[15] 1. State whether each of the following is **true** or **false**. **Justify your answer!**

a. Any finite language over the alphabet {a,b,c, … ,z} is context free.

True.

Since it is the finite language, it must be regular.

So it is context-free. 3

b. Every context free language can be recognized by a deterministic push-down automaton (PDA).

3

False. $\{a^i b^j c^k \mid i,j,k \geq 0 \text{ and } i=j \text{ or } j=k\}$ is

non-deterministic

c. Union of any context free language and a regular language is context free.

True. Since a regular language must be context-free.

3

So the union of two contex-free language is still context-free.

d. Pumping lemma is used to prove that a language is regular or context free.

3

False. It is used to prove the language is not

c. Union of any context free language and a regular language is context free.

True. Since a regular language must be context-free.

So the union of two contex-free language is still context-free.

3

d. Pumping lemma is used to prove that a language is regular or context free.

False. It is used to prove the language is not regular or context-free

3

e. The set of context free languages is closed under operation of intersection.

False.

$L_1 = \{a^i b^j c^k \mid i < j\}$

3

$L_2 = \{a^i b^j c^k \mid i < k\}$

but $L_1 \cap L_2$ is not context free.

[16] 2. Consider the following context-free grammar G

$$S \rightarrow 0S1 \mid 1S0 \mid SS \mid M$$
$$M \rightarrow R1 \mid 1R \mid R$$
$$R \rightarrow 1R \mid \varepsilon$$

Provide brief answers to the following questions:

[2] a. List all variables and all terminals of G.

Variables: {S, M, R} 2

Terminals: {1, 0}

[2] b. List two strings in L(G).

01, 10 2

[2] c. List two strings not in L(G).

2

00, 0

[2] d. List two sentential forms of G.

2

0S1, 1S0

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[2] d. List two sentential forms of G.

0S1 , 1S0

2

[4] d. Give two **different** leftmost derivations of the string **1110001** in G.

S → SS → 1S0S → 11S00S → 111S000S → 111M000S → 111R000S → 111000S
→ 111000M → 111000R1 → 111000ε1 → 1110001

S → SS → 1S0S → 11S00S → 11M00S → 111R00S → 111ε00S → 11100S
→ 11100S1 → 111000M1 → 111000R1 → 111000ε1 → 1110001

2

[4] e. Give a description of L(G) in English.

L(G) consists of 0 and 1. and the number of 1's is not less than the number of 0's

4

[10] **3.** Convert grammar G from the previous question into Chomsky normal form (the grammar is repeated here for convenience):

$S \to 0S1 \mid 1S0 \mid SS \mid M$
$M \to R1 \mid 1R \mid R$
$R \to 1R \mid \varepsilon$

①
$S_0 \to S$
$S \to 0S1 \mid 1S0 \mid SS \mid M$
$M \to R1 \mid 1R \mid R$
$R \to 1R \mid \varepsilon$

② Eliminate $R \to \varepsilon$
$S_0 \to S$
$S \to 0S1 \mid 1S0 \mid SS \mid M$
$M \to R1 \mid 1R \mid R \mid 1 \mid \varepsilon$
$R \to 1R \mid 1$

Eliminate $M \to \varepsilon$
$S_0 \to S$
$S \to 0S1 \mid 1S0 \mid SS \mid M \mid \varepsilon$
$M \to R1 \mid 1R \mid R \mid 1$
$R \to 1R \mid 1$

Eliminate $S \to \varepsilon$
$S_0 \to S \mid \varepsilon$
$S \to 0S1 \mid 1S0 \mid SS \mid 01 \mid 10 \mid S \mid M$
$M \to 1R \mid R1 \mid R \mid 1$
$R \to 1R \mid 1$

③ Eliminate $M \to R$
$S_0 \to S \mid \varepsilon$
$S \to 0S1 \mid 1S0 \mid SS \mid 01 \mid 10 \mid S \mid M$
$M \to 1R \mid R1 \mid 1$
$R \to 1R \mid 1$

Eliminate $S \to M$
$S_0 \to S \mid \varepsilon$
$S \to 0S1 \mid 1S0 \mid SS \mid 01 \mid 10 \mid S \mid 1R \mid R1 \mid 1$
$M \to 1R \mid R1 \mid 1$
$R \to 1R \mid 1$

Eliminate $S \to S$
$S_0 \to S \mid \varepsilon$
$S \to 0S1 \mid 1S0 \mid SS \mid 01 \mid 10 \mid 1R \mid R1 \mid 1$
$M \to 1R \mid R1 \mid 1$
$R \to 1R \mid 1$

Eliminate $S_0 \to S$
$S_0 \to 0S1 \mid 1S0 \mid SS \mid 01 \mid 10 \mid 1R \mid R1 \mid 1 \mid \varepsilon$
$S \to 0S1 \mid 1S0 \mid SS \mid 01 \mid 10 \mid 1R \mid R1 \mid 1$
$M \to 1R \mid R1 \mid 1$
$R \to 1R \mid 1$

