

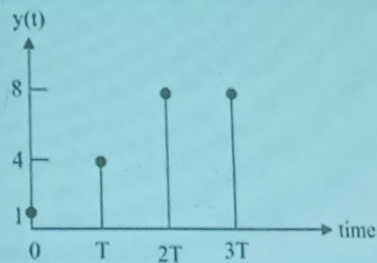
## Aula 06-09 finalizando Transformada Z

### Transformada Z inversa

- Utilizada para converter uma função no domínio Z para o domínio do tempo
- A técnica mais simples é convertendo a função, no domínio Z, para a soma de frações parciais

## FILTROS DIGITAIS

- Transformada Z inversa:
- Sinal em função do tempo:



$$y(t) = \sum_{n=0}^{\infty} y(nT)\delta(t - nT).$$

Passos para realizar a transformada Z inversa

$$\begin{aligned} Y(z) &= \frac{z}{(z-1)(z-2)} \\ \Downarrow \\ \frac{Y(z)}{z} &= \frac{1}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2} \\ \Downarrow \\ \frac{Y(z)}{z} &= \frac{-1}{z-1} + \frac{1}{z-2} \\ \Downarrow \\ Y(z) &= \frac{-z}{z-1} + \frac{z}{z-2} \end{aligned}$$

Pegando o resultado...

Anotação do professor

$$Y(z) = -\frac{z}{z-1} + \frac{z}{z-2} \Rightarrow Y(nT) = -1 + 2^n \Rightarrow Y(nT) = 2^n - 1$$

$$\frac{z}{z-e^{at}} = \sum_{n=0}^{\infty} e^{anT} z^{-n-1} \Rightarrow e^{at} = 2 = e^{0.693nT}$$

$$Y(t) = \sum_{n=0}^{\infty} Y(nT) \delta(t-nT)$$

$$Y(t) = Y(0) \delta(t) + Y(1T) \delta(t-T) + Y(2T) \delta(t-2T) + Y(3T) \delta(t-3T) + \dots$$

$$Y(t) = \delta(t) + 3 \delta(t-T) + 7 \delta(t-2T) + 15 \delta(t-3T) + \dots$$

$$\begin{cases} n=0 \Rightarrow Y(0) = -1 + 2^0 = 0 \\ n=1 \Rightarrow Y(1T) = -1 + 2^1 = 1 \\ n=2 \Rightarrow Y(2T) = -1 + 2^2 = 3 \\ n=3 \Rightarrow Y(3T) = -1 + 2^3 = 7 \end{cases}$$

Anotação do Cristiano



06/09/22

$$e^{-at} \Rightarrow \frac{1}{s+a} \Rightarrow \frac{z}{z-e^{-aT}} \Rightarrow e^{-anT}$$

$$I) Y(z) = \frac{z}{z-1} + \frac{z}{z-2} \cdot \frac{z}{z-e^{-aT}} = \frac{z}{z-1} + \frac{z}{z-2} \cdot \frac{z}{z-1} = \frac{z}{z-1} + \frac{z}{z-2}$$

$$Y(nT) = -1^n + \frac{z}{z-2} \cdot e^{at} = 2 \Rightarrow e^{-a(nT)} = 2$$

$$Y(nT) = -1^n + 2^n //$$

$$\boxed{Y(nT) = -1 + 2^n}$$

$$II) \begin{cases} n=0 \Rightarrow Y(0) = -1 + 2^0 = 0 \\ n=1 \Rightarrow Y(1T) = -1 + 2^1 = 1 \\ n=2 \Rightarrow Y(2T) = -1 + 2^2 = 3 \\ n=3 \Rightarrow Y(3T) = -1 + 2^3 = 7 \end{cases}$$

$$Y(x) = \sum_{n=0}^{\infty} Y(nT) \cdot \delta(x-nT)$$

$$Y(x) = Y(0) \cdot \delta(x) + Y(1T) \cdot \delta(x-T) + Y(2T) \cdot \delta(x-2T) + \dots$$

$$Y(x) = \delta(x-T) + 3 \cdot \delta(x-2T) + 7 \cdot \delta(x-3T) + \dots //$$

Pegando outro exemplo

Anotação do professor

$$\frac{Y(z)}{z} = \frac{1}{z(z-1)(z-2)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-2}$$

$$A = ? \quad \frac{1}{z(z-1)(z-2)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-2} \quad | z=0$$

$$A = \frac{1}{(\cancel{z-1})(\cancel{z-2})} \Big|_{z=0} \Rightarrow \boxed{A = -\frac{1}{2}}$$

$$B = ? \quad \frac{1}{z(z-1)(z-2)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-2} \quad | z=1$$

