Aula 20/09 Revisão para a prova 1 da N1
Prova 4 questões
Conteúdo:

- Revisão de filtros analógicos
- Filtros digitais
- Transformada de Laplace
- Transformada inversa de Laplace
- Transformada Z
- Transformada inversa Z

Lista de exercícios para a primeira avaliação para a N1

1. Aplique a transformada de Laplace para representar as FTs abaixo no domínio da frequência, no plano s.
   c) g(t) = 2*exp(-2*t);
   d) k(t) = 2*exp(-3*t) + 3*t;
   e) k(t) = 2*exp(-3*t) + 0.5*exp(-2*t);

   Solução

$$e^{-at} \xrightarrow[{[L]}]{} \frac{1}{s+a} \xrightarrow[{[Z]}]{} \frac{z}{z-e^{-aT}} \xrightarrow[{[Z^{-1}]}]{} e^{-anT}$$

01ª QUESTÃO:

APLICAR A TRANSFORMADA DE LAPLACE:

c) $g(t) = 2.e^{-2t}$

$$G(s) = 2 \times \frac{1}{s+2} \Rightarrow \boxed{G(s) = \frac{2}{s+2}}$$

d) $g(t) = 2.e^{-3t} + 3.t$

$$G(s) = 2.\frac{1}{s+3} + 3.\frac{1}{s^2}$$

G(s)

G(s)

E)

K

K

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$$G(s) = \frac{2}{s+3} + \frac{3}{s^2}$$

$$G(s) = \frac{2.s^2 + 3.(s+3)}{s^2.(s+3)} \Rightarrow \boxed{G(s) = \frac{2s^2+3s+9}{s^3+3s^2}}$$

E) $$K(t) = 2.e^{-3t} + 0{,}5.e^{-2t}$$

$$K(s) = 2 \times \frac{1}{s+3} + 0{,}5 \times \frac{1}{s+2}$$

$$K(s) = \frac{2}{s+3} + \frac{0{,}5}{s+2}$$

$$K(s) = \frac{2.(s+2) + 0{,}5.(s+3)}{(s+2).(s+3)}$$

$$K(s) = \frac{2{,}5s + 4 + 1{,}5}{s^2+5s+6}$$

$$\boxed{K(s) = \frac{2{,}5s+5{,}5}{s^2+5s+6}}$$

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Lista p/ prova 1

1) c)

$$e^{-at} \xrightarrow[{[L]}]{} \frac{1}{s+a} \xrightarrow[{[z]}]{} \frac{z}{z-e^{-ak}} \xrightarrow[{[z^{-1}]}]{} e^{-anT}$$

$$G(s) = 2 \times \frac{1}{s+2} \Rightarrow \boxed{G(s) = \frac{2}{s+2}} //$$

d) $G(s) = 2 \times \frac{1}{s+3} + 3 \times \frac{1}{s^2}$ → Func. rampa

$$G(s) = \frac{2}{s+3} + \frac{3}{s^2}$$

$$G(s) = \frac{2 \cdot s^2 + 3 \cdot (s+3)}{s^2 \cdot (s+3)} \rightarrow G(s) = \frac{2s^2 + 3s + 9}{s^3 + 3s^2} //$$

e) $K(s) = 2 \times \frac{1}{s+3} + 0,5 \cdot \frac{1}{s+2}$

$$K(s) = \frac{2}{s+3} + \frac{0,5}{s+2} \rightarrow \boxed{K(s) = \frac{2,5s + 5,5}{s^2 + 5s + 6}} //$$

$$K(s) = \frac{2(s+2) + 0,5(s+3)}{(s+2) \cdot (s+3)}$$

$$K(s) = \frac{2,5s + 4 + 1,5}{s^2 + 5s + 6}$$

2. Aplique a transformada de Laplace inversa para representar as Fts abaixo no domínio do tempo.
   a) $F(s) = 2/(s+4)$;

   c) $K(s) = 3/((s+2)*(s+3))$;
   d) $H(s) = 1/(s+1) + 3/(s+2)$;
   e) $F(s) = 2/((s+1)*(s+2)*(s+3))$;

$$e^{-at} \xrightarrow{[L]} \frac{1}{S+a} \xrightarrow{[Z]} \frac{z}{z-e^{-aT}} \xrightarrow{[z^{-1}]} e^{-anT}$$

$[L^{-1}]$

01ª QUESTÃO:

=> TRANSFORMADA DE LAPLACE INVERSA:

A) $F(S)=\dfrac{2}{S+4} \Rightarrow F(S)=2\cdot\dfrac{1}{S+4}$

$$\boxed{f(t)=2.e^{-4t}}$$

C) $K(S)=\dfrac{3}{(S+2).(S+3)}=\dfrac{A}{S+2}+\dfrac{B}{S+3}$

$A=?$ $\dfrac{3.\cancel{(S+2)}}{\cancel{(S+2)}.(S+3)}=\dfrac{A.\cancel{(S+2)}}{\cancel{S+2}}+\dfrac{B.\overbrace{(S+2)}^{0}}{S+3}\Bigg|_{S=-2}$