④ Add Rules
$U \to 1$
$X \to 0S$
$Y \to S0$
$Z \to 0$

10

$\therefore$ $S_0 \to XU \mid UY \mid SS \mid 01 \mid 10 \mid UR \mid RU \mid 1 \mid \varepsilon$
$S \to XU \mid UY \mid SS \mid 01 \mid 10 \mid UR \mid RU \mid 1$
$M \to UR \mid RU \mid 1$
$R \to UR \mid 1$
$U \to 1$
$X \to \cancel{Z}S$
$Y \to S\cancel{Z}$
$Z \to 0$

[6] **4.** Convert grammar G from the previous question to an equivalent PDA

$S \to 0S1 \mid 1S0 \mid SS \mid M$
$M \to R1 \mid 1R \mid R$

Eliminate M → ε

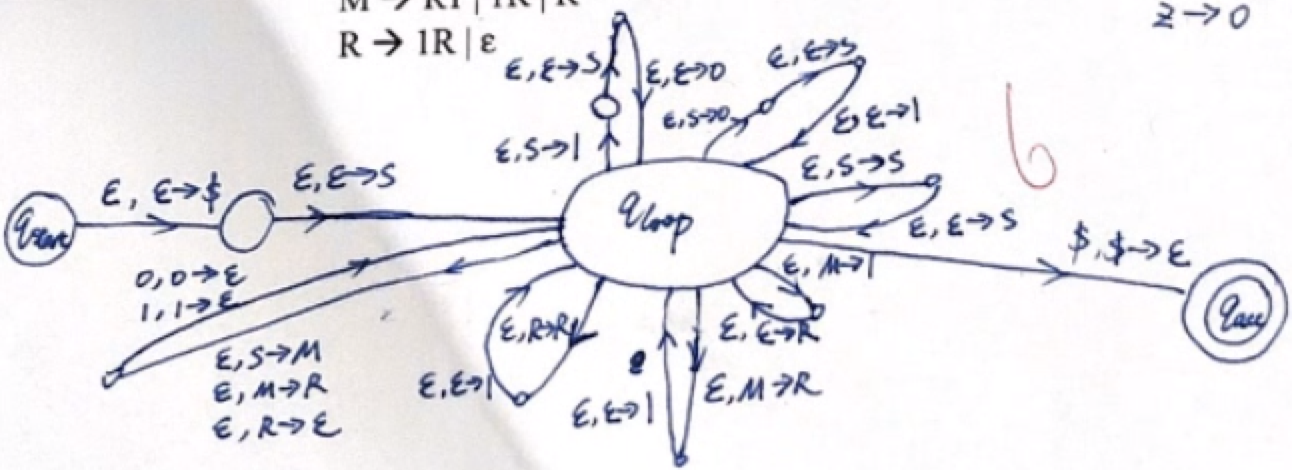
$S_0 \to S$
$S \to 0S1 \mid 1S0 \mid SS \mid M \mid \varepsilon$
$M \to R1 \mid 1R \mid R \mid 1$
$R \to 1R \mid 1$

Eliminate S → M
$S_0 \to S \mid \varepsilon$
$S \to 0S1 \mid 1S0 \mid SS \mid 01 \mid 10 \mid S \mid 1R \mid R1 \mid 1$
$M \to 1R \mid R1 \mid 1$
$R \to 1R \mid 1$

$X \to 0S$
$Y \to S0$
$Z \to 0$
$\therefore\ S_0 \to XU \mid UY \mid SS \mid 01 \mid 10 \mid UR \mid RU \mid 1 \mid \varepsilon$
$S \to XU \mid UY \mid SS \mid 01 \mid 10 \mid UR \mid RU \mid 1$
$M \to UR \mid RU \mid 1$
$R \to UR \mid 1$
$U \to 1$
$X \to ZS$
$Y \to SZ$
$Z \to 0$

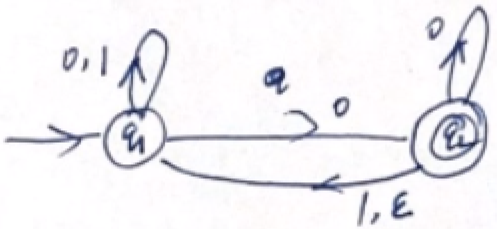
[6] **4.** Convert grammar G from the previous question to an equivalent PDA

S→ 0S1 | 1S0 | SS | M
M → R1 | 1R | R
R → 1R | ε



6

[8] 5. The formal description of a NFA is $(\{q_1, q_2\}, \{0,1\}, \delta, q_1, \{q_2\})$, where δ is given by the following table. Give the context free grammar that defines the same language as the one accepted by this NFA.



2

	0	1	ε
q_1	$\{q_1, q_2\}$	$\{q_1\}$	Ø
q_2	$\{q_2\}$	$\{q_1\}$	$\{q_1\}$

$S \rightarrow q_1$

$q_1 \rightarrow 0q_1 \mid 0q_2 \mid 1q_1$

$q_2 \rightarrow 1q_1 \mid 0q_2 \mid \cancel{q_2 q_1}$

1