$$B = \frac{1}{z(z-2)} \Big|_{z=1} \Rightarrow \boxed{B = -1}$$

$$C = ? \Rightarrow \frac{\cancel{z-2}}{z(z-1)(\cancel{z-2})} = \frac{A \cdot \overset{00}{\cancel{z-2}}}{\cancel{z}} + \frac{B \cdot \overset{00}{\cancel{z-2}}}{\cancel{z-1}} + \frac{C \cdot (\cancel{z-2})}{(z-2)} \Big|_{z=2}$$

$$C = \frac{1}{z(z-1)} \Big|_{z=2} \Rightarrow C = \frac{1}{2(1)} = \frac{1}{2}$$

$$\left. \begin{aligned} \frac{Y(z)}{z} &= \frac{1/2}{z} - \frac{1}{z-1} + \frac{1/2}{z-2} \\ Y(z) &= \frac{1/2 \cdot \cancel{z}}{\cancel{z}} - \frac{\cancel{z}}{z-1} + \frac{1}{2} \cdot \frac{\cancel{z}}{z-2} \\ Y(z) &= \frac{1}{2} - \frac{z}{z-1} + \frac{1}{2} \cdot \frac{z}{z-2} \end{aligned} \right\} \begin{array}{l} Y(n) \\ Y(z) \end{array}$$

$$\frac{C \cdot (z-2)}{(z-2)} \Big|_{z=2} \quad e^{-at} \Rightarrow \frac{1}{s+a} \Rightarrow \frac{z}{z-e^{-aT}} \Rightarrow \frac{z}{z-1} \Rightarrow 1 = e^{-aT}$$

$$\frac{z}{z-1} \Rightarrow 1 = e^{-aT}$$

$$\frac{1}{s} \Rightarrow \frac{z}{z-1}$$

$$\begin{cases} Y(nT) = a - 1 + \frac{2^n}{2} \\ Y(nT) = a - 1 + 2^{n-1} \Rightarrow Y(nT) = a - 1 + 2^{n-1} \end{cases}$$

$$n=0 \Rightarrow Y(0) = \frac{1}{2} - 1 + 2^{-1} \Rightarrow Y(0) = \frac{1}{2} - 1 + \frac{1}{2}$$

$$Y(0) = 0;$$

$$Y(T) = -1 + 2^0 \Rightarrow Y(T) = 0$$

$$Y(2T) = -1 + 2^1 \Rightarrow Y(2T) = 1$$

$$Y(3T) = -1 + 2^2 \Rightarrow Y(3T) = 3$$

$$Y(4T) = -1 + 2^3 \Rightarrow Y(4T) = 7$$



$$y(t) = \sum_{n=0}^{\infty} y(nT) \delta(t-nT)$$

$$y(t) = \cancel{y(0)} \cdot \delta(t) + \cancel{y(T)} \cdot \delta(t-T) + y(2T) \cdot \delta(t-2T) + y(3T) \cdot \delta(t-3T) + \dots$$

$$y(t) = \delta(t-2T) + 3 \cdot \delta(t-3T) + 7 \cdot \delta(t-4T) + \dots$$

Anotação do Cristiano

06/09/22

$$e^{-at} \Rightarrow \frac{1}{s+a} \Rightarrow \frac{z}{z-e^{-aT}} \Rightarrow e^{-anT}$$

$$I) Y(z) = \frac{z}{z-1} + \frac{z}{z-2} \cdot \frac{z}{z-e^{-aT}} \Rightarrow e^{-anT}$$

$$Y(nT) = -1^n + \frac{z}{z-2} \quad \because (e^{-aT}) = 2 \Rightarrow e^{-anT}$$

$$Y(nT) = -1^n + 2^n //$$

$$X(nT) = -1 + 2^n //$$

$$II) \begin{cases} n=0 \Rightarrow Y(0) = -1 + 2^0 = 0 \\ n=1 \Rightarrow Y(1) = -1 + 2^1 = 1 \\ n=2 \Rightarrow Y(2) = -1 + 2^2 = 3 \\ n=3 \Rightarrow Y(3) = -1 + 2^3 = 7 \end{cases}$$

$$Y(x) = \sum_{n=0}^{\infty} Y(nT) \cdot \delta(x-nT)$$

$$Y(x) = Y(0) \cdot \delta(x) + Y(1T) \cdot \delta(x-T) + Y(2T) \cdot \delta(x-2T) + \dots$$

$$Y(x) = \delta(x-T) + 3 \cdot \delta(x-2T) + 7 \cdot \delta(x-3T) + \dots //$$

$$EX2) \frac{Y(z)}{z} = \frac{1}{z \cdot (z-1) \cdot (z-2)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-2}$$