$A=\dfrac{3}{S+3}\Bigg|_{S=-2} \Rightarrow \boxed{A=3}$

$\dfrac{3.(S+}{(S+2).(}$

$B=$

$K(S)$

$K($

$\longrightarrow e^{-anT}$

$[z^{-1}]$

$$\frac{3.(s+3)}{(s+2).\cancel{(s+3)}} = \frac{A.\cancel{(s+3)}}{\cancel{s+2}} + \frac{B.(s+3)}{s+3} \Bigg|_{s=-3}$$

$$B = \frac{3}{s+2} \Bigg|_{s=-3} \Rightarrow \boxed{B=-3}$$

$$K(s) = \frac{3}{s+2} - \frac{3}{s+3} \Rightarrow K(s) = 3.\frac{1}{s+2} - 3.\frac{1}{s+3}$$

$$K(t) = 3.e^{-2t} - 3.e^{-3t}$$

$$e^{-at} \xrightarrow{[L]} \frac{1}{s+a} \xrightarrow{[Z]} \frac{z}{z - e^{-aT}} \xrightarrow{[Z^{-1}]}$$

$$\frac{1}{s+a} \xrightarrow{[L^{-1}]} e^{-at}$$

01ª QUESTÃO:

=> TRANSFORMADA DE LAPLACE INVERSA:

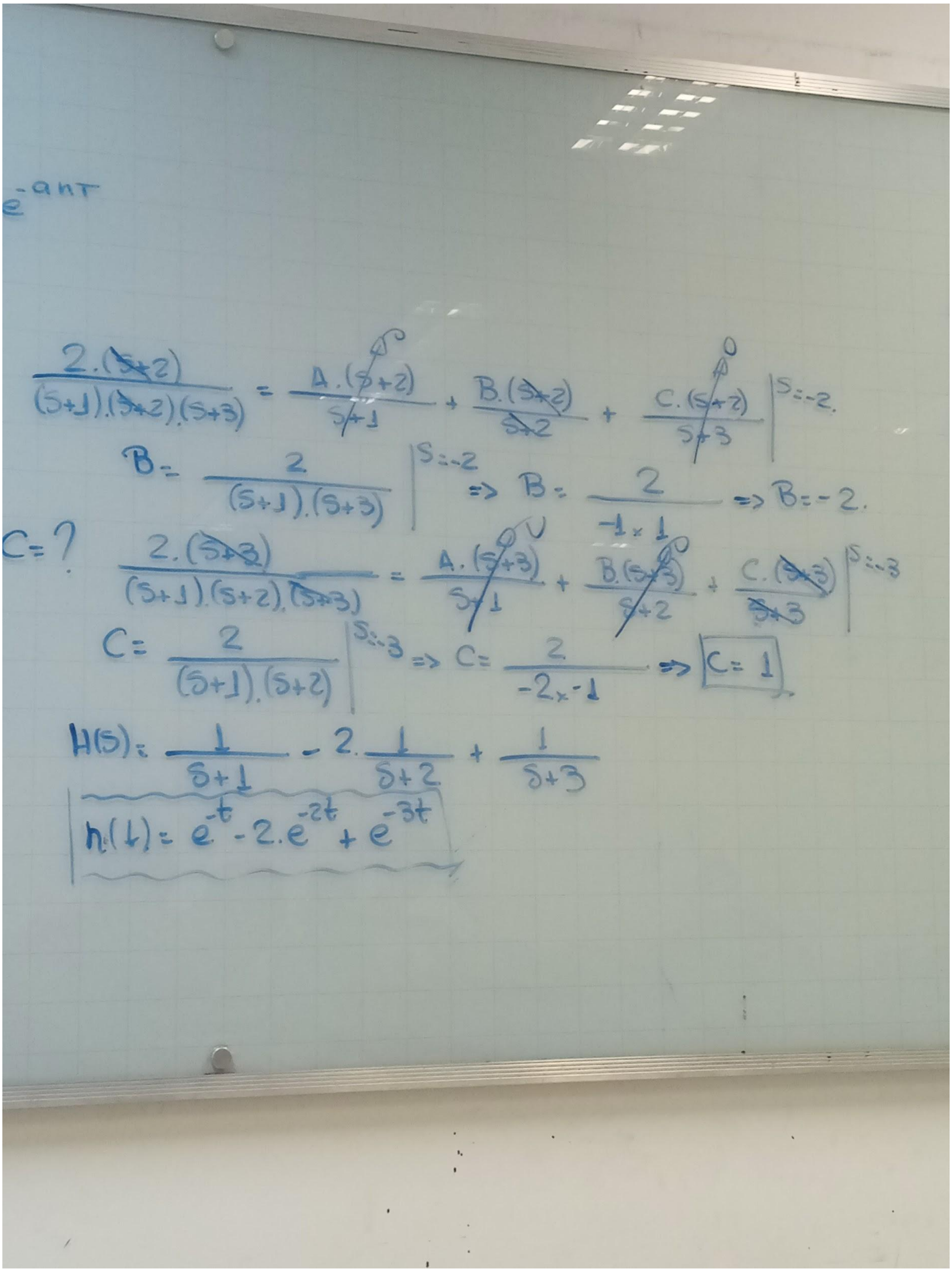
D) $H(s) = \frac{1}{s+1} + \frac{3}{s+2}$

