In this test, $\Sigma = \{0, 1\}$, x is a string.

[10] 1. State whether each of the following is **true** or **false**. **Justify your answer!**

a. Any language over the alphabet {b} is regular.

True. Any language over an alphabet with only on element is regular, it can be decribed with a DFA.

b. There are finitely many finite languages.

False. There are infinitely many finite languages.

ex?

1

c. Union of any two languages over alphabet {0, 1} is regular.

False. If one of the languages is non-regular then the union is non-regular.

$\{1^n0^n\} \cup \{1^*0^*\} = \{1^*0^*\}$

non reg reg reg

d. Pumping lemma is used to prove that a language is regular.

False. Pumping lemma is used to create a counterexample to prove that a language is non-regular.

2

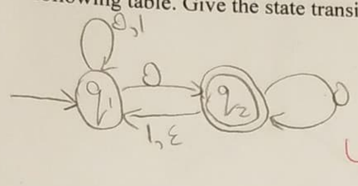
e. Single state NFA can recognize only finite languages.

False.

1

[4] 2. The formal description of a NFA is $(\{q_1, q_2\}, \{0,1\}, \delta, q_1, \{q_2\})$, where δ is given by the following table. Give the state transition diagram of this NFA.

	0	1	ε
q_1	$\{q_1, q_2\}$	$\{q_1\}$	∅
q_2	$\{q_2\}$	$\{q_1\}$	$\{q_1\}$



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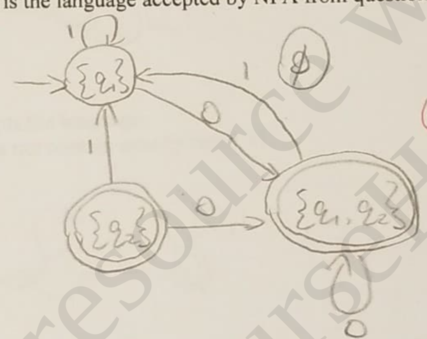
[4] 3. Give a DFA for L, where L is the language accepted by NFA from question 2.

	0	1
∅	∅	∅
→ $\{q_1\}$	$\{q_1,q_2\}$	$\{q_1\}$
* $\{q_2\}$	$\{q_1,q_2\}$	$\{q_1\}$
* $\{q_1,q_2\}$	$\{q_1,q_2\}$	$\{q_1\}$



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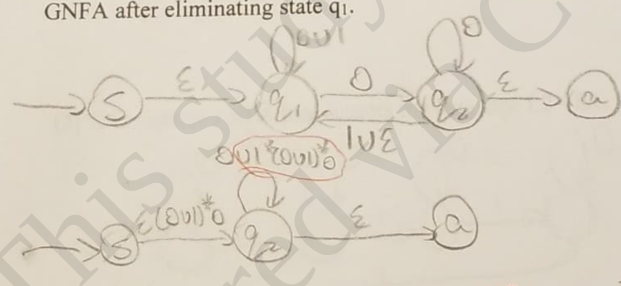
[4] 4. Give regular expression generating the language, recognized by NFA in question 2. Use GNFA and state elimination technique to find it. Show your work, in particular show GNFA after eliminating state q_1.



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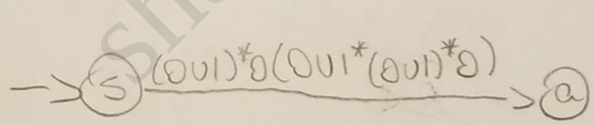
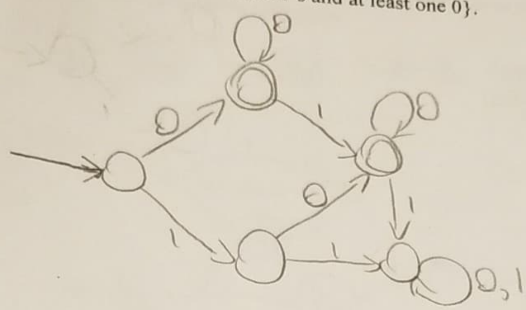
The regular expression generating the langu is $(0\cup1)^*0(0\cup1^*(0\cup$



[4] 5. Construct a DFA that accepts the language:
L1 = { x | x has at most one 1's and at least one 0}.



