4. a) $F(z) = \dfrac{z}{z-4}$

Aplicando a fórmula: $\dfrac{z}{z-e^{aT}} \rightarrow e^{-akT}$

$\rightarrow y(nT) = 4^n$

$\rightarrow$ Achando os primeiros termos:

$Y(0) = 4^0 = 1$

$Y(1) = 4^1 = 4$

$Y(2) = 4^2 = 16$

$Y(3) = 4^3 = 64$

$\rightarrow$ Substituindo na fórmula: $\sum_{n=0}^{\infty} y(nT)\,\delta(t-nT)$

$y(t) = y(0)\,\delta(t-0T) + y(1)\,\delta(t-1T) + y(2)\,\delta(t-2T) + y(3)(t-3T)$

Substituindo:

$y(t) = \delta(t) + 4\delta(t-T) + 16(t-2T) + 64(t-3T)$

c) $K(z) = \dfrac{z}{(z-1)(z-3)}$

Dividindo tudo por z:

$\dfrac{K(z)}{z} = \dfrac{1}{z(z-1)(z-3)}$

Aplicando frações parciais

$\dfrac{K(z)}{z} = \dfrac{1}{z(z-1)(z-3)} = \dfrac{A}{z} + \dfrac{B}{z-1} + \dfrac{C}{z-3}$

Descobrindo A:

$x(z) \rightarrow \dfrac{1}{(z-1)(z-3)} = \dfrac{A \cdot z}{z} + \dfrac{B \cdot z}{(z-1)} + \dfrac{C \cdot z}{(z-3)}$

adotando $z=0$:

$\dfrac{1}{(0-1)(0-3)} = A + 0 + 0 \rightarrow A = \dfrac{1}{3}$

Descobrindo B:

$x(z-1)$

$\dfrac{1}{z(z-3)} = \dfrac{A \cdot (z-1)}{z} + \dfrac{B(z-1)}{z-1} + \dfrac{C(z-1)}{z-3}$

$z=1$:

$\dfrac{1}{1(1-3)} = 0 + B + 0 \rightarrow B = -\dfrac{1}{2}$

Descobrindo C:

$x(z-3)$

$\dfrac{1}{z(z-1)} = \dfrac{A(z-3)}{z} + \dfrac{B(z-3)}{z-1} + \dfrac{C(z-3)}{z-3}$

$z=3$: $\dfrac{1}{3(3-1)} = 0 + 0 + C \rightarrow C = \dfrac{1}{6}$

Logo: $\dfrac{K(z)}{z} = \dfrac{1}{3} \cdot \dfrac{1}{z} - \dfrac{1}{2}\dfrac{1}{z-1} + \dfrac{1}{6} \cdot \dfrac{1}{z-3}$

$$K(z) = \frac{1}{3} \cdot \cancelto{1}{\frac{z}{z}} - \frac{1}{2} \cdot \frac{z}{z-1} + \frac{1}{6} \cdot \frac{z}{z-3}$$

$$y(nT) = \underbrace{\frac{1}{3} - \frac{1}{2}}_{} \cdot \cancelto{1}{1^n} + \frac{1}{6} \cdot 3^n$$

$$y(nT) = -\frac{1}{6} + \frac{1}{6} \cdot 3^n$$

$$y(0) = -\frac{1}{6} + \frac{1}{6}\, 3^0 = 0$$

$$y(1) = \frac{1}{6} \cdot 3^1 = \frac{1}{2}$$

$$y(2) = \frac{1}{6} \cdot 3^2 = \frac{3}{2}$$

$$y(3) = \frac{1}{6}\, 3^3 = \frac{9}{2}$$

$$y(t) = \cancelto{0}{0\,\delta(t-0T)} + \frac{1}{2}\delta(t-T) + \frac{3}{2}\delta(t-2T) + \frac{9}{2}(t-3T) + \dots$$