$$\boxed{h(t) = e^{-t} + 3.e^{-2t}}$$

E) $H(s) = \frac{2}{(s+1).(s+2).(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$

$$A = ? \quad \frac{2.\cancel{(s+1)}}{\cancel{(s+1)}.(s+2).(s+3)} = \frac{A.\cancel{(s+1)}}{\cancel{s+1}} + \cancel{\frac{B.(s+1)}{s+2}} + \cancel{\frac{C.(s+1)}{s+3}} \Bigg|_{s=-1}$$

$$A = \frac{2}{\underbrace{(s+2)}_{1}.\underbrace{(s+3)}_{2}} \Bigg|_{s=-1} \Rightarrow \boxed{A = 1}$$



Extra: questão de uma função do segundo grau com transformada de Laplace

$$\xleftarrow{[L^{-1}]}$$

$$e^{-at} \xrightarrow[{[L]}]{} \frac{1}{s+a} \xrightarrow[{[Z]}]{} \frac{z}{z-e}$$

01ª QUESTÃO:

=> TRANSFORMADA DE LAPLACE:

$$h(t) = 2.t^2 + 3t + 4$$

$$H(s) = 2.\frac{2}{s^3} + 3.\frac{1}{s^2} + 4.\frac{1}{s}$$

$$H(s) = \frac{4}{s^3} + \frac{3}{s^2} + \frac{4}{s}$$

$$H(s) = \frac{4+3s+4s^2}{s^3} \Rightarrow \boxed{H(s) = \frac{4s^2+3s+4}{s^3}}$$

