CS5170/6070: Assignment 3

Q1 Devise context-free grammars for the following languages:

Q1 $L_1 = \{0^n 1^m : n \geq m \geq 0\}^*$

A1 The grammar first introduces a zero for every one, and then introduces extra zeros. It then encapsulates it with productions for generating the Kleene-* of the language.

$$S \rightarrow AS | \lambda$$
$$A \rightarrow 0A1 | 0A | \lambda$$

Q2 L_2 is the language of all binary non-palindromes.

A2 The productions first generate pairs of matching symbols, and then introduces a single pair of mismatched bits. It then generates an arbitrary string in the middle (which we don't care about!).

$$S \rightarrow 0S0 | 1S1 | 0A1 | 1A0$$
$$A \rightarrow 0A | 1A | \lambda$$

Q3 $L_3 = \{0^i 1^j 2^k 3^\ell : j = i + k + \ell \text{ and } i, j, k, \ell \geq 0\}$

A3 The grammar first introduces as many ones as zeros (in the right order). Then, it introduces as many new ones as there are threes (again, in the right order), and finally introduces a one for every 2.

$$S \rightarrow AB$$
$$A \rightarrow 0A1 | \lambda$$
$$B \rightarrow 1B3 | 1C2 | \lambda$$
$$C \rightarrow 1C2 | \lambda$$

- $L_4 = \{0^i 1^j : 2 \leq \frac{i}{j} \leq 5 \text{ and } i, j \geq 1\}$

A4 For every 1 on the right side, introduce 2, 3, 4, or 5 zeros on the left side.

$$S \rightarrow 00S1 | 000S1 | 0000S1 | 00000S1 | 001 | 0001 | 00001 | 000001$$

Q5 $L_5 = \{0^i 1^m 2^k : i, m, k \geq 0 \text{ and } \max\{i, k\} \geq 2m\}$

A5 Devise two parts of the grammar. One checks if $i \geq 2m$ and the other checks if $k \geq 2m$. If either succeeds, it must be the case that $\max\{i, k\} \geq 2m$.

$$S \rightarrow A | C$$
$$A \rightarrow A2 | B$$
$$B \rightarrow 00B1 | 0B | \lambda$$
$$C \rightarrow 0C | D$$
$$D \rightarrow 1D22 | D2 | \lambda$$

Q6 $L_6 = \{0^i 1^j 2^k 3^\ell : i + j = 2k + \ell \text{ and } i, j, k, \ell \geq 0\}$.

A6 Let's break the language into two buckets. One where $i \geq \ell$, and the other where $i < \ell$. Variable A will generate strings of the first bucket, and B of the second bucket.

$$S \rightarrow A | D$$
$$A \rightarrow 0A3 | B$$
$$B \rightarrow 00B2 | 01C2 | C$$
$$C \rightarrow 11C2 | \lambda$$
$$D \rightarrow 0D3 | E$$
$$E \rightarrow 1E3 | F$$
$$F \rightarrow 11F2 | \lambda.$$