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[4] 6. Construct a DFA that accepts the language:
L2 = { x | x is any string that does not contain exactly two 1's}.

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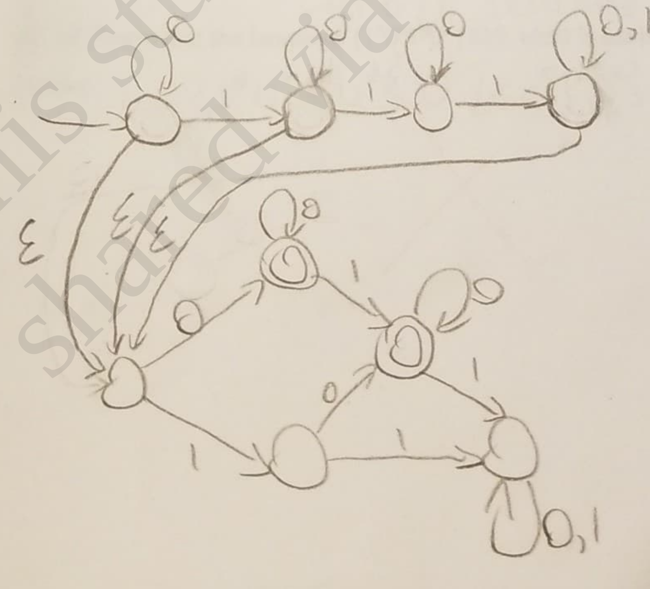
[2] 7. Construct a NFA that accepts the language **L2 ◦ L1**.



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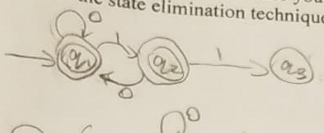
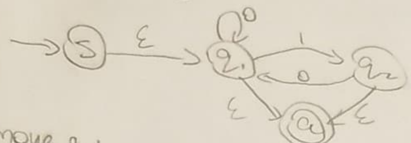
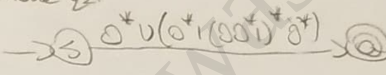
[4] 8. Write a regular expression for the set of strings of 0's and 1's with no two consecutive 1's. Hint: in case you do not know how to start, construct an NFA, and then use the state elimination technique.



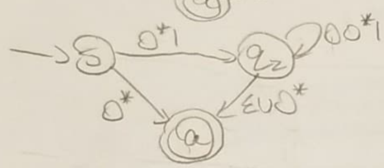
← q_3 doesn't matter, it has no outgoing edges



remove q_2:



remove q_1:



The regular expression is

4 $0^* \cup (0^*1(00^*1)^*0^*)$

[5] 9. Prove that the language of all strings having twice as many 0s as 1s is not regular. Use pumping lemma.

Assume this language is regular. Take $s = 0^P 1^{P/2}$. $xy = 0^P$, $y = 0^k$, $0 < k \leq P$. Then $|y| > 0$ and $|xy| \leq P$. But $xyyz = 0^{P+k} 1^{P/2}$ will not be accepted since $(P+k)/2 \neq P/2$, so there are more than twice as many 0s as 1s. Thus the language is non-regular.

5

$L = \{1^k 0^n 1^n 0^k \mid k \geq 0, n \geq 0\}$

[3] 10. Prove that the language $\{1^k0^n1^n0^k \mid k\geq 0, n\geq 0\}$ is not regular. Do not use pumping lemma.

$L = \{1^*0^*1^*0^*\} \cup \{s \in \{1^*, 0^*\} \mid \#1s = \#0s\}$

X