = -1 + 2^n}$ //

II)

$\begin{cases} n=0 \Rightarrow Y(0) = -1 + 2^0 = 0 \\ n=1 \Rightarrow Y(1) = -1 + 2^1 = 1 \\ n=2 \Rightarrow Y(2T) = -1 + 2^2 = 3 \\ n=3 \Rightarrow Y(3T) = -1 + 2^3 = 7 \\ \vdots \end{cases}$

$Y(t) = \sum_{n=0}^{\infty} Y(nT) \cdot \delta(t-nT)$

$Y(t) = Y(0) \cdot \delta(t) + Y(1T) \cdot \delta(t-T) + Y(2T)\delta(t-2T) + \dots$

$Y(t) = \delta(t-T) + 3 \cdot \delta(t-2T) + 7\delta(t-3T) + \dots$ //

Ex2)

$\frac{Y(z)}{z} = \frac{1}{z \cdot (z-1) \cdot (z-2)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-2}$

I) A = ? e x(z)

$\frac{z}{z \cdot (z-1) \cdot (z-2)} = \frac{A \cdot z}{z} + \frac{B \cdot z}{z-1} + \frac{C \cdot z}{z-2}$ | $z=0$

$A = \frac{1}{(z-1) \cdot (z-2)} \Rightarrow \boxed{A = \frac{1}{2}}$

II) $B = ?$ e $\times(z-1)$

$$\frac{z-1}{z\cdot(z-1)\cdot(z-2)} = \frac{A\cdot(z-1)}{z} + \frac{B\cdot(z-1)}{(z-1)} + \frac{C(z-1)}{z-2}\Bigg|_{z=1}$$

$$B = \frac{1}{z\cdot(z-2)}\Bigg|_{z=1} \Rightarrow \boxed{B=-1}$$

III) $C = ?$ e $\times(z-2)$

$$\frac{z-2}{z\cdot(z-1)\cdot(z-2)} = \frac{A\cdot(z-2)}{z} + \frac{B\cdot(z-2)}{(z-1)} + \frac{C(z-2)}{(z-2)}\Bigg|_{z=2}$$

$$C = \frac{1}{z\cdot(z-1)}\Bigg|_{z=2} \Rightarrow \boxed{C=\frac{1}{2}}$$

IV) Representando as funções de transferência por frações parciais

* Termo independente.

$$\frac{Y(z)}{z} = \frac{1/2}{z} - \frac{1}{z-1} + \frac{1/2}{z-2}$$

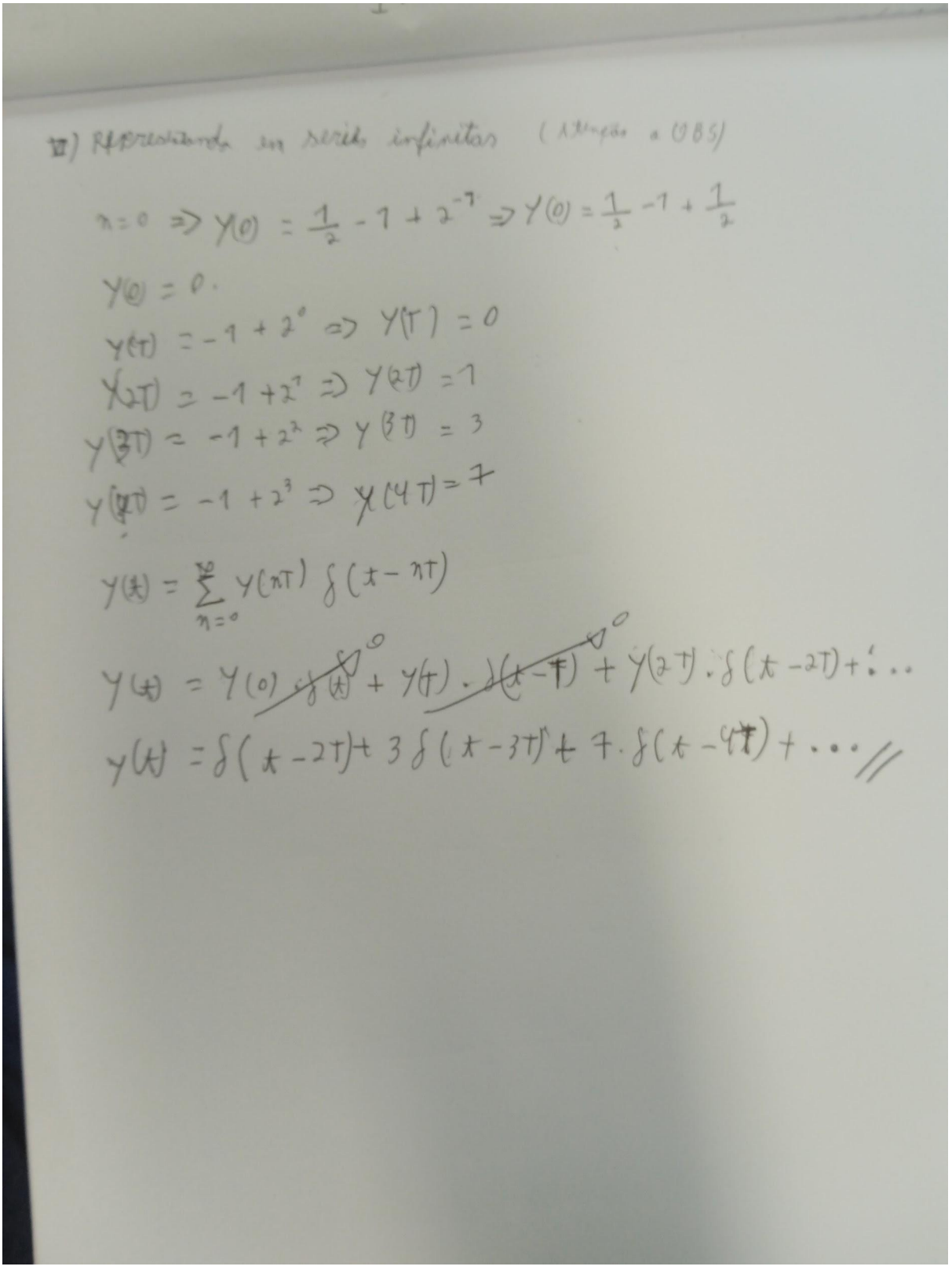
$$Y(z) = \frac{1/2\cdot z}{z} - \frac{z}{z-1} + \frac{1}{2}\cdot\frac{z}{z-2}$$