$$I) A=? \quad x(z)$$

$$\frac{z}{z \cdot (z-1) \cdot (z-2)} = \frac{A \cdot z}{z} + \frac{B \cdot z}{z-1} + \frac{C \cdot z}{z-2} \Big|_{z=0}$$

$$A = \frac{1}{(z-1) \cdot (z-2)} \Rightarrow \boxed{A = \frac{1}{2}}$$

II)  $B = ?$  e  $x(z-1)$

$$\frac{z-1}{z \cdot (z-1) \cdot (z-2)} = \frac{A \cdot \cancel{(z-1)}}{\cancel{z}} + \frac{B \cdot \cancel{(z-1)}}{\cancel{(z-1)}} + \frac{C \cdot \cancel{(z-1)}}{z-2} \Big|_{z=1}$$

$$B = \frac{1}{z \cdot (z-2)} \Big|_{z=1} \rightarrow \boxed{B = -1}$$

III)  $C = ?$  e  $x(z-2)$

$$\frac{z-1}{z \cdot (z-1) \cdot (z-2)} = \frac{A \cdot \cancel{(z-1)}}{\cancel{z}} + \frac{B \cdot \cancel{(z-2)}}{\cancel{(z-1)}} + \frac{C \cdot \cancel{(z-1)}}{\cancel{(z-2)}} \Big|_{z=2}$$

$$C = \frac{1}{z \cdot (z-1)} \Big|_{z=2} \Rightarrow \boxed{C = \frac{1}{2}}$$

IV) Representando as funções de transferência por frações parciais

\* Termos independentes

$$\frac{Y(z)}{z} = \frac{1/2}{z} - \frac{1}{z-1} + \frac{1/2}{z-2}$$

$$Y(z) = \frac{1/2 \cdot z}{z} - \frac{z}{z-1} + \frac{1}{2} \cdot \frac{z}{z-2}$$

$$Y(z) = \frac{1}{2} - \frac{z}{z-1} + \frac{1}{2} \cdot \frac{z}{z-2}$$

V) Representando em forma discreta, para  $n$  amostras

$$Y(nT) = a - 1^n + \frac{2^n}{2}$$

$$Y(nT) = a - 1^n + 2^{n-1} \Rightarrow \boxed{Y(nT) = a - 1 + 2^{n-1}}$$

iii) Representando em série infinita (Além da OBS)

$$n=0 \Rightarrow Y(0) = \frac{1}{2} - 1 + 2^{-1} \Rightarrow Y(0) = \frac{1}{2} - 1 + \frac{1}{2}$$

$$Y(0) = 0.$$

$$Y(T) = -1 + 2^0 \Rightarrow Y(T) = 0$$

$$Y(2T) = -1 + 2^1 \Rightarrow Y(2T) = 1$$

$$Y(3T) = -1 + 2^2 \Rightarrow Y(3T) = 3$$

$$Y(4T) = -1 + 2^3 \Rightarrow Y(4T) = 7$$

$$Y(x) = \sum_{n=0}^{\infty} Y(nT) \delta(x - nT)$$

$$Y(x) = Y(0) \cdot \delta(x) + Y(T) \cdot \delta(x - T) + Y(2T) \cdot \delta(x - 2T) + \dots$$

$$Y(x) = \delta(x - 2T) + 3\delta(x - 3T) + 7 \cdot \delta(x - 4T) + \dots //$$

Obs:

Termo independente: só influenciará para quando  $Y(0)$  ou seja  $Z=0$ , mas para os outros termos não precisará ser considerado por ser considerado um "Ponto de Partida". Para  $Y(T)$ ,  $Y(2T)$  em diante, ele será desconsiderado para os cálculos.



Cobrar o professor do exemplo prático desse caso no Matlab quinta

Ex2:

Anotação do professor

EXERCÍCIO: UTILIZE A TRANSFORMADA Z PARA

$$\frac{1}{s+a} \Rightarrow \frac{z}{z-e^{-at}}$$

REPRESENTAR A SEGUINTE FT NO DOMÍNIO Z.