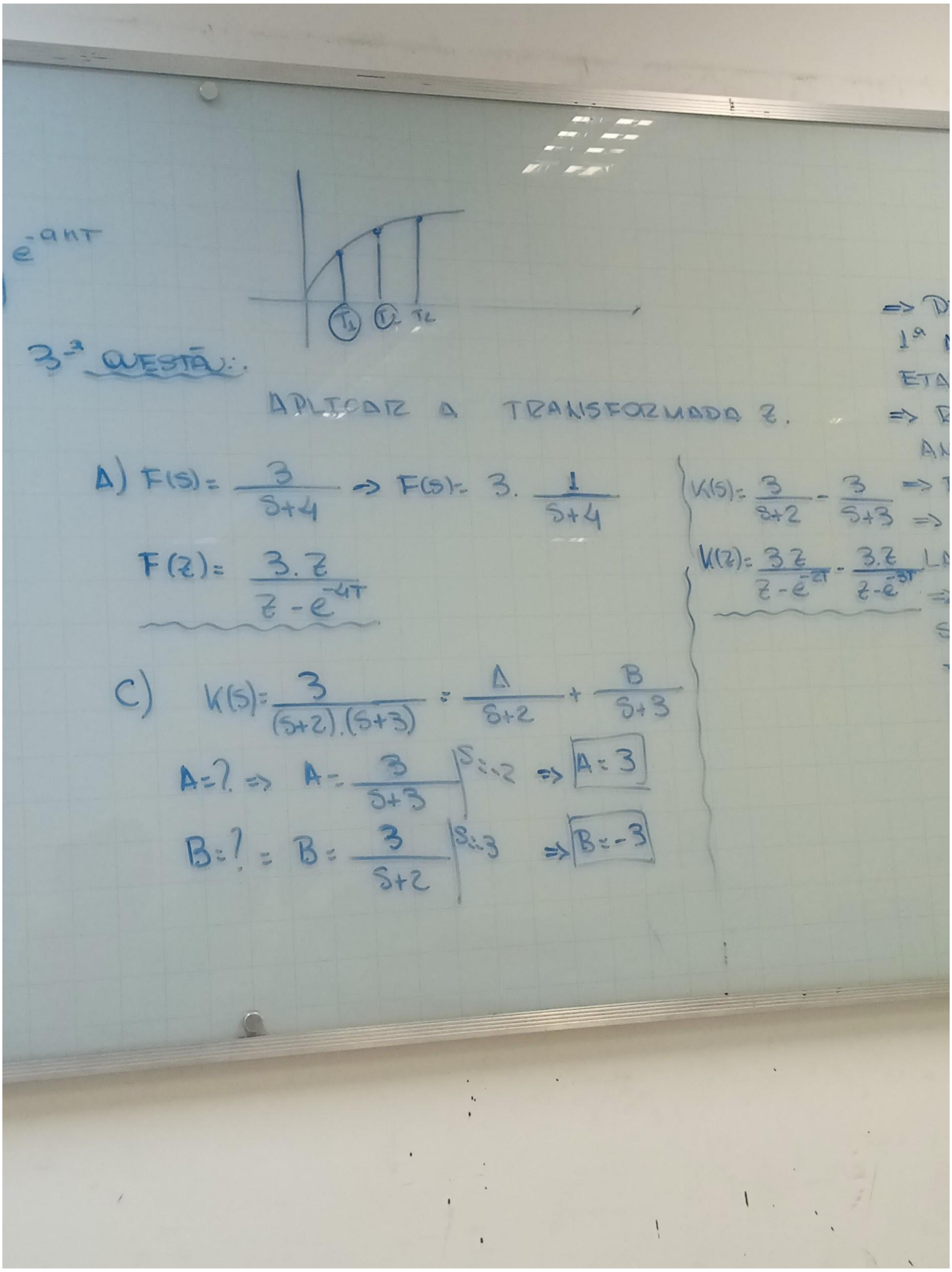
Extra) $[L] = ?$

$$h(t) = 2 \cdot t^2 + 3t + 4$$

$$H(s) = 2 \cdot \frac{2}{s^3} + 3 \cdot \frac{1}{s^2} + 4 \cdot \frac{1}{s}$$

$$H(s) = \frac{4}{s^3} + \frac{3}{s^2} + \frac{4}{s^1} \; //$$

3. Aplique a transformada Z para representar as FTs abaixo no plano Z.
   a) F(s) = 3/(s+4);
   c) K(s) = 3/((s+2)*(s+3));

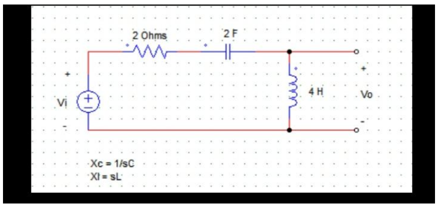


4. Aplique a transformada Z inversa para representar as FTs abaixo em tempo discreto (amostras).
   a) F(z) = z/(z-4);
   b) F(z) = z/(z-1) + z/(z-0.2);

c) K(z) = z/((z-1)*(z-3));

5. Determine, no domínio da frequência (plano S), a relação Vo(s)/Vi().

Dica para resolver: basta resolver as reatância (capacitiva e indutiva) no plano S



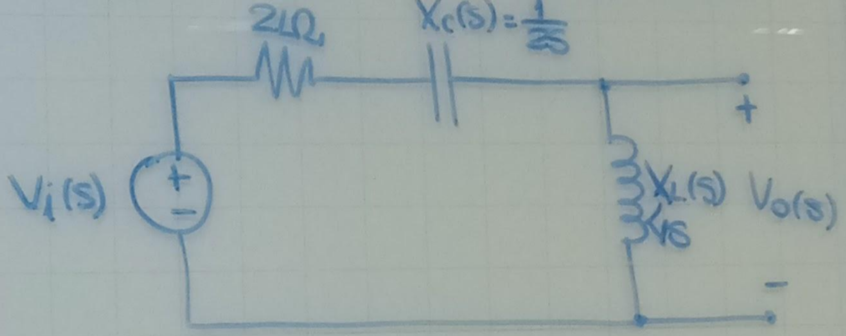
Resposta: 8s2/(4s + 1 + 8s2)