$$Y(z) = \boxed{\frac{1}{2}} - \frac{z}{z-1} + \frac{1}{2}\cdot\frac{z}{z-2}$$

V) Representando em tempo discreto, para n amostras

$$y(nT) = a - 1^n + \frac{2^n}{2}$$

$$y(nT) = a - 1^n + 2^{n-1} \Rightarrow \boxed{y(nT) = a - 1 + 2^{n-1}}$$



Obs:
Termo independente: só influenciará para quando Y(0) ou seja Z= 0, mas para os outros termos não precisará ser considerado por ser considerado um "Ponto de Partida". Para Y(T), Y(2T) em diante, ele será desconsiderado para os cálculos.

Cobrar o professor do exemplo prático desse caso no Matlab quinta

Ex2:

Anotação do professor

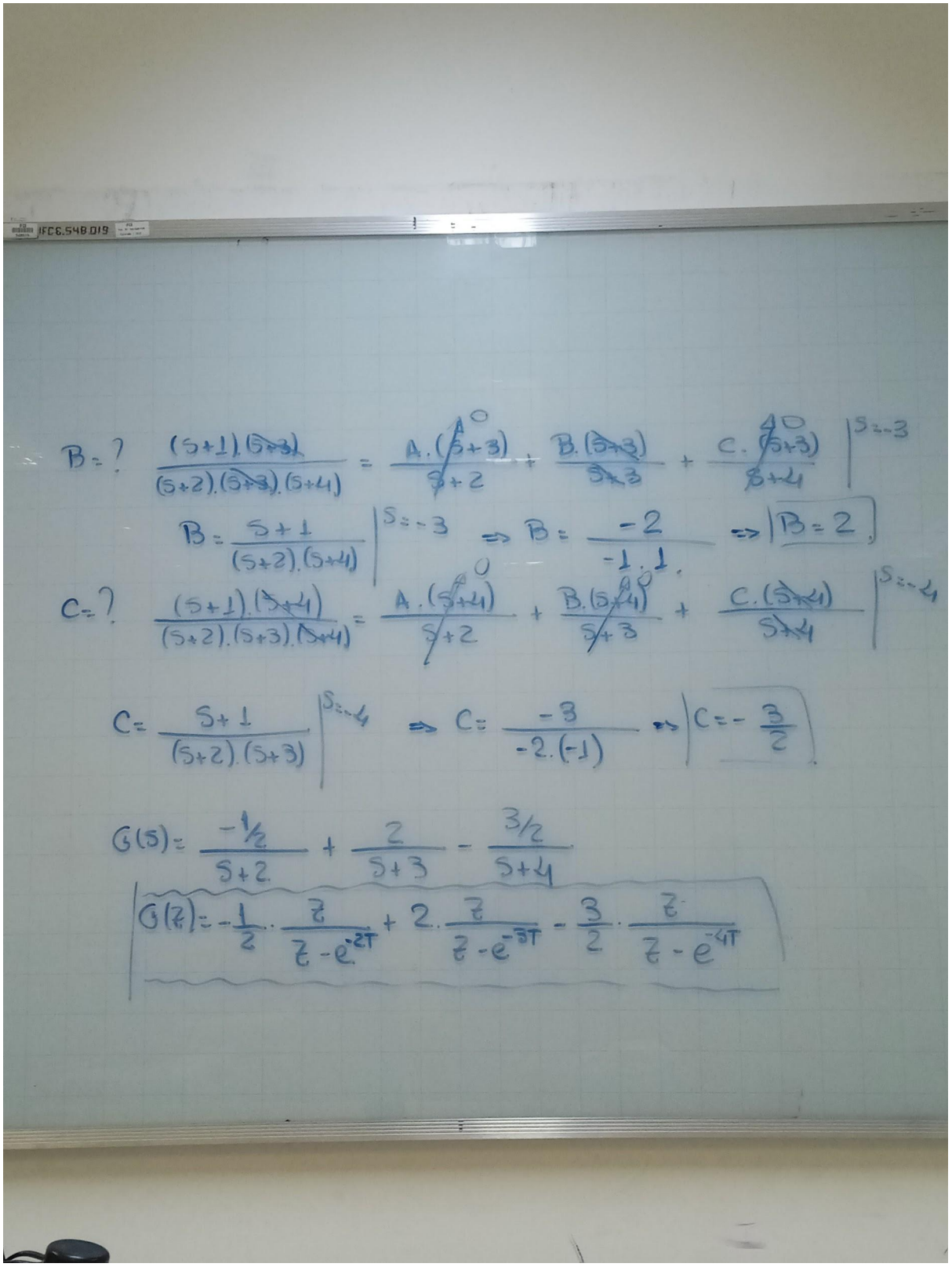
$\frac{1}{s+a} \Rightarrow \frac{z}{z-e^{-aT}}$