$$G(s) = \frac{s+1}{(s+2)(s+3)(s+4)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s+4}$$

$$\frac{(s+1) \cdot (\cancel{s+2})}{(\cancel{s+2})(s+3)(s+4)} = \frac{A \cdot (\cancel{s+2})}{s+2} + \frac{B(s+2)}{s+3} + \frac{C(s+2)}{\cancel{s+4}} \Big|_{s=-2}$$

$$A = \frac{s+1}{(s+3)(s+4)} \Big|_{s=-2} \Rightarrow A = \frac{-1}{1 \cdot 2} \Rightarrow \boxed{A = -\frac{1}{2}}$$

$$B = ? \quad \frac{(s+1)(\cancel{s+3})}{(s+2)(\cancel{s+3})(s+4)} = \frac{A(\cancel{s+3})}{s+2} + \frac{B(\cancel{s+3})}{s+3} + \frac{C(\cancel{s+3})}{s+4} \quad | s=-3$$

$$B = \frac{s+1}{(s+2)(s+4)} \quad | s=-3 \Rightarrow B = \frac{-2}{-1 \cdot 1} \Rightarrow \boxed{B=2}$$

$$C = ? \quad \frac{(s+1)(\cancel{s+4})}{(s+2)(s+3)(\cancel{s+4})} = \frac{A(\cancel{s+4})}{s+2} + \frac{B(\cancel{s+4})}{s+3} + \frac{C(\cancel{s+4})}{s+4} \quad | s=-4$$

$$C = \frac{s+1}{(s+2)(s+3)} \quad | s=-4 \Rightarrow C = \frac{-3}{-2 \cdot (-1)} \Rightarrow \boxed{C = -\frac{3}{2}}$$

$$G(s) = \frac{-\frac{1}{2}}{s+2} + \frac{2}{s+3} - \frac{3/2}{s+4}$$

$$\boxed{G(z) = -\frac{1}{2} \cdot \frac{z}{z-e^{-2T}} + 2 \cdot \frac{z}{z-e^{-3T}} - \frac{3}{2} \cdot \frac{z}{z-e^{-4T}}}$$

Ex2

$$G(s) = \frac{s+1}{(s+2) \cdot (s+3) \cdot (s+4)} = \frac{A}{(s+2)} + \frac{B}{(s+3)} + \frac{C}{(s+4)}$$

I)  $A = ? \times (s+2)$

$$\frac{(s+1) \cdot \cancel{(s+2)}}{\cancel{(s+2)} \cdot (s+3) \cdot (s+4)} = \frac{A \cdot \cancel{(s+2)}}{\cancel{(s+2)}} + \frac{B \cdot \cancel{(s+2)}}{\cancel{(s+2)} \cdot (s+3)} + \frac{C \cdot \cancel{(s+2)}}{\cancel{(s+2)} \cdot (s+4)} \quad | s = -2$$

$$A = \frac{(s+1)}{(s+3) \cdot (s+4)} \quad | s = -2 \Rightarrow A = \frac{-1}{1 \cdot 2} \Rightarrow \boxed{A = -\frac{1}{2}}$$

II)  $B = ? \times (s+3)$

$$\frac{(s+1) \cdot \cancel{(s+3)}}{(s+2) \cdot \cancel{(s+3)} \cdot (s+4)} = \frac{A \cdot \cancel{(s+3)}}{(s+2)} + \frac{B \cdot \cancel{(s+3)}}{\cancel{(s+3)}} + \frac{C \cdot \cancel{(s+3)}}{\cancel{(s+3)} \cdot (s+4)} \quad | s = -3$$

$$B = \frac{(s+1)}{(s+2) \cdot (s+4)} \quad | s = -3 \Rightarrow B = \frac{-2}{-1 \cdot 1} \Rightarrow \boxed{B = 2}$$

III)  $C = ? \times (s+4)$

$$\frac{(s+1) \cdot \cancel{(s+4)}}{(s+2) \cdot (s+3) \cdot \cancel{(s+4)}} = \frac{A \cdot \cancel{(s+4)}}{(s+2)} + \frac{B \cdot \cancel{(s+4)}}{(s+3)} + \frac{C \cdot \cancel{(s+4)}}{\cancel{(s+4)}} \quad | s = -4$$

$$C = \frac{(s+1)}{(s+2) \cdot (s+3)} \quad | s = -4 \Rightarrow C = \frac{-3}{(-2) \cdot (-1)} \Rightarrow \boxed{C = \frac{3}{2}}$$

IV)



$$\text{IV)} \quad G(s) = \frac{-\frac{1}{2}}{(s+2)} + \frac{2}{(s+3)} + \frac{-\frac{3}{2}}{(s+4)}$$

$$G(s) = \frac{-1}{2} \cdot \frac{1}{(s+2)} + \frac{2}{(s+3)} + \frac{-3}{2} \cdot \frac{1}{(s+4)}$$

$$G(t) = \frac{-1}{2} \cdot \frac{z}{z-e^{-2T}} + 2 \cdot \frac{z}{z-e^{-T}} + \frac{-3}{2} \cdot \frac{z}{z-e^{-4T}}$$