Resolução

$aT \xrightarrow[{[z^{-1}]}]{} e^{-anT}$

$$X_c(S) = \frac{1}{SC} \Rightarrow X_c(S) = \frac{1}{2S}$$

$$X_L(S) = L.S \Rightarrow X_L(S) = 4.S$$



⇒ APLI

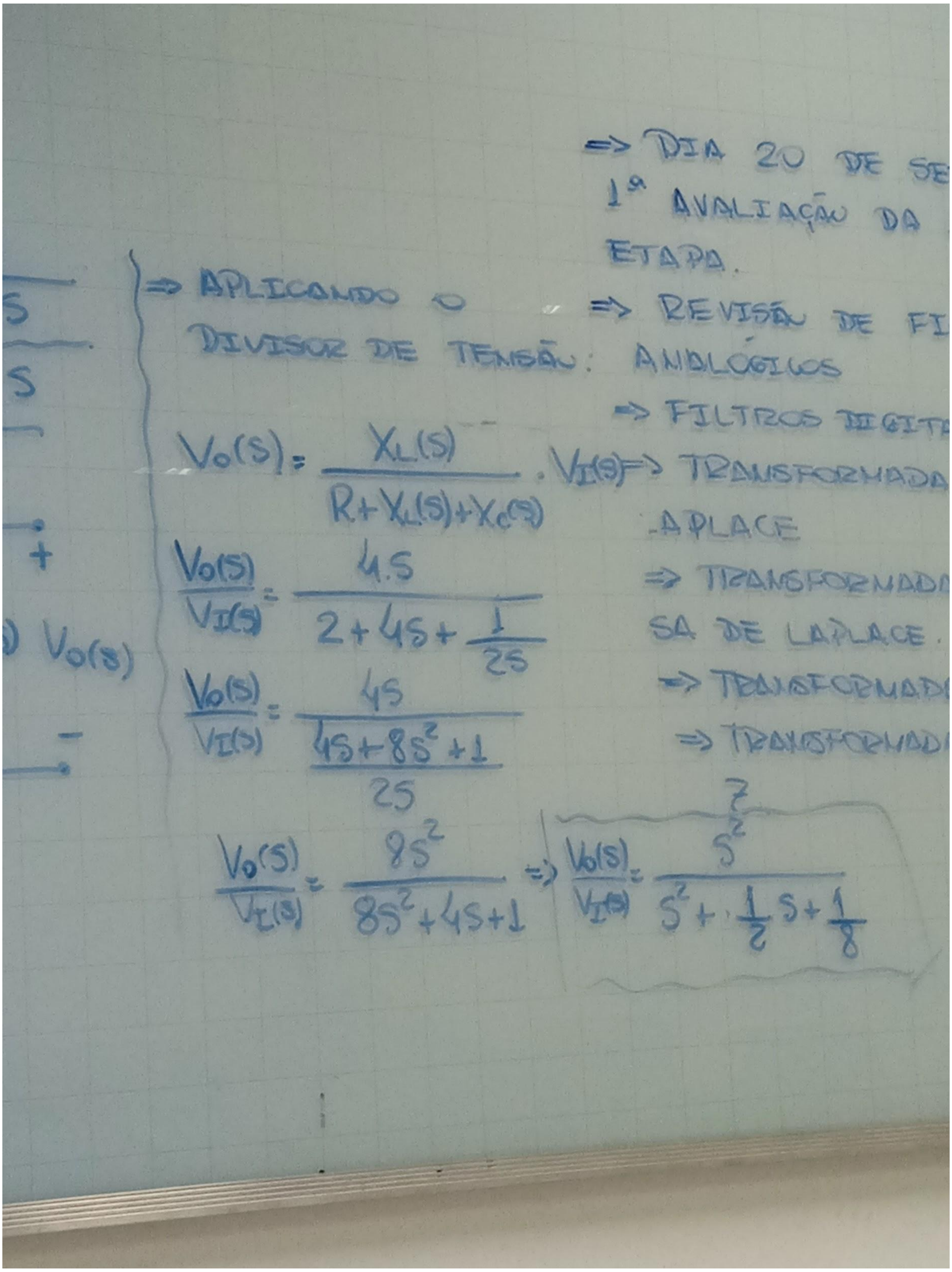
DIVI

$V_o(S)$

$\frac{V_o(S)}{V_I(S)} =$

$\frac{V_o(S)}{V_I(S)} =$

$\frac{V_o(S}{V_I(S}$



Extra) Filtros Medianos(9ª questão)

$$e^{-ab} \xrightarrow{[L]} \frac{1}{s+a} \xrightarrow{[Z]} \frac{z}{z-e^{-aT}} \xrightarrow{[Z}$$

$$\frac{1}{s+a} \xrightarrow{[L^{-1}]} e^{-ab}$$

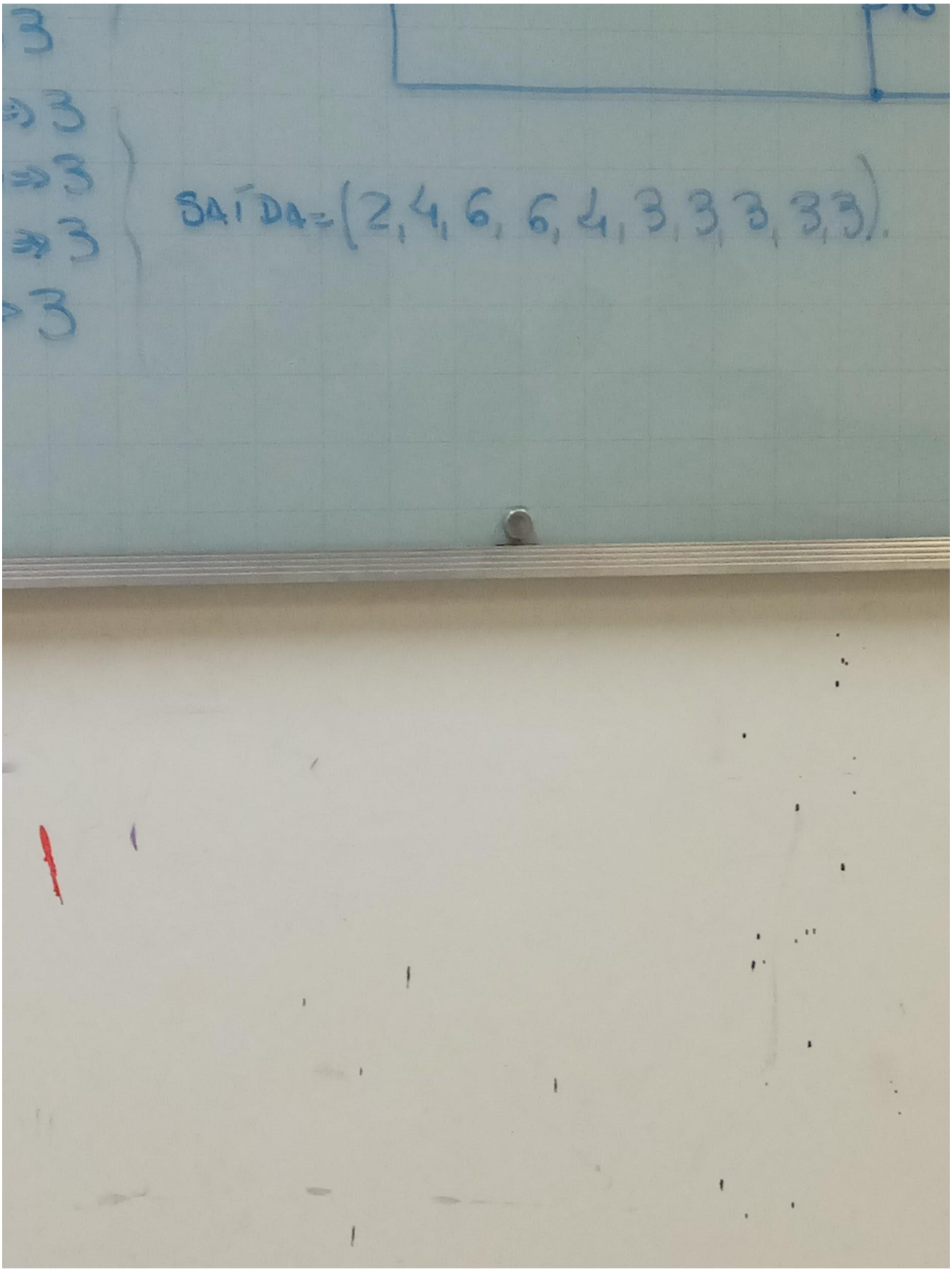
09ª QUESTÃO:

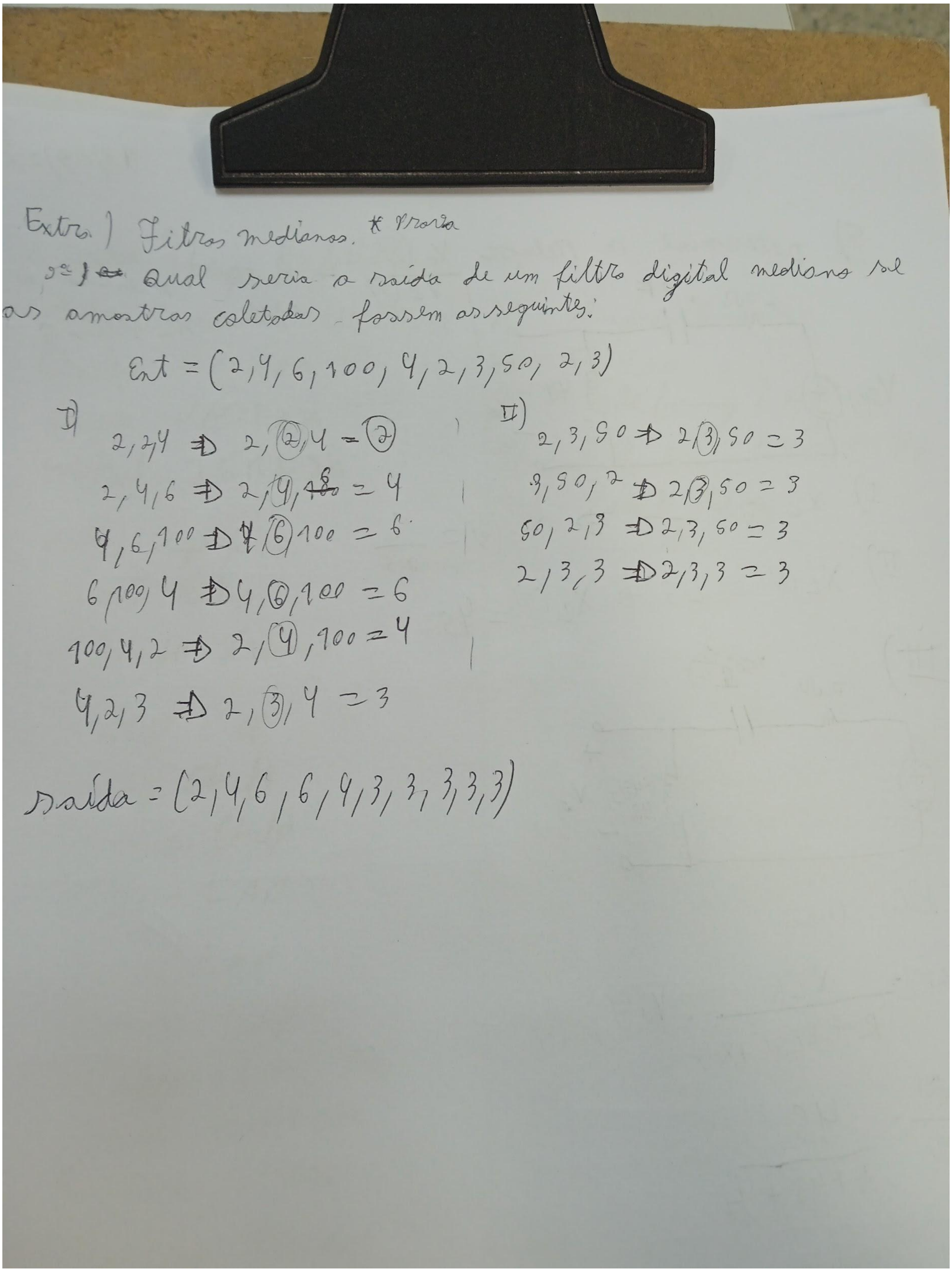
QUAL SERIA A SAÍDA DE UM FILTRO DIGITAL MEDIANO SE AS AMOSTRAS COLETADAS FOSSEM AS SEGUINTES:

ENT = (2, 4, 6, 100, 4, 2, 3, 50, 2, 3)

2, 2, 4 ⇒ 2, 2, 4 = 2
2, 4, 6 ⇒ 2, 4, 6 = 4
4, 6, 100 ⇒ 4, 6, 100 = 6
6, 100, 4 ⇒ 4, 6, 100 = 6
100, 4, 2 ⇒ 2, 4, 100 = 4

4, 2, 3 ⇒ 2, 3, 4 ⇒ 3
2, 3, 50 ⇒ 2, 3, 50 ⇒ 3
3, 50, 2 ⇒ 2, 3, 50 ⇒ 3
50, 2, 3 ⇒ 2, 3, 50 ⇒ 3
2, 3, 3 ⇒ 2, 3, 3 ⇒ 3


SAÍDA = (2, 4, 6, 6, 4, 3, 3, 3, 3, 3).



Extra) Filtros medianos. * Prova

2ª) Qual seria a saída de um filtro digital mediano se as amostras coletadas fossem as seguintes:

$$Ent = (2, 4, 6, 100, 4, 2, 3, 50, 2, 3)$$

I)

$2, 2, 4 \Rightarrow 2, 2, 4 = 2$

$2, 4, 6 \Rightarrow 2, 4, \cancel{100}\ 6 = 4$

$4, 6, 100 \Rightarrow \cancel{4}\ 4, 6, 100 = 6$

$6, 100, 4 \Rightarrow 4, 6, 100 = 6$

$100, 4, 2 \Rightarrow 2, 4, 100 = 4$

$4, 2, 3 \Rightarrow 2, 3, 4 = 3$

II)

$2, 3, 50 \Rightarrow 2, 3, 50 = 3$

$3, 50, 2 \Rightarrow 2, 3, 50 = 3$

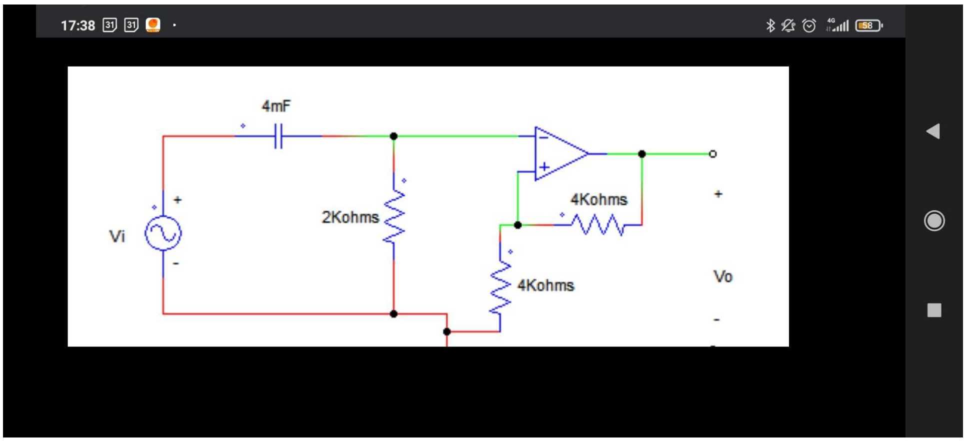
$50, 2, 3 \Rightarrow 2, 3, 50 = 3$

$2, 3, 3 \Rightarrow 2, 3, 3 = 3$

$$saída = (2, 4, 6, 6, 4, 3, 3, 3, 3, 3)$$

6. Nos filtros abaixo, determine o tipo de filtro e a relação Av = Vo/Vi.

C.