EXERCÍCIO: UTILIZE A TRANSFORMADA Z PARA REPRESENTAR A SEGUINTE FT NO DOMÍNIO Z.

$$G(s) = \frac{s+1}{(s+2).(s+3).(s+4)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s+4}$$

$$\frac{(s+1).(s+2)}{(s+2).(s+3).(s+4)} = \frac{A.(s+2)}{s+2} + \frac{B(s+2)}{s+3} + \frac{C.(s+2)}{s+4} \Big|_{s=-2}$$

$$A = \frac{s+1}{(s+3).(s+4)}\Big|_{s=-2} \Rightarrow A = \frac{-1}{1.2} \Rightarrow \boxed{A = -\frac{1}{2}}$$



Anotação do Cristiano

Ex:

$$G(s) = \frac{s+1}{(s+2)\cdot(s+3)\cdot(s+4)} = \frac{A}{(s+2)} + \frac{B}{(s+3)} + \frac{C}{(s+4)}$$

I) $A = ?$ e $\times(s+2)$

$$\frac{(s+1)\cdot(s+2)}{(s+2)\cdot(s+3)\cdot(s+4)} = \frac{A(s+2)}{(s+2)} + \frac{B(s+2)}{(s+3)} + \frac{C(s+2)}{(s+4)} \Big|_{s=-2}$$

$$A = \frac{(s+1)}{(s+3)\cdot(s+4)}\Big|_{s=-2} \Rightarrow A = \frac{-1}{1\cdot 2} \Rightarrow \boxed{A = -\frac{1}{2}}$$

II) $B = ?$ e $\times(s+3)$

$$\frac{(s+1)\cdot(s+3)}{(s+2)\cdot(s+3)\cdot(s+4)} = \frac{A\cdot(s+3)}{(s+2)} + \frac{B\cdot(s+3)}{(s+3)} + \frac{C(s+3)}{(s+4)} \Big|_{s=-3}$$

$$B = \frac{(s+1)}{(s+2)(s+4)}\Big|_{s=-3} \qquad B = \frac{-2}{-1\cdot 1} \Rightarrow \boxed{B = 2}$$

III) $C = ?$ e $\times(s+4)$

$$\frac{(s+1)\cdot(s+4)}{(s+2)\cdot(s+3)\cdot(s+4)} = \frac{A\cdot(s+4)}{s+2} + \frac{B\cdot(s+4)}{(s+3)} + \frac{C\cdot(s+4)}{(s+4)} \Big|_{s=-4}$$

$$C = \frac{(s+1)}{(s+2)\cdot(s+3)}\Big|_{s=-4} \quad C = \frac{-3}{(-2)\cdot(-1)} \Rightarrow \boxed{C = \frac{3}{2}}$$

IV)

IV)

$$G(s) = \frac{-\frac{1}{2}}{(s+2)} + \frac{2}{(s+3)} + \frac{-\frac{3}{2}}{(s+4)}$$

$$G(s) = -\frac{1}{2} \cdot \frac{1}{(s+2)} + \frac{2}{(s+3)} + -\frac{3}{2} \cdot \frac{1}{(s+4)}$$

$$G(t) = -\frac{1}{2} \cdot \frac{z}{z-e^{-2T}} + 2 \cdot \frac{z}{z-e^{-3T}} - \frac{3}{2} \cdot \frac{z}{z-e^{-4T}}$$