17:38
4mF
Vi
+
-
2Kohms
4Kohms
4Kohms
+
Vo
-

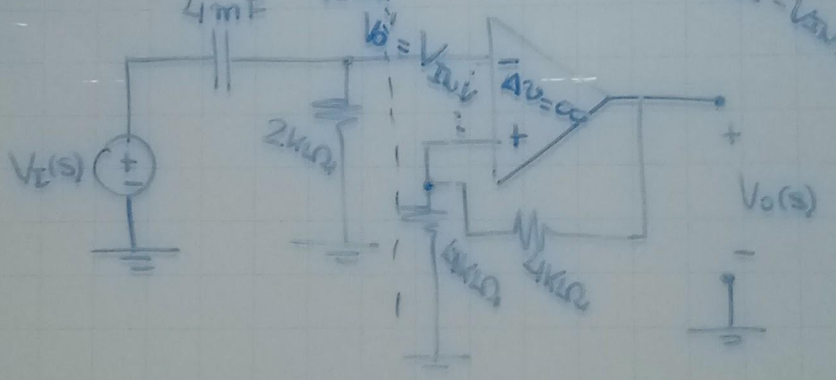
$$e^{-at} \xrightarrow{[L]} \frac{1}{s+a} \xrightarrow{[Z]} \frac{z}{z - e^{-aT}}$$

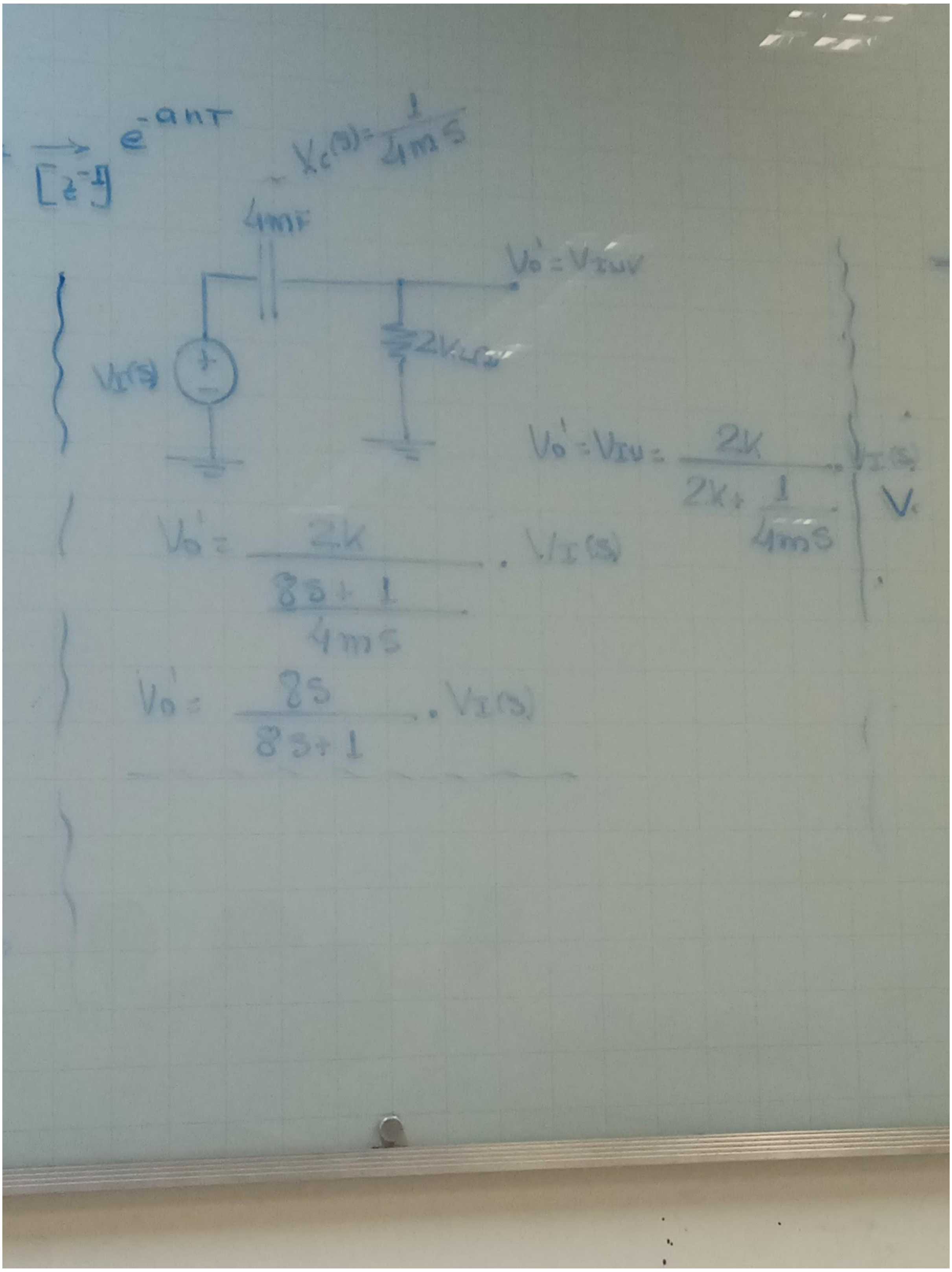
$[L^{-1}]$

$A_v = \frac{V_o}{V_I} = V_{INI} - V_{INV}$

06ª QUESTÃO: $\frac{V_0(s)}{V_I(s)} = ?$

c)




e^-anT
Vc(s) = 1/4ms
4mF
Vo' = VIUV
VI(s)
2K
Vo' = VIU = 2K/(2K + 1/4ms) . VI(s)
Vo' = 2K/((8s+1)/4ms) . VI(s)
Vo' = 8s/(8s+1) . VI(s)

7.